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Modeling an expert system for learning to solve mathematical problems

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Abstract— **Teaching and learning mathematics has always posed a challenge for educators and learners. Solving mathematical problems, in particular, requires a complex combination of cognitive skills, problem-solving strategies, and effective pedagogical approaches. In this context, recent advancements in cognitive models, problem-solving models, and didactic models offer new opportunities to enhance the teaching and learning of mathematics. The integration of these models enables the design of learning environments that promote more effective acquisition of problem-solving skills.**

In this work, we propose an innovative approach to predict an expert system model aimed at aiding in the development of mathematical problem-solving skills. Drawing on a comparative and cross-disciplinary analysis of advancements in cognitive models, problem-solving models, and didactic models, our goal is to design an expert system capable of understanding the mental processes involved in problem-solving, offering effective strategies for approaching different categories of mathematical problems, and providing personalized and tailored instruction to meet the individual needs of learners.

We will describe the key concepts and methodologies used in each domain, as well as our approach to integrating and synergizing these perspectives. Finally, we will discuss the potential implications of our predictive model for improving the teaching and learning of mathematics.

Keywords— Problem solving; Expert system; Modeling; Skills; Cognitive; Metacognitive

I. Introduction

The teaching of mathematics is at the heart of educational concerns, being fundamental to the development of learners' cognitive and analytical skills. In this context, advances in cognitive science, problem-solving theory, and the didactics of mathematics education are opening up new perspectives for improving pedagogical practices. In particular, the integration of technology and expert systems in mathematics education offers innovative opportunities to support the learning process.

Cognitive sciences provide an in-depth theoretical framework for understanding the mental processes involved in mathematical problem solving. These advances make it possible to explore the underlying mechanisms of mathematical thinking, from problem perception to decision-making during problem solving. At the same time, problem-solving theory offers structured methodological approaches to effectively tackle mathematical challenges. It proposes formal strategies for breaking down complex problems, using search algorithms, and applying heuristics to arrive at solutions.

In addition, advances in the didactics of mathematics education highlight the importance of designing effective pedagogical strategies to support student learning. These strategies aim to create stimulating learning environments, adapted to learners' individual needs, and to promote their engagement and motivation. Thus, the integration of these three areas offers a comprehensive framework for the design of innovative and effective teaching methods in the field of mathematics.

In this context, the use of technology and expert systems represents a major breakthrough. Expert systems are computer programs capable of reproducing human expertise in a specific field. When applied to mathematics teaching, these systems can offer personalized, adaptive support to learners, identifying their individual needs and offering relevant solutions and advice.

This article sets out to explore how advances in cognitive science, problem-solving theory and mathematics education didactics can be integrated in the design of a predictive expert system model for the development of mathematical problem-solving skills. By examining the theoretical foundations and practical applications of this approach, we hope to contribute to the evolution of mathematics teaching practices and the improvement of learner success in this crucial area.

II. Literature review

Several articles propose innovative didactic strategies, including the flipped classroom, for teaching mathematics, highlighting the use of online platforms. These approaches are seen as crucial, particularly in the context of the pandemic, and are particularly suited to the new generation, accustomed to digital environments. As well as enhancing mathematical understanding, these strategies also promote the development of social skills by encouraging interaction and socialization in the learning process.

Take, for example, Khan Academy's individual initiative, which has grown into a global platform with over 150 professionals dedicated to providing free quality education. It integrates pedagogical theories such as connectivism and meaningful learning, using technology to teach mathematics [1]. Lessons are designed to be interactive and adaptive, taking into account each learner's level of understanding. According to Tang and colleagues [2], Khan Academy's pedagogical approach draws on research in cognitive science and didactics to deliver effective, personalized learning.

Students prepare the material at home and tackle the most complex concepts in class, optimizing classroom time, meeting individual student needs, and promoting collaborative work. The methodology used is problem-solving, supported by recent research highlighting its importance [3].

Khan Academy offers various resources, such as videos, articles and mathematical exercises, enabling users to learn at their own pace. A study conducted by Ruiz and colleagues in 2023 [4], assessing Khan Academy's impact on mathematics learning among UNDAC students, revealed significant improvements in mathematical performance, with 70% of students achieving high levels of mathematical proficiency and expressing high satisfaction with the platform. In addition, the study showed that Khan Academy has a significant and positive influence on mathematics learning among university students, promoting not only mastery of content but also autonomy in learning. These results support the adoption of innovative educational technologies, particularly in STEM fields.

Coursera is another leading online learning platform, offering a wide range of courses, specializations and degree programs from the world's top universities and educational institutions. Founded in 2012 by Stanford University professors Andrew Ng and Daphne Koller, Coursera aims to provide accessible, high-quality education to learners around the world.
Its courses are developed with best practices in teaching and learning in mind, incorporating elements of cognitive science and learning theory [5]. The platforms also use data analysis techniques to assess the effectiveness of teaching methods and adapt them accordingly [6].

In addition to individual courses, Coursera offers specializations, which are series of courses designed to help learners master a specific skill or field.
The platform allows learners to study at their own pace while offering options for scheduled sessions with deadlines. It has a dynamic community of learners from all over the world and offers learning solutions tailored to businesses, governments and non-profit organizations. Overall, Coursera offers high-quality, accessible and flexible education to a wide range of learners [7].

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According to Richard and colleagues [8], TURING's design is based on problem-solving theory, integrating geometric problem-solving strategies, offering structured methods for tackling mathematical challenges, and encouraging students to develop analytical and reasoning skills. On the didactics of teaching by integrating recent research in mathematics didactics with the possibilities of computerized human learning environments. It also proposes a progression adapted to each student's level and provides personalized feedback to encourage learning. And also on artificial intelligence, using machine learning and data analysis algorithms to personalize each student's learning experience, identifying their needs, preferences, and errors. It also adjusts the content and difficulty of exercises according to the student's performance.

DreamBox, meanwhile, is an adaptive math learning platform designed for students from kindergarten to college level. Using adaptive learning algorithms, the platform personalizes each student's learning path according to his or her skill level, strengths and weaknesses.

DreamBox's pedagogical content is based on research in neuroscience and the psychology of learning, aimed at maximizing teaching effectiveness [9].

Lessons are presented in the form of interactive games to make learning mathematics more attractive and engaging. DreamBox provides immediate feedback to students throughout their activities, encouraging reflection and independent learning. In addition, the platform provides teachers with tools to track student progress, enabling them to monitor performance and provide targeted support.

DreamBox aims to provide a personalized and effective mathematical learning experience by combining adaptability, playful engagement, immediate feedback and progress tracking all by often incorporating problem-solving elements and didactic strategies based on best practice in mathematics education [10].

Although many e-learning platforms incorporate principles from cognitive science, problem-solving theory and educational didactics into their designs, few are specifically designed to draw explicitly on these three fields in a cross-comparative way.

In this study, our aim is to design a predictive model of an expert system designed to support the development of problem-solving skills. To this end, we draw on recent advances in cognitive science, problem-solving theory and mathematics education didactics.

III. Methodology

To achieve our goal, we adopted a method of cross-comparative analysis of the main information processing models and emerging cognitive models most cited in the literature. This method consists of eight steps (Figure 1).

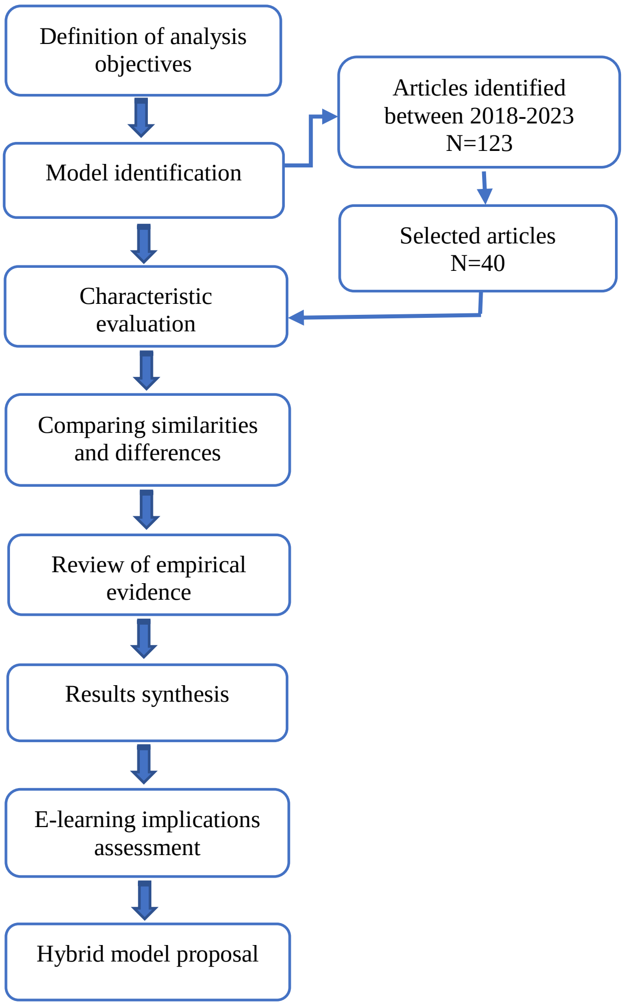

Figure 1. Comparative and cross-analysis methodology

The comparative and cross-analysis method is a research approach widely used in various fields, including cognitive science and adaptive e-learning [11],[12].

The comparative and cross-analysis of different cognitive models, problem-solving models and associated didactic models is an essential approach to better understand the underlying mental processes and mechanisms involved in cognitive problem-solving processes [13], [14], [15], [16].

Furthermore, the aim of our analysis is to compare the different models in order to deduce the characteristics of a hybrid model that combines the strengths of each model, and which can form the basic foundation for the design of an expert system to help develop mathematical problem-solving skills.

We identified 123 Scopus, Web Sciences, Ebsco and direct sciences research articles dealing with the involvement of information processing and emerging models in the design and evaluation of e-learning systems between the years 2018 and 2023. We have selected 40 articles explicitly involving these models.

Each model is examined in detail to understand its fundamental principles, basic assumptions, operating mechanisms, strengths and weaknesses, and potential applications in the design of online courses and e-learning systems. The characteristics of each model are compared to identify similarities and differences between them. The comparison is made at the level of the model's structure, the cognitive processes involved and its practical applications in the design of e-learning systems and experiences.

Empirical evidence from experimental research, case studies or other sources such as meta-analysis studies is analyzed to assess the validity and applicability of different models in real e-learning contexts. The results of the analysis are synthesized to provide an overview of similarities, differences and general trends among the models examined. The practical and theoretical implications of the results are discussed, including their implications for deriving a hybrid model for the design of an adaptive, aligned and effective e-learning system.

IV. Result and Discussion

To deduce an expert system model that helps develop problem-solving skills, we examined cognitive models, problem-solving models and mathematics didactics models in a comparative and cross-referenced way.

Our comparative and cross-analysis is presented in the following table:

TABLE I. COMPARATIVE AND CROSS-ANALYSIS OF COGNITIVE MODEL, PROBLEM-SOLVING MODEL AND MATHEMATICS DIDACTICS MODEL

	Cognitive models	**Problem-solving models**	**Didactic models**
Features	- Analysis of the mental processes involved in problem solving, - Identification of cognitive biases and limitations in decision-making and problem-solving.	- Provides formal structures for describing problem-solving processes, - Identifies heuristic strategies for efficiently solving different types of problems.	- Focus on methods and strategies for effectively teaching problem-solving skills - Use of active learning techniques to engage learners in the problem-solving process and promote knowledge retention
Objective	Understanding how individuals approach problems	Describe effective problem-solving steps and strategies	Teach and learn mathematics effectively, with an emphasis on problem solving
Main focus	Mental processes involved in problem solving (decision-making, memory, learning, perception)	Steps and strategies for solving a problem	Methods for teaching and learning mathematics
Main approaches	Analysis of cognitive processes involved in problem solving	Breaking down the problem, using heuristics, thinking about alternative solutions	Presentation of mathematical concepts, guidance through problems, promotion of understanding and independent solving.
Key theories	Gestalt problem-solving theory, Newell and Simon problem-solving theory	Step-by-step approach to problem solving. Polya's problem-solving method, Search algorithms	Problem-based learning theory, inquiry-based learning, constructivist teaching methods
Methodology	Cognitive psychology	Structured approaches	Pedagogy and didactics
Similarities	In-depth understanding of resolution mechanisms	Common goal of supporting the development of problem-solving skills	Interest in teaching and learning to solve problems
Differences	Focuses on mental processes	Focuses on resolution steps and strategies	Focusing on the design and implementation of teaching strategies
Practical test	- Analysis of cognitive performance in problem solving using experimental tasks and psychological tests - Cartographic maps - Brain imaging techniques to observe the brain regions activated during problem solving.	- Develop algorithmic models to solve specific problems - Design laboratory experiments in which participants are confronted with problems of various kinds and observe their solving strategies.	- Conduct longitudinal studies to evaluate the effectiveness of different problem-solving teaching methods on skill acquisition - Design curricula based on real cases or simulations to encourage active learning and problem-solving by students
Contribution to the expert system model	Insights into the cognitive processes involved in problem solving	Problem-solving strategies and methods to be integrated into the expert system	Pedagogical techniques to guide learners through the problem-solving process
Integration with AI	Analysis of mental processes to adapt assistance	Use of algorithms to provide resolution suggestions	Support tailored to individual user needs

From this table, we can see that each model brings a unique perspective to the understanding and development of problem-solving skills:

Cognitive models focus on the mental processes involved in problem-solving, offering an in-depth understanding of underlying mechanisms such as perception, attention, memory and decision-making. These models provide an essential theoretical basis for understanding how individuals approach and solve problems, while identifying the cognitive biases and limitations that can affect their performance.

On the other hand, problem-solving models offer formal structures and methodological approaches for efficiently solving different types of problems. Whether through search algorithms or heuristic strategies.

These models provide concrete tools to guide the resolution process and overcome potential obstacles. Their use extends to various technical fields, where they are applied to solve specific problems in computer science, engineering and other disciplines.

As for didactic models, they focus on methods and strategies for effectively teaching problem-solving skills. Using pedagogical approaches such as problem-based learning and constructivist methods. These models encourage the active engagement of learners in the learning process. They also promote the use of innovative educational technologies, such as simulations and serious games, to create interactive and stimulating learning environments.

The similarities between these models lie in their common aim of improving individuals' understanding and performance in problem solving, although they focus on different aspects of this process. Cognitive models provide a theoretical framework for understanding the underlying mechanisms of problem solving, while problem-solving models offer methodological approaches to solving these problems. Didactic models, meanwhile, provide strategies for teaching these skills effectively.

The combination of these perspectives offers a holistic approach to the development of an expert system aimed at improving problem-solving skills that includes both the cognitive processes involved in problem-solving and the best practices for teaching these skills. Cognitive models provide a theoretical framework for understanding how individuals approach and solve problems, which can guide the design of algorithms and problem-solving strategies in the expert system.

Problem-solving models offer concrete methods for structuring the solving process, identifying key steps and effective techniques for overcoming obstacles.

Didactic models help design learning environments and teaching strategies that promote the acquisition and development of problem-solving skills in expert system users.

As a result, integrating these perspectives into the design of an expert system creates a powerful learning tool that provides personalized and effective support to improve users' problem-solving skills.

This expert system becomes more than just a problem-solving tool; it becomes an interactive, adaptive learning environment. Thanks to its ability to understand users' cognitive processes, apply varied problem-solving approaches and provide effective teaching strategies, the expert system is able to help users develop and improve their problem-solving skills significantly.

Indeed:

Adaptation to individual needs represents a crucial element in the design of a model that draws on advances in cognitive science, problem-solving theory and the didactics of mathematics teaching. This capability enables the expert system to adjust its teaching strategies and problem difficulty levels according to the specific characteristics and skills of each learner, thus offering differentiated and personalized learning.

To implement this adaptation, the model could make use of various techniques. Firstly, it could collect and analyze data on the learner's past performance, such as answers to exercises, response times, frequent errors, and levels of confidence in answers. This information would help to understand each learner's strengths and weaknesses.

In addition, before learning begins, the model could administer a diagnostic assessment to evaluate the learner's skill level in different mathematical areas. Based on the results, the system could identify gaps and specific needs for each learner.

Once this data has been collected, the expert system can adapt its teaching strategies by proposing methods and examples that match each learner's learning preferences and cognitive styles. For example, some learners might benefit from visual teaching methods, while others might prefer more conceptual approaches.

Finally, depending on the learner's level of competence and comfort with mathematical concepts, the model could adjust the difficulty of the problems presented. Less advanced learners could start with simpler problems and gradually progress to higher levels of difficulty as they gain confidence and competence.

On the other hand, this system will predict the steps involved in solving mathematical problems. This capability represents a crucial aspect in the design of the expert system. This feature enables the expert system to anticipate the strategies that learners are likely to adopt when tackling mathematical problems, based on cognitive models and predictive analyses.

To implement this prediction, the model could take advantage of advances in cognitive science to understand the mental processes involved in mathematical problem solving. By analyzing data on learners' past performance, such as their choices and approaches when solving problems, the system could identify recurring patterns and trends in their solving approaches.

Based on this information, the model could use predictive analysis techniques to anticipate the specific steps learners are likely to take when faced with similar problems in the future. For example, it could predict whether a learner will first analyze the problem, identify relevant data, choose a solution strategy, or perform specific calculations.

This predictive ability could also be fueled by cognitive models that describe the underlying mental processes involved in mathematical problem solving.

Feedback and interactive support are also essential elements in fostering the learning and development of mathematical problem-solving skills. This feature aims to provide immediate, personalized feedback to learners throughout their solving process, identifying errors and offering suggestions for improving their understanding and skills.

To implement this interactive feedback system, the model could be designed to analyze learners' actions and responses in real time as they solve mathematical problems. It could identify common errors, conceptual gaps and missed steps in their solving process.

Based on this analysis, the expert system could then provide immediate and specific feedback to each learner, highlighting mistakes made and explaining misunderstood concepts. It could also offer suggestions for correcting errors, by proposing alternative resolution approaches, additional explanations or illustrative examples.

Finally, evaluation and monitoring of learners' progress in problem solving is a key element of this system. This feature aims to track learners' performance over time, assess their progress in problem solving and identify areas where improvement is needed.

To implement this assessment, the model could collect and analyze data on learners' performance when solving mathematical problems. This data could include information on answers provided, response times, levels of success, and solving approaches used.

By tracking learners' performance over time, the model could identify trends and patterns in their problem-solving progress. It could highlight areas where learners show consistent improvement, as well as those where difficulties persist.

By identifying areas where improvement is needed, the model could provide specific feedback and suggestions to help learners strengthen their skills in these areas. It could also recommend additional learning activities or complementary resources to support their development.

In addition, tracking learners' progress would enable the model to provide teachers and educators with information on the individual needs of each learner, so that teaching and pedagogical interventions could be better tailored to each student's specific needs.

Overall, by combining these features, the model becomes a powerful tool to support the teaching and learning of mathematics by offering individualized support and resources tailored to the specific needs of each learner. This opens up new possibilities for more effective, inclusive and learner-centered mathematics education.

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