

Lecture 2: Descriptive Statistics

Center · Spread · Quantiles · Shape · ECDF · Anscombe · Simpson

You get a spreadsheet with 10,000 rows...

What do you look at first?

And what can fool you?

Before any model, any test, any estimation — **look at the data.**

Goals of Descriptive Statistics

Summarize **center**, **spread**, and **shape**

Detect **outliers**, missing data, impossible values

Compare groups or time periods visually

Generate **hypotheses** before testing them

Descriptive \neq inferential: we're describing *this sample*, not yet the population.

Measures of Center

Sample Mean

$$\bar{X} = \frac{1}{n} \sum X_i$$

Uses all data

Minimizes
squared error

Sensitive
to outliers

Sample Median

Middle value

Robust (50%
breakdown)
Ignores mag-
nitudes

Resists outliers

Mode

Most fre-
quent value

Best for cat-
egorical
Can be non-unique
Minimizes 0–1 loss

Trimmed Mean

Drop
top/bottom $k\%$

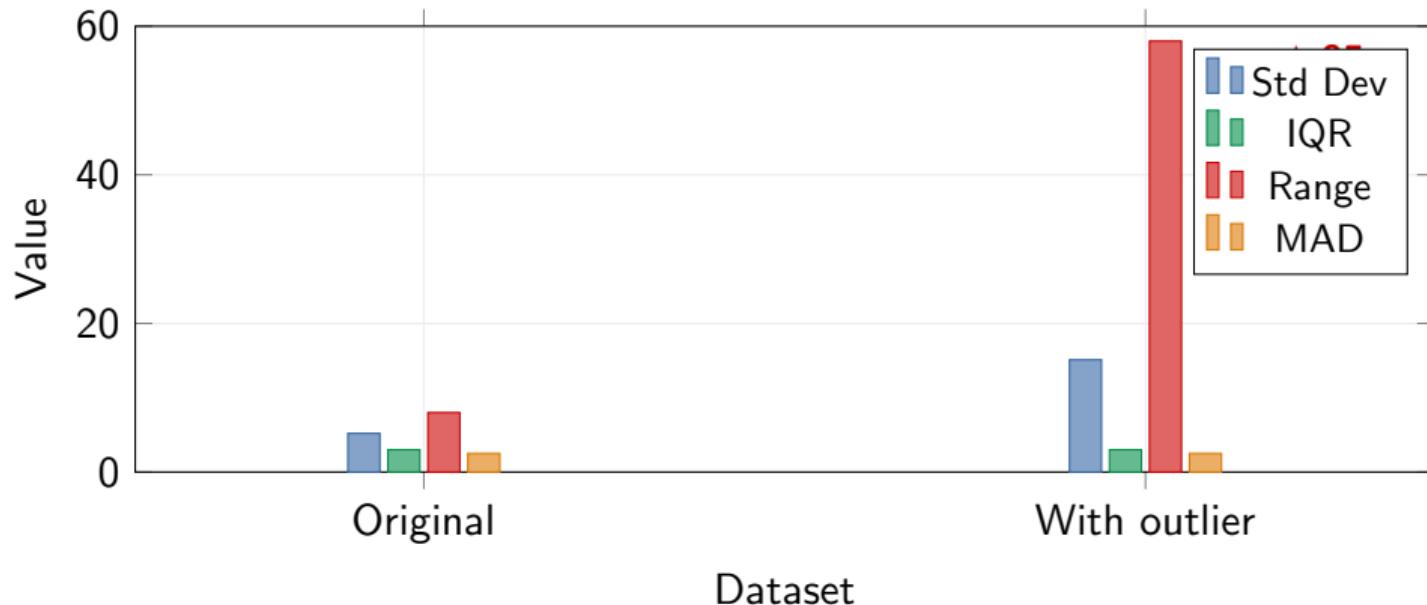
Compromise:
mean \leftrightarrow median
Tunable ro-
bustness

Measures of Spread

| Measure | Formula | Properties |
|----------------|--|---|
| Variance S^2 | $\frac{1}{n-1} \sum (X_i - \bar{X})^2$ | Uses all data; $n-1 =$ Bessel's correction (unbiased) |
| Std Dev S | $\sqrt{S^2}$ | Same units as data |
| Range | $\max - \min$ | Simple; extremely fragile |
| IQR | $Q_3 - Q_1$ | Middle 50%; robust |
| MAD | $\text{med } X_i - \text{med} $ | Most robust; companion to median |

Why $n - 1$? We used up one “degree of freedom” estimating \bar{X} .
This makes S^2 **unbiased**: $\mathbb{E}[S^2] = \sigma^2$.

Robust vs Non-Robust: Visual



One outlier: Range explodes, Std Dev triples. IQR and MAD don't budge.

How a Histogram Is Built

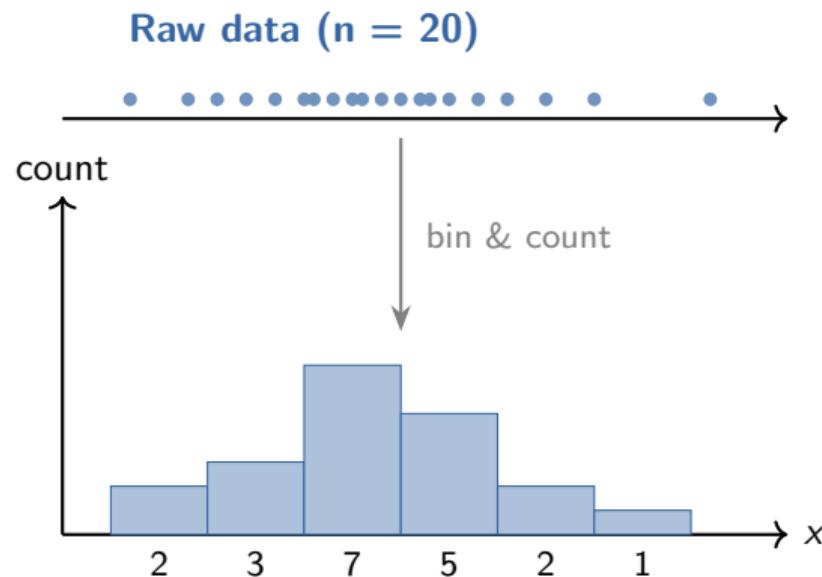
Recipe:

1. Choose **bins**: equal-width intervals covering the data range
2. Count observations in each bin
3. Draw bars — height = count (or density)

Density form:

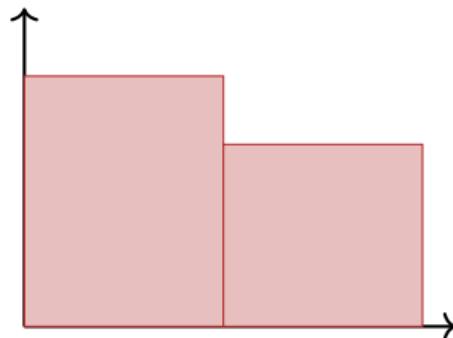
$$\text{height} = \frac{\text{count}}{n \times \text{bin width}}$$

so total area = 1 (comparable across bin widths).



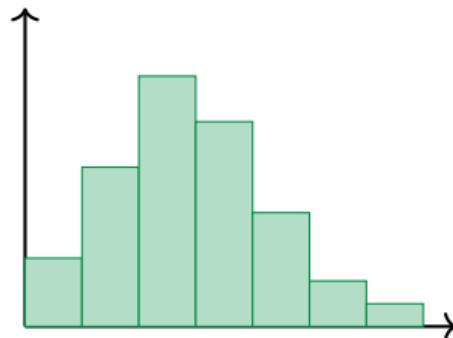
Bin Width Matters

Too few bins (2)



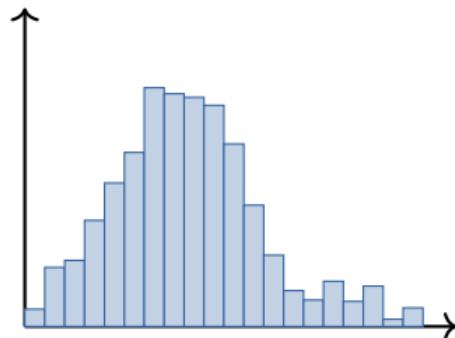
Hides all structure

Good bin width



Shape is clear

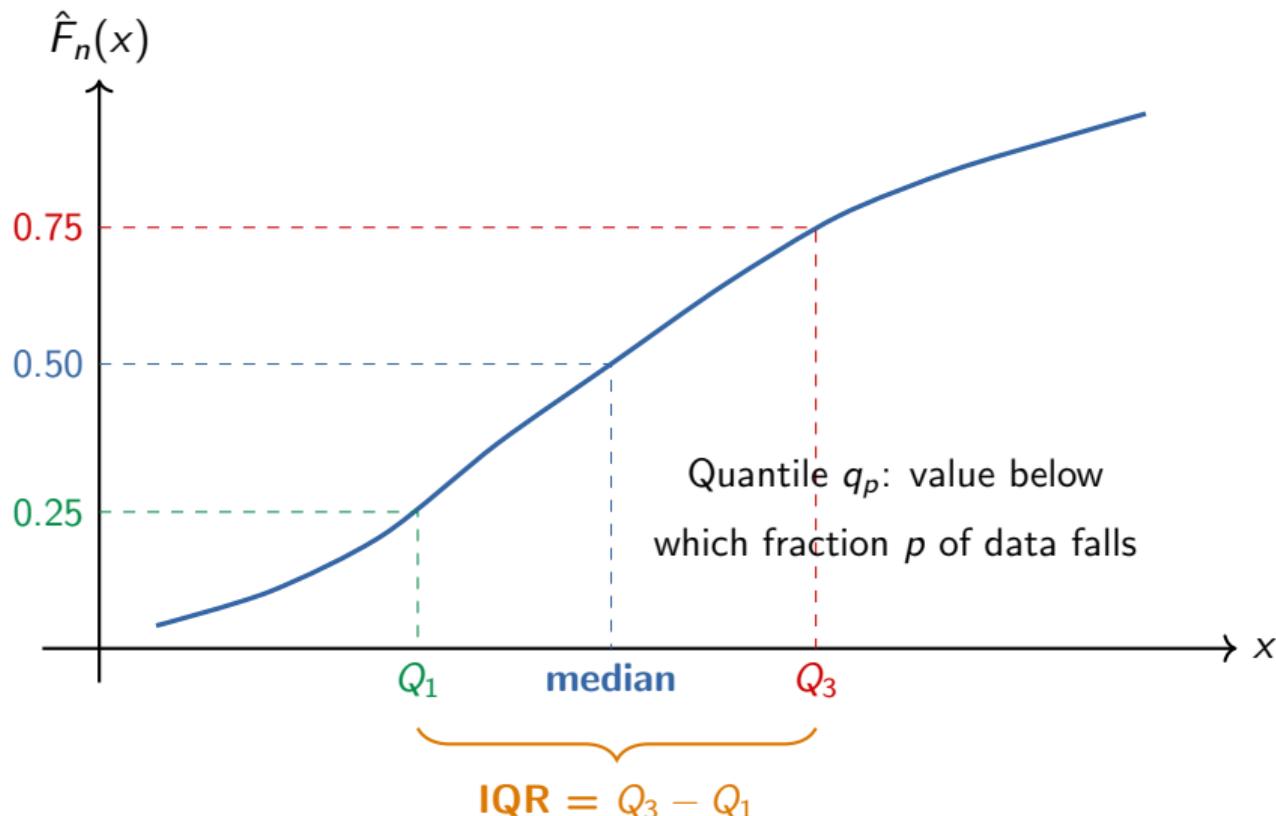
Too many bins (20)



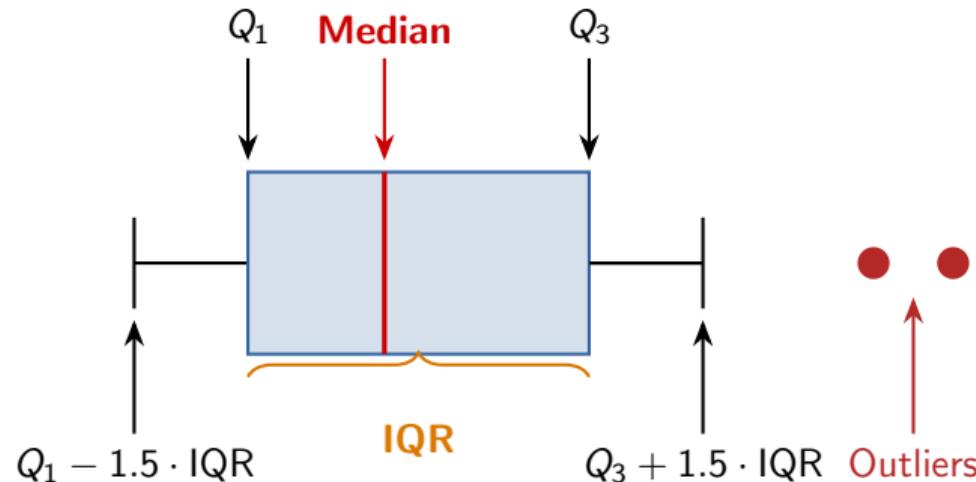
Too noisy

Common rules: Sturges ($k = 1 + \log_2 n$), Freedman–Diaconis ($h = 2 \cdot \text{IQR} \cdot n^{-1/3}$).
In practice: try several and look.

Quantiles and Percentiles



Boxplot Anatomy



Five-number summary: min, Q_1 , median, Q_3 , max — the boxplot visualizes exactly this.

Strengths:

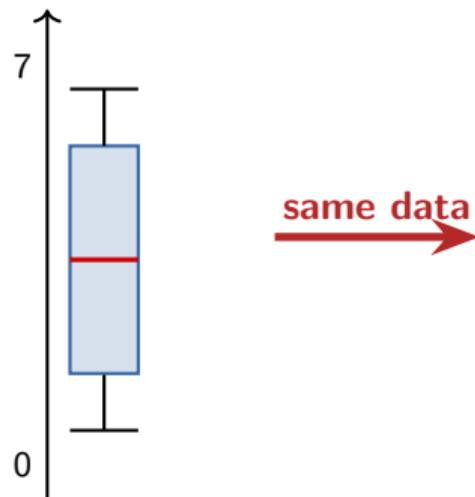
- Compact group comparison
- Shows center, spread, outliers

Weakness:

- Hides multimodality!
- Pair with histogram or violin

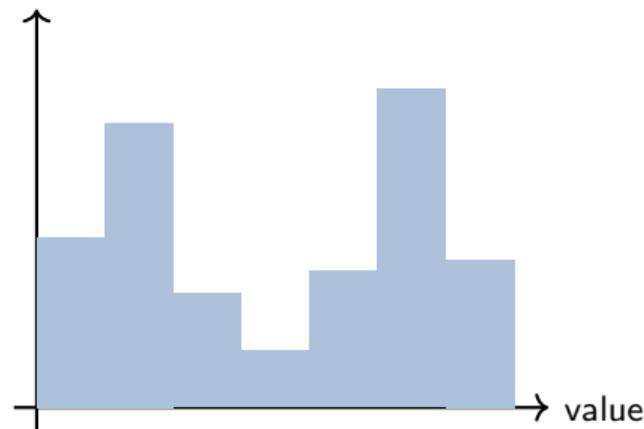
Boxplot Hides Bimodality

Boxplot



Looks unimodal

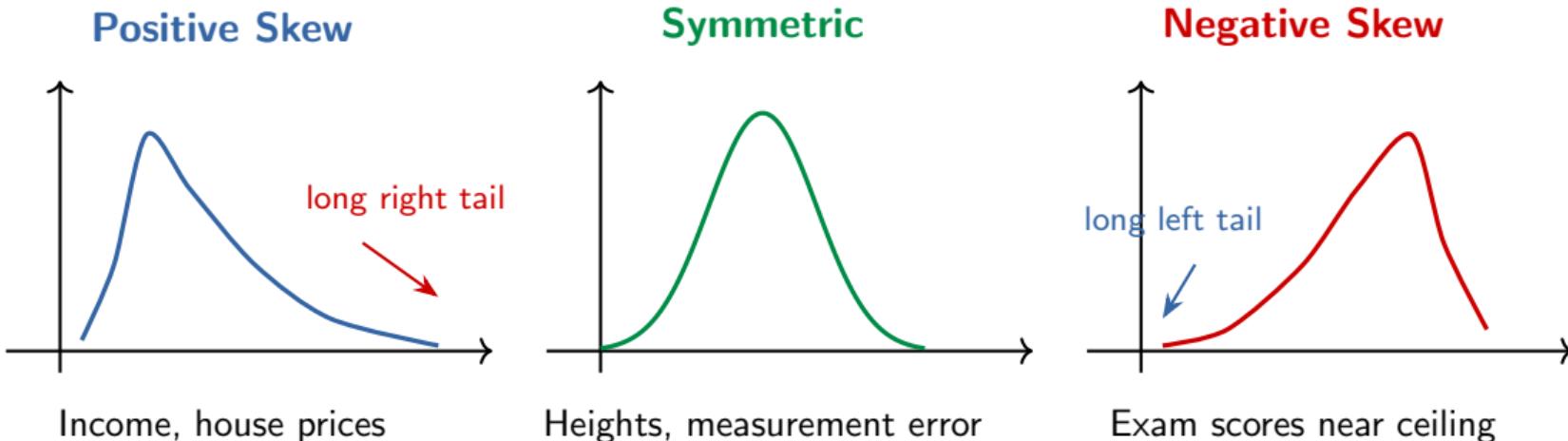
Histogram



Two distinct groups!

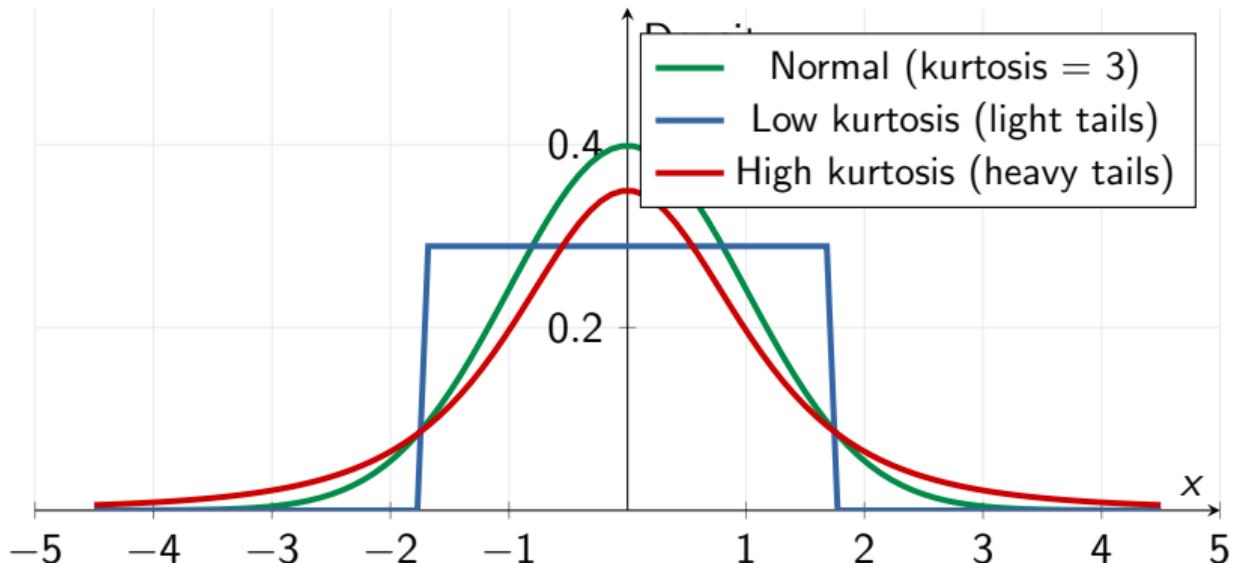
Always pair boxplots with histograms or violin plots.

Skewness: Measuring Asymmetry



$$\text{Skewness} = \frac{1}{n} \sum_{i=1}^n \left(\frac{X_i - \bar{X}}{S} \right)^3$$

Kurtosis: Tail Heaviness

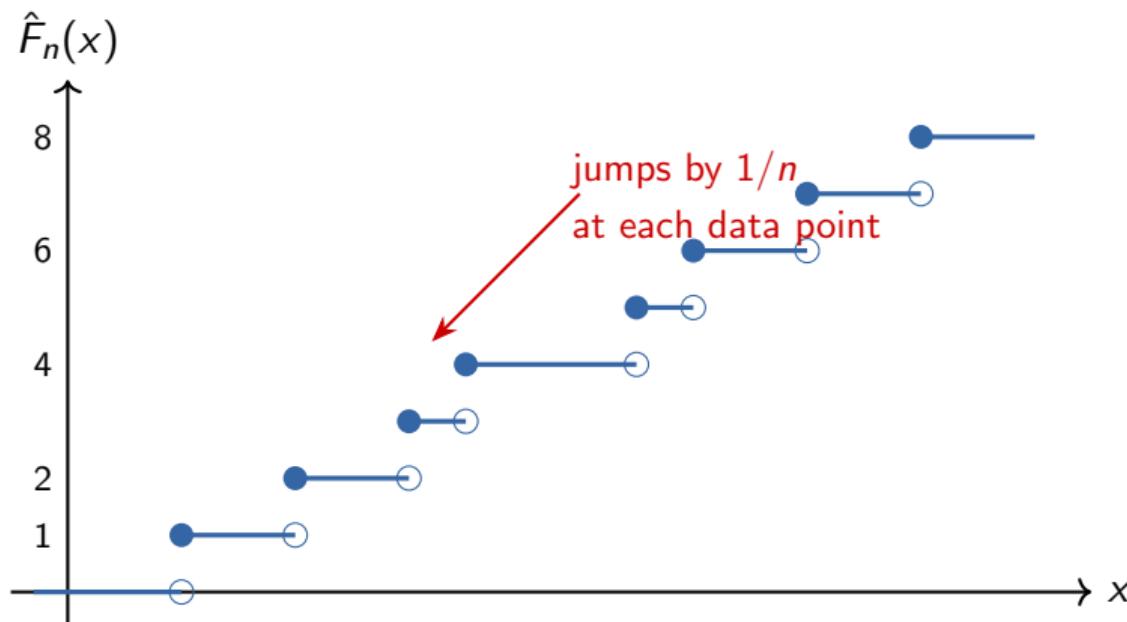


$$\text{Kurtosis} = \frac{1}{n} \sum_{i=1}^n \left(\frac{X_i - \bar{X}}{S} \right)^4 \quad \text{Excess kurtosis} = \text{Kurt} - 3$$

Normal has kurtosis = 3 (excess = 0). Most software reports **excess kurtosis**.
Financial returns have high kurtosis — assuming normality underestimates risk.

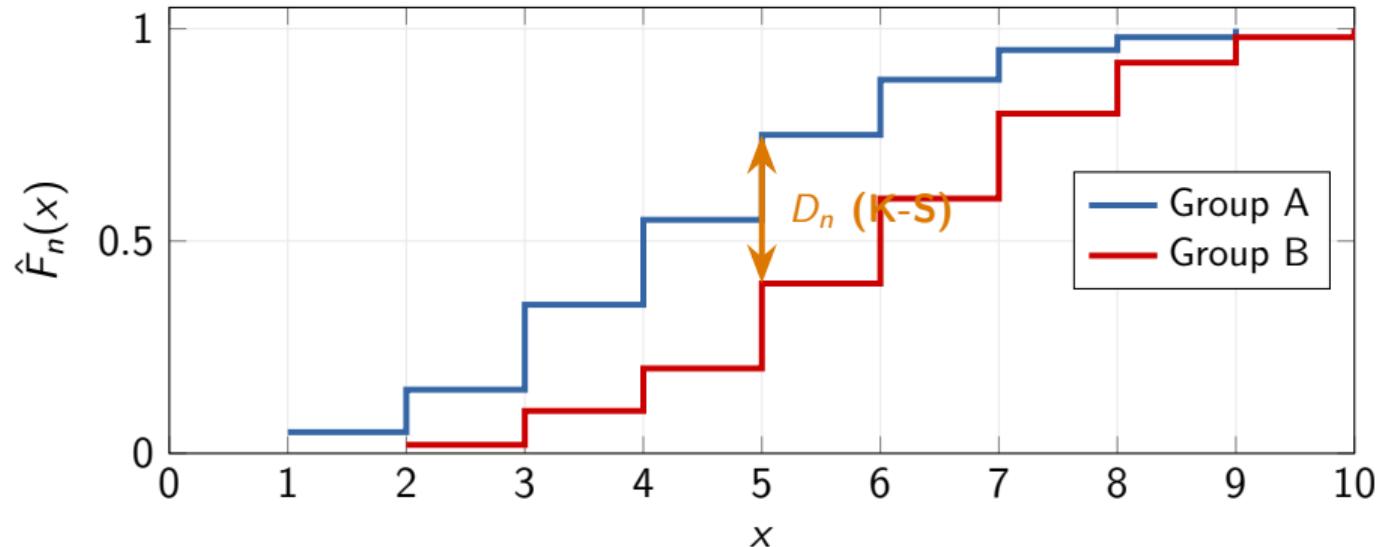
The Empirical CDF

$$\hat{F}_n(t) = \frac{1}{n} \#\{X_i \leq t\} = \frac{\text{number of observations} \leq t}{n}$$



Example: $n = 8$ observations

ECDF: Why It's Powerful



- ▶ No bin-width choice (unlike histograms) — the ECDF is **parameter-free**
- ▶ Biggest gap = **Kolmogorov–Smirnov statistic D_n**
- ▶ Glivenko–Cantelli: $\hat{F}_n \rightarrow F$ uniformly as $n \rightarrow \infty$

Choosing the Right Plot

What do I want to see?

Best plot

Distribution shape of one variable

Histogram / density plot

Compare distributions across groups

Boxplot / violin plot

Relationship between two variables

Scatter plot

Full distribution, no bin choices

ECDF

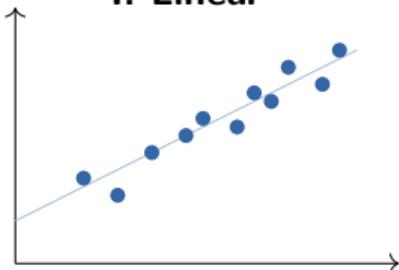
Change over time

Line plot / time series

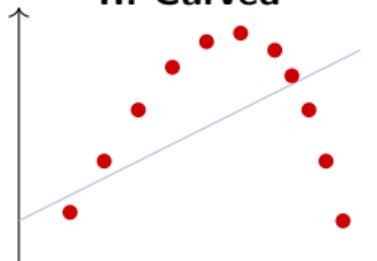
Rule of thumb: always start with a histogram + scatter plot matrix. Then refine.

Anscombe's Quartet (1973)

I: Linear



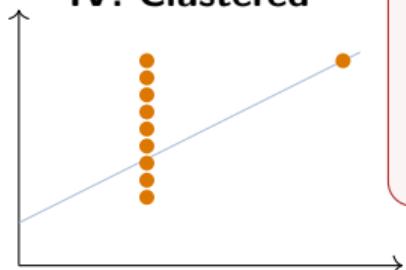
II: Curved



III: Outlier



IV: Clustered

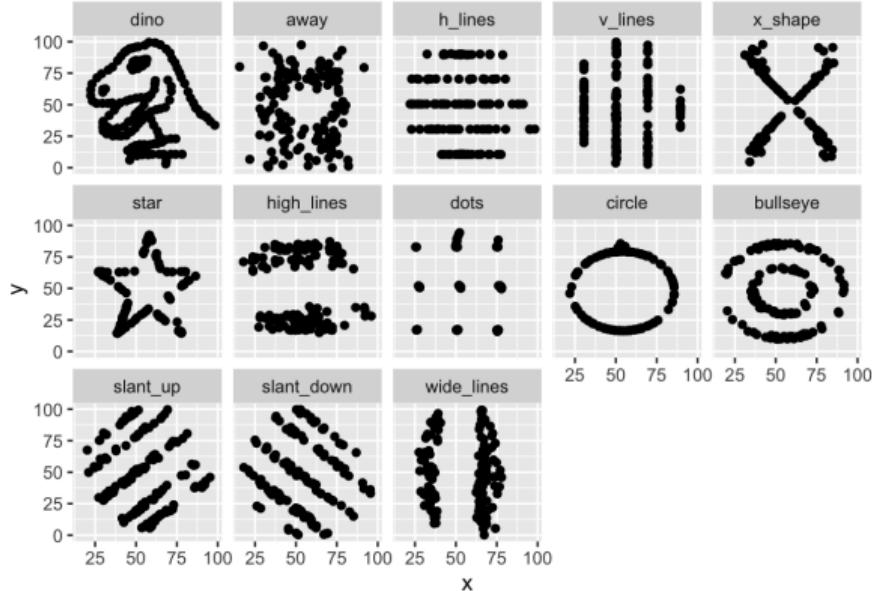


All four datasets:

$$\begin{aligned}\bar{x} &= 9, \bar{y} \approx 7.5 \\ S_x^2 &= 11, S_y^2 \approx 4.13 \\ r &\approx 0.816 \\ \hat{y} &= 3 + 0.5x\end{aligned}$$

Identical statistics.
Wildly different data.

The Datasaurus Dozen (2017)



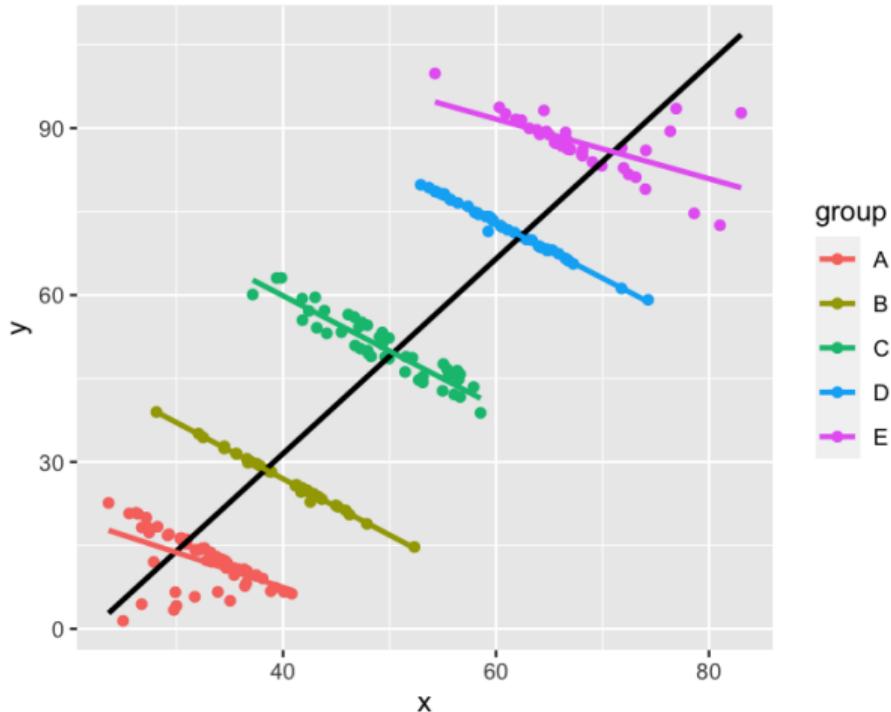
13 datasets, all with:

- Same \bar{x} , \bar{y}
- Same S_x , S_y
- Same correlation r

Yet shapes include a **dinosaur**, a star, parallel lines, a circle...

**Never trust summary statistics alone.
Always plot your data.**

Simpson's Paradox



Each colored group trends **down**, yet the aggregate trend goes **up**.
How? The groups have **different sizes and positions**.

Simpson's Paradox: UC Berkeley Admissions (1973)

12,763 applicants to UC Berkeley graduate programs.

Aggregate data:

| | Applied | Admitted |
|-------|---------|----------|
| Men | 8,442 | 44% |
| Women | 4,321 | 35% |

9 percentage points gap!
Lawsuit filed for gender bias.

By department:

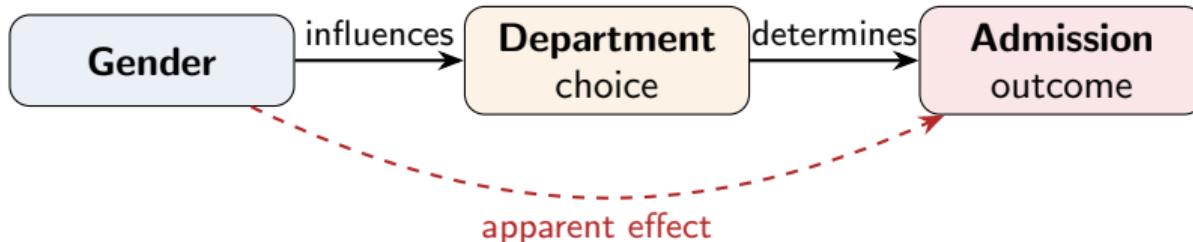
| | Rate | Women | Men |
|--------|------|-------|-----|
| Dept A | easy | 82% | 62% |
| Dept B | easy | 68% | 63% |
| Dept C | hard | 34% | 35% |
| Dept D | hard | 7% | 6% |

Women admitted at equal or
higher rates in each dept!

How is this possible?

Simpson's Paradox: Why It Happens

confounding variable



The key: Women disproportionately applied to **hard** departments (low acceptance for everyone). Men disproportionately applied to **easy** departments.

When you aggregate, the **different weights** reverse the trend:

- ▶ Women: ~80% applied to hard depts → low overall rate
- ▶ Men: ~80% applied to easy depts → high overall rate

General lesson: a trend in every subgroup can **reverse** when subgroups are combined.

Always ask: *is there a hidden variable that changes the group sizes?*

Homework

1. Go over the **Data Visualization** topic:

https://hayktarkhanyan.github.io/python_math_ml_course/python_libs/06_data_viz.html

2. Pick a dataset (e.g. from **Kaggle** or <http://armstat.am/>) and **explore** it:

compute summary statistics, build histograms, boxplots, scatter plots, ECDF

3. Come up with your own examples of:

- ▶ **Survivorship bias**
- ▶ **Simpson's paradox**

Questions?