UTS

Stochastic Forecast for the South Korean National Pension System

Master of Data Science in Quantitative Finance
Hayoung Lee

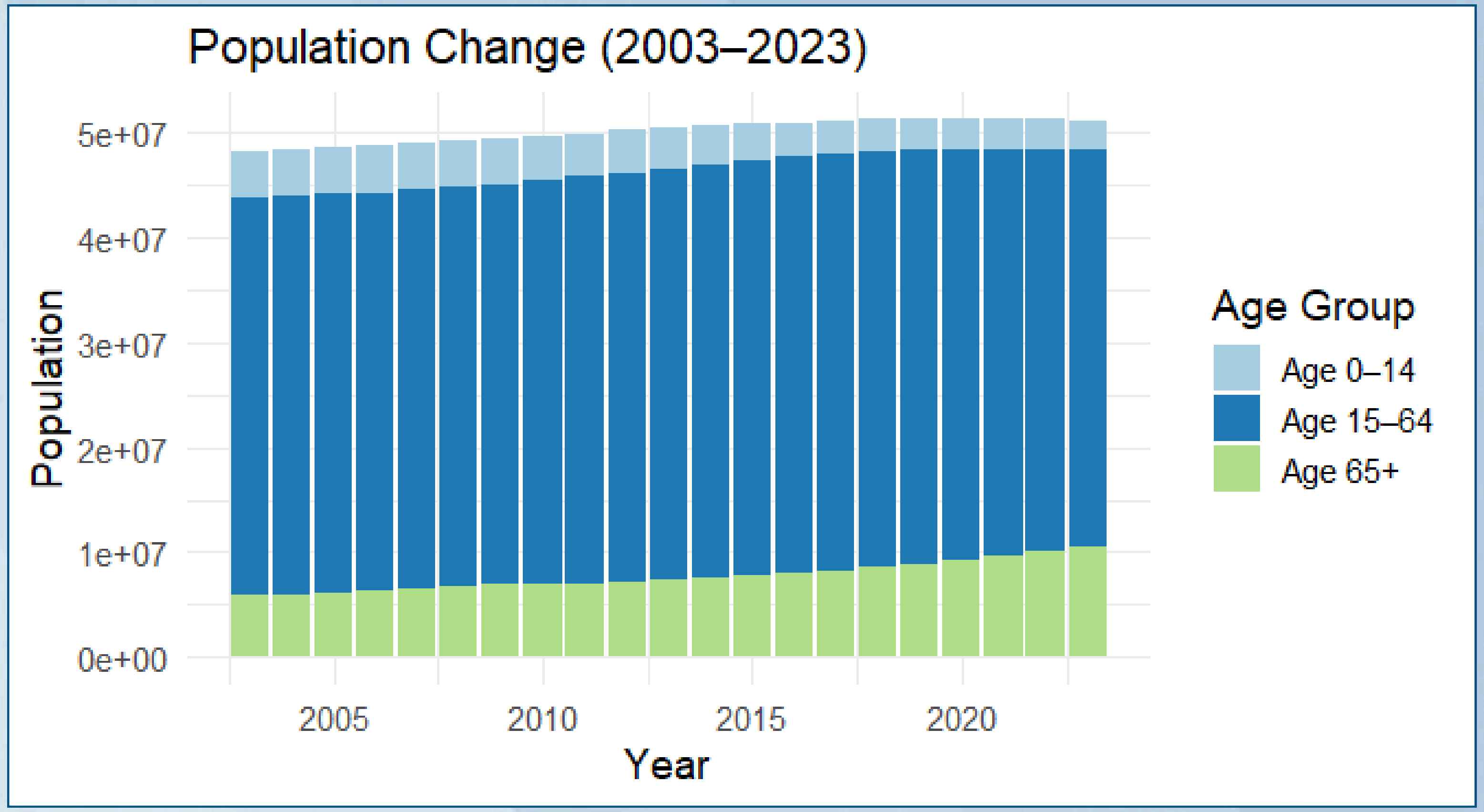
UTS

Overview

Introduction

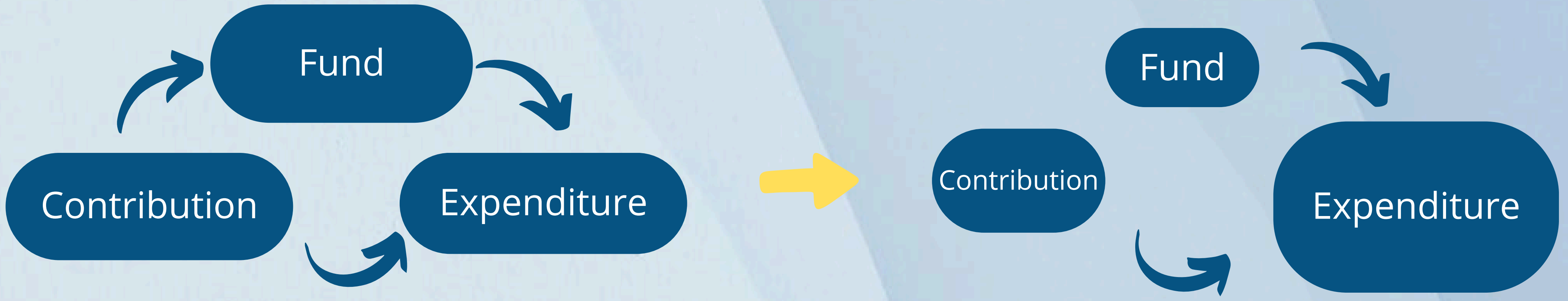
Demographic Changes in Korea

- Increasing life expectancy
- Declining fertility
- Shriking population



Korean National Pension System

- Decreasing contributions
- Increasing pension expenditure
- Declining fund
- Risk of depletion



Research Problems

1. Can we project population **stochastically** using fertility model and mortality model?

2. What are **insights** when **uncertainty** is included in pension fund forecast?

3. What is the effect of **pension reform** on **delaying fund depletion**?

Research Gap

1. First study to combine the **Generalied-Log-Gamma (GLG) fertility model** and the **affine mortality model** to simulate **population** with **uncertainty**

2. This study simulates the **fund stochastically** under **various pension reform policies** and **economic conditions**

Objectives of Research

1. To forecast **survival probabilities** using an affine mortality model that better captures **complexity of mortality rate**

2. To provide insight into how **pension reforms** could impact **financial sustainability of the fund**

Literature Review

1. **Partially funded systems** (Barr and Diamond, 2009) require regular actuarial valuation, but rely on deterministic model, which **are not enough to capture long-term uncertainties** (Baek, 2016)

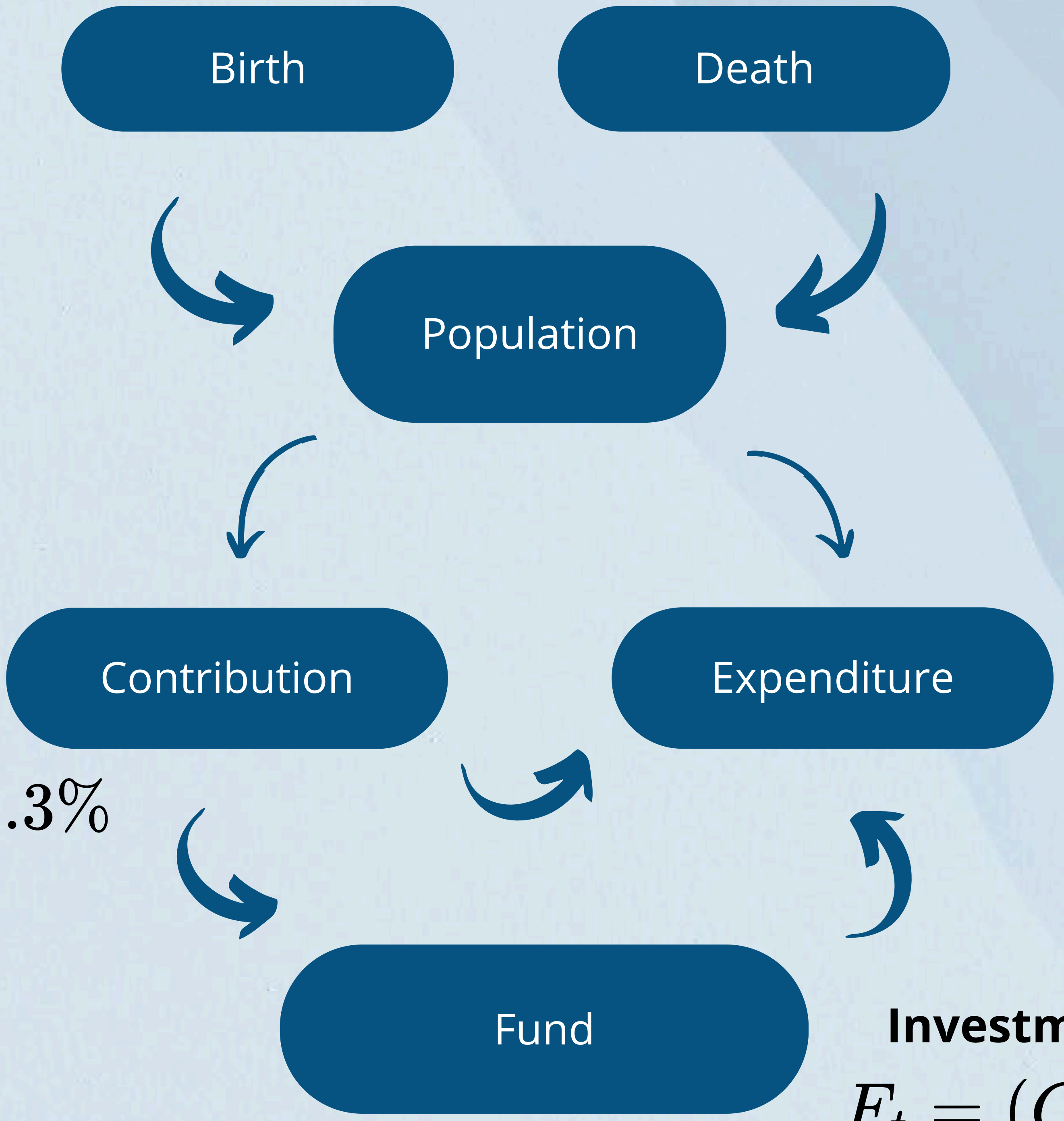
2. Recent studies apply **stochastic models to the National Pension** (D et al., 2003; Yuehong and Xianglian, 2016; Ivan, 2020). Main variables (fertility rate, mortality rate, wage growth rate, investment rate) are treated as stochastic.

3. Park et al. (2013) shows **GLG model** (Kaneko, 2003) provides a good fit for South Korean ASFR.

4. **Affine mortality model** treat mortality intensity as a stochastic process. (Dahl, 2004) Recent works (Ungolo et al., 2024) have applied it to cohort data via the AffineMortality R package. (Francesco et al., 2023)

Model Framework - Valuation Model

UTS



Contribution Rate
wage growth rate
Age 18~59 Population
Participation Rate

$$C_t = CR_t \times W_{t,a} \times P_{t,a} \times 73.3\%$$

Replacement Rate
wage growth rate
Age over 65 Population
Recipient Rate

$$E_t = RR_t \times W_{t,b} \times P_{t,b} \times 51.2\%$$

Investment return

$$F_t = (C_t - E_t) + F_{t-1} \times (1 + r_t)$$

Model Framework - Fertility Rate Model

Objective	Predict the **number of births** per year
Data	Period **ASFR* by birth order (1, 2, 3, 4, 5+)** (**2000–2023**, Human Fertility Database) Age Specific Fertility Rate (ASFR) - number of births given by women of a specific age divided by the total population of women in that age group.
Assumption	• Only for Women aged **15–49** - international standard • At birth, there are about 106 males for every 100 females
Model	Generalized Log-Gamma (GLG) Model $F_i(a+1) - F_i(a)$ → **ASFR of i-th child at age a** $F_i(x; C_i, \lambda, \mu, b) = C_i G(x; \lambda, \mu, b)$ **where** $G(x) = 1 - I\left(\lambda^{-2}, \lambda^{-2}\exp\left(\lambda\frac{x-\mu}{b}\right)\right)$ cumulative fertility function — Log-Gamma distribution
Estimation	Nonlinear least squares using **nlsLM()** in R (minpack.lm, Elzhov et al., 2023)
Optimization	Levenberg–Marquardt algorithm
Simulation	Simulate ASFR treating **parameters** as **Random walk** time series

Model Framework - Mortality Rate Model

Objective	Predict **the number of people who survive** in a given year
Data	Period death rate (2003–2023, Human Mortality Database)
Assumption	• Model only for **ages 40–109** - mortality patterns • Rates for ages 0–39 - Fixed at 2023
Affine mortality model	• Blackburn-Sherris (BS) model (Blackburn and Sherris, 2013) • Arbitrage-Free Nelson-Siegel (AFNS) model (Christensen et al., 2011)
Estimation	Univariate Kalman filter using **AffineMortality** R package (Francesco et al., 2023)
Optimization	Coordinate ascent algorithm (group-wise log-likelihood maximization)
Simulation	Simulate **X(t)** as a **Markov process**

	BS model	**AFNS model**
Latent factor X(t)	$(X_1(t) \quad X_2(t) \quad X_3(t))^T$	$(L(t) \quad S(t) \quad C(t))^T$
Dynamics	$dX(t) = -\Delta X(t)dt + \Sigma dW^Q(t)$	

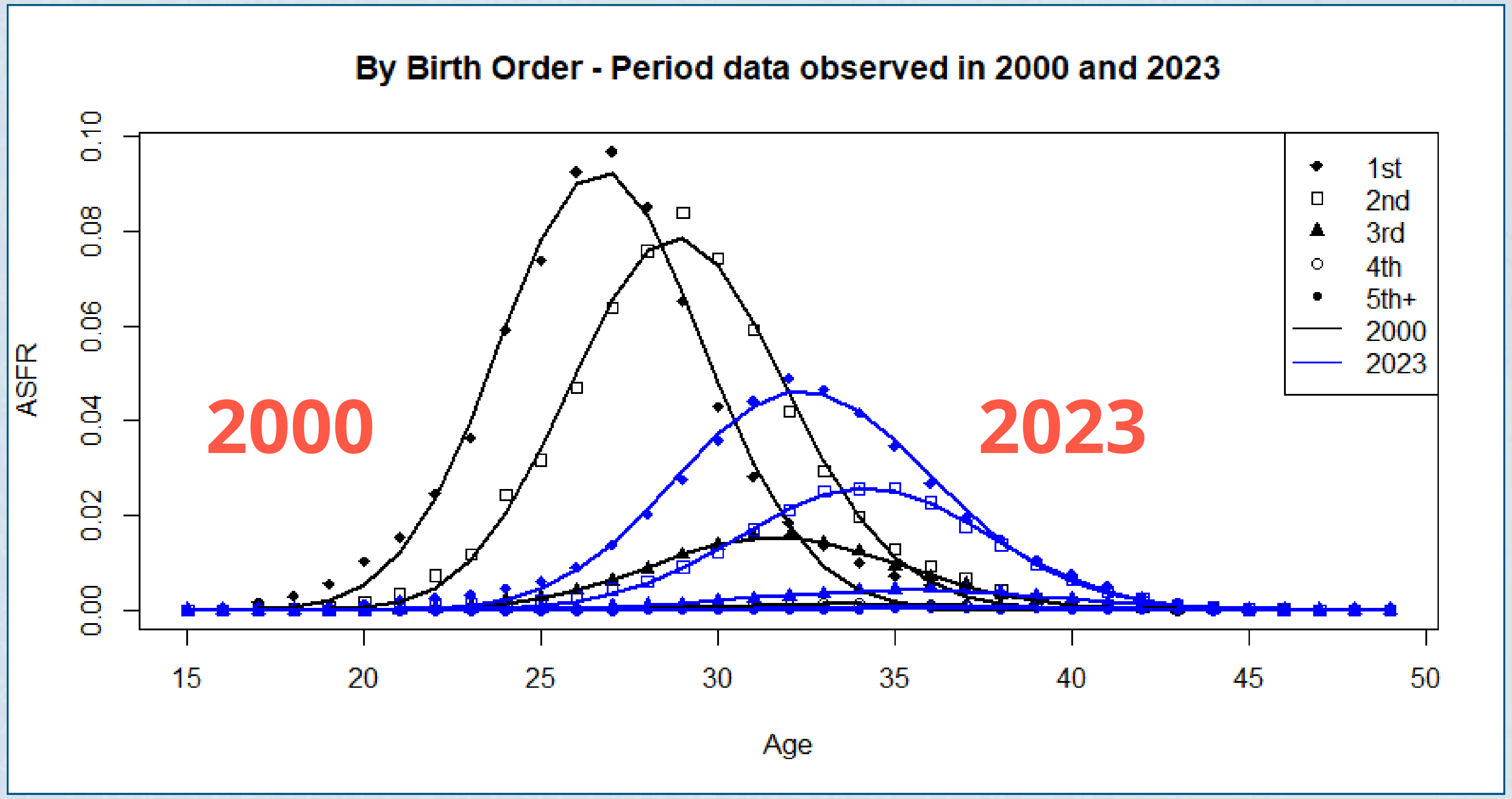
$$S_a(t,T) := \mathbb{E}^Q\left[e^{-\int_t^T \mu_a(s)\,ds} \middle| \mathcal{G}_t\right] = e^{A^Q(t,T)+B^Q(t,T)^T X(t)}$$

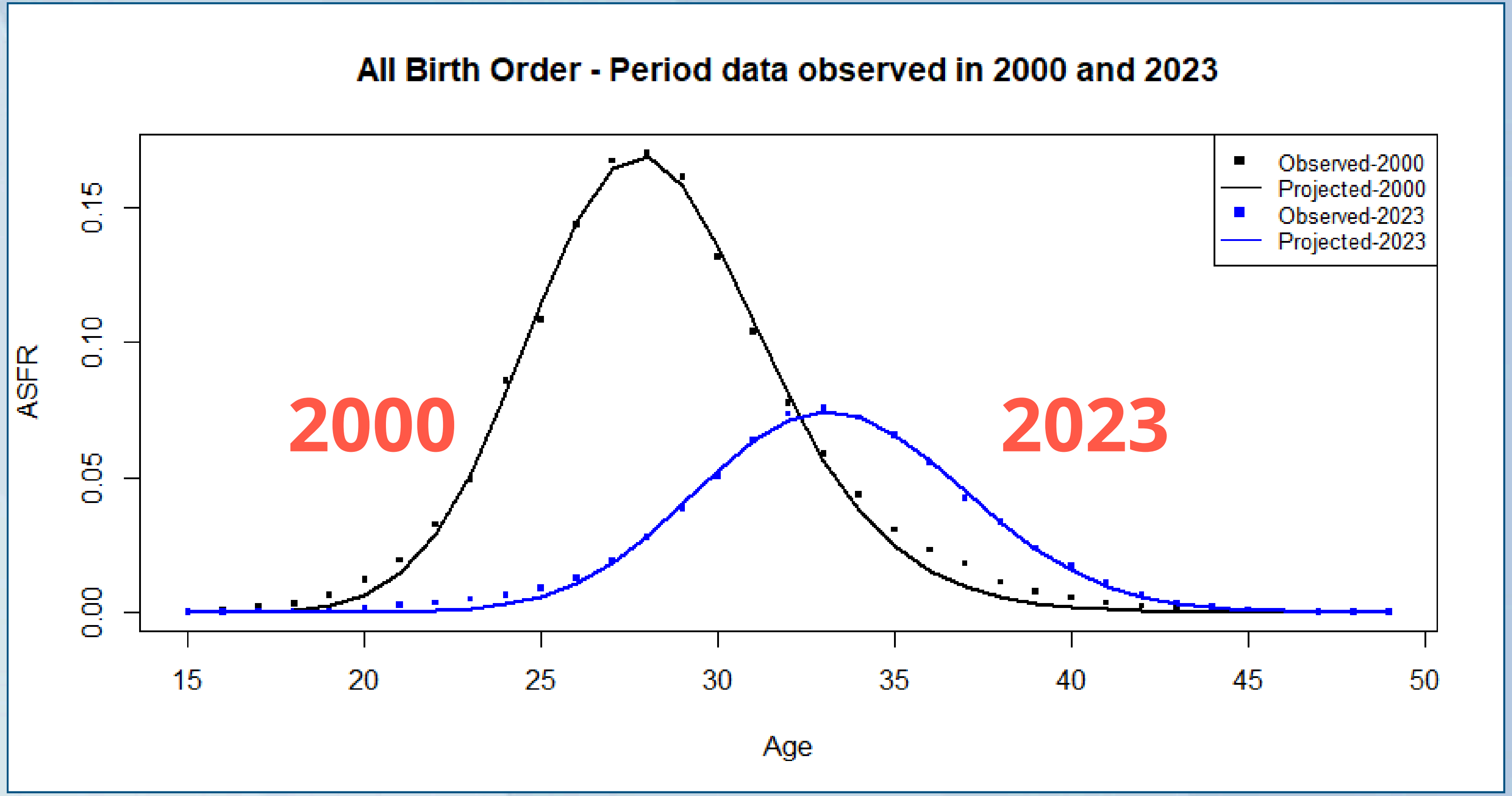
Survival probability from t to T for an individual aged a at time t

Parameter Estimation - Fertility Rate Model

UTS

Comparison of ASFR (2000 vs 2023)





- The distribution shifted **rightward** - Delayed having birth
- From the thrid birth, ASFR drop significantly
- **Underestimation** occurs at both younger (<22) and older age (>35)

  → Log Gamma distribution drops steeply in tails

C, λ, μ, b

- **4 parameters** assumes to follow a **random walk** and are simulated using **Monte Carlo**
- (200 iterations) for **80-year** projections

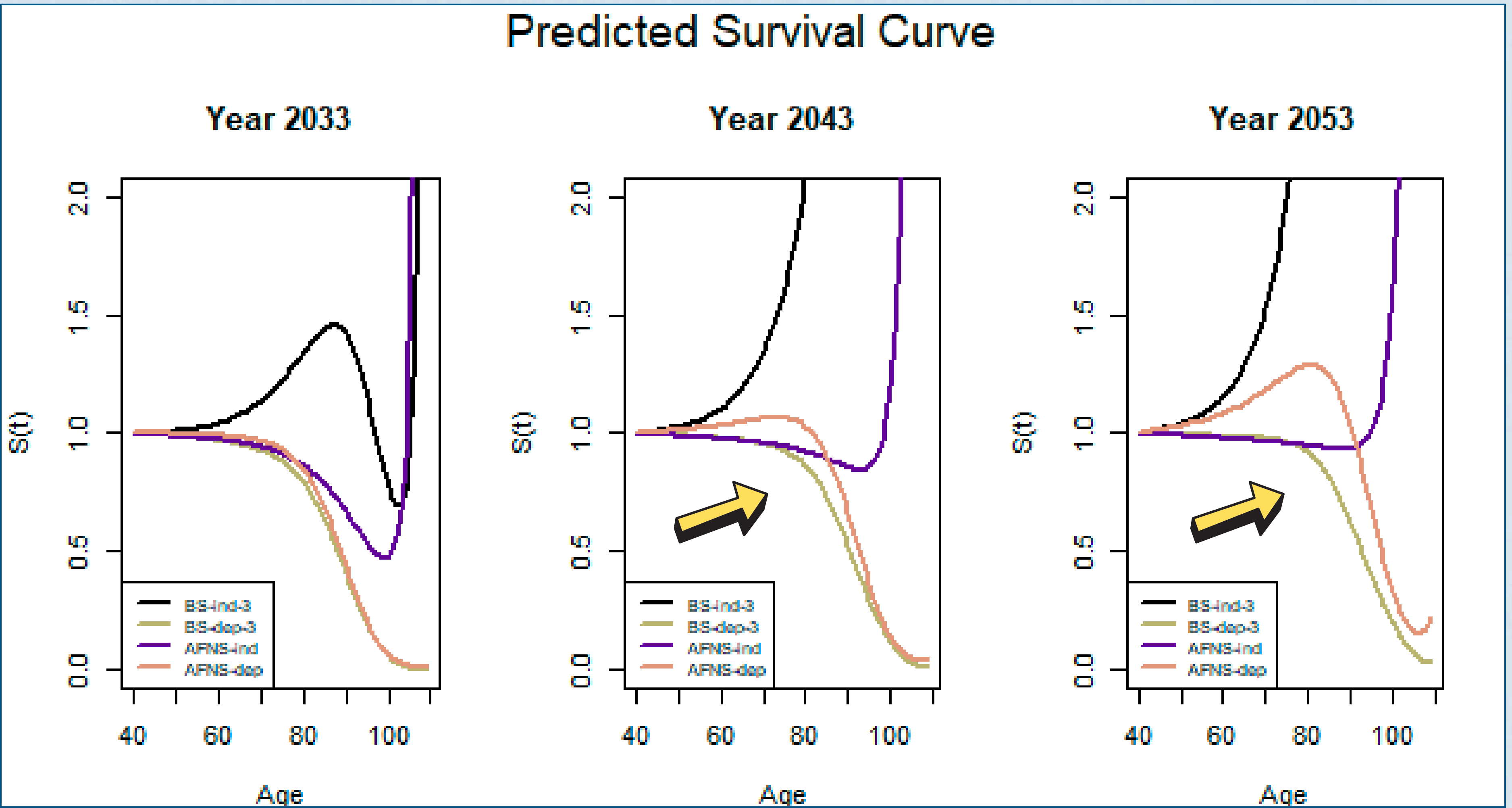
Model Selection - Mortality Rate Model

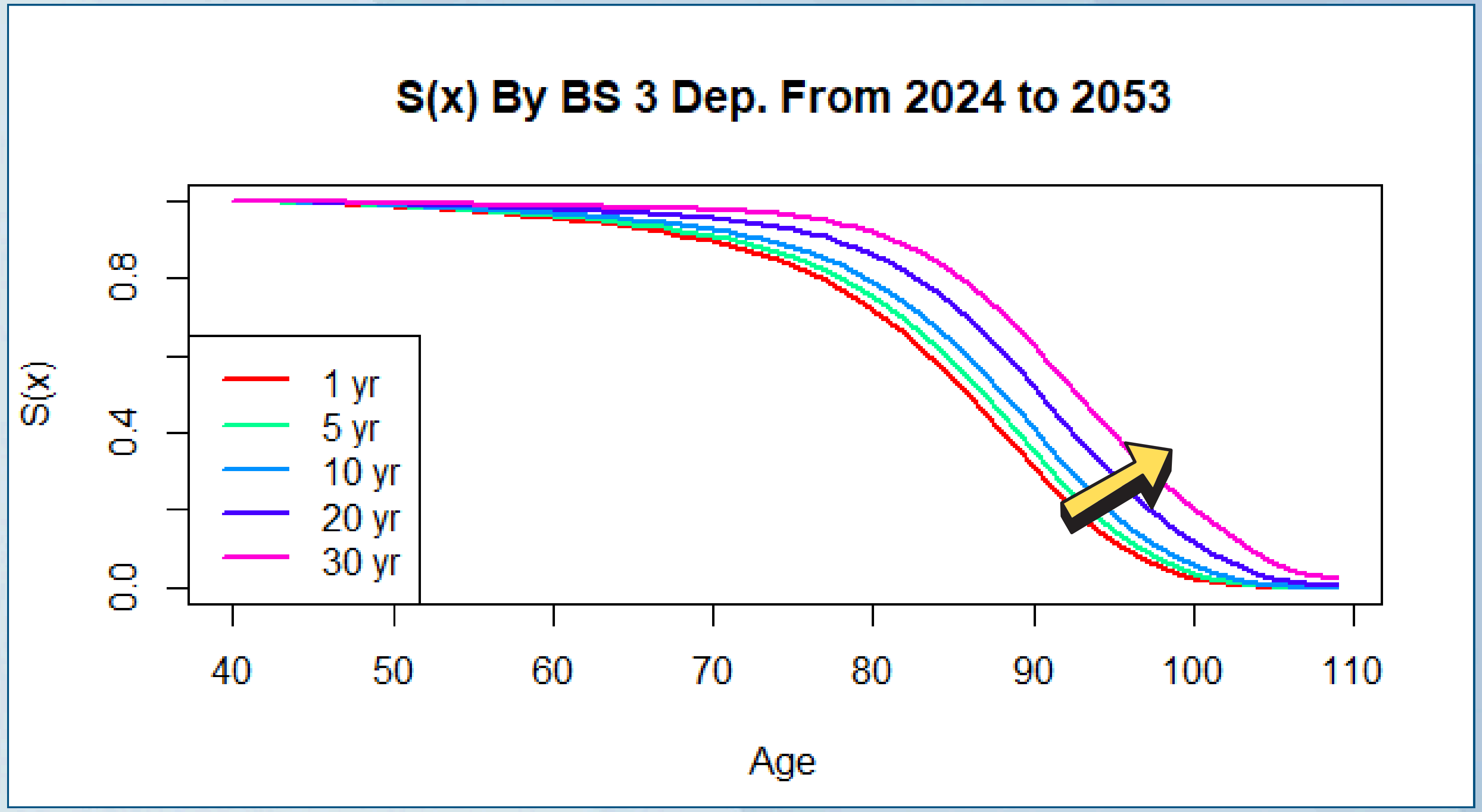
In-sample Performance

Model	AIC	BIC	RMSE(E-06)
BS 3 ind.	-21557	-21477	324
BS 3 dep.	-22916	-22805	9
AFNS ind	-20270	-20202	100
AFNS dep	-22198	-22113	24

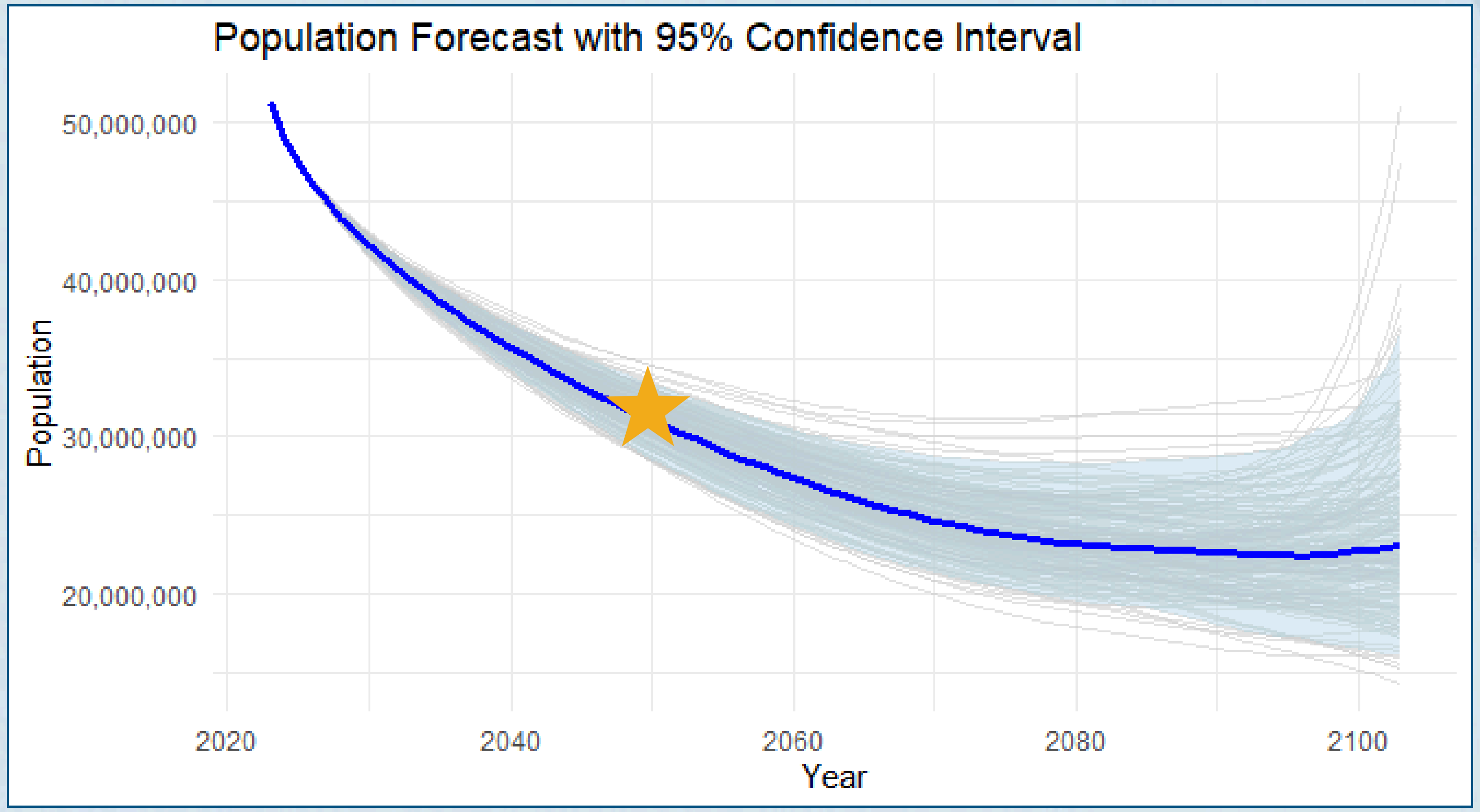
- The **BS Model with 3 dependent factors** shows better in-sample performance and stable long-term dynamics
- Others perform poorly in long-term forecasting due to limited observations
- The selected model shows increasing survival probabilities over time

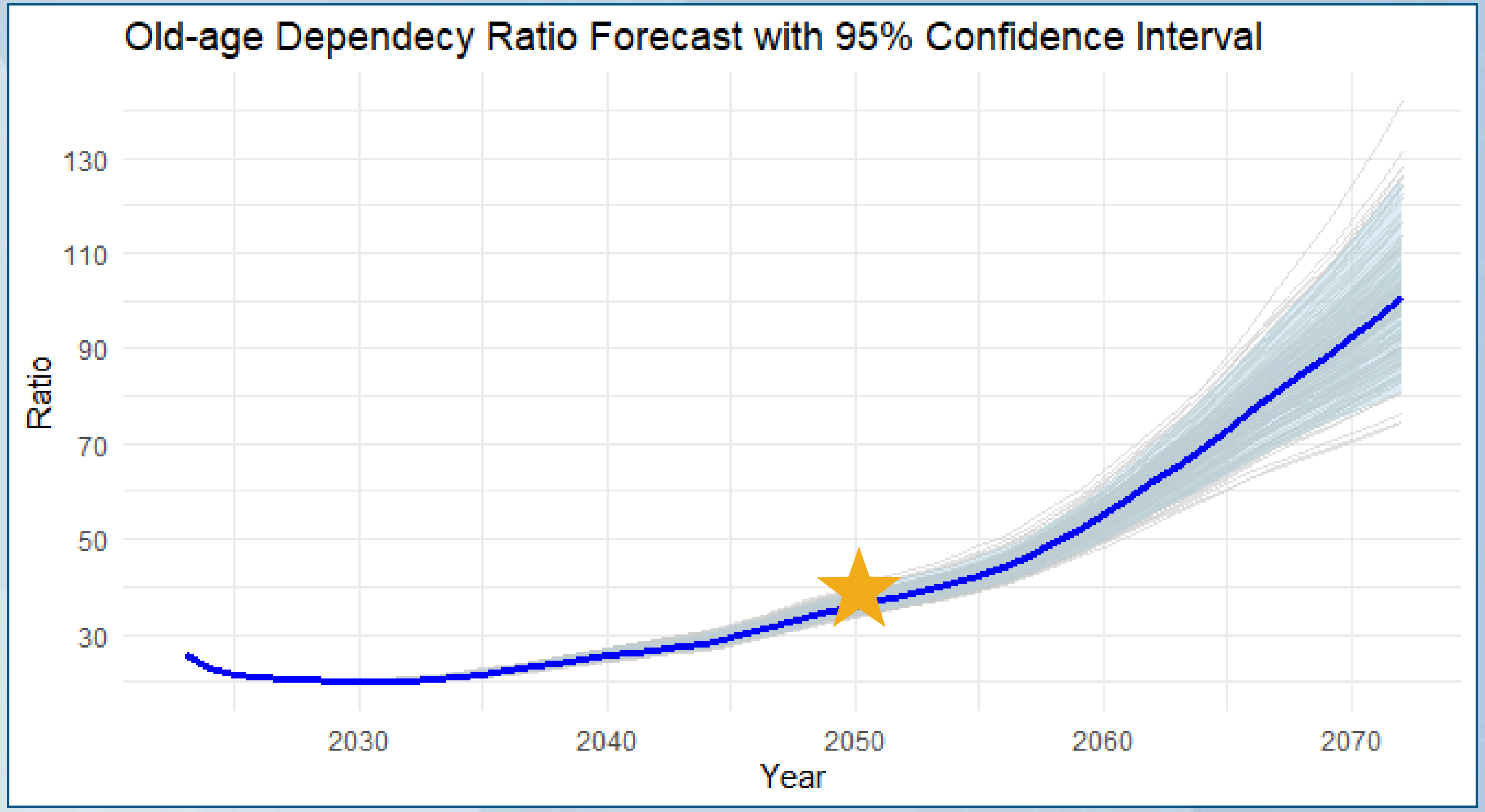
Forecast Performance





Simulation - Population Forecast



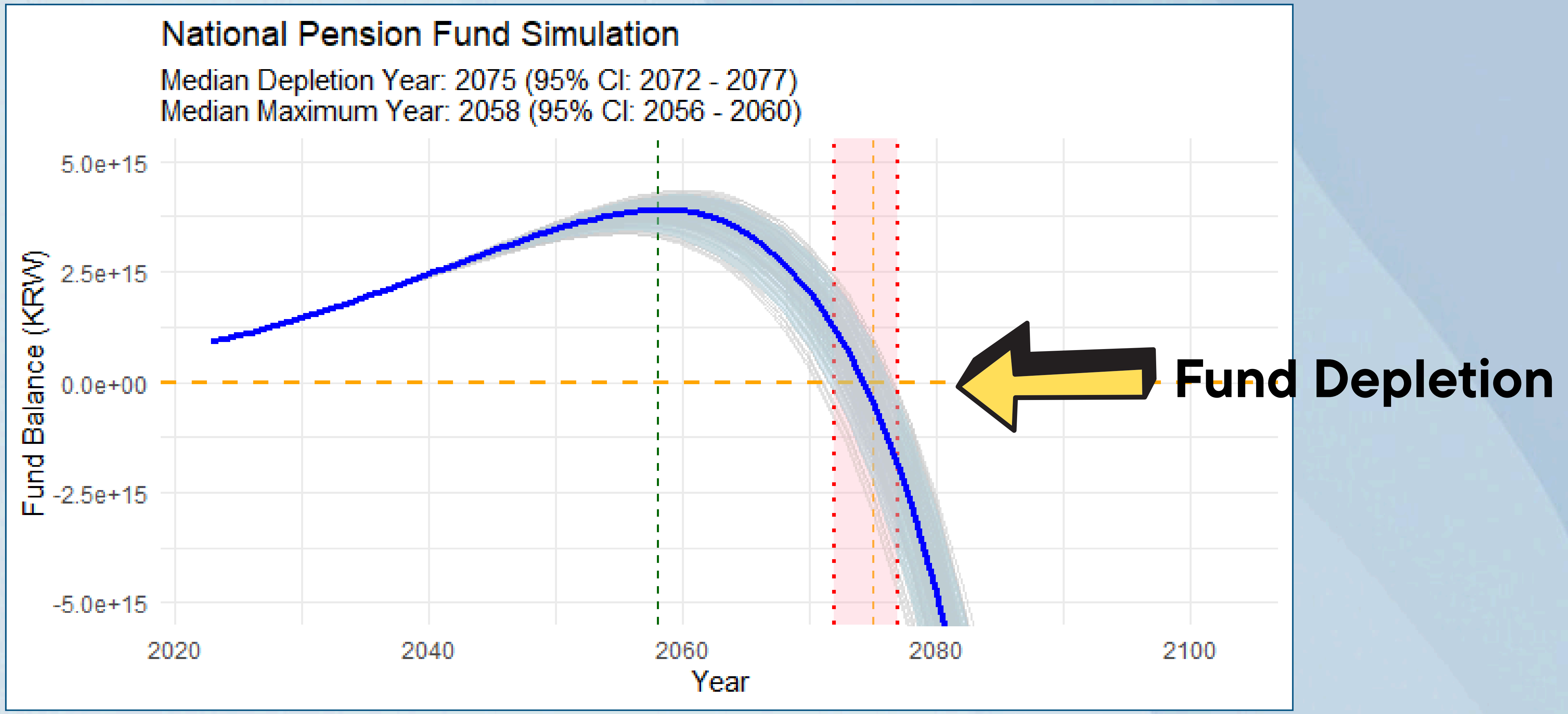


Over the next **30 years**,

- Korea's population is projected to decline from **50 million to 30 million**
- **The old-age dependency ratio** is projected to rise to 40, meaning every 100 working-age people will support around 40 elderly people

  → Increasing the **burden on the working-age population**
- The confidence interval widens over time, reflecting **increasing uncertainty in long-term forecasts**

Simulation - Fund Forecast



- In 2023, the pension fund was KRW 950 trillion (AUD 1.06 trillion)
- Under the **current system** with wage growth rate of 3.7% and investement return of 4%, the fund is projected to reach its **peak in 2058**, then **fully depleted by 2075** (**95% confidence interval: 2072–2077**)

→ After depletion, **government subsidies** may be required leading to higher taxes

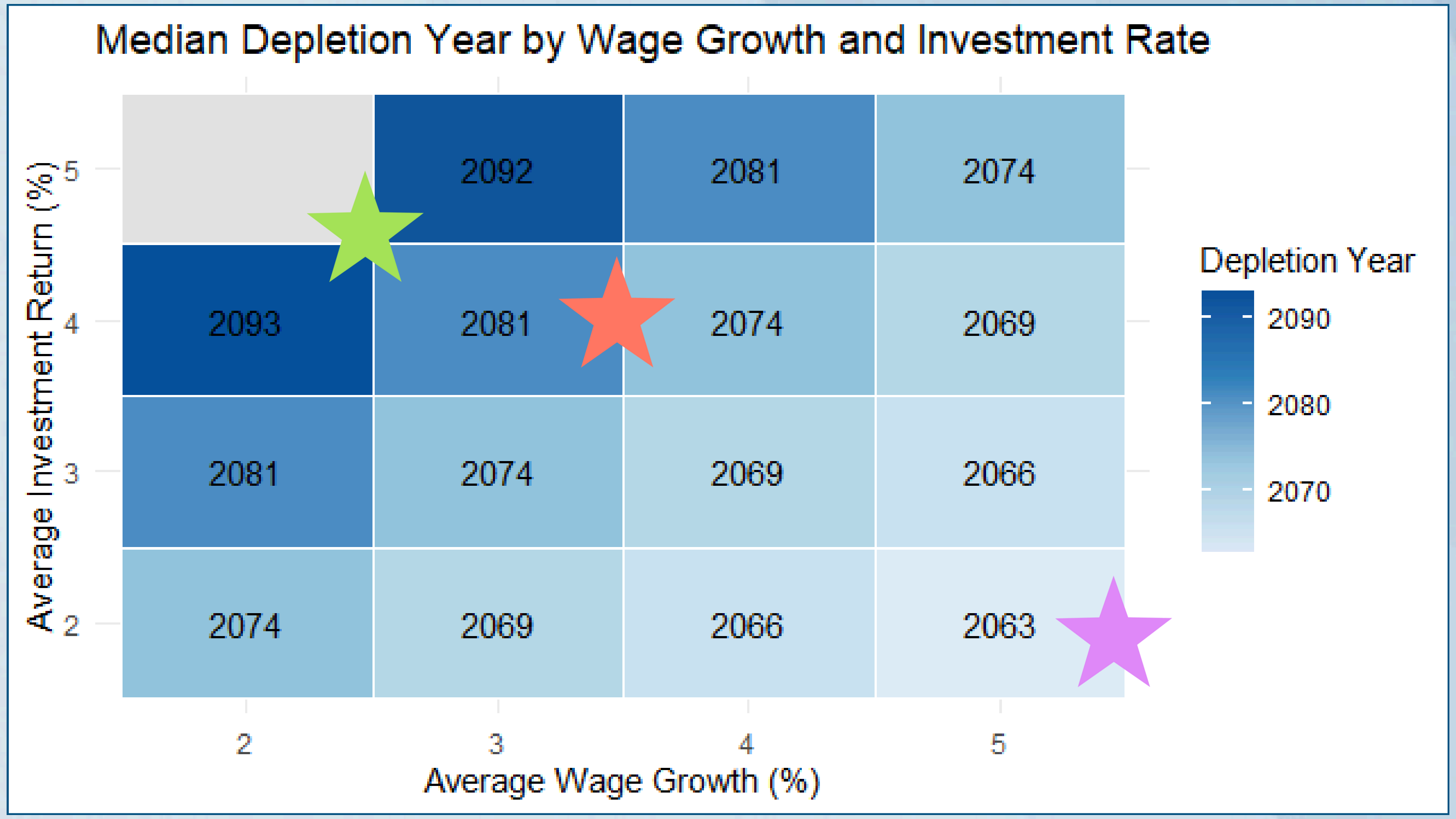
→ **Delaying depletion** through pension reforms is important

Comparison with NPS forecast

		NPS	This study
Framework	Fertility Model	GLG Model	
	Mortality Model	Li-Lee-Gerland (2013)	Affine Mortality
In 2070	Population	37.7m	21.4m (95% CI: 21.4m-28.8m)
	Old-age Dependency Ratio	62	92.3 (95 % CI: 96.6-113)
Depletion Year		2057	2075 (95% CI: 2072-2077)

- Population - did not take migration into account
- Fund - fixed economic variables may be optimistic,
- Used oversimplified long-term actuarial valuation formula

Sensitivity Analysis - Economic variables



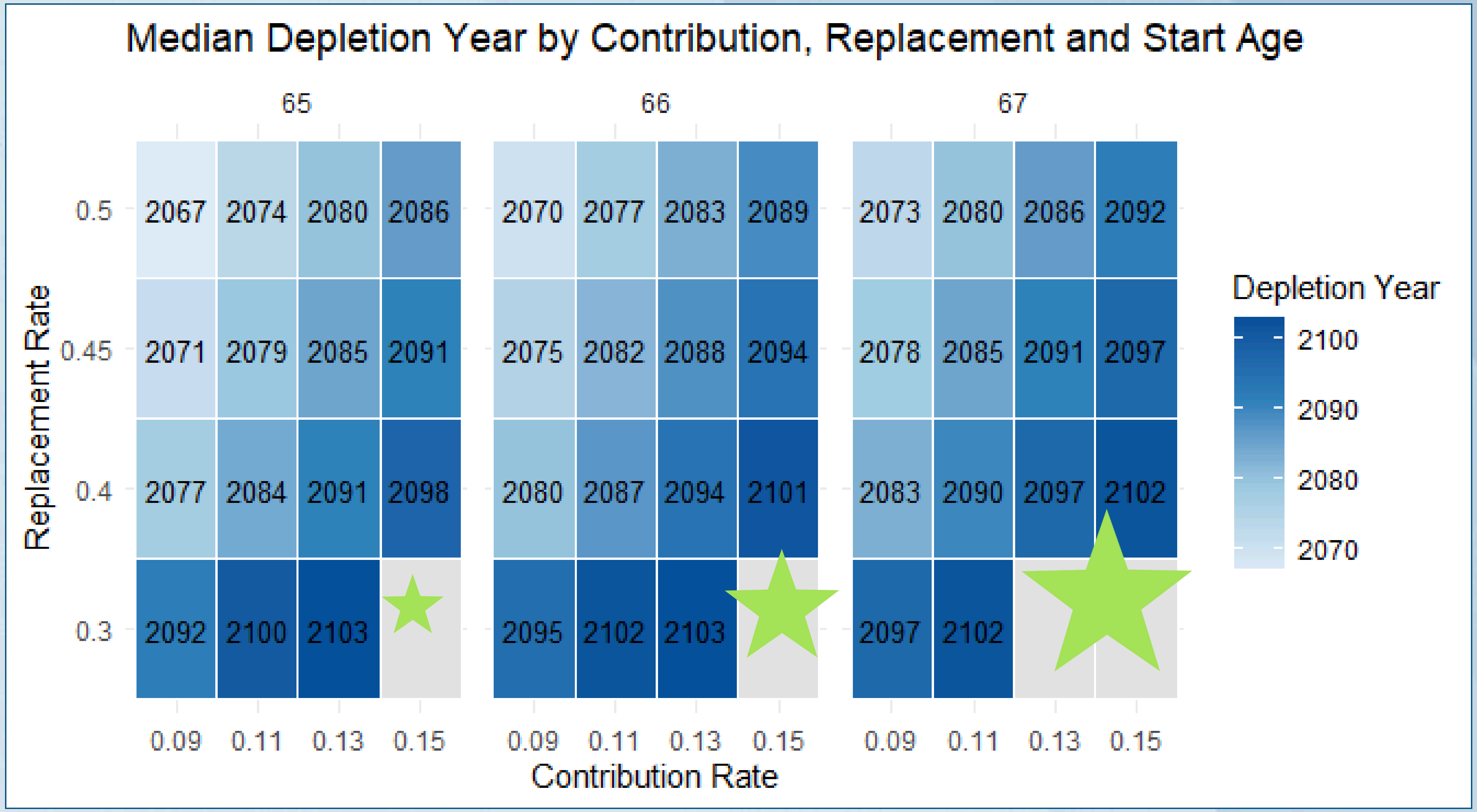
- It shows how the median depletion year varies depending on wage growth and investemnt return rates

Optimistic scenario :
Wage growth ↓ & Return ↑ → later depletion

Pessimistic scenario :
Wage growth ↑ & Return ↓ → earlier depletion

- As the investment return increases by 1 percentage point, the depletion year is delayed more significantly compared to wage growth rate

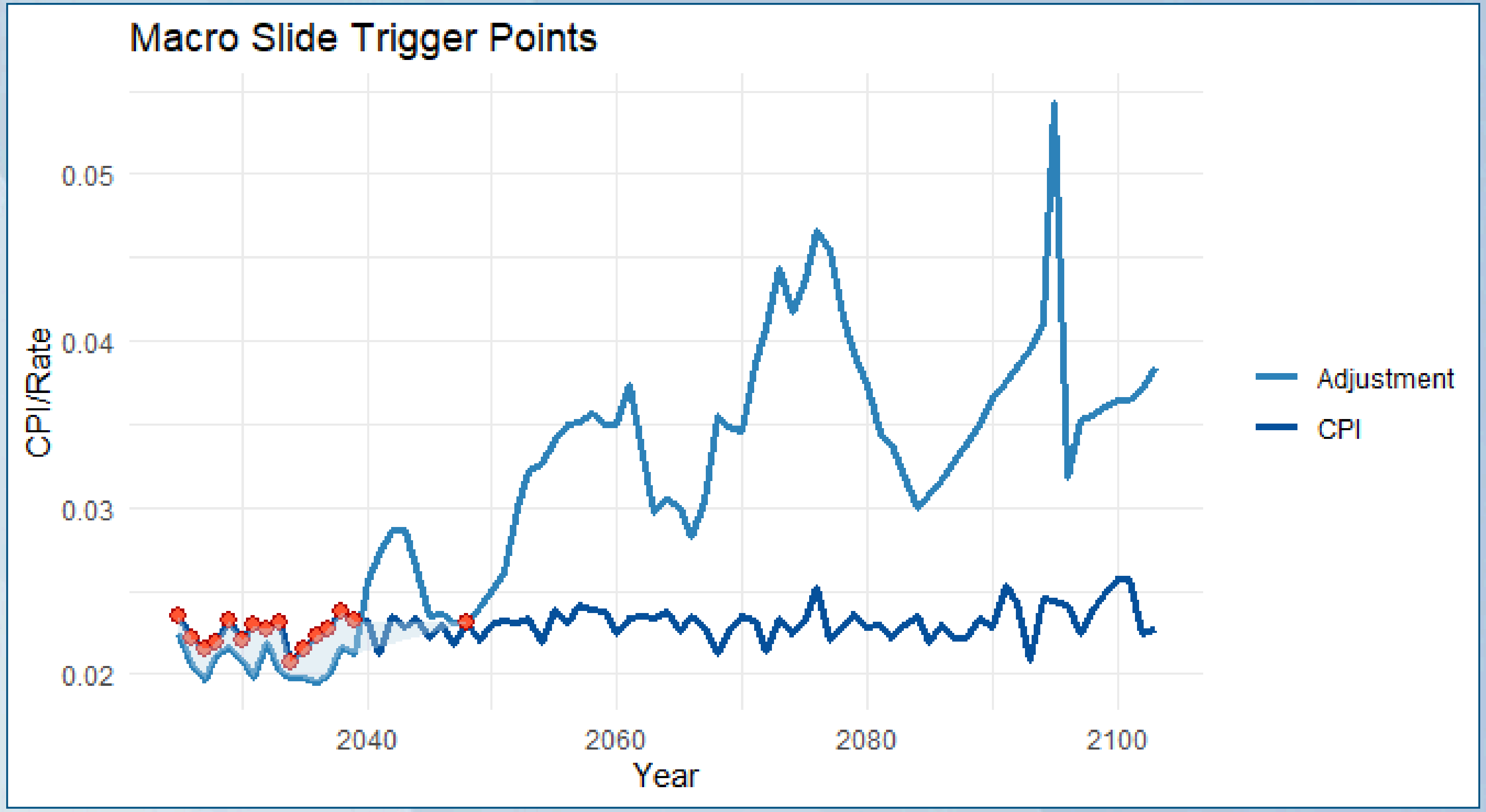
Pension Reform Scenarios



- It shows how the depletion year changes under **different pension reform scenarios**
- Contribution Rate ↑ Replacement rate ↓ Pension age ↑ → delay fund depletion
- **Raising the contribution rate** has the greater impact on delaying fund depletion among all policy options

Pension Reform - Macro Slide

- The macro slide system **automatically reduces pension benefits** when inflation (CPI) rises more than demographic changes
- CPI is modeled using ARMA(1, 1) and simulated.
- The macro slide is triggered at the initial stage
- It does **not significantly change** the depletion year, but leads to wider confidence intervals and one year delay in depletion.



Depletion Year

	Mean	Median	Lower_95	Upper_95
No Macroslide	2075	2076	2073	2078
With Macroslide	2076	2076	2073	2079

Summary

- **GLG fertility model and Affine mortality model** were used to stochastically forecast **population and fund dynamics** with **confidence intervals** for better **risk assessment.**
- Under the current system, the fund is projected to be depleted in 2075 (95% confidence interval: 2072–2077)
- **Sensitivity analysis** shows that how changes in wage growth and investment return can affect **sustainability**.
- **Pension reforms** such as higher contribution rate, lower replacement rate, delayed pension age can **delay depletion.**
- **Increasing the investment return** and **raising the contribution rate** should be prioritized over other policy options.
- Macro slide system has modest effects but contributes to delaying depletion.

Future work

- Apply cohort-based models for both fertility and mortality when complete cohort data becomes available
- Simulate economic variables using stochastic models instead of using fixed assumptions
- Incorporate more complex pension benefit formula to enhance accuracy

Reference

UTS

- Barr, N., & Diamond, P. (2009). Reforming pensions: Principles, analytical errors and policy directions. International social security review, 62 (2), 5–29.
- Baek, H. (2016). Enhancing the reliability of the long-term financial projections on the national pension: Introduction to the stochastic models of oasdi and cpp. Health and Welfare Policy Forum, 234, 21–33
- Blackburn, C., & Sherris, M. (2013). Consistent dynamic affine mortality models for longevity risk applications. Insurance: Mathematics and Economics, 53(1), 64–73.
- Christensen, J. H., Diebold, F. X., & Rudebusch, G. D. (2011). The affine arbitrage-free class of nelson–siegel term structure models. Journal of Econometrics, 164(1), 4–20.
- D, L. R., W, A. M., & Shripad, T. (2003). Stochastic forecasts of the social security trust fund. Michigan Retirement Research Center Research Paper No. WP, 43
- Dahl, M. (2004). Stochastic mortality in life insurance: Market reserves and mortality-linked insurance contracts. Insurance: mathematics and economics, 35(1), 113–136.
- Elzhov, T. V., Mullen, K. M., Spiess, A.-N., & Bolker, B. (2023). Minpack.lm: R interface to the levenberg-marquardt nonlinear least-squares algorithm found in minpack, plus support for bounds [R package version 1.2-4]. https://CRAN.R-project.org/package=minpack.lm
- Francesco, U., M, G. L. P. D., Michael, S., & Yuxin, Z. (2023). Affinemortality: An r package for estimation, analysis, and projection of affine mortality models. Annals of Actuarial Science, 1–26.
- Ivan, T. (2020). A stochastic forecast for the croatian pension system. Public Sector Economics, 44(1), 41–98.
- Kaneko, R. (2003). Elaboration of the coale-mcneil nuptiality model as the generalized log gamma distribution: A new identity and empirical enhancements. Demographic Re search, 9, 223–262.
- National Pension Service. (2023). National Pension Fund Report 2023. National Pension Research Institute.
- Park, Y., Kim, M., & Kim, S. (2013). Probabilistic fertility models and the future population structure of korea. The Korean Association for Survey Research, 14, 49–78.
- Ungolo, F., Garces, L. P. D. M., Sherris, M., & Zhou, Y. (2024). Estimation, comparison, and projection of multifactor age–cohort affine mortality models. North American Actuarial Journal, 28(3), 570–592.
- Yuehong, T., & Xianglian, Z. (2016). Stochastic forecast of the financial sustainability of basic pension in china. Sustainability, 8(1), 46.

THANK YOU