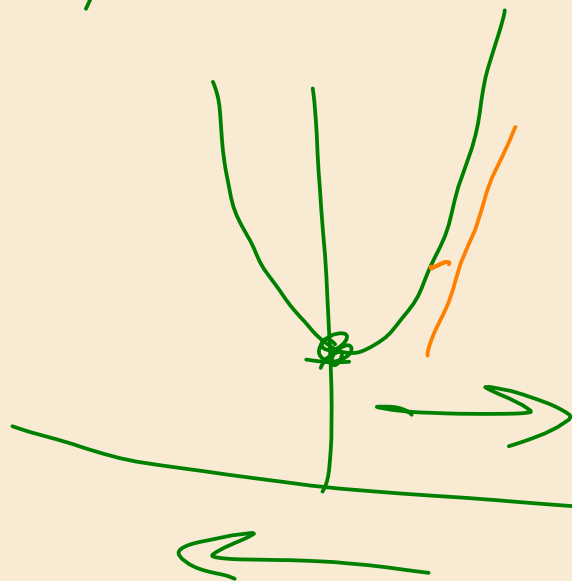


$$f(x) = 3 + 3x^2$$



$$f(x, y) = x + y^2$$



$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Partial der
 $f(x, y)$ mit $h \rightarrow 0$
 mit

$$\lim_{h \rightarrow 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$\frac{\partial}{\partial x} \sqrt{x^2 + y^2}$$

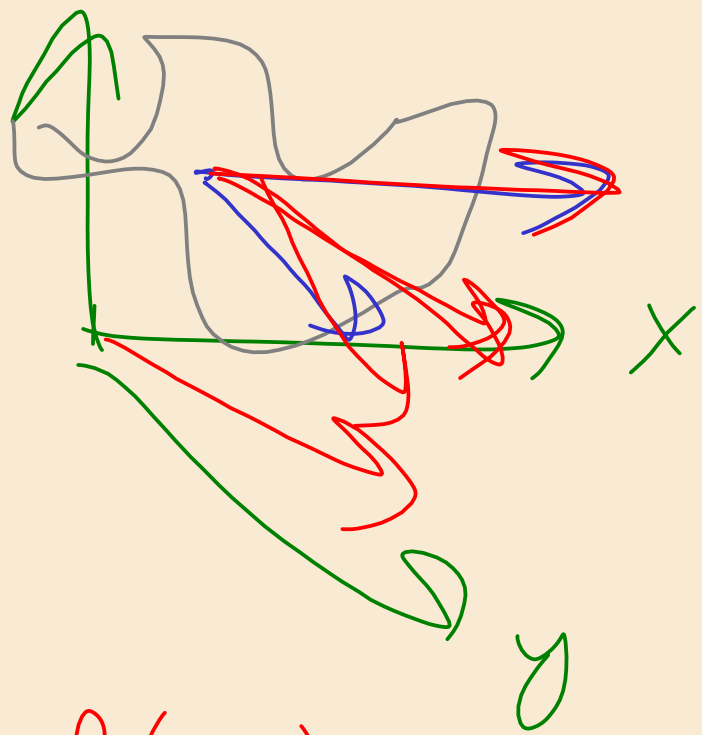
$$x^2 + 50y \quad \frac{\partial f}{\partial x} = 2x = 0$$

$$f'_x = 2x \quad f'_y = 2y$$

$$x^2 + y^2 + 2xy \quad (x+y)^2 \xrightarrow{2 \times 8} 16 \rightarrow 16$$

$$f_x = 2x + 2y$$

$$f_y = 2y + 2x$$



der ext:

f_x

f_y

$\frac{1}{2} f_{x+y}$

$$\frac{\partial f(x,y) + g(x,y)}{\partial x} = \frac{\partial f(x,y)}{\partial x} + \frac{\partial g(x,y)}{\partial x}$$

$$(cf)' = c f' + f c'$$

$$\frac{\partial}{\partial x} f \cdot y = \frac{\partial f}{\partial x} \cdot y + f \cdot \frac{\partial y}{\partial x}$$

$$\frac{\partial f}{\partial x} \boxed{2x^2(4x+6y)} = \cancel{4x^3} + \boxed{12x^2y}$$

$2x^2 \cdot 4 + (4x+6y) \cdot 2x$

$$\frac{\partial f}{\partial y} = 0 \cdot (4x+6y) + \cancel{2x^3} \cdot \boxed{12x^2}$$

Chain

partial. d. x

$$\begin{bmatrix} f' \\ f_x \\ f_y \end{bmatrix}$$

gradient

$$\nabla f = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} f' \\ f_x \\ f_y \end{bmatrix}$$

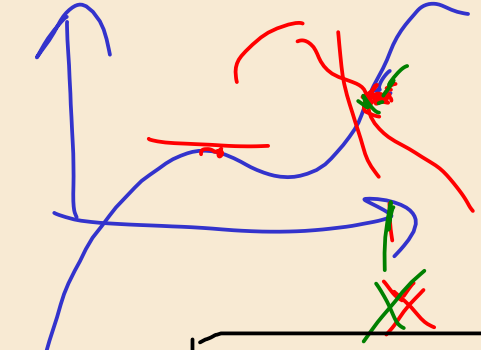
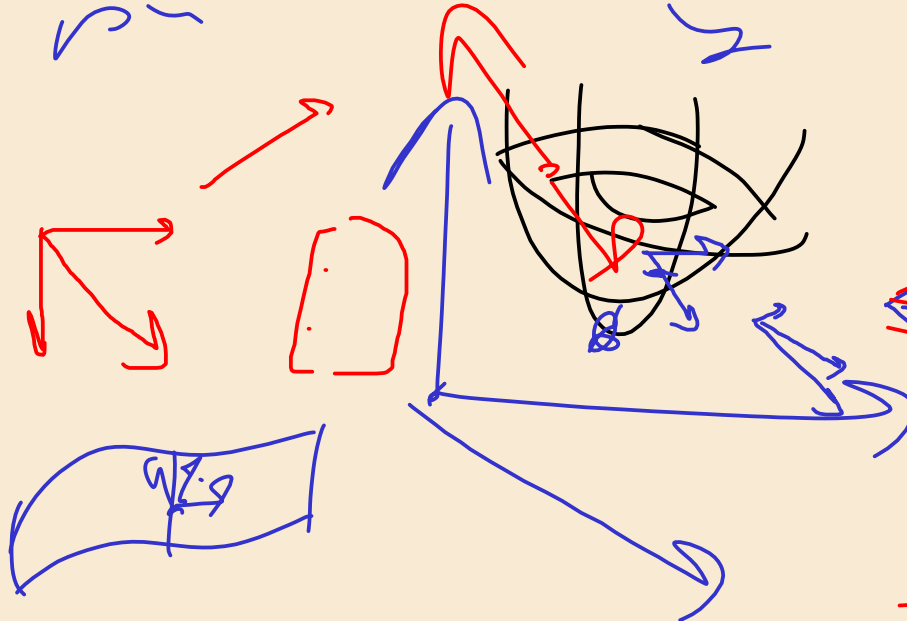
$$-\nabla f$$

$$\nabla f = 0$$

$$x^0 = 1$$

$$e^x$$

$$3x^2 \quad 3_0 = 0$$



$$\begin{bmatrix} f_x = 0 \\ f_y = 0 \end{bmatrix}$$

$$f(x) > 0$$

$$f(x) < 0$$

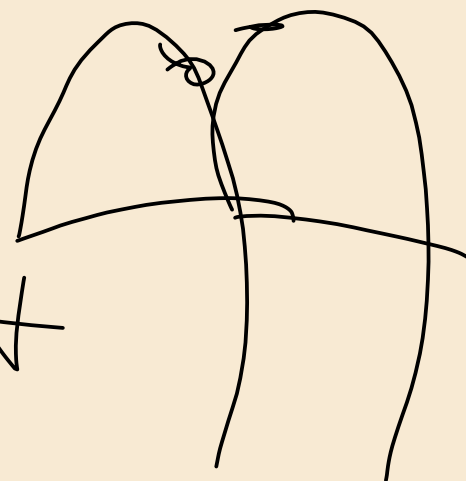
$$f(x) = 0$$

$$f(x)$$



5 old old

posit



$$x^2 + 3xy - y^2$$

$$f'_x = 2x + 3y - 0$$

$$f'_y = 0 + 3x - 2y$$

$$\nabla f = \begin{bmatrix} 2x + 3y \\ 3x - 2y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x + 3y = 0$$

$$3x - 2y = 0$$

$$\boxed{2x = -3y}$$

$$3x = 2y$$

$$2x = -3.0$$

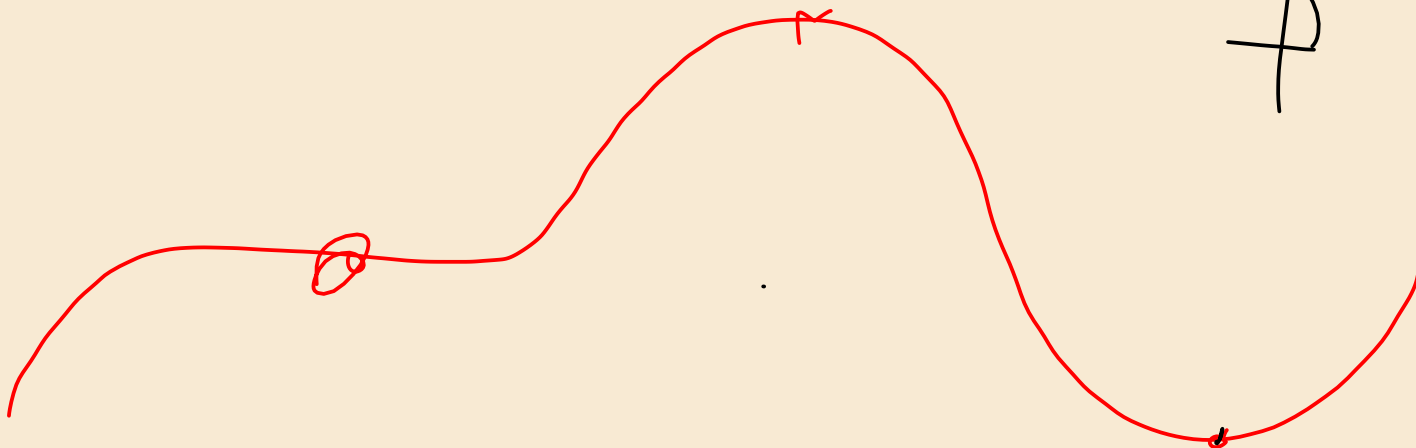
$$1.5 \cdot 2x = 0$$

$$1.5 \cdot 3y = 2y$$

$$= -1.5 \cdot 3y = 2y$$

$$= -4.5y = 0 \Rightarrow y = 0$$

$$f''(x) > 0$$



$$\nabla f = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

$$\begin{bmatrix} 2x + 3y \\ 3x - 2y \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

$$f_{xx} > 0$$

$$D_{-} f_{xx} f_{yy} - f_{xy} f_{xy} > 0$$

$$\begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix}$$

$$f''(x) > 0$$

$$f_{xx} = 2 > 0$$

$$D = -4 - 9 = -13$$

$$D < 0 \rightarrow \text{Saddle Point}$$

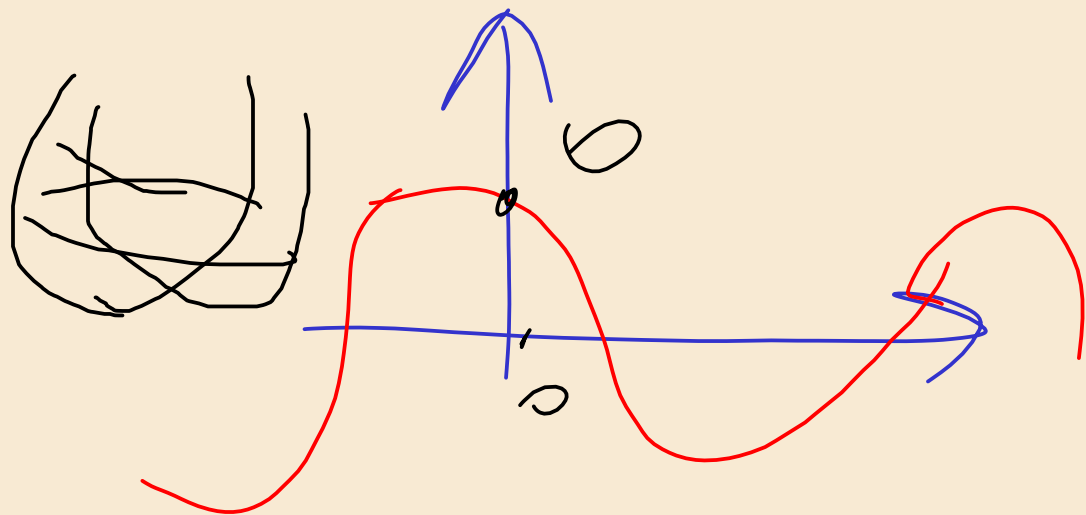
$$D > 0, f_{xx} > 0 \rightarrow \text{local min}$$

$$D > 0, f_{xx} < 0 \rightarrow \text{local max}$$

$2x$	$3y$
$\sin x$	$\sin x \cos y$

$s \sim 1 - r^2$

level curve



(0,0)

$$a_0 + a_1 x + a_2 x^2$$

$$f(0) = P(0) + a_2 x^2$$

$$f'(0) =$$

$$f(x) \approx f(0) + \frac{f'(0)}{1!} (x-0) + \frac{f''(0)}{2!} (x-0)^2$$

$$f(a) + \nabla f(a)(x-a) +$$

$$+ \frac{1}{2} \nabla^2 f(a) (x-a)^2$$

$$\begin{bmatrix} \vdots \end{bmatrix}_{n \times 1} \begin{bmatrix} \end{bmatrix}_{n \times 1}$$

$$(x-a)^2 = (x-a) \frac{1}{(x-a)}$$

$$\begin{bmatrix} a & b & c \\ \hline d & e & f \end{bmatrix} \xrightarrow{\sim} \begin{bmatrix} d & e \\ c & f \end{bmatrix}$$

$$\begin{bmatrix} \end{bmatrix}_{n \times m} \otimes \begin{bmatrix} \end{bmatrix}_{k \times l}$$

$n = k$