Overview

What is Deep Learning?

Single neuron

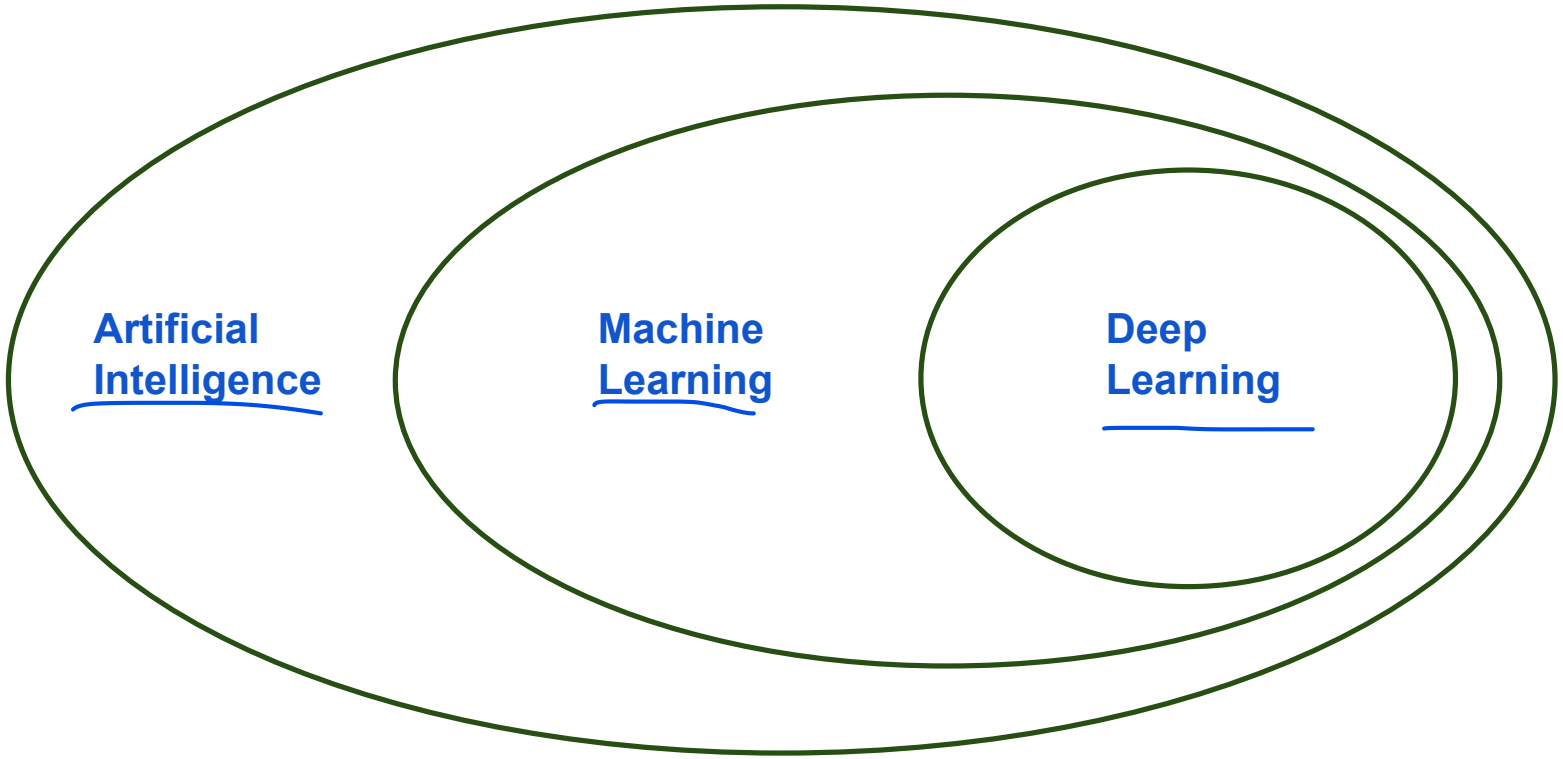
Single layer network

Multi-Layer network

Parameter learning

Further reading

WHAT IS DEEP LEARNING



- Deep learning is a subield of ML based on artificial neural networks.

DEEP LEARNING AND NEURAL NETWORKS

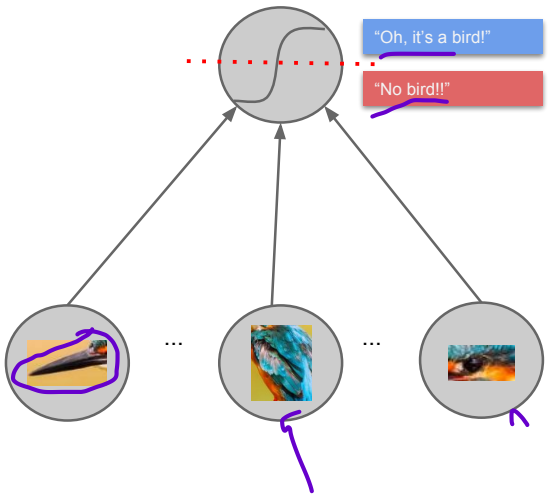
- Deep learning itself is not *new*:
  - Neural networks have been around since the 70s.
  - *Deep* neural networks, i.e., networks with multiple hidden layers, are not much younger.

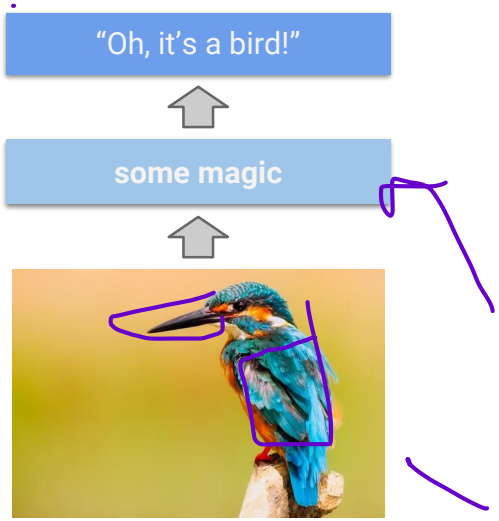
- Why everybody is talking about deep learning now:
  1. Specialized, powerful hardware allows training of huge neural networks to push the state-of-the-art on difficult problems.
  2. Large amount of data is available.
  3. Special network architectures for image/text data.
  4. Better optimization and regularization strategies.

IMAGE CLASSIFICATION WITH NEURAL NETWORKS

"Machine learning algorithms, inspired by the brain, based on learning multiple levels of representation/abstraction."

Y. Bengio





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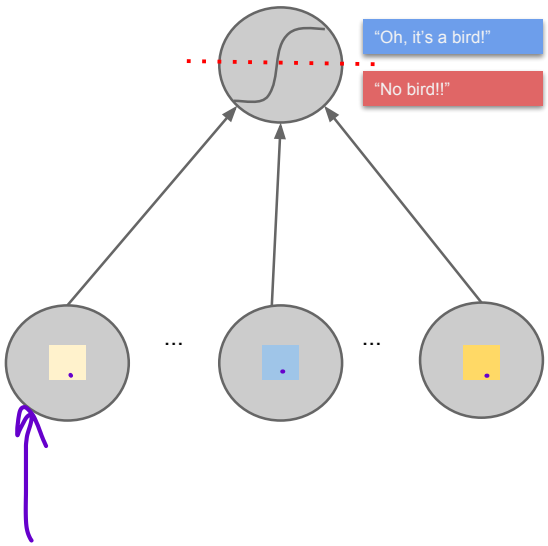

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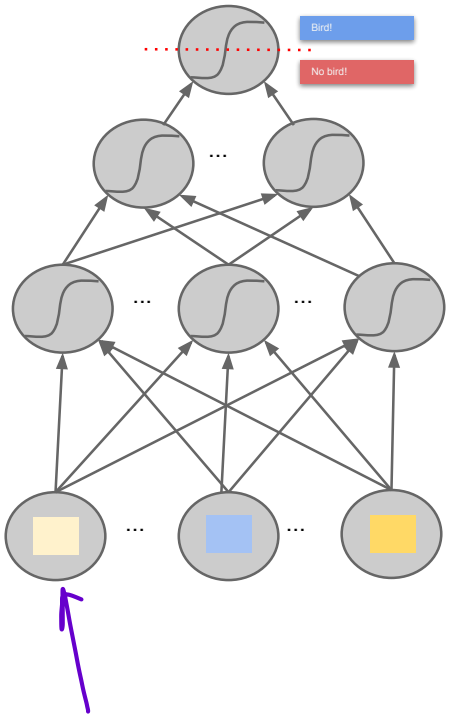

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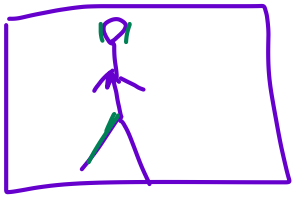
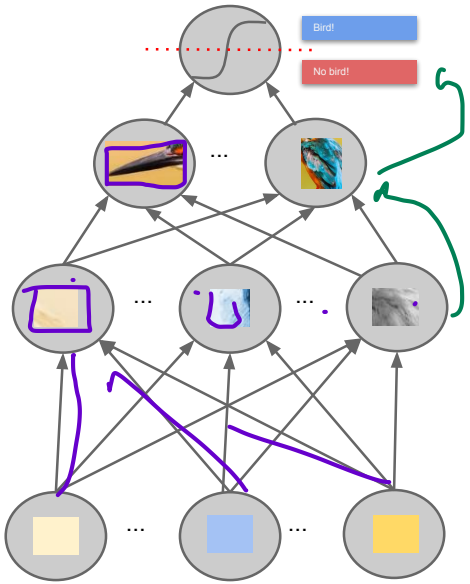
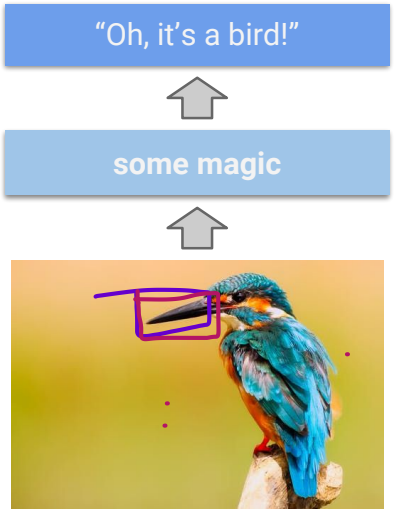






POSSIBLE USE-CASES

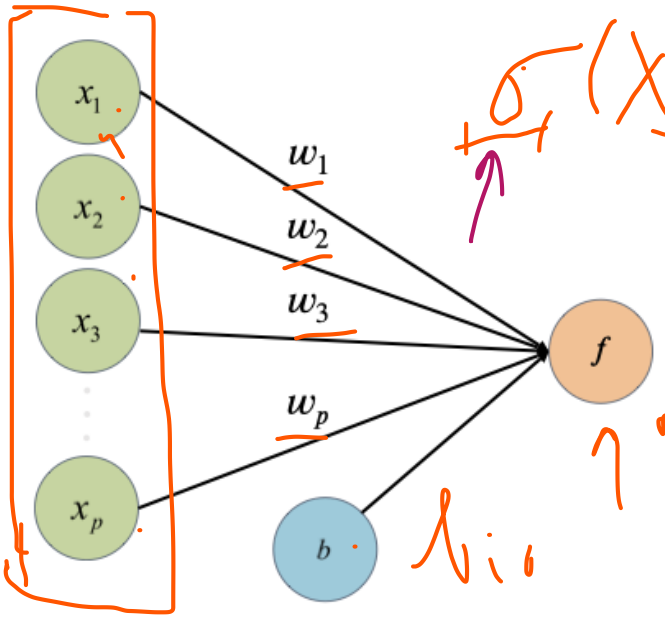
Deep learning can be extremely valuable if the data has these properties:

- It is high dimensional.
- Each single feature itself is not very informative but only a combination of them might be.
- There is a large amount of training data.

This implies that for tabular data, deep learning is rarely the correct model choice.

- Without extensive tuning, models like random forests or gradient boosting will outperform deep learning most of the time.
- One exception is data with categorical features with many levels.

A SINGLE NEURON



Perceptron with **input features** $x_1, x_2, \ldots, x_p$, **weights** $w_1, w_2, \ldots, w_p$, **bias term** b, and **activation function** τ.

- The perceptron is a single artificial neuron and the basic computational unit of neural networks.
- It is a weighted sum of input values, transformed by τ:

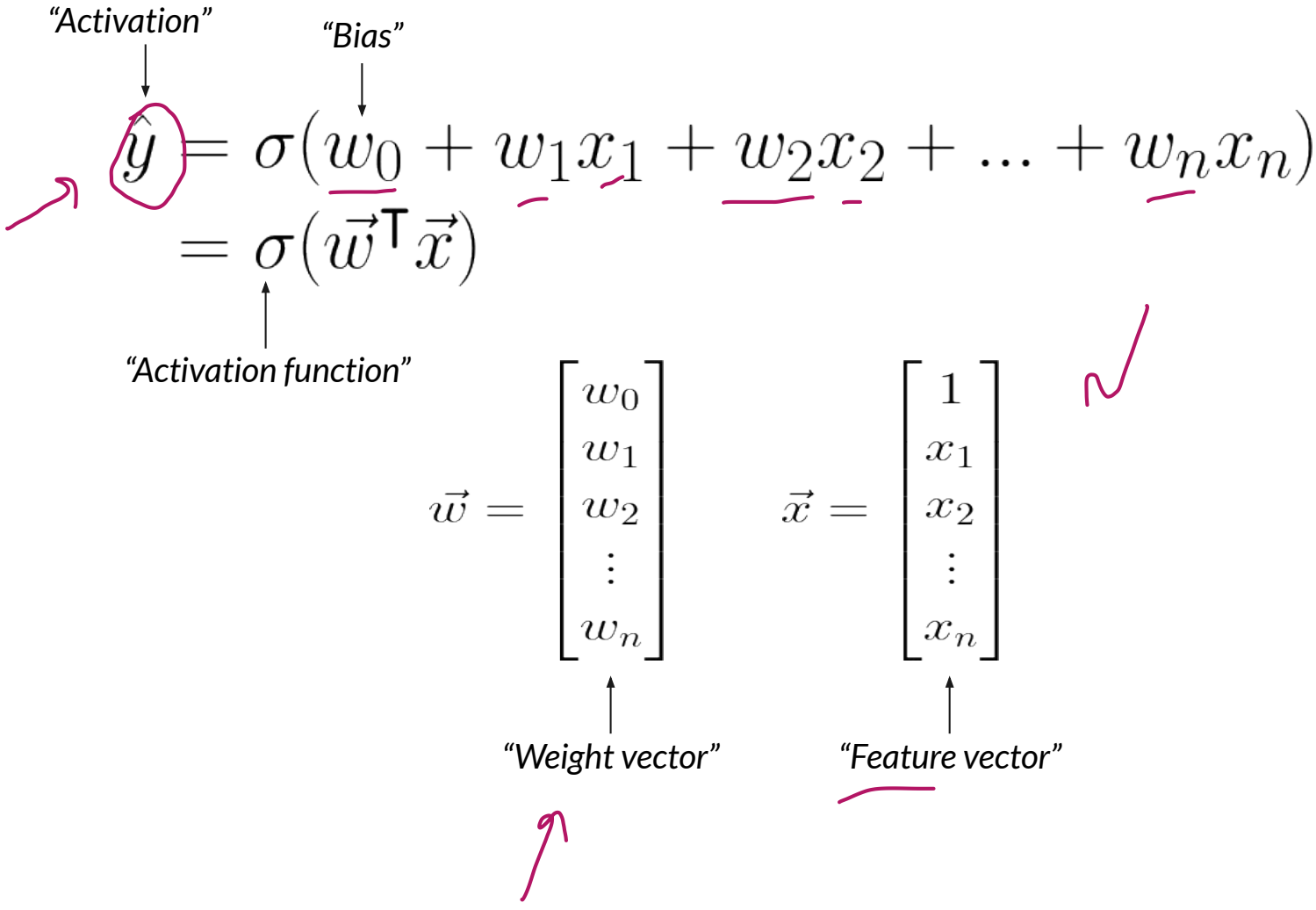
$$f(x) = \tau(w_1 x_1 + \ldots + w_p x_p + b) = \tau(\mathbf{w}^T \mathbf{x} + b)$$

$$\hat{y} = \sigma(w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_n x_n)$$

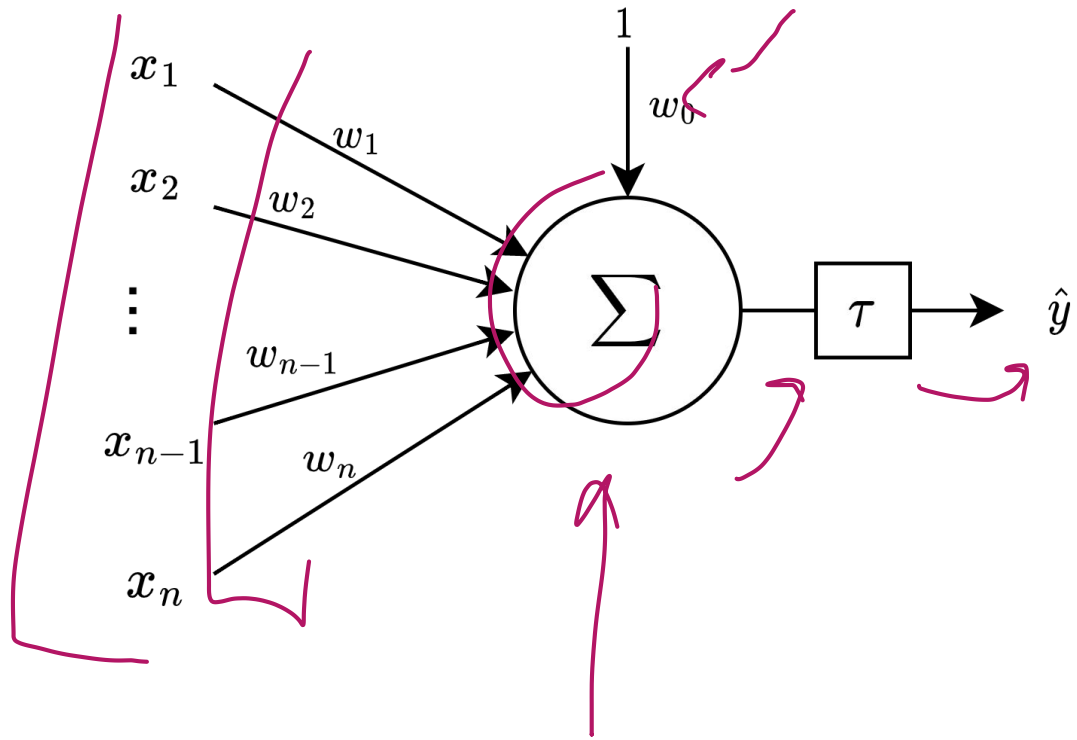
$l(y, \hat{y})$

$$\hat{y} = \sigma(w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_n x_n)$$
$$= \sigma(\vec{w}^\intercal \vec{x})$$

$$\vec{w} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$


"Activation"
"Bias"
$\hat{y} = \sigma(w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_n x_n)$
$= \sigma(\vec{w}^\mathsf{T} \vec{x})$
"Activation function"
$\vec{w} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$
$\vec{x} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$
"Weight vector"
"Feature vector"

$$\hat{y} = \tau(w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_n x_n)$$



A SINGLE NEURON

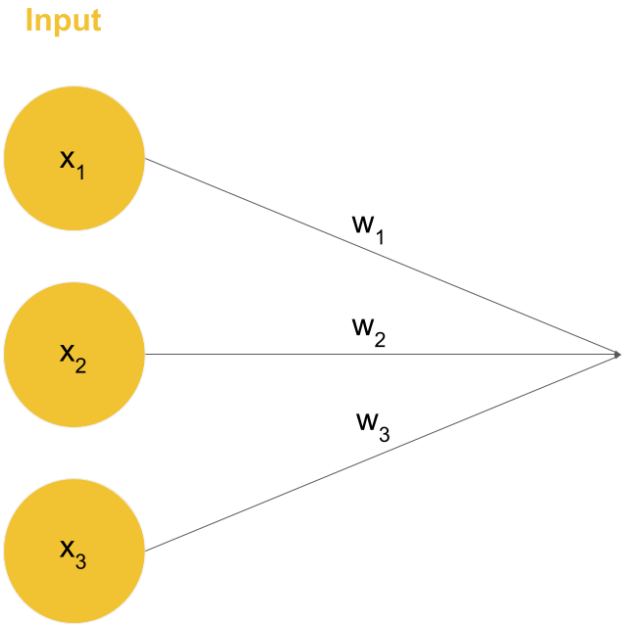
We consider a perceptron with 3-dimensional input, i.e.
$f(\mathbf{x}) = \tau(w_1 x_1 + w_2 x_2 + w_3 x_3 + b)$.

- Input features **x** are represented by nodes in the "input layer".

- In general, a p-dimensional input vector **x** will be represented by p nodes in the input layer.

A SINGLE NEURON

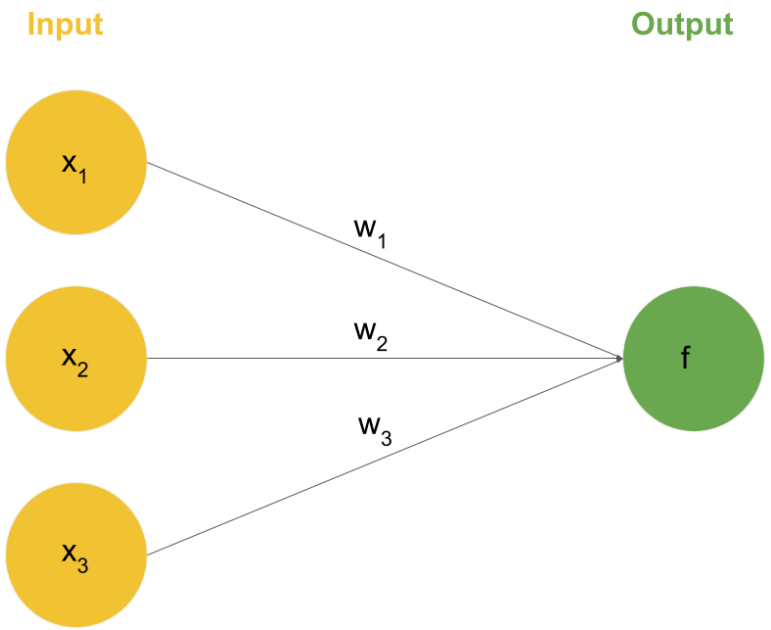
- Weights **w** are connected to edges from the input layer.



- The bias term *b* is implicit here. It is often not visualized as a separate node.

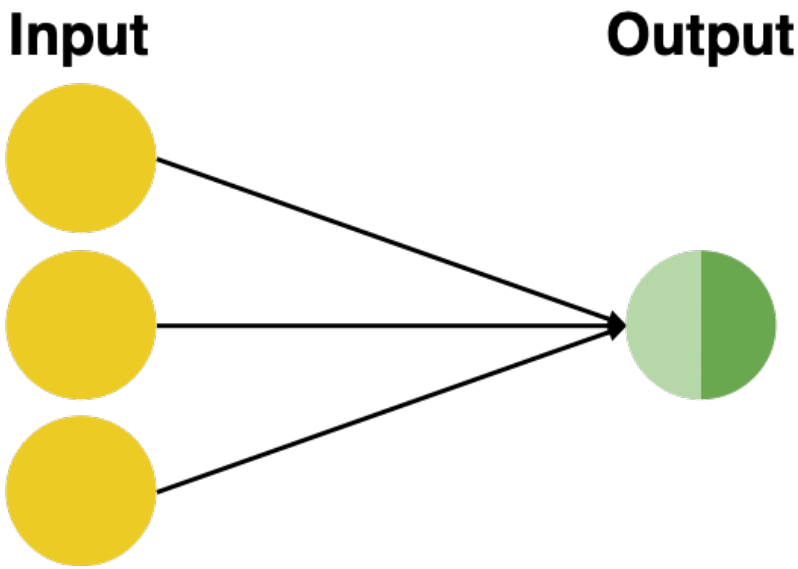
A SINGLE NEURON

- The computation $\tau(w_1 x_1 + w_2 x_2 + w_3 x_3 + b)$ is represented by the neuron in the "output layer".



A SINGLE NEURON

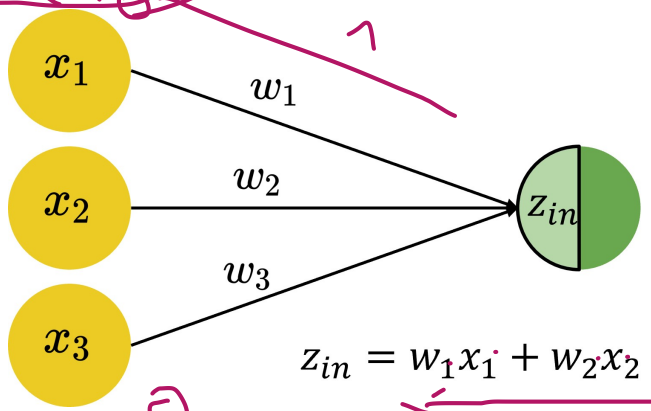
- You can picture the input vector being "fed" to neurons on the left followed by a sequence of computations performed from left to right. This is called a **forward pass**.



A SINGLE NEURON

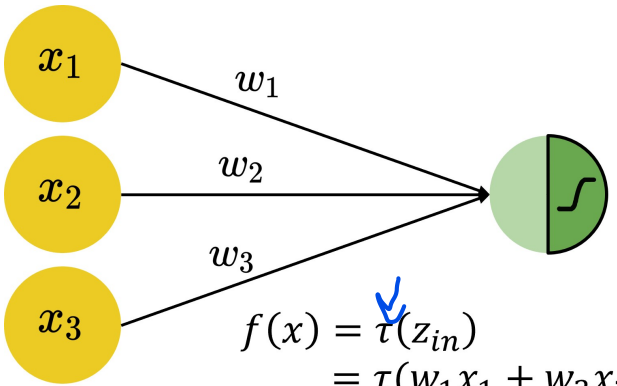
A neuron performs a 2-step computation:

1. **Affine Transformation:** weighted sum of inputs plus bias.



$$z_{in} = w_1x_1 + w_2x_2 + w_3x_3 + b$$

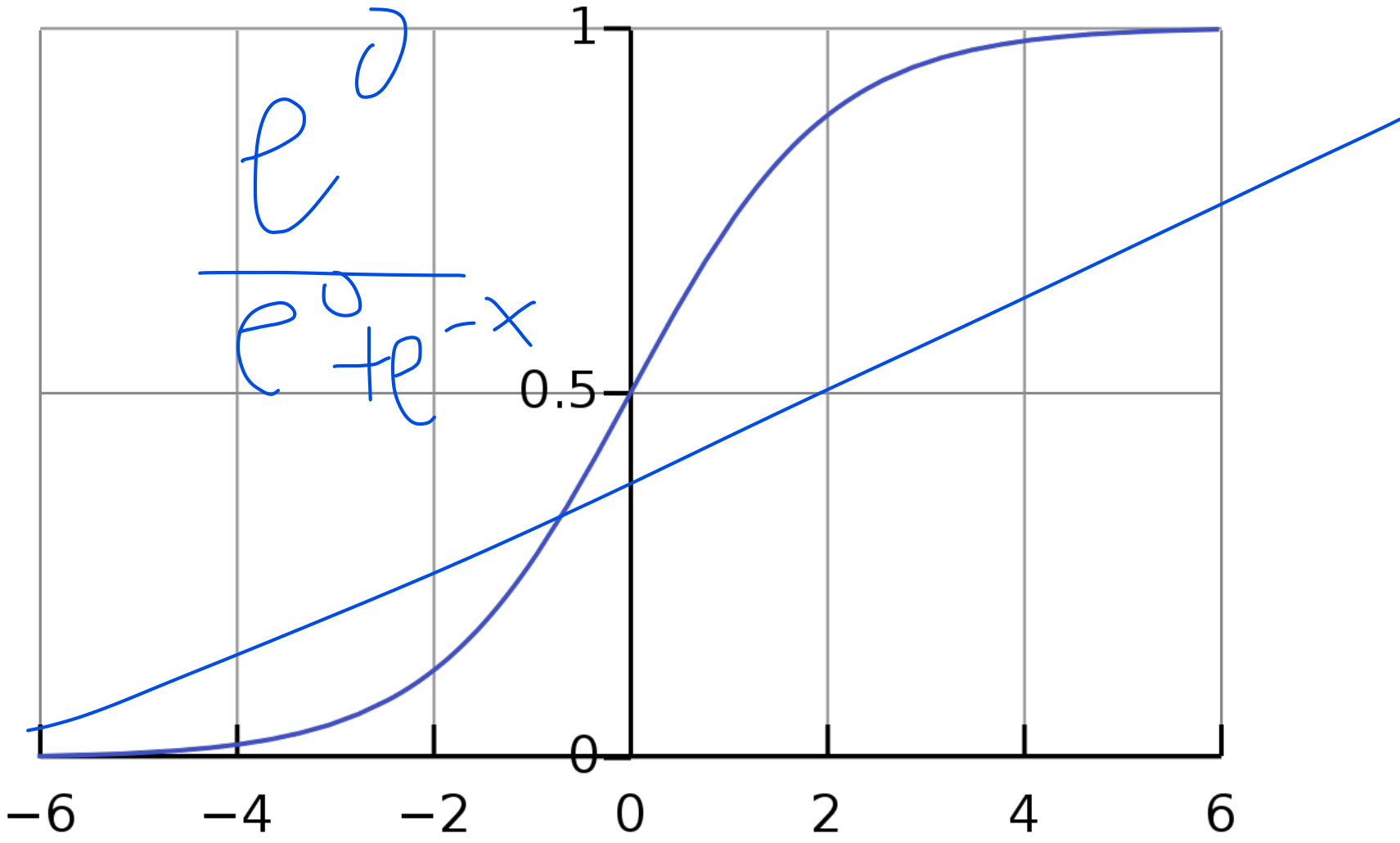
2. **Non-linear Activation:** a non-linear transformation applied to the weighted sum.



$$f(x) = \tau(z_{in})$$
$$= \tau(w_1x_1 + w_2x_2 + w_3x_3 + b)$$

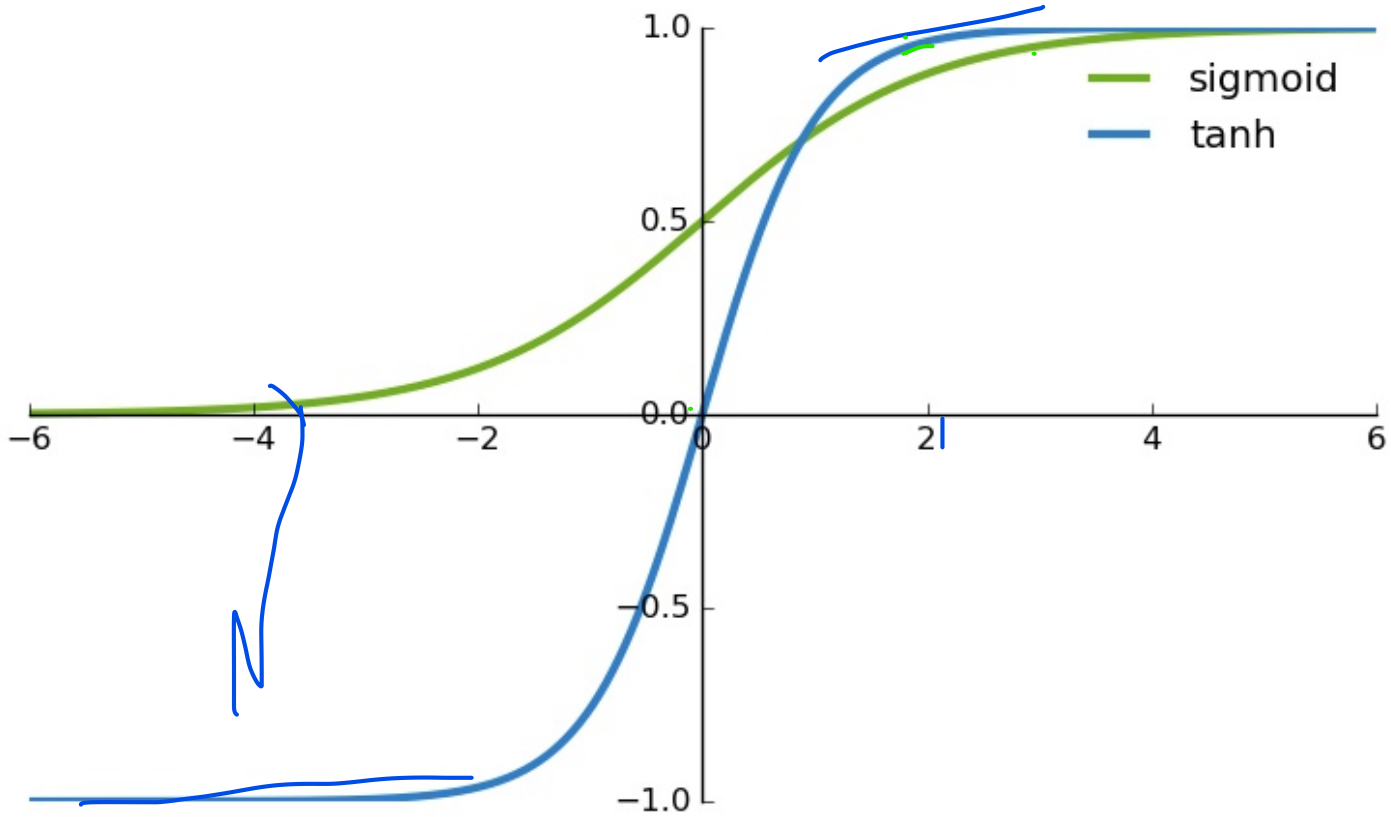
Activation functions

1. Sigmoid: $\sigma(x) = \dfrac{1}{1 + e^{-x}}$



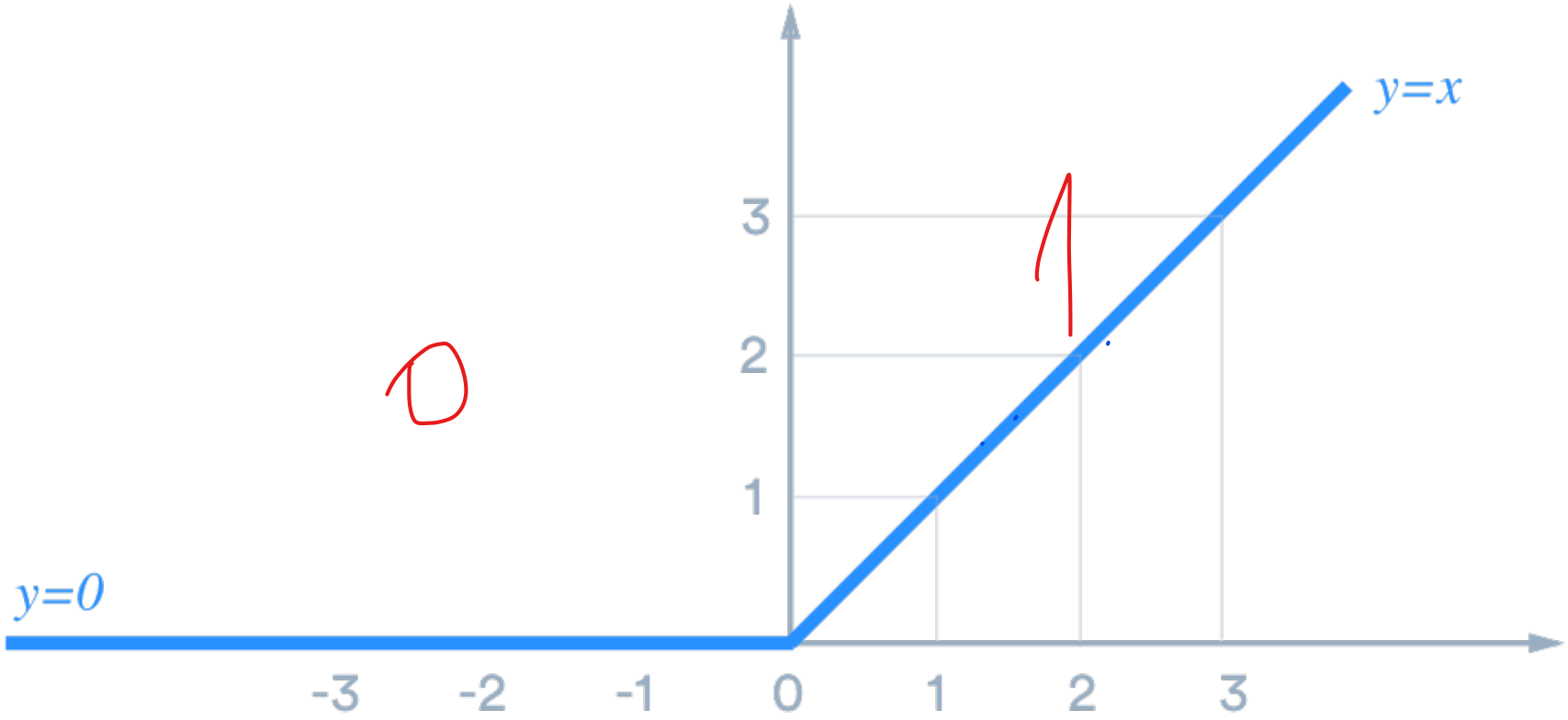
Activation functions

2. Tanh: $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$



Activation functions

3. Rectified linear unit: $ReLU(x) = \max(0, x)$



A SINGLE NEURON: OPTIMIZATION

- To optimize this model, we minimize the empirical risk

$$\mathcal{R}_{\text{emp}} = \frac{1}{n} \sum_{i=1}^{n} L\left(y^{(i)}, f\left(\mathbf{x}^{(i)}\right)\right),$$

where $L(y, f(\mathbf{x}))$ is a loss function. It compares the network's predictions $f(\mathbf{x})$ to the ground truth y.

- For regression, we typically use the L2 loss (rarely L1):

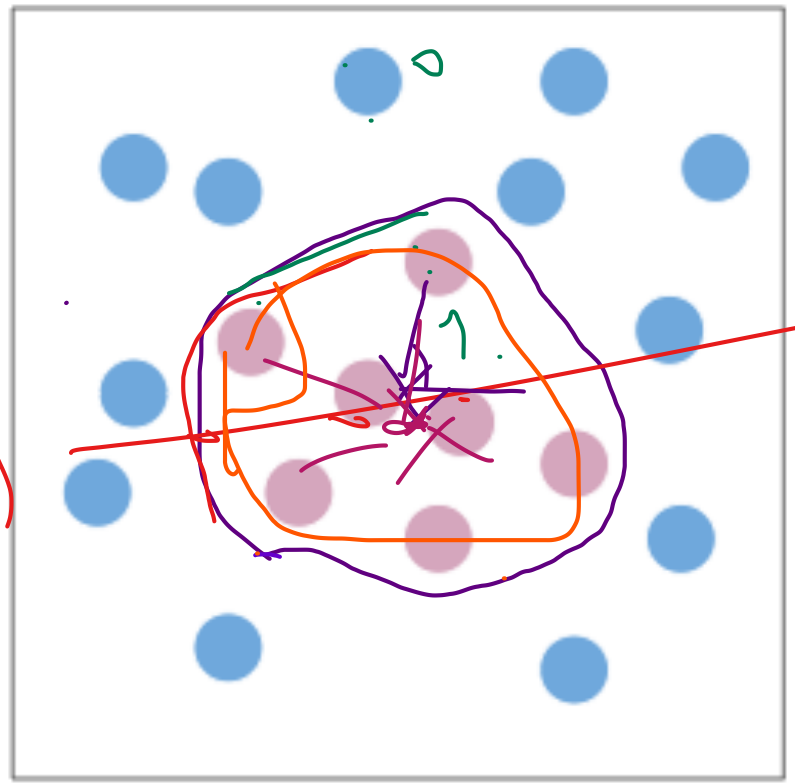
$$L(y, f(\mathbf{x})) = \frac{1}{2}(y - f(\mathbf{x}))^2$$

- For binary classification, we typically apply the cross entropy loss (also known as Bernoulli loss):

$$L(y, f(\mathbf{x})) = -(y \log f(\mathbf{x}) + (1 - y) \log(1 - f(\mathbf{x})))$$

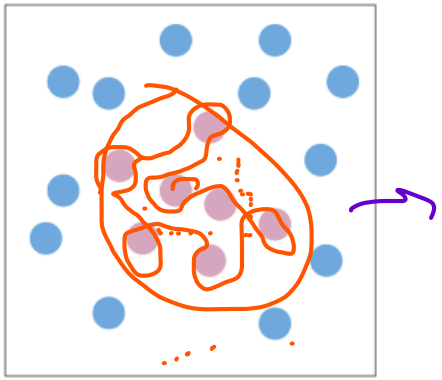
MOTIVATION

Can a single neuron perform binary classification of these points?

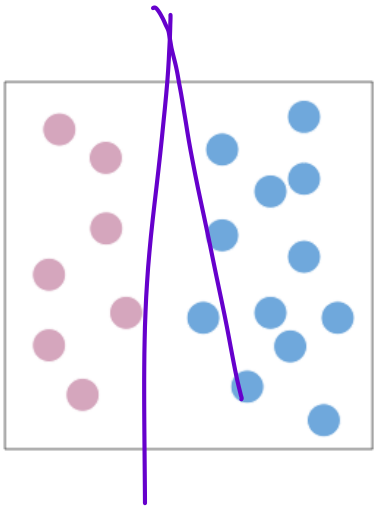


MOTIVATION

- As a single neuron is restricted to learning only linear decision boundaries, its performance on the following task is quite poor:

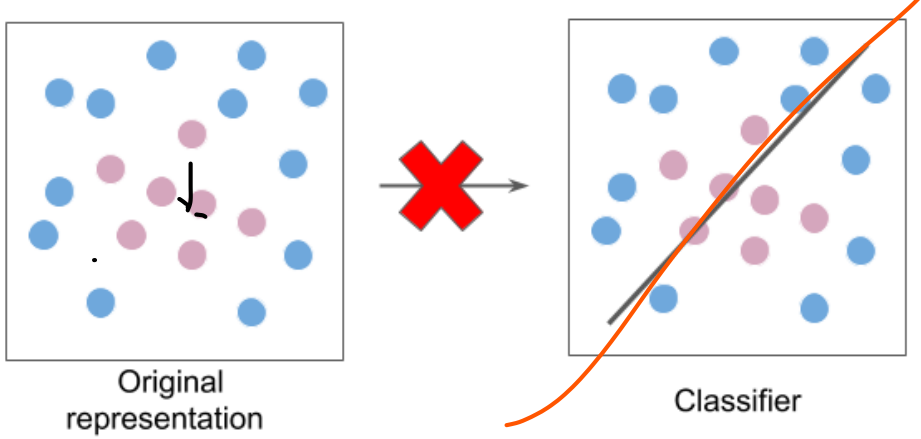


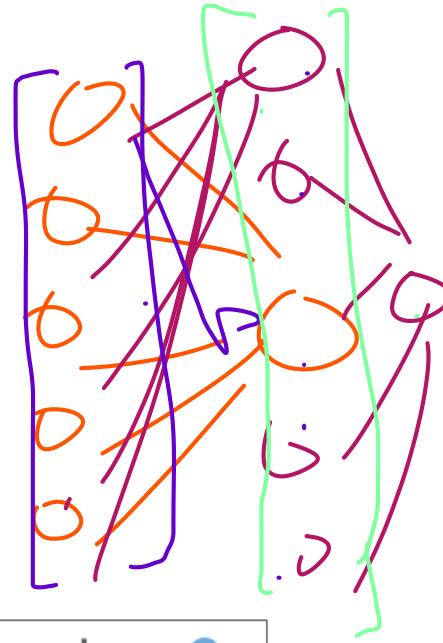
- However, the neuron can easily separate the classes if the original features are transformed (e.g., from Cartesian to polar coordinates):



MOTIVATION

- Instead of classifying the data in the original representation,

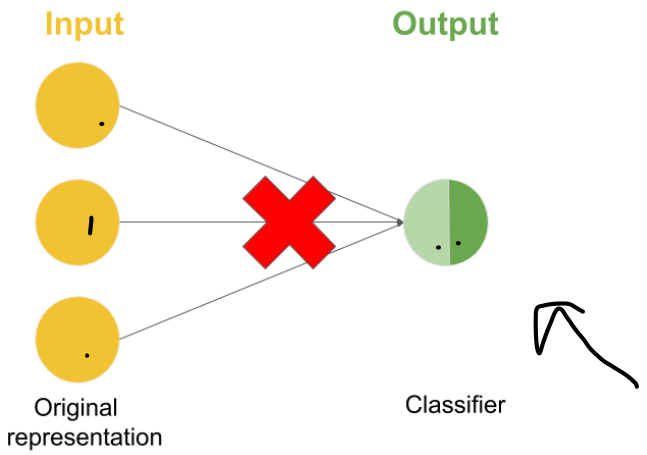




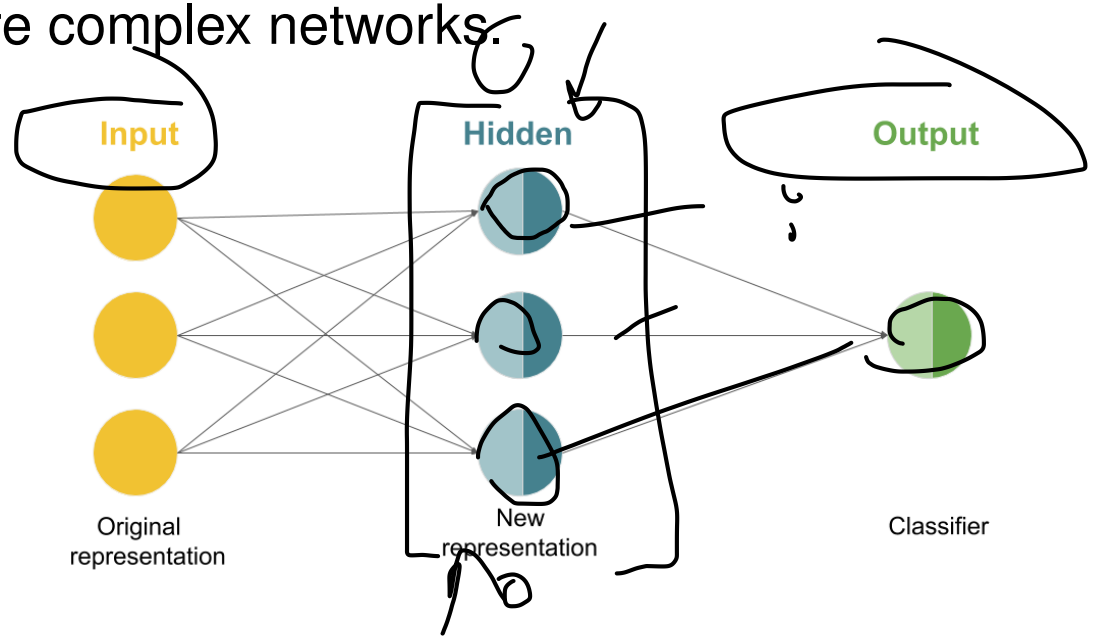
- we classify it in a new feature space.

MOTIVATION

- Analogously, instead of a single neuron,



- we use more complex networks.

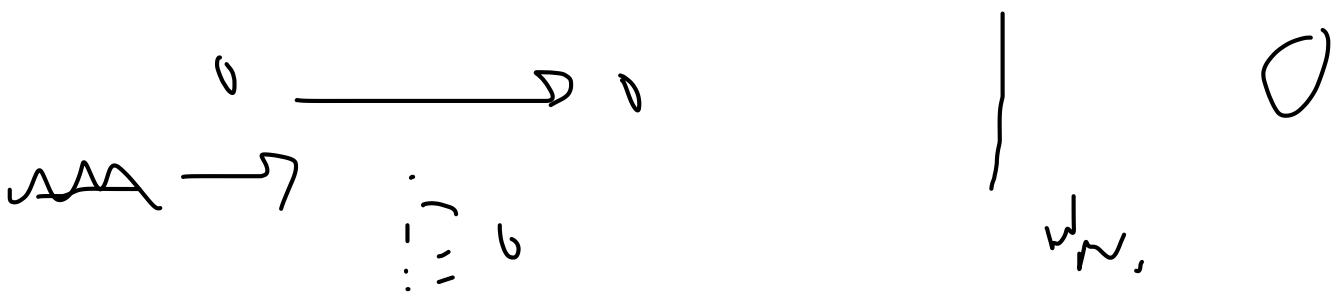


REPRESENTATION LEARNING

- It is *very* critical to feed a classifier the "right" features in order for it to perform well.

- Before deep learning took off, features for tasks like machine vision and speech recognition were "hand-designed" by domain experts. This step of the machine learning pipeline is called **feature engineering**.

- DL automates feature engineering. This is called **representation learning**.



SINGLE HIDDEN LAYER NETWORKS

Single neurons perform a 2-step computation:

1. **Affine Transformation:** a weighted sum of inputs plus bias.
2. **Activation:** a non-linear transformation on the weighted sum.

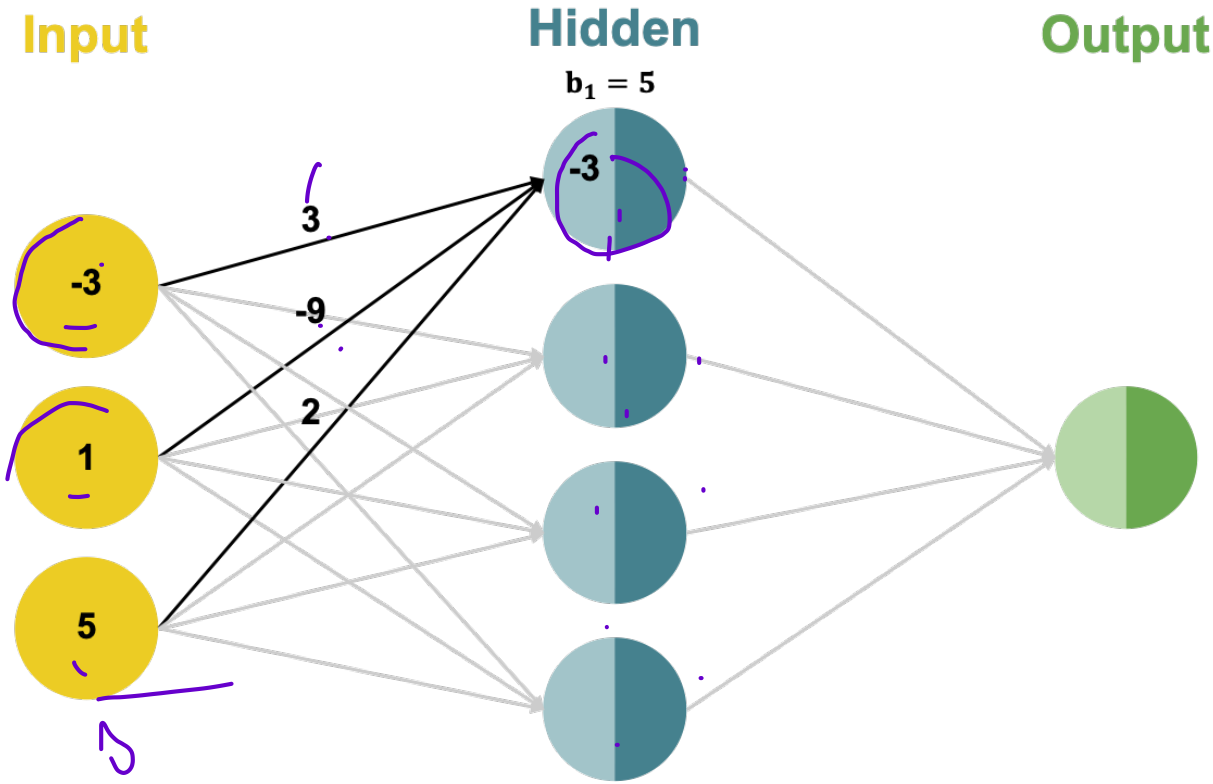
Single hidden layer networks consist of two layers (without input layer):

1. **Hidden Layer:** having a set of neurons.
2. **Output Layer:** having one or more output neurons.

- Multiple inputs are simultaneously fed to the network.
- Each neuron in the hidden layer performs a 2-step computation.
- The final output of the network is then calculated by another 2-step computation performed by the neuron in the output layer.

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:

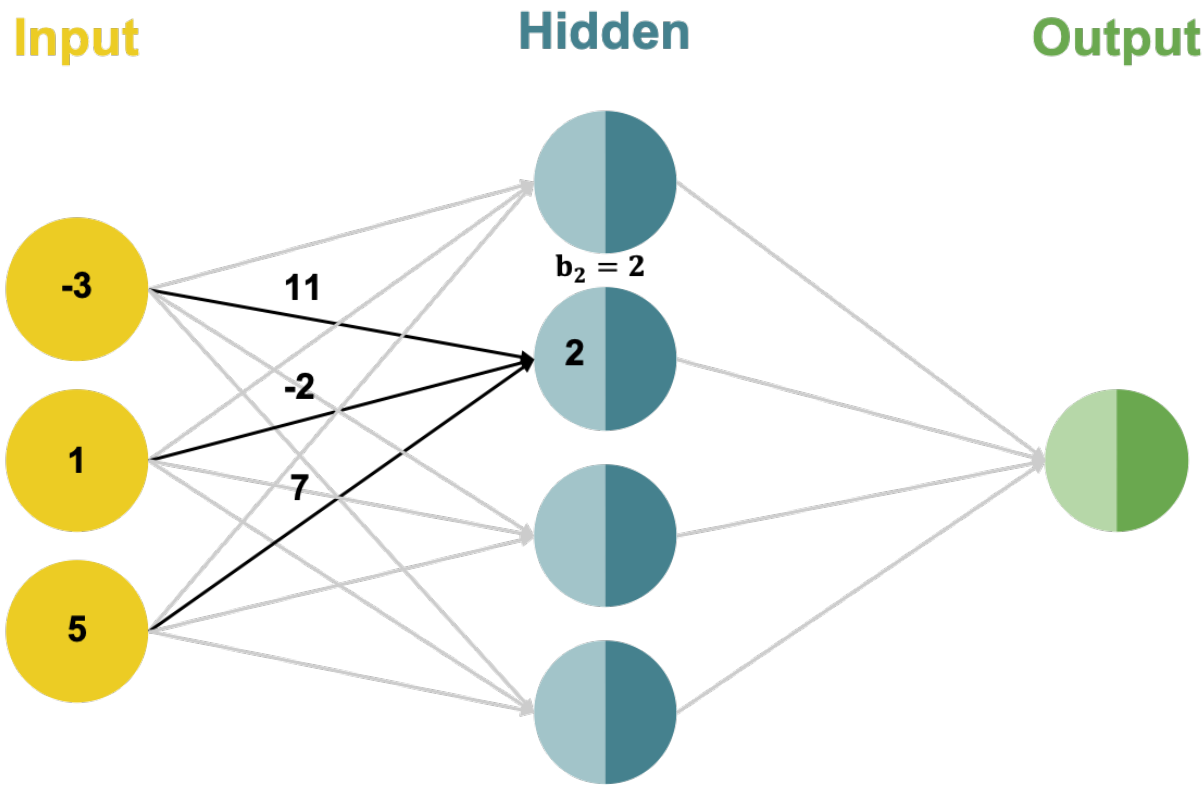


$$z_{\text{in}}^{(1)} = w_{11}x^{(1)} + w_{21}x^{(2)} + w_{31}x^{(3)} + b_1$$

$$z_{\text{in}}^{(1)} = 3 * (-3) + (-9) * 1 + 2 * 5 + 5 = -3$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

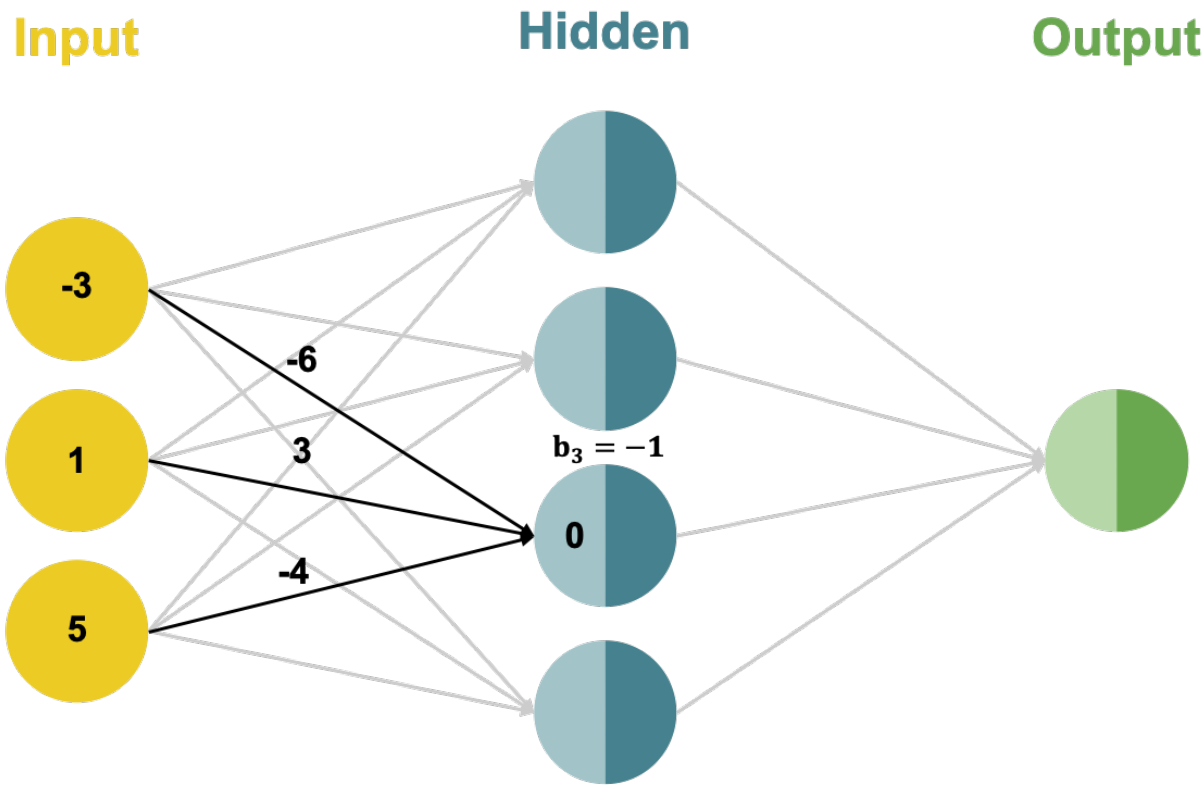
Each neuron in the hidden layer performs an **affine transformation** on the inputs:



$$z_{\text{in}}^{(2)} = w_{12}x^{(1)} + w_{22}x^{(2)} + w_{32}x^{(3)} + b_2$$
$$z_{\text{in}}^{(2)} = 11 * (-3) + (-2) * 1 + 7 * 5 + 2 = 2$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

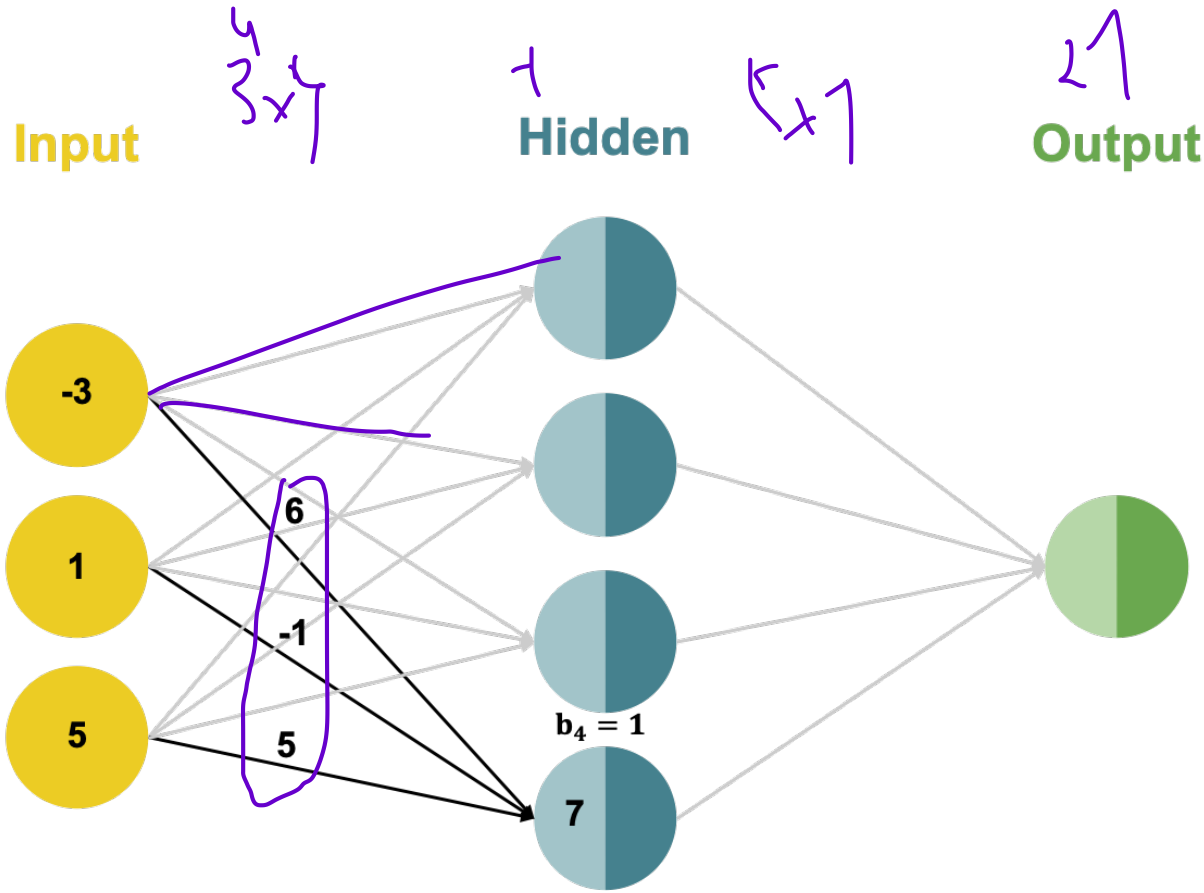
Each neuron in the hidden layer performs an **affine transformation** on the inputs:



$$z_{\text{in}}^{(3)} = w_{13}x^{(1)} + w_{23}x^{(2)} + w_{33}x^{(3)} + b_3$$
$$z_{\text{in}}^{(3)} = (-6) * (-3) + 3 * 1 + (-4) * 5 - 1 = 0$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

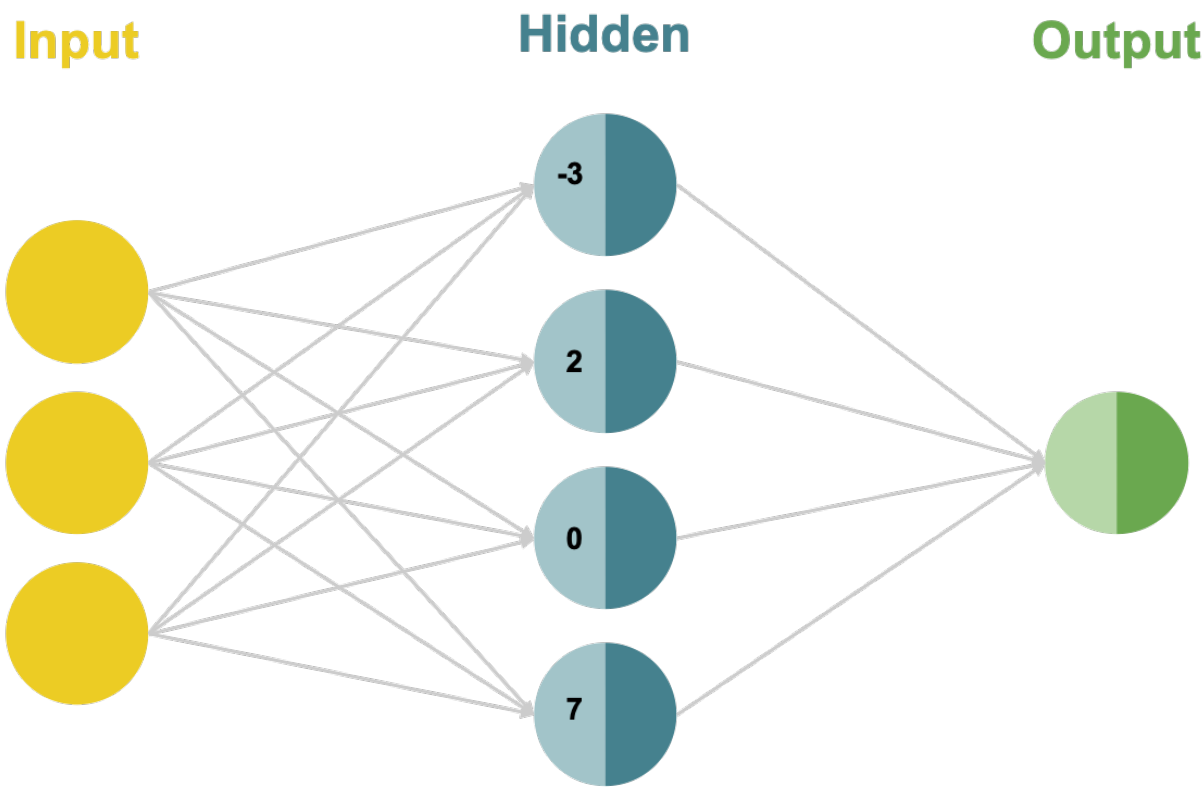
Each neuron in the hidden layer performs an **affine transformation** on the inputs:



$$z_{\text{in}}^{(4)} = w_{14}x^{(1)} + w_{24}x^{(2)} + w_{34}x^{(3)} + b_4$$
$$z_{\text{in}}^{(4)} = 6 * (-3) + (-1) * 1 + 5 * 5 + 1 = 7$$

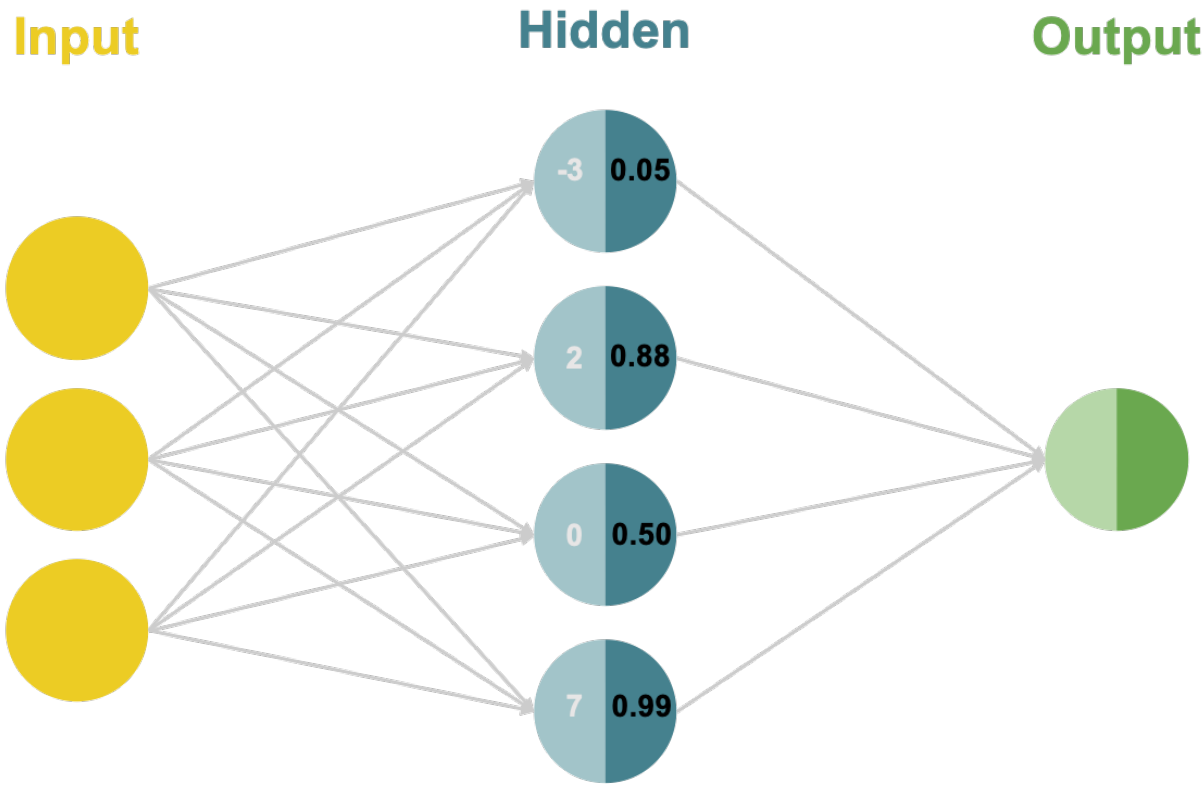
SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:



SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

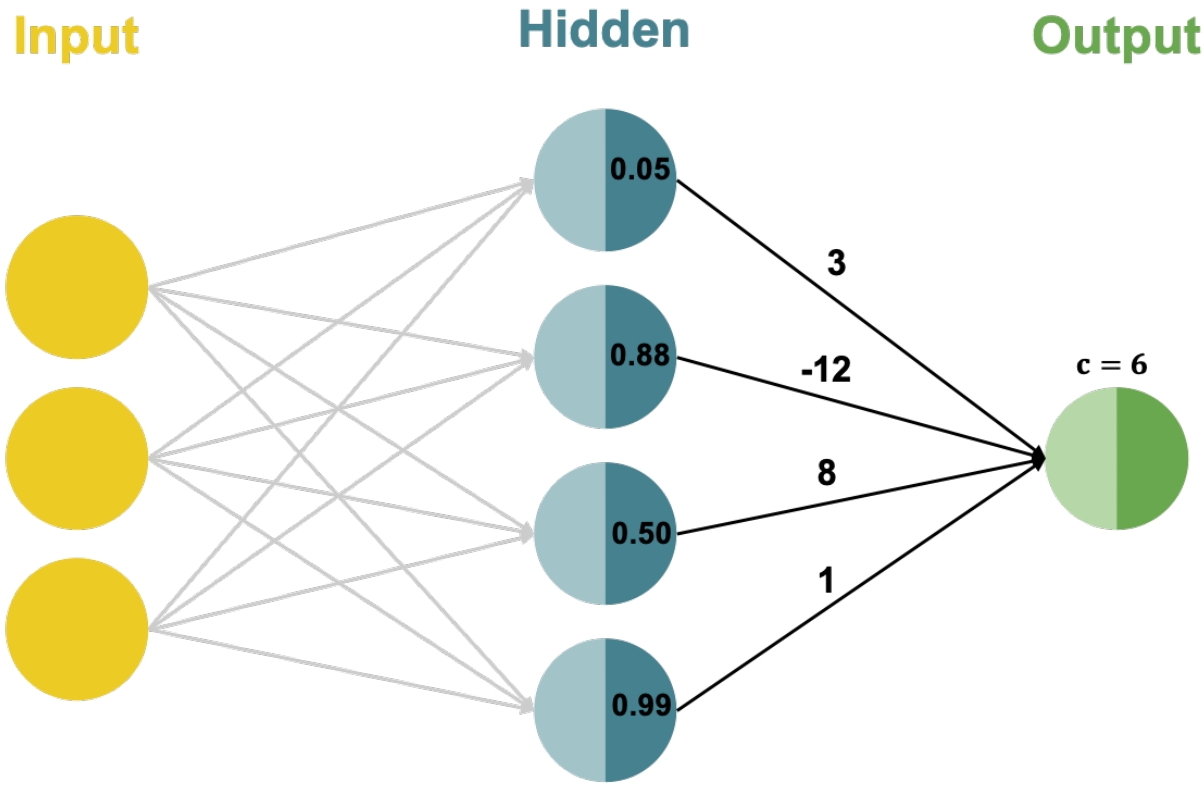
Each hidden neuron performs a non-linear **activation** transformation on the weight sum:



$$z_{\text{out}}^{(i)} = \sigma\left(z_{\text{in}}^{(i)}\right) = \frac{1}{1+e^{z_{\text{in}}^{(i)}}}$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

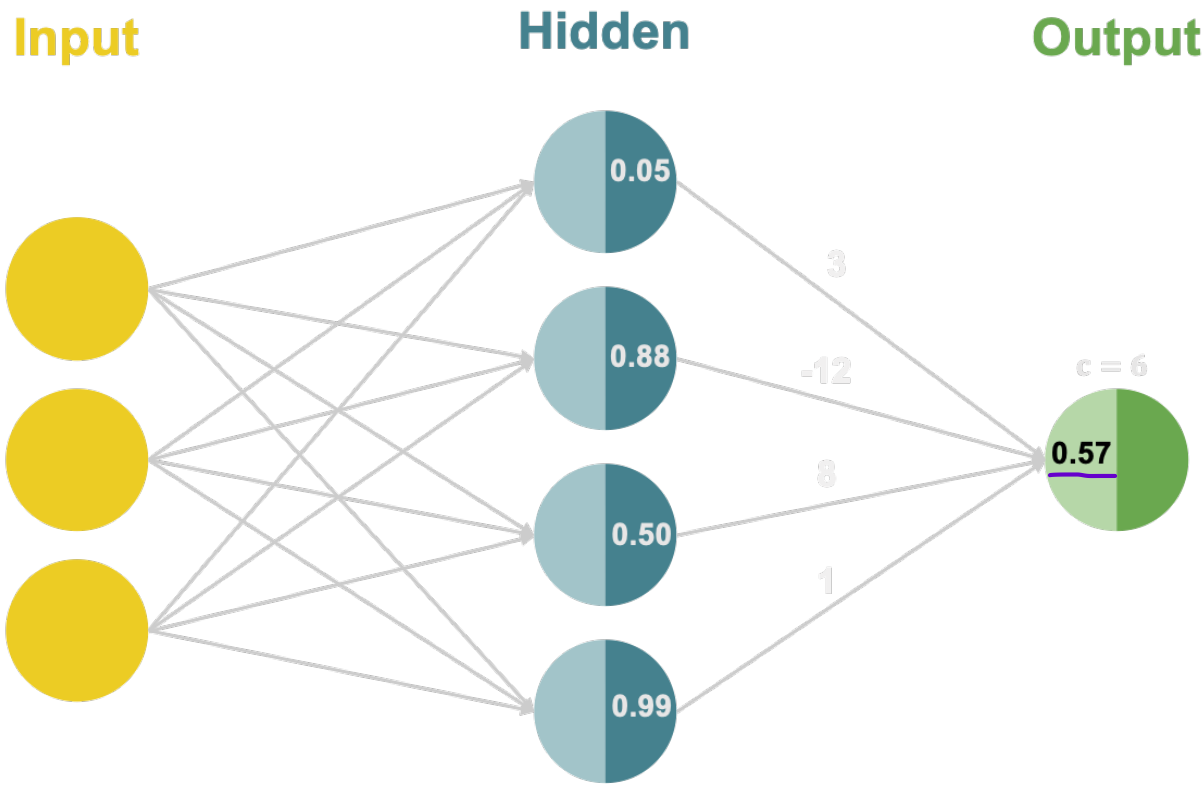
The output neuron performs an **affine transformation** on its inputs:



$$f_{\text{in}} = u_1 z_{\text{out}}^{(1)} + u_2 z_{\text{out}}^{(2)} + u_3 z_{\text{out}}^{(3)} + u_4 z_{\text{out}}^{(4)} + c$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

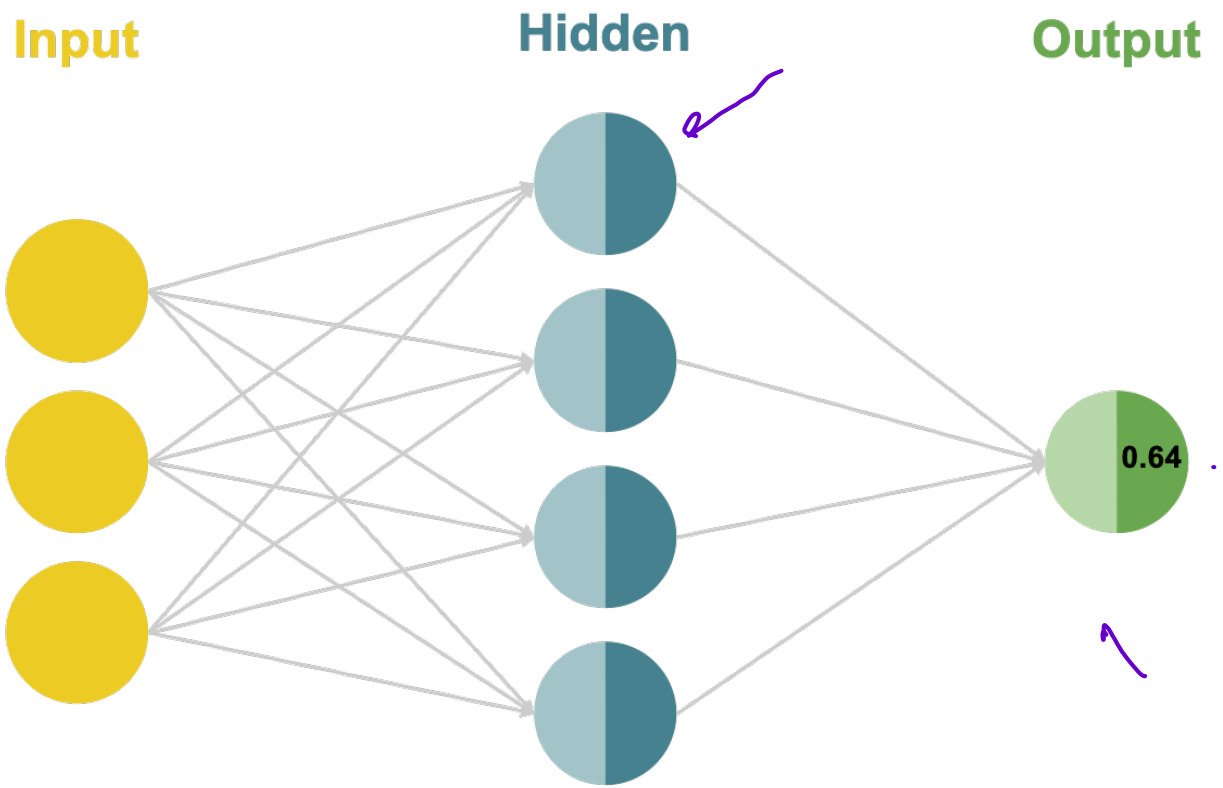
The output neuron performs an **affine transformation** on its inputs:



$$f_{\text{in}} = u_1 z_{\text{out}}^{(1)} + u_2 z_{\text{out}}^{(2)} + u_3 z_{\text{out}}^{(3)} + u_4 z_{\text{out}}^{(4)} + c$$
$$f_{\text{in}} = 3 * 0.05 + (-12) * 0.88 + 8 * 0.50 + 1 * 0.99 + 6 = 0.57$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

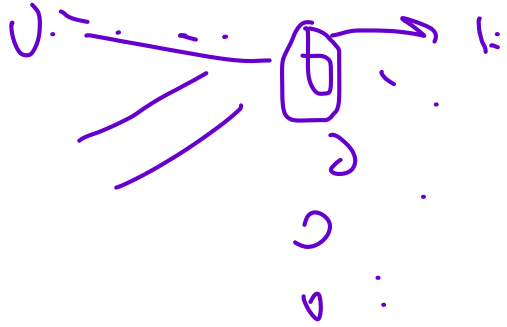
The output neuron performs a non-linear **activation** transformation on the weight sum:



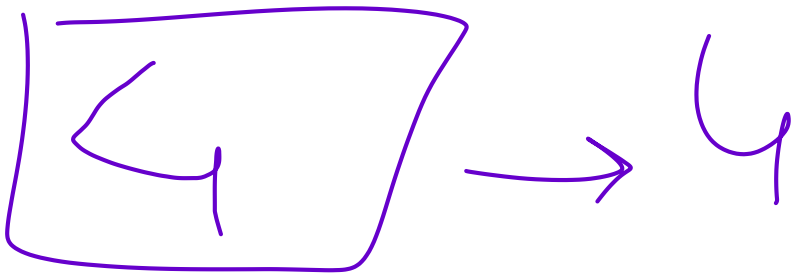
$$f_{\text{out}} = \sigma(f_{\text{in}}) = \frac{1}{1+e^{f_{\text{in}}}}$$

$$f_{\text{out}} = \frac{1}{1+e^{0.57}} = 0.64$$

MULTI-CLASS CLASSIFICATION



- We have only considered regression and binary classification problems so far.
- How can we get a neural network to perform multiclass classification?



MULTI-CLASS CLASSIFICATION

- The first step is to add additional neurons to the output layer.
- Each neuron in the layer will represent a specific class (number of neurons in the output layer = number of classes).

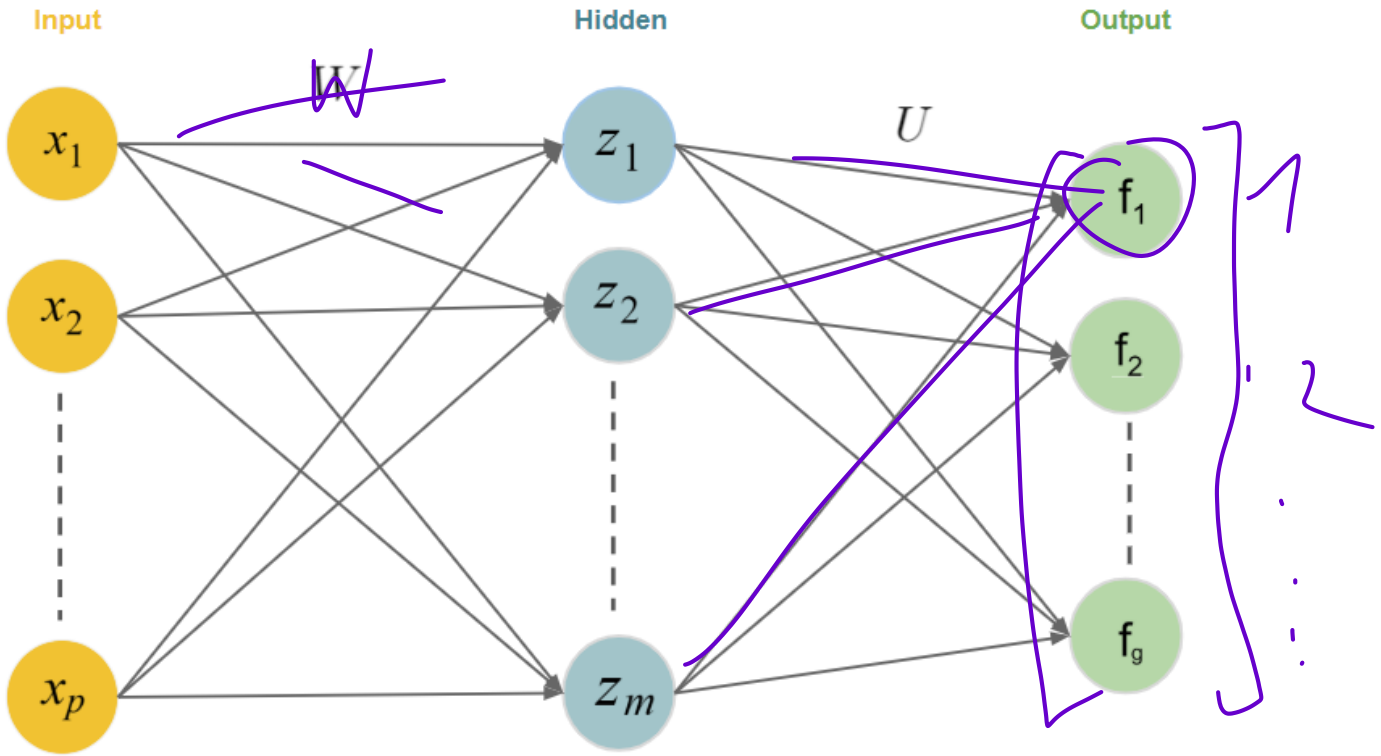


Figure: Structure of a single hidden layer, feed-forward neural network for g-class classification problems (bias term omitted).

MULTI-CLASS CLASSIFICATION

Notation:

- For g-class classification, g output units:

$$\mathbf{f} = (f_1, \ldots, f_g)$$

- m hidden neurons $z_1, \ldots, z_m$, with

$$z_j = \sigma(\mathbf{W}_j^T \mathbf{x}), \quad j = 1, \ldots, m.$$

- Compute linear combinations of derived features z:

$$f_{in,k} = \boldsymbol{U}_k^T \mathbf{z}, \quad \mathbf{z} = (z_1, \ldots, z_m)^T, \quad k = 1, \ldots, g$$

MULTI-CLASS CLASSIFICATION

- The second step is to apply a **softmax** activation function to the output layer.

- This gives us a probability distribution over g different possible classes:

$$f_{out,k} = \tau_k(f_{in,k}) = \frac{\exp(f_{in,k})}{\sum_{k'=1}^{g} \exp(f_{in,k'})}$$

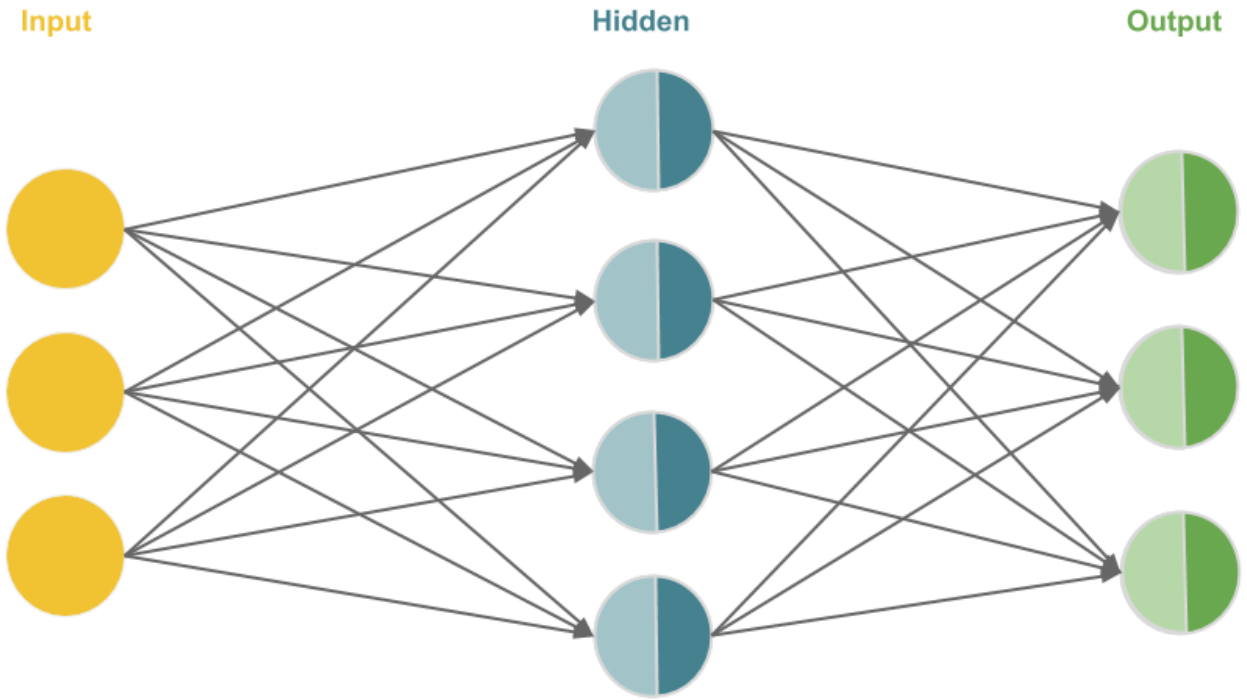
- This is the same transformation used in softmax regression!

- Derivative $\frac{\partial \tau(\mathbf{f}_{in})}{\partial \mathbf{f}_{in}} = \text{diag}(\tau(\mathbf{f}_{in})) - \tau(\mathbf{f}_{in})\tau(\mathbf{f}_{in})^T$

- It is a "smooth" approximation of the argmax operation, so $\tau((1, 1000, 2)^T) \approx (0, 1, 0)^T$ (picks out 2nd element!).

MULTI-CLASS CLASSIFICATION: EXAMPLE

Forward pass (Hidden: Sigmoid, Output: Softmax).



$$\begin{pmatrix} 3 & -9 & 2 \\ 11 & -2 & 7 \\ -6 & 3 & -4 \\ 6 & -1 & 5 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ -1 \\ 1 \end{pmatrix}$$

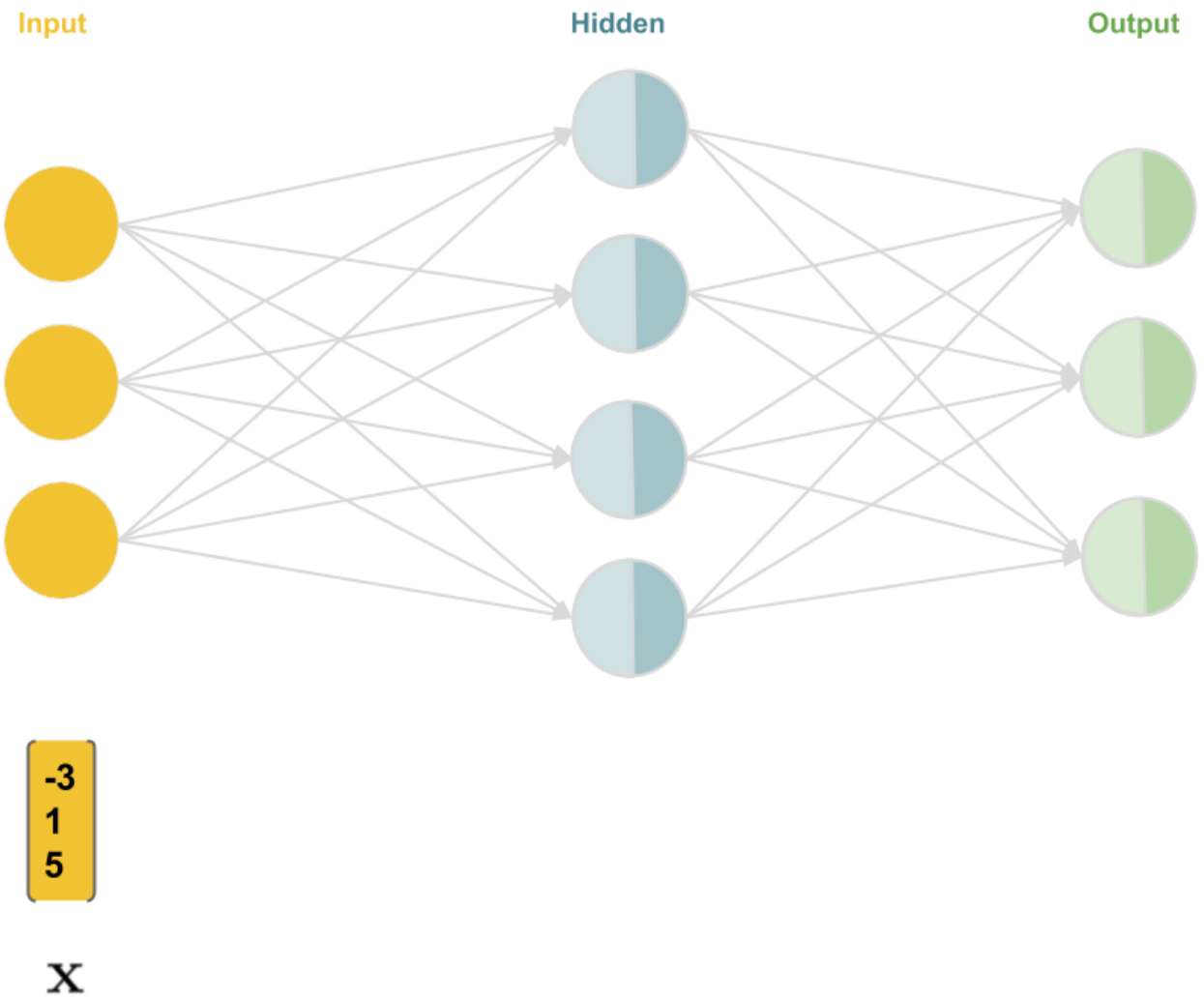
W^T $\mathbf{b}$

$$\begin{pmatrix} 3 & -12 & 8 & 1 \\ 2 & -3 & 9 & 1 \\ -5 & 1 & -1 & 7 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \\ -8 \end{pmatrix}$$

U^T $\mathbf{c}$

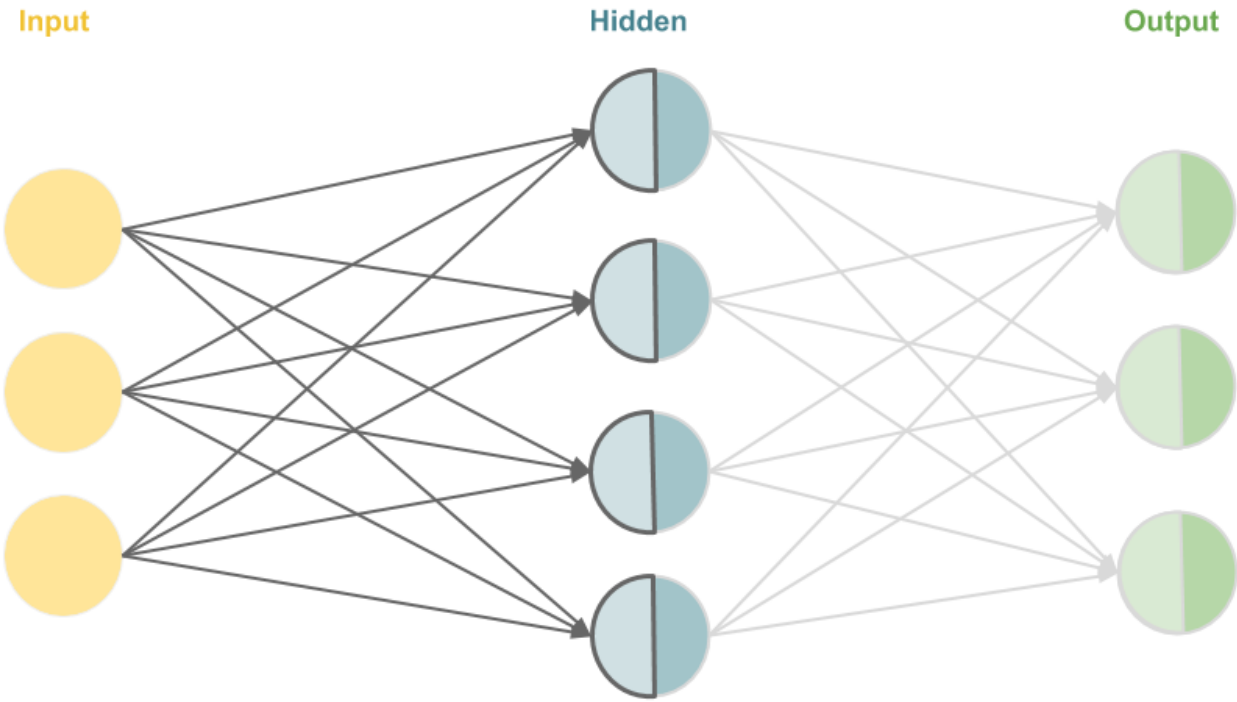
MULTI-CLASS CLASSIFICATION: EXAMPLE

Forward pass (Hidden: Sigmoid, Output: Softmax).



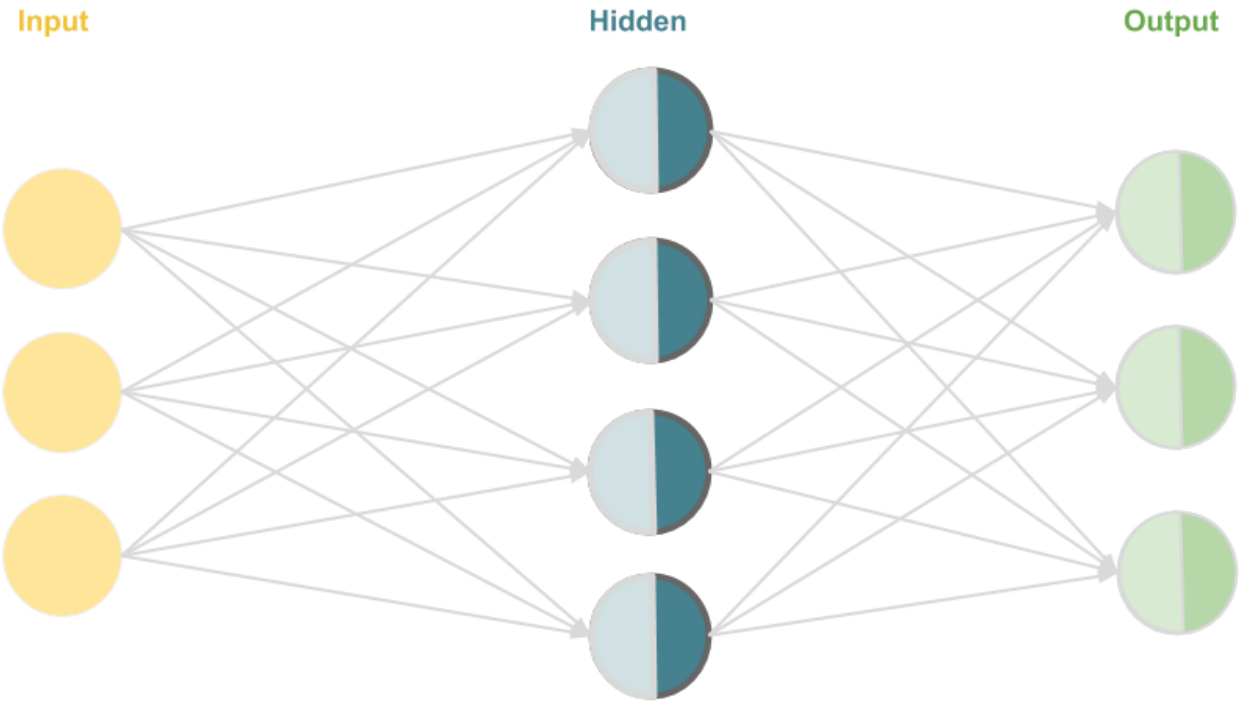
MULTI-CLASS CLASSIFICATION: EXAMPLE

Forward pass (Hidden: Sigmoid, Output: Softmax).



MULTI-CLASS CLASSIFICATION: EXAMPLE

Forward pass (Hidden: Sigmoid, Output: Softmax).

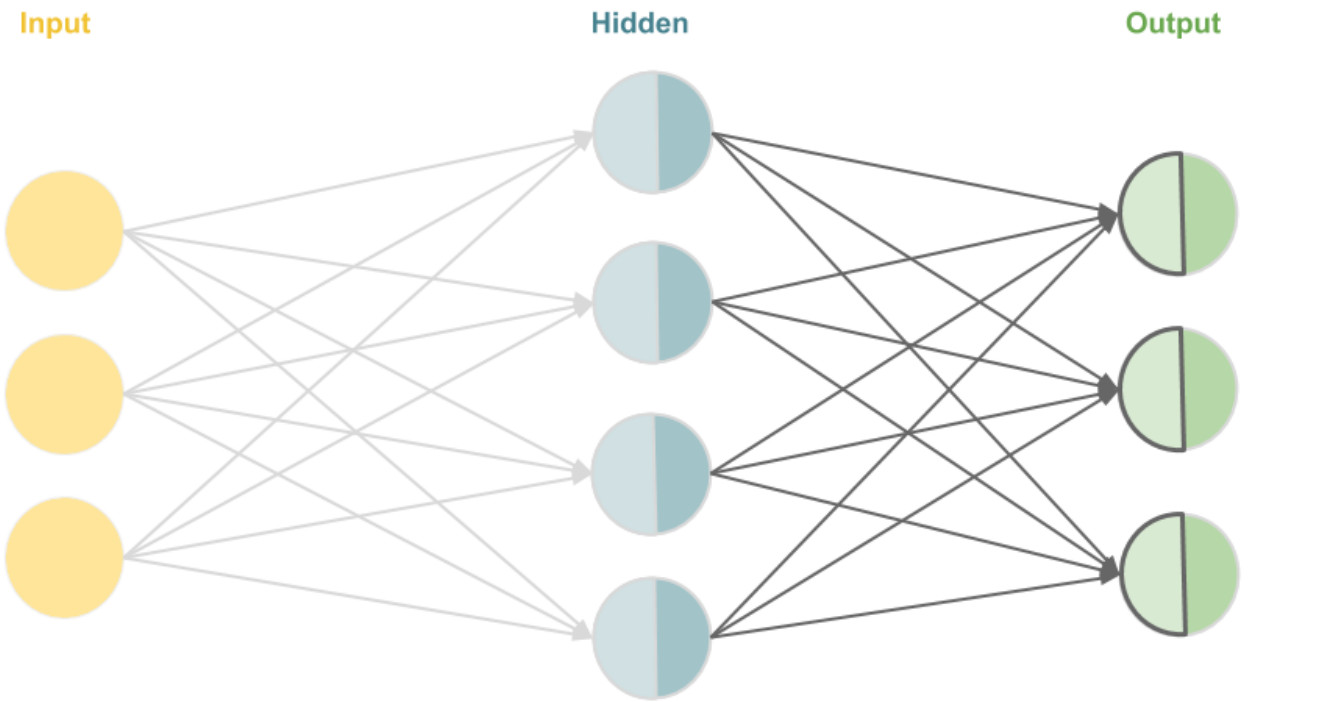


$$\begin{bmatrix} -3 \\ 1 \\ 5 \end{bmatrix} \quad \begin{bmatrix} -3 \\ 2 \\ 0 \\ 7 \end{bmatrix} \quad \begin{bmatrix} 1/(1+\exp(-(-3))) \\ 1/(1+\exp(-2)) \\ 1/(1+\exp(-0)) \\ 1/(1+\exp(-7)) \end{bmatrix}$$

$$\mathbf{x} \qquad \mathbf{z}_{in} \qquad \mathbf{z} = \mathbf{z}_{out} = \sigma(\mathbf{z}_{in})$$

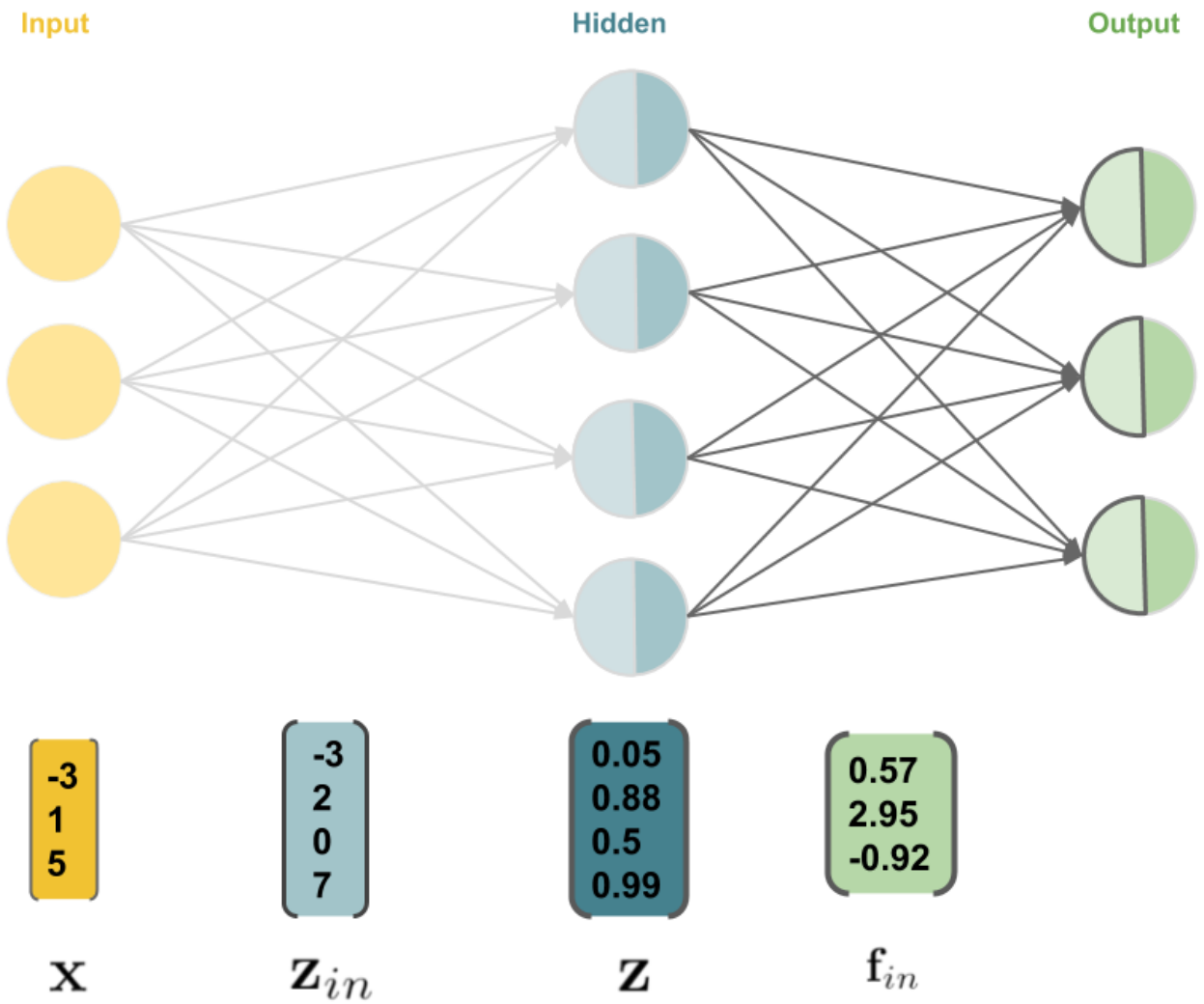
MULTI-CLASS CLASSIFICATION: EXAMPLE

Forward pass (Hidden: Sigmoid, Output: Softmax).



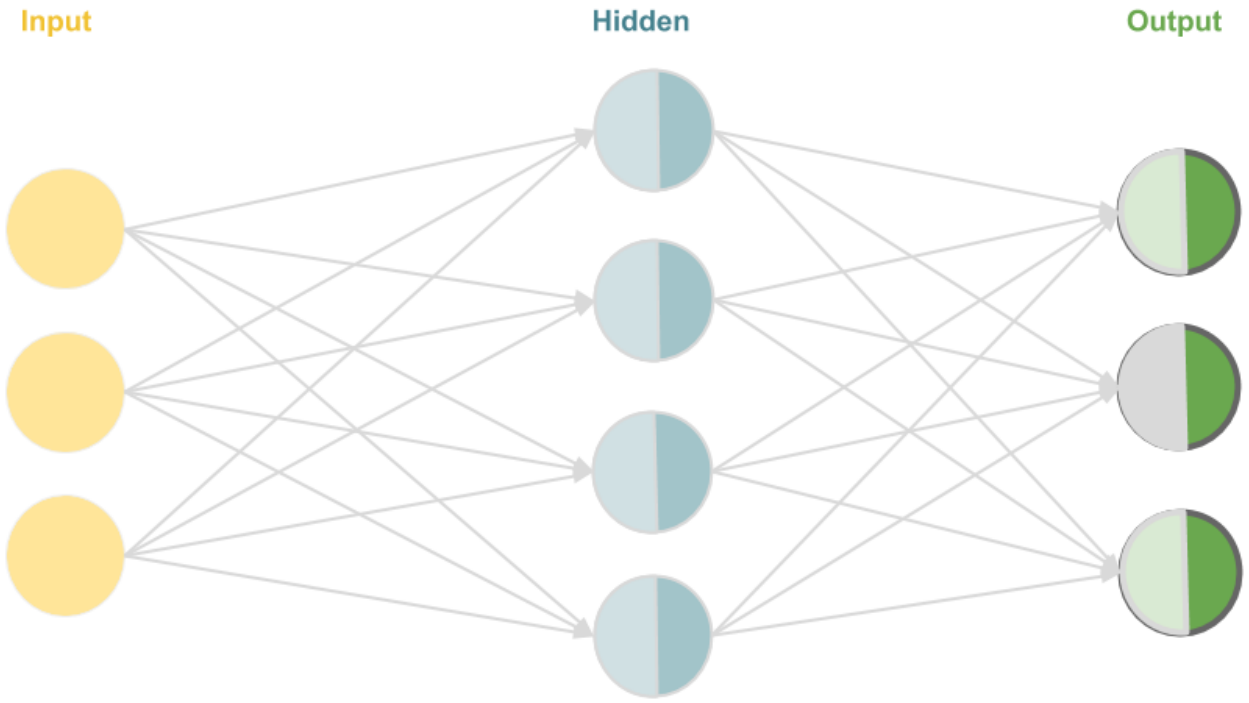
MULTI-CLASS CLASSIFICATION: EXAMPLE

Forward pass (Hidden: Sigmoid, Output: Softmax).



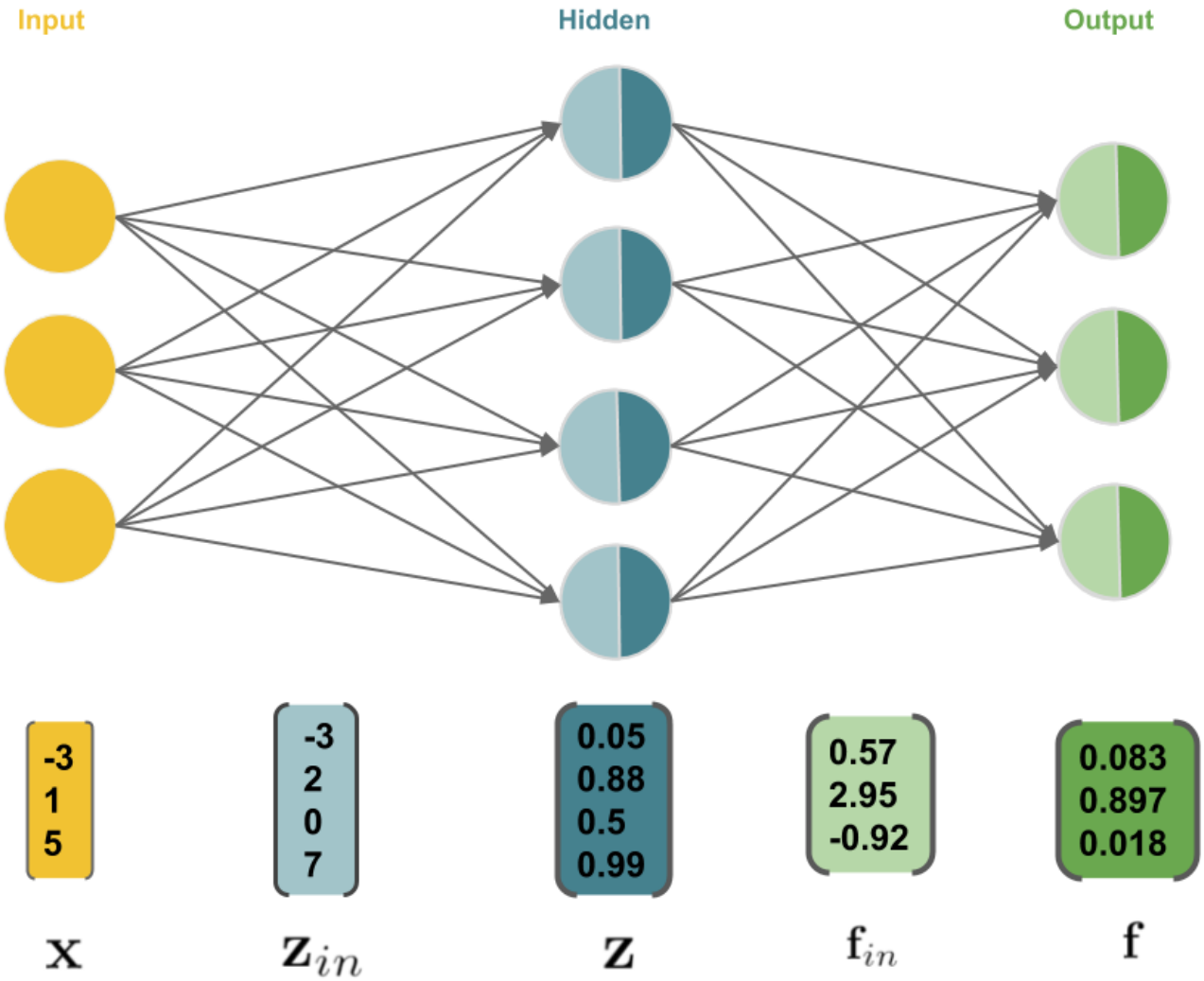
MULTI-CLASS CLASSIFICATION: EXAMPLE

Forward pass (Hidden: Sigmoid, Output: Softmax).



MULTI-CLASS CLASSIFICATION: EXAMPLE

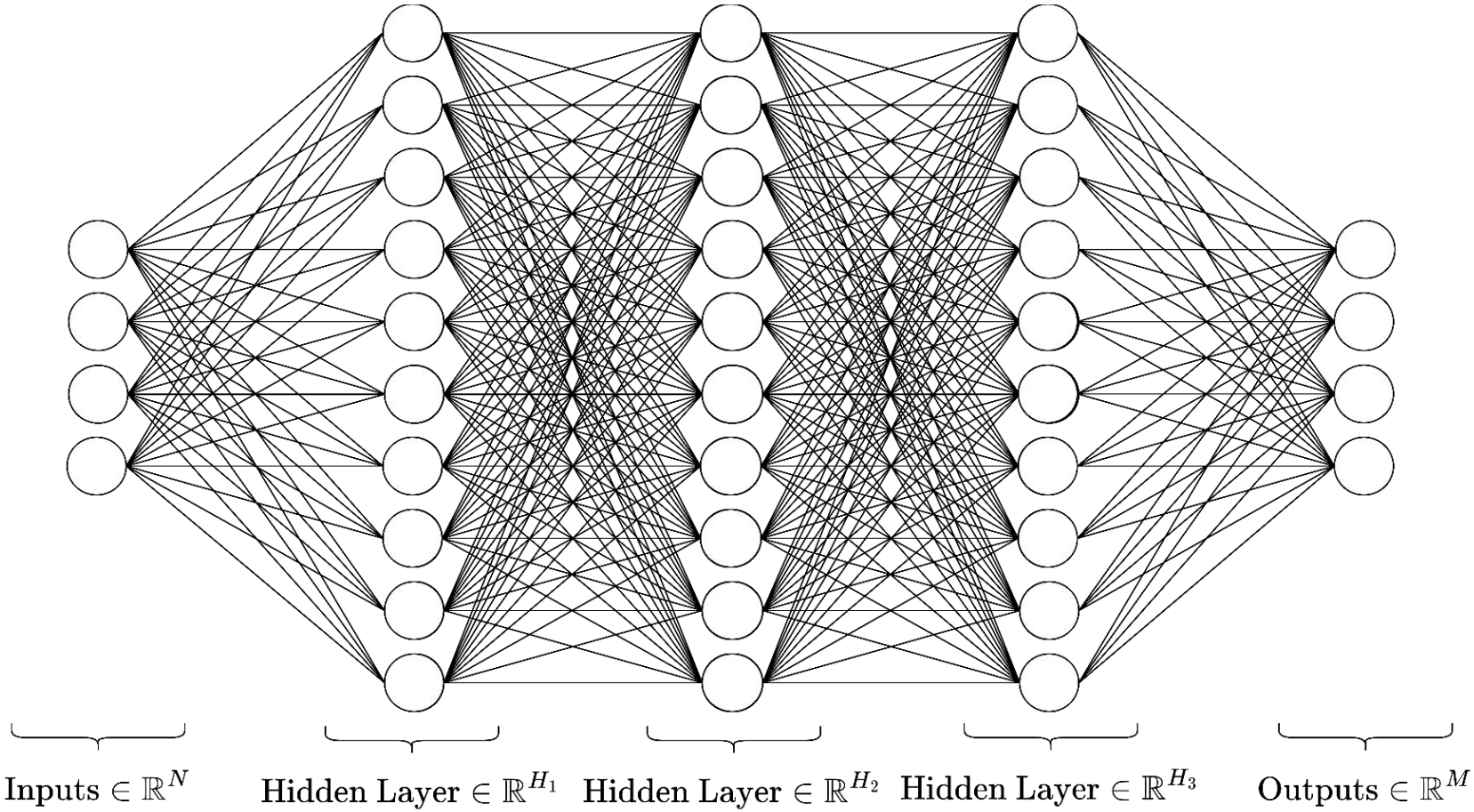
Forward pass (Hidden: Sigmoid, Output: Softmax).

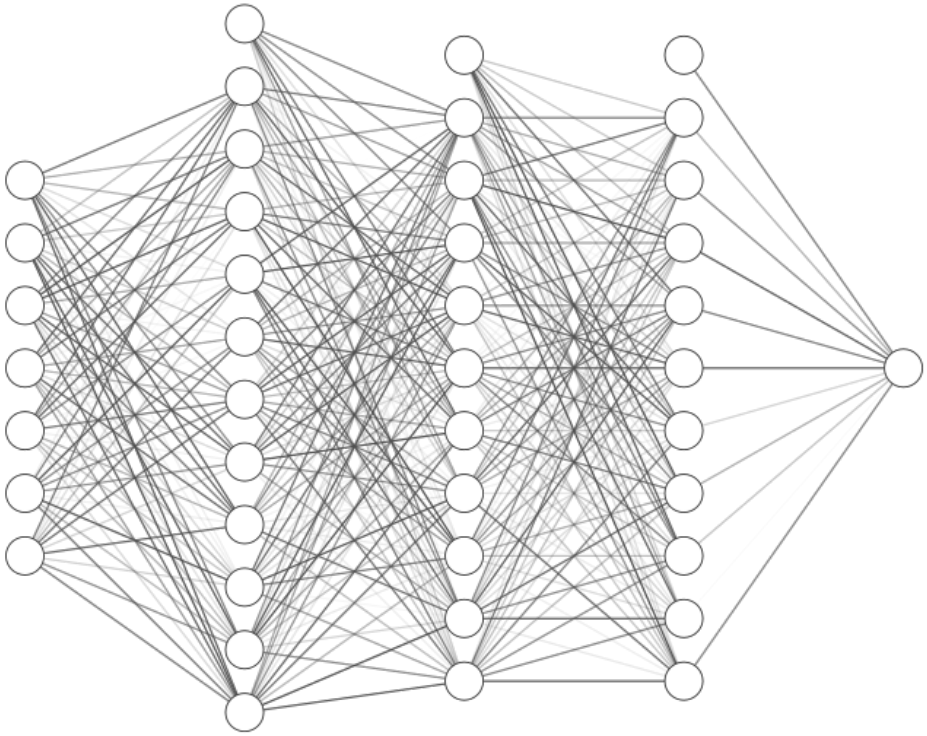


FEEDFORWARD NEURAL NETWORKS

- We will now extend the model class once again, such that we allow an arbitrary amount l of hidden layers.

- The general term for this model class is (multi-layer) **feedforward networks** (inputs are passed through the network from left to right, no feedback-loops are allowed)


Inputs $\in \mathbb{R}^N$
Hidden Layer $\in \mathbb{R}^{H_1}$
Hidden Layer $\in \mathbb{R}^{H_2}$
Hidden Layer $\in \mathbb{R}^{H_3}$
Outputs $\in \mathbb{R}^M$



A multi-layer perceptron is a series of affine transformations of an input vector, each of which is wrapped in a non-linear activation function.

$$\mathcal{N} : \mathbb{R}^N \to \mathbb{R}^M$$
$$N, M \in \mathbb{N}$$

(*Translation:* an MLP is a fancy function)

FEEDFORWARD NEURAL NETWORKS

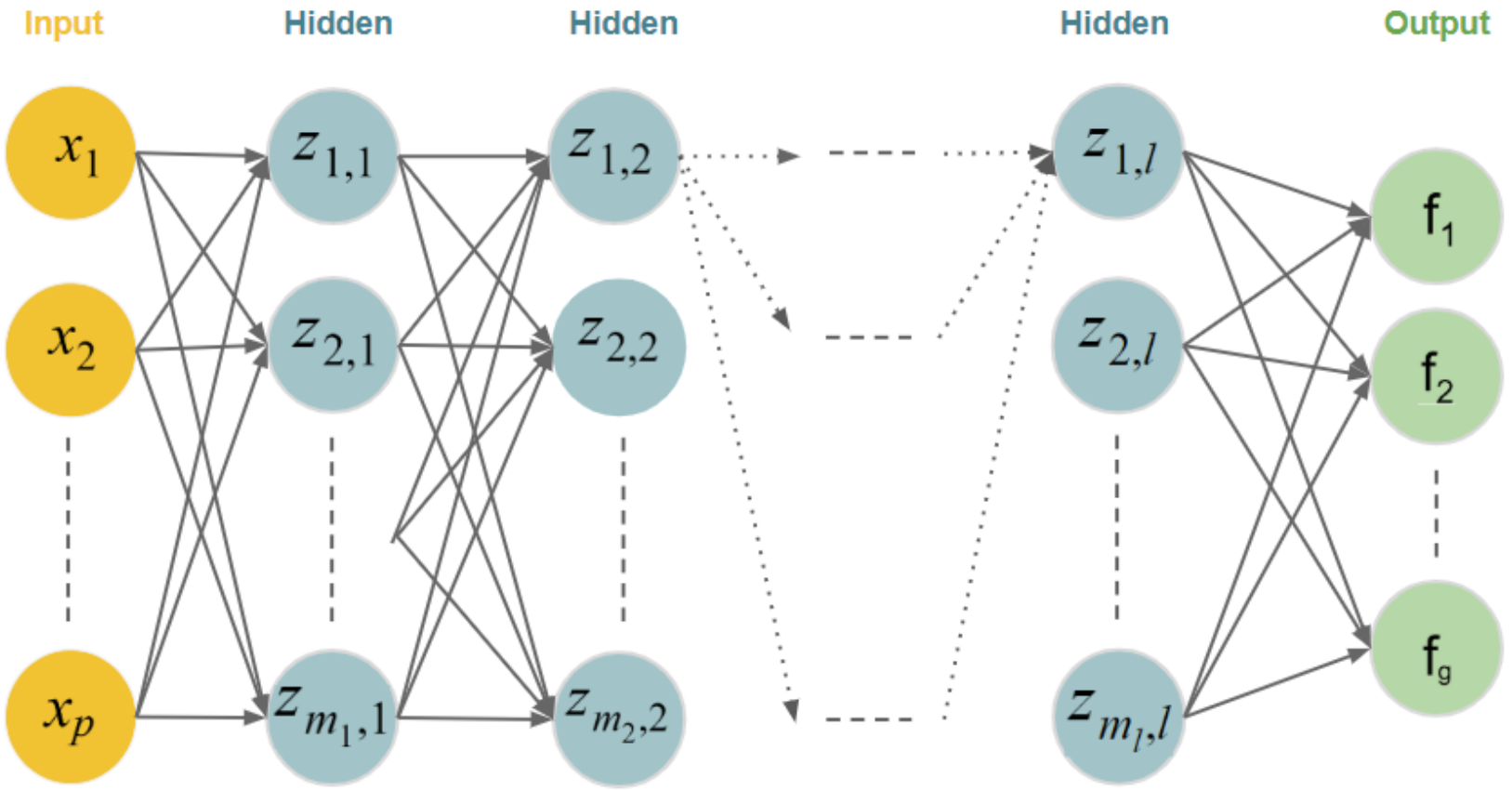
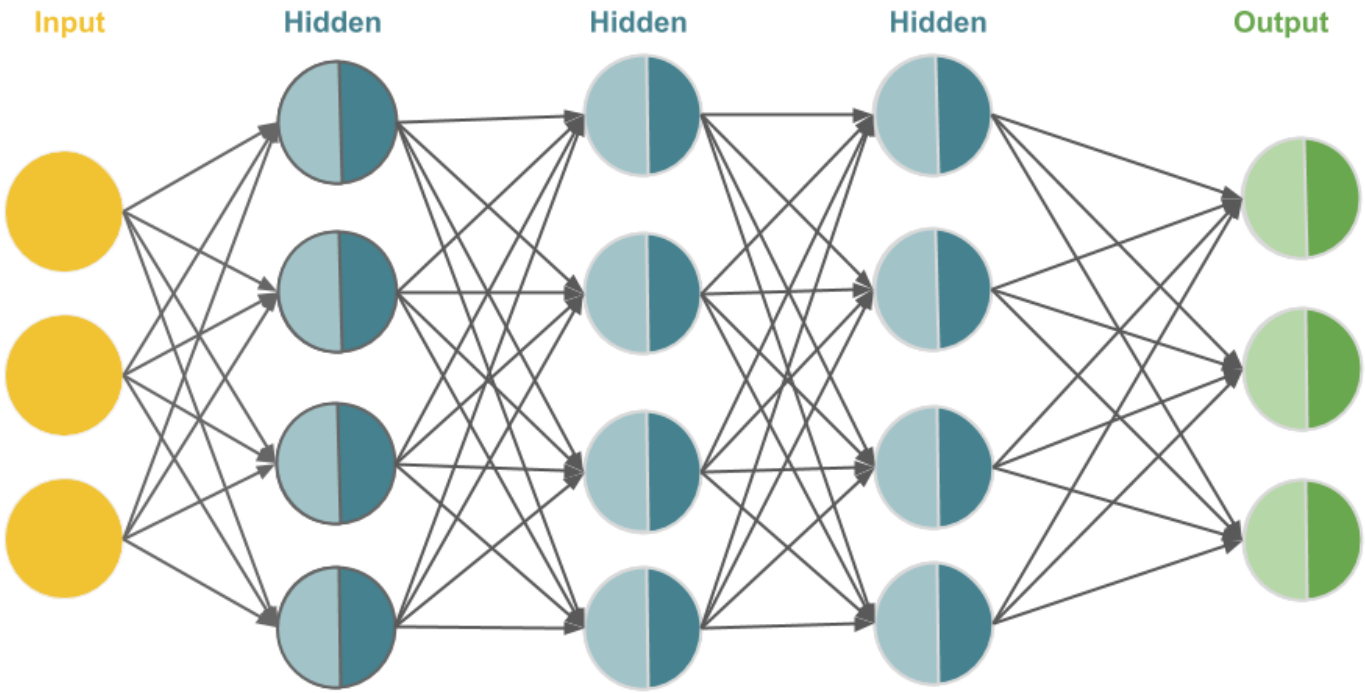


Figure: Structure of a deep neural network with *l* hidden layers (bias terms omitted).

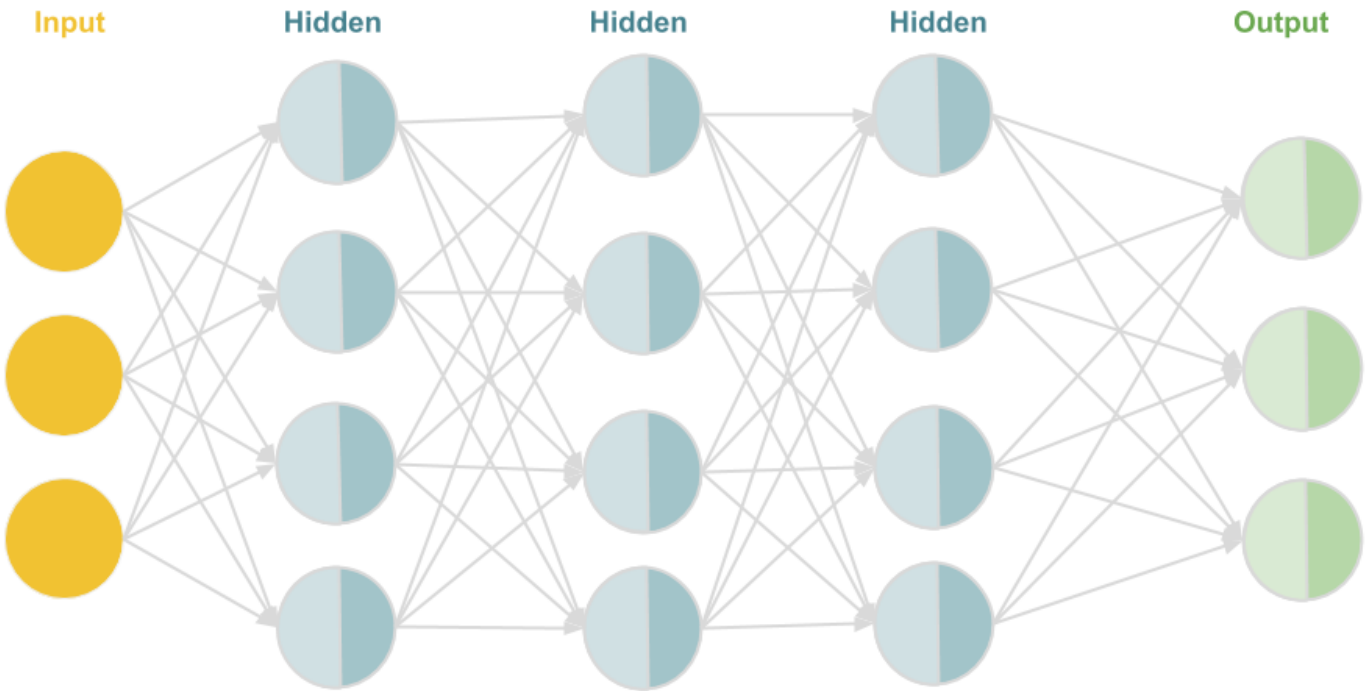
FEEDFORWARD NEURAL NETWORKS: EXAMPLE



$$\begin{bmatrix} 13 & -9 & 2 \\ -8 & 0 & 3 \\ 4 & -1 & 5 \\ -3 & 12 & 7 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 2 \\ 11 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & -4 & 1 \\ 0 & 11 & 2 & -14 \\ -1 & 5 & -2 & 16 \\ 0 & -9 & -3 & 4 \end{bmatrix} \begin{bmatrix} -5 \\ 3 \\ 1 \\ -8 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & -18 & -7 \\ 3 & -4 & 8 & 0 \\ -2 & 1 & 21 & 5 \\ 2 & -2 & 11 & -13 \end{bmatrix} \begin{bmatrix} 4 \\ -6 \\ 1 \\ -17 \end{bmatrix} \quad \begin{bmatrix} 9 & 3 & -1 & -4 \\ -8 & -2 & 14 & 3 \\ 13 & 2 & -9 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -4 \\ -30 \end{bmatrix}$$

$$(W^{(1)})^T \quad \mathbf{b}^{(1)} \quad (W^{(2)})^T \quad \mathbf{b}^{(2)} \quad (W^{(3)})^T \quad \mathbf{b}^{(3)} \quad U^T \quad \mathbf{c}$$

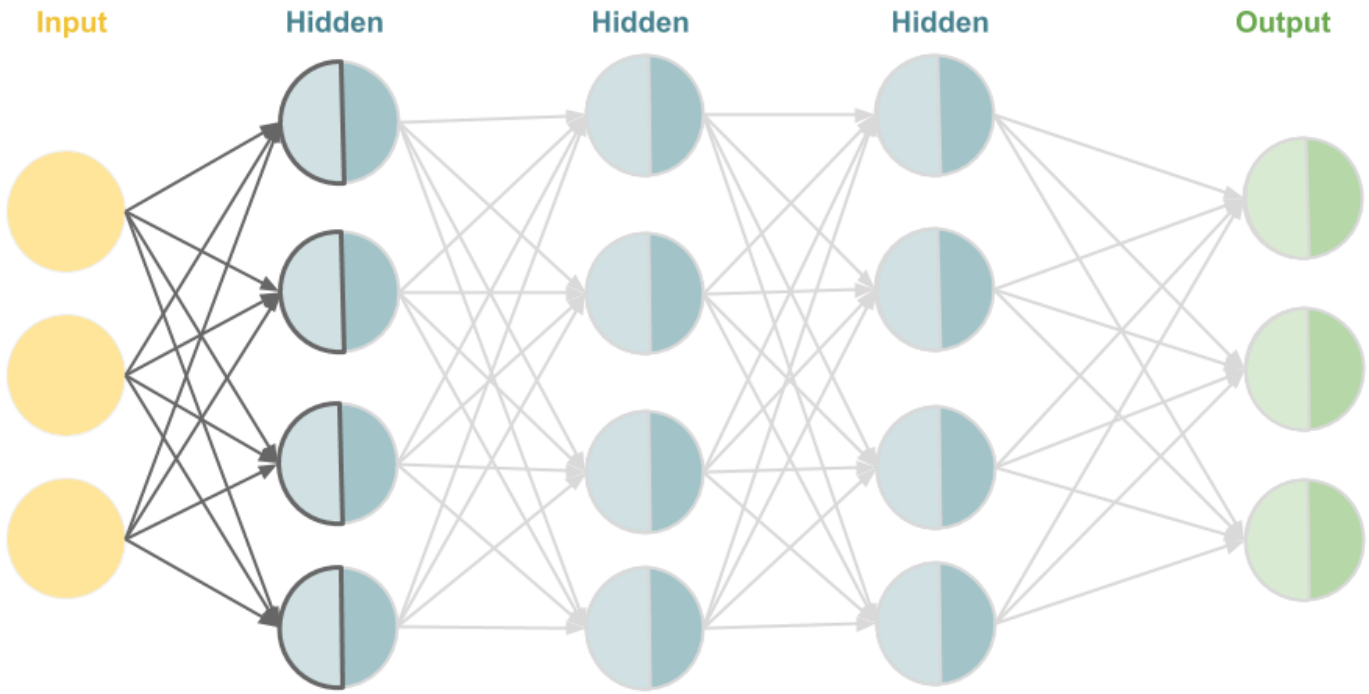
FEEDFORWARD NEURAL NETWORKS: EXAMPLE



7
1
-4

$\mathbf{x}$

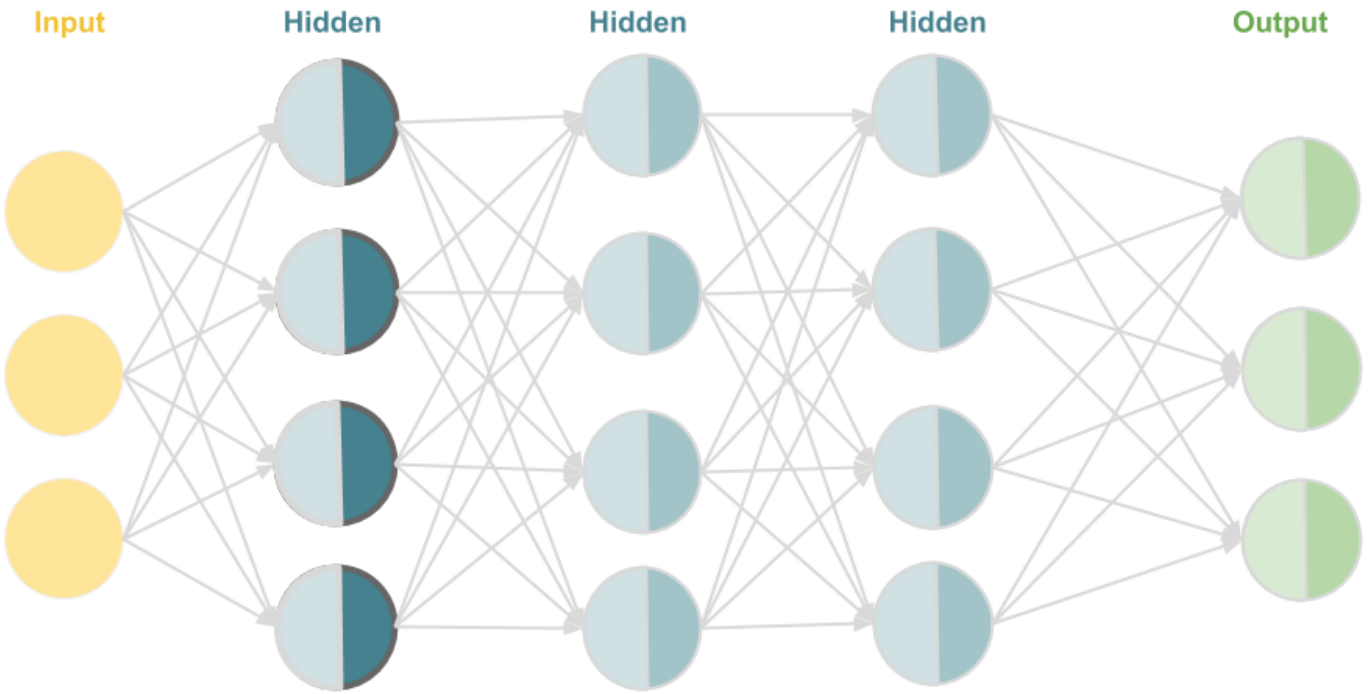
FEEDFORWARD NEURAL NETWORKS: EXAMPLE



$$\begin{matrix}7\\1\\-4\end{matrix}\begin{pmatrix}7*13 + 1*(-9) + (-4)*2 + 5\\7*(-8) + 1*0 + (-4)*3 + (-2)\\7*4 + 1*(-1) + (-4)*5 + 2\\7*(-3) + 1*12 + (-4)*7 + 11\end{pmatrix}$$

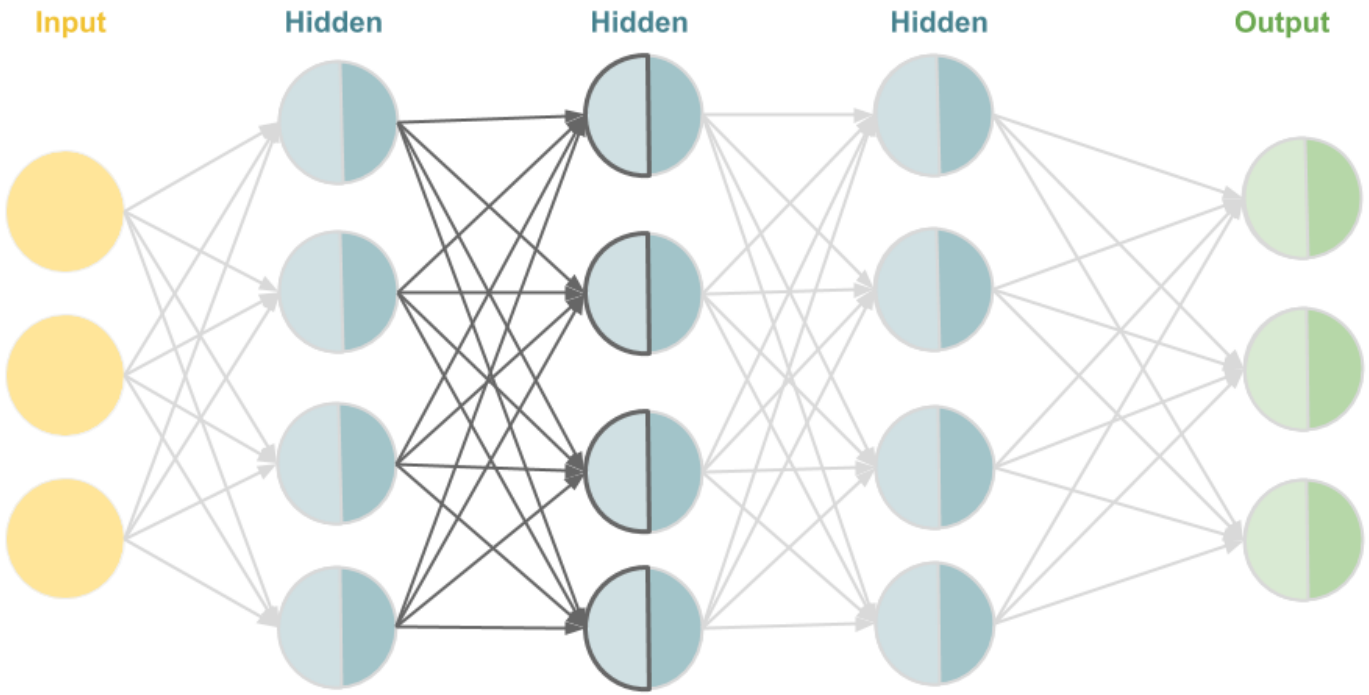
$\mathbf{x}$ $\quad \mathbf{z}_{in}^{(1)} = W^{(1)T}\mathbf{x} + \mathbf{b}^{(1)}$

FEEDFORWARD NEURAL NETWORKS: EXAMPLE



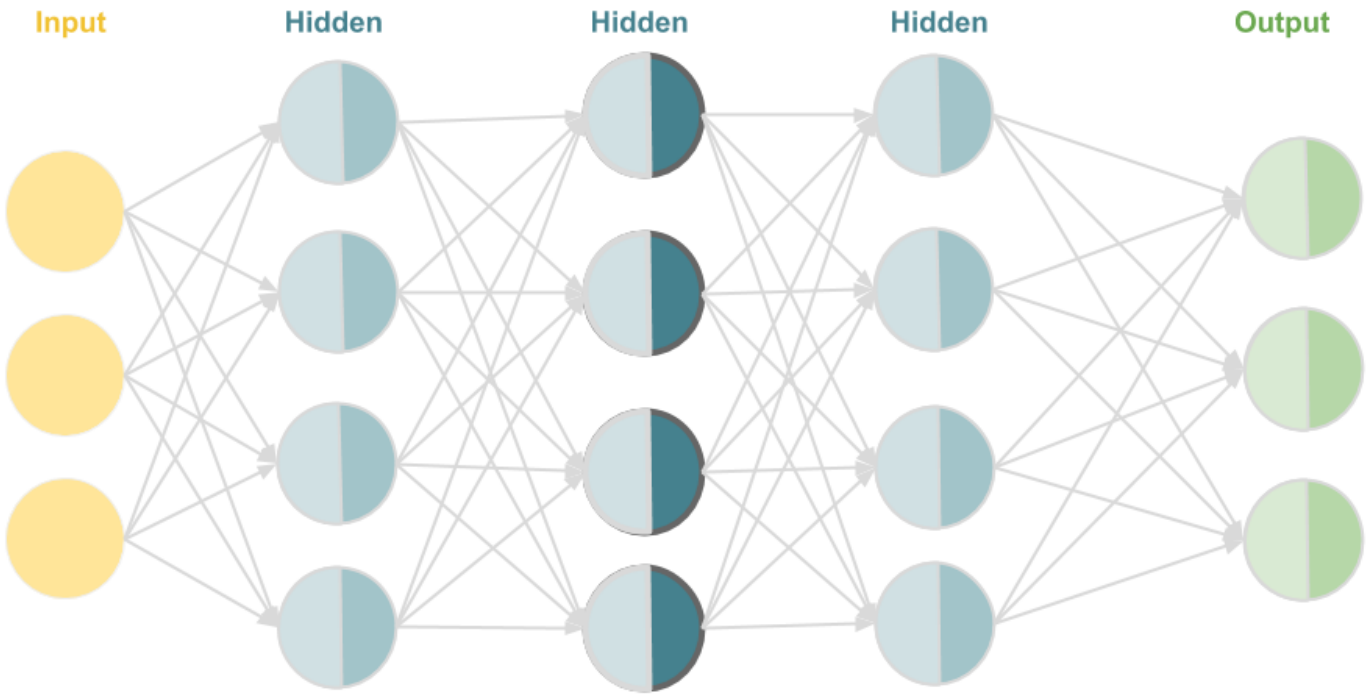
$\mathbf{x}$	$\mathbf{z}_{in}^{(1)}$	$\mathbf{z}^{(1)} = \mathbf{z}_{out}^{(1)} = \sigma(\mathbf{z}_{in}^{(1)})$
7 1 -4	79 -70 9 -26	max(0, 79) max(0, -70) max(0, 9) max(0, -26)

FEEDFORWARD NEURAL NETWORKS: EXAMPLE



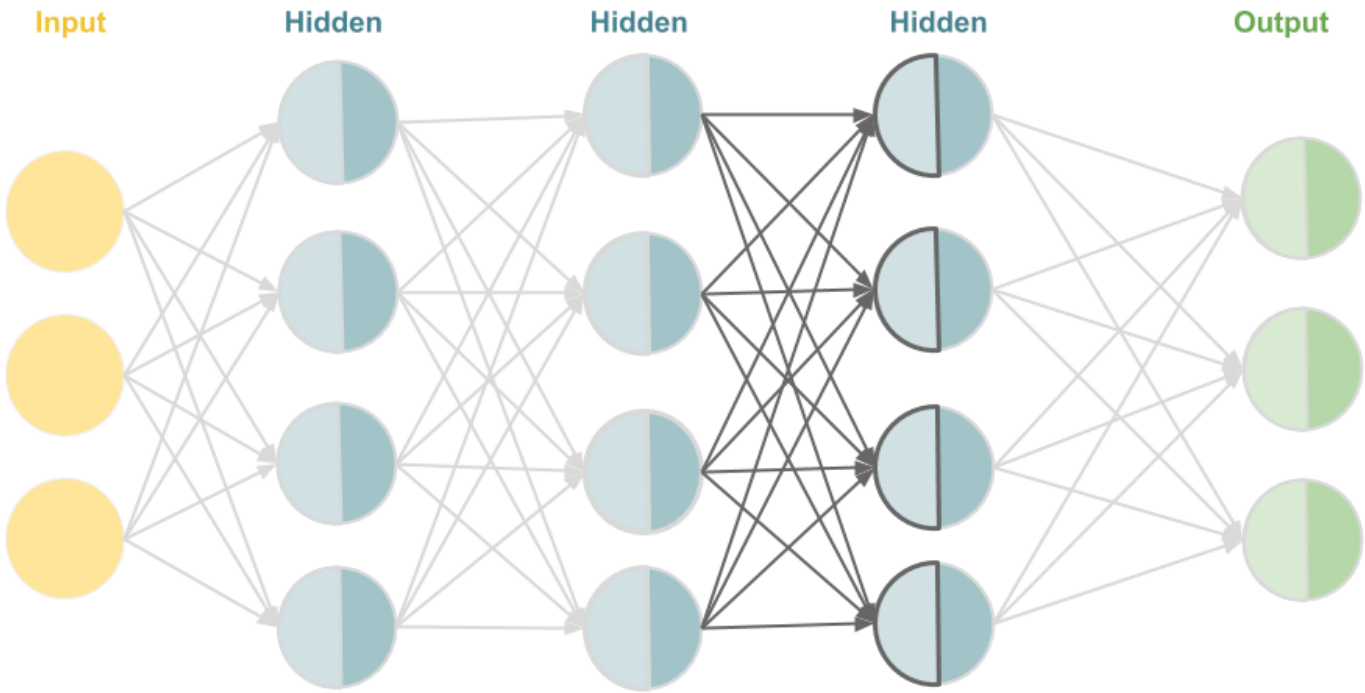
$\mathbf{x}$	$\mathbf{z}_{in}^{(1)}$	$\mathbf{z}^{(1)}$	$\mathbf{z}_{in}^{(2)} = W^{(2)T}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$
7 1 -4	79 -70 9 -26	79 0 9 0	79*1 + 0*0 + 9*(-4) + 0*1 + (-5) 79*0 + 0*11 + 9*2 + 0*(-14) + 3 79*(-1) + 0*5 + 9*(-2) + 0*16 + 1 79*0 + 0*(-9) + 9*(-3) + 0*4 + (-8)

FEEDFORWARD NEURAL NETWORKS: EXAMPLE



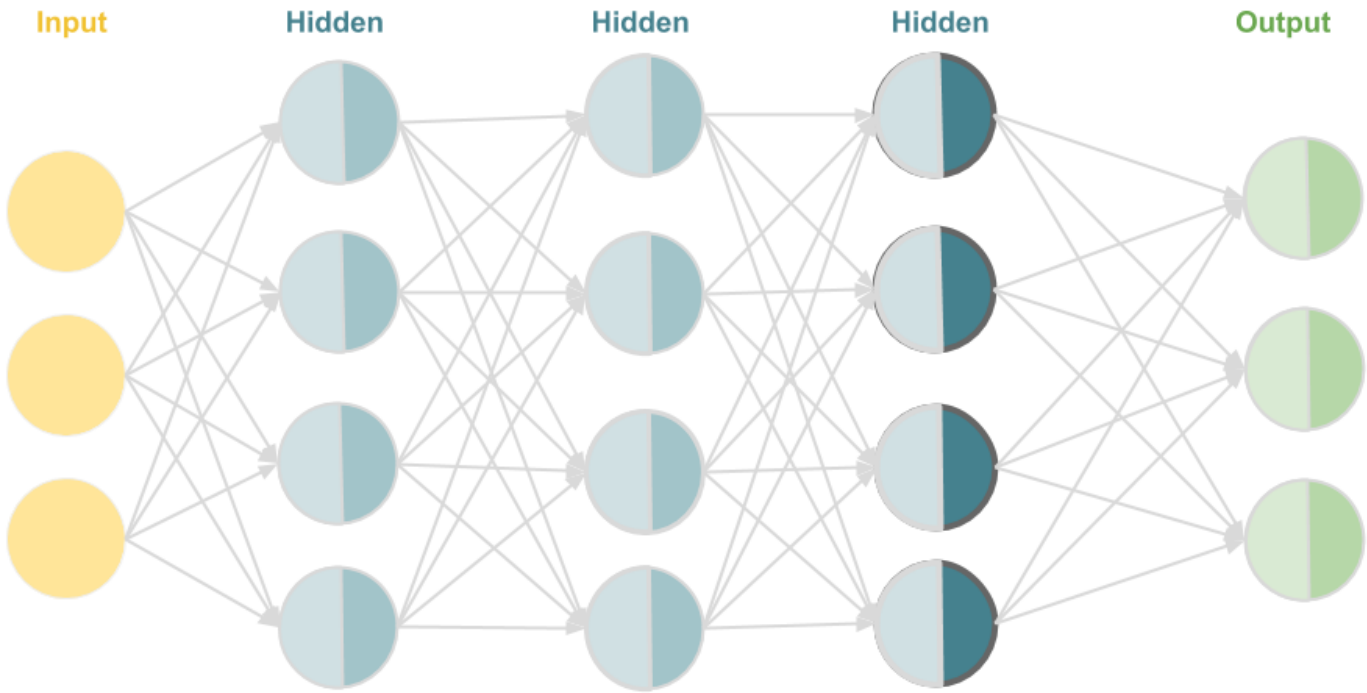
$\mathbf{x}$	$\mathbf{z}_{in}^{(1)}$	$\mathbf{z}^{(1)}$	$\mathbf{z}_{in}^{(2)}$	$\mathbf{z}^{(2)} = \mathbf{z}_{out}^{(2)} = \sigma(\mathbf{z}_{in}^{(2)})$
7 1 -4	79 -70 9 -26	79 0 9 0	38 21 -96 -35	max(0, 38) max(0, 21) max(0, -96) max(0, -36)

FEEDFORWARD NEURAL NETWORKS: EXAMPLE



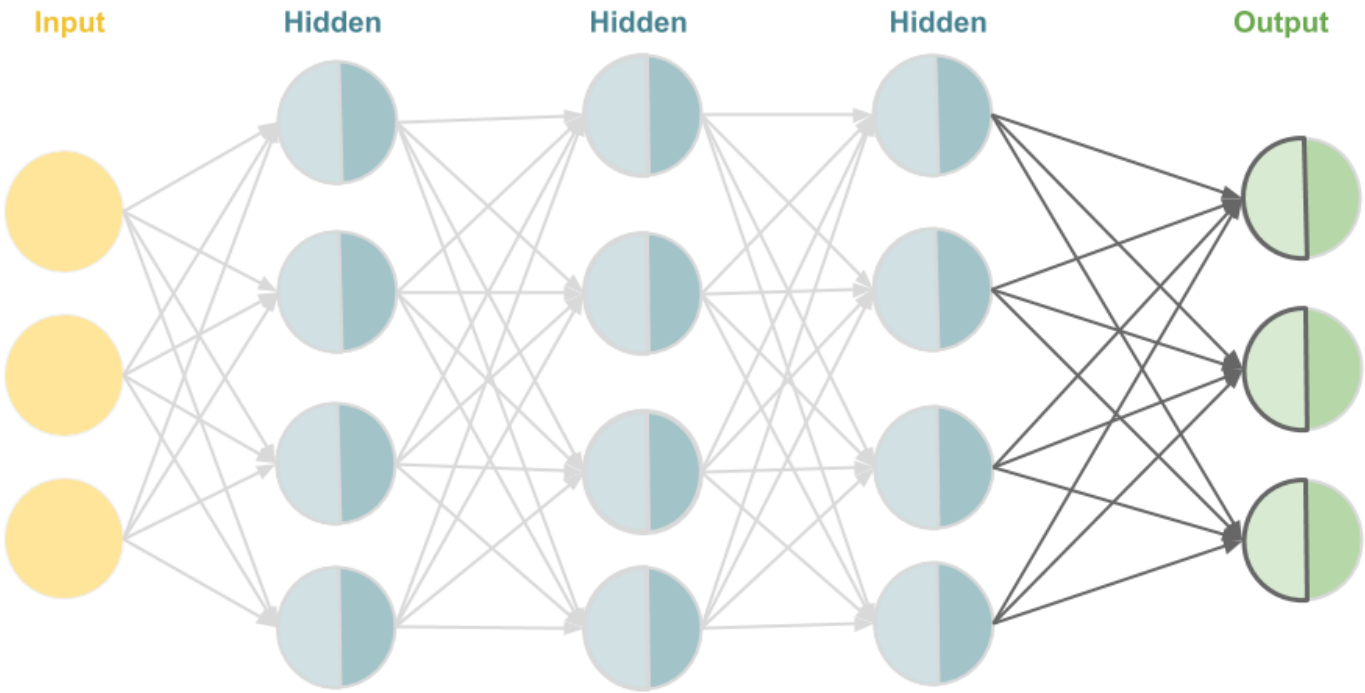
$\mathbf{x}$	$\mathbf{z}_{in}^{(1)}$	$\mathbf{z}^{(1)}$	$\mathbf{z}_{in}^{(2)}$	$\mathbf{z}^{(2)}$	$\mathbf{z}_{in}^{(3)} = W^{(3)T}\mathbf{z}^{(2)} + \mathbf{b}^{(3)}$
7	79	79	38	38	38*1 + 21*(-2) + 0*(-18) + 0*(-7) + 4
1	-70	0	21	21	38*3 + 21*(-4) + 0*8 + 0*0 + (-6)
-4	9	9	-96	0	38*(-2) + 21*1 + 0*21 + 0*5 + 1
	-26	0	-35	0	38*2 + 21*(-2) + 0*11 + 0*(-13) + (-17)

FEEDFORWARD NEURAL NETWORKS: EXAMPLE



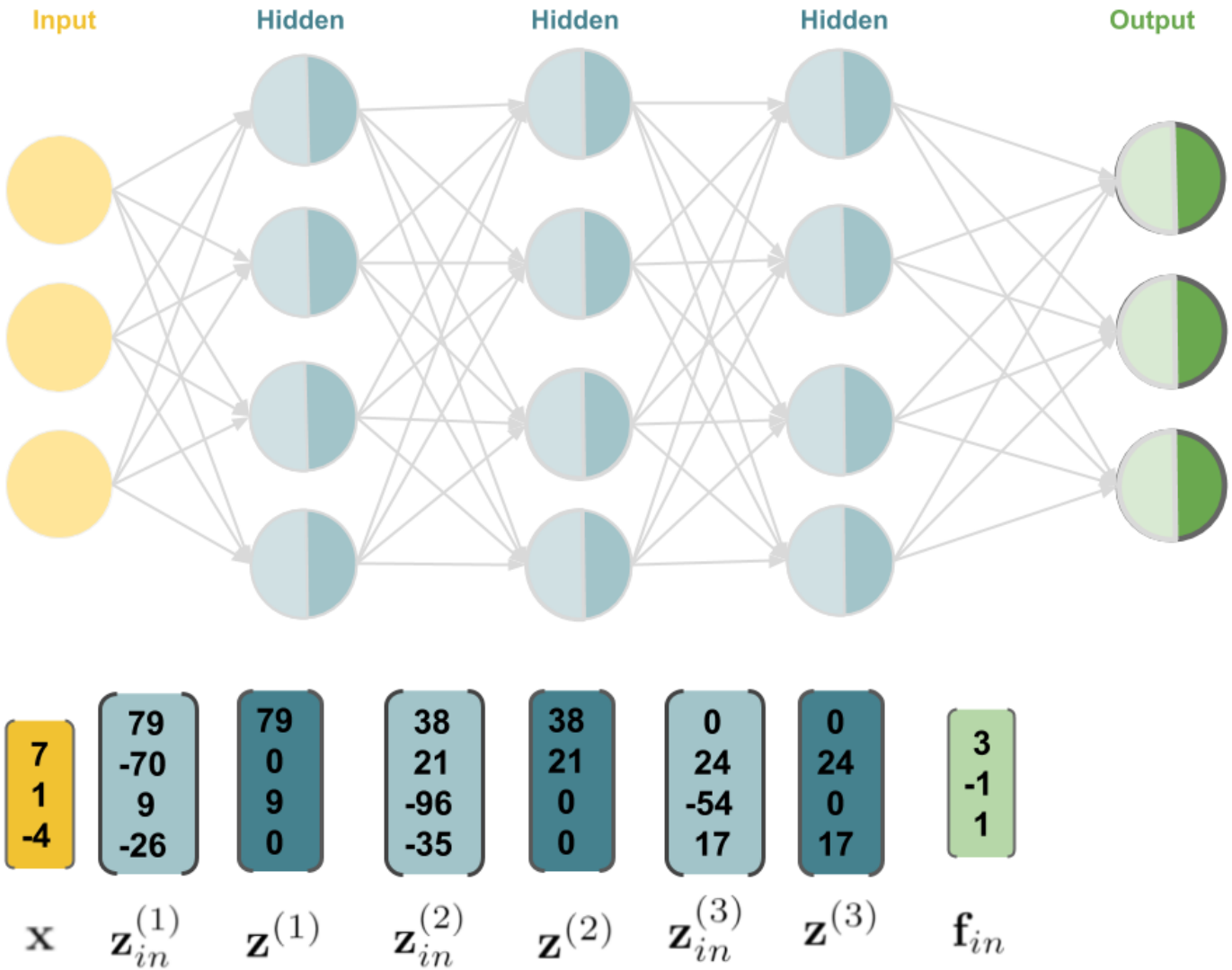
$\mathbf{x}$	$\mathbf{z}_{in}^{(1)}$	$\mathbf{z}^{(1)}$	$\mathbf{z}_{in}^{(2)}$	$\mathbf{z}^{(2)}$	$\mathbf{z}_{in}^{(3)}$	$\mathbf{z}^{(3)} = \mathbf{z}_{out}^{(3)} = \sigma(\mathbf{z}_{in}^{(3)})$
7	79	79	38	38	0	max(0, 0)
1	-70	0	21	21	24	max(0, 24)
-4	9	9	-96	0	-54	max(0, -54)
	-26	0	-35	0	17	max(0, 17)

FEEDFORWARD NEURAL NETWORKS: EXAMPLE

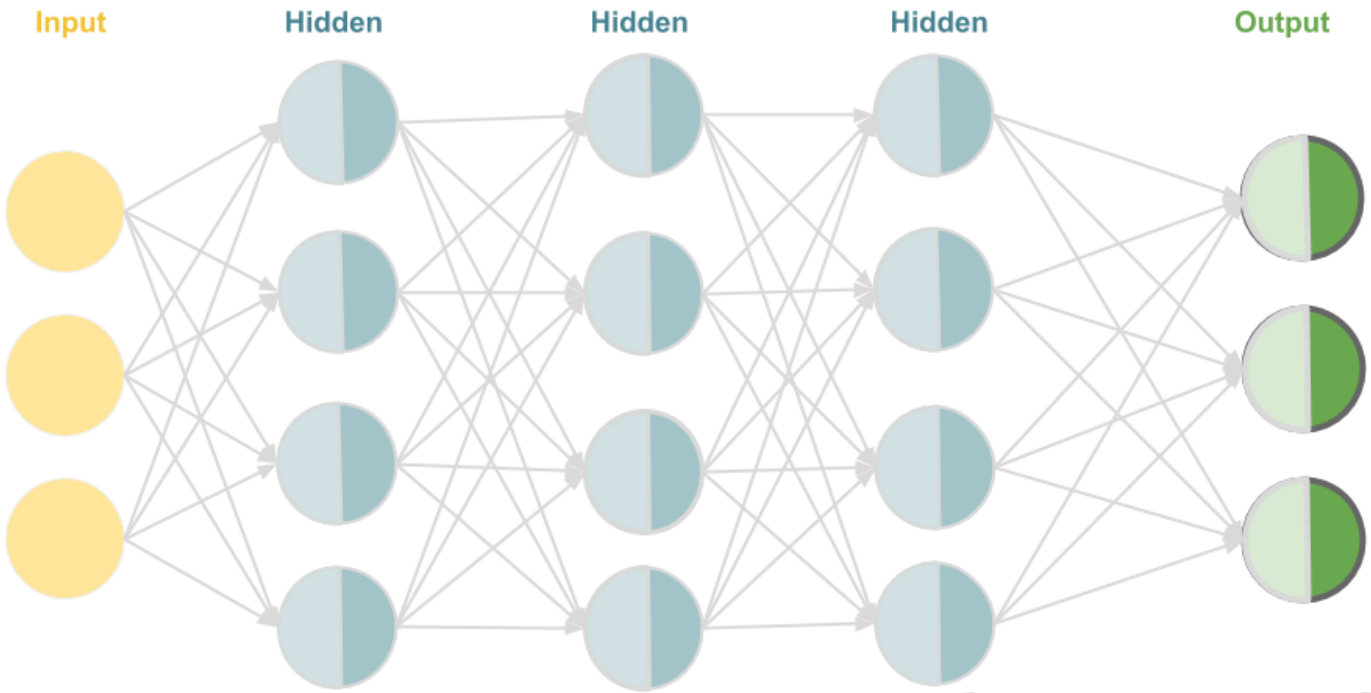


$\mathbf{x}$	$\mathbf{z}_{in}^{(1)}$	$\mathbf{z}^{(1)}$	$\mathbf{z}_{in}^{(2)}$	$\mathbf{z}^{(2)}$	$\mathbf{z}_{in}^{(3)}$	$\mathbf{z}^{(3)}$	$\mathbf{f}_{in} = U^T\mathbf{z}^{(3)} + \mathbf{c}$
7	79	79	38	38	0	0	24*3 + 17*(-4) + (-1)
1	-70	0	21	21	24	24	24*(-2) + 17*3 + (-4)
-4	9	9	-96	0	-54	0	24*2 + 17*(-1) + (-30)
	-26	0	-35	0	17	17	

FEEDFORWARD NEURAL NETWORKS: EXAMPLE



FEEDFORWARD NEURAL NETWORKS: EXAMPLE



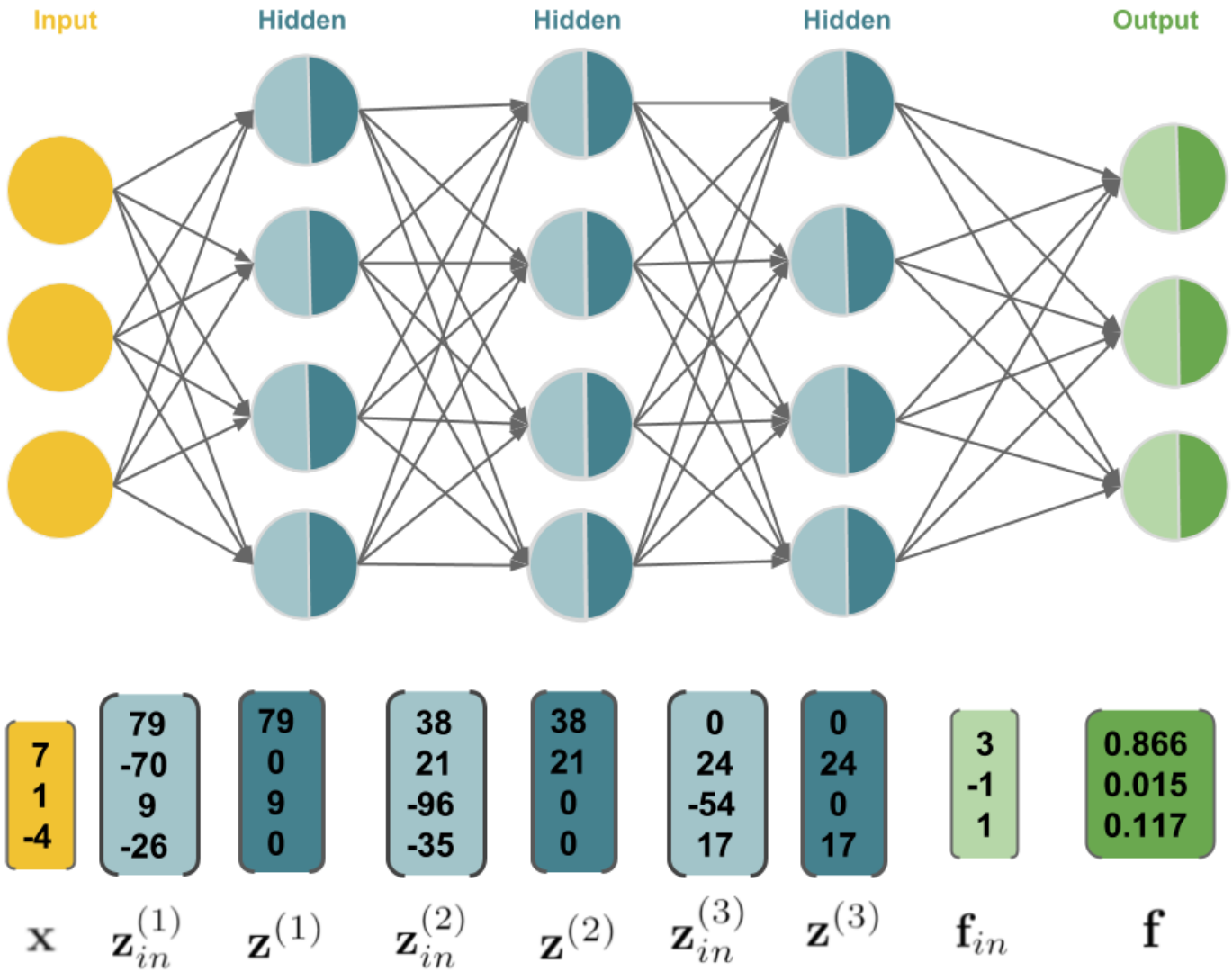
$$\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$\mathbf{f}_{in}$

$$\begin{bmatrix} \frac{\exp(3)}{\exp 3+\exp(-1)+\exp(1)} \\ \frac{\exp(-1)}{\exp 3+\exp(-1)+\exp(1)} \\ \frac{\exp(1)}{\exp 3+\exp(-1)+\exp(1)} \end{bmatrix}$$

$$\mathbf{f} = \mathbf{f}_{out} = \tau(\mathbf{f}_{in})$$

FEEDFORWARD NEURAL NETWORKS: EXAMPLE



WHY ADD MORE LAYERS?

- Multiple layers allow for the extraction of more and more abstract representations.
- Each layer in a feed-forward neural network adds its own degree of non-lnearity to the model.

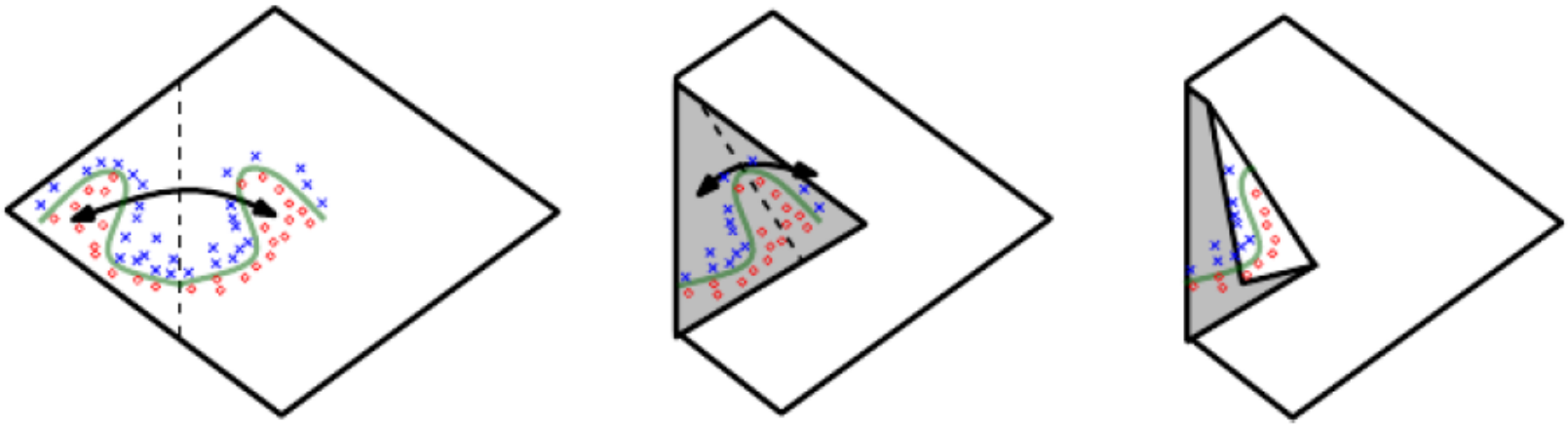
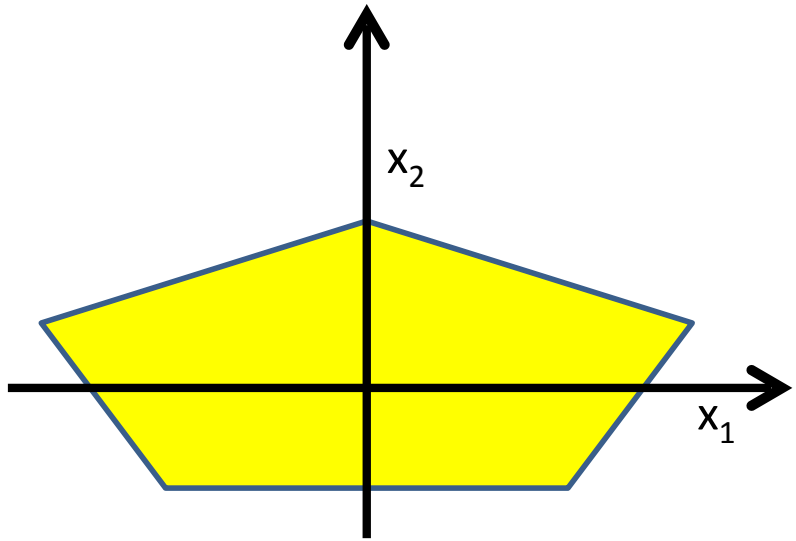


Figure: An intuitive, geometric explanation of the exponential advantage of deeper networks formally (Montúfar et al. (2014)).

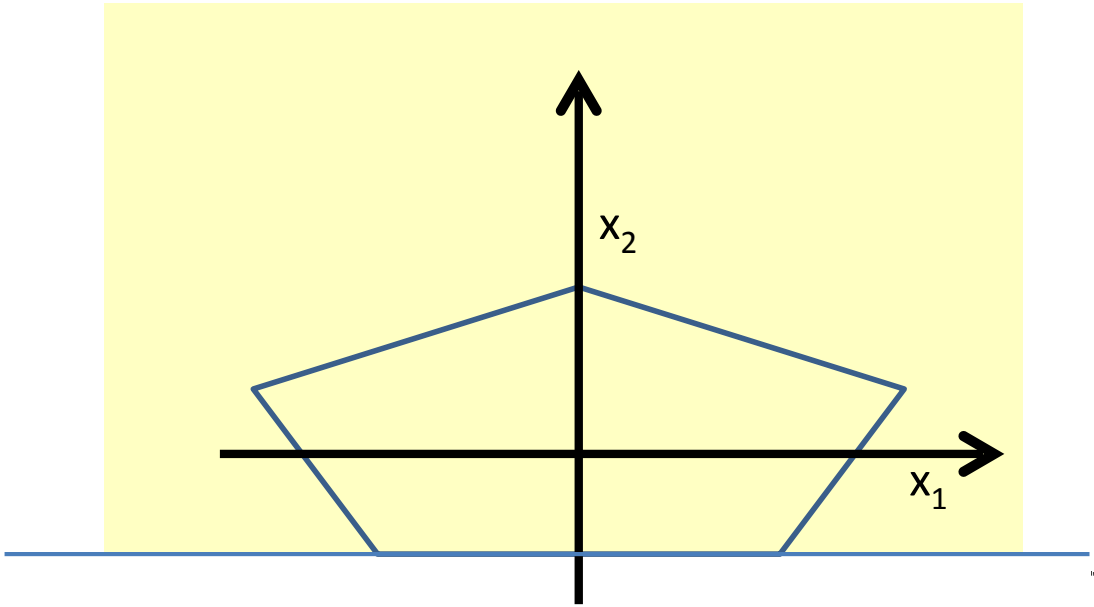
Composing complicated "decision" boundaries

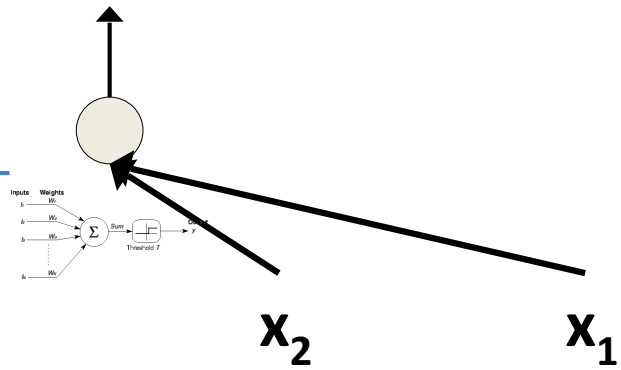


Can now be composed into "networks" to compute arbitrary classification "boundaries"

- Build a network of units with a single output that fires if the input is in the coloured area

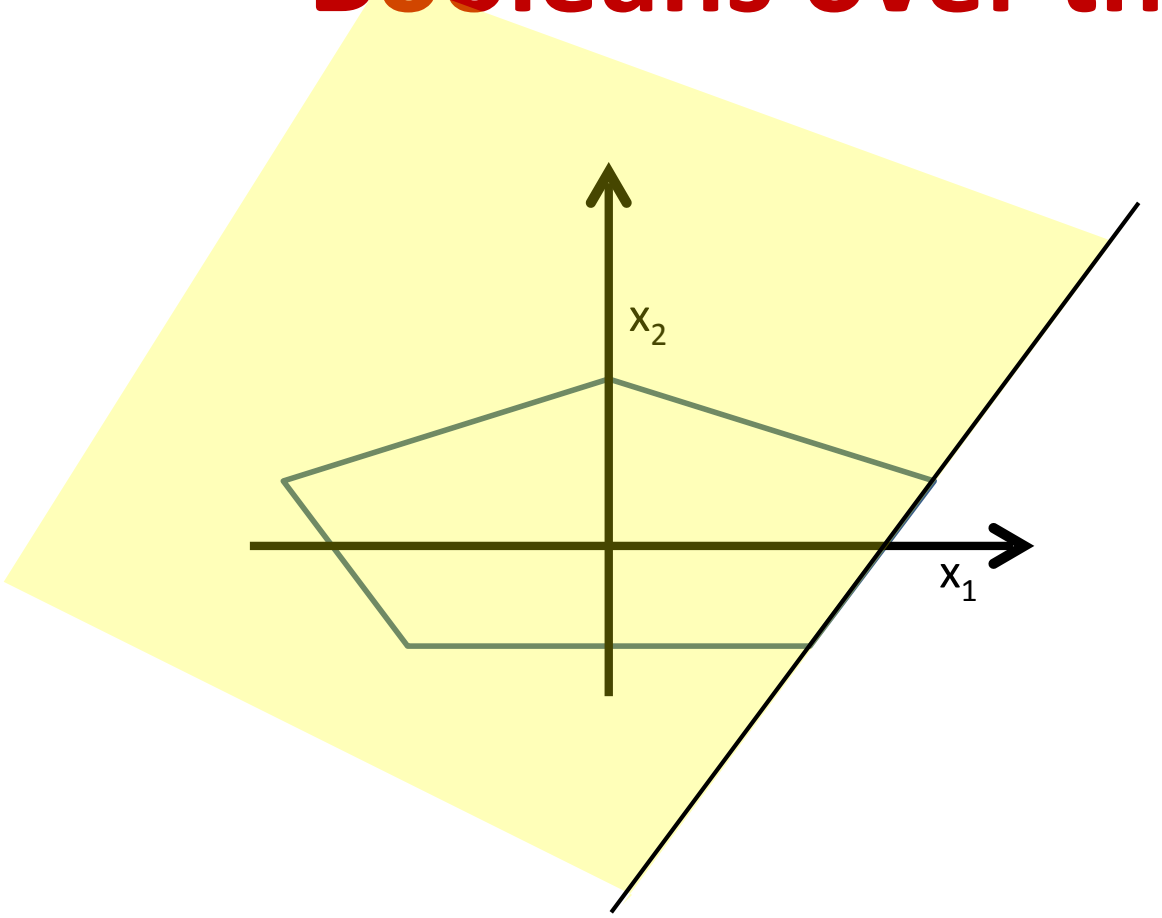
Booleans over the reals

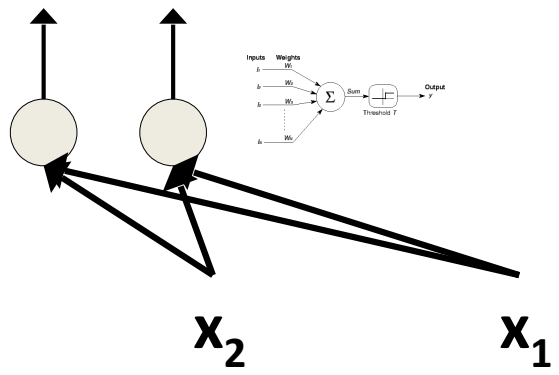




- The network must fire if the input is in the coloured area

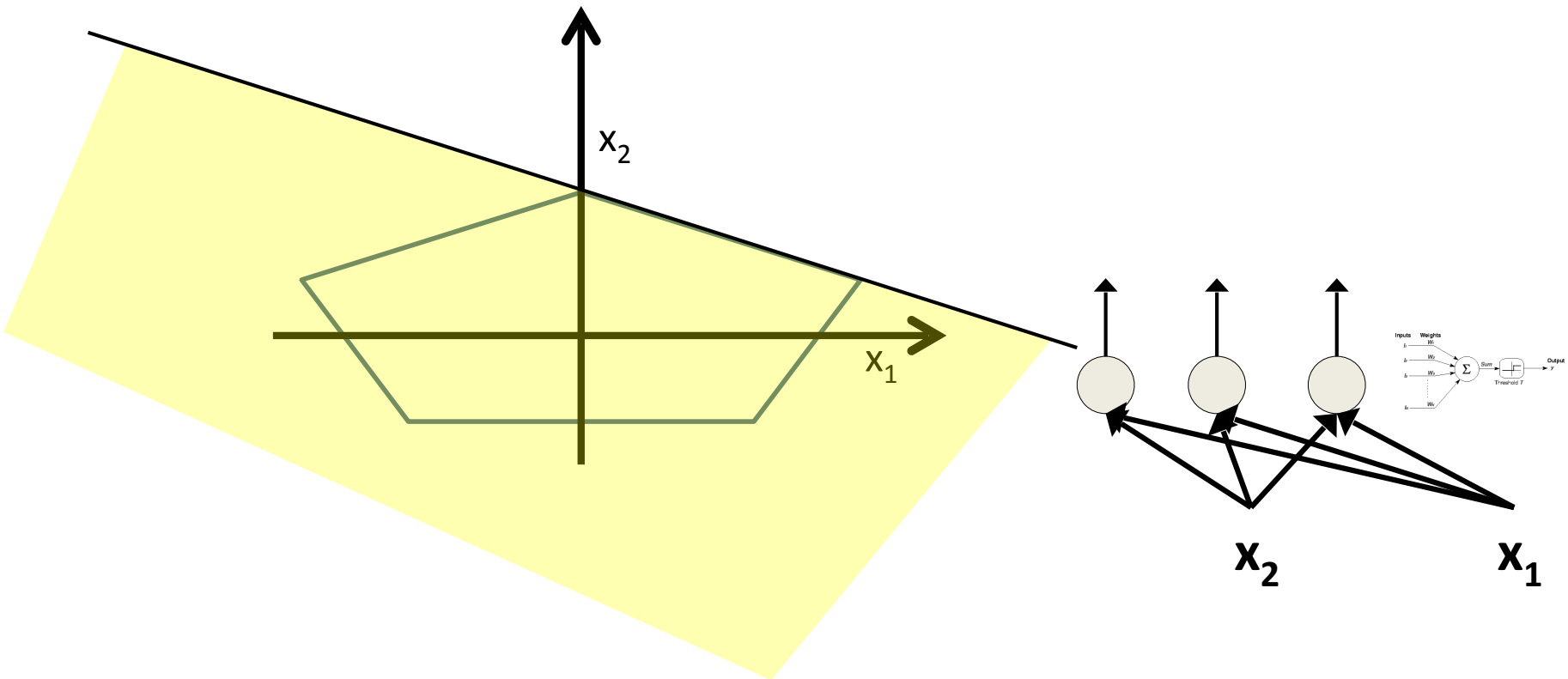
Booleans over the reals





- The network must fire if the input is in the coloured area

Booleans over the reals



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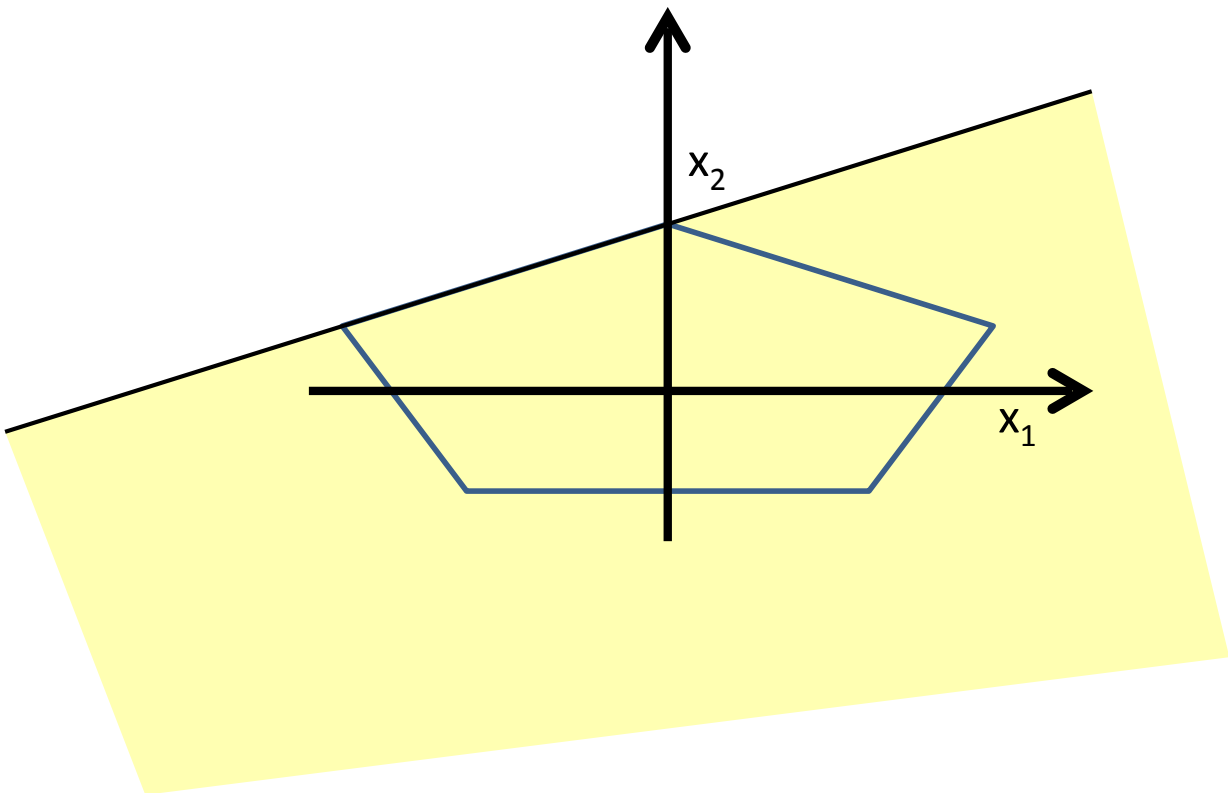
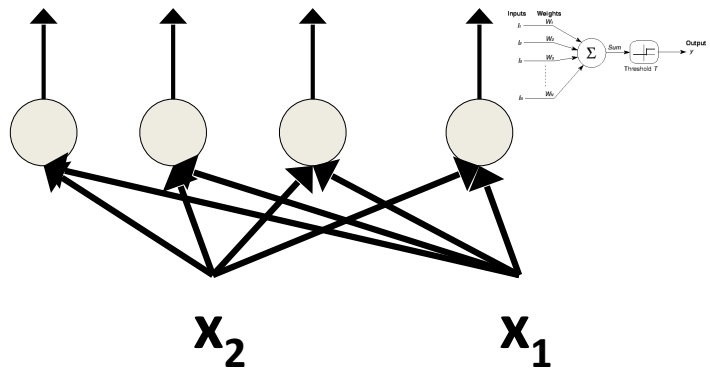
Booleans over the reals





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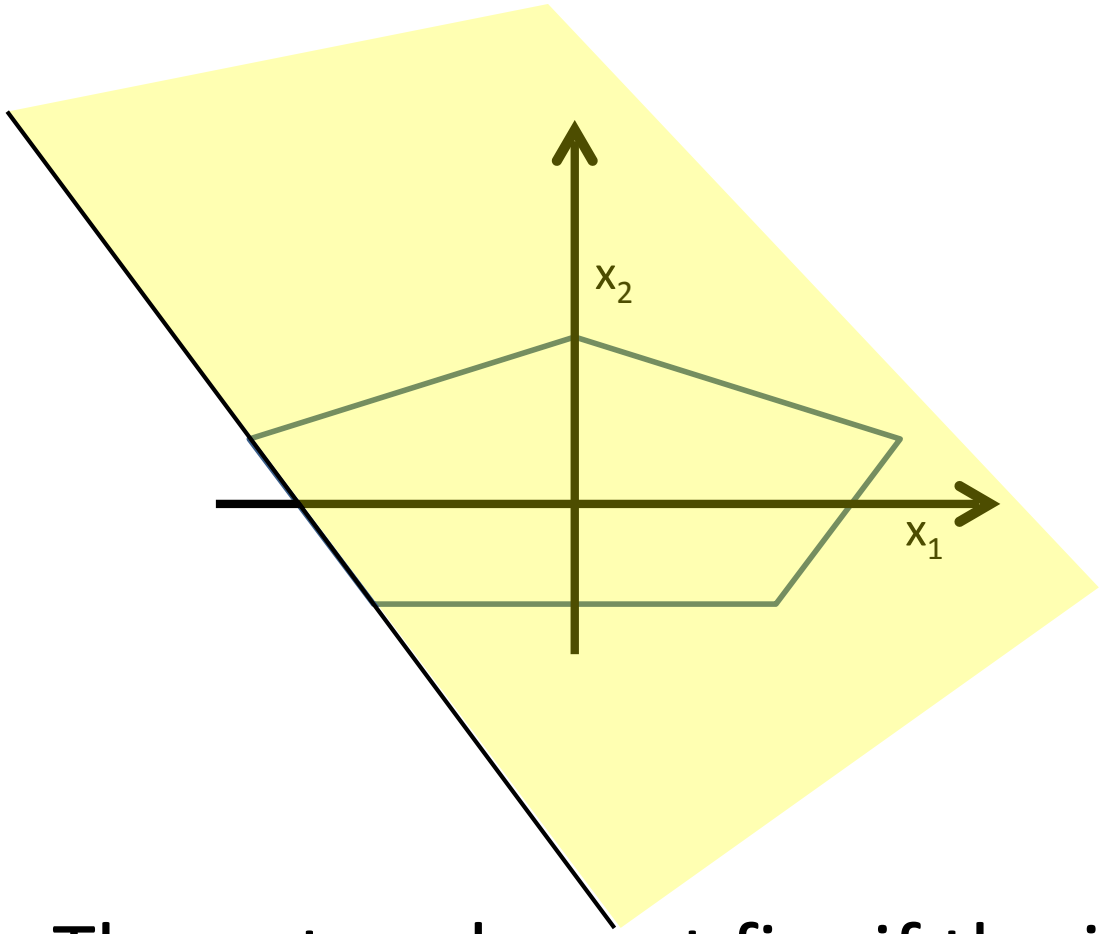
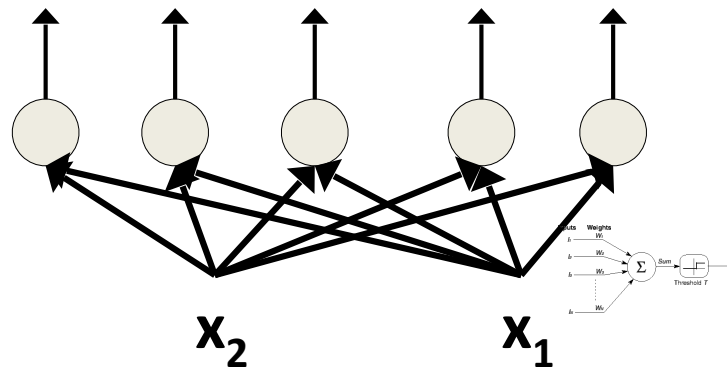
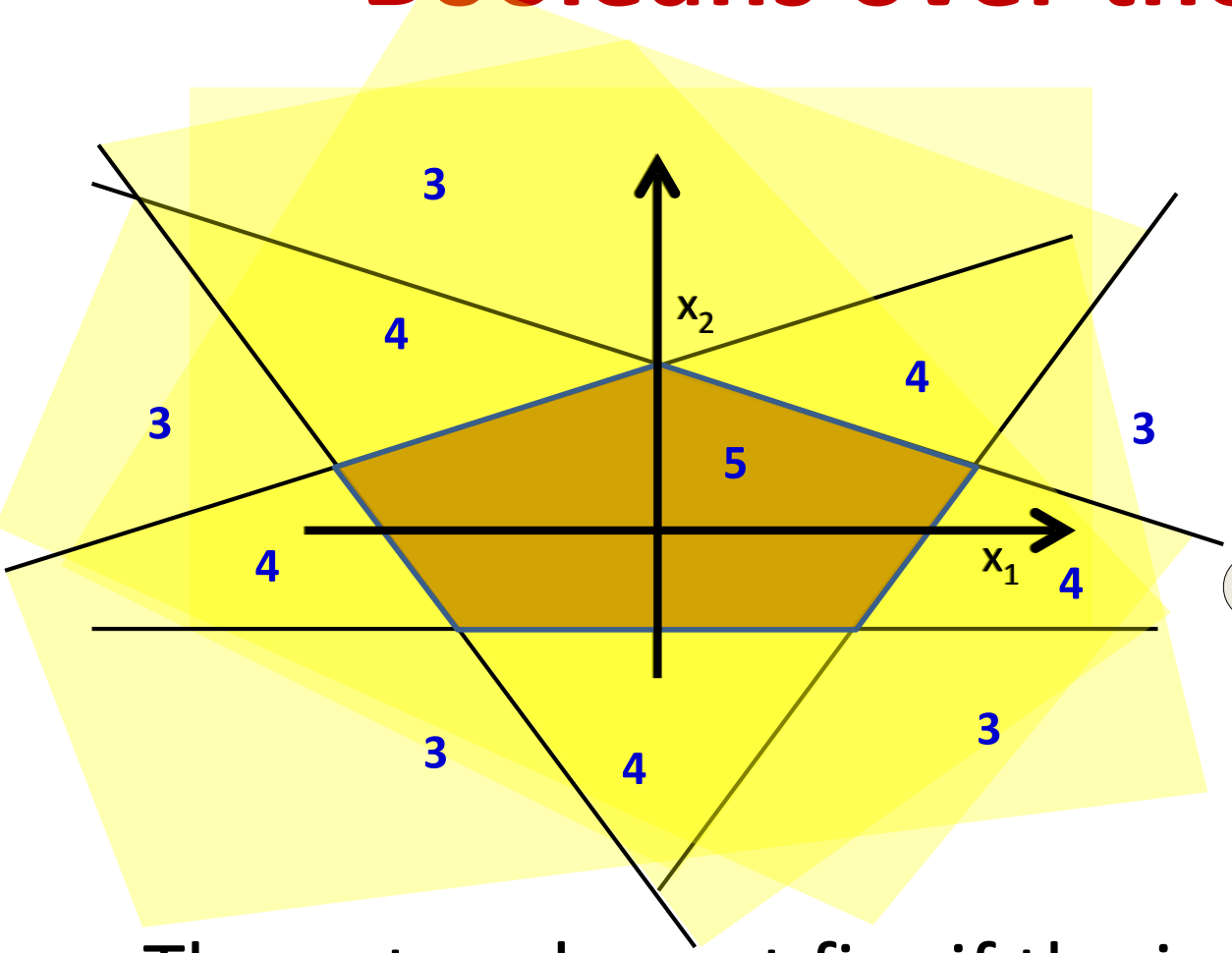
Booleans over the reals

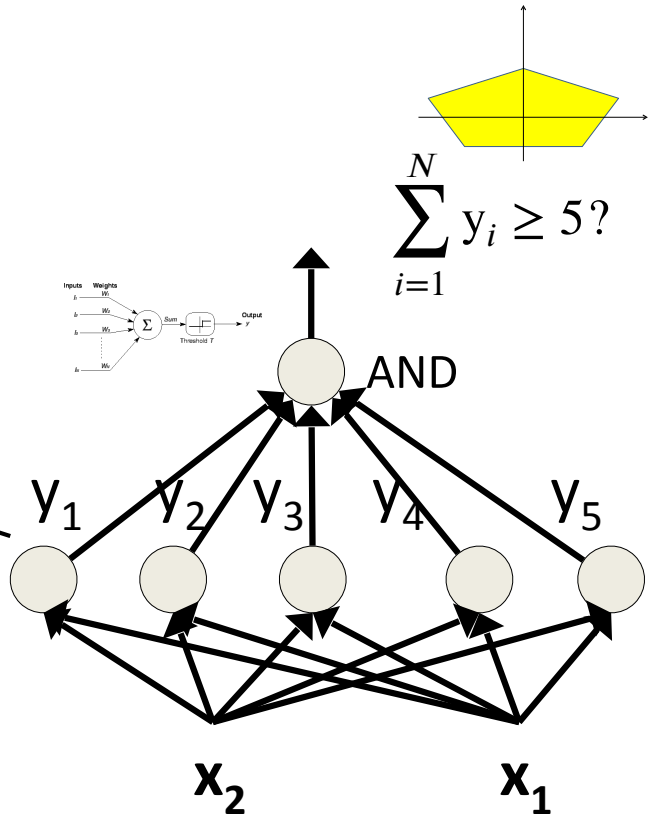




- The network must fire if the input is in the coloured area

Booleans over the reals





- The network must fire if the input is in the coloured area

Implementing learning: Gradient Descent

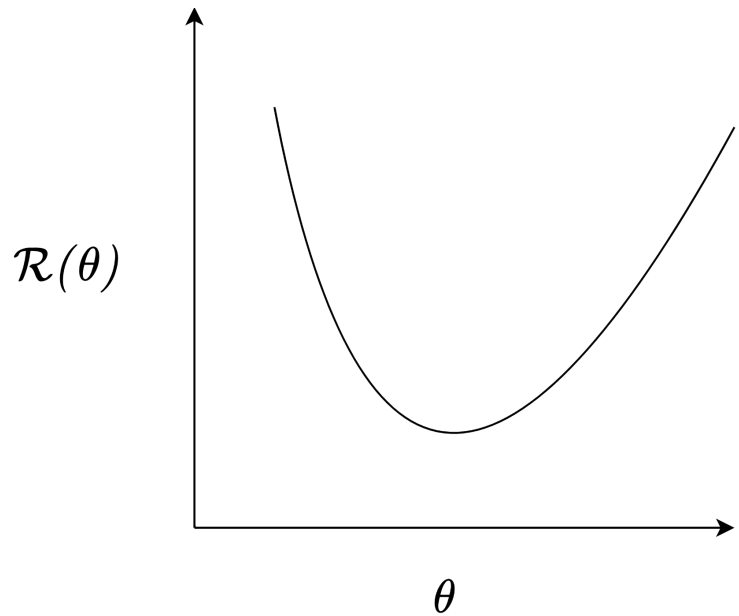
Given:

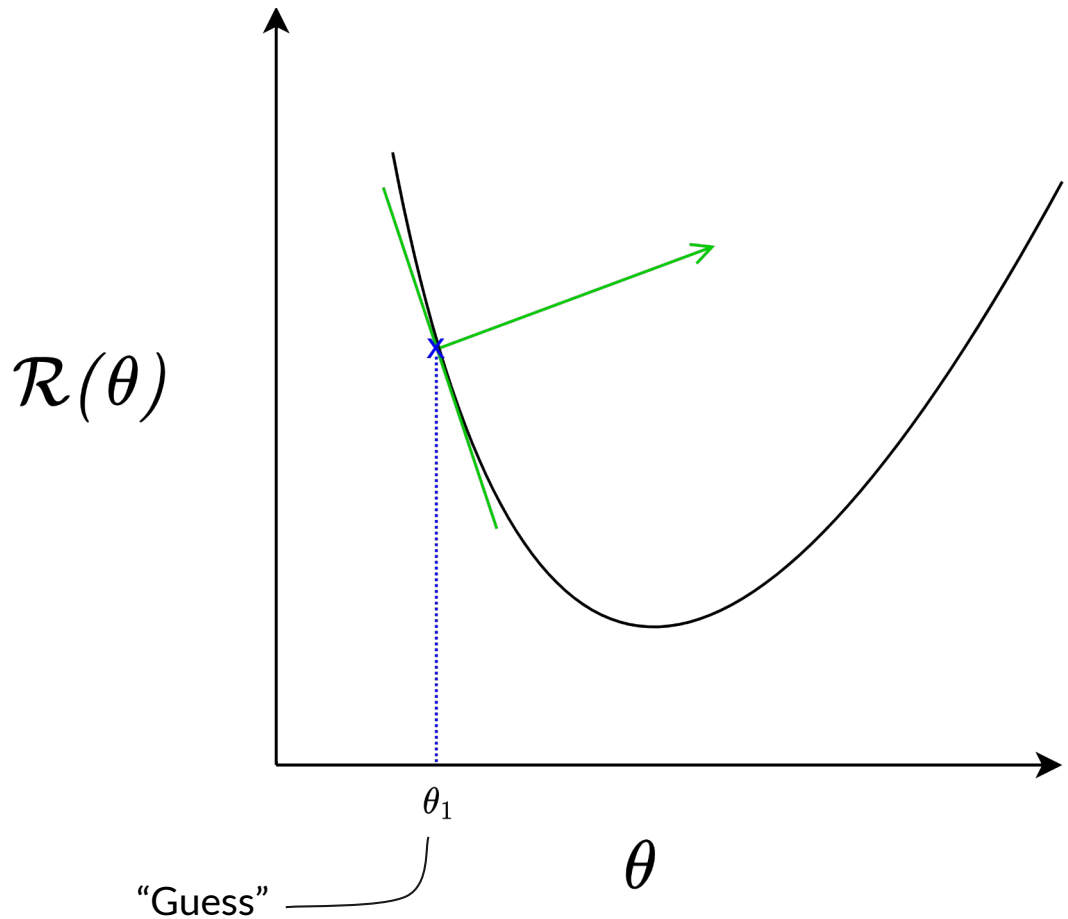
- Family of parameters Θ (e.g. possible weights of a NN)
- Differentiable risk function $\mathcal{R}(\theta)$

Goal:

$$\theta_{opt} = \underset{\theta \in \Theta}{\operatorname{argmin}}\ \mathcal{R}(\theta)$$

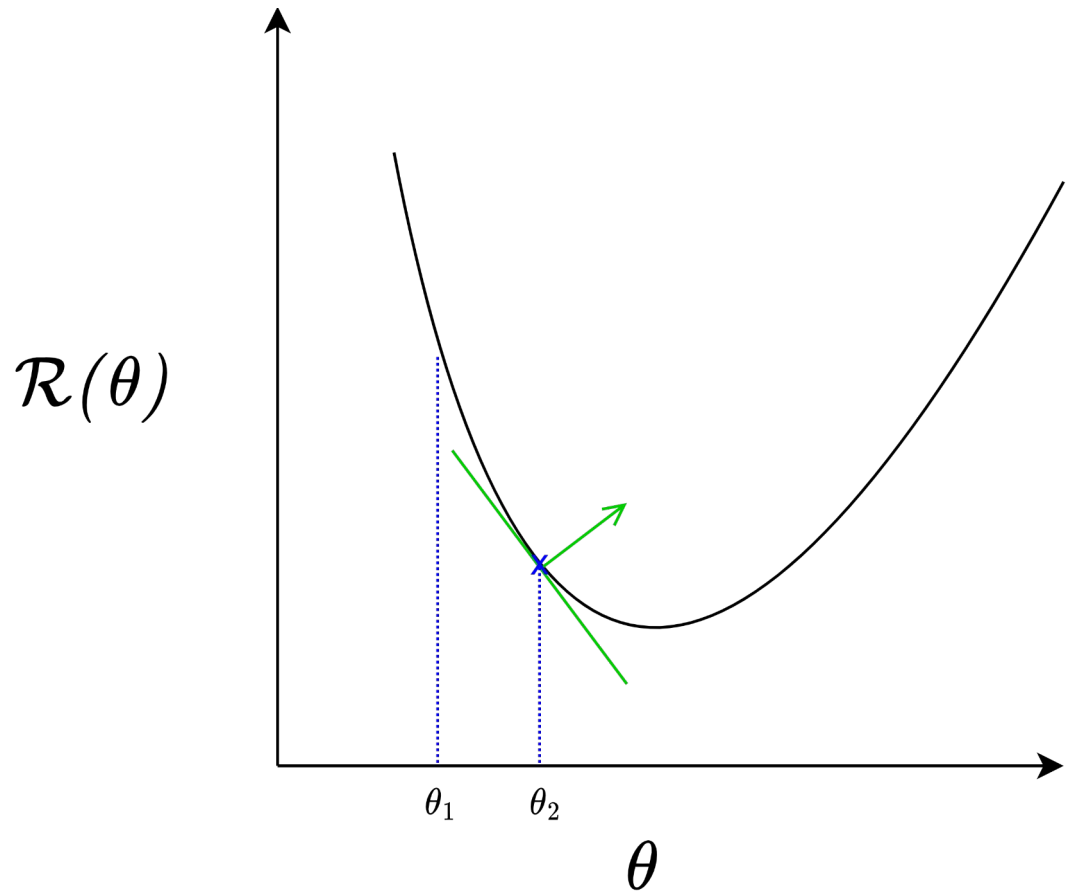
Backprop: Gradient descent



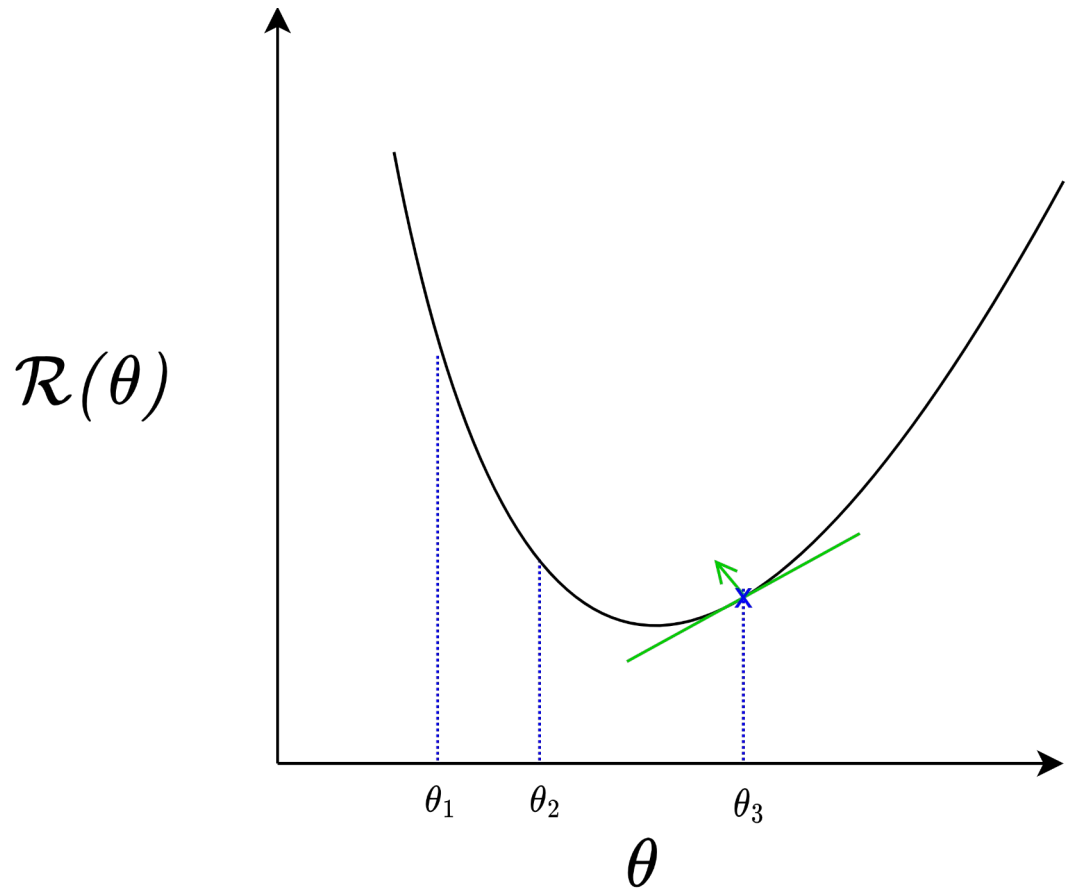


"*Learning rate*"

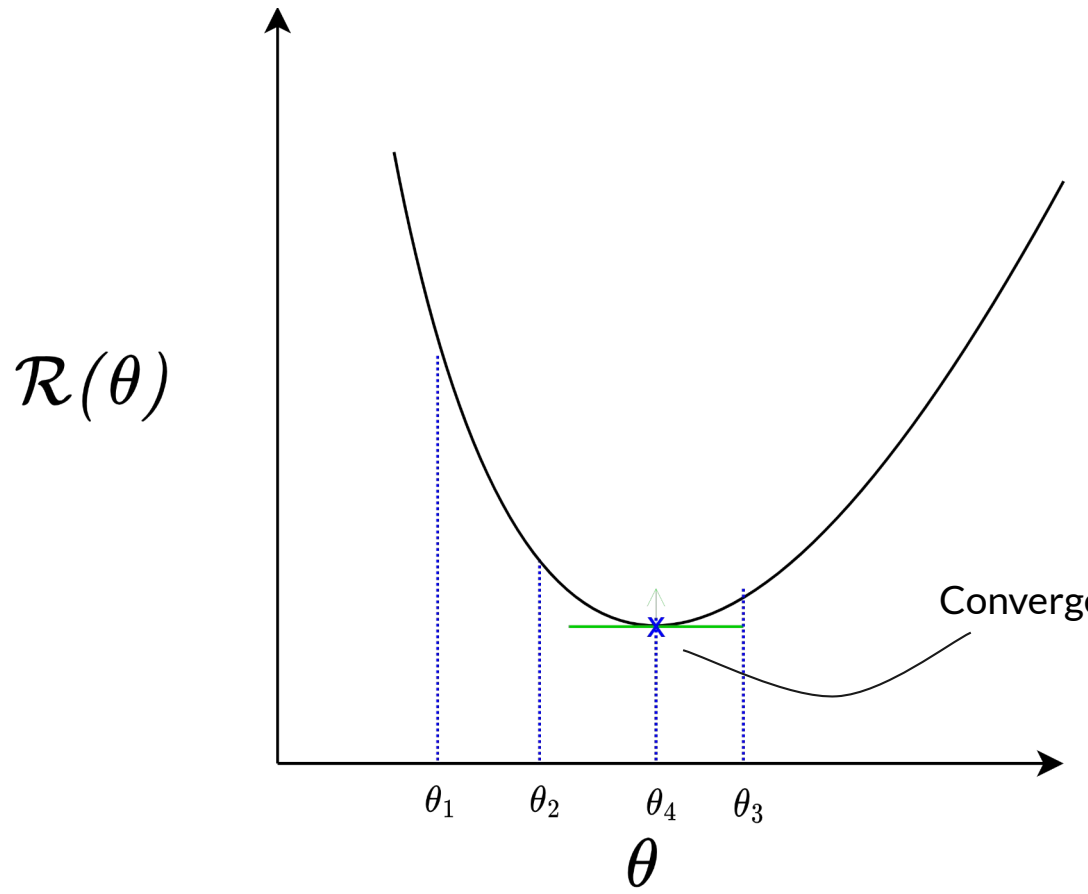
$$\theta_i = \theta_i - \alpha \frac{\partial}{\partial \theta_i} \mathcal{R}(\theta_i)$$



$$\theta_i = \theta_i - \alpha \frac{\partial}{\partial \theta_i} \mathcal{R}(\theta_i)$$



$$\theta_i = \theta_i - \alpha \frac{\partial}{\partial \theta_i} \mathcal{R}(\theta_i)$$

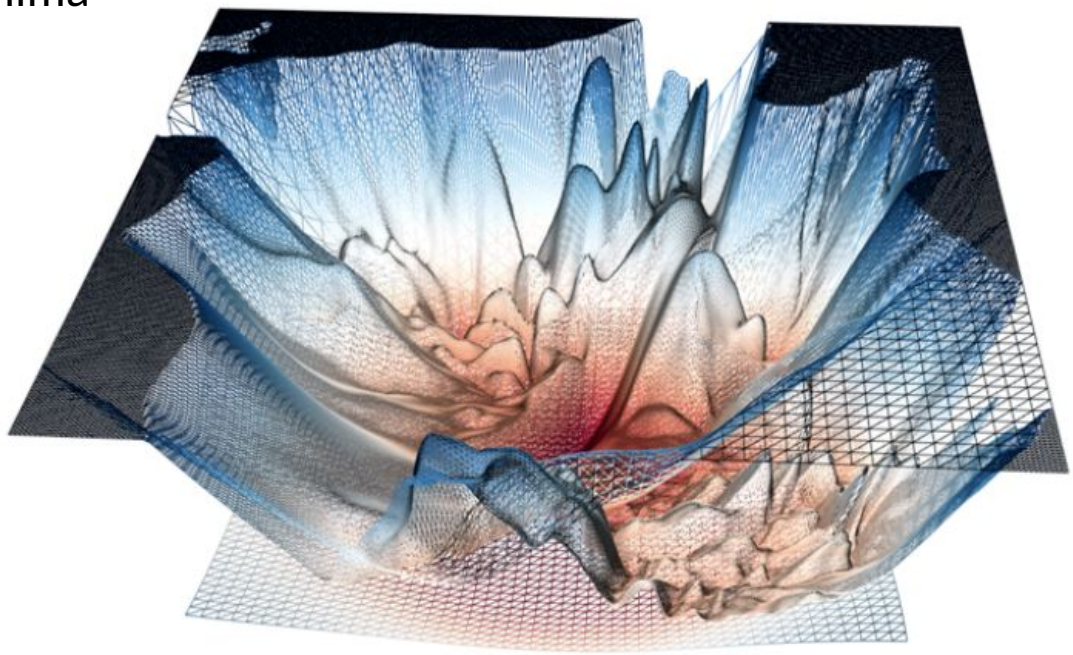


$$\theta_i = \theta_i - \alpha \frac{\partial}{\partial \theta_i} \mathcal{R}(\theta_i)$$

Converges to local minima

A “real” loss landscape:

- Many (many many) local minima
- Saddle points



http://www.telesens.co/2019/01/16/neural-network-loss-visualization/

Moreover, as the number of the parameters grows with the network depth, the computations become exponentially large. How to deal with it?

(see in the next lecture)

Further reading

- Amazing short lecture series by 3Blue1Brown, chapter: 1, 2, 3
- Neural Network interactive playground: https://playground.tensorflow.org
- Visual explanation of Neural Networks: https://mlu-explain.github.io/neural-networks