PHY 294 Lecture Notes

Hei Shing Cheung

Quantum and Thermal Mechanics, Winter 2025 PHY294

The up-to-date version of this document can be found at `https://github.com/HaysonC/skulenotes`

Chapter 1

Quantum Mechanics

1.1 The Schrödinger Equation

1.1.1 Review of Wavefunctions and Operators

Definiton 1.1.1.1 (The Schrödinger Equation)**.** The time-dependent Schrödinger equation for a single non-relativistic particle in one dimension is given by

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) \tag{1.1}$$

where $\Psi(x,t)$ is the wavefunction of the particle, $V(x)$ is the potential energy, m is the mass of the particle, and $\hbar$ is the reduced Planck's constant.

We denote the the momentum operator as

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \tag{1.2a}$$

and the Hamiltonian operator as the sum of kinetic and potential energy operators:

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \tag{1.2b}$$

and the energy operator as

$$\hat{E} = i\hbar\frac{\partial}{\partial t} \tag{1.2c}$$

Definiton 1.1.1.2 (Expectation Value)**.** The expectation value of an operator $\hat{O}$ in a state described by the wavefunction $\Psi(x,t)$ is given by

$$\langle\hat{O}\rangle = \int_{-\infty}^{\infty}\Psi^*(x,t)\hat{O}\Psi(x,t)\,dx \tag{1.3}$$

where $\Psi^*(x,t)$ is the complex conjugate of the wavefunction.

Example 1.1.1.3. The expectation value of the momentum operator $\hat{p}$ is given by

$$\langle \hat{p} \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t) \left(-i\hbar \frac{\partial}{\partial x} \right) \Psi(x,t)\, dx \tag{1.4}$$

We also write this as:

$$\langle \hat{p} \rangle = -i\hbar \langle \Psi | \frac{\partial}{\partial x} | \Psi \rangle \tag{1.5}$$

Definiton 1.1.1.4 (Infinite Square Well Potential)**.** The infinite square well potential is defined as

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < a \\ \infty & \text{otherwise} \end{cases} \tag{1.6}$$

where a is the width of the well.

The normalized stationary state wavefunctions for the infinite square well are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad \text{for } n = 1, 2, 3, \ldots \tag{1.7}$$

with corresponding energy eigenvalues

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2} \tag{1.8}$$

Thus, the momentum eigenvalues are

$$p_n = \pm \frac{n\pi\hbar}{a} \tag{1.9}$$

Then, according to uncertainty principle, the uncertainty in position Δx and uncertainty in momentum Δp satisfy

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{1.10}$$

Chapter 2

Thermal Mechanics