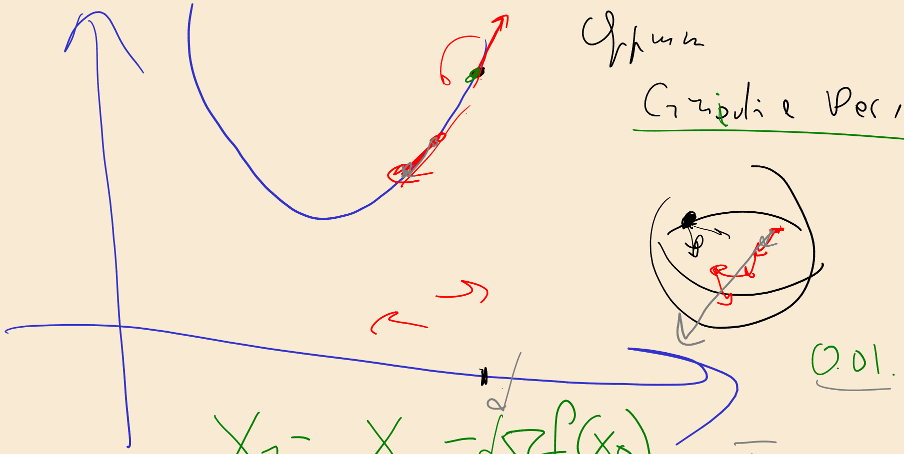
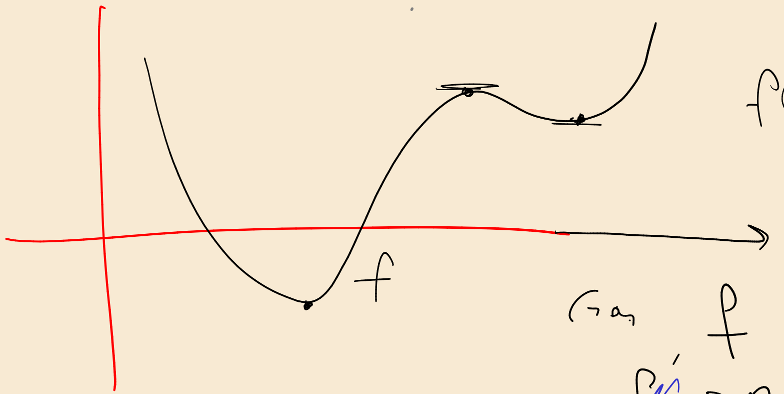


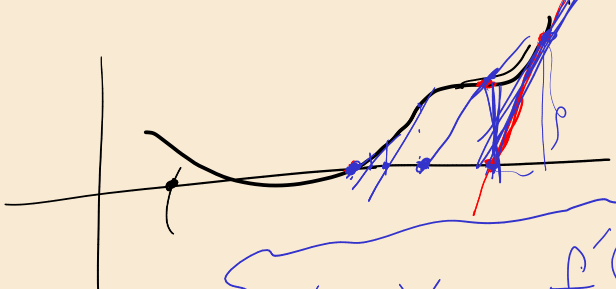
$$X_1 = X_0 - \alpha \nabla f(X_0)$$

$$X_2 = X_1 - \alpha \nabla f(X_1)$$

0.01



$$f'(x) = 0$$



$$f' \to 0$$

$$X_{k+1} = X_k - \frac{f'(X_k)}{f''(X_k)}$$

$$y = f'$$

$$\frac{g}{g'} \qquad \frac{f'}{f''}$$

$f(x) = e^x - x$

$x_0 = 1$

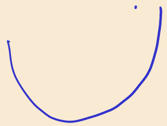
$f'(x) = e^x - 1$

$x_1 = 1 - \frac{e^1 - 1}{e^1} = 1.5$

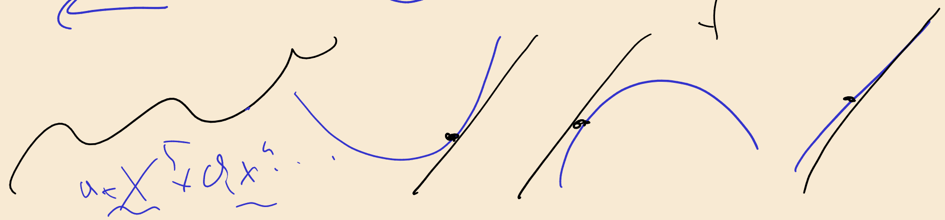
$f''(x) = e^x$

$x_2 = 1.5 - \frac{e^{1.5} - 1}{e^{1.5}}$

$\vdots$



f''



$$f(x_k + p) \approx f(x_k) + \nabla f(x_k)^T p + \frac{1}{2} p^T H_k p$$

$$a_0 + b x + c^2 x^2 = 0$$

$$-\frac{b}{c^2} = -b \cdot c^{-2}$$

$$P = -H^{-1} \cdot \nabla f$$

Suppose you know that $2^{10} = 1024$. Moreover, you are able to recall that $\ln 2 \approx 0.69 \approx 0.7$.

2.1) Can you deduce from this how large the logarithm with basis 2 of 1000 is approximately, i.e., $\log_2 1000$? *You may use Newton's method, or simply reason verbally and provide an estimate.* *3 points*

$\log_2 1000$ $\quad 2^{10} = 1024$ $\quad a^b$

$2^y = 1000$ $\quad \log 8 = x$ $\quad 2^x = 8$ $\quad \frac{f'}{f''}$

$f(y) = 2^y - 1000 = 0$

2^y

$\frac{1024.7}{10}$

$y_{k+1} = y_k - \frac{f(y^k)}{f'(y^k)}$ $\quad \frac{f'}{f''}$ $\quad x^n$ $\quad a^x$

$= \frac{1024-1000}{1024\cdot 0.7} \approx 10 - 0.03302 = 9.966$ $\quad 9.97$

$f'(y) = 2^y \ln 2$

$$\boxed{y_{k+1} = y_k - \frac{2^{y_k} - 1000}{2^{y_k} \cdot \ln 2}}$$

$e^x = e^x \ln e$

$4^x = 4^x \cdot \ln 4$

$a^x = a^x \ln a$

$V(t) = \max_x f(x,t)$

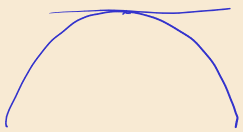
$f'(x(t)) = f'_x \cdot x'_t$

$f(x,3)$

$x^*(t)$

$f(x,3.001)$

t

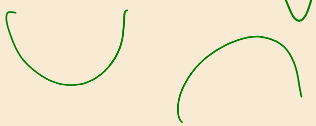


$V(t)$

$V'(t) = \frac{\partial f}{\partial t}(x^*(t),t) \cdot \frac{\partial x}{\partial t}$

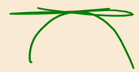
$f' := \frac{f(x+h)-f(x)}{h}$

0



$V'(t) = \cancel{f_x(x^*,t)\, x^{*\prime}(t)} + \underline{f'_t(x^*,t)}$

$f'_x(x^*,t) \approx 0$

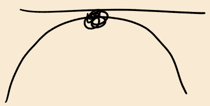


$f(x,a) = \underline{-(x-a)^2 + a}$

$-(x-2)^2+2$

$V(a) = \max_x f(x,a)$

$V(1) =$



$f(x,a)$

$\frac{\partial f}{\partial x} = \underline{-2(x-a)} = 0 \Rightarrow x = a$ | $V(a) = f(a,a) = -(a-a)^2 + a = a$

$$V'(a) = \frac{\partial f}{\partial a}(x, a) = a' = 1$$

$$-(x-a)^2 + a$$

$$\frac{\partial f}{\partial a} = -2(x-a)\cdot -1 + 1$$

$$2((x-a) + 1 = 1$$

f

$\underline{3+\ln x-\alpha x}$ $\quad f(x,\underline{\alpha})$

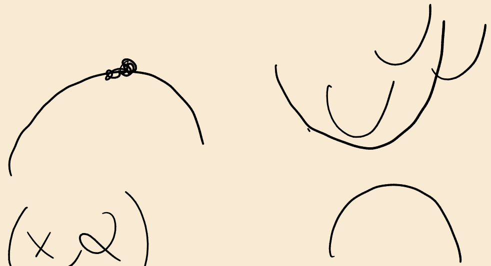
$f'(x)=\frac{1}{x}-\alpha=0$ $\quad 3+\ln x-2x$

$x^*(\alpha)=\frac{1}{\alpha}$

$(x^{-1})'=$

$=-1x^{-1-1}=$

$f''=-\frac{1}{x^2}<0$



$V(\alpha)=\max f(x,\alpha)$ $\quad \alpha:0$

$x^*(\alpha)=\frac{1}{\alpha}$

$3+\ln\alpha^{(-1)}-\alpha\cdot\frac{1}{\alpha}=3-\ln\alpha-1$

$2-\ln\alpha$

$$\underline{3+\ln x-\alpha x} \quad \frac{\partial f}{\partial \alpha} = -\alpha \quad \Big| \quad V(\alpha) = -\frac{1}{\alpha}$$

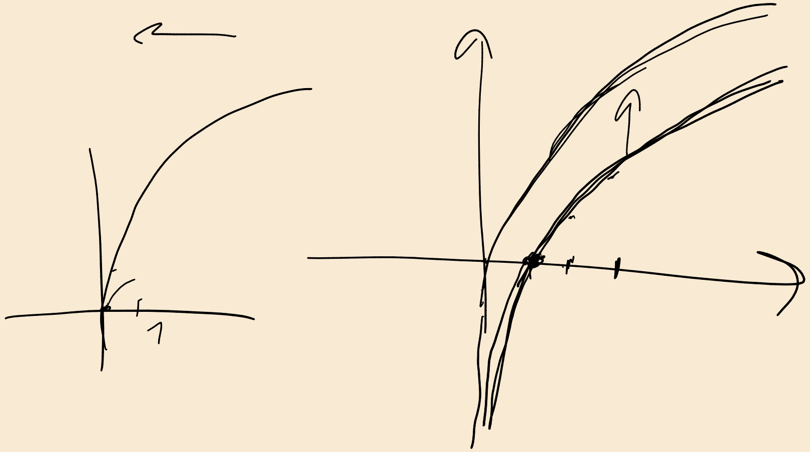
$$V(\alpha) = 2 - \ln \alpha > 0$$

$$\alpha^{\log_a b} = b$$

$$e^{\ln \alpha} < e^{2} \Rightarrow \alpha <$$

$$\alpha < e^2$$

$$\longrightarrow \quad \underline{3+\ln x}-\alpha x$$



$$\frac{1}{\ }$$

$\ln 1$

$e^x = 1$

$\ln e^2 = 2$

0.8