CIV 102 Lecture Notes

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"Structural Engineering is the art and science of designing and molding structures with economy and elegance so that they can safely resist the force that they are subjected"
- Prof. Evan Bentz, 2024

1 Force, Moment, and Geometry

Moment for One Force The moment due to only one force is:

$$\mu = \text{moment} = \vec{F} \times \vec{d_\perp} \tag{1}$$

Where:

$$\vec{d_\perp} = \text{The perpendicular displacement to the center of rotation}$$

Centroid The centroid of a shape with multiple geometries is calculated by:

$$\bar{y} = \frac{\sum_i A_i y_i}{\sum_i A_i} \tag{2}$$

Parallel axis Theorem The moment of inertia is calculated by the following

$$I = \sum_i I_i + \sum_i A_i d_i^2 \tag{3}$$

Where:

$$d_i = \text{The displacement of } \bar{y}_i \text{ to } \bar{y}$$

First Moment of Area (Q) It is expressed as.

$$Q = \sum_i A_i \cdot d_i \tag{4}$$

2 Truss

Area Design area against tension/compression by:

$$A \geq \frac{2F}{\sigma_y} \tag{5}$$

Moment of Intertia Design MOI against Euler's bucking by:

$$I \geq \frac{3FL^2}{\pi^2 E} \tag{6}$$

Radius of Gyration Design radius of gyration against slenderness ratio by:

$$r \geq \frac{L}{200} \tag{7}$$

3 Beam

Navier's equation

$$\sigma = \frac{My}{I} \tag{8}$$

Curvature Equation

$$\phi = \frac{M}{EI} \tag{9}$$

MAT 1 The change in slope between two points is given by the first moment area theorem:

$$\Delta_{AB} = \theta_B - \theta_A = \int_A^B \phi(x)\, dx \tag{10}$$

MAT 2 The deviation of point A from the tangent drawn at point B is given by the second moment area theorem:

$$EIt_{A/B} = \int_B^A xM(x)\, dx = \bar{x}_{AB} \int_B^A M(x)\, dx \tag{11}$$

Shear Stress Given by Jourawski's equation:

$$\tau = \frac{VQ}{Ib} \tag{12}$$

4 Virtual Work

Work In elastic deformation, for internal energy, we have

$$W_{\text{int}} = V\frac{\sigma\epsilon}{2} = \frac{P\Delta}{2}$$

In Hookes Law, for external energy, we have:

$$W_{\text{ext}} = F\Delta r$$

Change of length For a change of length, we have

$$\Delta = \frac{PL}{EA} \tag{13}$$

Deflection to calculate bean deflection, we sum the total virtual force multiplied by extension (worked done by victual forces, virtual work):

$$F^{\star}\Delta_{\hat{r}} = \sum_i P_i^{\star}\Delta_i \tag{14}$$

5 Vibraion

5.1 Free Vibration

Since for change of length, we have:

$$\Delta = \frac{PL}{EA}$$

This could be rewritten as:

$$P = \frac{EA}{L}\Delta$$

For stiffness k, this could be molded as a simple harmonic motion with:

$$k = \frac{EA}{L} \tag{15}$$

For Truss We can use the method of virtual load to determine Δ_0.

For Beam We can use the method of MAT to determine Δ_0.

Point Load The natural frequency is:

$$f_n = \frac{15.76}{\sqrt{\Delta_0}} \tag{16}$$

Uniform Load The natural frequency is

$$f_n = \frac{17.76}{\sqrt{\Delta_0}} \tag{17}$$

Dynamic Amplification Factor Denoted as DAF, for forced vibration at a frequency f, DAF is computed as:

$$\mathrm{DAF} = \frac{1}{\sqrt{(1-(\frac{f}{f_n})^2)^2 + (2\beta\frac{f}{f_n})^2}} \tag{18}$$

Amplification Members would be subject to loads DAF $\times P$ when the load is vibrating.

6 Shear and Local Buckling

Table 1: Summary of plate buckling failure modes

Failure Mode	**Failure Condition**	**Equation**
Buckling of the compressive flange between the webs	$\sigma = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	$\sigma = \frac{My}{I}$
Buckling of the tips of the compressive flange	$\sigma = \frac{0.425\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	$\sigma = \frac{My}{I}$
Buckling of the webs due to the flexural stresses	$\sigma = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	$\sigma = \frac{My}{I}$
Shear buckling of the webs	$\tau = \frac{5\pi^2 E}{12(1-\mu^2)}\left(\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right)$	$\tau = \frac{VQ}{Ib}$

7 Concrete

7.1 Material Properties

Tensile Strength The compressive strength and the tensile strength of concrete is related as follows:

$$f'_t = 0.33\sqrt{f'_c} \tag{19}$$

Young's Modulus The compressive strength and the Young's modulus of concrete is related as follows:

$$E_c = 4730\sqrt{f'_c} \tag{20}$$

Typical Values:
Steel's Young's modulus is usually $E_s = 200,000$ MPa and the yield strength is $f_y = 400$ MPa.

The modular ratio n is given as:

$$n = \frac{E_s}{E_c} \tag{21}$$

The quantity of longitudinal reinforcement ρ is given as:

$$\rho = \frac{A_s}{bd} \tag{22}$$

where:

$$A_s = \text{The area of the steel reinforcements.}$$
$$b = \text{The width of the cross-sectional region of interest.}$$
$$d = \text{The distance from the edge of the region of interest to the opposing reinforcements.}$$

k, Scaling Factor of extreme compression fiber to the neutral axis is given as:

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{23}$$

j, Scaling factor of the flexural lever is given as:

$$j = 1 - \frac{1}{3}k \tag{24}$$

Typical Values:
The scaler factors are usually $k = \frac{3}{8}$ and $j = \frac{7}{8}$.

7.2 Flexural Stress Analysis

In Reinforcement The stress is given by:

$$f_s = \frac{M}{A_s jd} \tag{25}$$

In Concrete The stress is given by:

$$f_c = \frac{k}{1-k} \cdot \frac{M}{nA_s jd} \tag{26}$$

7.3 Shear Stress Analysis

Maximum Shear Stress in Cracked Concrete The maximum shear stress v in a cracked concrete member's web is given by:

$$v = \frac{V}{b_w jd} \tag{27}$$

where b_w is the effective web width.

Buckling Shear Stress The shear stress $v_{\max}$ that causes buckling from diagonal compression is:

$$v_{\max} = 0.25 f'_c \tag{28}$$

Steps for checking Shear Stress Steps by step, if the shear strength does not pass the conditions below:

- **Concrete Crushing Limit** Concrete will crush when:

$$V \geq \min(V_r, V_{\max}) \tag{29}$$

  where:

$$V_{\max} = 0.25 f'_c b_w jd \tag{30}$$

- **Shear Strength of the Member** The shear strength V_r of the member is:

$$V_r = V_c + V_s \tag{31}$$

- **Safety Factor for Design** For design purposes, select V_r such that:

$$V_r = 0.5V_c + 0.5V_s \leq 0.5V_{\max} \tag{32}$$

Then we pass on another case below if it involves V_r

- **Without Reinforcement** If no shear reinforcement is present, the shear strength V_c of the concrete is:

$$V_c = 230\sqrt{f'_c} + 0.9 d b_w jd \tag{33}$$

- **With Minimum Reinforcement** When using shear reinforcement (stirrups), the shear strength V_c of the concrete is:

$$V_c = 0.18\sqrt{f'_c} b_w jd \tag{34}$$

  This equation is valid if:

$$\begin{aligned} \frac{A_v f_y}{b_w s} &\geq 0.06\sqrt{f'_c} \\ \Leftrightarrow s &\leq \frac{A_v f_y}{0.06 \cdot b_w \cdot \sqrt{f'_c}} \end{aligned} \tag{35}$$

  where:

$$\begin{aligned} A_v &= \text{The effective area of stirrups; and} \\ s &= \text{Spacing between stirrups} \end{aligned}$$

- **With Additional Reinforcement** If shear reinforcement is used, the maximum shear force V_s carried is:

$$V_s = \frac{A_v f_y jd}{s} \cot(35^\circ) \tag{36}$$

Design Suggestions for Safety If a design is unsafe, consider the following:

- If $V \geq 0.5V_{\max}$, resize the cross-section.
- If $V \geq 0.5V_c$, add reinforcements.
- If $V \geq 0.5V_c + 0.5V_s$, adjust the spacing s:

$$s = \frac{0.5A_v f_y jd \cot(35^\circ)}{V - 0.5 \times 0.18\sqrt{f_c'} b_w jd} \tag{37}$$

A Formula Sheet

Geometry

Area of a Circle:

$$A = \pi r^2 \tag{1}$$

Circumference of a Circle (in terms of radians):

$$C = 2\pi r \quad \text{(radians in a full circle)} \tag{2}$$

Pythagorean Theorem:

$$c^2 = a^2 + b^2 \tag{3}$$

Area of a Triangle:

$$A = \frac{1}{2}bh \tag{4}$$

Arc Length of a Circle (radians):

$$s = r\theta \quad \text{(where } \theta \text{ is in radians)} \tag{5}$$

Area of a Sector (radians):

$$A = \frac{1}{2}r^2\theta \quad \text{(where } \theta \text{ is in radians)} \tag{6}$$

Volume of a Sphere:

$$V = \frac{4}{3}\pi r^3 \tag{7}$$

Forces

Force:

$$\vec{F} = m\vec{a} \tag{8}$$

Friction:

$$F_f \leq \mu R \tag{9}$$

Moment:

$$\mu = \text{moment} = \vec{F} \times \vec{d}_\perp \tag{10}$$

Load

$$F = \int wdl \tag{11}$$

Momentum and Impulse

$$\Delta P = \Delta mv = F\Delta T \tag{12}$$

Energy

Moment-Energy

$$E = \frac{p^2}{2m} \tag{13}$$

Kinetic, SHM

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2) = -\frac{1}{2}kx^2 \tag{14}$$

Strain Energy

$$\int \sigma d\epsilon \tag{15}$$

Materials

Stress:

$$\sigma = \frac{F}{A} \tag{16}$$

Strain (linear):

$$\epsilon = \frac{\Delta L}{L_0} \tag{17}$$

Young's Modulus (Elastic Modulus):

$$E = \frac{\sigma}{\epsilon} \tag{18}$$

Materials

Spring Constant

$$F = -kx \tag{19}$$

$$k = \frac{EA}{Ł} \tag{20}$$

B Commonly Used Unit Conversions

Force

1 lbf (pound-force) = 4.44822 N
1 N (newton) = 0.224809 lbf

Length

1 km = 1000 m
1 m = 100 cm = 1000 mm
1 inch = 2.54 cm
1 foot = 12 inches = 0.3048 m
1 yard = 3 feet = 0.9144 m
1 mile = 5280 feet = 1.60934 km
1 nm (nautical mile) = 1852 m

Mass

1 ton (metric) = 1000 kg
1 kg = 1000 g
1 g = 1000 mg
1 lb (pound) = 0.453592 kg
1 oz (ounce) = 28.3495 g

Speed

1 km/h = 0.621371 miles/hour
1 m/s = 3.6 km/h

Time

1 hour = 60 minutes
1 minute = 60 seconds
1 day = 24 hours
1 year ≈ 365.25 days

Temperature

°F to °C:

$$T(^\circ\text{C}) = \frac{5}{9}(T(^\circ\text{F}) - 32) \tag{1}$$

°C to °F:

$$T(^\circ\text{F}) = \frac{9}{5}T(^\circ\text{C}) + 32 \tag{2}$$

K to °C:

$$T(^\circ\text{C}) = T(K) - 273.15 \tag{3}$$

°C to K:

$$T(K) = T(^\circ\text{C}) + 273.15 \tag{4}$$

C Commonly Used Constants

Gravitational Constant

$g = 9.81\,\mathrm{m/s^2}$
Standard acceleration due to gravity.

Density of Water

$\rho_{\text{water}} = 1000\,\mathrm{kg/m^3}$
At 4°C.

Density of Air

$\rho_{\text{air}} = 1.225\,\mathrm{kg/m^3}$
At sea level and 15°C.

Young's Modulus for Steel

$E_{\text{steel}} = 200\,\mathrm{GPa} = 200 \times 10^9\,\mathrm{N/m^2}$
Elastic modulus for structural steel.

Poisson's Ratio for Steel

$\nu_{\text{steel}} = 0.3$
Typical Poisson's ratio for steel.

Young's Modulus for Concrete

$E_{\text{concrete}} = 25\,\mathrm{GPa} = 25 \times 10^9\,\mathrm{N/m^2}$
Typical elastic modulus for concrete.

Poisson's Ratio for Concrete

$\nu_{\text{concrete}} = 0.2$
Typical Poisson's ratio for concrete.

Boltzmann Constant

$k = 1.38 \times 10^{-23}\,\mathrm{J/K}$
Used in thermodynamics.

Gas Constant

$R = 8.314\,\mathrm{J/mol{\cdot}K}$
Universal gas constant.

Speed of Light

$c = 3 \times 10^8\,\mathrm{m/s}$
Speed of light in a vacuum.

Atmospheric Pressure

$P_{\text{atm}} = 101.325\,\mathrm{kPa} = 101325\,\mathrm{N/m^2}$
Standard atmospheric pressure at sea level.

Coefficient of Thermal Expansion for Steel

$\alpha_{\text{steel}} = 12 \times 10^{-6}\,\mathrm{K^{-1}}$
Thermal expansion coefficient for steel.

Coefficient of Thermal Expansion for Concrete

$\alpha_{\text{concrete}} = 10 \times 10^{-6}\,\mathrm{K^{-1}}$
Thermal expansion coefficient for concrete.

D Special Trigonometric Relationships

Sine ($\sin\theta$)

θ	$\sin\theta$
$0°$	0
$15°$	$\frac{\sqrt{6}-\sqrt{2}}{4}$
$30°$	$\frac{1}{2}$
$45°$	$\frac{\sqrt{2}}{2}$
$60°$	$\frac{\sqrt{3}}{2}$

Tangent ($\tan\theta$)

θ	$\tan\theta$
$0°$	0
$15°$	$2-\sqrt{3}$
$30°$	$\frac{1}{\sqrt{3}}$
$45°$	1
$60°$	$\sqrt{3}$

Cosine ($\cos\theta$)

θ	$\cos\theta$
$0°$	1
$15°$	$\frac{\sqrt{6}+\sqrt{2}}{4}$
$30°$	$\frac{\sqrt{3}}{2}$
$45°$	$\frac{\sqrt{2}}{2}$
$60°$	$\frac{1}{2}$

Trigonometric Identities

Pythagorean: $\sin^2\theta + \cos^2\theta = 1$

also: $\sec^2\theta - 1 = \tan^2\theta$

Sine: $\sin(\theta + 180°) = -\sin\theta$

also: $\sin(90° - \theta) = \cos\theta$

Cosine: $\cos(\theta + 180°) = -\cos\theta$

Tangent: $\tan(\theta + 180°) = \tan\theta$