

$$f(x,y) := x^2 + xy + \frac{y^2}{2} + 2x$$

$$\nabla f(x,y) = \begin{bmatrix} \cancel{2x+y+2} \\ \cancel{x+y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x+y=0 \Rightarrow x=-y$$

$$\frac{\partial f}{\partial x} = x$$

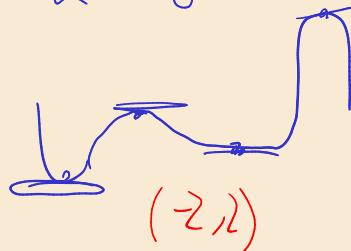
$$2x+y = -2$$

$$x = -2$$

$$-2y+y = -2 \Rightarrow y=2$$

$(-2, 2)$ ungewöhnlich

$$\nabla^2 f(x,y) = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$



$$\det(H) := (2-1) \cdot 1^2 = 1 > 0$$

$$2 > 0 \Rightarrow \text{global minimum}$$

$$\begin{bmatrix} > \\ > \end{bmatrix} \rightarrow \begin{array}{l} \rightarrow \\ \rightarrow \end{array}$$

$$f''(x) > 0 \Rightarrow x$$

$$\begin{array}{c} \geq \rightarrow \leftarrow \rightarrow \\ \searrow \nearrow \end{array}$$

$$f(x,y) = e^{2x+y} - x^2 - 2y$$

$$\nabla f(x,y) = \begin{bmatrix} 2e^{2x+y} - 2x \\ e^{2x+y} - 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4e^{2x+y} - 2 & 2e^{2x+y} \\ 2e^{2x+y} & e^{2x+y} \end{bmatrix} \xrightarrow{\text{Solve system}} \begin{cases} x=2 \\ y=\ln 2 \end{cases}$$

$$\begin{bmatrix} 4 \cdot 2 - 2 & 2 \cdot 2 \\ 2 \cdot 2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 4 & 2 \end{bmatrix} \cdot e^{4+\ln 2} = 2^2 \cdot \ln 2$$

$\Delta = 12 - 16 = -4 < 0$

$b > 0$

$$\ln_2 b = c \Rightarrow b = a^c \Rightarrow 4 + \ln 2 = 2^c$$

$$g = \ln 2 - 4$$

$$\ln e^x = x \quad e^{4+\ln 2} = e^4$$

$$1 \quad 3xy - x^2y \sim$$

$$\nabla f = \begin{bmatrix} 3y^2 - 2x \\ 3x^2 - 2y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix} \geq \begin{bmatrix} 3y = 2x \\ 3x = 2y \end{bmatrix} \quad \begin{array}{l} 2x + 3x = 15 \\ 5x = 15 \\ x = 3 \end{array}$$

$4 - 9 = -5 < 0$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ saddle point

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

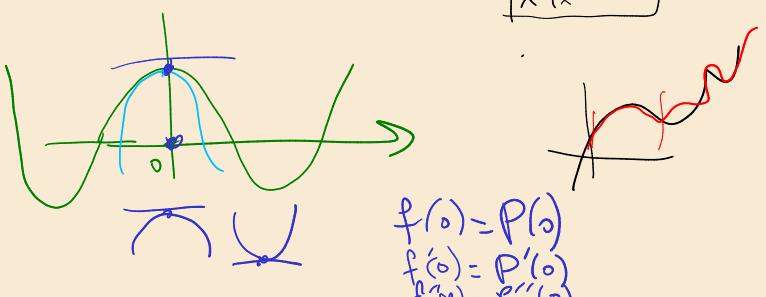
$$f_{xx} \cdot f_{yy} - f_{xy} f_{yx} > 0$$

$$\begin{array}{c} f_{xx} > 0 \\ x=1 \\ \text{sunk} \end{array} \quad \begin{array}{c} f_{yy} > 0 \\ y=1 \\ \text{win} \end{array}$$

$$\begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix} \quad 1 - 2 > 0$$

$$D < 0 \rightarrow \text{mbo}$$

$$f(x) \approx P(x)$$



$$\begin{aligned} f(0) &= P(0) \\ f'(0) &= P'(0) \\ f''(0) &= P''(0) \end{aligned}$$

$$P(x) = f(0) + \underline{f'(0)x} + \frac{f''(0)}{2!}x^2 +$$

$$\cancel{\frac{1}{2!}x^2} + \dots + \frac{f'''(0)}{3!}x^3$$

$$\frac{f(a)}{e^x}, a=0 + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 \dots$$

$$e^x \approx \underbrace{1+x+\frac{1}{2!}(x-\cancel{x})^2}_{\cancel{x}} + \frac{x^3}{3!}$$

$$\sqrt{1+x} \quad (1+x)^{\frac{1}{2}} = \boxed{\frac{1}{2}(1+x)^{-\frac{1}{2}}}$$

$$P(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{2}\sqrt{1+x}$$

$$\boxed{f'(a)(x-a)} = \left(\frac{1}{2}(1+x)^{-\frac{1}{2}} \right)' =$$

$$\boxed{\frac{1}{2!} - \frac{1}{4}x^2} = \frac{1}{2}(1+x)^{-\frac{1}{2}}$$

$$\frac{1}{3!} \cdot \frac{1}{8}x^3 = \frac{1}{2} \cdot -\frac{1}{2}(1+x)^{-\frac{3}{2}}$$

$$\frac{1}{1 \cdot 2 \cdot 3} \frac{3}{8}x^3 = \frac{1}{2} \cdot -\frac{1}{2} \cdot -\frac{3}{2}(1+x)^{-\frac{5}{2}}$$

$$\frac{1}{16}x^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{8}(1+x)^{-\frac{5}{2}}$$