# ECE159 Cheat Sheet

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Fundamentals of Electric Circuits, Winter 2024 ECE 159

## Basic Concepts

*Foundation for all circuit analyses.*

### Voltage, Current & Power

$$V = \int_C \mathbf{E}\cdot d\mathbf{l} = \frac{dW}{dq}, \quad I = \frac{dq}{dt}, \quad P = VI$$

$$W = \int_{t_0}^{t_1} P(t)\,dt$$

### Fields & Energy Density

$$F = qE, \quad U_E = \tfrac{1}{2}CV^2, \quad u_E = \tfrac{1}{2}\varepsilon_r\varepsilon_0 E^2$$

$$U_B = \tfrac{1}{2}Li^2, \quad \nu_B = \tfrac{1}{2\mu_r\mu_0}B^2$$

## Basic DC Analysis

*Use for resistive network calculations.*

### Ohm's Law

$$V = IR$$

### Resistors

$$R_{eq} = \sum R_i, \quad \frac{1}{R_{eq}} = \sum \frac{1}{R_i}, \quad P = I^2R = \frac{V^2}{R}$$

### Division Rules

$$V_i = V\frac{R_i}{R_{eq}}, \quad I_i = I\frac{G_i}{G_{eq}} = I\frac{1/R_i}{\sum 1/R_i}$$

### Kirchhoff's Laws

$$\sum I_{in} = \sum I_{out} \quad \text{(KCL)}, \quad \sum V = 0 \quad \text{(KVL)}$$

### Maximum Power Transfer

*Select load to maximize delivered power.*

$$R_L = R_{th}, \quad P_{\max} = \frac{V_{th}^2}{4R_{th}}$$

*For AC circuit*

$$Z_L = Z_{\text{th}}^{\star}$$

$$P_{\text{avg}} = \frac{|V_{\text{th}}|^2}{8R_L} \quad P_{\max} = \frac{|V_{\text{th}}|^2}{4R_L}$$

## Operational Amplifiers

*Assume ideal unless specs given.*

### Ideal Assumptions

$$V_+ = V_-, \quad I_+ = I_- = 0$$

### Common Configs

$$\begin{aligned} &\text{Inverting: } V_{out} = -\frac{R_f}{R_{in}}V_{in}, \\ &\text{Non-inv.: } V_{out} = \big(1 + \tfrac{R_f}{R_{in}}\big)V_{in}, \\ &\text{Buffer: } V_{out} = V_{in}. \end{aligned}$$

## First-Order Circuits

*Transient response: exponential approach to steady state.*

### RC (Voltage)

$$v(t) = v_\infty + (v_0 - v_\infty)e^{-t/RC}, \quad \tau = RC$$

### RL (Current)

$$i(t) = i_\infty + (i_0 - i_\infty)e^{-tR/L}, \quad \tau = \tfrac{L}{R}$$

## Capacitor

*Blocks DC, smoothing/filtering.*

$$C = \frac{Q}{V}, \quad X_C = \frac{1}{\omega C}$$

$$i(t) = C\frac{dV}{dt},\ V(t) = \frac{1}{C}\int i(t)\,dt$$

$$E_C = \tfrac{1}{2}CV^2$$

## Inductor

*Passes DC, used in chokes/filters.*

$$X_L = \omega L$$

$$v(t) = L\frac{di}{dt},\ i(t) = \frac{1}{L}\int v(t)\,dt$$

$$E_L = \tfrac{1}{2}LI^2$$

## Power

*Compute p(t) in time domain, P in steady-state AC.*

### General

$$p(t) = v(t)i(t), \quad p_R = i^2R = \tfrac{v^2}{R}$$

### AC Average

$$P = \tfrac{1}{2}V_pI_p\cos\theta = V_{rms}I_{rms}\cos\theta$$

### Product-to-Sum

$$\cos A\cos B = \tfrac{1}{2}[\cos(A-B) + \cos(A+B)]$$

## Complex & Reactive Power

*Use phasors for AC analysis, P and Q form a right angled $\Delta$*

$$S = P + jQ, \quad |S| = \sqrt{P^2+Q^2},$$

$$Q = V_{rms}I_{rms}\sin\theta,$$

$$pf = \cos\theta = \frac{P}{|S|}$$

*In General, we have (bold = complex):*

$$\begin{aligned} S = P + jQ &= \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^* \\ &= V_{\text{rms}}I_{\text{rms}}\angle(\theta_v - \theta_i) \\ &= \frac{1}{2}\mathbf{V}_m\mathbf{I}_m^* \\ &= \mathbf{Z}I_{\text{rms}}^2 \\ &= \frac{V_{\text{rms}}^2}{\mathbf{Z}^*} \\ &= \frac{\mathbf{Z}}{|Z|}|S| \end{aligned}$$

## Power Factor

### Power Factor

$\theta = \theta_v - \theta_i$
Leading pf: $\theta < 0$ ; Lagging pf: $\theta > 0$

### PFC

*Add C to // to the load adjust reactive power.*

$$Q_{old} = P\tan(\cos^{-1} pf_{old}),$$

$$Q_{new} = P\tan(\cos^{-1} pf_{new}),$$

$$Q_{corr} = Q_{old} - Q_{new}$$

$$X_C = \frac{V_{rms}^2}{Q_{corr}}, C = \frac{1}{\omega X_C}, Z_{corr} = \frac{1}{j\omega C}$$

## Impedance & Resonance

*Series/parallel combinations; LC resonance.*

$$Z_{ser} = \sum Z_i, \frac{1}{Z_{par}} = \sum \frac{1}{Z_i}, f_0 = \frac{1}{2\pi\sqrt{LC}}$$

*Db formula.*

$$H_{\text{dB}} = 20\log_{10} H$$

## Mutual Inductance

*Phasor-domain relations and dot-convention.*

$$V_1 = j\omega L_1 I_1 + j\omega M\,I_2,$$
$$V_2 = j\omega M\,I_1 + j\omega L_2 I_2$$

### Dot Convention:

- If $I_1$ enters the dotted terminal of coil 1, the induced voltage in coil 2 at its dotted terminal is $+j\omega M I_1$.
- If $I_1$ enters the undotted end, the induced voltage at the dotted end of coil 2 is $-j\omega M I_1$.
- Polarity: "dots" mark the same instantaneous polarity of induced voltages.

## Maths

*Useful tricks and identities for signal and circuit math.*

### Trig Conversions

$$\sin(x) = \cos\left(\frac{\pi}{2} - x\right), \quad -\sin(x) = \cos\left(x + \frac{\pi}{2}\right)$$

$$\cos(x) = \sin\left(\frac{\pi}{2} - x\right), \quad -\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

### Cramer's Rule (2x2)

*Solve linear systems:* $Ax = b$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$

$$\Delta = ad - bc, \quad x = \frac{ed - bf}{\Delta}, \quad y = \frac{af - ec}{\Delta}$$

### Complex Number Tricks

- $z = a + jb \quad$ (Rectangular)
- $z = re^{j\theta} = r(\cos\theta + j\sin\theta)$
- $|z| = \sqrt{a^2 + b^2}, \quad \theta = \tan^{-1}(b/a)$
- $z^\star = a - jb \quad$ (Conjugate)
- $\frac{1}{j} = -j, \quad j^2 = -1$
- Multiplying: $z_1 z_2 = r_1 r_2 e^{j(\theta_1 + \theta_2)}$
- Dividing: $\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$