

Small Area Estimation—New Developments and Directions

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Summary

The purpose of this paper is to provide a critical review of the main advances in small area estimation (SAE) methods in recent years. We also discuss some of the earlier developments, which serve as a necessary background for the new studies. The review focuses on model dependent methods with special emphasis on point prediction of the target area quantities, and mean square error assessments. The new models considered are models used for discrete measurements, time series models and models that arise under informative sampling. The possible gains from modeling the correlations among small area random effects used to represent the unexplained variation of the small area target quantities are examined. For review and appraisal of the earlier methods used for SAE, see Ghosh & Rao (1994).

Key words: Best linear unbiased prediction; Cross-sectional correlations; Empirical Bayes; Hierarchical Bayes; Informative sampling; Mixed models; Time series models.

1 Introduction

Small area estimation (SAE) is a topic of great importance due to the growing demand for reliable small area statistics even when only very small samples are available for these areas. Over the years, many statistical agencies have introduced vigorous programs to meet this demand. Extensive research on the theoretical and practical aspects of SAE is carried out and many international conferences and workshops are held in order to share the results of this research effort. Interest in small area estimation methods has further enhanced in recent years due to the tendency of many European countries to base future censuses on administrative record systems. Recognizing the inaccuracies of the administrative data and the fact that even the richest records cannot cover all the detailed information required for small census tracts, the idea is to test, correct and supplement the administrative information by sample data.

The problem of SAE is twofold. First is the fundamental question of how to produce reliable estimates of characteristics of interest, (means, counts, quantiles, etc.) for small areas or domains, based on very small samples taken from these areas. The second related question is how to assess the estimation error. Note in this respect that except in rare cases, sampling designs and in particular sample sizes are chosen in practice so as to provide reliable estimates for aggregates of the small areas such as large geographical regions or broad demographic groups. Budget and other constraints usually prevent the allocation of sufficiently large samples to each of the small areas. Also, it is often the case that domains of interest are only specified after the survey has already been designed and carried out. Having only a small sample (and possibly an empty sample) in a given area, the only possible solution to the estimation problem is to *borrow information* from other related data sets. Potential data sources can be divided into two broad categories:

- Data measured for the characteristics of interest in other 'similar' areas,
- Data measured for the characteristics of interest on previous occasions.

The methods used for SAE can be divided accordingly by the related data sources that they employ, whether cross-sectional (from other areas), past data or both. A further division classifies the methods by the type of inference: 'design based', 'model dependent' (with sub-division into the frequentist and Bayesian approaches), or the combination of the two. In what follows I describe briefly two actual applications of SAE that illustrate what the areas or domains might be, the quantities of interest and the kind of concomitant variables that are used for estimation. Other examples are mentioned in subsequent sections.

A—Estimation of illicit drug use—Over the last six years, the Research Triangle Institute (RTI) in the U.S. is engaged in producing small area estimates of drug use for all the 50 states and the District of Columbia. Estimates of rates of use (prevalence), numbers of users and other measures related to the use of illicit drugs, alcohol and tobacco are derived for 32 demographic cells within 300 regions (sub-states), based on the National Household Survey on Drug Abuse (NHSDA). The cells are defined by age, race and sex. The total sample size of the 1999 survey is about 67500 persons, with an average of 75 persons per age group. The estimates are derived by use of logistic models that contain 12–25 regressor variables with fixed coefficients and state and sub-state random effects with age group specific elements as the independent variables (see Section 4). The regressor variables comprise person and block group level demographic variables, census tract level demographic and socio-economic status variables, and county level rates of drug-related arrests and deaths. The auxiliary information is available from various administrative records. For a detailed description of the NHSDA and the other data sources, the modeling process and estimates released, see the web site: <http://www.samhsa.gov/oas/NHSDA/1999/Table%20of%20Contents.htm>. The article by Folsom *et al.* (1999) expands on methodological issues.

B—Estimation of Small Area Employment—The Bureau of Labor Statistics (BLS) in the U.S. is running a monthly survey of businesses within states, called the Current Employment Statistics (CES) survey (also known as the payroll survey). The survey is designed to produce monthly estimates of employment for major industry divisions within states and large metropolitan statistical areas. Monthly estimates for major industry divisions are, however, desired also for about 320 local labor market areas, defining a total of over than 2500 nonempty small domains, with many of the domains having only 10 or fewer responding units. The direct estimates obtained from the CES are therefore very erratic. Administrative data of total employment within the same domains are obtained from state unemployment insurance reports, collected for virtually all the businesses, but these reports, known as the ES202 data become available only with a time lag of 6–12 months. Recent studies (Harter *et al.*, 1999) suggest that the direct small domain CES estimates can be improved upon by estimates of the form,

$$\text{employment in CES sample} + \text{non-sample predicted employment} + \text{"add-ons"}.$$

The first component of this estimate is the total measured employment for businesses in the CES sample in the month of investigation. The second component is a Ratio predictor of total employment in non-sample businesses of the form, $\hat{E}_{\bar{s}} = X_{\bar{s}} \hat{R}_I$ where $X_{\bar{s}}$ denotes the small domain ES202 benchmark employment for non-sample businesses and \hat{R}_I is the ratio between the state-wide employment estimated from the CES sample and the state-wide employment obtained from the ES202 data in the corresponding industry. This predictor is an example of a 'synthetic estimate' (see Section 2) in that the ratio between the CES and ES202 estimates at the state level is applied to the ES202 data locally. The supplemental "add-ons" adjust for employment that cannot be assigned to an industry division through specific firms. Enhancements to these small domain estimates are

currently investigated.

The purpose of this paper is to discuss some of the recent developments in SAE. The main emphasis is on general methodological issues rather than on detailed technical solutions. For these, as well as some other recent applications of SAE, the reader is referred to the new review article by Rao (1999), which is an update of the more extensive review of Ghosh & Rao (1994). Another recent review of SAE methods is the article by Marker (1999).

Section 2 illustrates the important role of administrative information for SAE and introduces the family of synthetic regression estimators. Section 3 reviews several cross-sectional models in common use for continuous measurements. These models are extended to handle discrete measurements in Section 4 and to account for time series relationships in Section 5. The analysis in Sections 3–5 assumes implicitly that the sampling process can be ignored for inference. Section 6 considers the case of informative sampling under which the sample data no longer represent the population model. Section 7 examines the importance of modeling the correlations among the random area effects. I conclude with brief remarks in Section 8.

2 The Importance of Concomitant Administrative Data, Synthetic Estimation

In this and the next three sections I assume for convenience that the sample is selected by simple random sampling without replacement. The possible implications of the use of complex sampling schemes with unequal selection probabilities are discussed in Section 6. Let y define the characteristic of interest and denote by y_{ij} the outcome value for unit j belonging to area i , $i = 1, \dots, m$; $j = 1, \dots, N_i$, where N_i is the area size. Let $s = s_1 \cup \dots \cup s_m$ signify the sample where s_i of size n_i defines the sample observed for area i . Notice that the n_i 's are random unless a separate sample with fixed sample size is taken in every area. Suppose that the objective is to estimate the true area mean $\bar{Y}_i = \sum_{j=1}^{N_i} y_{ij}/N_i$. If no auxiliary information is available, the ordinary *direct* design unbiased estimator and its design variance over the *randomization distribution* (the distribution induced by the random selection of the sample with the population values held fixed), conditional on the realized sample sizes are given by

$$\bar{y}_i = \sum_{j=1}^{n_i} y_{ij}/n_i ; \quad V_D[\bar{y}_i|n_i] = (\tilde{S}_i^2/n_i)[1 - (n_i/N_i)] = S_i^{*2} \quad (2.1)$$

where $\tilde{S}_i^2 = \sum_{j=1}^{N_i} (y_{ij} - \bar{Y}_i)^2/(N_i - 1)$. Clearly, for small n_i the variance will be large unless the variability of the y -values is sufficiently small. Suppose, however, that in addition to measuring y , values x_{ij} of p concomitant variables $x(1), \dots, x(p)$ are also known for each of the sample units and that the row area means $\bar{X}_i = \sum_{j=1}^{N_i} x_{ij}/N_i$ are likewise known. Such information may be obtained from a recent census or some other administrative records, see the examples in the introduction. In this case, a more efficient design unbiased estimator is the regression estimator,

$$\bar{y}_{reg,i} = \bar{y}_i + (\bar{X}_i - \bar{x}_i)' \beta_i ; \quad V_D(\bar{y}_{reg,i}|n_i) = S_i^{*2}(1 - R_i^2) \quad (2.2)$$

where $\bar{x}_i = \sum_{j=1}^{n_i} x_{ij}/n_i$, and β_i and R_i are correspondingly the vector of regression coefficients and the multiple correlation coefficient between y and $x(1), \dots, x(p)$ computed from all the N_i measurements in area i . Thus, by use of the concomitant variables, the variance is reduced by the factor $(1 - R_i^2)$, illustrating very clearly the importance of using auxiliary information with good prediction power for SAE. Other well-known uses of auxiliary information for direct estimation are the Ratio estimator and Poststratification.

The problem with the use of $\bar{y}_{reg,i}$ is that in practice the coefficients β_i are seldom known. Replacing β_i by its ordinary least square estimator from the sample s_i is not effective because of the small sample size. If, however, the β_i 's are known to be 'similar' across the small areas and likewise for the 'intercepts' ($\bar{Y}_i - \bar{X}_i' \beta_i$), a more stable estimator is the *synthetic regression* estimator

$\bar{y}_{reg,i}^{syn} = (\bar{y} - \bar{x}'b) + \bar{X}'_i b$, where $(\bar{y}, \bar{x}') = \sum_{i=1}^m \sum_{j=1}^{n_i} (y_{ij}, x'_{ij}) / \sum_{i=1}^m n_i$ are the global sample means and $b = [\sum_{i=1}^m \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)(x_{ij} - \bar{x}_i)']^{-1} \sum_{i=1}^m \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)(y_{ij} - \bar{y}_i)$ is the pooled regression estimator computed likewise from all the samples s_i . In the special case of a single concomitant variable and 'zero intercepts', $\bar{y}_{reg,i}^{syn}$ is replaced by the *synthetic Ratio* estimator $\bar{y}_{ratio,i}^{syn} = \bar{X}_i \bar{y} / \bar{x}$.

This is basically the ratio predictor $\hat{E}_{\tilde{r}}$ used in Example B of the introduction.

The term "synthetic" refers to the fact that an estimator computed from a large domain is used for each of the separate areas comprising that domain, assuming that the areas are 'homogeneous' with respect to the quantity that is estimated. Thus, synthetic estimators already borrow information from other 'similar areas'. Another, even simpler example is the use of the global mean \bar{y} for estimating each of the small area means when no auxiliary information is available. The article by Marker (1999) contains a thorough discussion of synthetic estimation with many examples. Ghosh & Rao (1994) provide model-based justifications for some of the synthetic estimators in common use.

The prominent advantage of synthetic estimation is the potential for substantial variance reduction but it can lead to severe biases if the assumption of homogeneity within the larger domain is violated. For example, the (unconditional) design bias of the synthetic regression estimator is approximately $Bias_D(\bar{y}_{reg,i}^{syn}) \cong \bar{Y} - \bar{X}B - (\bar{Y}_i - \bar{X}'_i B)$, where \bar{Y} and \bar{X} are the true large domain means of y and x and B is the corresponding vector of regression coefficients. The bias can be large unless the intercept and slope coefficients are similar across the areas. Note again that the design bias is computed with respect to the randomization distribution (repeated sampling). Bias reduction under this distribution (but at the expense of increased variance) can be achieved by the use of *composite estimators*. A composite estimator is a weighted sum of the area direct estimator (has small or no bias but large variance) and the synthetic estimator (has small variance but possibly large bias). Thus, denoting more generally by θ_i the small area characteristic of interest, a composite estimator has the general form,

$$\hat{\theta}_{com,i} = w_i \tilde{\theta}_i + (1 - w_i) \hat{\theta}_i^{syn} \quad (2.3)$$

where $\tilde{\theta}_i$ is the direct estimator and $\hat{\theta}_i^{syn}$ is the synthetic estimator. The question underlying the use of composite estimators is the choice of the weights w_i . Ideally, the weights should be selected so as to minimize the mean square error (MSE) but this is problematic since the MSE of the synthetic estimator is generally unknown because of its bias. One simple alternative therefore is to set $w_i = f_i = (n_i / N_i)$ so that more weight is given to the direct unbiased estimator as the sampling fraction f_i increases. However, the sampling fractions are ordinarily very small and hence the use of this weight practically implies the use of the synthetic estimator, irrespective of the variability of the outcome variable. Other plausible specifications of the weights are discussed in Ghosh & Rao (1994), Thomsen & Holmøy (1998) and Marker (1999). Optimal choices of the weights under mixed linear models are considered in Section 3.

A common feature of the estimators considered in this section is that they are 'model free' in the sense that no explicit model assumptions are used for their derivation, and the variance and bias are computed with respect to the randomization distribution. The article by Marker (1999) contains a historical survey of design-based estimators with many references. In the rest of the paper I consider model dependent estimators.

3 Cross-sectional Models for Continuous Measurements

SAE is widely recognized as one of the few problems in survey sampling where the use of models is often inevitable. The specification of an appropriate working model permits the construction of correspondingly efficient estimators and the computation of variances and confidence intervals, which may not be feasible under the randomization distribution. (With the very small sample sizes often encountered in practice, large sample normal theory does not apply.) The models reviewed in

this and the next sections are ‘mixed effects models’, containing fixed and random effects. Some special features of the application of these models for SAE problems are summarized at the end of the section.

One of the simplest models in common use is the ‘nested error unit level regression model’, employed originally by Battese *et al.* (1988) for predicting areas under corn and soybeans in 12 counties of the state of Iowa in the U.S. Suppose that the values of concomitant variables $x(1), \dots, x(p)$ are known for every unit in the sample and that the true area means of these variables are also known. Denoting by x_{ij} the concomitant values for unit j in area i , the model has the form,

$$y_{ij} = x'_{ij}\beta + u_i + \varepsilon_{ij} \quad (3.1)$$

where u_i and ε_{ij} are mutually independent error terms with zero means and variances σ_u^2 and σ_ε^2 respectively. The random term u_i represents the joint effect of area characteristics that are not accounted for by the concomitant variables. (In Battese *et al.*, 1988 y_{ij} is the reported number of hectares of corn (or soybeans) in sample segment j of county i and $x'_{ij} = (x_{1ij}, x_{2ij})$ denotes the numbers of pixels classified as corn and soybeans from satellite pictures. The satellite information is known for both the sample and nonsample segments.) Under the model the true small area means are $\bar{Y}_i = \bar{X}'_i\beta + u_i + \bar{\varepsilon}_i$ but since $\bar{\varepsilon}_i = \sum_{j=1}^{N_i} \varepsilon_{ij} / N_i \cong 0$ for large N_i , the target parameters are ordinarily defined as $\theta_i = \bar{X}'_i\beta + u_i$. For known variances ($\sigma_u^2, \sigma_\varepsilon^2$), the Best Linear Unbiased Predictor (BLUP) of θ_i under the model is,

$$\hat{\theta}_i = \gamma_i \left[\bar{y}_i + (\bar{X}_i - \bar{x}_i)' \hat{\beta}_{GLS} \right] + (1 - \gamma_i) \bar{X}'_i \hat{\beta}_{GLS} \quad (3.2)$$

where $\hat{\beta}_{GLS}$ is the (optimal) Generalized Least Square (GLS) estimator of β computed from all the observed data and $\gamma_i = \sigma_u^2 / (\sigma_u^2 + \sigma_\varepsilon^2 / n_i)$. For areas k with no samples, $\hat{\theta}_k = \bar{X}'_k \hat{\beta}_{GLS}$. The coefficient γ_i is a “shrinkage factor” providing a trade-off between the (usually large) variance of the regression predictor $\bar{y}_i + (\bar{X}_i - \bar{x}_i)' \hat{\beta}_{GLS}$, and the bias of the synthetic estimator $\bar{X}'_i \hat{\beta}_{GLS}$ for given value θ_i . (The synthetic estimator and hence the BLUP are biased when conditioning on θ_i or equivalently on u_i . The two predictors are unbiased unconditionally under the model since $E(\bar{Y}_i) = \bar{X}'_i\beta$.) The predictor $\hat{\theta}_i$ has the structure of the composite estimator $\hat{\theta}_{com,i}$ defined by (2.3). However, the weight γ_i is chosen in an optimal way under the model so that it accounts for the magnitude of the differences between the area effects u_i . Thomsen (*in* Ghosh & Rao, 1994), and Thomsen & Holmøy (1998) comment that predictors of the form (3.2) tend to over-estimate area means with small random effects and under-estimate area means with large effects such that the variation between the predictors is smaller than the variation between the true means. This is clear considering the structure of these predictors but it raises the question of the appropriate loss function in a particular application.

The BLUP $\hat{\theta}_i$ is also the Bayesian predictor (posterior mean) under normality of the error terms and diffuse prior for β . In practice, however, the variances σ_u^2 and σ_ε^2 are seldom known. A common procedure is to replace them in the BLUP formula by standard variance components estimates like Maximum Likelihood Estimators (MLE), Restricted MLE (REML) or Analysis of Variance (ANOVA) estimators. The resulting predictors are known as the Empirical BLUP (EBLUP) or Empirical Bayes (EB) predictors, see, for example, Prasad & Rao (1990) for details. Alternatively, Hierarchical Bayes (HB) predictors can be developed by specifying prior distributions for β and the two variances and computing the posterior distribution $f(\theta_i | y, X)$ given all the observations in all the areas, see, e.g., Datta & Ghosh (1991). The actual application of this approach can be quite complicated since the posterior mean, $E(\theta_i | y, X)$ has generally no close form. Recent studies use the Gibbs sampler (Gelfand & Smith, 1990) or other Markov Chain Monte Carlo (MCMC) techniques for stochastic simulation. The HB approach is very general and also very appealing since it produces the posterior variances associated with the point predictors (see below), but it requires in addition to the specification of the prior distributions good computing skills and intensive computations.

A somewhat different model from (3.1) discussed extensively in the literature is the ‘area level random effects model’, which is used when the concomitant information is only at the area level. Let x_i represent this information. The model, used originally by Fay & Herriot (1979) for the prediction of mean per capita income (PCI) in small geographical areas within counties (less than 500 persons) is defined as,

$$\tilde{\theta}_i = \theta_i + e_i ; \quad \theta_i = x_i' \beta + u_i \quad (3.3)$$

where $\tilde{\theta}_i$ denotes the direct sample estimator (for example, the sample mean \bar{y}_i), so that e_i represents in this case the sampling error, assumed to have zero mean (see comment below) and known design variance $Var_D(e_i) = \sigma_{D_i}^2$, (S_i^{*2} if $\tilde{\theta}_i = \bar{y}_i$, equation 2.1). The model (3.3) integrates therefore a model dependent random effect u_i and a sampling error e_i with the two errors being independent. (In Fay & Herriot $\tilde{\theta}_i$ is the direct sample estimate of mean PCI in ‘local government unit’ i and x_i contains data on average county PCI, tax returns and values of housing. All the variables are measured in the log scale.) The BLUP under this model is,

$$\hat{\theta}_i = \gamma_i \tilde{\theta}_i + (1 - \gamma_i) x_i' \hat{\beta}_{GLS} = x_i' \hat{\beta}_{GLS} + \gamma_i (\tilde{\theta}_i - x_i' \hat{\beta}_{GLS}) \quad (3.4)$$

which again is a composite estimator with weight $\gamma_i = \sigma_u^2 / (\sigma_{D_i}^2 + \sigma_u^2)$. In practice, the variances σ_u^2 and $\sigma_{D_i}^2$ are usually unknown and they are replaced by sample estimates, yielding in turn the corresponding EBLUP predictors. Arora & Lahiri (1997) show that if the variances $\sigma_{D_i}^2$ are considered random with a non-degenerate prior, then for known β and σ_u^2 the Bayesian predictor has a smaller MSE than the corresponding BLUP.

Comment: The target area quantity θ_i is often a nonlinear function of the area mean \bar{Y}_i so that the direct estimator $\tilde{\theta}_i$ is a nonlinear function of the sample mean \bar{y}_i . For example, Fay & Herriot (1979) use the log transformation for estimating area per capita incomes. The problem arising in the case of nonlinear transformations is that the assumption $E_D(e_i) = 0$ (design unbiasedness of the direct estimator) may not hold even approximately if the sample sizes are too small, requiring instead the use of Generalized Linear Models (see Section 4).

As defined in Section 1, an important aspect of SAE is the assessment of the prediction errors. This problem does not exist in principle under the full Bayesian paradigm, which produces the posterior variances of the target quantities around the HB predictors (the posterior means). However, as already stated, the implementation of this approach requires the specification of prior distributions and the computations can become very intensive. Assessment of the prediction errors under the EBLUP and EB approaches is also complicated because of the added variability induced by the estimation of the model parameters. To illustrate the problem, consider the model defined by (3.3) and suppose that the design variances $\sigma_{D_i}^2$ are known. (Stable variance estimators are often calculated as $\hat{\sigma}_{D_i}^2 = \hat{\sigma}_D^2 / n_i$ with $\hat{\sigma}_D^2$ computed from all the data or obtained from other sources. The estimators $\hat{\sigma}_{D_i}^2$ are treated as the true variances.) If β and σ_u^2 were also known, the variance of the BLUP (or HB under normality assumptions) is, $Var[\hat{\theta}_i(\sigma_u^2, \beta)] = \gamma_i \sigma_{D_i}^2 = g_{1i}$. Under the EBLUP and EB approaches, β and σ_u^2 are replaced by sample estimates and a naïve variance estimator is obtained by replacing σ_u^2 by $\hat{\sigma}_u^2$ in g_{1i} . This estimator ignores the variability of $\hat{\sigma}_u^2$ and hence underestimates the true variance. Prasad & Rao (1990), extending the work of Kackar & Harville (1984) approximate the true prediction MSE of the EBLUP under normality of the two error terms and for the case where σ_u^2 is estimated by the ANOVA (fitting of constants) method as,

$$MSE [\hat{\theta}_i(\hat{\sigma}_u^2, \hat{\beta})] = E [\hat{\theta}_i(\hat{\sigma}_u^2, \hat{\beta}) - \theta_i]^2 = g_{1i} + g_{2i} + g_{3i} \times Var(\hat{\sigma}_u^2) \quad (3.5)$$

where $g_{2i} = (1 - \gamma_i)^2 x_i' \text{Var}(\hat{\beta}_{GLS}) x_i$ and $g_{3i} = [\sigma_{Di}^4 / (\sigma_{Di}^2 + \sigma_u^2)^3]$. The term g_{2i} is the excess in MSE due to estimation of β and $g_{3i} \times \text{Var}(\hat{\sigma}_u^2)$ is the excess in MSE due to estimation of σ_u^2 . The neglected term in the approximation is of order $o(1/m)$ with m denoting the number of sampled areas. Note that the leading term in (3.5) is $g_{1i} = \gamma_i \sigma_{Di}^2$ such that for large m and small values γ_i , $MSE[\hat{\theta}_i(\hat{\sigma}_u^2, \hat{\beta})] \ll \sigma_{Di}^2 = \text{Var}_D(\tilde{\theta}_i)$, illustrating the possible gains from using the model dependent predictor. Building on the approximation (3.5), Prasad & Rao (1990) develop a MSE estimator with bias of order $o(1/m)$ as,

$$\widehat{MSE}[\hat{\theta}_i(\hat{\sigma}_u^2, \hat{\beta})] = g_{1i}(\hat{\sigma}_u^2) + g_{2i}(\hat{\sigma}_u^2) + 2g_{3i}(\hat{\sigma}_u^2) \times \widehat{Var}(\hat{\sigma}_u^2) \quad (3.6)$$

where $g_{ki}(\hat{\sigma}_u^2)$ is obtained from g_{ki} by substituting $\hat{\sigma}_u^2$ for σ_u^2 , $k = 1, 2, 3$. A similar estimator is developed for the nested error regression model (3.1). Lahiri & Rao (1995) show that the estimator (3.6) is robust to departures from normality of the random area effects u_i (but not the sampling errors e_i).

Datta & Lahiri (2000) extend the results of Prasad and Rao to other variance components estimators and general linear mixed models of the form,

$$Y_i = X_i \beta + Z_i u_i + e_i = \xi_i + e_i, \quad i = 1, \dots, m. \quad (3.7)$$

In (3.7), Y_i is a vector observation of order n_i , β is a vector of fixed coefficients, Z_i is a fixed matrix of order $n_i \times d$ and u_i and e_i are independent vector random effects and residual terms of orders d and n_i respectively. It is further assumed that $u_i \sim N_d(0, G_i)$, $e_i \sim N_{n_i}(0, \Sigma_i)$, with G_i and Σ_i being functions of some vector parameter λ . The model contains as special cases familiar models used for SAE like the models defined by (3.1) and (3.3), the model considered by Moura & Holt (1999) that permits the regression coefficients in (3.1) to depend stochastically on area level auxiliary variables, and the two-folded nested error regression model studied by Stukel & Rao (1999). Explicit expressions of the BLUP (or posterior means) of linear combinations of the fixed and random effects and their variances (posterior variances) are derived for the case of known λ using results from Henderson (1975). The authors develop MSE estimators with bias of order $o(1/m)$ for the EBLUP obtained when estimating λ by MLE or REML.

The MSE approximations discussed so far are under the frequentist approach. The use of the EB approach again requires appropriate measures of errors. Kass & Steffey (1989) develop first and second order EB MSE estimators for a general two-stage hierarchical model by approximating the posterior distribution of the hyper-parameters indexing the second stage model (β and λ in the notation of 3.7) by the normal distribution. Singh *et al.* (1998) study modifications to the Kass–Steffey approximations and propose Monte Carlo alternatives to some of the frequentist and EB measures of error.

Comment: The MSE approximations under both the frequentist and the EB approaches involve bias corrections of desired order and as a result, the use of these approximations improves the coverage properties of standard confidence intervals for the small area quantities. However, as found empirically by Singh *et al.* (1998), the addition of bias corrections can inflate the variance of the MSE approximations to a degree where it offsets the reduction in the bias.

We conclude this section by pointing out the special features of the use of mixed effects models for SAE. The most prominent feature is that in SAE the target parameters are the realized (actual) small area quantities, which renders the problem into a prediction problem. In other familiar applications of these models like in animal breeding or in education, the prime interest is in inference on the fixed regression coefficients and the variance components, which is only an intermediate step in SAE. The fact that the target quantities in SAE are the random area realizations complicates the evaluation of the prediction MSE very substantially since the unknown variance components are replaced by

sample estimates. Indeed, the major developments in the estimation of prediction MSE for mixed effect models over the last decade have originated and evolved in the SAE context.

A second important feature of the application of mixed effects models for SAE is the kind of data available for analysis. On the one hand there often exists a large volume of administrative data at the unit and/or the area level. On the other hand, agencies collecting the data are often reluctant to release the micro data for secondary analysis so that the data available for the response variable consists of only the direct estimators that are subject to sampling errors that need to be modeled. The sampling designs used to select the sample may dictate the use of weighted estimators, which further complicates the modeling process and the evaluation of the prediction errors. This issue is discussed in section 6.

4 Models for Discrete Measurements

Recent research in SAE with some real breakthroughs focuses on situations where the measurements y_{ij} are categorical or discrete and the small area quantities of interest are proportions or counts. In such cases, the mixed linear models considered before are no longer applicable. MacGibbon & Tomberlin (1989) consider the following model for the case of binary measurements,

$$\begin{aligned} \Pr(y_{ij} = 1|p_{ij}) &= p_{ij} ; \Pr(y_{ij} = 0|p_{ij}) = (1 - p_{ij}) \\ \log it(p_{ij}) &= \log[p_{ij}/(1 - p_{ij})] = x'_{ij}\beta + u_i ; u_i \sim N(0, \sigma_u^2). \end{aligned} \quad (4.1)$$

The outcomes y_{ij} are assumed to be conditionally independent and likewise for the random effects u_i . The purpose is to predict the true area proportions $p_i = \sum_{j=1}^{N_i} y_{ij}/N_i$. In an application considered by the authors y_{ij} defines labour force participation for sample individual j in county i and x_{ij} defines 'sex' and 'age'. Assuming a diffuse prior for β and known variance $\sigma_u^2 = \text{Var}(u_i)$, the authors approximate the joint posterior distribution of β and $\{u_i, i = 1, \dots, m\}$ given the data by the multivariate normal distribution with mean equal to the mode of the true posterior and covariance matrix equal to the inverse information matrix evaluated at the mode. Denoting by $\tilde{\beta}$ and \tilde{u}_i the respective modes, p_{ij} is estimated as $\tilde{p}_{ij} = [1 + \exp\{-(x'_{ij}\tilde{\beta} + \tilde{u}_i)\}]^{-1}$ and for small sampling fractions n_i/N_i , $\hat{p}_i = \sum_{j=1}^{N_i} \tilde{p}_{ij}/N_i$. The case of unknown σ_u^2 is treated by repeating the same analysis with σ_u^2 set to its MLE, thus yielding the corresponding EB predictor. A naïve estimator for the variance of the empirical predictor is obtained by use of Taylor expansion but this estimator ignores the variance component resulting from the estimation of σ_u^2 . Farrell, MacGibbon & Tomberlin (1997) show that the naïve variance estimator can be improved by use of a parametric Bootstrap procedure proposed by Laird & Louis (1987).

The model defined by (4.1) has been studied more recently by Jiang & Lahiri (1998). The authors derive the Best Predictor (BP) of $\log it(p_{ij})$ for given values (σ_u^2, β) as $\widehat{\log it(p_{ij})} = x'_{ij}\beta + E(u_i | \sum_{j=1}^{n_i} y_{ij})$, with the conditional expectation obtained as a ratio of two one-dimensional integrals. The estimator has generally no closed form because of the complex integration involved, but for the special case where $x_{ij} = x_i$ for all j and $\sigma_u^2 \rightarrow \infty$, $E(u_i | \sum_{j=1}^{n_i} y_{ij}) \cong [\log it(\bar{y}_i) - x'_i\beta]$. Notice that in this case $\widehat{\log it(p_{ij})} = \log it(\bar{y}_i)$. An Empirical BP (EBP) is obtained by replacing β and σ_u^2 by sample estimates obtained by the *method of moments*. An approximation to the MSE of the EBP with bias of order $o(1/m)$ is developed.

Malec *et al.* (1997) likewise consider Bernoulli outcomes imposing the same probabilities of 'ones' for all individuals j in area i belonging to the same socio-economic/demographic class k and the same 'cluster' c (homogeneous geographic unit). Specifically, the model assumes,

$$\Pr(y_{ijk} = 1) = p_{kc} ; \log it(p_{kc}) = x'_k\beta_c, \beta_c \sim N(z_c\eta, \Gamma) \quad (4.2)$$

where x_k is a vector of regressor values defining the k -th class and z_c represents cluster level covariates. The random coefficients β_c are independent between the clusters. The authors compare several predictors of the totals $\theta_i = \sum_c \sum_k \sum_{j=1}^{N_{ikc}} y_{ijkc}$ where N_{ikc} is the number of individuals in class k belonging to cluster c of area i . Model dependent predictors are derived by evaluation of the conditional expectations $E(p_{kc}|y, x)$. For application of the Hierarchical Bayes (HB) approach, the hyper-parameters (η, Γ) are assigned the improper prior $p(\eta, \Gamma) = \text{constant}$. The computation of the HB predictors is carried out by use of the Gibbs sampler. The authors consider also several ‘time saving’ approximations to the HB predictors, obtained by replacing some of the conditional densities underlying the use of the Gibbs sampler by normal densities. Empirical Bayes (EB) predictors are obtained by computing (numerically) the posterior expectations $E(p_{kc}|y, \eta, \Gamma)$ with (η, Γ) assumed known (set at their MLE). Other estimators considered are ‘synthetic’ estimators obtained by setting $\beta_c \equiv z_c \eta$ ($\Gamma = 0$) and $p(\eta) = \text{constant}$ in the equation defining the HB point estimator, and *direct* design based estimators. Empirical results based on simulated and real data indicate that unless the sample sizes within the areas are sufficiently large, the exact HB predictors dominate the other estimators both with respect to point estimation and variance measures. It should be noted, however, that no bias corrections that account for parameter estimation have been applied when estimating the variances of the EB estimators. As noted before, the HB approach ‘automatically’ accounts for all sources of error.

Ghosh *et al.* (1998) develop a general methodology for HB inference under the Generalized Linear Model (GLM) with random effects that includes the model (4.1) as a special case. The GLM (McCullagh & Nelder, 1989) is defined as,

$$f(y_{ij}|\theta_{ij}, \phi_{ij}) = \exp \{ [(y_{ij}\theta_{ij} - b(\theta_{ij})) / \phi_{ij}] + a(\phi_{ij}) + c(y_{ij}, \phi_{ij}) \} \quad (4.3)$$

where $a(\cdot) > 0$, $b(\cdot)$ and $c(\cdot)$ are known functions. As easily verified, $E(y_{ij}) = \partial b(\theta_{ij}) / \partial \theta_{ij}$; $\text{Var}(y_{ij}) = \{\partial^2 [b(\theta_{ij})] / \partial \theta_{ij}^2\} \phi_{ij}$. The scale parameters ϕ_{ij} are assumed known. In order to borrow information across the areas, the ‘natural parameters’ θ_{ij} are modeled as $h(\theta_{ij}) = x'_{ij}\beta + u_i + \varepsilon_{ij}$ where $h(\cdot)$ is a strictly increasing *link function* with u_i and ε_{ij} playing a similar role as in the mixed linear model. The class of HB models studied by Ghosh *et al.* (1998) consists of the following equations (see below for an application):

$$\begin{aligned} y_{ij}|\theta_{ij}, \beta, u_i, \sigma_u^2, \sigma_\varepsilon^2 &\stackrel{\text{ind}}{\sim} \text{GLM} \\ h(\theta_{ij})|\beta, u_i, \sigma_u^2, \sigma_\varepsilon^2 &\stackrel{\text{ind}}{\sim} N(x'_{ij}\beta + u_i, \sigma_\varepsilon^2); u_i|\beta, \sigma_u^2, \sigma_\varepsilon^2 \stackrel{\text{ind}}{\sim} N(0, \sigma_u^2) \\ \beta &\sim \text{Uniform}(R^k); (\sigma_u^2)^{-1} \sim \text{Gamma}(a, b); (\sigma_\varepsilon^2)^{-1} \sim \text{Gamma}(c, d), \end{aligned} \quad (4.4)$$

with $\beta, \sigma_u^2, \sigma_\varepsilon^2$ mutually independent and (a, b, c, d) denoting fixed parameter values. The joint posterior distribution of the parameters θ_{ij} or functions of them (like the means $E(y_{ij}|\theta_{ij})$) is evaluated by the Gibbs sampler. It is essential, however, to ensure that the resulting posterior is *proper* and the authors establish sufficient conditions to this effect. In the special case of the model (4.1), the conditions reduce to the requirement that the measurements y_{ij} for a given area i are not all zeroes or ones. When the y_{ij} are Poisson (see below), the condition is $y_i = \sum_{j=1}^{n_i} y_{ij} > 0$.

As an example, consider the model applied by Ghosh *et al.* (1998) for estimating lung cancer death counts in 115 counties of the State of Missouri in the U.S. for the period 1972–1981. Let y_{ias} define the lung cancer death count in cell (a, s) of county i , where a and s define age/sex groups.

The model assumes,

$$\begin{aligned}
 y_{ias} | \lambda_{ias} &\overset{ind}{\sim} \text{Poisson}(\lambda_{ias}) \\
 \log(\lambda_{ias}) &= \log(r N_{ias}) + x_s \beta_1 + x_a \beta_2 + x_s x_a \beta_3 + u_i + \varepsilon_{ias} \\
 u_i | \{u_k, k \neq i\} &\sim N(\bar{u}_{-i}, \sigma_u^2 / m_i); \varepsilon_{ias} \sim N(0, \sigma_\varepsilon^2) \\
 \beta &\sim \text{Uniform}(R^3); (\sigma_u^2)^{-1} \sim \text{gamma}(.01, .01); (\sigma_\varepsilon^2)^{-1} \sim \text{gamma}(1, 1)
 \end{aligned} \tag{4.5}$$

where N_{ias} is the mid-period population size in cell ias , r is the statewide lung cancer rate, x_s is a sex indicator variable and x_a defines three age groups taking the values $(-1, 0, 1)$. The model for the area effects u_i accounts for spatial clustering by defining the means \bar{u}_{-i} to be the average of the random effects $\{u_k, k \neq i\}$ that are “neighbors” of u_i , with m_i denoting the number of these neighbors. (Two counties are defined as neighbors if and only if they are physically adjacent to each other.) The distribution postulated for the random effects has a ‘Conditional Autoregressive’ structure; see Clayton & Kaldor (1987) for discussion and uses of this kind of model. As implied by the second and third equations, the model dependent estimate of λ_{ias} borrows strength from other area estimates and from estimates of other cells within the same area. Rao (1999) reviews several new studies with interesting variations of the model (4.4).

5 Time Series Models

The models and estimators considered so far borrow strength from administrative data and neighboring areas. Another valuable source of information when available is data measured for the target characteristics on previous occasions. The (direct) estimators obtained from the different surveys are usually correlated even when independent samples are selected on different occasions because of the correlations among the true area characteristics over time, giving rise to time series modeling. A typical time series model fitted to survey data consists of two parts:

- A model fitted to the population (area) quantity of interest,
- A model fitted to the sampling errors.

Consider the following general class of *state-space* models for a single area i with the index t designating time. Below we consider time series models that borrow also strength across areas.

$$\begin{aligned}
 y_{ti} &= \theta_{ti} + e_{ti} = x'_{ti} \beta_{ti} + e_{ti} \\
 \beta_{ti} &= T_i \beta_{t-1,i} + \eta_{ti}; e_{ti} \sim \text{ARMA}(a, b)
 \end{aligned} \tag{5.1}$$

where $\beta_{ti}(p \times 1)$ is a random *state vector*, $T_i(p \times p)$ is a fixed *transition matrix* and e_{ti} and $\eta_{ti} = (\eta_{ti1}, \dots, \eta_{tip})'$ are independent random errors with $\text{Var}(e_{ti}) = \sigma^2$ and $V(\eta_{ti}) = Q$ respectively. It is assumed also that $E(\eta_{ti} \eta'_{t-j,i}) = 0$ for $j > 0$. In (5.1), y_{ti} is the (direct) sample estimate for area i at time t and $\theta_{ti} = x'_{ti} \beta_{ti}$ is the target quantity, modeled as a linear combination of known concomitant variables with random coefficients, so that $e_{ti} = (y_{ti} - \theta_{ti})$ is the sampling error. The notation $\text{ARMA}(a, b)$ defines the Auto-Regressive Moving Average model of order (a, b) , (Box & Jenkins 1976). As can be seen, the model (5.1) accounts for the time series relationships between the true area quantities via the model postulated for the state vectors and for the autocorrelations between the sampling errors. Note in this regard that the sampling errors may be correlated even when there is no sample overlap. This is so because in repeated surveys it is often the case that units joining the sample are from the same small geographical areas (like census tracts) as units leaving the sample.

The following models can be represented in the form (5.1), see Pfeiffermann & Burck (1990) for details.

1. The *area level random effects* model (3.3); the model defined by (5.1) can easily be extended to the case where individual measurements y_{tij} with corresponding covariate values x_{tij} are available, in which case the model contains also the *unit level regression* model (3.1) as a special case.
2. The *random coefficient regression* model, $\beta_{tik} = \beta_k + \eta_{tik}$. This is one of the models for which Prasad & Rao (1990) developed the EBLUP MSE bias corrections. By imposing $\text{Var}(\eta_{tij}) = 0$ for some j , the corresponding regression coefficient is set fixed over time.
3. The *first order autoregression* model, $(\beta_{tik} - \beta_k) = \phi_k(\beta_{t-1,ik} - \beta_k) + \eta_{tik}$. By setting $\phi_k = 1$, the model simplifies to the *random walk* model.
4. As last example consider the model used in the U.S. for the production of all the major state employment and unemployment statistics. The model is fitted separately for each of the 50 states and the District of Columbia, see Tiller (1992) for details. In what follows y_{ti} is the direct estimate (say, the observed unemployment rate) in state i at month t , L_t is a trend component with (random) slope R_t and S_t is a seasonal effect.

Observation equation

$$y_{ti} = x_{ti}\alpha_{ti} + L_{ti} + S_{ti} + e_{ti}; e_{ti} \sim AR(15) \quad (5.2a)$$

Model for regression coefficients

$$\alpha_{ti} = \alpha_{t-1,i} + \eta_{ati} \quad (5.2b)$$

Model for Trend component

$$L_{ti} = L_{t-1,i} + R_{t-1,i} + \eta_{Lti}; R_{ti} = R_{t-1,i} + \eta_{Rti} \quad (5.2c)$$

Model for Seasonal effects

$$\begin{aligned} S_{jti} &= \cos \omega_j S_{j,t-1,i} + \sin \omega_j S_{j,t-1,i}^* + \eta_{Sjti} \\ S_{jti}^* &= -\sin \omega_j S_{j,t-1,i} + \cos \omega_j S_{j,t-1,i}^* + \eta_{Sjti}^* \\ S_{ti} &= \sum_{j=1}^6 S_{jti} \\ \omega_j &= 2\pi j/12, j = 1, \dots, 6 \end{aligned} \quad (5.2d)$$

The error terms η_{ati} , η_{Lti} , η_{Rti} , η_{Sjti} , η_{Sjti}^* are independent white noise. Note that the observed series consists of only the direct estimates y_{ti} . The model postulated for the trend is a local approximation to a linear trend. The AR(15) model accounts for the short and long term autocorrelations between the sampling errors as implied by the particular rotating panel sampling scheme underlying the U.S. Labor Force Survey. The covariate x_{ti} represents the ‘number of unemployed workers claiming unemployment insurance’. Pfeiffermann *et al.* (1998a) fit a similar model for the production of labor force statistics and trend estimates in small regions of Australia. Binder & Dick (1989) and Bell & Hillmer (1990) fit Multiplicative Seasonal ARIMA models (Box & Jenkins, 1976) to the “regression residuals” $(\theta_{ti} - x'_{ti}\gamma)$ where θ_{ti} defines as before the true area quantity of interest and x_{ti} represents area level covariate variables. The fitting of these models requires differencing of the observed series to guarantee stationarity. The area estimates are obtained by signal extraction.

The model (5.1) contains in general as unknown hyper-parameters the variances and covariances in Q , the parameters of the ARMA model of the sampling error and possibly also some of the elements of the transition matrices T_t . Because of possible identification problems and in order to simplify the maximization of the likelihood, it is customary to estimate the ARMA parameters based on external estimates of the variance and autocorrelations of the sampling error. Pfeiffermann *et al.* (1998a) develop a simple method of estimating the sampling error autocorrelations for rotating panel sampling designs. The other parameters are estimated by the values maximizing the likelihood when fixing the ARMA parameters at their estimated values. The likelihood function is conveniently

obtained by use of the *Kalman filter*, which for known hyper-parameters yields the BLUP of the state vector and the corresponding prediction error variance-covariance matrix for every time t . See Harvey (1989) for details.

Like with cross-sectional models, an important issue when fitting state-space models with estimated parameters is how to obtain reliable estimates for the MSE of the state vector predictors $\hat{\beta}_{ti}$, and hence for the MSE of the small area predictors $\hat{\theta}_{ti} = x'_{ti}\hat{\beta}_{ti}$. A “naïve” MSE estimator is obtained by running the Kalman filter with the estimated parameter values and using the variance-covariance matrix produced by the filter, but this again results in underestimation of the true MSE. Ansley & Kohn (1986) develop a bias correction to the naïve MSE estimator by expanding the MLE around the true parameters. The bias correction is derived under the frequentist approach but it is shown to have also a Bayesian interpretation. Hamilton (1986) adopts the Bayesian perspective and proposes a Monte Carlo method that consists of generating samples from a normal approximation to the posterior distribution of the unknown parameters. The normal approximation is obtained by fixing the mean at the MLE (or REML) and the covariance matrix at the inverse information matrix evaluated at the MLE. Quenneville & Singh (2000) show that both approaches have a bias of order $O(1/n)$ where n is the length of the series. The authors propose a second order approximation that reduces the bias to the order of $o(1/n)$. This approximation again assumes normality of the posterior distribution of the parameters and requires an evaluation of the information matrix. Pfeffermann & Tiller (2001) propose a simple parametric bootstrap method that yields frequentist MSE estimates with bias of order $O(1/n^2)$. The use of this approach does not assume normality of the unknown parameters or their estimators and it does not require evaluation of the inverse information matrix. This matrix is often unstable, particularly when the number of unknown parameters is large.

Evaluation of the prediction MSE of the small area predictors $\hat{\theta}_{ti} = x'_{ti}\hat{\beta}_{ti}$ creates no extra problem under the full HB modeling approach because the prediction MSE is defined by the posterior variance. The use of this approach requires, however, the specification of prior distributions for all the unknown hyper-parameters and as mentioned before, the computations can be very intensive; see, for example, Ghosh *et al.* (1996) and Datta *et al.* (1999). The latter study again considers estimation of unemployment rates in states of the U.S. but uses a different model from the model defined by (5.2). In particular, this model combines cross-sectional information across states (see also below).

The models considered so far pertain to time series observed in a single area. In practice, data are often available for many small areas simultaneously, although possibly for only few time points. In such cases it is desired to borrow information both cross-sectionally and over time. The combined modeling of cross-sectional and time series data is a classical problem in econometrics; see, e.g., Johnson (1977, 1980) for annotated bibliographies, but the econometric literature does not address SAE problems. One way of borrowing information cross-sectionally and over time within the framework of the model (5.1) is by including in the model both contemporary random effects and time varying effects. Rao & Yu (1994) consider the model

$$y_{ti} = \theta_{ti} + e_{ti} = [x'_{ti}\beta + u_i + v_{ti}] + e_{ti} \quad (5.3)$$

where, using previous notation, y_{ti} is the direct estimator for area i at time t , e_{ti} is the sampling error (assumed to be correlated over time with known variances and covariances), u_i is the contemporary random effect and v_{ti} is a time effect, assumed to evolve as in an AR(1) process. The random errors u_i , v_{ti} , e_{ti} are independent within and between the areas for any given time point t . The authors derive the EBLUP and valid (correct order) MSE estimators. Datta *et al.* (1999) extend the model (5.3) by permitting random regression coefficients β_i instead of the fixed vector β and adding extra random terms to account for seasonal variations. The authors develop an appropriate HB methodology for point and MSE estimation using the Gibbs sampler.

An alternative way of accounting for cross-sectional relationships under the model (5.1) is by permitting corresponding components of the error terms η_{ti} pertaining to different areas i to be

correlated. Pfeiffermann & Burck (1990) derive the explicit expression of the small area predictor obtained this way for the case where the state vector is a random walk and the sampling errors are uncorrelated, illustrating how the time series and cross-sectional data combine to strengthen the direct estimator. The authors apply the model for predicting mean house sale prices in cells defined by size (number of rooms) and cities. Ghosh *et al.* (1996) consider a more extreme case by which the state vectors are the same across the areas ($\beta_{it} \equiv \beta_t \sim \text{random walk}$), and apply the HB methodology with appropriate prior distributions using the Gibbs sampler. The specific problem considered is the prediction of median income of four person families in states of the U.S. An interesting feature of this study is that the direct estimators y_{it} are multivariate, containing also the median income estimates of three and five person families (with the other model components modified accordingly). Since the three contemporary direct estimates are correlated, including all of them in the model potentially improves the prediction compared to modeling of only the univariate estimators. In this particular application the best predictors were actually obtained when using a bivariate model with the direct median estimates of four and five person families as the input response data.

6 Accounting for Sampling Effects

The models and inference procedures considered so far assume implicitly simple random sampling designs. In practice, the sample selection process often involves unequal selection probabilities at least at some stages of the selection process. When the selection probabilities are not related to the values of the response variable, the models holding for the population hold also for the sample data and the sample selection process can be ignored. If, however, the selection probabilities are related to the response variable values even after conditioning on the values of the explanatory variables included in the model, the sampling design becomes *informative* and the model holding for the sample data differs from the model holding in the population. Ignoring the sampling process in such cases may yield biased predictors for the target characteristics of interest.

As a simple (but extreme) example of an informative sampling scheme suppose that the outcomes y_{ij} in a given area i are binary with $\Pr(y_{ij} = 1) = p$. Let the sample be selected with probabilities, $\Pr(j \in s_i | y_{ij} = k) = \pi_k$, $k = 0, 1$. Thence, by application of Bayes rule, $\Pr(y_{ij} = 1 | j \in s_i) = \pi_1 p / [\pi_1 p + \pi_2(1 - p)] = p_s$. Clearly, $p \neq p_s$ unless $\pi_1 = \pi_2$ and inference on p that ignores the selection scheme actually means inference on p_s . For recent discussion of informative probability sampling with key references, see Pfeiffermann *et al.* (1998b).

A common approach to account for possible sampling effects is to weight the sample measurements by the sampling weights, defined here as the inverse of the sample inclusion probabilities. (In practice the weights are often modified to account for missing data and post-stratification adjustments.) In the context of small area estimation, Kott (1989) and Prasad & Rao (1999) consider the simple 'unit level random effect model', $y_{ij} = \mu + u_i + \varepsilon_{ij} = \theta_i + \varepsilon_{ij}$, with the same assumptions on u_i and ε_{ij} as in (3.1), and propose to replace the simple means \bar{y}_i by the weighted means $\bar{y}_{iw} = \sum_{j=1}^{n_i} w_{ij} y_{ij} / \sum_{j=1}^{n_i} w_{ij}$ as the input *direct* estimators where $w_{ij} = 1/\pi_{ij}$; $\pi_{ij} = P(j \in s_i)$. The estimator \bar{y}_{iw} is approximately design unbiased (over repeated sampling) and consistent for the corresponding area mean θ_i . The predictor $\hat{\theta}_i$ in both articles has the form of a composite estimator, $\hat{\theta}_i = \hat{\alpha}_i \bar{y}_{iw} + (1 - \hat{\alpha}_i) \hat{\mu}_w$, where $\hat{\mu}_w$ is a weighted average of the unweighted sample means \bar{y}_i in Kott (1989), and a weighted average of the weighted means \bar{y}_{iw} in Prasad & Rao (1999). The coefficients $\hat{\alpha}_i$ and the vector coefficients defining $\hat{\mu}_w$ are functions of the sampling weights and the unweighted ANOVA estimators of the model variances. Prasad & Rao (1999) derive MSE estimators of correct order under the assumption that the model holds for the sample data (implying *de facto* non-informative sampling). The authors extend the results to the unit level regression model defined by (3.1). Arora & Lahiri (1997) model the weighted area means in a HB analysis that uses the Gibbs sampler to obtain the posterior means and variances. The authors allow for different variances $k_i \sigma_i^2 = \text{Var}(\bar{y}_{iw} | \theta_i)$ in different areas, postulating

the general prior $\sigma_i^{-2} \sim \text{gamma}(a, b)$, but like in Prasad & Rao (1999), the (known) coefficients k_i are computed under the population model that is assumed for the individual measurements. An empirical investigation with data collected in the U.S. family expenditure survey illustrates the better performance of the HB estimators compared to the EBLUP and the direct weighted means.

The use of the estimators \bar{y}_{iw} protects against informative sampling *within* the areas. Furthermore, as n_i increases, the weight $\hat{\alpha}_i$ attached to the direct estimator increases so that the estimator $\hat{\theta}_i$ is design consistent for θ_i . Note also that by estimating μ as a weighted average of the weighted means \bar{y}_{iw} like in Prasad & Rao (1999), the estimator $\hat{\theta}_i$ is approximately unbiased for θ_i if the expectation is taken over both the randomization distribution and the model. The estimator is neither approximately design unbiased nor approximately model unbiased under informative sampling. Folsom *et al.* (1999) incorporate the sampling weights for defining the conditional posterior densities of the fixed and random effects in a logistic mixed model (extension of the model 4.1). The resulting densities are employed for a survey weighted HB analysis using the Gibbs sampler. The model is used for estimating illicit drug use in the U.S. (first example in Section 1).

Weighting the sample measurements by the sampling weights does not protect against informative sampling of the areas when only some of the areas are selected to the sample. Note in this regard that under the models (3.1) and (3.3) considered in section 3, the EBLUP or EB predictors for areas not included in the sample are the synthetic estimators $\bar{X}'_i \hat{\beta}_{GLS}$ and $x'_i \hat{\beta}_{GLS}$ respectively. These estimators may be severely biased if the selection of the areas is informative, since it is no longer necessarily true that $E(u_i | i \notin s) = 0$, a condition validating the use of the synthetic estimators under noninformative sampling. For the model (3.1) (with possibly more than one random effect), Pfeffermann *et al.* (1998c) propose a weighting system that yields consistent estimators for all the model parameters, but that article does not address the prediction of small area means.

A different way of protecting against informative sampling is to base the inference on the *sample* probability density function (pdf), defined for *single stage* sampling as,

$$f_s(y_k|x_k) = f(y_k|x_k, k \in s) = \Pr(k \in s | y_k, x_k) f_p(y_k|x_k) / \Pr(k \in s | x_k) \quad (6.1)$$

where $f_p(y_k|x_k)$ defines the population pdf (Pfeffermann *et al.* 1998b). Denote the sample inclusion probabilities by $\pi_k = \Pr(k \in s)$. These probabilities depend in general on all the population values of y and x , and possibly also on the population values of design variables used for the sample selection but not included in the model. However, by viewing the population values as random realizations under the model, the probabilities π_k are likewise random and the pdf (6.1) can be written alternatively as,

$$f_s(y_k|x_k) = E_p(\pi_k | y_k, x_k) f_p(y_k|x_k) / E_p(\pi_k | x_k). \quad (6.2)$$

Note that $E_p(\pi_k | y_k, x_k)$ is generally not the same as $\pi_k = \Pr(k \in s)$, which may depend on all the population measurements. It follows from (6.2) that for a given population pdf, the sample pdf is fully specified by the expectation $E_p(\pi_k | y_k, x_k) = 1 / E_s(w_k | y_k, x_k)$, where E_s defines the expectation with respect to the sample distribution. The sample expectation can be identified and estimated from the sample data and knowledge of the sampling design, see Pfeffermann & Sverchkov (1999) for discussion and examples. Furthermore, Pfeffermann *et al.* (1998b) establish that for independent population measurements, the sample measurements are asymptotically independent with respect to the sample distribution under commonly used sampling schemes for selection with unequal probabilities. The asymptotic independence assumes $N \rightarrow \infty$ with n held fixed. (N and n are the population and sample sizes respectively.)

How can the sample distribution be used for SAE? Consider for convenience the unit level random effect model $y_{ij} = \mu + u_i + \varepsilon_{ij} = \theta_i + \varepsilon_{ij}$ mentioned earlier in this section. The corresponding sample model is,

$$f_s(y_{ij}|\theta_i) = E_p(\pi_{j|i} | y_{ij}, \theta_i) f_p(y_{ij}|\theta_i) / E_p(\pi_{j|i} | \theta_i) \quad (6.3a)$$

$$f_s(\theta_i) = E_p(\pi_i|\theta_i) f_p(\theta_i) / E_p(\pi_i) \quad (6.3b)$$

where $\pi_{j|i}$ is the conditional sample selection probability given that area i is in the sample, $\pi_i = \Pr(i \in s)$ and $f_p(y_{ij}|\theta_i)$ and $f_p(\theta_i)$ are the corresponding population densities. By modelling the expectations defining the sample distributions in (6.3a) and (6.3b), one can obtain the ‘model dependent’ predictors of the small area means, employing the methods discussed in previous sections. As a simple illustration suppose that the selection of the areas is carried out with probabilities π_i such that $E(\pi_i|\theta_i) = \Pr(i \in s) = k \times \exp(a_0 + a_1\theta_i)$ where k is some constant and that the sampling process within the areas is noninformative. Simple calculations show that for known variances (σ_u^2, σ_e^2), the optimal (BLUP or HB) predictors of the area means for areas in the sample and areas not in the sample are correspondingly,

$$\begin{aligned} \hat{\theta}_{s,i} &= E(\theta_i | i \in s, y) = \hat{\theta}_i + a_1 \sigma_e^2 \gamma_i \\ \hat{\theta}_{c,i} &= E(\theta_i | i \notin s, y) = \hat{\theta}_i - \frac{a_1 \sigma_e^2 \gamma_i k \exp(a_0 + d_i)}{1 - k \exp(a_0 + d_i)} \end{aligned} \quad (6.4)$$

where y denotes the sample data, $\gamma_i = \sigma_u^2 / (\sigma_u^2 + \sigma_e^2 / n_i)$, $d_i = a_1 \hat{\theta}_i + (a_1^2 / 2) \sigma_e^2 \gamma_i$ and $\hat{\theta}_i = \gamma_i \bar{y}_i + (1 - \gamma_i) \bar{y}$ is the optimal predictor under noninformative selection of the areas (see Section 3). As clearly indicated by (6.4), the optimal predictors for areas in the sample are higher than the optimal predictors under noninformative sampling since under the selection process considered the selected areas tend to have larger means. Conversely, the optimal predictors for areas not in the sample are lower than the optimal predictors under noninformative sampling. Thus, the use of the predictors $\hat{\theta}_i$ that ignore the sampling process yields biased predictions in this case. We are currently studying the application of this approach in conjunction with the model (3.1) using data from the Brazilian ‘Basic Education Evaluation Study’.

Comment: Application of the EB or HB approaches under informative sampling does not require changing the prior distributions since they refer to the population model. Ghosh (1999) explores the properties of the EB predictor obtained by use of the sample distribution for the case of the Fay–Herriot model (3.3).

7 The Importance of Modeling the Correlations Among Random Area Effects

Many of the models in common use for SAE assume that the random small area effects are independent between the areas. In practice, it would often be reasonable to assume that the random effects associated with ‘neighboring areas’ by some distance measure (not necessarily geographical) are correlated, with the correlations decaying to zero as the distance increases. Such models are very common in *spatial analysis* (see, e.g., Cressie 1993), but are not in wide use for SAE problems. As few exceptions we mention the articles by Datta & Ghosh (1991), Datta *et al.* (1999), Ghosh *et al.* (1996), Ghosh *et al.* (1998), Rao & Yu (1994) and Pfeffermann & Burck (1990) that have been reviewed in previous sections.

How much can be gained by accounting for the existing correlations among the area effects? In order to study this question, we consider two simple examples. In the first example we let the *working model* be defined by (3.3) but with no covariate variables and with the added simplification of equal sample sizes, $n_i = n$. Thus, the working model is,

$$\bar{y}_i = \mu + u_i + e_i = \theta_i + e_i; \quad \text{Var}_D(e_i) = \sigma_e^2 / n = \sigma^2; \quad \text{Var}(u_i) = \sigma_u^2 \quad (7.1)$$

with (u_i, e_i) being independent within and between the areas. Let σ^2 and σ_u^2 be known but μ unknown. Under this model, the optimal predictor of the small area mean is,

$$\hat{\theta}_i = \gamma \bar{y}_i + (1 - \gamma) \bar{y}; \quad \bar{y} = \sum_{i=1}^m \bar{y}_i / m; \quad \gamma = \sigma_u^2 / (\sigma_u^2 + \sigma^2). \quad (7.2)$$

Suppose now that under the *correct model* $\text{Corr}(u_i, u_k) = \rho > 0$ for $i \neq k$. Simple calculations show that for this model the optimal predictor and its MSE are,

$$\theta_i^* = \tilde{\gamma} \bar{y}_i + (1 - \tilde{\gamma}) \bar{y}; \quad \tilde{\gamma} = [\sigma_u^2(1 - \rho)] / [\sigma_u^2(1 - \rho) + \sigma^2] \quad (7.3)$$

$$\begin{aligned} \text{MSE}(\theta_i^*) &= \tilde{\gamma}^2 \delta^2 + (1 - \tilde{\gamma}^2) [\delta^2 + (m - 1) \rho \sigma_u^2] / m \\ &+ (1 - 2\tilde{\gamma}) \sigma_u^2 - 2(1 - \tilde{\gamma}) [\sigma_u^2 + (m - 1) \rho \sigma_u^2] / m \end{aligned} \quad (7.4)$$

where $\delta^2 = (\sigma_u^2 + \sigma^2) = \text{Var}(\bar{y}_i)$. The 'working predictor' $\hat{\theta}_i$ is unbiased under the correct model and its MSE is obtained from (7.4) by replacing $\tilde{\gamma}$ by γ .

The question arising is: what is the loss from using $\hat{\theta}_i$ instead of θ_i^* ? We mention first that $\lim_{m \rightarrow \infty} \text{MSE}(\theta_i^*) / \text{MSE}(\hat{\theta}_i) = [\tilde{\gamma} / \gamma] / [1 - \rho(1 - \gamma)]$, with the limit decreasing to 0 as ρ increases to 1, showing that with many areas and large cross-sectional correlations, the loss in efficiency can be substantial. Table 1 shows the relative efficiencies $R = \text{MSE}(\theta_i^*) / (\text{MSE} \hat{\theta}_i)$ for selected numbers of areas and different values of ρ and $L = (\sigma^2 / \sigma_u^2)$.

Table 1

Relative efficiencies of working predictor compared with the optimal predictor under equal correlations model.

		$L = 0.1$	$L = 1.0$	$L = 10$	$L = 100$
$m = 5$	$\rho = .25$	1.00	0.99	1.00	1.00
	$\rho = .50$	0.97	0.93	0.99	1.00
	$\rho = .75$	0.88	0.80	0.98	1.00
$m = 40$	$\rho = .25$	0.99	0.98	1.00	1.00
	$\rho = .50$	0.96	0.90	0.97	1.00
	$\rho = .75$	0.85	0.67	0.92	1.00
$m = 100$	$\rho = .25$	0.99	0.98	0.99	1.00
	$\rho = .50$	0.96	0.89	0.97	1.00
	$\rho = .75$	0.85	0.65	0.88	1.00

For the second example we use the same working model but let $\text{Corr}(u_i, u_k) = \rho^{|i-k|}$ to represent the true correlations among the random area effects. This correlation pattern corresponds to the case where the means \bar{y}_i refer to different time points and the area effects follow the AR(1) model,

$$u_i = \rho u_{i-1} + \varepsilon_i; \quad \text{Var}(\varepsilon_i) = \sigma_\varepsilon^2, \quad \text{Var}(u_i) = \sigma_u^2 = \sigma_\varepsilon^2 / (1 - \rho^2). \quad (7.5)$$

If $i = m$ signifies the last time point with observations, the optimal predictor of θ_m under this model is easily obtained by application of the recursive Kalman filter equations (Harvey, 1989). For 'large m ' the MSE of the optimal predictor can be computed explicitly by solving the corresponding Riccati equation. We find that

$$\text{MSE}(\theta_m^*) = \left\{ \left[(1 - \rho^2)^2 \delta^4 + 4\sigma_u^2 \sigma^2 \rho^2 (1 - \rho^2) \right]^{1/2} - (1 - \rho^2) \delta^2 \right\} / 2\rho^2. \quad (7.6)$$

As in the previous example, the working predictor $\hat{\theta}_i$ defined by (7.2) is unbiased under the model (7.5) and as $m \rightarrow \infty$, $\text{MSE}(\hat{\theta}_i) \rightarrow \gamma \sigma^2$ under both the working model and the true model. Table

2 shows the relative efficiencies $R = MSE(\theta_i^*)/(MSE\hat{\theta}_i)$ for large m and different values of ρ and $L = (\sigma^2/\sigma_u^2)$.

Table 2

Relative efficiencies of working predictor compared with optimal predictor under Auto-regression model, large m .

	$L = 0.1$	$L = 1$	$L = 10$	$L = 100$
$\rho = 0.25$	0.99	0.98	0.99	1.00
$\rho = 0.50$	0.97	0.93	0.97	1.00
$\rho = 0.75$	0.91	0.80	0.91	0.99
$\rho = 0.95$	0.66	0.48	0.66	0.93

The first notable (but expected) result emerging from the two tables is that for fixed ratios L , R decreases as ρ increases, implying that the loss from using the working predictor increases. Yet, unless $\rho > 0.5$, $R \geq 0.90$. Another less expected result is that increasing L does not necessarily imply a corresponding decrease in R . In fact, in both tables R decreases as L increases from 0.1 to 1 but then it increases when further increasing L . (It is easily shown that for the AR(1) model and also for the limiting case $m \rightarrow \infty$ under the equal correlations model, $R(L) = R(1/L)$ for all L , with the minimum value of R attained at $L = 1$ for which $\gamma = 0.5$.) This result is explained by the fact that as σ^2 increases in relation to σ_u^2 , more weight is assigned to the synthetic estimator \bar{y} which is common to both the working predictor and the optimal predictor under the true model so that the differences between the MSE of the two predictors diminish. The results presented in Table 2 for the AR(1) model are 'one sided' in the sense that the 'borrowing of strength' is only from previous occasions. We studied also the 'two sided' case where data collected before and after the time point of interest are used for deriving the optimal predictor and obtained very similar results.

The overall conclusion from this study is that unless the correlations between the area random effects are large, the loss in efficiency from using the working model is small. Notice also that for small values of ρ the MSE of the working predictor $\hat{\theta}_i$ under the correct model is similar to the MSE evaluated under the working model. Clearly, the results of this rather limited study need to be tested under different models.

8 Concluding Remarks

This article attempts to overview the main topics in SAE research in recent years, emphasizing the new models and inference methods with particular attention to point estimation and MSE evaluations. Two important issues not considered are model selection and model validation. As mentioned before, SAE is one of the few fields in survey sampling where it is widely recognized that the use of model dependent inference is often inevitable. Given the growing use of small area statistics and their immense importance, it is imperative to develop efficient tools for the selection of models and the ascertainment of their goodness of fit. This is a difficult problem because small area models contain assumptions on unobservable random effects, which are therefore difficult to verify. We mention also that under informative sampling discussed in Section 6, the observed data no longer represent the population model, making the model selection and validation even harder. Ghosh & Rao (1994) discuss a few formal and informal diagnostic tests that have been developed until that year. Recently, Jiang, Lahiri & Wu (1999) developed a 'chi-square statistic' for testing the distribution of the combined error components. A third related issue is how to secure the robustness of the estimators after a model and inference method have been selected. Several sensitivity analyses are reported in the literature but more theoretical research is needed for studying the robustness of the SAE methods

in common use. With the extensive research on SAE taking place in so many countries, I expect many new innovative studies of these and other issues in the near future.

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Résumé

Cette communication a pour but de fournir une revue des principaux développements des méthodes d'estimation pour les statistiques des petites régions, des années récentes. Nous discutons également des développements précédents, qui serviront d'arrière-plan pour les études nouvelles. La revue se concentre sur les méthodes basées sur les modèles et met le point surtout sur la prédiction ponctuelle des quantités des petites régions et l'estimation des erreurs carrées moyennes. Les nouveaux modèles considérés sont des modèles employés pour les mesures discrètes, des modèles de séries temporelles et des modèles produits sous l'échantillonnage informatif. On examine aussi les bénéfices possibles de l'emploi des modèles pour décrire les corrélations entre les effets aléatoires des petites régions, afin de représenter la variation inexpliquée entre ces quantités. Une revue et une évaluation des méthodes employées antérieurement pour les statistiques des petites régions sont présentées par Ghosh & Rao (1994).

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