$f(x,y) = 4 + 2x - x^2 - \frac{y^2}{2}$

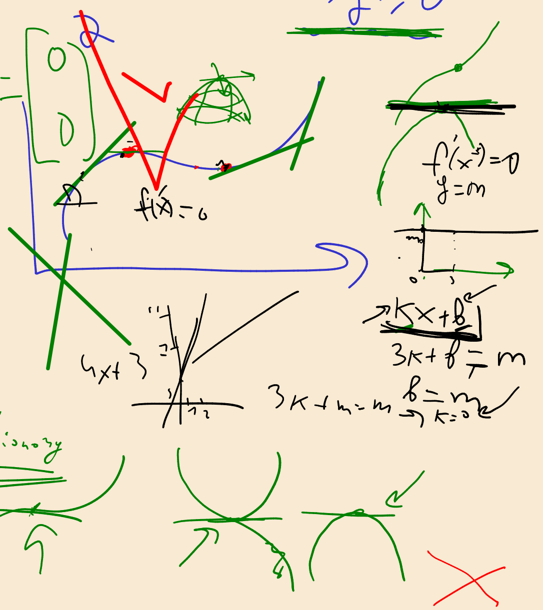
$x, y \geq 0$

$$\nabla f(x,y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-2x \\ -y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x = 1 \\ y = 0 \end{cases}$$

stationary



$f'(x) = 0$

$f'(x^*) = 0$

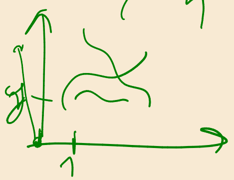
$y = m$

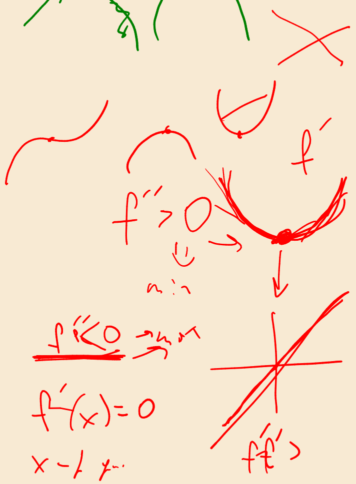
$kx + b$

$3k + b = m$

$3k + m = m$, $b = m$, $k = 0$

$4x + 3$





$f'' > 0$ min

$f'' < 0$ max

$f'(x) = 0$

x_1

$f''(x_1) \to$

negative defin

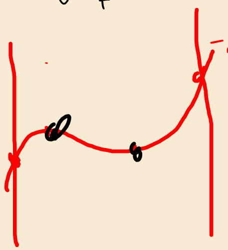
$$\nabla f = \begin{bmatrix} 2-2x \\ -y \end{bmatrix}$$

$$H = \nabla^2 f = \begin{bmatrix} \boxed{-2} & 0 \\ 0 & \boxed{-1} \end{bmatrix}$$

$\nabla f \cdot i$

$-2 < 0$

$-2 \cdot -1 - 0 \cdot 0 = 2 > 0$



$x, y \geq 0$

$x = 0$, $y = 0$

$x \to \infty$, $y \to \infty$

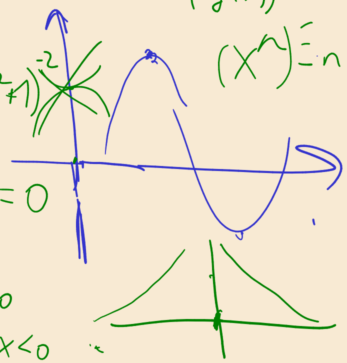
$$(x^2+1)^{-1} = 2x \cdot -1 \cdot (x^2+1)^{-2} = -\frac{2x}{(x^2+1)^2} = 0$$

$f'_y(x)$

$D(f) = (-\infty, \infty)$

$(x^n)' = n x^{n-1}$

$\frac{1}{x^2+1} \xrightarrow[x \to \infty]{} 0$



$x \to \infty$ $(x^2+1)^{-1} \to$

$x \to -\infty$

$x > 0 \to$

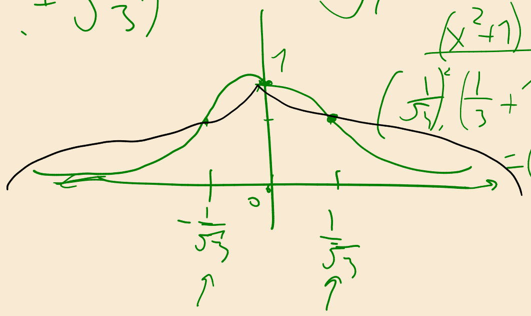
$x = 0$

$x < 0$

$$-2x\,(x^2+1)^{-2} = -2(x^2+1)^{-2} + (-2x)\left((x^2+1)^{-2}\right)'$$

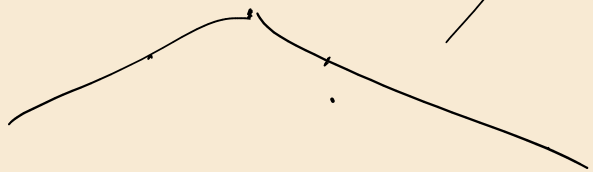
$$(f\cdot g)' = f'g + g'f$$

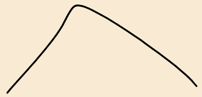
$$\pm\frac{1}{\sqrt{3}}$$



$$(x^2+1)^{-1}$$

$$\left(\frac{1}{\sqrt{3}}\right)^2 \quad \left(\frac{1}{3}+1\right)^{-1} = \left(\frac{4}{3}\right)^{-1} = \frac{3}{4}$$





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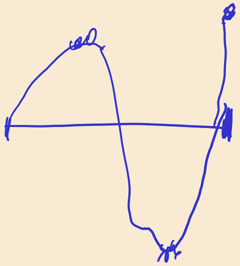

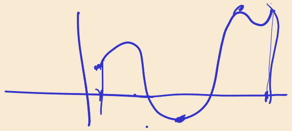

[0, 1]

$\frac{1}{x}$