Math 4423 Assignment 1

* I submit as LaTeX for the bonus points XD

$Q1)$

$H_0: m = 5$

$H_1: m> 5$

$\alpha = 0.05$

$Sorted\space Statistics: 1,2,3,4,4,7,9,10,12,15$

$a)\space \text{Sign Test}$

$S=\sum_{i=1}^{10}I{X_i>m}=5$

$S \sim Bin(10, \frac{1}{2})$

P-Value

$P(S\geq 5\mid H_0)$

$=P(Bin(10, \frac{1}{2})\geq5)$

$=\binom{10}{5}(\frac{1}{2})^{10}+\binom{10}{5}(\frac{1}{2})^{10}+\binom{10}{7}(\frac{1}{2})^{10}+\binom{10}{8}(\frac{1}{2})^{10}+\binom{10}{9}(\frac{1}{2})^{10}+\binom{10}{10}(\frac{1}{2})^{10}$

$=(\frac{1}{2})^{10}\times{(252+210+120+45+10+1)}$

$=0.623046875$

$\gt 0.05$

$\text{We do not have enough evidence to reject the null hypothesis}$

$b)\space \text{Sign Test with Normal Approximation}$

$S=\sum_{i=1}^{10}I{X_i>m}=5$

$S \sim Bin(10, \frac{1}{2})$

$\text{By Central Limit Theorem:}$ $$\frac{S-np}{\sqrt{np(1-p)}}\to N(0,1)\space \text{for Large n}$$

$\text{P-Value}\ P(S\geq 5\mid H_0)\ =P(Bin(10, \frac{1}{2})\geq5)\ \approx 1-\Phi(\frac{5+\frac{1}{2}-\frac{10}{2}}{\sqrt{10}/2}) \\approx 1-\Phi(0.316227766) \\approx 0.1255 \\gt 0.05 $

$\text{We do not have enough evidence to reject the null hypothesis}$

$c)\space \text{Wilxocon Sign Rank Text}$

X_i	**X_1**	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**X_7**	**X_8**	**X_9**	**X_{10}**
obs	3	4	7	10	4	12	1	9	2	15
X_i-m_0	-2	1	2	5	-1	7	-4	4	-3	10
$\text{abs of }{X_i-m_0}$	2	1	2	5	1	7	4	4	3	10
$\text{R of } X_i-m_0$	$\frac{3+4}{2}$	$\frac{1+2}{2}$	$\frac{3+4}{2}$	8	$\frac{1+2}{2}$	9	$\frac{6+7}{2}$	$\frac{6+7}{2}$	5	10
$\text{Signs of } X_i-m_0$	-	+	+	+	-	+	-	+	-	+

X_i	**X_1**	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**X_7**	**X_8**	**X_9**	**X_{10}**
R_i	-3.5	1.5	3.5	8	-1.5	9	-6.5	6.5	-5	10

$W=\sum_{i=1}^{10}R_iI_i\ =1.5+3.5+8+9+6.5+10\=38.5 \\text{By looking up the table: We know that the critical value is 10 when n=10, }\alpha=0.05\ \because 38.5>10\ \therefore \text{We do not have enough evidence to reject the null hypothesis}$

$d) \space\text{Wilxocon Sign Rank Test with Normal Approximation}$

$W=\sum_{i=1}^{10}R_iI_i\ =1.5+3.5+8+9+6.5+10\=38.5$

$\text{P-Value}\ P(W\geq 38.5)\ \approx 1-\Phi(\frac{38.5-\frac{10(10+1)}{2}-0.5}{\sqrt{\frac{1}{24}(10)(10+1)(2\times10+1)}})\ \approx0.6423\gt0.05\ \therefore \text{We do not have enough evidence to reject the null hypothesis}$

$e)\text{ Parametric t-test}\ \text{We Assume the data follows Normal Distribution}$

$X_1..X_{10} \sim i.i.d\space N(\mu, \sigma ^ 2)$

$ \begin{cases} H_0: m = 5\ H_1: m> 5 \end{cases}\ $

T-Statistics $$T=\frac{\sqrt{n}(\bar{X}-5)}{\sigma}$$ $$\hat{\sigma}=\sqrt{\frac{1}{n-1}\sum{(X_i-\bar{X})^2}}$$ $t_{obs}=\frac{\sqrt{10}(6.7-5)}{4.667856991}=1.151678818$

$T\sim t(n-1)\sim t(9)$

$\text{P-Value}\ P(T\geq1.1517)\ \approx 0.13956\ \gt0.05\ \therefore \text{We do not have enough evidence to reject the null hypothesis} $

$\text{f) Yes, five approaches reach the same conclusion of not rejecting the null hypothesis of the median being 5 among the population}$

$\text{Q2) Exact Distribution of Wilcoxon sign Rnak Test}$

n=5						**W**	**Prob**
signs of R_i	1	2	3	4	5		
	+	+	+	+	+	15	1/32
	-	+	+	+	+	14	1/32
	+	-	+	+	+	13	1/32
	-	-	+	+	+	12	1/32
	+	+	-	+	+	12	1/32
	-	+	-	+	+	11	1/32
	+	-	-	+	+	10	1/32
	-	-	-	+	+	9	1/32
	+	+	+	-	+	11	1/32
	-	+	+	-	+	10	1/32
	+	-	+	-	+	9	1/32
	-	-	+	-	+	8	1/32
	+	+	-	-	+	8	1/32
	-	+	-	-	+	7	1/32
	+	-	-	-	+	6	1/32
	-	-	-	-	+	5	1/32
	+	+	+	+	-	10	1/32
	-	+	+	+	-	9	1/32
	+	-	+	+	-	8	1/32

n=5						W	Prob
	-	-	+	+	-	7	1/32
	+	+	-	+	-	7	1/32
	-	+	-	+	-	6	1/32
	+	-	-	+	-	5	1/32
	-	-	-	+	-	4	1/32
	+	+	+	-	-	6	1/32
	-	+	+	-	-	5	1/32
	+	-	+	-	-	4	1/32
	-	-	+	-	-	3	1/32
	+	+	-	-	-	3	1/32
	-	+	-	-	-	2	1/32
	+	-	-	-	-	1	1/32
	-	-	-	-	-	0	1/32

$\text{From the Derived Exact Distribution:}\ P(W\leq3)=5/32=0.15625\ P(W\geq8)=16/32=0.5 $

$Q3)$

Statistics					
X	17.2	21.6	19.5	19.0	22.0
Y	18.3	20.8	20.9	21.2	22.7
Z=X-Y	-1.1	0.8	-1.4	-2.2	-0.7

$\text{a) By Order Statistics,}\ \hat{m_0}=z_{(3)}=-1.1\ P(S<K_{\alpha})=\frac{0.05}{2}-0.025\ \sum_{i=0}^{k_{\alpha}-1}\binom{5}{i}=0.025\ k_{\alpha}=1 \text{ is closest to 0.025}\ \therefore \text{CI of median=}[z_{(1)}, z_{(5-1+1)}]=[-2.2, 0.8] $

$b) \text{ Hodgers-Lehmann estimator}\ \hat{}m_{HL}=\text{MD of all Walsh Average}\ \text{Walsh Average for {-2.2, -1.4, -1.1, -0.7, 0.8}:}\ {-2.2, -1.8, -1.65, -1.45, -1.4, -1.25, -1.1, -1.05, -0.9, -0.7, -0.7, -0.3, -0.15, 0.05, 0.8}\ \hat{m_{HL}}=X_{(\frac{15+1}{2})}=X_{(8)}=-1.05\ \text{By Tukey's method of CI}\ P(W<K_{\alpha})=0.05/2\ \text{when } k_{\alpha} \text{is 1, it is closest to 0.025,}\ \therefore \text{the median 95% CI is [-2.2, 0.8]} $