

$$f(x,y) = x^2 + xy + \frac{y^2}{2} + 2x \quad \boxed{\text{Hesse}}$$

$$\nabla f(x,y) = \begin{bmatrix} 2x+y+2 \\ x+y \end{bmatrix} \stackrel{!}{=} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \left(\frac{1}{x}\right) \quad \nabla_x$$

$$x+y=0 \Rightarrow x=-y$$

$$\begin{array}{c|c} x & y \\ \hline x & y \end{array}$$

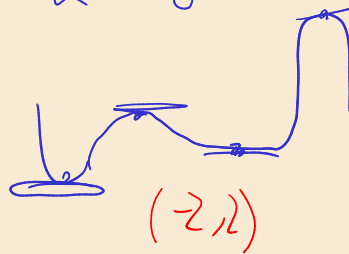
$$2x+y=-2$$

$$x=-2$$

$$-2y+y=-2 \Rightarrow y=2$$

$$(-2, 2) \text{ ungerade}$$

$$\nabla^2 f(x,y) = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$



$$\det(H) = (2-1) - 1^2 = 1 > 0$$

$$2 > 0$$

\Rightarrow 2 Punkte
ungerade

$$f''(x) > 0 \Rightarrow x$$

$\begin{bmatrix} > \end{bmatrix} \rightarrow + \rightarrow$
 $\rightarrow \min$

$\geq \rightarrow - \rightarrow$
 $\rightarrow \max$
 $< \rightarrow \min$

$$f(x,y) = e^{2x+y} - x^2 - 2y$$

$$\nabla f(x,y) = \begin{bmatrix} 2e^{2x+y} - 2x \\ e^{2x+y} - 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{c|c} 4e^{2x+y} - 2 & 2e^{2x+y} \\ \hline 2e^{2x+y} & e^{2x+y} \end{array} \right]$$

$\begin{matrix} \nearrow \rightarrow x \cdot y & y \cdot x \\ \nearrow \rightarrow \end{matrix}$

\rightarrow saddle \Rightarrow

$$\boxed{x=2}$$

$$\begin{bmatrix} 4 \cdot 2 - 2 & 2 \cdot 2 \\ 2 \cdot 2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 4 & 2 \end{bmatrix} \cdot e^{4+y} = 2 \begin{pmatrix} 2 \\ \ln 2 - 4 \end{pmatrix}$$

$$12 - 16 = -4 < 0$$

$$6 > 0$$

$$\ln_2 b = c \Rightarrow b = a^c \rightarrow 4+y = \ln 2$$

$$\boxed{y = \ln 2 - 4}$$

$$\ln e^x = x \quad e^{4+y} = e^{\ln 2 - 4}$$

$$3xy - x^2 - y^2$$

$$\nabla f = \begin{bmatrix} 3y - 2x \\ 3x - 2y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix} \Rightarrow \begin{cases} 3y = 2x \\ -2y = 3x \end{cases} \begin{matrix} 2x \cdot 1.5 = 3x \\ 1.5 = 1.5 \end{matrix}$$

$$4.5y = 3x$$

$$3x = 2y$$

$$4.5y = 2y$$

$$4.5y - 2y = 0$$

$$2.5y = 0$$

$$y = 0$$

$$x = 0$$

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$$

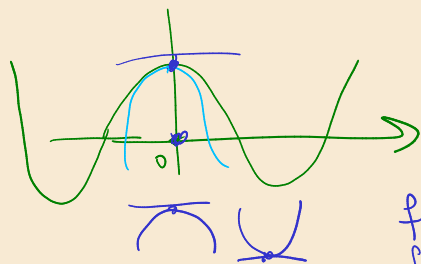
$$f_{xx} \cdot f_{yy} - f_{xy}^2 > 0$$

$$\begin{matrix} f_{xx} > 0 \\ \swarrow \text{sub} \\ \searrow \end{matrix} \begin{matrix} \text{local min} \\ \text{local max} \end{matrix}$$

$$\begin{matrix} -1 & 3 \\ -2 & -1 \end{matrix} \begin{matrix} 1-7 < 0 \end{matrix} \rightarrow \text{sub}$$

$$D < 0 \rightarrow \text{sub}$$

$$f(x) \approx P(x)$$



$$\begin{aligned} f(0) &= P(0) \\ f'(0) &= P'(0) \\ f''(0) &= P''(0) \end{aligned}$$



$$P(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 +$$



$$1, \dots + \frac{f'''(0)}{3!}x^3$$

$$\frac{f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2}{e^x}, a=0$$

$$e^x \approx 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3$$

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} = \frac{1}{2}(1+x)^{-\frac{1}{2}}$$

$$P(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{2\sqrt{1+x}}$$

$$\frac{f'(a)(x-a)}{\left(\frac{1}{2}(1+x)^{-\frac{1}{2}}\right)'} =$$

$$\frac{\frac{1}{2!} - \frac{1}{4}x^2}{\frac{1}{2}(1+x)^{-\frac{1}{2}}} =$$

$$\frac{\frac{1}{3!} - \frac{1}{8}x^3}{\frac{1}{2} \cdot -\frac{1}{2}(1+x)^{-\frac{3}{2}}} =$$

$$\frac{\frac{1}{12}x - \frac{1}{8}x^3}{\frac{1}{16}x^3} = \frac{\frac{1}{2} \cdot -\frac{1}{2} \cdot -\frac{3}{2}(1+x)^{-\frac{5}{2}}}{\frac{1}{8}(1+x)^{-\frac{5}{2}}}$$