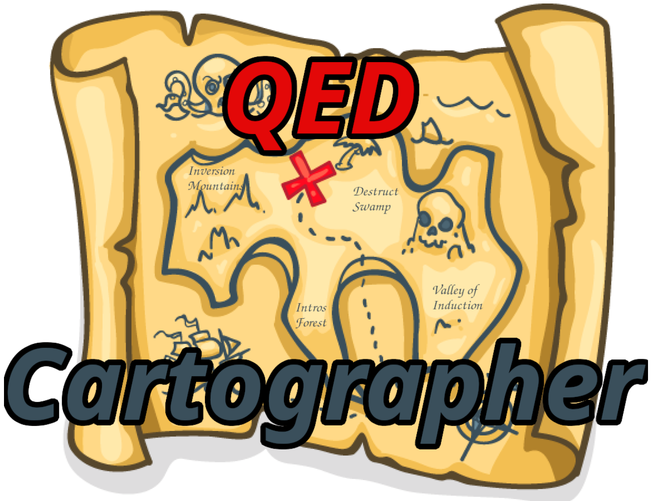
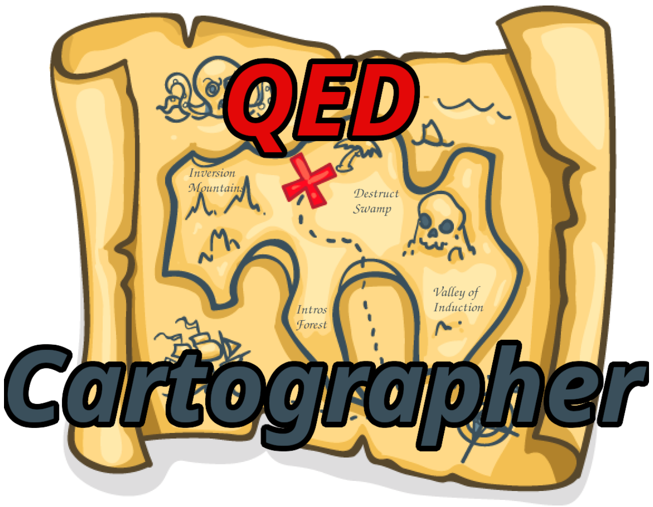


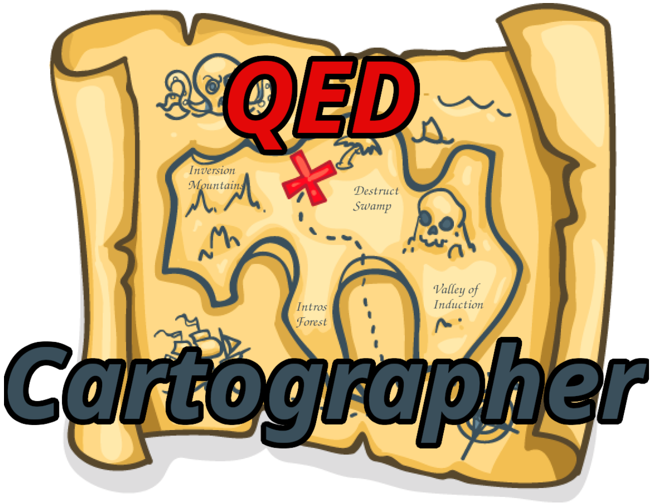
QED Cartographer

Alex Sanchez-Stern, Abhishek Varghese, Zhanna Kaufman,
Dylan Zhang, Talia Ringer, Yuriy Brun



QED Cartographer

Automating Formal Verification with
Reward-Free Reinforcement Learning



Automating Formal Verification with Reward-Free Reinforcement Learning

Formal Verification

writing proofs about programs!

Formal Verification

writing proofs about programs!

more reliability than testing!

☺

Formal Verification

writing proofs about programs!

more reliability than testing!

☺

labor intensive

☹

Formal Verification

writing proofs about programs!

more reliability than testing!

☺

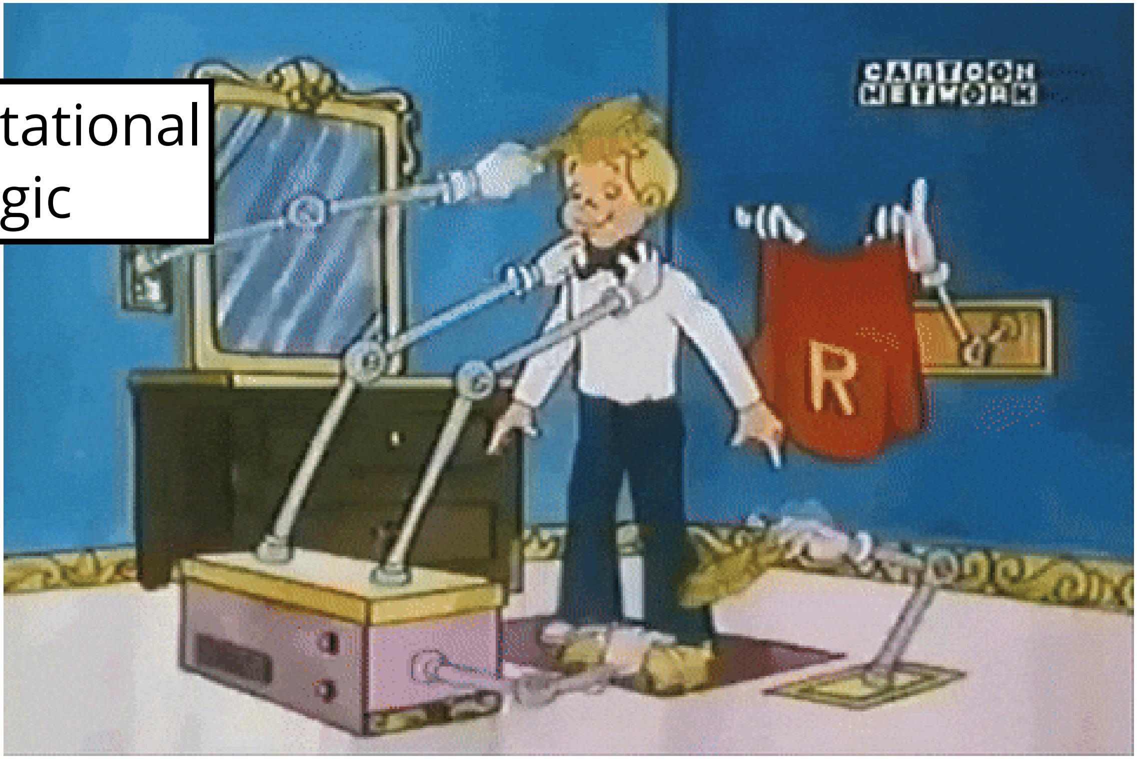
labor intensive

requires specialized expertise

☹

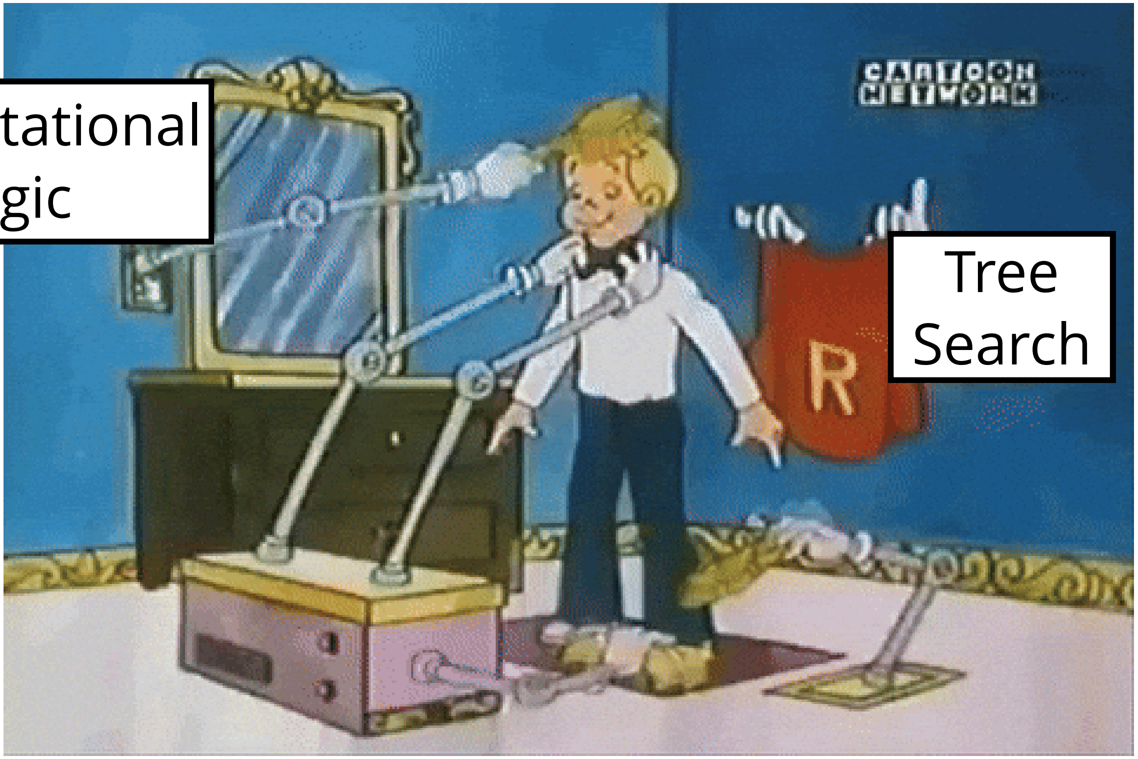
Automating Formal Verification

Automating Formal Verification



Computational Logic

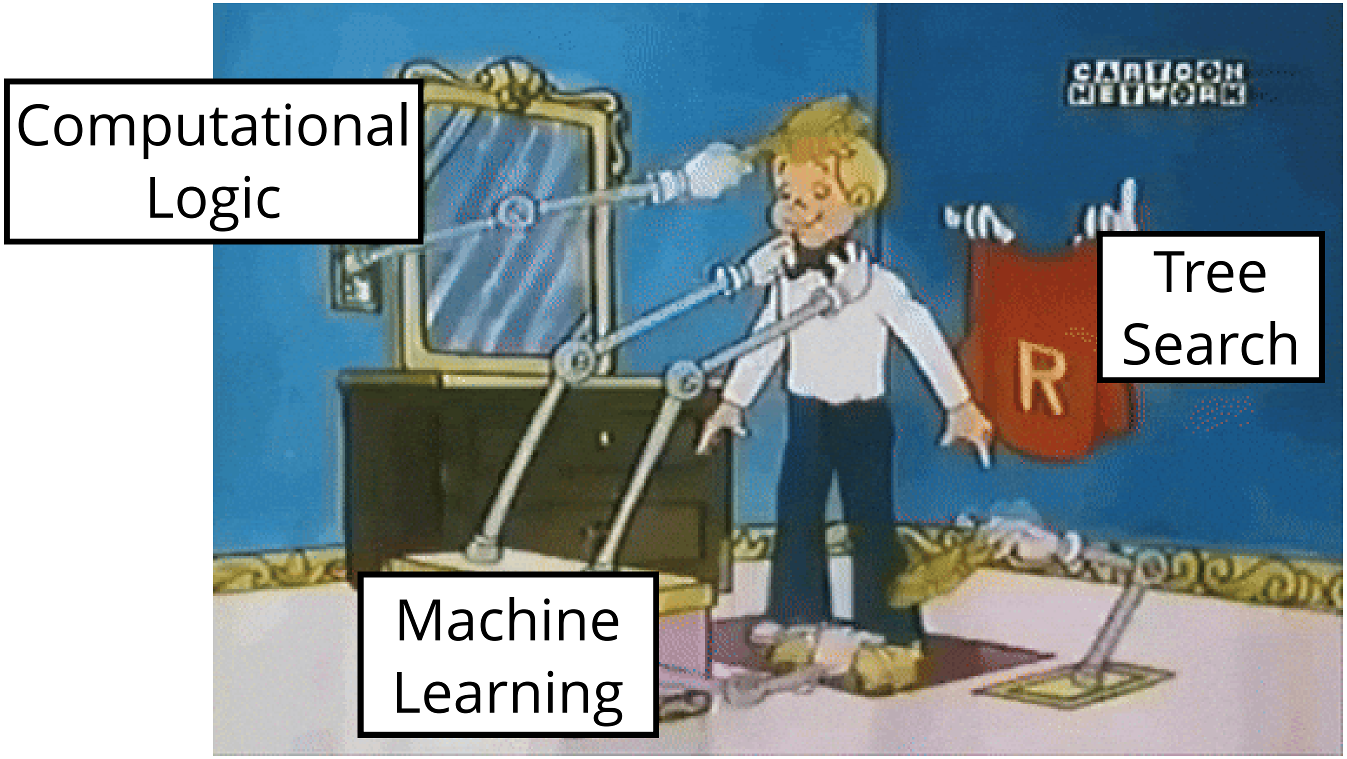
Automating Formal Verification



Computational Logic

Tree Search

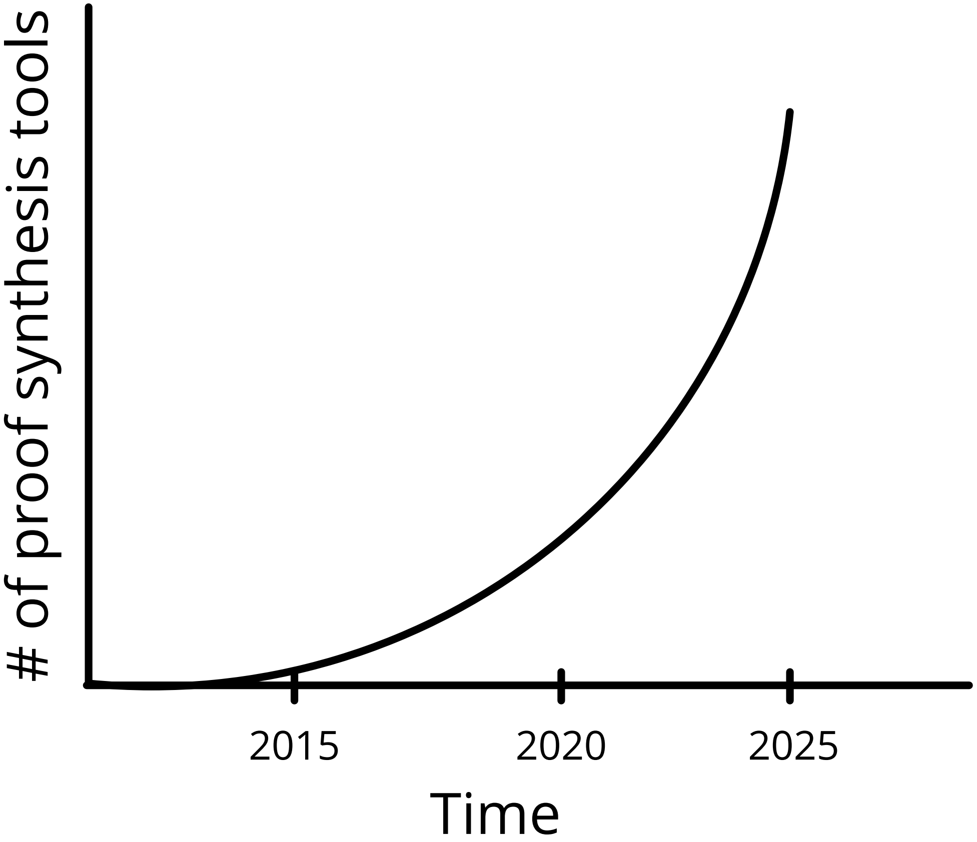
Automating Formal Verification



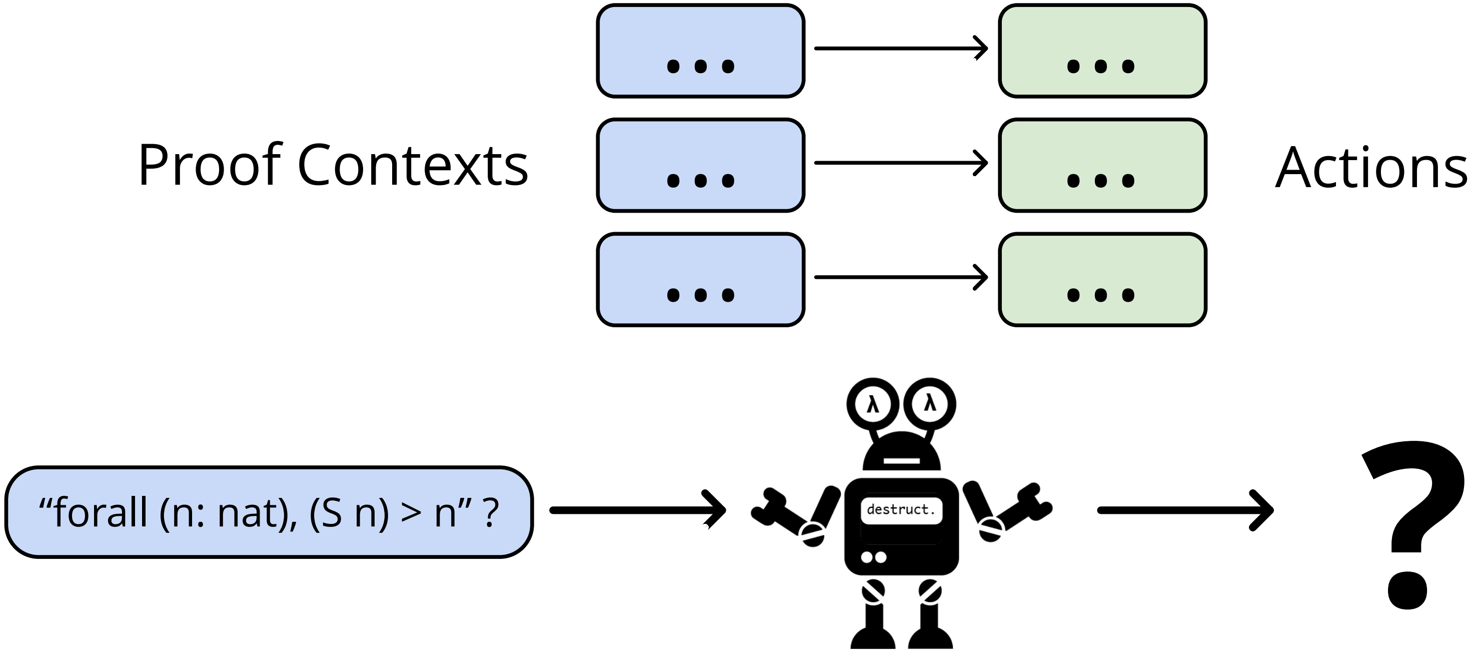
Computational Logic

Tree Search

Machine Learning



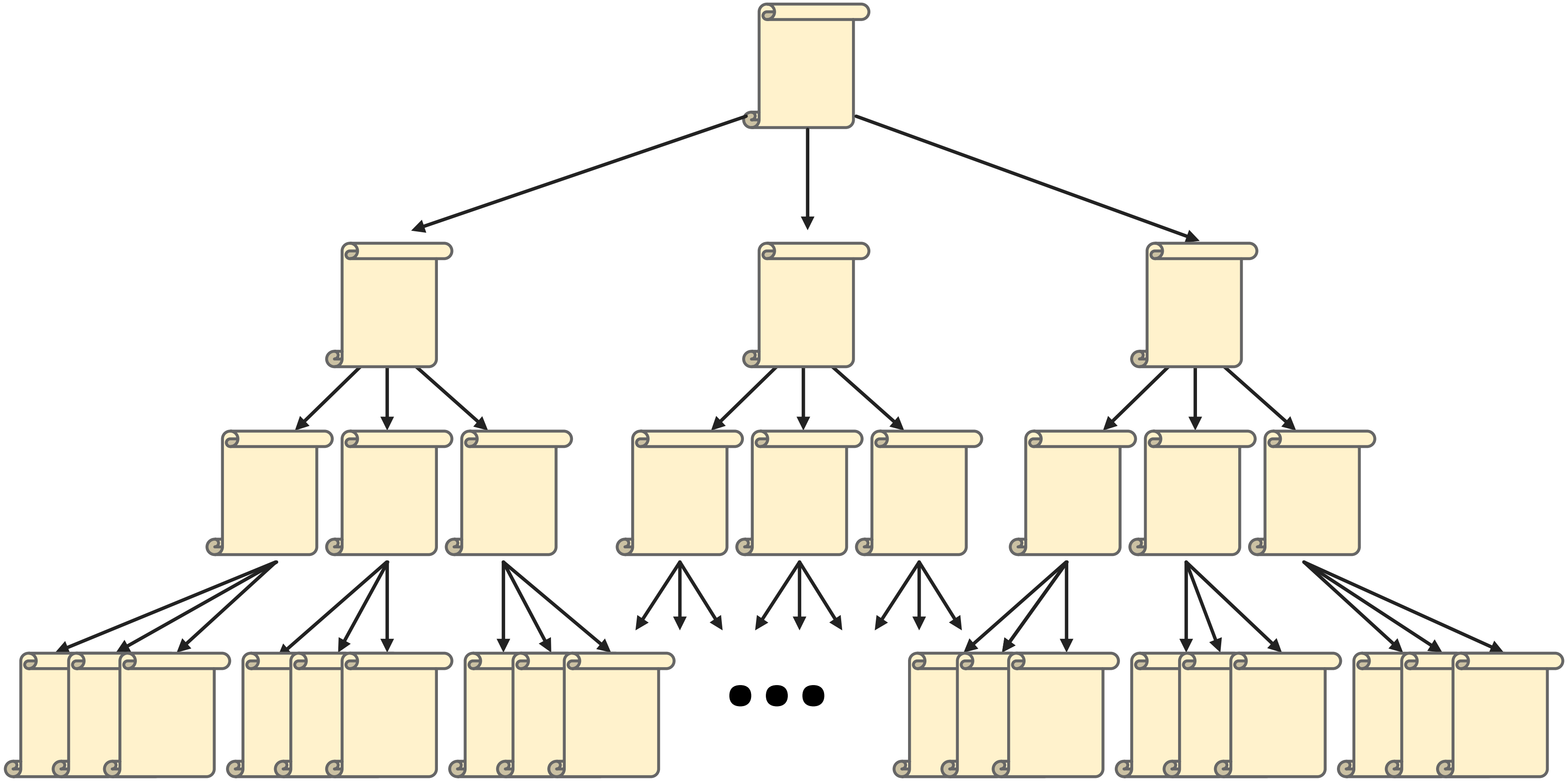
# of proof synthesis tools
2015
2020
2025
Time

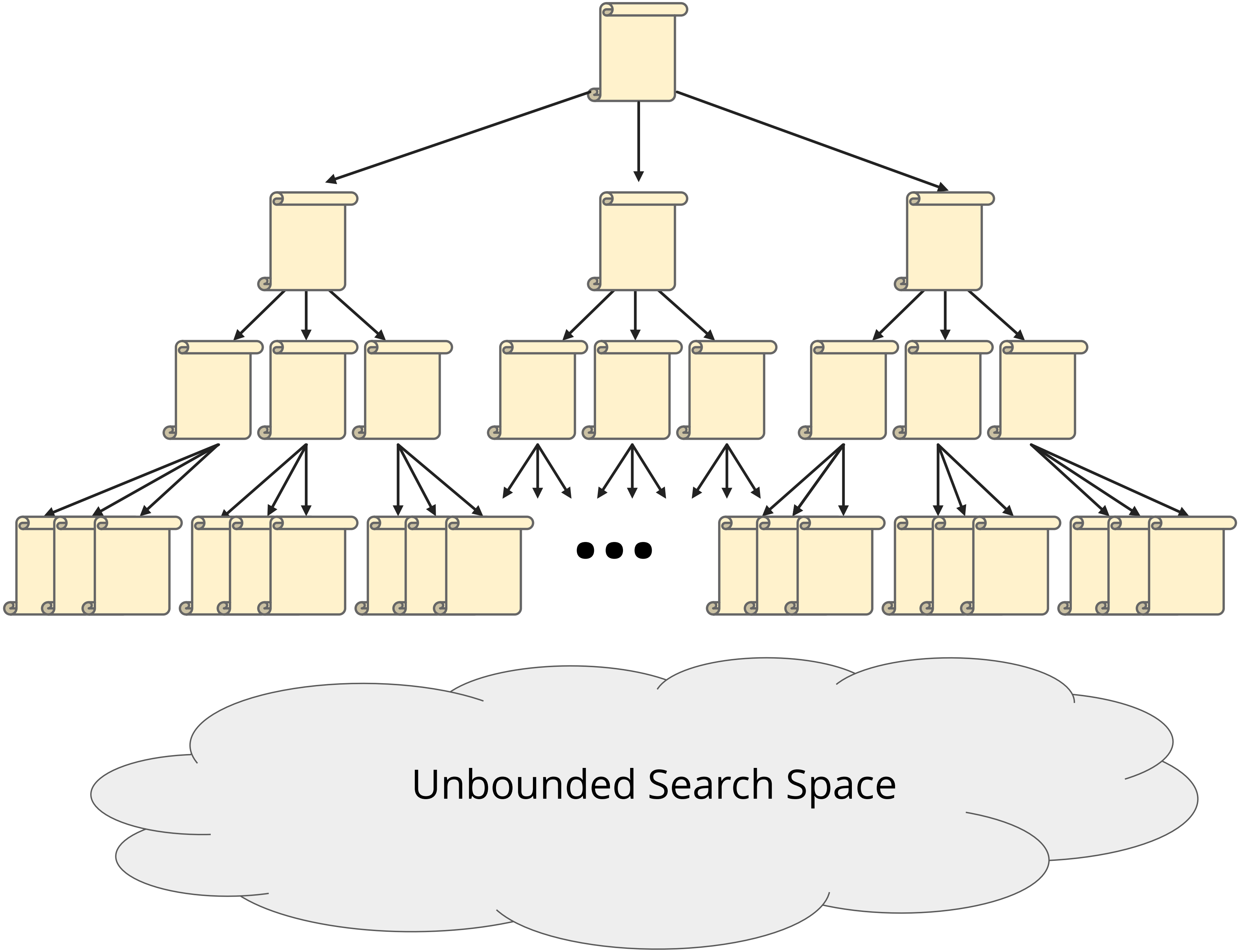


Proof Contexts

Actions

“forall (n: nat), (S n) > n” ?

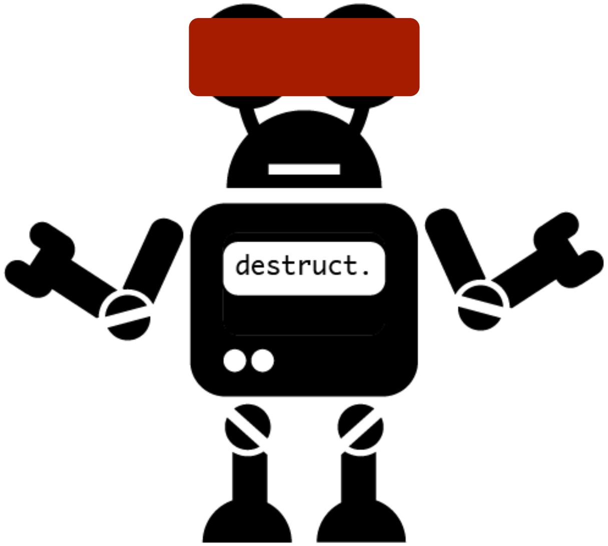



Unbounded Search Space

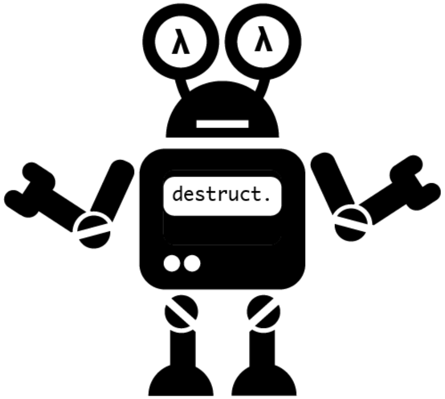
There are some techniques that can help us prune the search tree

There are some techniques that can help us prune the search tree

It's hard to explore when you don't know where you are!



We can search more efficiently if we can evaluate proof states



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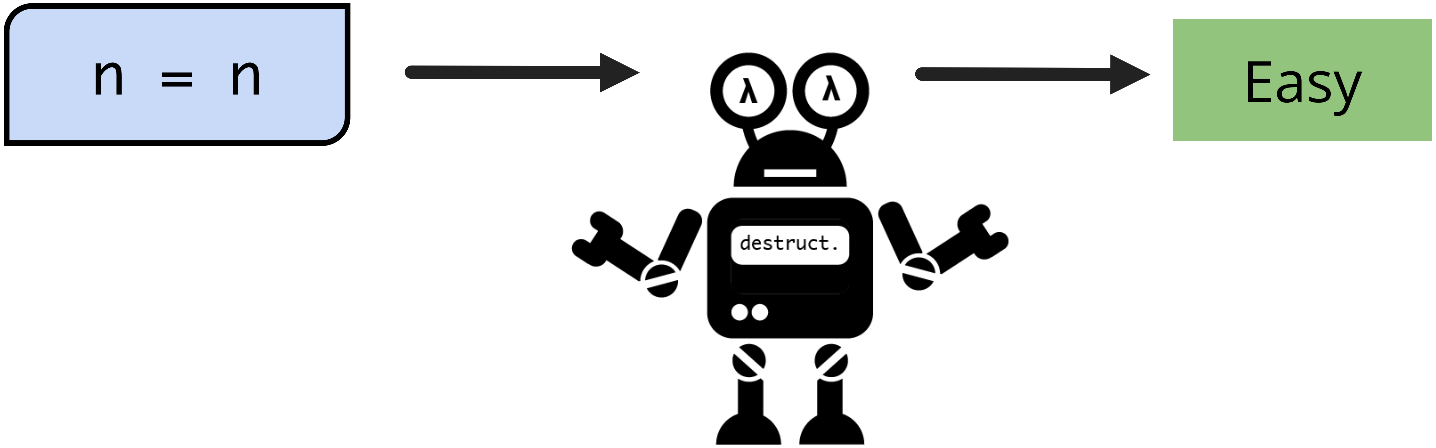
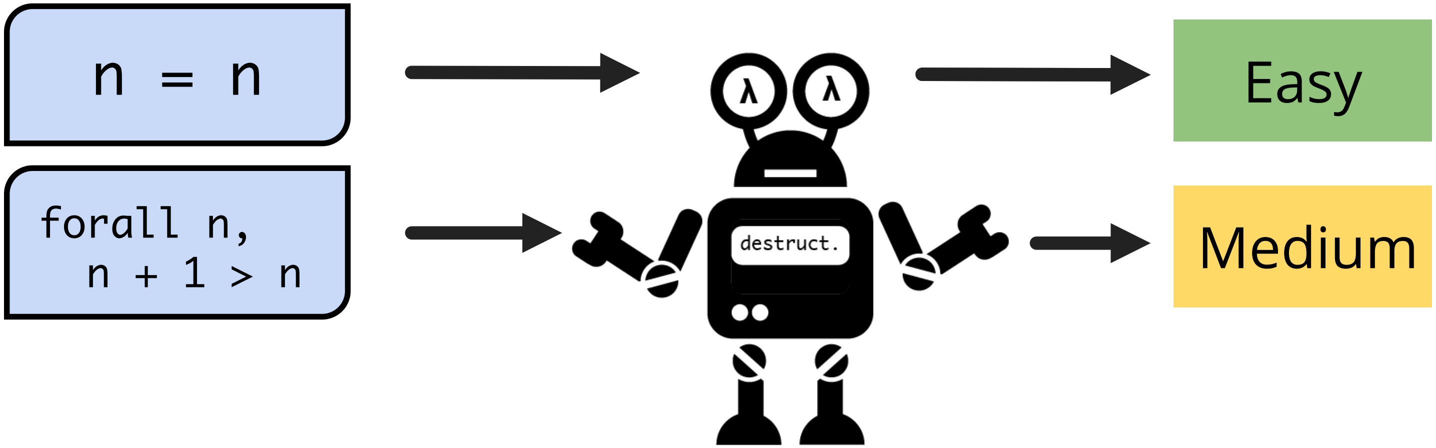


n = n

Easy

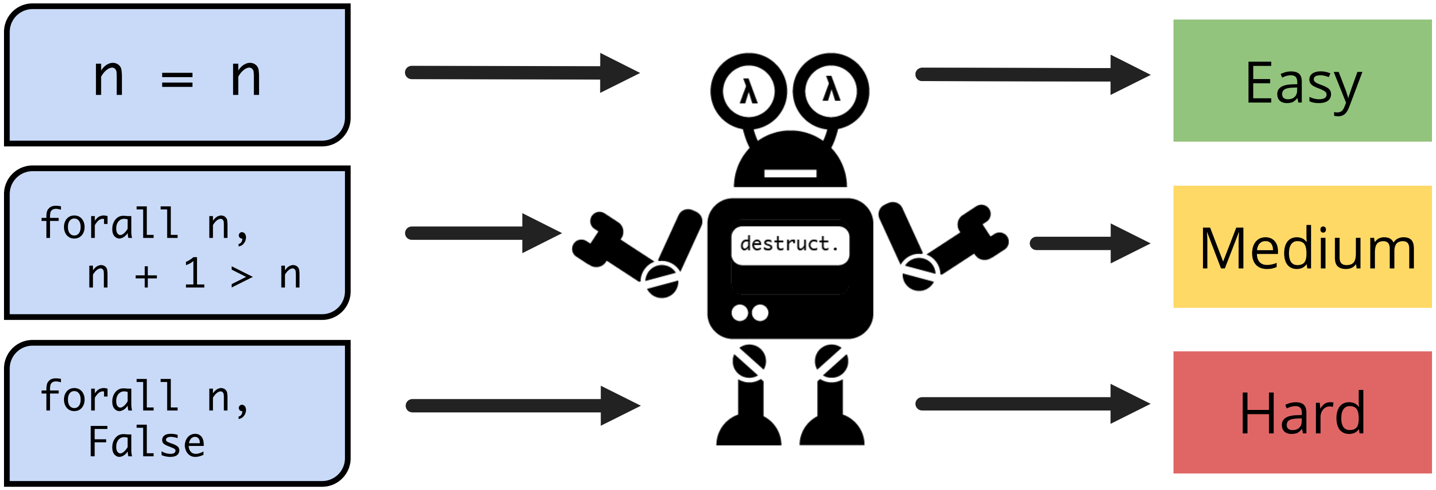
We can search more efficiently if we can evaluate proof states



```
n = n
```

```
forall n,
  n + 1 > n
```

We can search more efficiently if we can evaluate proof states



```
n = n
```

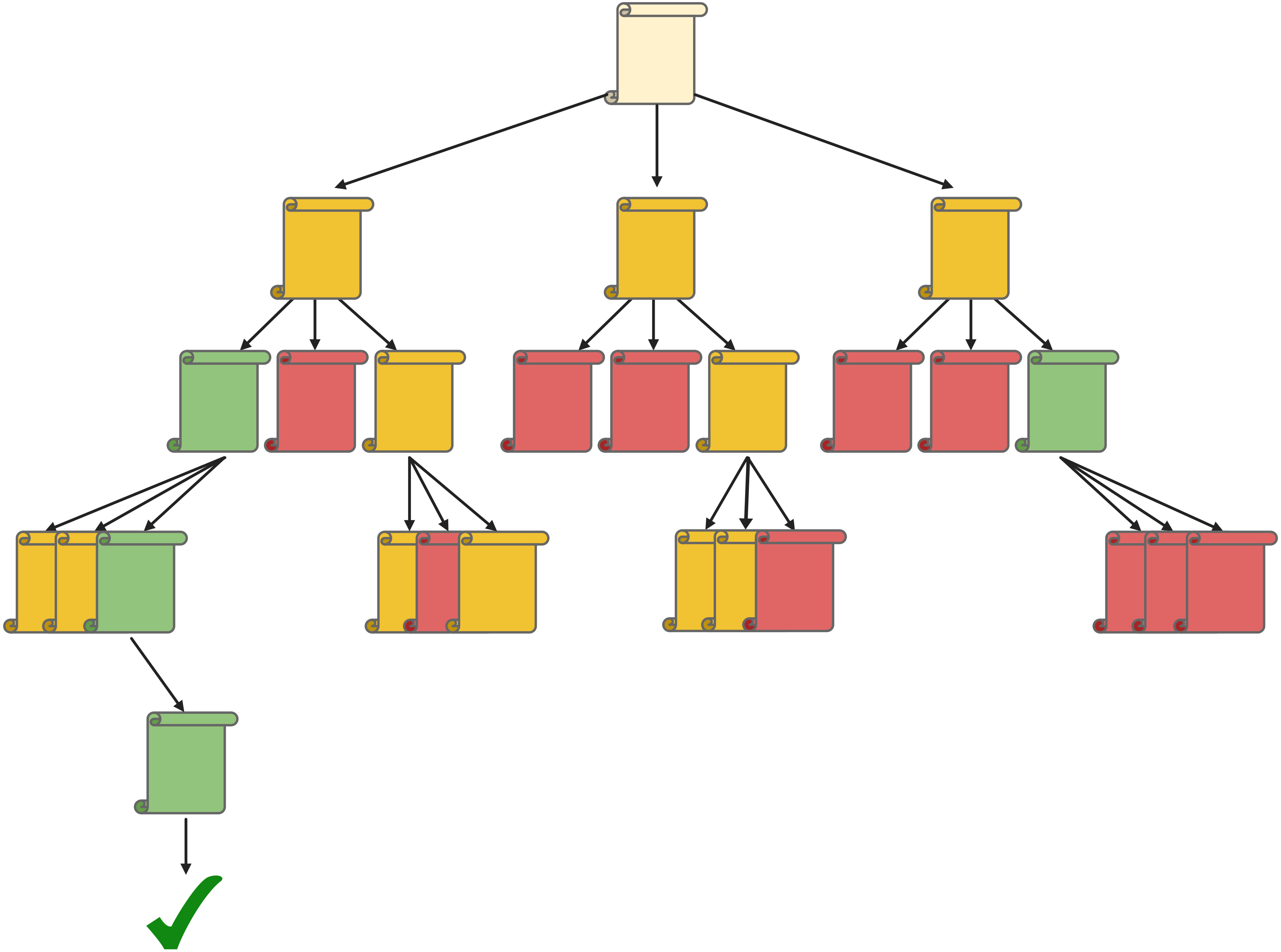
```
forall n,
  n + 1 > n
```

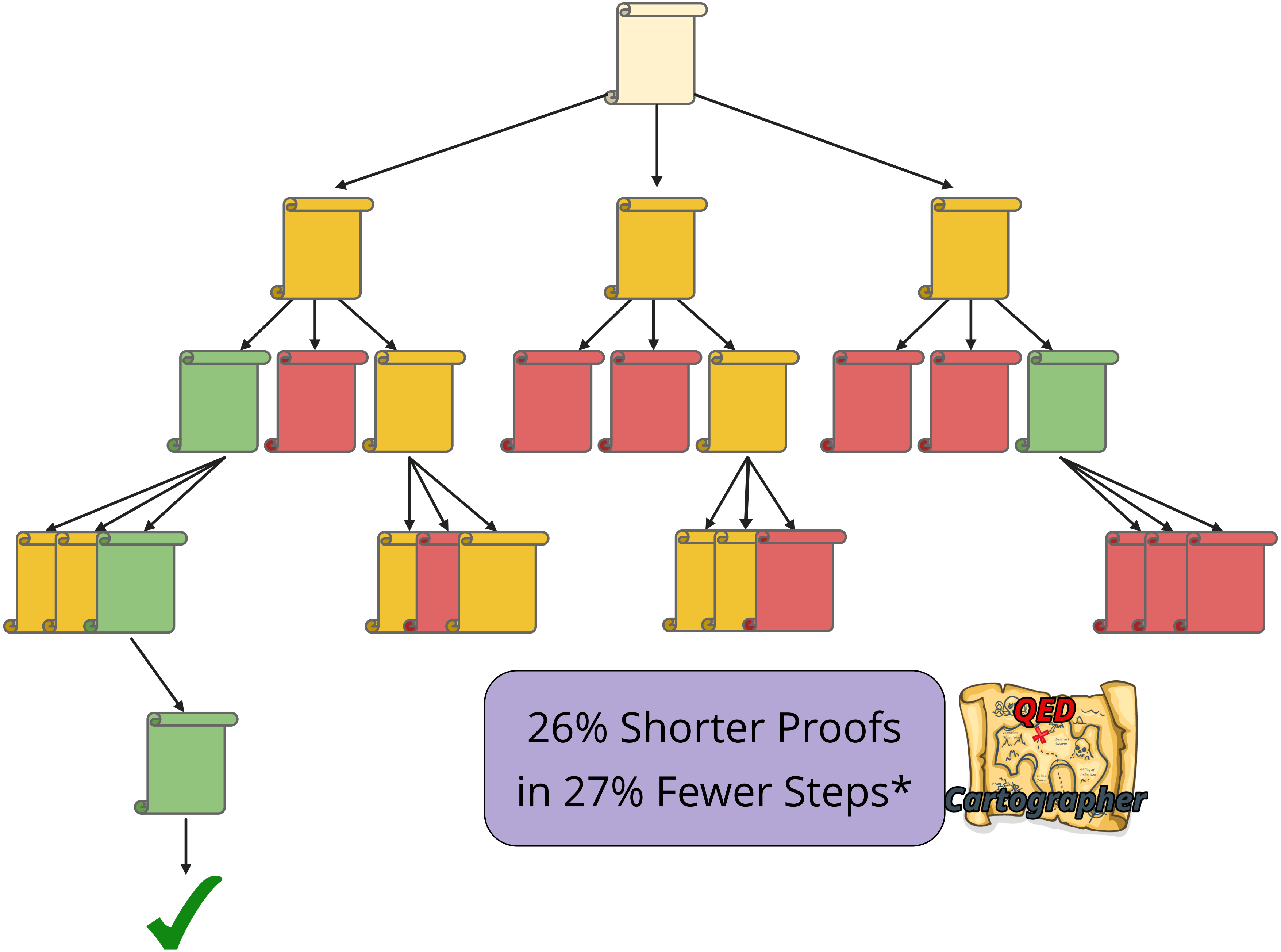
```
forall n,
  False
```

Easy

Medium

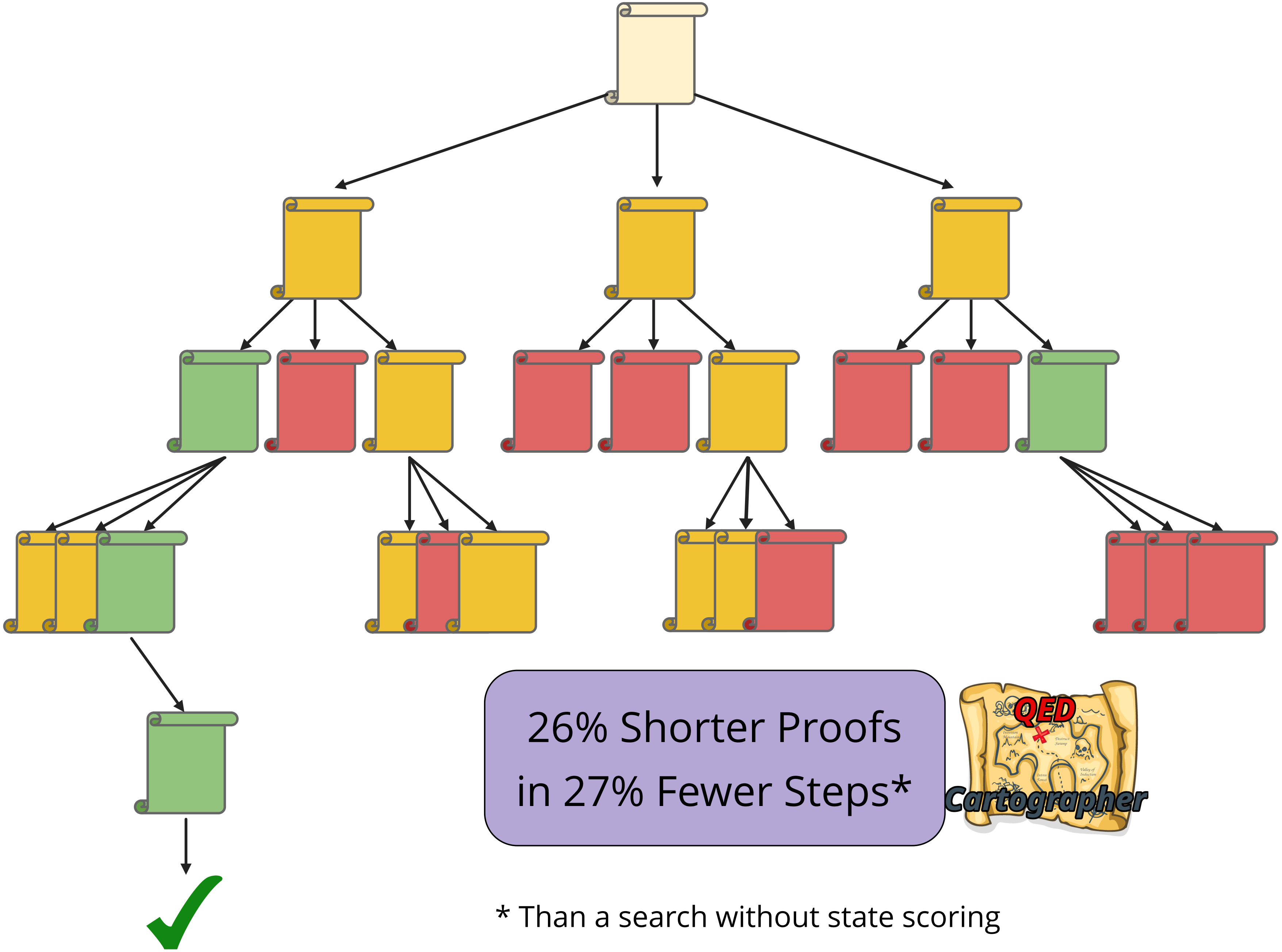
Hard





26% Shorter Proofs
in 27% Fewer Steps*

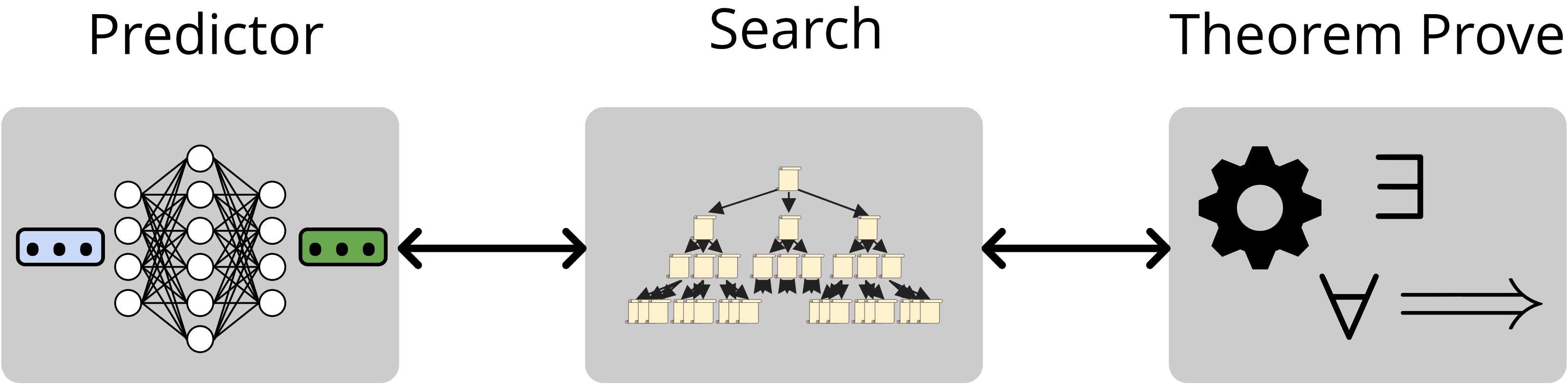
QED
Cartographer

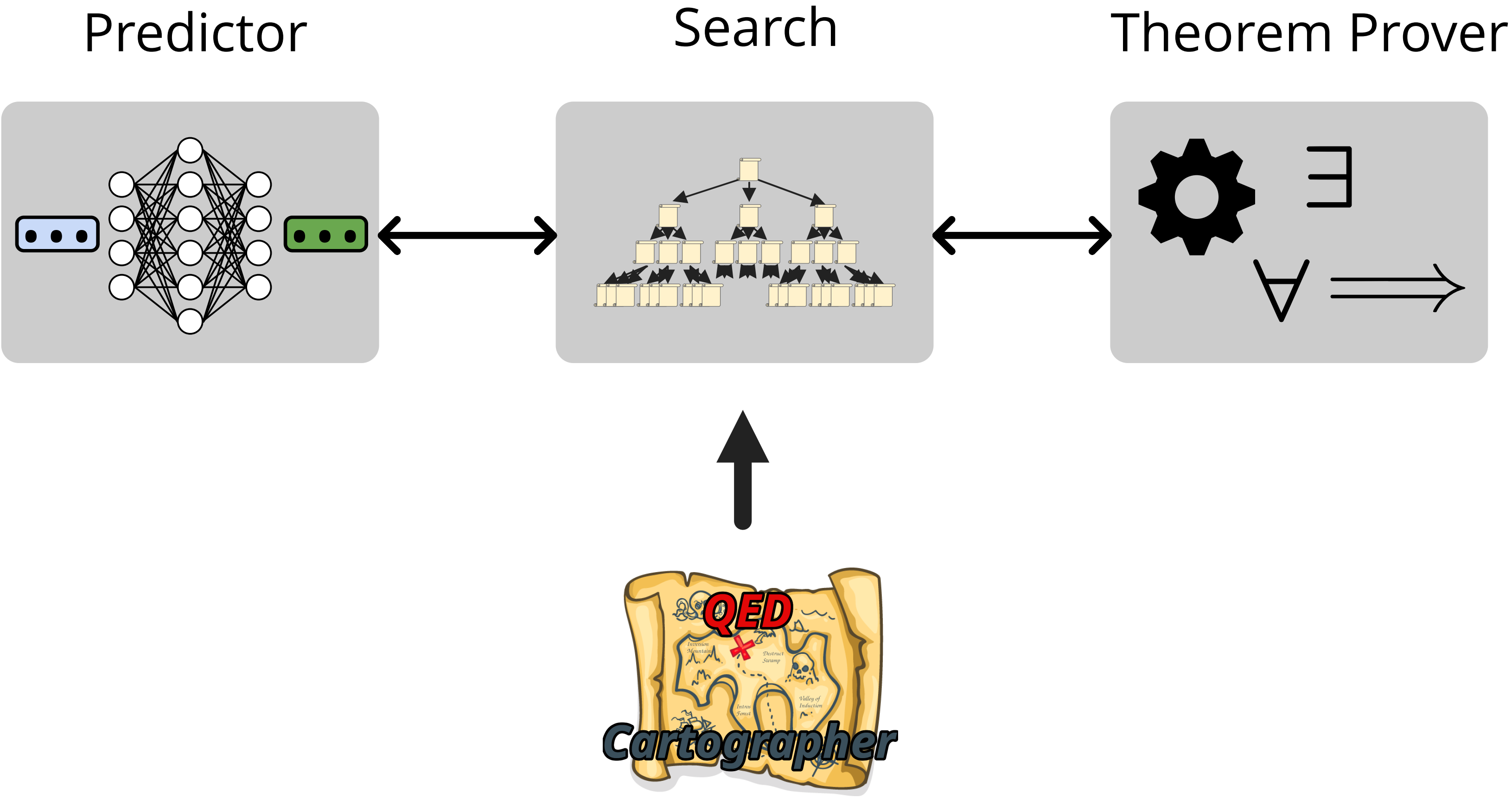


26% Shorter Proofs
in 27% Fewer Steps*

QED
Cartographer

* Than a search without state scoring


Predictor
Search
Theorem Prover
∃
∀ ⟹

Predictor
Search
Theorem Prover
∃
∀ ⟹
QED
Inversion Mountains
Destruct Swamp
Intros Forest
Valley of Induction
Cartographer

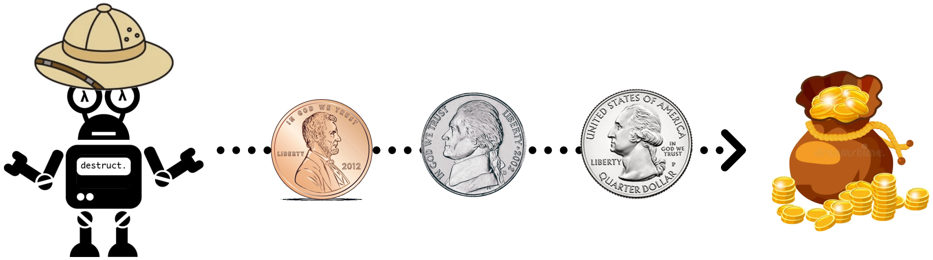
"Reward-free Reinforcement Learning"

"Reward-free $\underbrace{\text{Reinforcement Learning"}}_{\text{In particular, V-learning}}$

This Talk

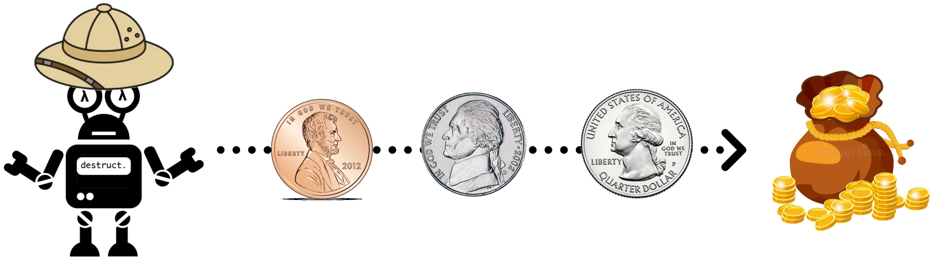
This Talk

V-Learning

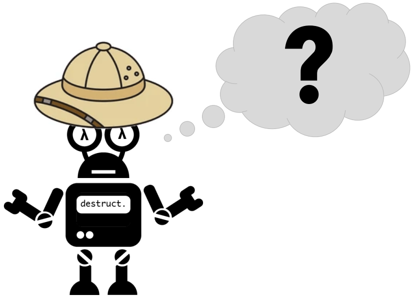


This Talk

V-Learning

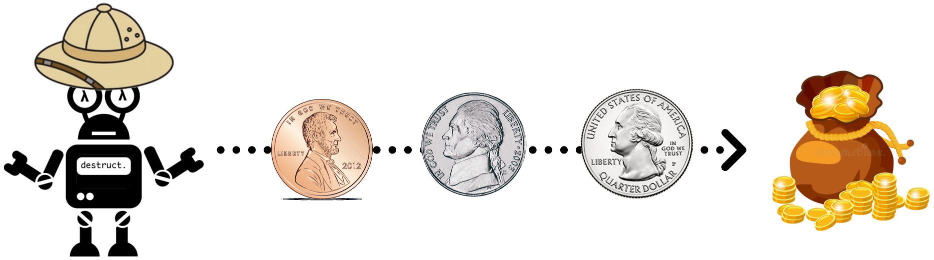


Limitations in Proofs

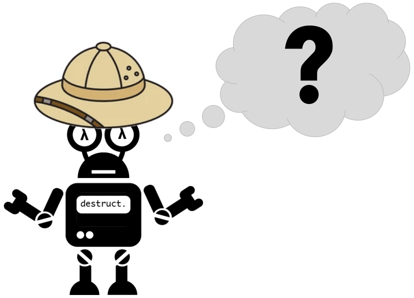


This Talk

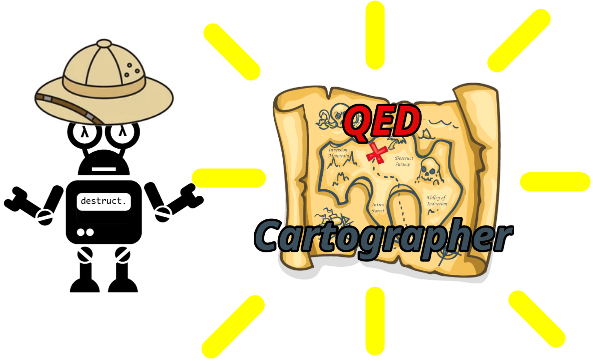
V-Learning



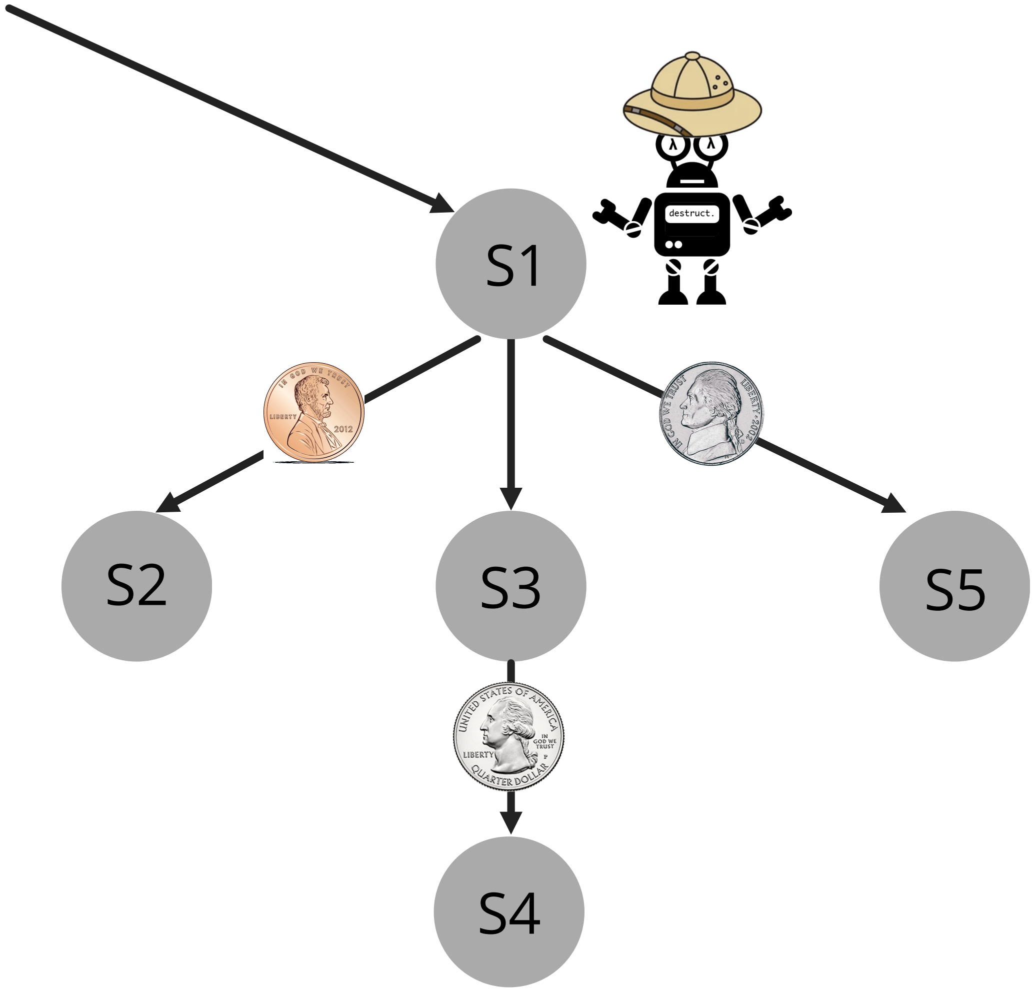
Limitations in Proofs



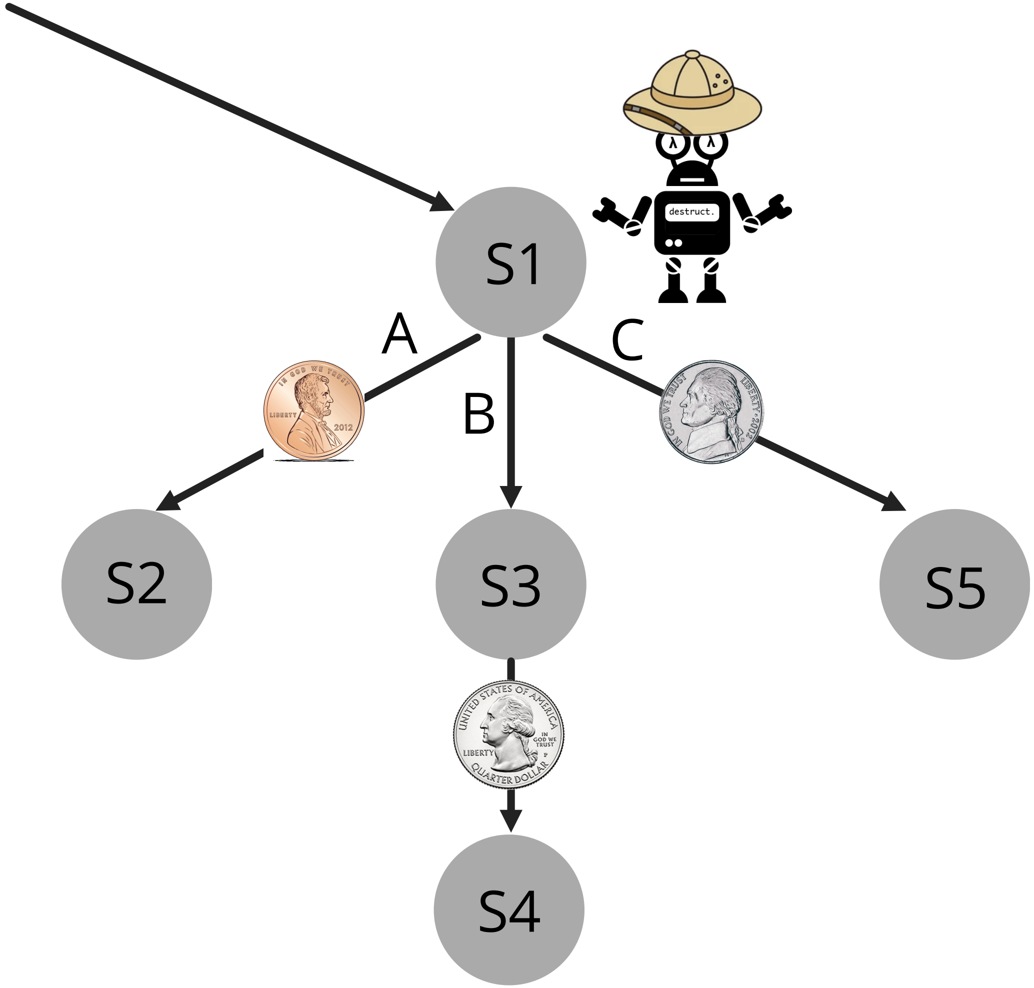
Adapting to Proofs



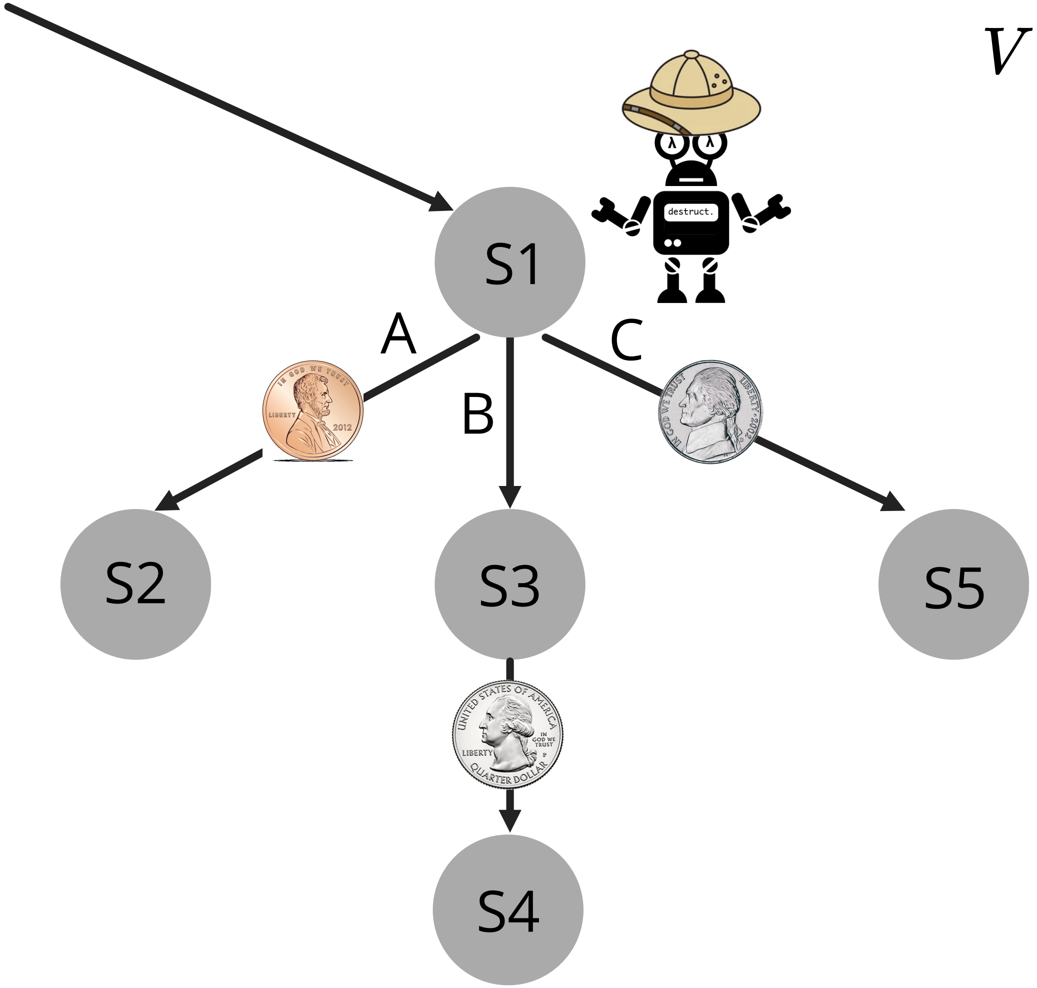
Classic V-Learning



Classic V-Learning



Classic V-Learning



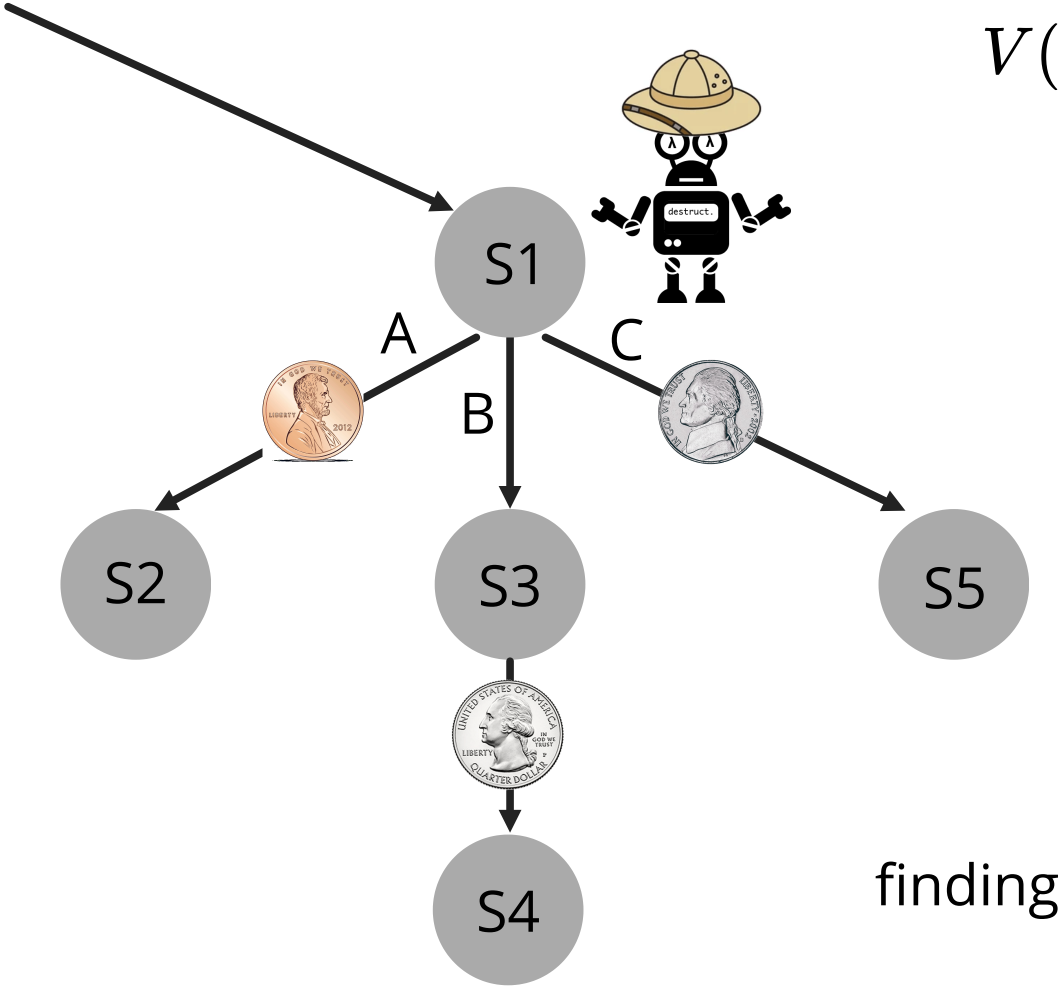
$V(S1) = \max($

$+ \quad V(S2),$

$($ $+ \quad V(S4))\,,$

$+ \quad V(S5))$

Classic V-Learning



$$V(S1) = \max($$

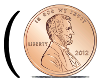
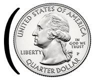
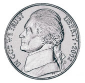
$+ \gamma V(S2),$

$\gamma($ $+ \gamma V(S4))$,

$+ \gamma V(S5))$

γ = time discount

finding rewards sooner is better!

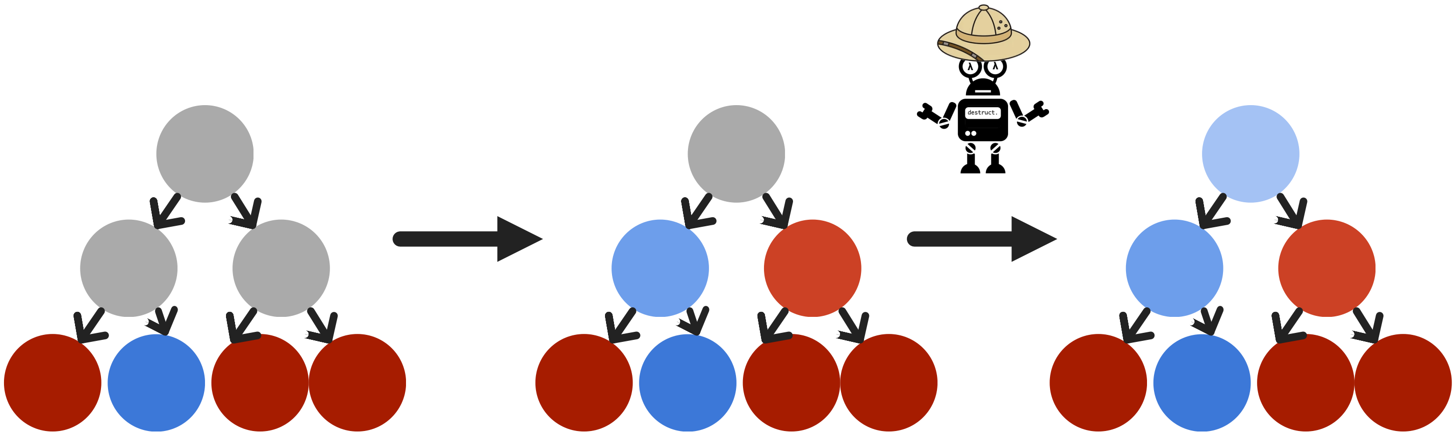
$V(S1) = \max($  $+\gamma V(S2),\ \gamma\,($  $+\gamma V(S4)),$  $+\gamma V(S5))$

$V(S1) = \max($ $+\gamma V(S2),\ \ \gamma\,($ $+\gamma V(S4))\,,$ $+\gamma V(S5))$

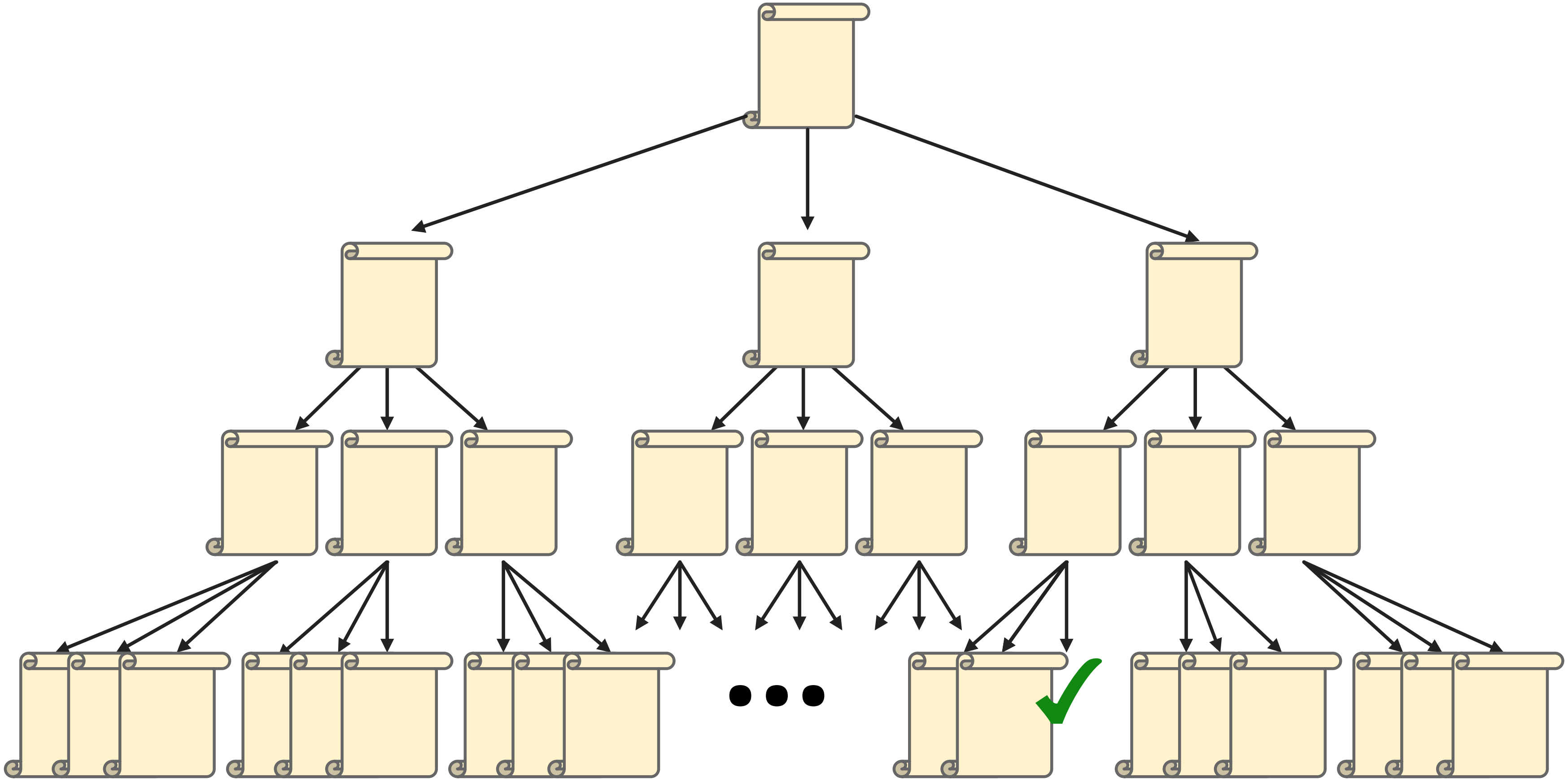
Generalizes to

$$V(S) = \max_{a\in\text{actions}(S)}\left(R(S,a) + \gamma V(\text{next-state}(S,a))\right.$$

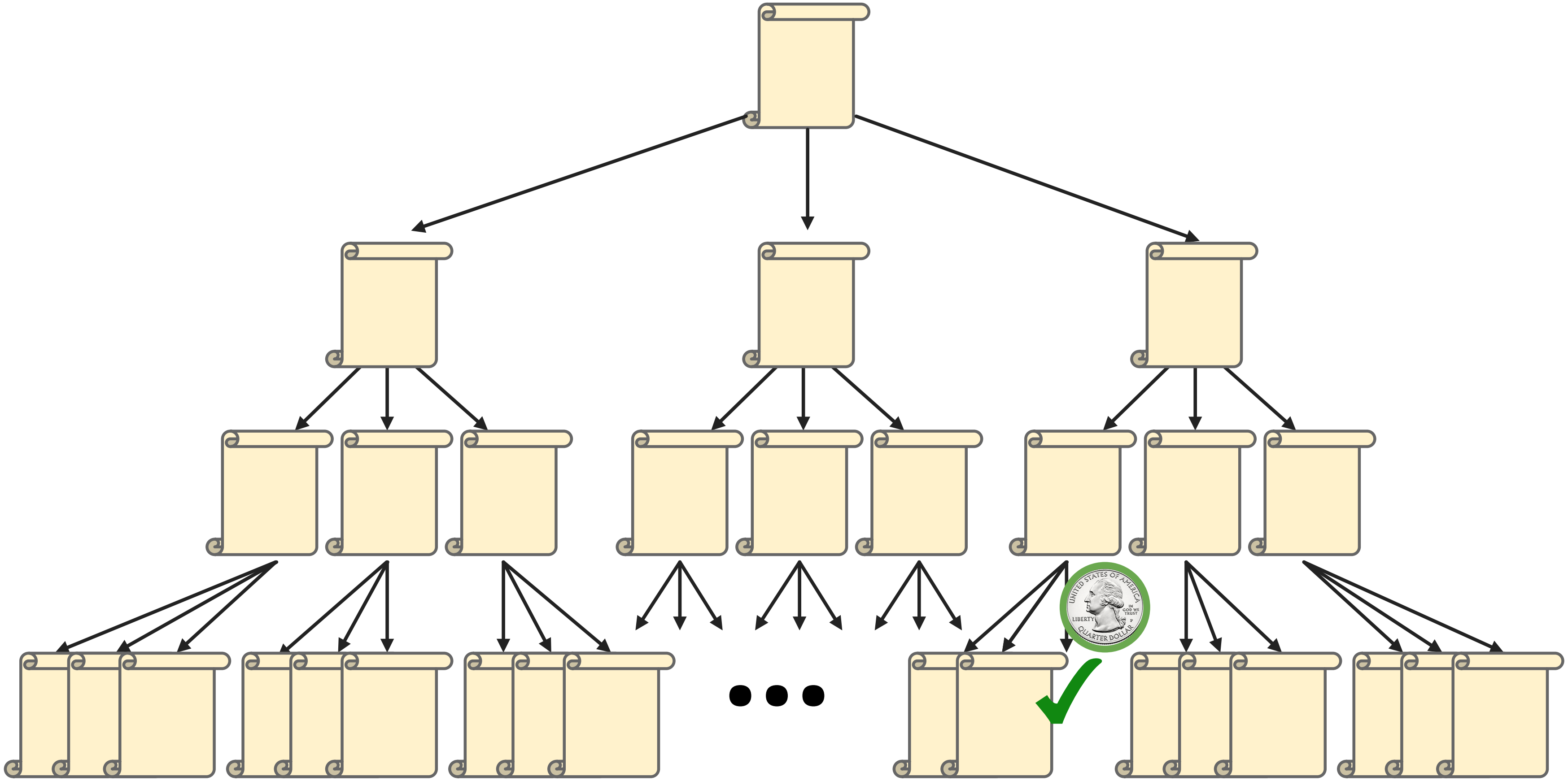
V-Learning in Practice: Iterative Updates



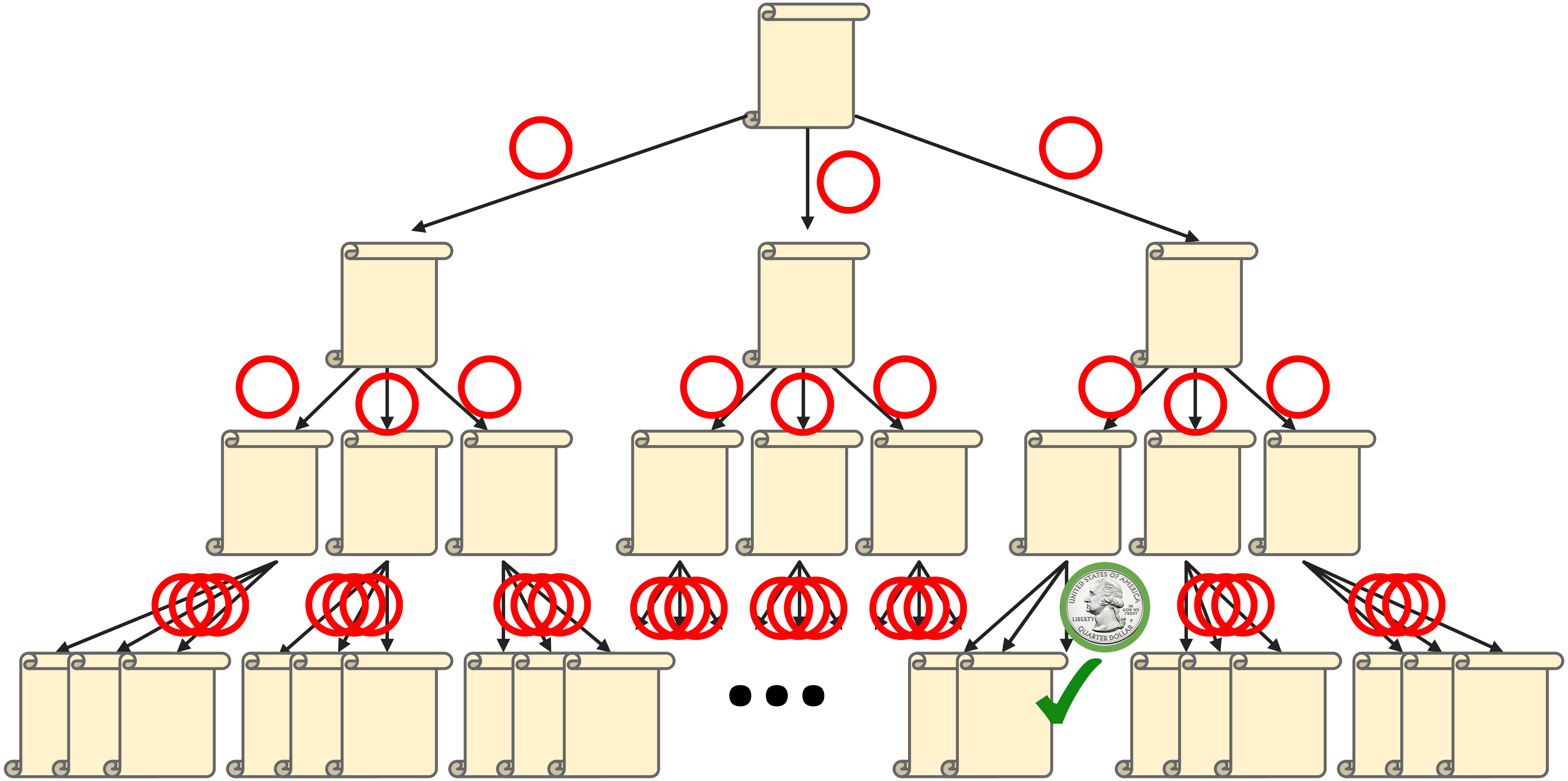
V-Learning in Proofs: The Sparse Reward Problem



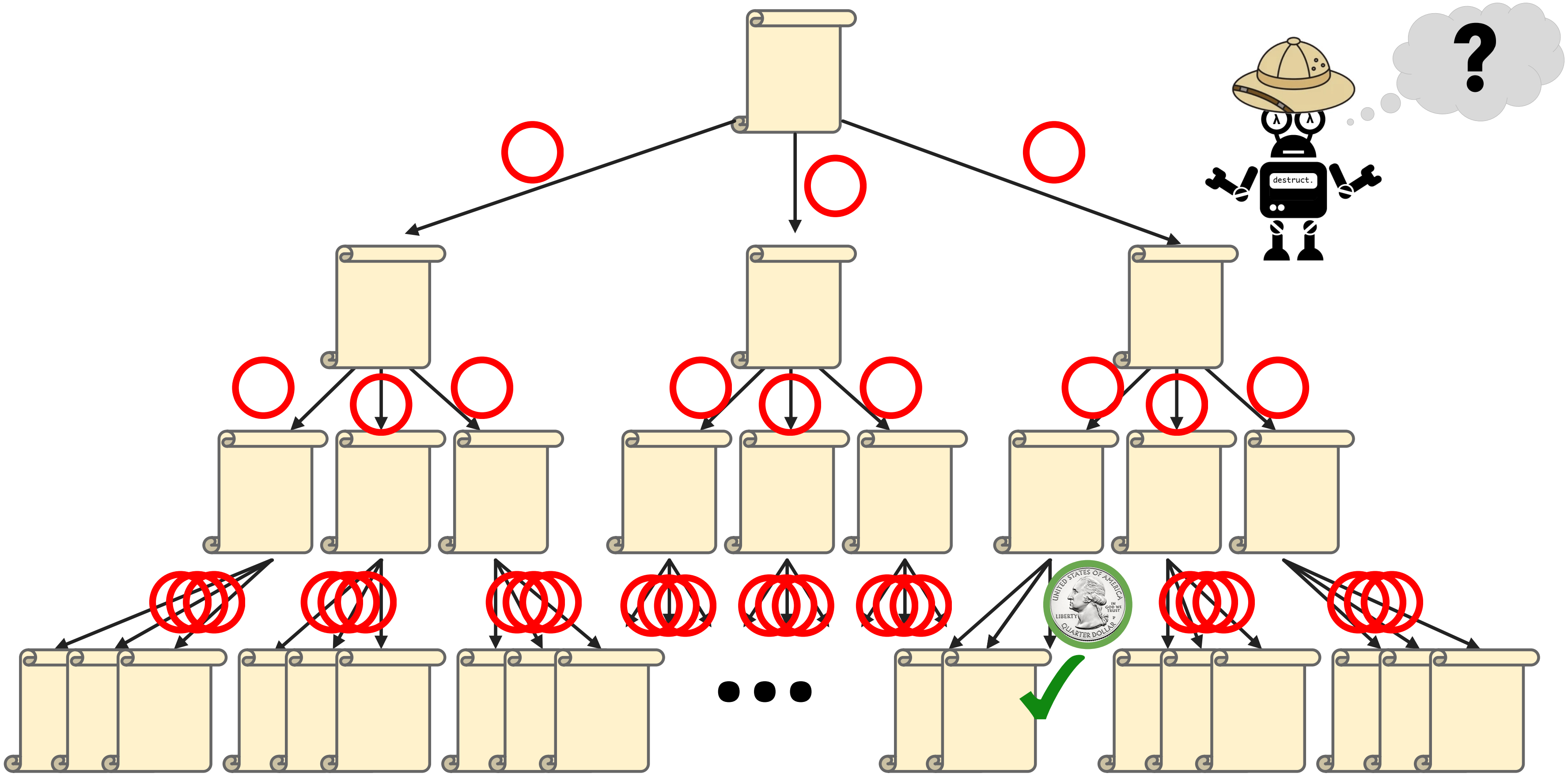
V-Learning in Proofs:
The Sparse Reward Problem



V-Learning in Proofs:
The Sparse Reward Problem



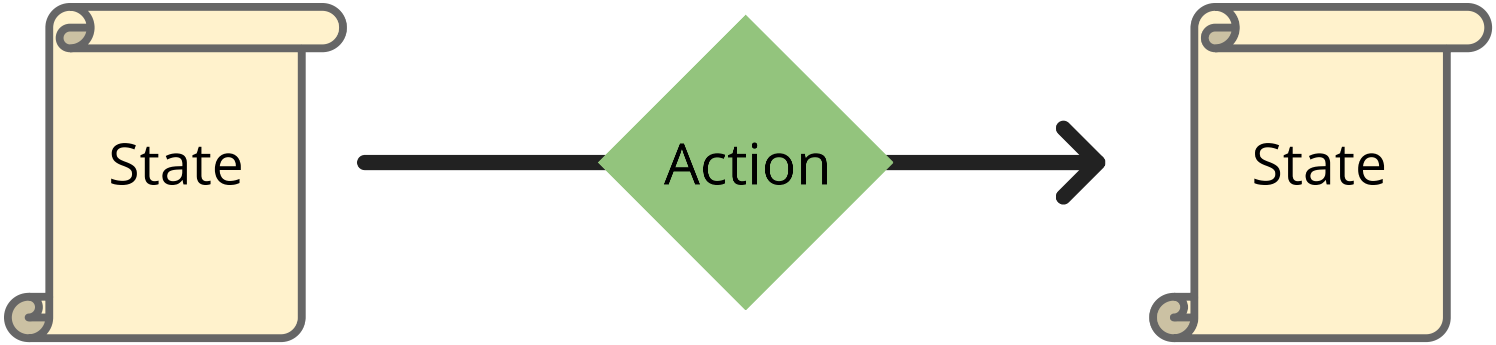
V-Learning in Proofs:
The Sparse Reward Problem

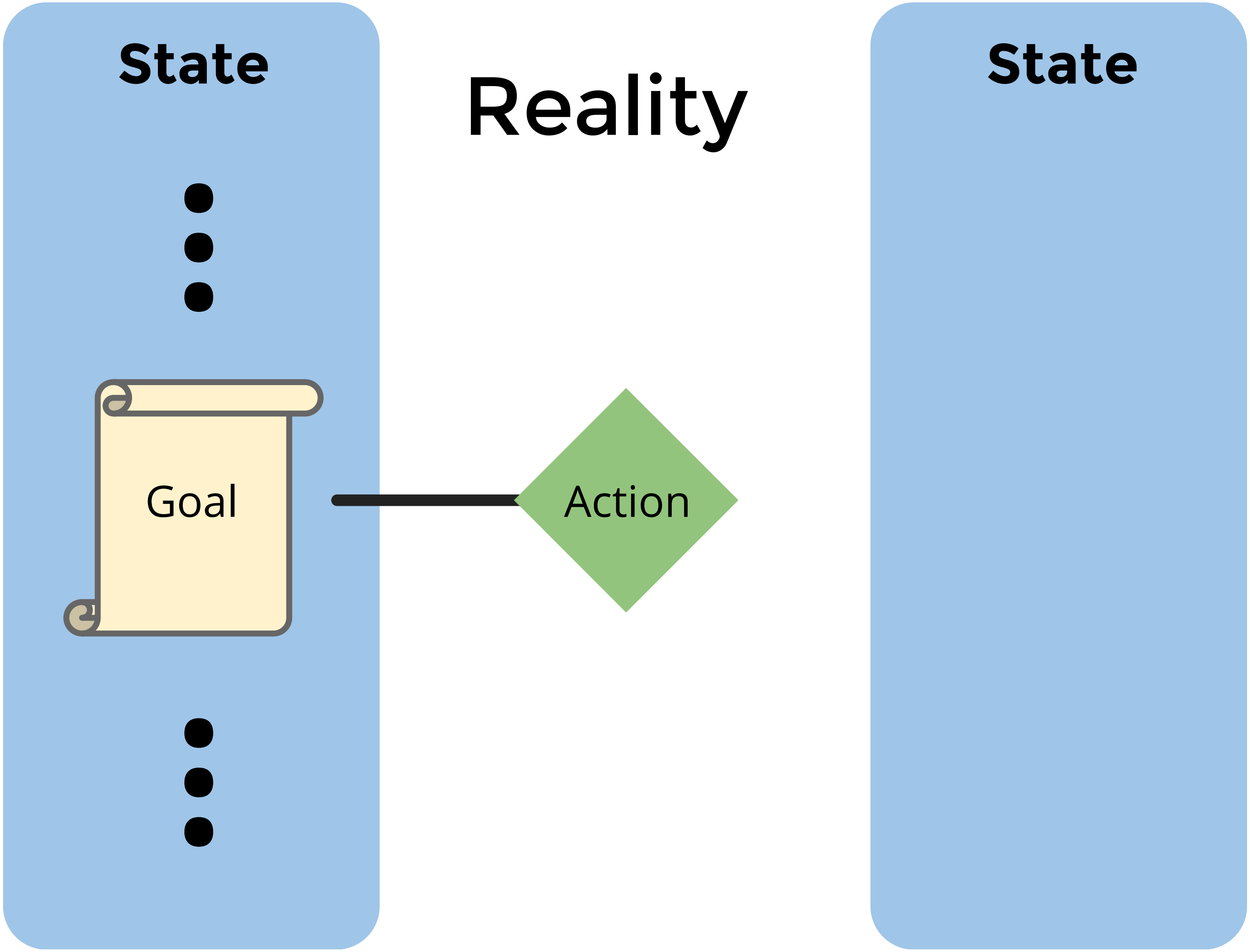


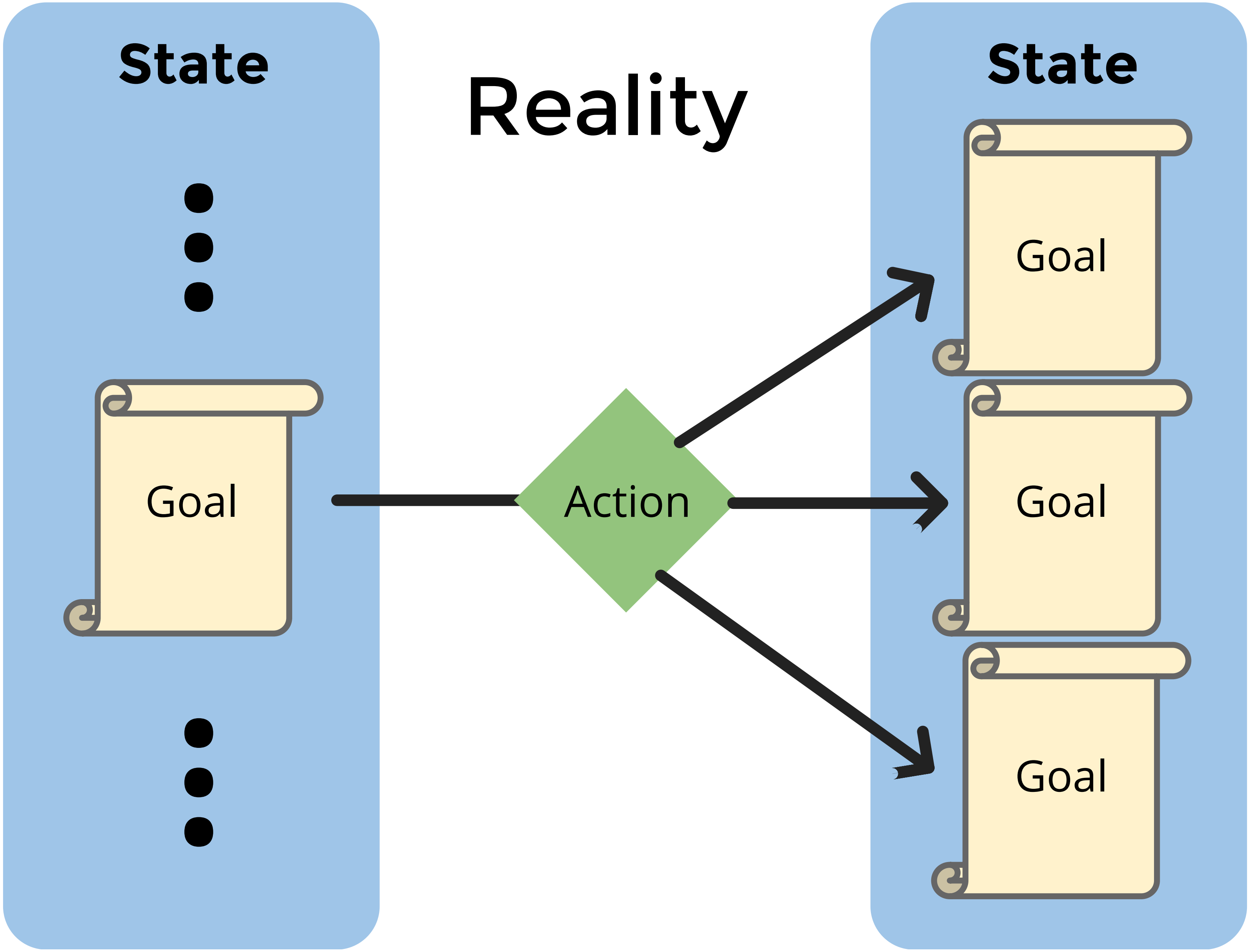
Insight:

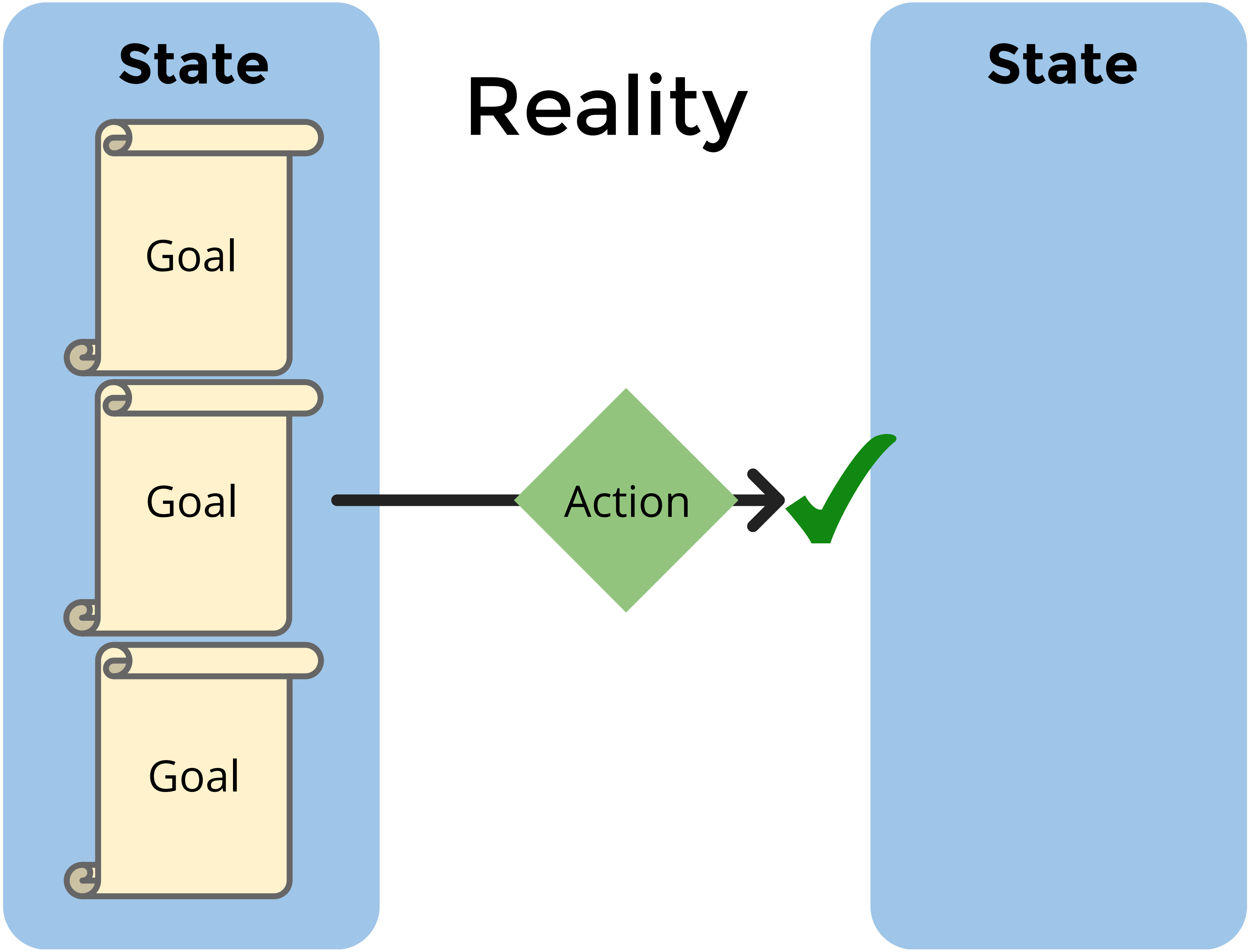
Proofs have Useful Structure

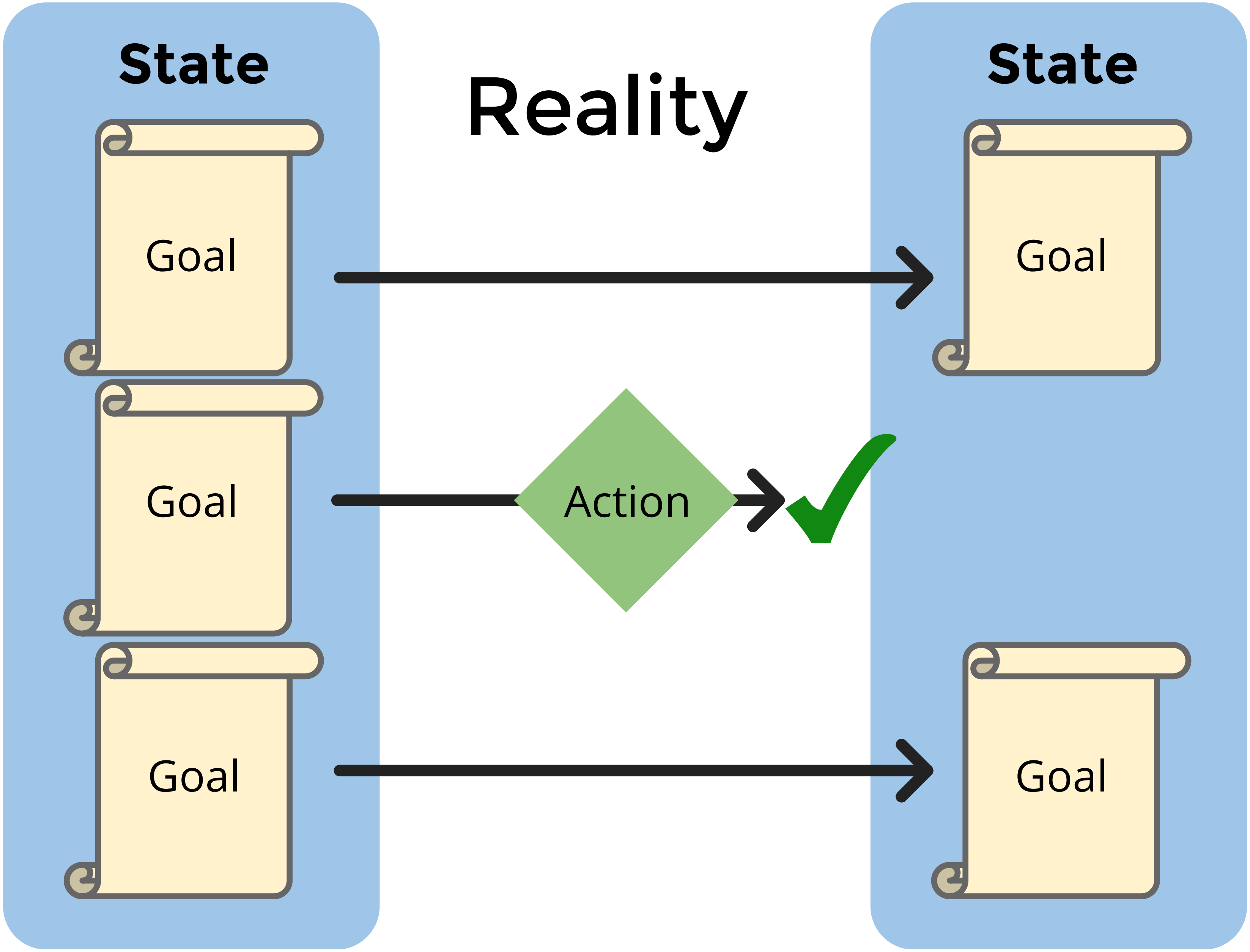
Abstraction




State
Reality
State
Goal
Action

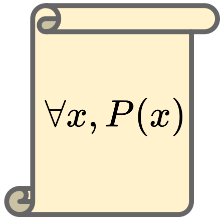

State
Reality
State
Goal
Goal
Action
Goal
Goal


State
Reality
State
Goal
Goal
Goal
Action
✓

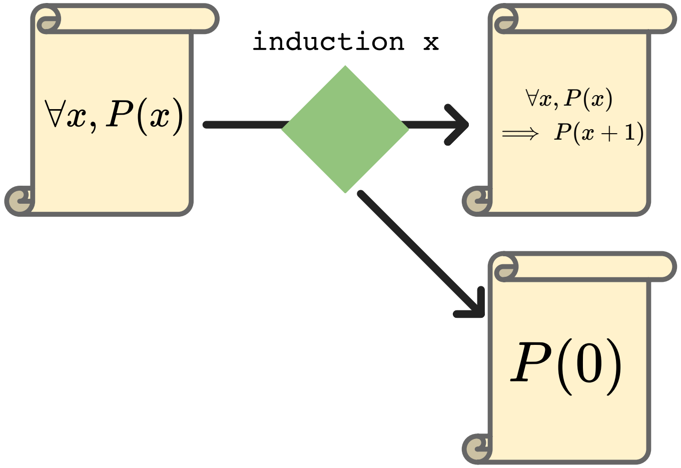

State
Reality
State
Goal
Goal
Goal
Action
✓
Goal
Goal

Adding extra rewards can lead to bad behavior

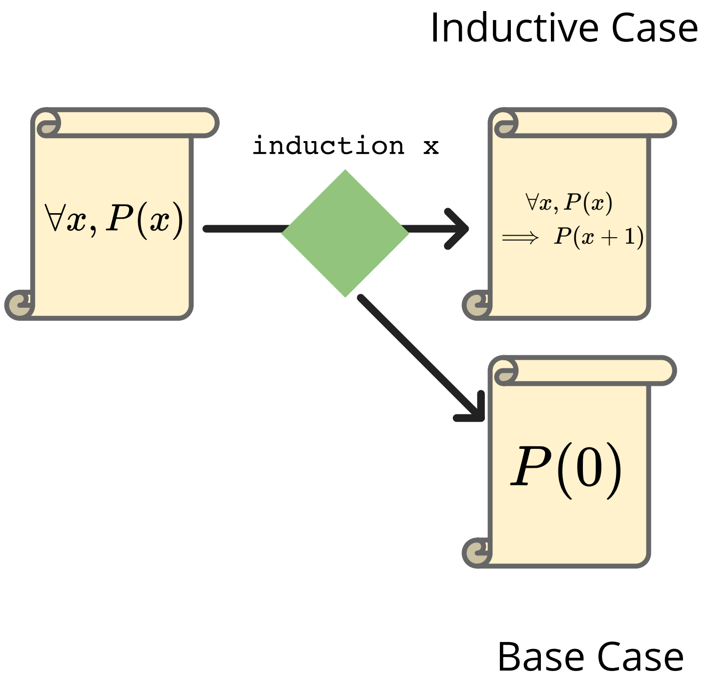
Adding extra rewards can lead to bad behavior



Adding extra rewards can lead to bad behavior



Adding extra rewards can lead to bad behavior



Inductive Case

$\forall x, P(x)$

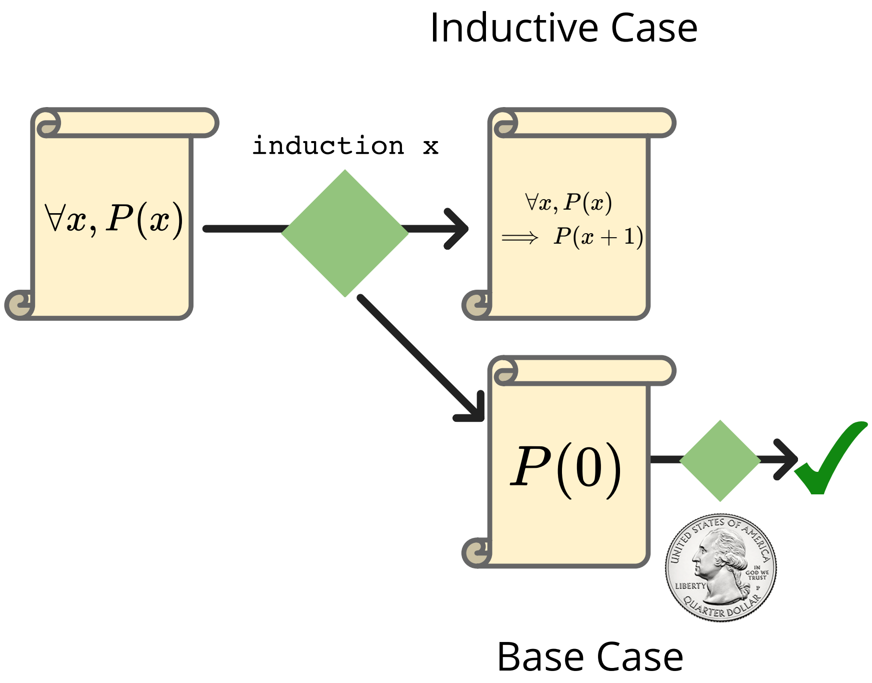
`induction x`

$\forall x, P(x) \implies P(x+1)$

$P(0)$

Base Case

Adding extra rewards can lead to bad behavior



Inductive Case

$\forall x, P(x)$

`induction x`

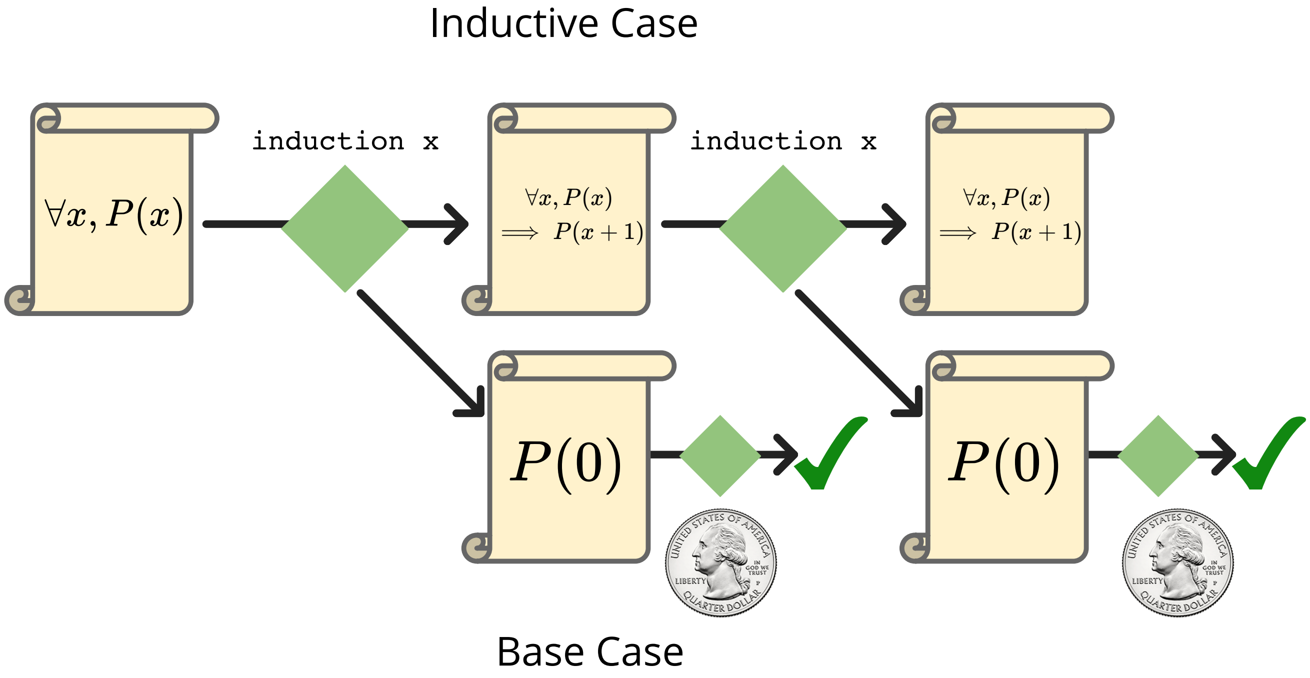
$\forall x, P(x) \implies P(x+1)$

$P(0)$

✓

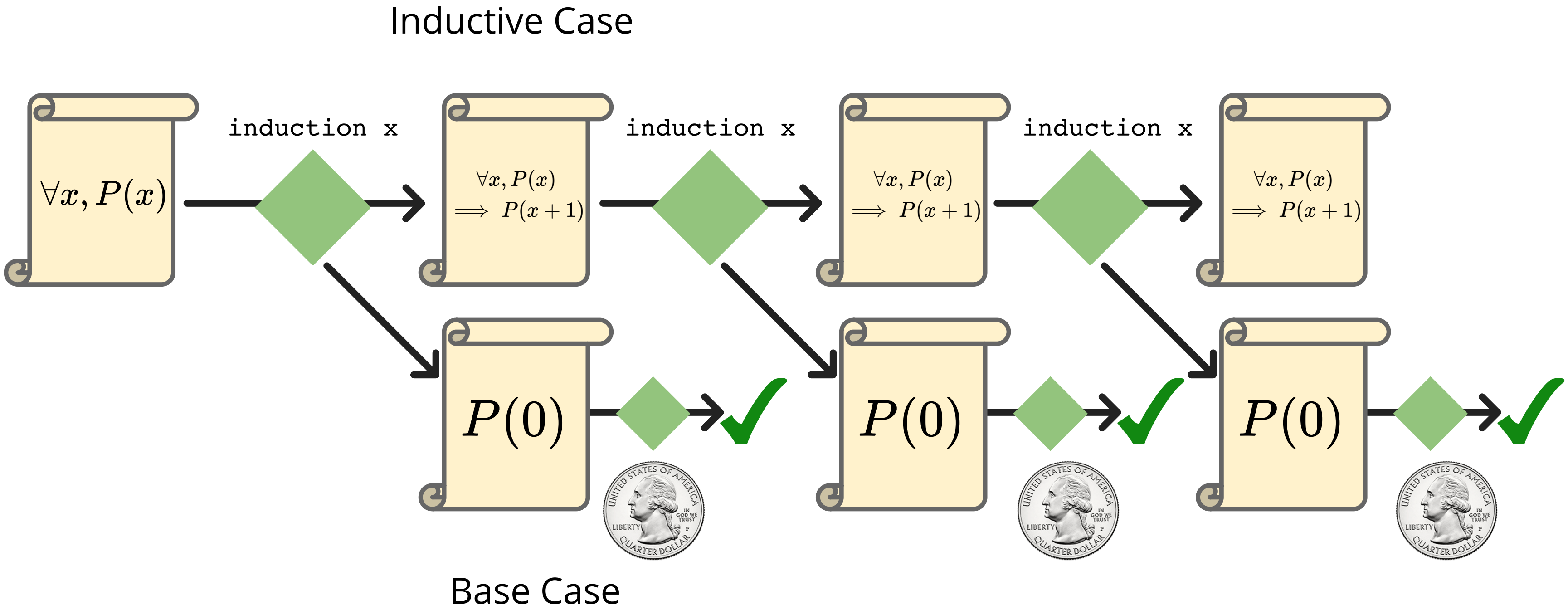
Base Case

Adding extra rewards can lead to bad behavior

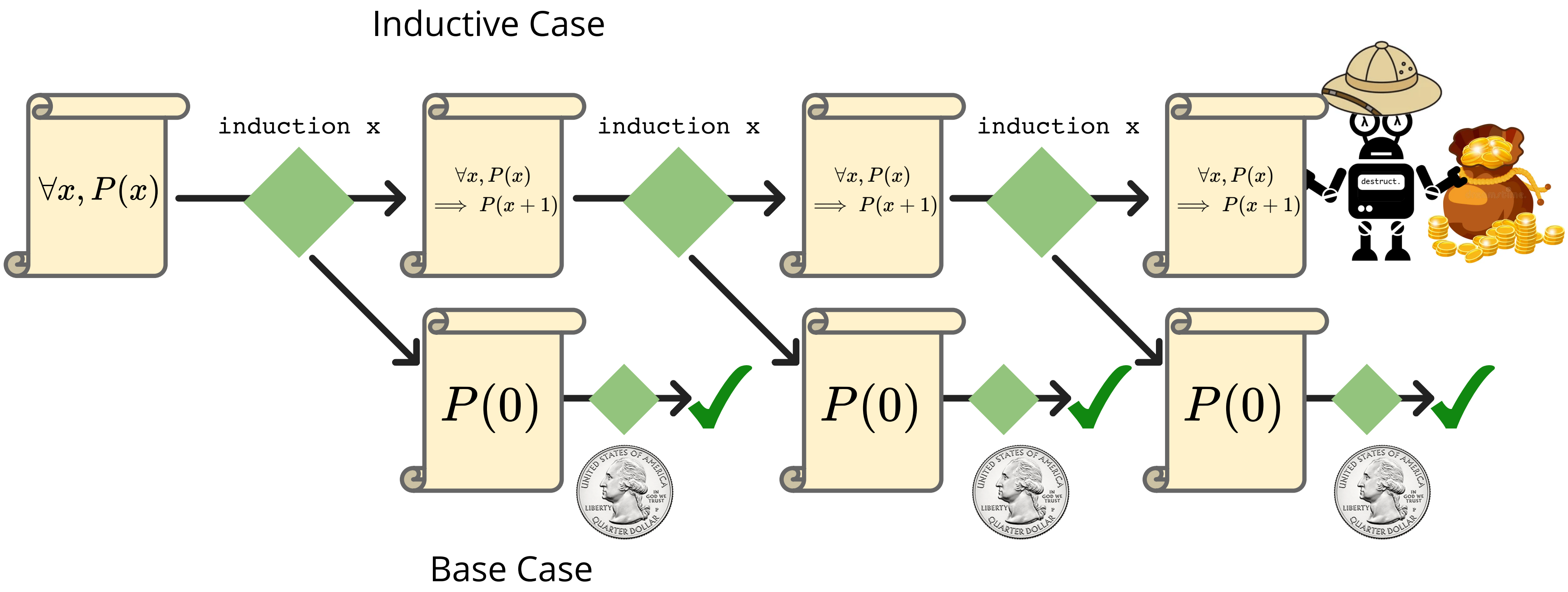

Inductive Case

Base Case

Adding extra rewards can lead to bad behavior


Inductive Case
induction x
induction x
induction x
$\forall x, P(x)$
$\forall x, P(x) \implies P(x+1)$
$\forall x, P(x) \implies P(x+1)$
$\forall x, P(x) \implies P(x+1)$
$P(0)$
$P(0)$
$P(0)$
✓
✓
✓
Base Case

Adding extra rewards can lead to bad behavior



Inductive Case

$\forall x, P(x)$

`induction x`

$\forall x, P(x) \implies P(x+1)$

`induction x`

$\forall x, P(x) \implies P(x+1)$

`induction x`

$\forall x, P(x) \implies P(x+1)$

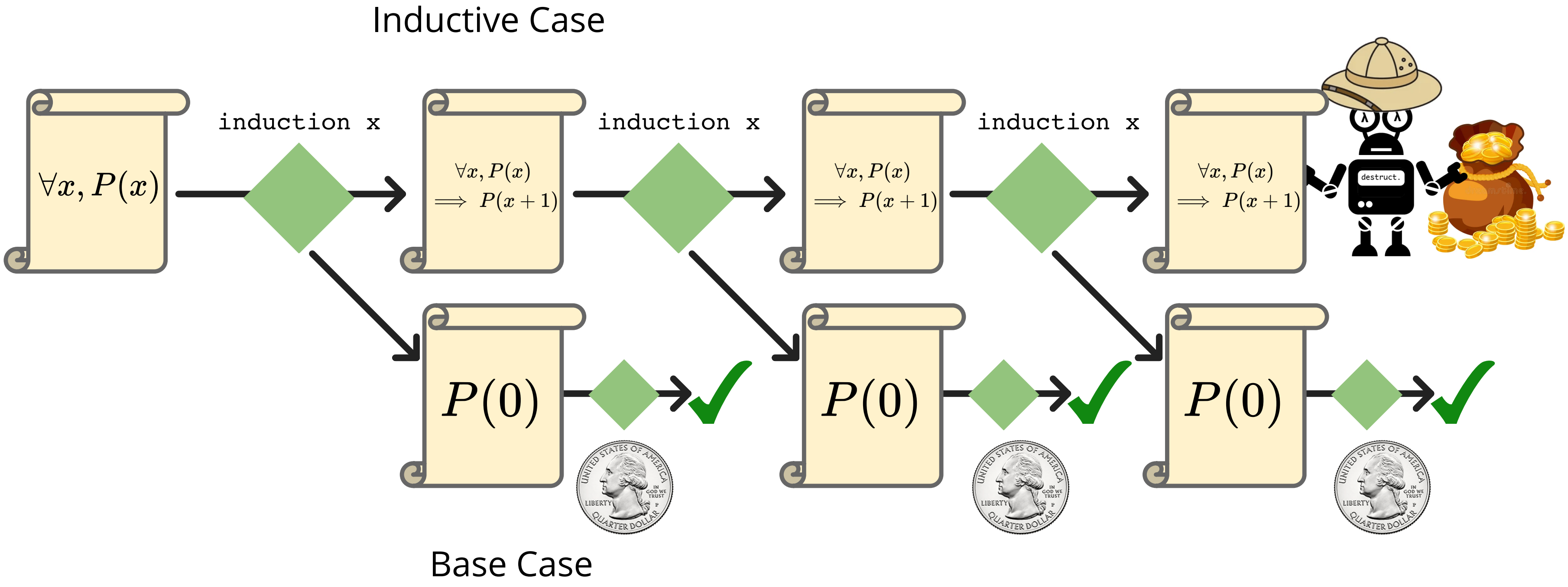
$P(0)$ ✓

$P(0)$ ✓

$P(0)$ ✓

Base Case

Adding extra rewards can lead to bad behavior



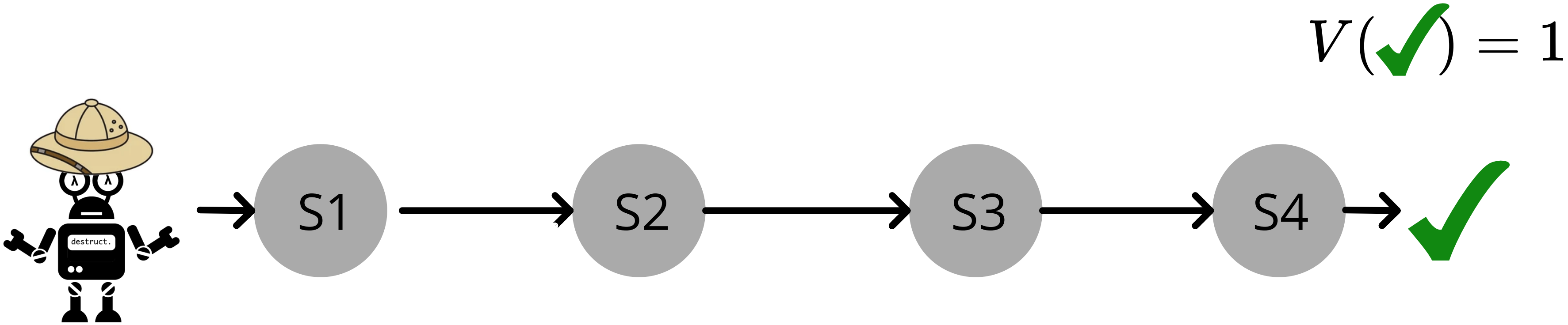
Reward-free doesn't have this problem!

What We Need

A new update equation that accounts for the branching structure of proofs

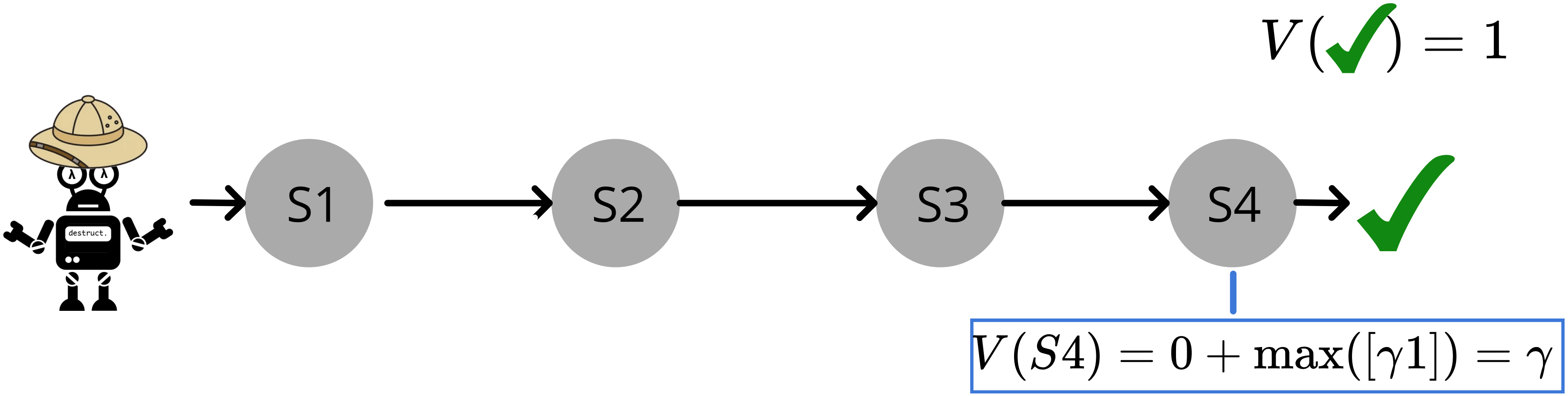
Assumptions:

- The state of a completed proof has value 1



Assumptions:

- The state of a completed proof has value 1

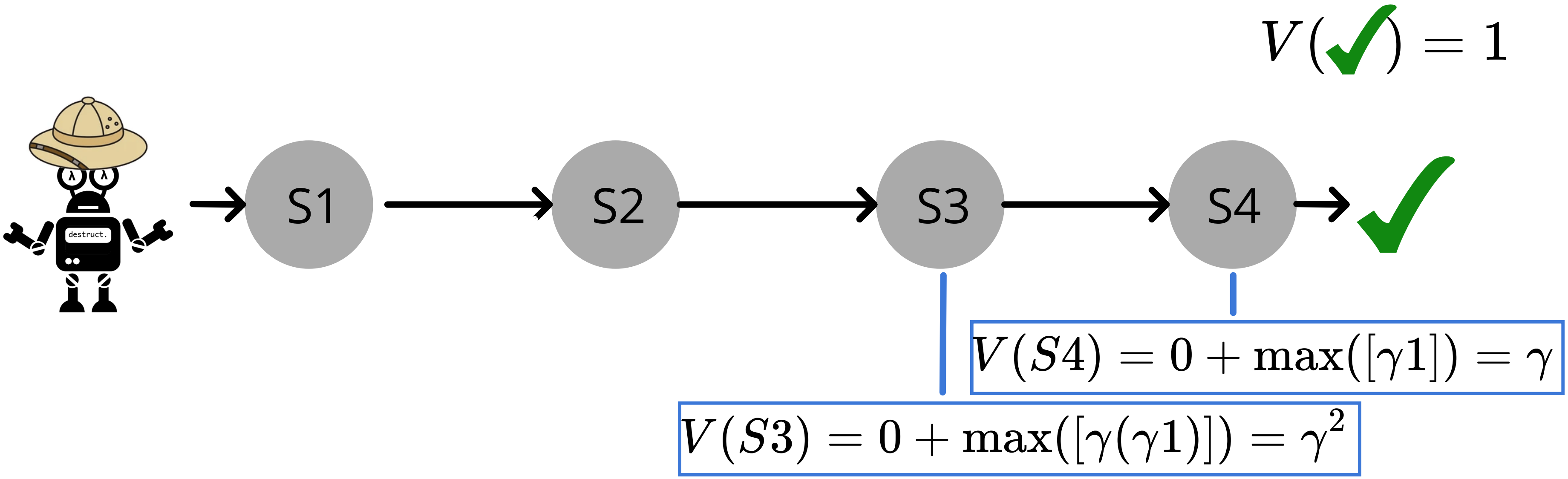


$$V(\checkmark) = 1$$

$$\boxed{V(S4) = 0 + \max([\gamma 1]) = \gamma}$$

Assumptions:

- The state of a completed proof has value 1



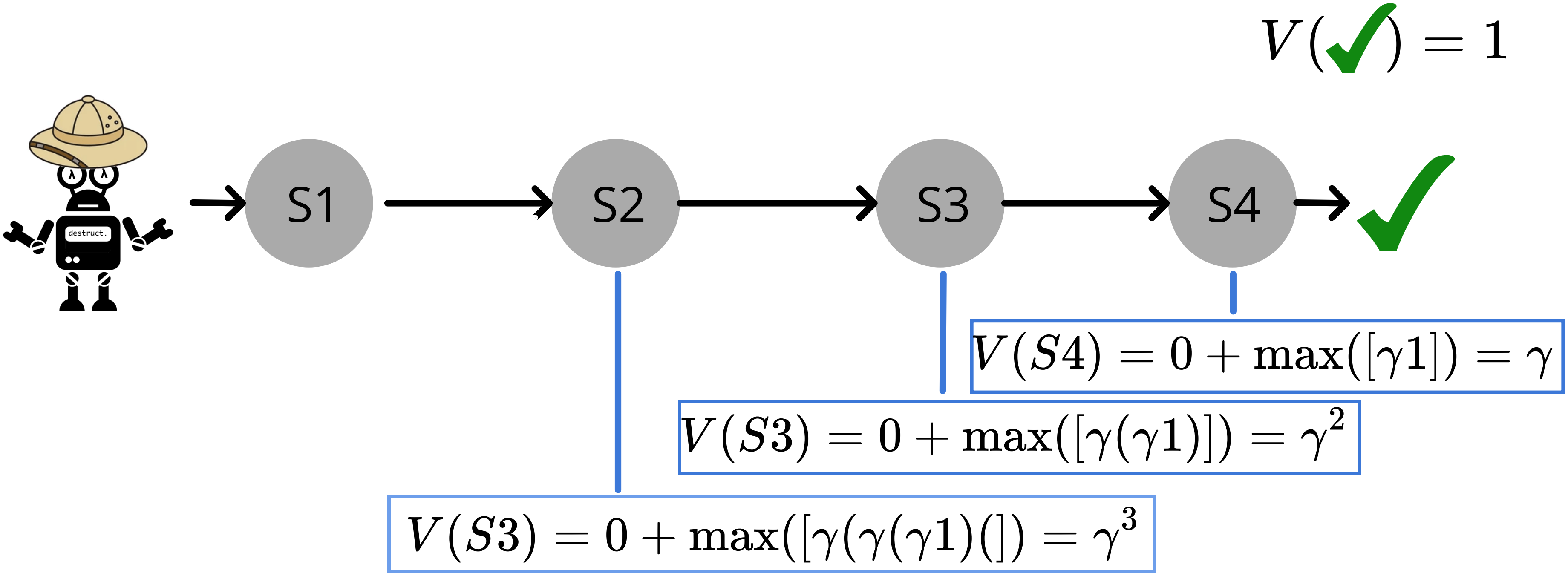
$$V(\checkmark) = 1$$

$$V(S4) = 0 + \max([\gamma 1]) = \gamma$$

$$V(S3) = 0 + \max([\gamma(\gamma 1)]) = \gamma^2$$

Assumptions:

- The state of a completed proof has value 1



$V(\checkmark) = 1$

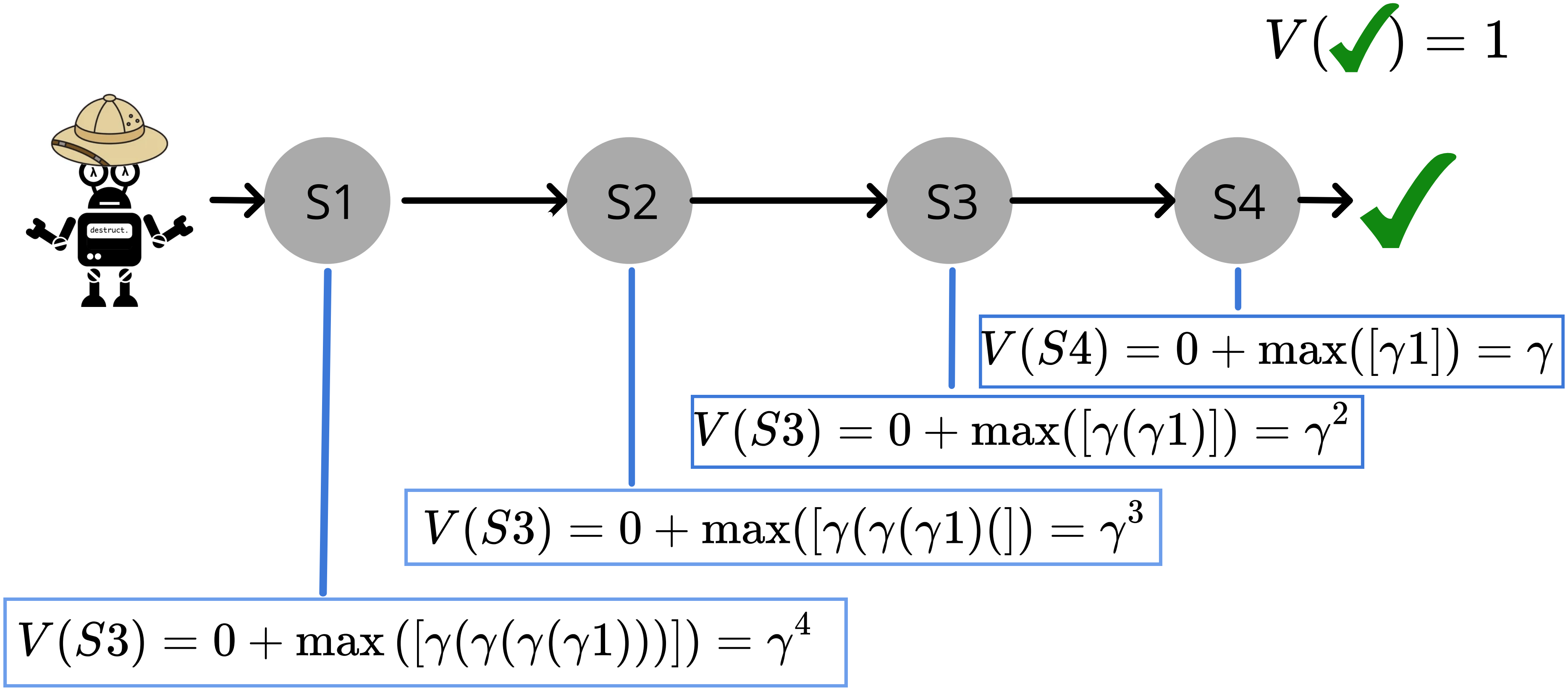
$V(S4) = 0 + \max([\gamma 1]) = \gamma$

$V(S3) = 0 + \max([\gamma(\gamma 1)]) = \gamma^2$

$V(S3) = 0 + \max([\gamma(\gamma(\gamma 1)(]) = \gamma^3$

Assumptions:

- The state of a completed proof has value 1



$$V(\checkmark) = 1$$

$$V(S4) = 0 + \max([\gamma 1]) = \gamma$$

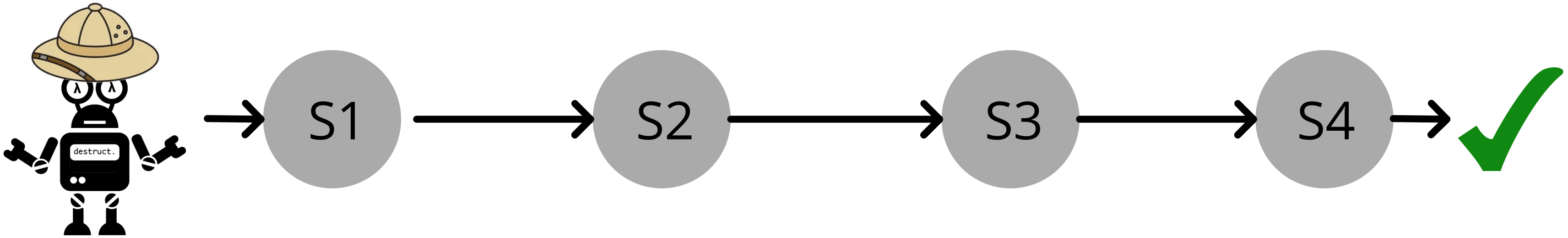
$$V(S3) = 0 + \max([\gamma(\gamma 1)]) = \gamma^2$$

$$V(S3) = 0 + \max([\gamma(\gamma(\gamma 1)(]) = \gamma^3$$

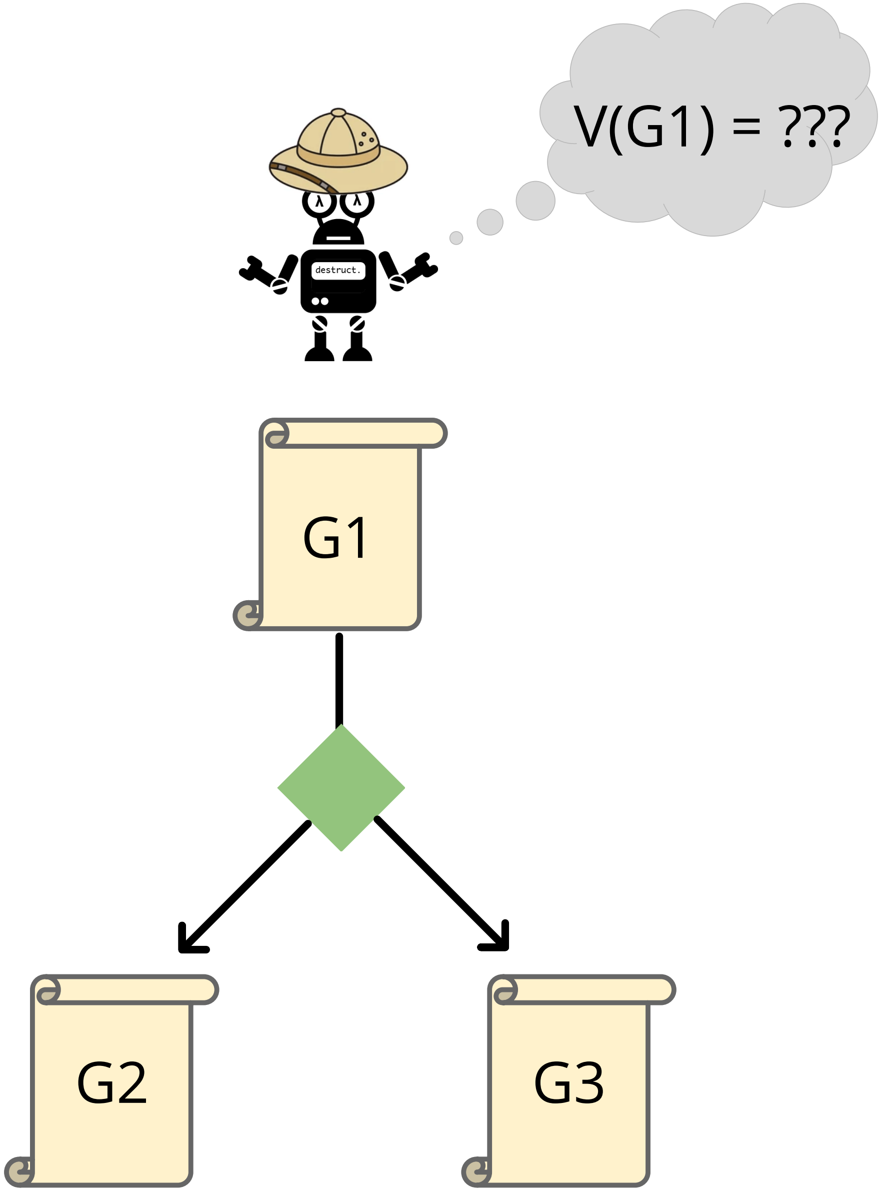
$$V(S3) = 0 + \max([\gamma(\gamma(\gamma(\gamma 1)))]) = \gamma^4$$

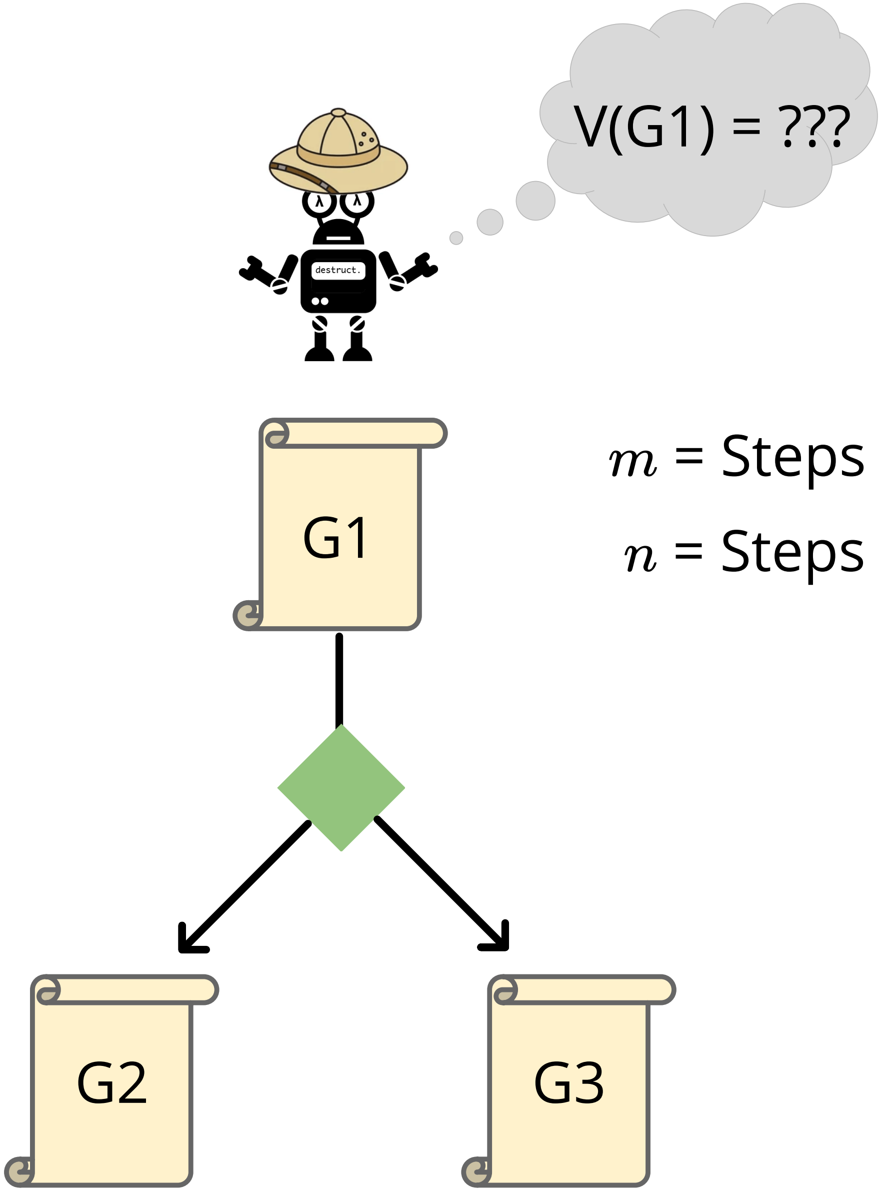
Assumption: The state of a completed proof has value 1

$V(\checkmark) = 1$



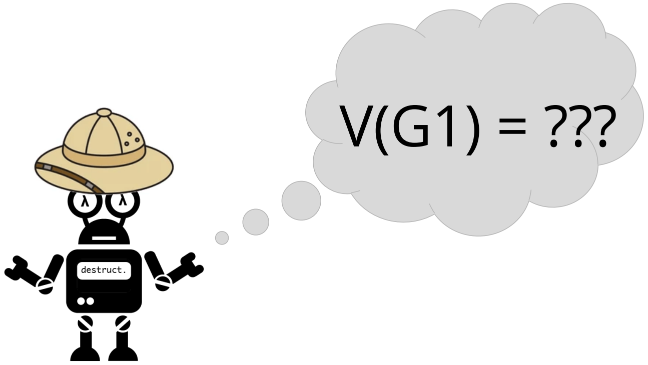
$$V(S) = \gamma^{(\text{number of steps left})}$$

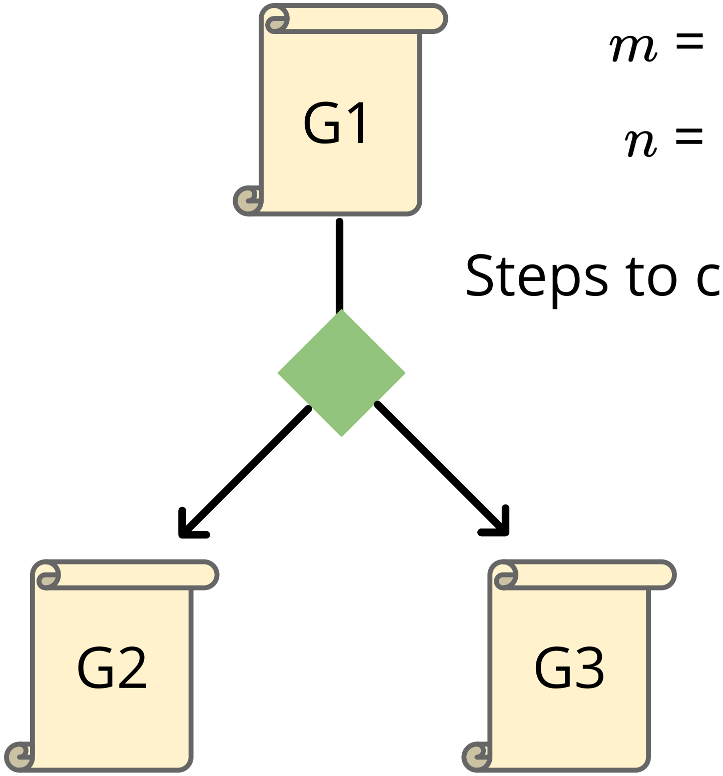

V(G1) = ???
destruct.
G1
G2
G3



m = Steps to complete proof from G2

n = Steps to complete proof from G3

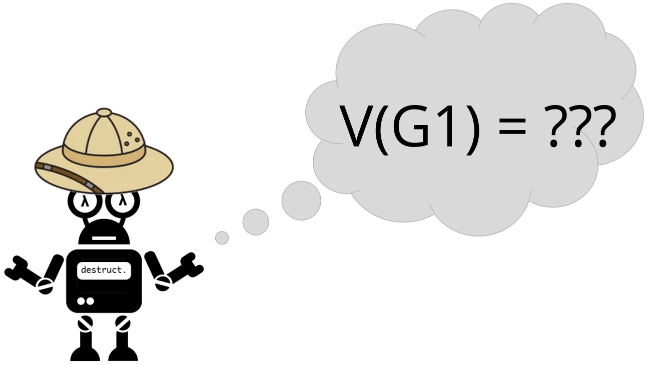


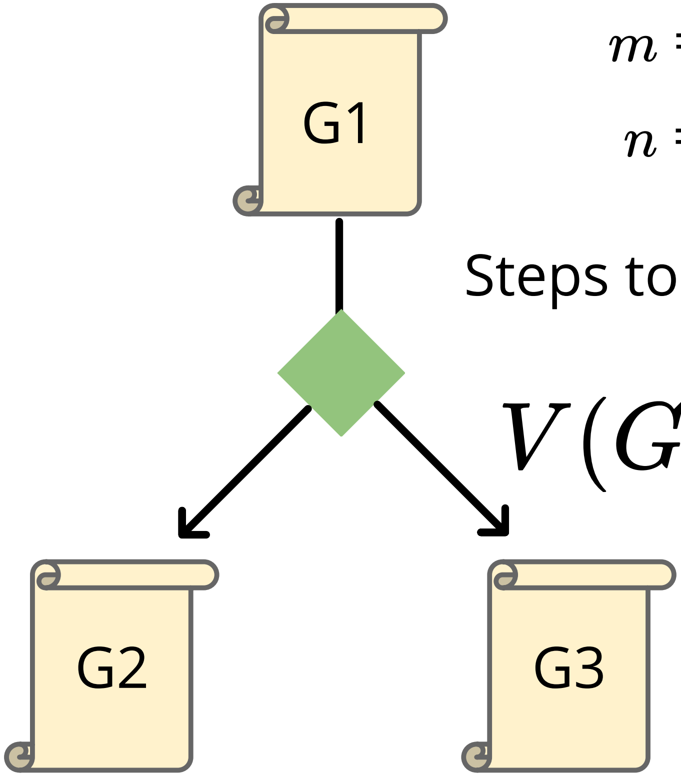


m = Steps to complete proof from G2

n = Steps to complete proof from G3

Steps to complete proof from G1 = $m + n + 1$





m = Steps to complete proof from G2

n = Steps to complete proof from G3

Steps to complete proof from G1 = $m + n + 1$

$$V(G1) = \gamma * V(G2) * V(G3)$$

Update Equation for Branch-Structured Proofs

$$V(G) = \max_{a \in \text{actions}(G)} \left(\gamma \prod_{G' \in \text{next-states}(G,a)} V(G') \right)$$

Update Equation for Branch-Structured Proofs

$$V(G) = \boxed{\max_{a \in \text{actions}(G)}} \left(\gamma \prod_{G' \in \text{next-states}(G,a)} V(G') \right)$$

Update Equation for Branch-Structured Proofs

$$V(G) = \max_{a \in \text{actions}(G)} \left(\gamma \prod_{\boxed{G' \in \text{next-states}(G,a)}} V(G') \right)$$

Update Equation for Branch-Structured Proofs

$$V(G) = \max_{a \in \text{actions}(G)} \left(\gamma \boxed{\prod_{G' \in \text{next-states}(G,a)} V(G')} \right)$$

Update Equation for Branch-Structured Proofs

$$V(G) = \max\nolimits_{a \in \text{actions}(G)} \left(\boxed{\gamma} \prod\nolimits_{G' \in \text{next-states}(G,a)} V(G') \right)$$

Update Equation for Branch-Structured Proofs

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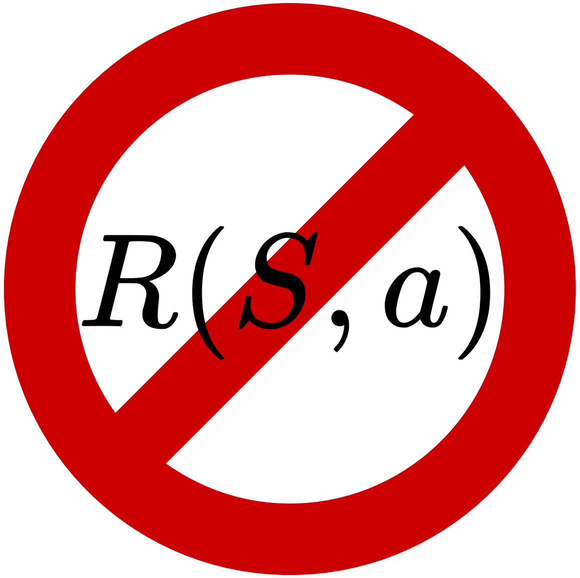
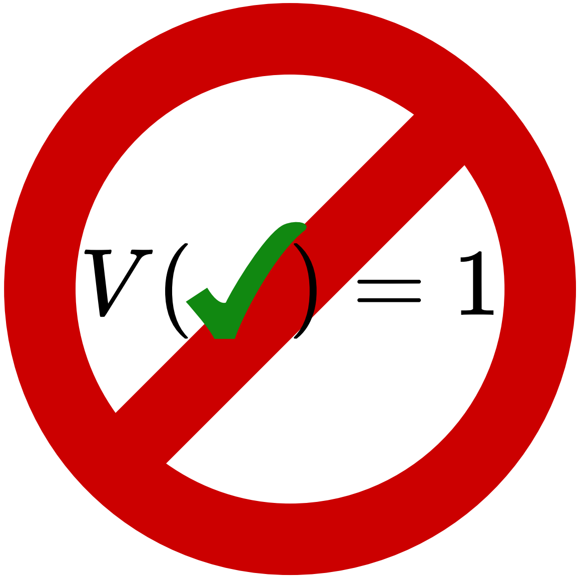
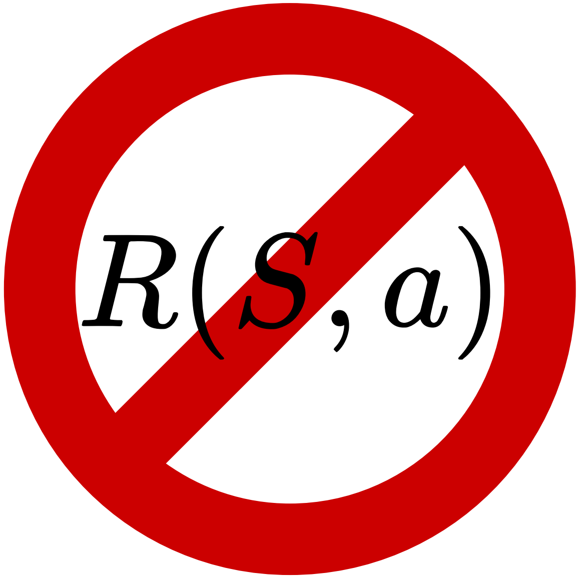


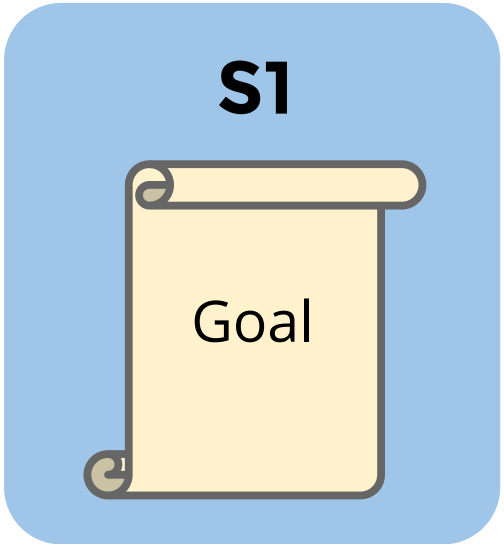
Update Equation for Branch-Structured Proofs

$$V(G) = \text{max}_{a\in\text{actions}(G)} \left(\gamma \prod_{G'\in\text{next-states}(G,a)} V(G')\right)$$

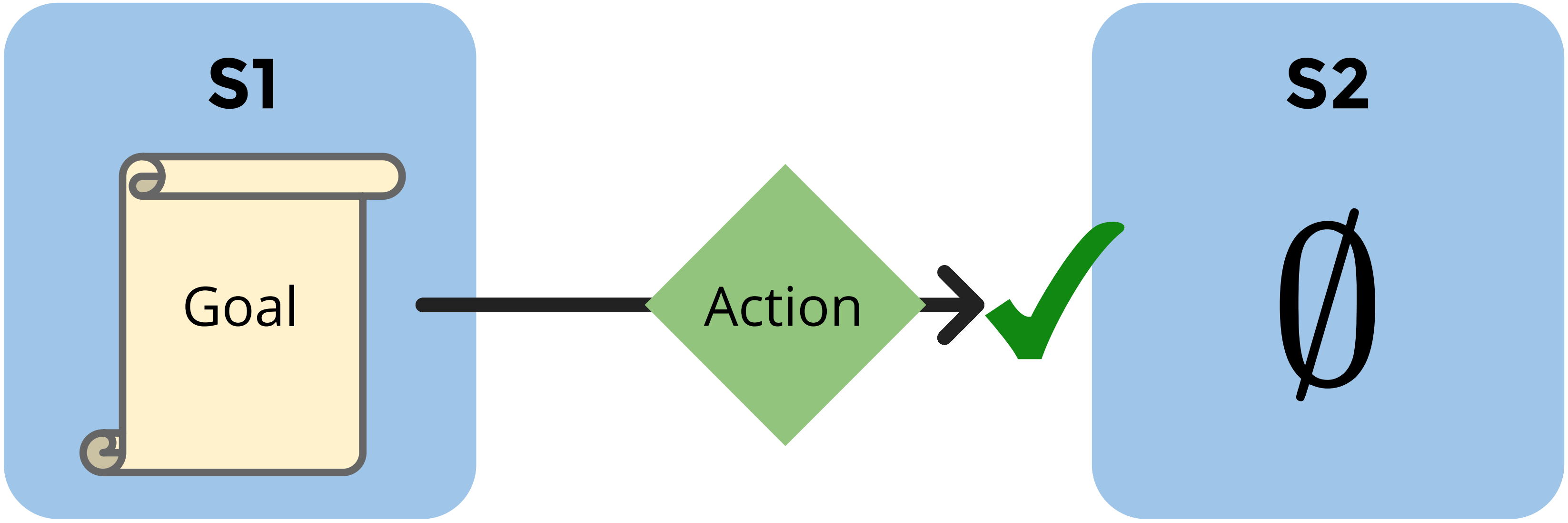




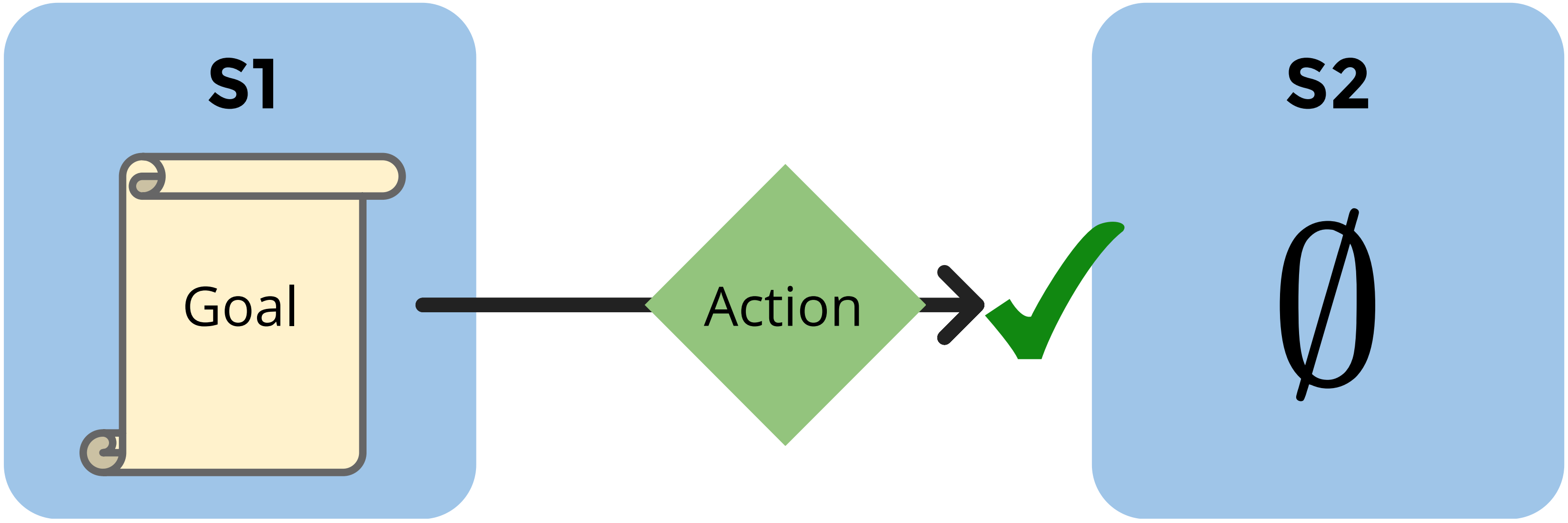
Don't need one!



Don't need one!

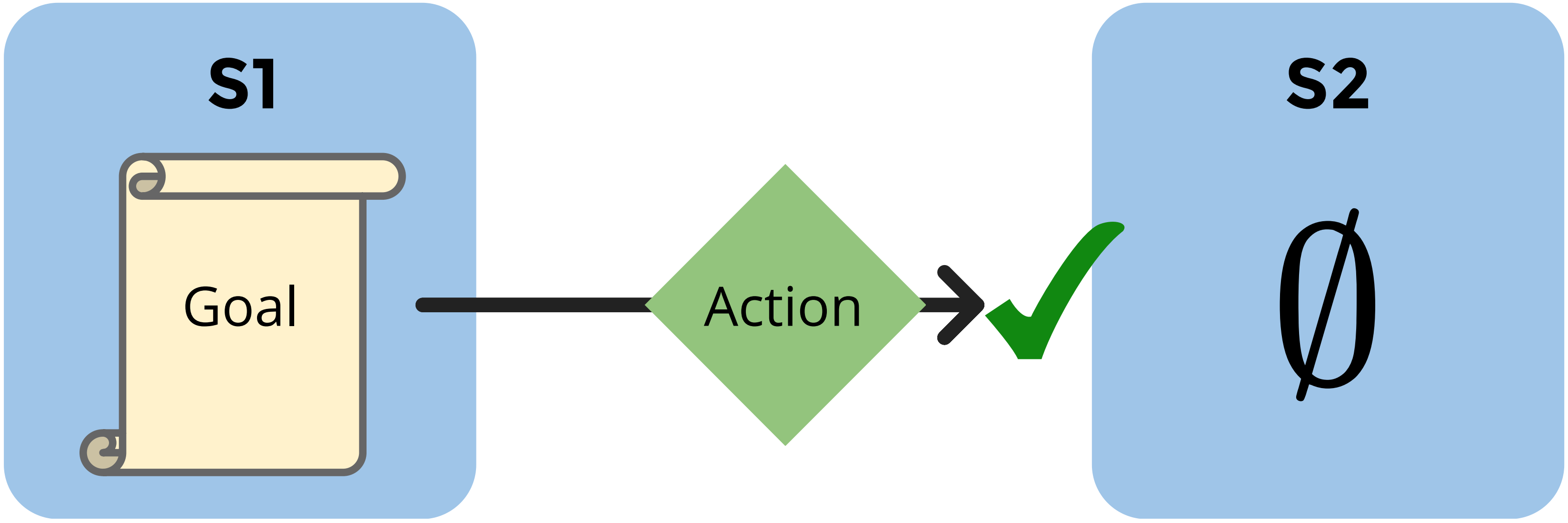


Don't need one!



$$V(G) = \max_{a \in \text{actions}(G)} \left(\gamma \prod_{G' \in \text{next-states}(G,a)} V(G') \right)$$

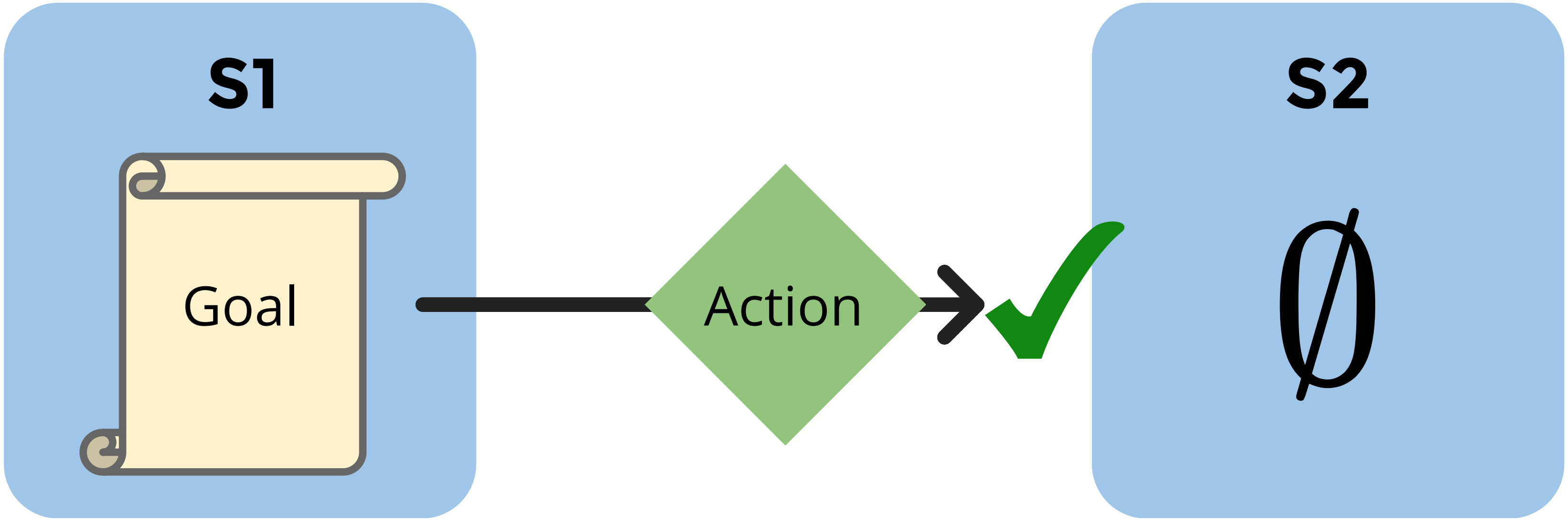
Don't need one!



$$V(G) = \max_{a \in \text{actions}(G)} \left(\gamma \prod_{G' \in \text{next-states}(G,a)} V(G') \right)$$

$$\gamma \prod_{G' \in \emptyset} V(G')$$

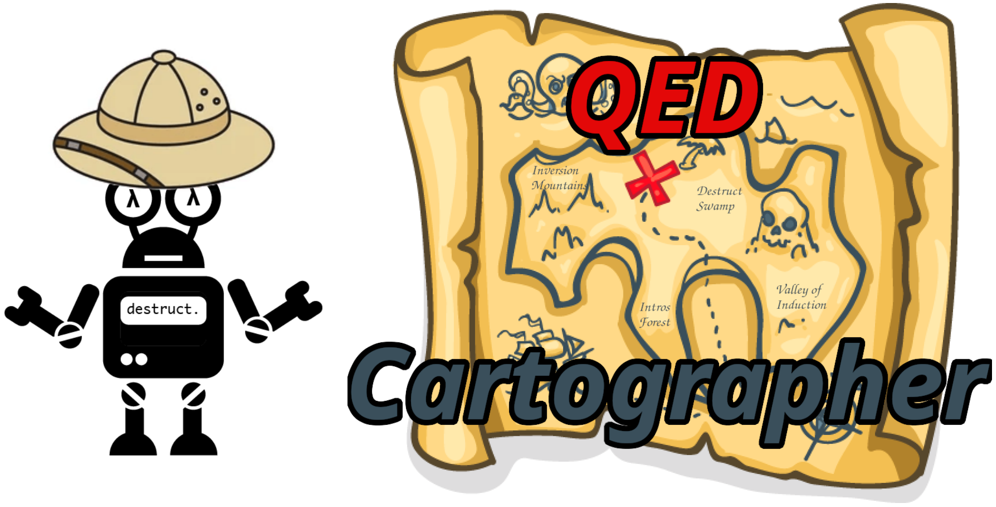
Don't need one!



$$V(G) = \max_{a \in \text{actions}(G)} \left(\gamma \prod_{G' \in \text{next-states}(G,a)} V(G') \right)$$

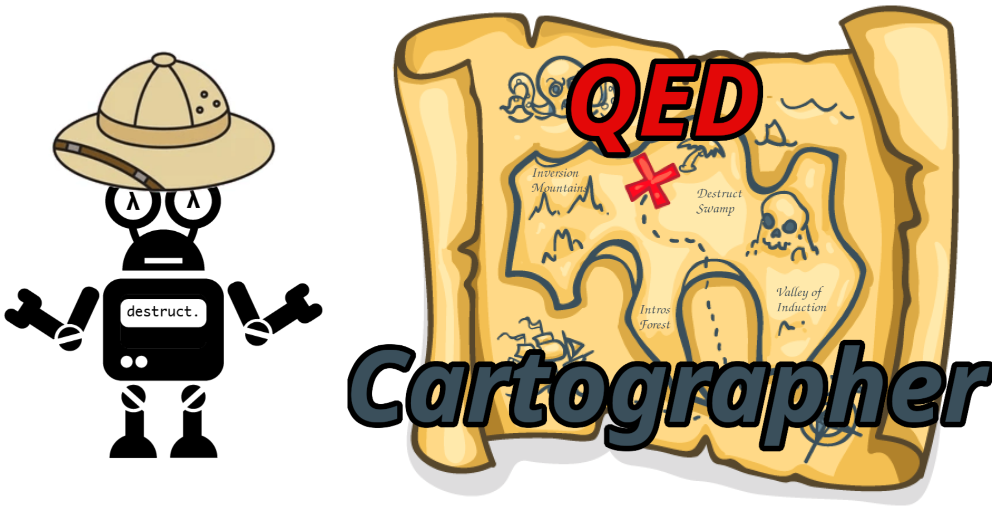
$$\gamma \prod_{G' \in \emptyset} V(G')$$

$$\gamma(1)$$



QED Cartographer

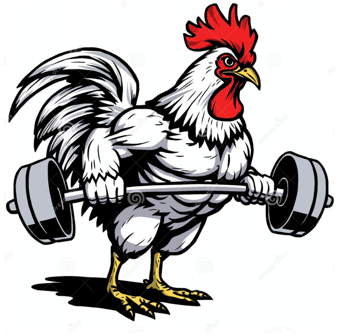
Automating Formal Verification with Reward-Free Reinforcement Learning



Automating Formal Verification with
Reward-Free Reinforcement Learning

26% Shorter Proofs
in 27% Fewer Steps

Benchmark: CoqGym



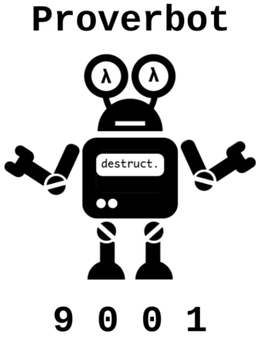
124 Coq Projects

68,501 Theorems

85/15 train-test split

Baseline: Proverbot9001 (updated)

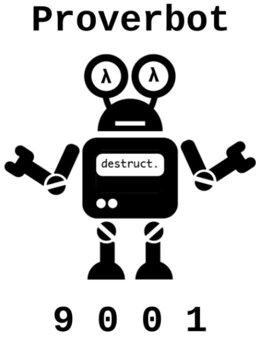
QEDCartographer, except without state scoring-based search

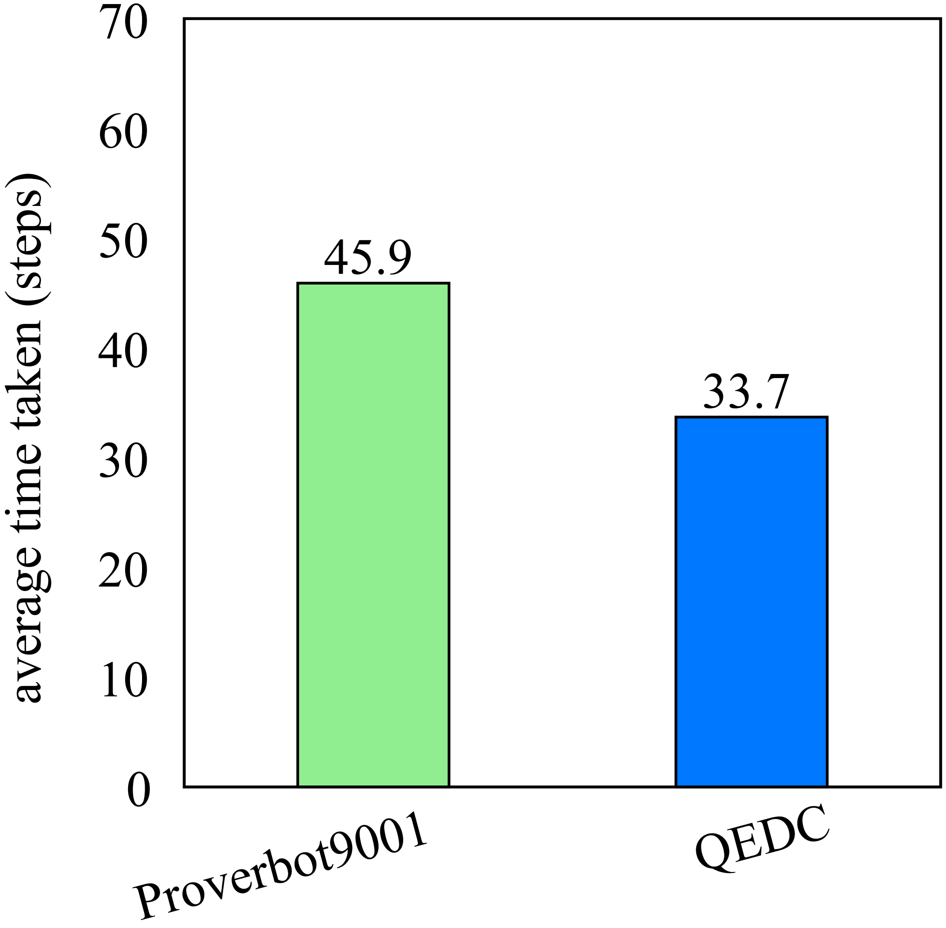


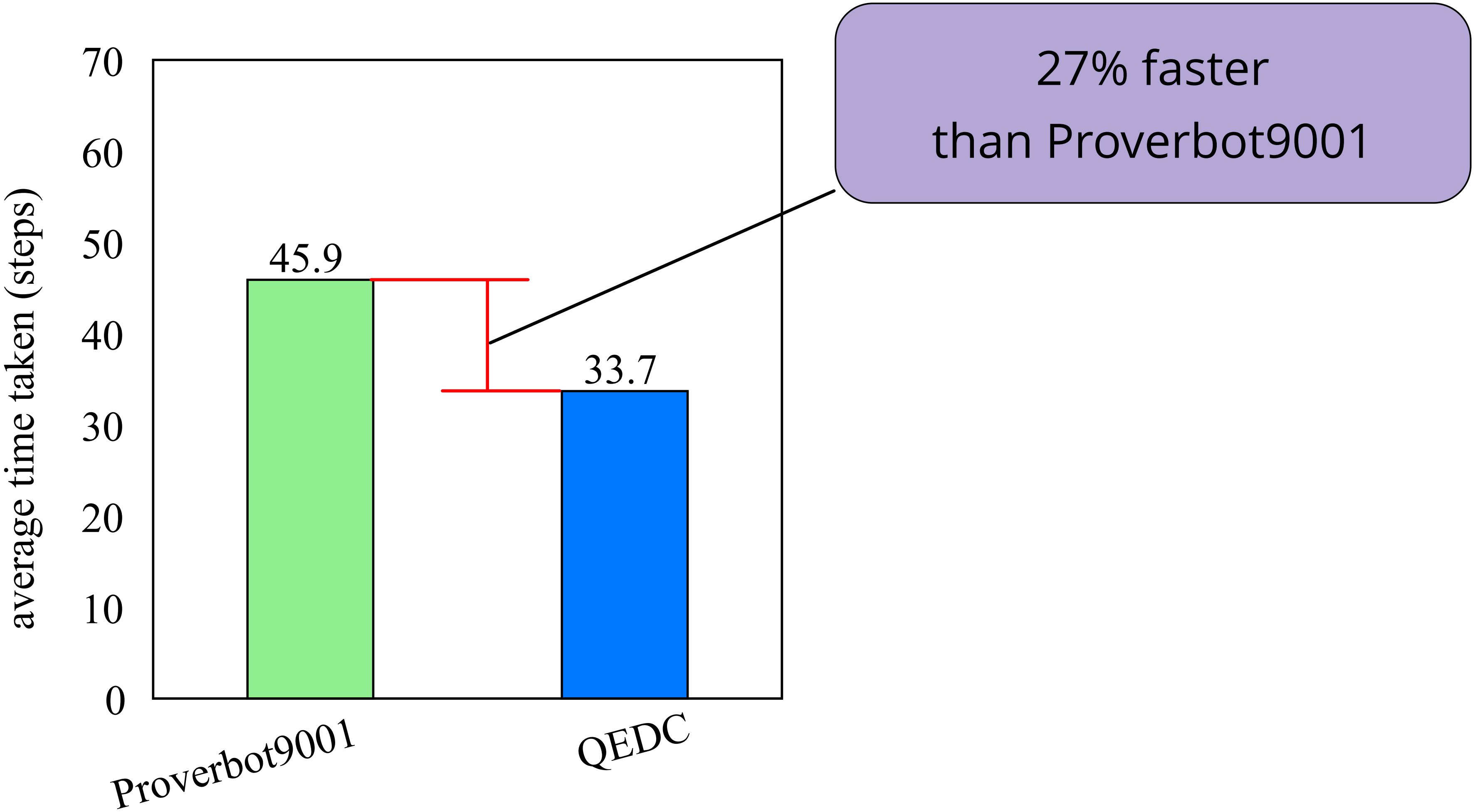
Baseline: Proverbot9001 (updated)

QEDCartographer, except without state scoring-based search

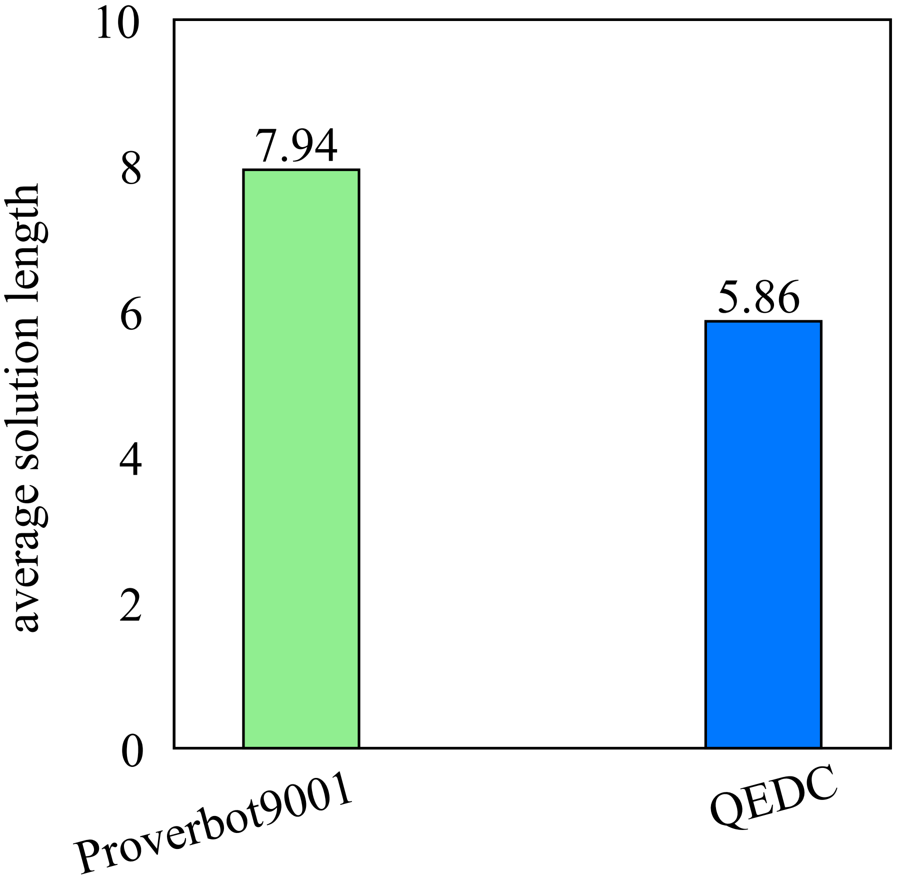
Uses a variant of depth-first search instead



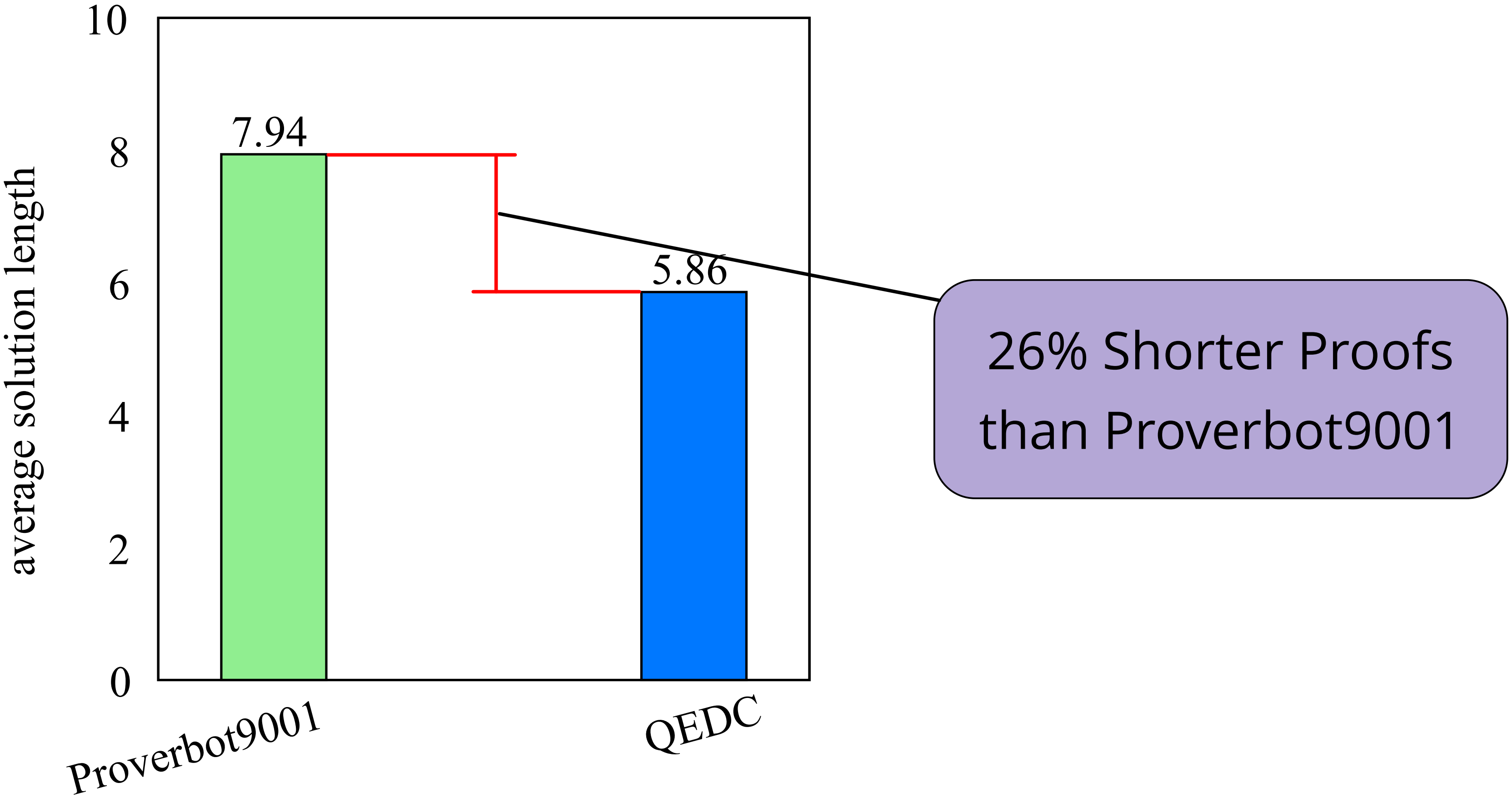

average time taken (steps)
70
60
50
40
30
20
10
0
45.9
33.7
Proverbot9001
QEDC



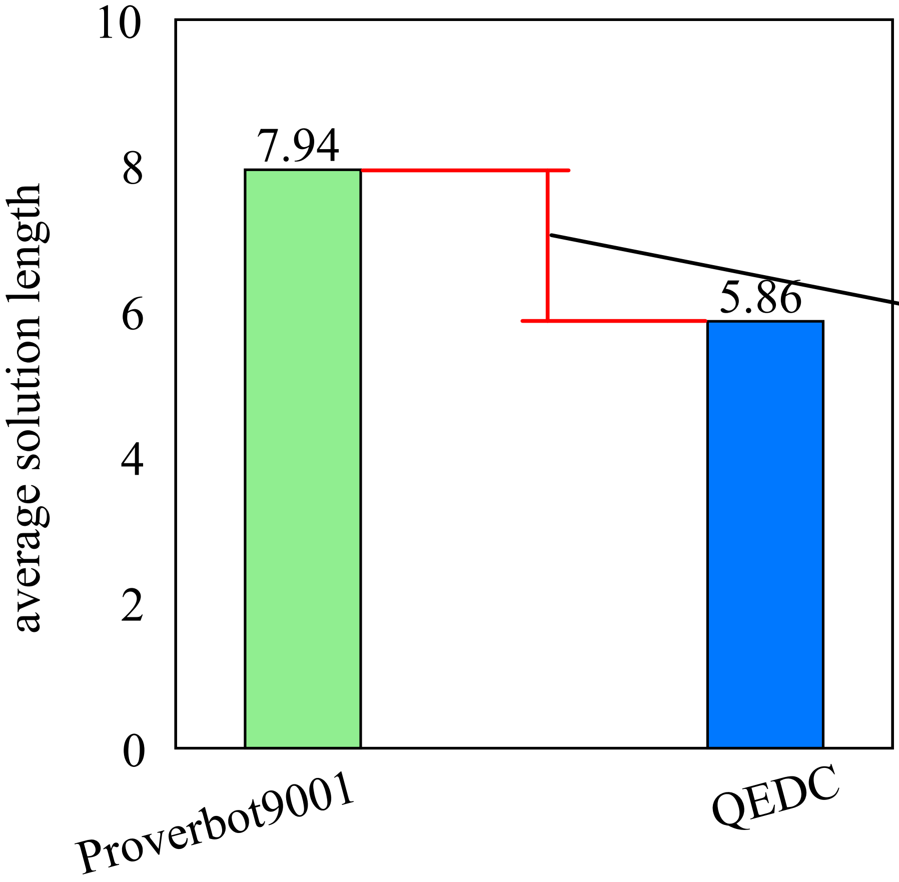
27% faster
than Proverbot9001



average solution length
10
8
6
4
2
0
7.94
5.86
Proverbot9001
QEDC



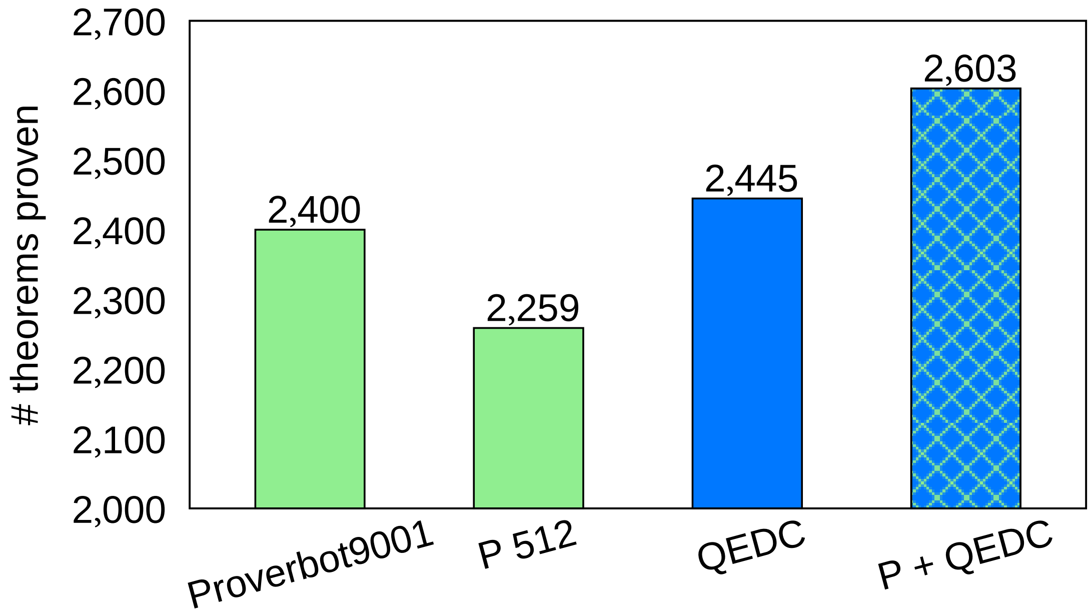
26% Shorter Proofs than Proverbot9001



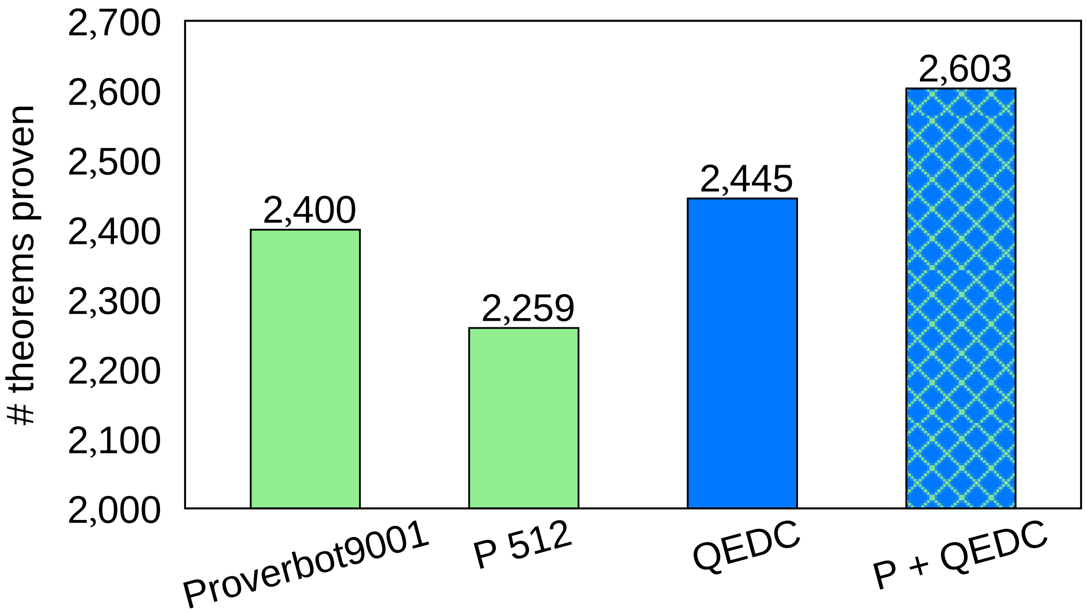
26% Shorter Proofs than Proverbot9001

Shorter proofs are:

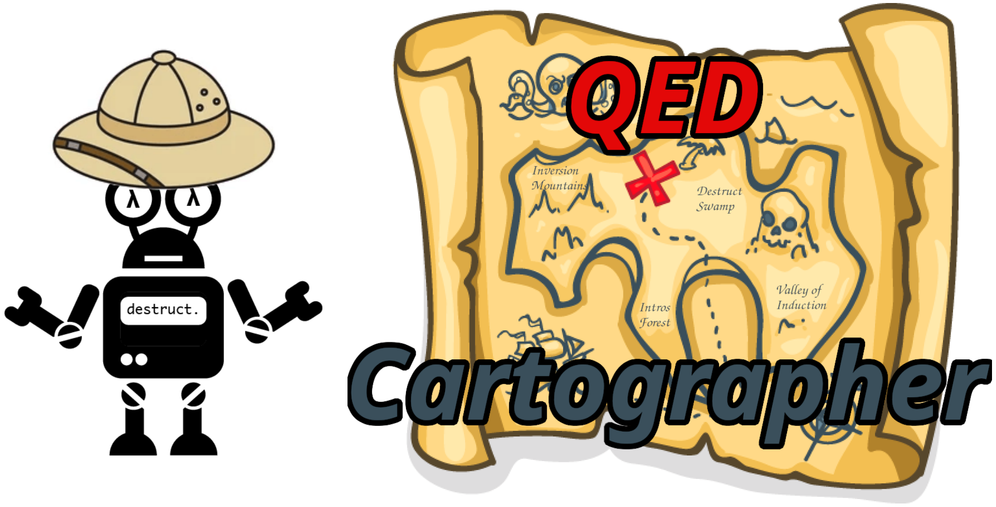
- More readable
- More maintainable
- Less expensive to check



# theorems proven
2,700
2,600
2,500
2,400
2,300
2,200
2,100
2,000
2,400
2,259
2,445
2,603
Proverbot9001
P 512
QEDC
P + QEDC



Proves slightly more theorems, and proves complementary theorems



QED Cartographer

Automating Formal Verification with
Reward-Free Reinforcement Learning

Uses a new V-value equation for branching goal structure

Makes producing verified-correct code easier and faster

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