MAT 185 Lecture Notes

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1 Vector Space

Definiton 1.0.1 (Vector Space in $\mathbb{R}$)**.** A vectors space,V, over $\mathbb{R}$ is a collection of **object** $\mathbf{v} \in V$ s.t. the follow axioms are followed

1. **Closure Under Addition** $\mathbf{x}, \mathbf{y} \in V \implies \mathbf{x} + \mathbf{y} \in V$
2. **Order of Addition** $\mathbf{x}, \mathbf{y}, \mathbf{z} \in V \implies (\mathbf{x} + \mathbf{y}) + \mathbf{z} = \mathbf{x} + (\mathbf{y} + \mathbf{z}) \in V$
3. **Existence of Additional Identity** $\exists \mathbf{0} \in V$ s.t. $\mathbf{x} \in V \implies \mathbf{x} + \mathbf{0} = \mathbf{x}$
4. **Existence of Additional Inverse** $\forall \mathbf{x} \in V \exists \textbf{-}\mathbf{x} \in V$ s.t. $\mathbf{x} + \textbf{-}\mathbf{x} = \mathbf{0}$
5. **Closure under Scalar Multiplication** $\forall \mathbf{x} \in V$ and $\forall \alpha \in \mathbb{R} \implies \alpha \mathbf{x} \in V$
6. **Order of Scalar Multiplication** $\forall \mathbf{x} \in V$ and $\alpha, \beta \in \mathbb{R}$, $(\alpha\beta)\mathbf{x} \implies \alpha(\beta\mathbf{x})$
7. **Distributive Scalar Multiplication** $\forall \mathbf{x} \in V$ and $\alpha, \beta \in \mathbb{R}$, $(\alpha + \beta)\mathbf{x} = \alpha\mathbf{x} + \beta\mathbf{x}$
8. **Existence of Multiplicative Identity** $\forall \mathbf{x} \in V$, $1\mathbf{x} = \mathbf{x}$

Note It could be shown that the axiom imply the commutativity of in addition, namely $\forall \mathbf{x}, \mathbf{y} \in \mathbb{R}$, $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$