

From N-grams to LSTMs

N-grams · Word2Vec · RNNs · LSTMs · Motivation for Attention

The core problem

“The cat sat on the mat”



?

Representation

How do we turn words
into numbers that
capture meaning?

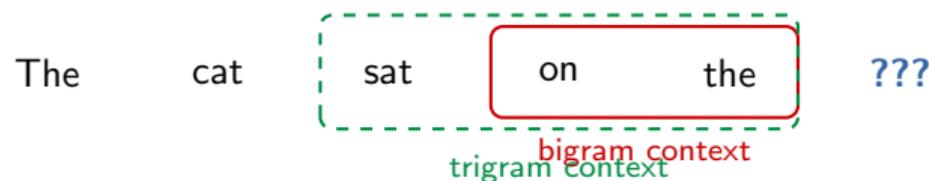
Sequence modeling

How do we capture con-
text and word order?



N-gram language models

$$P(w_t \mid w_{t-n+1}, \dots, w_{t-1}) = \frac{\text{count}(w_{t-n+1} \dots w_t)}{\text{count}(w_{t-n+1} \dots w_{t-1})}$$



Bigram:

$$P(\text{mat} \mid \text{the}) = \frac{\text{count}(\text{"the mat"})}{\text{count}(\text{"the"})}$$

Trigram:

$$P(\text{mat} \mid \text{on, the}) = \frac{\text{count}(\text{"on the mat"})}{\text{count}(\text{"on the"})}$$

Idea: estimate probabilities by counting how often word sequences appear in a large corpus

N-gram limitations

12 words apart — trigram window can't see "cat"

"The **cat** that sat on the mat near the door by the window **was** _____"

1. Fixed context window

A trigram only sees 2 previous words. Long-range dependencies are invisible.

2. Curse of dimensionality

Vocabulary $V = 50k \Rightarrow$ possible 5-grams: $50k^5 = 3 \times 10^{23}$. Most never observed.

3. Data sparsity

"armadillo aardvark" has count 0 in most corpora. Smoothing helps but doesn't solve it.

4. No generalization

Knowing "dog sat on" doesn't help predict after "cat sat on." Words are discrete symbols.

We need a way to represent words so that similar words share information.

One-hot encoding

Vocabulary

1. cat

2. dog

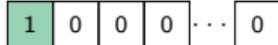
3. sat

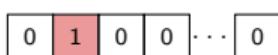
4. the

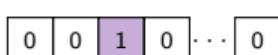
:

50k. zoo

One-hot vectors

cat = 

dog = 

sat = 

50,000 dimensions

No similarity

$\text{cat} \cdot \text{dog} = 0$

$\text{cat} \cdot \text{banana} = 0$

Every pair is equally different!

Huge & sparse

50k-dim vectors with exactly one non-zero entry

We need: dense, low-dimensional vectors
where similar words have similar representations

Word2Vec — the distributional hypothesis

“You shall know a word by the company it keeps”
— J. R. Firth, 1957

Words appearing in similar contexts have similar meanings:

“The movie was surprisingly **good** and **the audience** loved it”

“The movie was surprisingly **great** and **the audience** loved it”

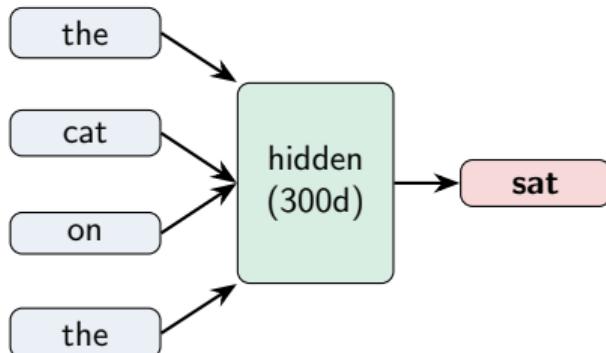
“The movie was surprisingly **excellent** and **the audience** loved it”

good, great, and excellent share context ⇒
their vectors should be close in embedding space

Word2Vec — CBOW & Skip-gram

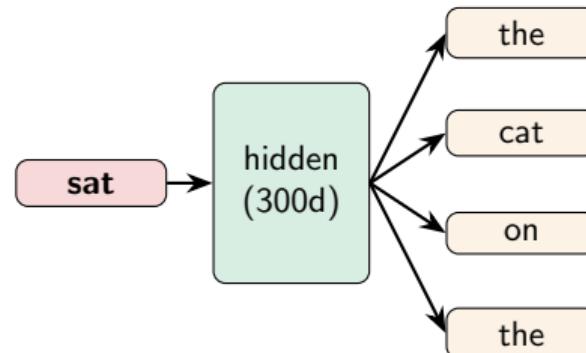
CBOW

Context → Center word



Skip-gram

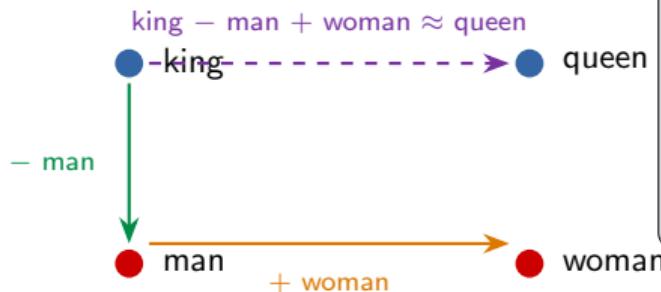
Center word → Context



Result: the hidden layer weights *are* the word embeddings — dense vectors (100–300 dims)

Word2Vec — emergent properties

Vector arithmetic in embedding space



More analogies:

Paris – France + Italy \approx Rome
bigger – big + small \approx smaller
walking – walk + swim \approx swimming

Critical limitation: one vector per word

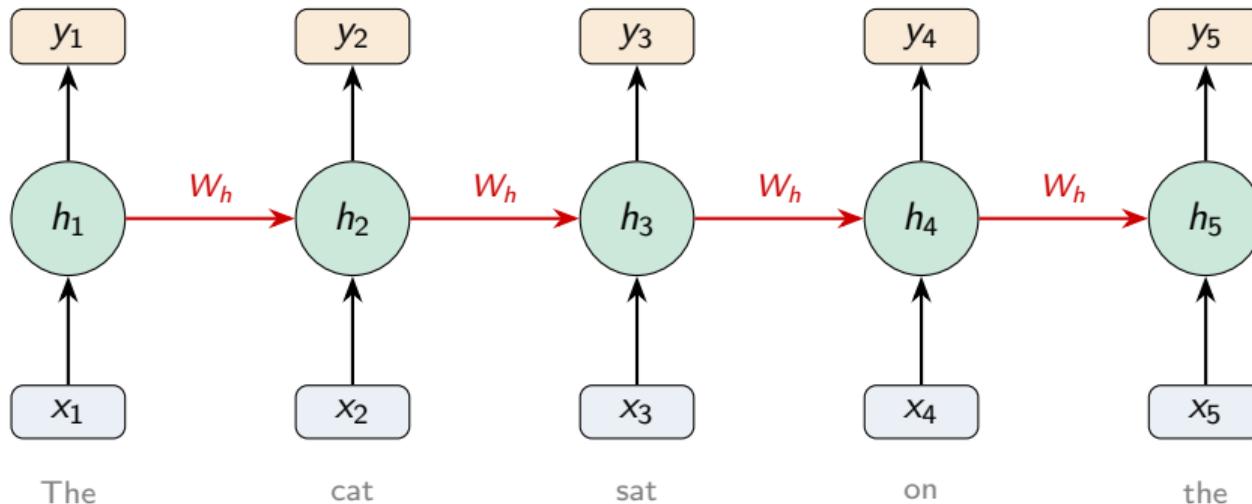
"I went to the river **bank**" vs. "I deposited money at the **bank**"

Both map to the *same* vector — no polysemy, no context-dependence

We need representations that change based on context. Enter: recurrent neural networks.

Recurrent Neural Networks (RNNs)

Unrolled RNN across time steps

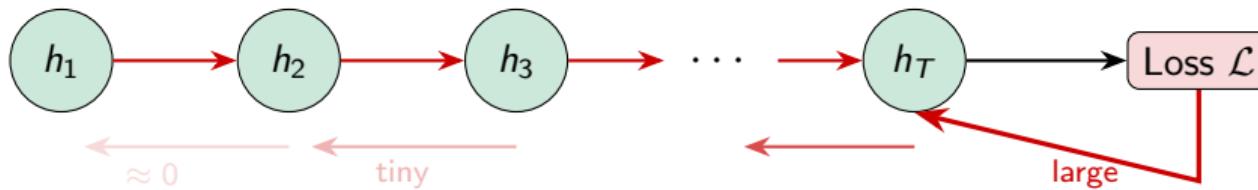


$$h_t = \tanh(W_h \cdot h_{t-1} + W_x \cdot x_t + b)$$

Key: hidden state h_t is a “memory” — no fixed window!

The vanishing gradient problem

Backpropagation through time



$$\frac{\partial \mathcal{L}}{\partial h_1} = \frac{\partial \mathcal{L}}{\partial h_T} \cdot \prod_{t=2}^T \frac{\partial h_t}{\partial h_{t-1}}$$

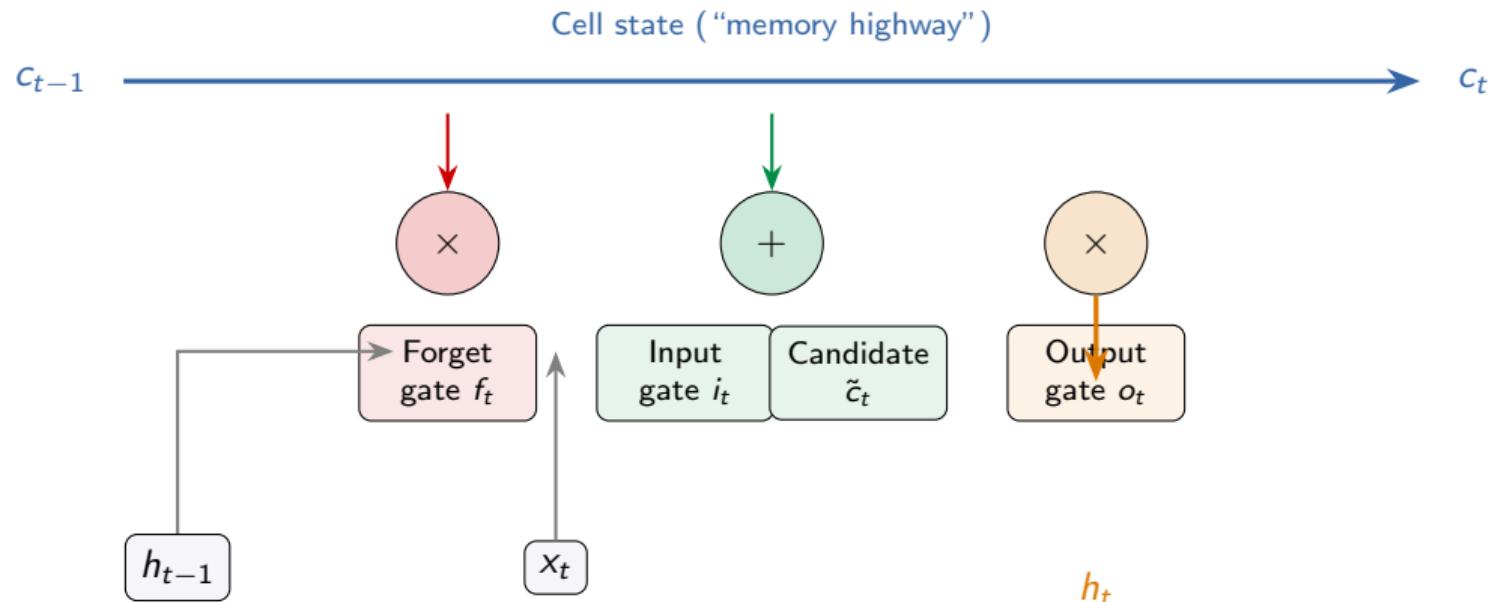
Product of many values $< 1 \Rightarrow$ gradient vanishes

“**The cat** that I saw yesterday in the garden near the old oak tree **was** cute”

The gradient from “**was**” can’t reach “**cat**” — the RNN *forgets* early tokens

Solution attempt: LSTM (1997) and GRU (2014)

LSTM — Long Short-Term Memory

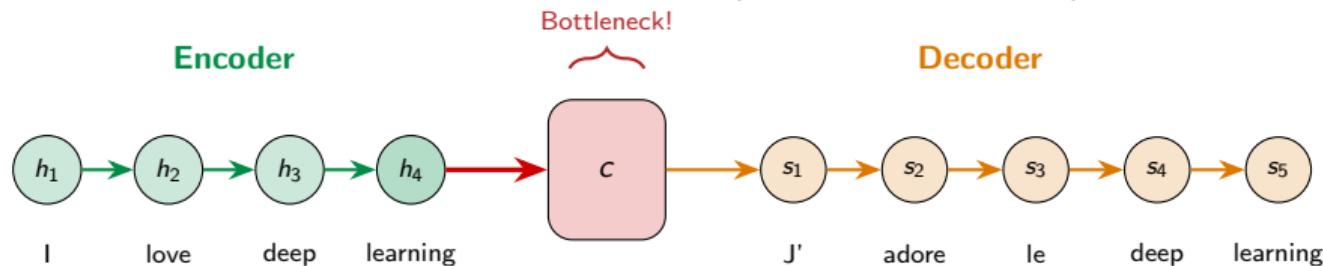


Cell state flows with minimal modification — gradient highway

Gates learn what to forget, store, and output at each step

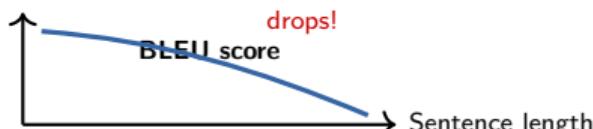
Sequence-to-sequence (Encoder–Decoder)

Machine translation with LSTMs (Sutskever et al., 2014)



The entire input sentence is compressed into a single fixed-size vector c !

Works for short sentences, but performance degrades for long ones.

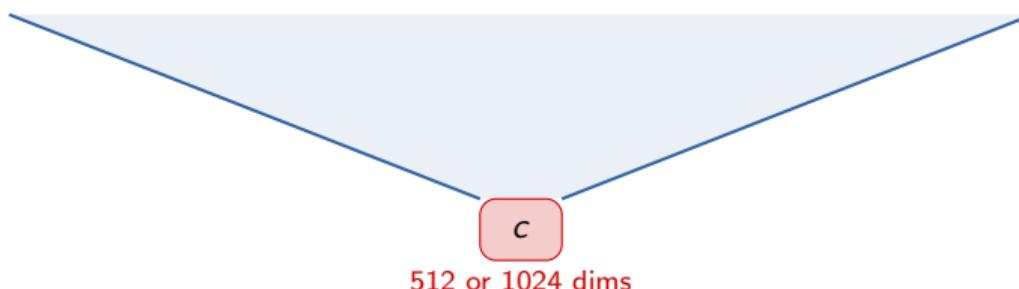


What if the decoder could look back at all encoder states, not just c ?

→ This is attention!

Why LSTMs are not enough — the bottleneck

"The quick brown fox jumped over the lazy dog that
was sleeping by the fireplace in the old cottage"



Information lost:

Early tokens get overwritten by later ones as h_t is updated sequentially

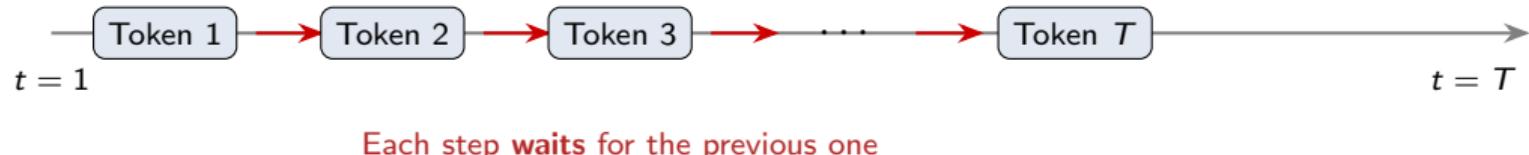
Fixed capacity:

Whether the input is 5 tokens or 500, it must fit in the same vector

Fundamental issue: you can't losslessly compress arbitrary-length sequences into a fixed-size vector

Why LSTMs are not enough — sequential processing

LSTM: must process tokens one at a time



What if we could process all tokens in parallel?



LSTM: $O(T)$ sequential steps
Can't utilize GPU parallelism
Training is *slow*

Transformer: $O(1)$ parallel steps
Fully utilizes GPU cores
Training is *fast*

Why LSTMs are not enough — long-range dependencies

“The **trophy** didn’t fit in the **suitcase** because **it** was too _____”

If **it** = **trophy**:

“it was too **big**”

If **it** = **suitcase**:

“it was too **small**”

Path length from “it” to its referent:

LSTM: $O(n)$ steps

Information must pass through
every intermediate hidden state

Signal degrades over distance

Attention: $O(1)$ steps

Direct connection be-
tween any two tokens

No degradation over distance

Attention: “let me directly look at any token I need” — regardless of distance

The stage is set

N-grams

- Fixed window
- No generalization
- Data sparsity
- Can't scale

Word2Vec

- Good embeddings
- But static (one vector per word)
- No sequence modeling

RNN / LSTM

- Sequential (\times parallel)
- Bottleneck vector
- Long-range deps still hard



We need: **parallel processing, direct access to all positions, contextual representations**

⇒ **Self-Attention** and the **Transformer** (Vaswani et al., 2017)

Word2Vec — training tricks

$$\text{Skip-gram objective: } \max_{\theta} \sum_{t=1}^T \sum_{-c \leq j \leq c, j \neq 0} \log P(w_{t+j} | w_t; \theta)$$

Problem: full softmax

$$P(w_O | w_I) = \frac{\exp(v'_{w_O} \cdot v_{w_I})}{\sum_{w=1}^V \exp(v'_{w'} \cdot v_{w_I})}$$

Sum over $V = 50k$ words per update!

Negative sampling

Instead of all V words, sample k random “negative” words

Binary classification:
real context vs. noise

$$\log \sigma(v'_{w_O} \cdot v_{w_I}) + \sum_{i=1}^k \mathbb{E}_{w_i \sim P_n} [\log \sigma(-v'_{w_i} \cdot v_{w_I})]$$

Subsampling

Downsample frequent words (“the”, “a”)

Window size

Larger window = more semantic; smaller = more syntactic

$$f(w)^{3/4}$$

Dimensionality

Typical: 100–300 dims; more data → more dims

Beyond Word2Vec — GloVe & FastText

GloVe (Pennington et al., 2014)

Idea: combine count-based and prediction-based methods

Build word co-occurrence matrix

X_{ij} , then factorize:

$$w_i^T \tilde{w}_j + b_i + \tilde{b}_j = \log X_{ij}$$

Global statistics + local context

Often matches or beats Word2Vec

FastText (Bojanowski et al., 2017)

Idea: represent words as bags of character n-grams

“where” → {⟨wh, whe, her, ere, re⟩}

Word vector = sum of n-gram vectors

Handles unseen words & morphology

Great for morphologically rich lan-

Comparison

	Word2Vec	GloVe	FastText
Training signal	Local context	Global co-occurrence	Local + subword
OOV words	No	No	Yes
Morphology	No	No	Yes
Speed	Fast	Fast	Medium

All three share the same limitation: static embeddings — one vector per word regardless of context

Further reading

N-grams & Statistical NLP

- Jurafsky & Martin, *Speech and Language Processing*, Ch. 3 — N-gram Language

Word Embeddings

- Mikolov et al. (2013), “Efficient Estimation of Word Representations in Vector Space”
- Mikolov et al. (2013), “Distributed Representations of Words and Phrases” (negative sampling)

RNNs & Sequence Models

- Hochreiter & Schmidhuber (1997), “Long Short-Term Memory”
- Cho et al. (2014), “Learning Phrase Representations using RNN Encoder-Decoder” (GRU)
- Sutskever et al. (2014), “Sequence to Sequence Learning with Neural Networks”
- Bahdanau et al. (2015), “Neural Machine Translation by Jointly Learning to Align and Translate”

Questions?

Next: The Transformer Architecture