

CIV 102 Lecture Notes

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CIV102

The up-to-date version of this document can be found at <https://github.com/HaysonC/skulenotes>

"Structural Engineering is the art and science of designing and molding structures with economy and elegance so that they can safely resist the force that they are subjected"

- Prof. EVAN BENTZ, 2024

1 Force, Moment, and Geometry

Moment for One Force The moment due to only one force is:

$$\mu = \text{moment} = \vec{F} \times \vec{d}_{\perp} \quad (1)$$

Where:

\vec{d}_{\perp} = The perpendicular displacement to the center of rotation

Centroid The centroid of a shape with multiple geometries is calculated by:

$$\bar{y} = \frac{\sum_i A_i y_i}{\sum_i A_i} \quad (2)$$

Parallel axis Theorem The moment of inertia is calculated by the following

$$I = \sum_i I_i + \sum_i A_i d_i^2 \quad (3)$$

Where:

d_i = The displacement of \bar{y}_i to \bar{y}

First Moment of Area (Q) It is expressed as.

$$Q = \sum_i A_i \cdot d_i \quad (4)$$

2 Truss

Area Design area against tension/compression by:

$$A \geq \frac{2F}{\sigma_y} \quad (5)$$

Moment of Inertia Design MOI against Euler's bucking by:

$$I \geq \frac{3FL^2}{\pi^2 E} \quad (6)$$

Radius of Gyration Design radius of gyration against slenderness ratio by:

$$r \geq \frac{L}{200} \quad (7)$$

3 Beam

Navier's equation

$$\sigma = \frac{My}{I} \quad (8)$$

Curvature Equation

$$\phi = \frac{M}{EI} \quad (9)$$

MAT 1 The change in slope between two points is given by the first moment area theorem:

$$\Delta_{AB} = \theta_B - \theta_A = \int_A^B \phi(x) dx \quad (10)$$

MAT 2 The deviation of point A from the tangent drawn at point B is given by the second moment area theorem:

$$EI t_{A/B} = \int_B^A x M(x) dx = \bar{x}_{AB} \int_B^A M(x) dx \quad (11)$$

Shear Stress Given by Jourawski's equation:

$$\tau = \frac{VQ}{Ib} \quad (12)$$

4 Virtual Work

Work In elastic deformation, for internal energy, we have

$$W_{\text{int}} = V \frac{\sigma \epsilon}{2} = \frac{P \Delta}{2}$$

In Hookes Law, for external energy, we have:

$$W_{\text{ext}} = F \Delta r$$

Change of length For a change of length, we have

$$\Delta = \frac{PL}{EA} \quad (13)$$

Deflection to calculate beam deflection, we sum the total virtual force multiplied by extension (worked done by virtual forces, virtual work):

$$F^* \Delta_{\hat{r}} = \sum_i P_i^* \Delta_i \quad (14)$$

5 Vibraion

5.1 Free Vibration

Since for change of length, we have:

$$\Delta = \frac{PL}{EA}$$

This could be rewritten as:

$$P = \frac{EA}{L} \Delta$$

For stiffness k , this could be molded as a simple harmonic motion with:

$$k = \frac{EA}{L} \quad (15)$$

For Truss We can use the method of virtual load to determine Δ_0 .

For Beam We can use the method of MAT to determine Δ_0 .

Point Load The natural frequency is:

$$f_n = \frac{15.76}{\sqrt{\Delta_0}} \quad (16)$$

Uniform Load The natural frequency is

$$f_n = \frac{17.76}{\sqrt{\Delta_0}} \quad (17)$$

Dynamic Amplification Factor Denoted as DAF, for forced vibration at a frequency f , DAF is computed as:

$$\text{DAF} = \frac{1}{\sqrt{(1 - (\frac{f}{f_n})^2)^2 + (2\beta \frac{f}{f_n})^2}} \quad (18)$$

Amplification Members would be subject to loads $\text{DAF} \times P$ when the load is vibrating.

6 Shear and Local Buckling

Table 1: Summary of plate buckling failure modes

Failure Mode	Failure Condition	Equation
Buckling of the compressive flange between the webs	$\sigma = \frac{4\pi^2 E}{12(1 - \mu^2)} \left(\frac{t}{b}\right)^2$	$\sigma = \frac{My}{I}$
Buckling of the tips of the compressive flange	$\sigma = \frac{0.425\pi^2 E}{12(1 - \mu^2)} \left(\frac{t}{b}\right)^2$	$\sigma = \frac{My}{I}$
Buckling of the webs due to the flexural stresses	$\sigma = \frac{6\pi^2 E}{12(1 - \mu^2)} \left(\frac{t}{b}\right)^2$	$\sigma = \frac{My}{I}$
Shear buckling of the webs	$\tau = \frac{5\pi^2 E}{12(1 - \mu^2)} \left(\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2 \right)$	$\tau = \frac{VQ}{Ib}$

7 Concrete

7.1 Material Properties

Tensile Strength The compressive strength and the tensile strength of concrete is related as follows:

$$f'_t = 0.33\sqrt{f'_c} \quad (19)$$

7.2 Flexural Stress Analysis

Young's Modulus The compressive strength and the Young's modulus of concrete is related as follows:

$$E_c = 4730\sqrt{f'_c} \quad (20)$$

Typical Values:

Steel's Young's modulus is usually $E_s = 200,000$ MPa and the yield strength is $f_y = 400$ MPa.

The modular ratio n is given as:

$$n = \frac{E_s}{E_c} \quad (21)$$

The quantity of longitudinal reinforcement ρ is given as:

$$\rho = \frac{A_s}{bd} \quad (22)$$

where:

A_s = The area of the steel reinforcements.

b = The width of the cross-sectional region of interest.

d = The distance from the edge of the region of interest to the opposing reinforcements.

k , Scaling Factor of extreme compression fiber to the neutral axis is given as:

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \quad (23)$$

j , Scaling factor of the flexural lever is given as:

$$j = 1 - \frac{1}{3}k \quad (24)$$

Typical Values:

The scaler factors are usually $k = \frac{3}{8}$ and $j = \frac{7}{8}$.

7.2 Flexural Stress Analysis

In Reinforcement The stress is given by:

$$f_s = \frac{M}{A_s j d} \quad (25)$$

In Concrete The stress is given by:

$$f_c = \frac{k}{1-k} \cdot \frac{M}{n A_s j d} \quad (26)$$

7.3 Shear Stress Analysis

Maximum Shear Stress in Cracked Concrete The maximum shear stress v in a cracked concrete member's web is given by:

$$v = \frac{V}{b_w j d} \quad (27)$$

where b_w is the effective web width.

Buckling Shear Stress The shear stress v_{\max} that causes buckling from diagonal compression is:

$$v_{\max} = 0.25 f'_c \quad (28)$$

Steps for checking Shear Stress Steps by step, if the shear strength does not pass the conditions below:

- **Concrete Crushing Limit** Concrete will crush when:

$$V \geq \min(V_r, V_{\max}) \quad (29)$$

where:

$$V_{\max} = 0.25 f'_c b_w j d \quad (30)$$

- **Shear Strength of the Member** The shear strength V_r of the member is:

$$V_r = V_c + V_s \quad (31)$$

- **Safety Factor for Design** For design purposes, select V_r such that:

$$V_r = 0.5 V_c + 0.5 V_s \leq 0.5 V_{\max} \quad (32)$$

Then we pass on another case below if it involves V_r

- **Without Reinforcement** If no shear reinforcement is present, the shear strength V_c of the concrete is:

$$V_c = 230 \sqrt{f'_c} + 0.9 d b_w j d \quad (33)$$

- **With Minimum Reinforcement** When using shear reinforcement (stirrups), the shear strength V_c of the concrete is:

$$V_c = 0.18 \sqrt{f'_c} b_w j d \quad (34)$$

This equation is valid if:

$$\begin{aligned} \frac{A_v f_y}{b_w s} &\geq 0.06 \sqrt{f'_c} \\ \Leftrightarrow s &\leq \frac{A_v f_y}{0.06 \cdot b_w \cdot \sqrt{f'_c}} \end{aligned} \quad (35)$$

where:

A_v = The effective area of stirrups; and

s = Spacing between stirrups

- **With Additional Reinforcement** If shear reinforcement is used, the maximum shear force V_s carried is:

$$V_s = \frac{A_v f_y j d}{s} \cot(35^\circ) \quad (36)$$

Design Suggestions for Safety If a design is unsafe, consider the following:

- If $V \geq 0.5V_{\max}$, resize the cross-section.
- If $V \geq 0.5V_c$, add reinforcements.
- If $V \geq 0.5V_c + 0.5V_s$, adjust the spacing s :

$$s = \frac{0.5A_v f_y j d \cot(35^\circ)}{V - 0.5 \times 0.18 \sqrt{f'_c} b_w j d} \quad (37)$$

A Formula Sheet**Geometry**

Area of a Circle:

$$A = \pi r^2 \quad (1)$$

Circumference of a Circle (in terms of radians):

$$C = 2\pi r \quad (\text{radians in a full circle}) \quad (2)$$

Pythagorean Theorem:

$$c^2 = a^2 + b^2 \quad (3)$$

Area of a Triangle:

$$A = \frac{1}{2}bh \quad (4)$$

Arc Length of a Circle (radians):

$$s = r\theta \quad (\text{where } \theta \text{ is in radians}) \quad (5)$$

Area of a Sector (radians):

$$A = \frac{1}{2}r^2\theta \quad (\text{where } \theta \text{ is in radians}) \quad (6)$$

Volume of a Sphere:

$$V = \frac{4}{3}\pi r^3 \quad (7)$$

Forces

Force:

$$\vec{F} = m\vec{a} \quad (8)$$

Friction:

$$F_f \leq \mu R \quad (9)$$

Moment:

$$\mu = \text{moment} = \vec{F} \times \vec{d}_\perp \quad (10)$$

Load

$$F = \int w dl \quad (11)$$

Momentum and Impulse

$$\Delta P = \Delta mv = F\Delta T \quad (12)$$

Energy

Moment-Energy

$$E = \frac{p^2}{2m} \quad (13)$$

Kinetic, SHM

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2) = -\frac{1}{2}kx^2 \quad (14)$$

Strain Energy

$$\int \sigma d\epsilon \quad (15)$$

Materials

Stress:

$$\sigma = \frac{F}{A} \quad (16)$$

Strain (linear):

$$\epsilon = \frac{\Delta L}{L_0} \quad (17)$$

Young's Modulus (Elastic Modulus):

$$E = \frac{\sigma}{\epsilon} \quad (18)$$

Materials

Spring Constant

$$F = -kx \quad (19)$$

$$k = \frac{EA}{L} \quad (20)$$

B Commonly Used Unit Conversions**Force**

1 lbf (pound-force) = 4.44822 N
 1 N (newton) = 0.224809 lbf

Length

1 km = 1000 m
 1 m = 100 cm = 1000 mm
 1 inch = 2.54 cm
 1 foot = 12 inches = 0.3048 m
 1 yard = 3 feet = 0.9144 m
 1 mile = 5280 feet = 1.60934 km
 1 nm (nautical mile) = 1852 m

Mass

1 ton (metric) = 1000 kg
 1 kg = 1000 g
 1 g = 1000 mg
 1 lb (pound) = 0.453592 kg
 1 oz (ounce) = 28.3495 g

Speed

1 km/h = 0.621371 miles/hour
 1 m/s = 3.6 km/h

Time

1 hour = 60 minutes
 1 minute = 60 seconds
 1 day = 24 hours
 1 year \approx 365.25 days

Temperature

$^{\circ}\text{F}$ to $^{\circ}\text{C}$:

$$T(^{\circ}\text{C}) = \frac{5}{9}(T(^{\circ}\text{F}) - 32) \quad (1)$$

$^{\circ}\text{C}$ to $^{\circ}\text{F}$:

$$T(^{\circ}\text{F}) = \frac{9}{5}T(^{\circ}\text{C}) + 32 \quad (2)$$

K to $^{\circ}\text{C}$:

$$T(^{\circ}\text{C}) = T(K) - 273.15 \quad (3)$$

$^{\circ}\text{C}$ to K:

$$T(K) = T(^{\circ}\text{C}) + 273.15 \quad (4)$$

C Commonly Used Constants

Gravitational Constant

$g = 9.81 \text{ m/s}^2$
Standard acceleration due to gravity.

Density of Water

$\rho_{\text{water}} = 1000 \text{ kg/m}^3$
At 4°C.

Density of Air

$\rho_{\text{air}} = 1.225 \text{ kg/m}^3$
At sea level and 15°C.

Young's Modulus for Steel

$E_{\text{steel}} = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2$
Elastic modulus for structural steel.

Poisson's Ratio for Steel

$\nu_{\text{steel}} = 0.3$
Typical Poisson's ratio for steel.

Young's Modulus for Concrete

$E_{\text{concrete}} = 25 \text{ GPa} = 25 \times 10^9 \text{ N/m}^2$
Typical elastic modulus for concrete.

Poisson's Ratio for Concrete

$\nu_{\text{concrete}} = 0.2$
Typical Poisson's ratio for concrete.

Boltzmann Constant

$k = 1.38 \times 10^{-23} \text{ J/K}$
Used in thermodynamics.

Gas Constant

$R = 8.314 \text{ J/mol}\cdot\text{K}$
Universal gas constant.

Speed of Light

$c = 3 \times 10^8 \text{ m/s}$
Speed of light in a vacuum.

Atmospheric Pressure

$P_{\text{atm}} = 101.325 \text{ kPa} = 101325 \text{ N/m}^2$
Standard atmospheric pressure at sea level.

Coefficient of Thermal Expansion for Steel

$\alpha_{\text{steel}} = 12 \times 10^{-6} \text{ K}^{-1}$
Thermal expansion coefficient for steel.

Coefficient of Thermal Expansion for Concrete

$\alpha_{\text{concrete}} = 10 \times 10^{-6} \text{ K}^{-1}$
Thermal expansion coefficient for concrete.

D Special Trigonometric Relationships**Sine ($\sin \theta$)**

θ	$\sin \theta$
0°	0
15°	$\frac{\sqrt{6}-\sqrt{2}}{4}$
30°	$\frac{1}{2}$
45°	$\frac{\sqrt{2}}{2}$
60°	$\frac{\sqrt{3}}{2}$

Tangent ($\tan \theta$)

θ	$\tan \theta$
0°	0
15°	$2 - \sqrt{3}$
30°	$\frac{1}{\sqrt{3}}$
45°	1
60°	$\sqrt{3}$

Cosine ($\cos \theta$)

θ	$\cos \theta$
0°	1
15°	$\frac{\sqrt{6}+\sqrt{2}}{4}$
30°	$\frac{\sqrt{3}}{2}$
45°	$\frac{\sqrt{2}}{2}$
60°	$\frac{1}{2}$

Trigonometric Identities

Pythagorean: $\sin^2 \theta + \cos^2 \theta = 1$

also: $\sec^2 \theta - 1 = \tan^2 \theta$

Sine: $\sin(\theta + 180^\circ) = -\sin \theta$

also: $\sin(90^\circ - \theta) = \cos \theta$

Cosine: $\cos(\theta + 180^\circ) = -\cos \theta$

Tangent: $\tan(\theta + 180^\circ) = \tan \theta$