PHY 293 Lecture Notes

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The up-to-date version of this document can be found at `https://github.com/HaysonC/skulenotes`

Chapter 1

Waves

1.1 Harmonic Oscillators

1.1.1 Governing Equations of Harmonic Oscillators

Types of Harmonic Oscillators There are three types of harmonic oscillators: simple, damped, and driven harmonic oscillators. Consider a simple one dimensional harmonic oscillator, they are defined by the following differential equations:

Definiton 1.1.1.1 (Simple Harmonic Oscillator)**.** A simple harmonic oscillator is described by Hooke's law:

$$m\frac{d^2x}{dt^2} + kx = 0 \tag{1.1}$$

where k is the spring constant, m is the mass, and x is the displacement from equilibrium.

Definiton 1.1.1.2 (Damped Harmonic Oscillator)**.** A damped harmonic oscillator is described by the following differential equation, by adding a damping term proportional to $\dot{x}$ to the simple harmonic oscillator equation:

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0 \tag{1.2}$$

where γ is the damping coefficient.

Definiton 1.1.1.3 (Driven Harmonic Oscillator)**.** A driven harmonic oscillator is described by the following differential equation, which includes an external driving force $F(t)$:

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F(t) \tag{1.3}$$

1.1.2 The Wave Equation

Definiton 1.1.2.1 (The Wave Equation)**.** The wave equation is a second-order linear partial differential equation that describes the propagation of waves, such as sound waves, light waves, and water waves, through a medium. In one dimension, it is given by the following PDE:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0 \tag{1.4}$$

where $u(x, t)$ is the wave function, c is the speed of the wave in the medium, x is the spatial coordinate, and t is time.

1.1.3 Simple Harmonic Motion

Definiton 1.1.3.1 (Simple Harmonic Motion)**.** You should have leaned the Hooke's law and Newton's second law, which gives us the equation of motion for a simple harmonic oscillator. The same with the equation (1.1), which can be rewritten as:

$$F = m\ddot{x} = -kx \tag{1.5}$$

By setting $\omega^2 = \frac{k}{m}$, ageneral solution can be written as:

$$x(t) = x_0 + A_1 \cos(\omega t) + A_2 \sin(\omega t) \tag{1.6}$$

where A are the constants determined by the IVP, ω is the angular frequency, and ϕ is the phase constant. x_0 is the equilibrium position where we generally set it to be 0. The unknown constant can be determined by knowing $x, \dot{x}$ at specific times.

Definiton 1.1.3.2 (Period, Frequency, and Angular Frequency)**.** The period T is the time it takes for one complete cycle of the motion, given by:

$$T = 2\pi \sqrt{\frac{m}{k}} \tag{1.7}$$

The frequency f is the number of cycles per unit time, given by:

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \tag{1.8}$$

The angular frequency ω is related to the frequency by:

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \tag{1.9}$$

Example 1.1.3.3. A simple harmonic oscillator consisting of mass $m = 11.0$ kg attached to a spring with spring constant $k = 201\ \mathrm{N\,m^{-1}}$. At time $t = 0$ s the oscillator is at position $x(0) = -0.207$ m and has velocity $v(0) = -1.33\ \mathrm{m\,s^{-1}}$. Determine all coefficients of the equation describing the position $x(t)$ of the oscillator as a function of time, assuming the offset is zero.

To solve for A_1 and A_2, while we assume $x_0 = 0$, we can use the initial conditions:

$$x(0) = A_1 \cos(0) + A_2 \sin(0) = A_1 = -0.207 \text{ m}$$
$$v(0) = -A_1\omega \sin(0) + A_2\omega \cos(0) = A_2\omega = -1.33 \text{ m s}^{-1}$$

We can find ω from the given m and k:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{201 \text{ N m}^{-1}}{11.0 \text{ kg}}} \approx 4.28 \text{ rad s}^{-1}$$

Therefore, we can solve for A_2:

$$A_2 = \frac{v(0)}{\omega} = \frac{-1.33 \text{ m s}^{-1}}{4.28 \text{ rad s}^{-1}} \approx -0.311 \text{ m}$$

Thus, the equation describing the position $x(t)$ of the oscillator as a function of time is:

$$x(t) = -0.207 \cos(4.28t) - 0.311 \sin(4.28t)$$

Theorem 1.1.3.4 (A Trigonometric Identity)**.** We can also express the solution in a more compact form using a single cosine function with a phase shift:

$$x(t) = A\cos(\omega t + \phi) \tag{1.10}$$

where:

$$A = \sqrt{A_1^2 + A_2^2} \tag{1.11a}$$
$$\phi = \tan^{-1}\left(\frac{A_2}{A_1}\right) = \tan^{-1}\left(\frac{-v(0)/\omega}{x(0)}\right) \tag{1.11b}$$

Proof. Let $A = \sqrt{A_1^2 + A_2^2}$ and some ϕ such that $\cos(\phi) = \frac{A_1}{A}$ and $\sin(\phi) = \frac{A_2}{A}$. Then, we can rewrite our original solution as:

$$\begin{aligned} x(t) &= A_1 \cos(\omega t) + A_2 \sin(\omega t) \\ &= A\cos(\phi)\cos(\omega t) + A\sin(\phi)\sin(\omega t) \\ &= A[\cos(\phi)\cos(\omega t) + \sin(\phi)\sin(\omega t)] \\ &= A\cos(\omega t - (-\phi)) \qquad \text{(by the cosine addition formula)} \\ &= A\cos(\omega t + \phi) \end{aligned}$$

□

Example 1.1.3.5. To determine the amplitude A and phase constant ϕ for the oscillator in the previous example, we can use the values of A_1 and A_2 we found:

$$A = \sqrt{(-0.207)^2 + (-0.311)^2} \approx 0.374 \text{ m}$$
$$\phi = \tan^{-1}\left(\frac{-0.311}{-0.207}\right) \approx 0.982 \pm n\pi \approx 4.12 \text{ rad} \quad (n = 1, \text{since both } A_1, A_2 < 0)$$

Therefore, the equation describing the position $x(t)$ of the oscillator as a function of time can also be written as:

$$x(t) = 0.374 \cos(4.28t + 4.12)$$

Chapter 2

Modern Physics