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Martin Rohrmeier
École Polytechnique Fédérale de Lausanne

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Towards a generative syntax of tonal harmony

Martin Rohrmeier*

Centre for Music & Science, Faculty of Music, University of Cambridge, 11 West Road, Cambridge CB3 9DP, UK

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This paper aims to propose a hierarchical, generative account of diatonic harmonic progressions and suggest a set of phrase-structure grammar rules. It argues that the structure of harmonic progressions exceeds the simplicity of the Markovian transition tables and proposes a set of rules to account for harmonic progressions with respect to key structure, functional and scale degree features as well as modulations. Harmonic structure is argued to be at least one subsystem in which Western tonal music exhibits recursion and hierarchical organization that may provide a link to overarching linguistic generative grammar on a structural and potentially cognitive level.

Keywords: generative grammar; harmony; recursion; music theory; tonal music; music syntax; music and language; context-free grammar; music cognition

MCS/CCS/AMS Classification/CR Category numbers: F1.1; F4.3

1. Introduction

In music theory, harmony is a well-researched area, and there are dozens of taxonomies for tonal harmony phenomena. Most theories concentrate on classifying chord types and deducing complex phenomena from basic (diatonic) chords (such as explaining the predominant Neapolitan chord in minor from a minor *IV* in which the minor sixth replaces the fifth and results in a first inversion chord). However, only few systems give a rule-based account of harmonic progressions, which are often described in terms of acceptable/unacceptable chord-to-chord progressions [1–3]. One seminal example constitutes Piston's table of usual root progressions [3] which has been found to accord with harmonic progressions in Bach's chorales as well as with experimental findings based on probe-chord studies [4–6] to some extent. From a computational perspective, Piston's table constitutes an early, intuitive version of a stochastic Markovian transition matrix.

The idea that tonal music and harmony are governed by an underlying structure, the complexity of which exceeds the simplicity of linear or Markovian approaches and links to generative approaches in linguistics [7–9], has been raised by various approaches [10]. Bernstein's famous Harvard lectures were motivated by drawing a relationship between music and Chomsky's formal grammars [11]. Similarly, the Generative Theory of Tonal Music (GTTM) [12] extends the

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$$
\begin{array}{l}
i - \quad - \quad - - - - - V - - - - - - \\
i - \quad - \quad - - - iv - V - - - - - - \\
i - \quad - \quad - i^6 - iv - V - - - - - - \\
i - vii^0 {}^6_5 - i^6 - iv - V - - - i - V \\
i - vii^0 {}^6_5 - i^6 - iv - V - V^6 - i - V
\end{array}
$$

Figure 1. Haydn, Sonata No. 33, II. Analysis of the levels of harmony according to Kostka and Payne.[1]

notions of the Schenkerian theory [13] by a recursive formalization (of the cognitive structure of an experienced listener) which directly links to the Chomskian generative programme [14,15]. Whereas the GTTM does not explicitly provide generative grammatical *rules*, other approaches specify context-free rules for subsystems [16–28]. For example, Steedman [18,19] designed a context-free grammar specifically for the delimited domain of Blues progressions and Baroni *et al*. [21] modelled melodic structure. Other computational approaches worked towards the formal implementation of the Schenkerian or GTTM reductions [29–32].

The notion that harmony is organized hierarchically may be best illustrated by an example given by Kostka and Payne, who described the hierarchical organization of '*levels of harmony*', which they compared to the syntactic organization of a linguistic sentence [33, Chapter 13]. In the first example of this chapter, they argued that the chord sequence of the beginning of the second movement of Haydn's sonata No. 33 could be hierarchically structured into chords of greater or lesser structural importance (Figure 1). Without bringing this idea into the context of context-free grammars, they implicitly suggested a structure that captures the spirit of context-free grammars.

Following Kostka and Payne's explanation, the diagram illustrates the skeletal structure of the phrase as well as the relative importance (fundamental or ornamental) and function of the chords. In the present example, the first part constitutes an arpeggiation from i to i^6 filled by $vii^0{}^6_5$. The second part constitutes a prolongation of the V, which is prepared by iv and prolonged by i (itself being prepared by the V^6 chord). According to Kostka and Payne, the idea of harmonic layers reveals that 'although each chord may be labelled with its own roman numeral, all chords are *not* equally important. In fact, not all chords with the same label (all V's, all I's) have identical uses. Some serve as starting points, some as goals, others as connectors, and so on' (p. 189).

Such a formalization implicitly represents tree-based dependency structures, recursion and syntactic derivations depending on functional harmonic categories. The purpose of this paper is to propose a formalization of this insight and to propose a core set of grammatical rules to cover the fundamental features of the recursive structure of tonal harmony. Whereas a full account of tonal harmony would require a large number of varying style-specific rules, this contribution focuses on a core set of rules describing core tonal phrases and the relationship between keys and modulations. The formalism strongly relates to the work done by Steedman [18,19]; whereas Steedman's grammar is very specific to the 12-bar Blues scheme and does not, for instance, include a general account of modulation, the proposed grammar aims to generalize to a larger set of tonal harmonic progressions. The formalism differs from theories such as the GTTM [12]. While the GTTM mainly discusses the core principles of tonal cognition without presenting explicit context-free production rules, this paper presents a set of rules that are explicitly designed to be computationally implementable and testable (cf. [34]).

2. Principles of organization

The relationships expressed by Kostka and Payne and others could be brought into a closer formalization and concrete syntactic rules based on two core principles, which shall be illustrated by an analysis of the chord sequence $C\ A^7\ Dm\ G\ C$.

$$C\ A^7\ Dm\ G\ C \tag{a}$$

$$C\ Dm\ G\ C \tag{b}$$

$$(*)\quad C\ A^7\ G\ C \tag{c}$$

$$(*)\quad C\ A^7\ Dm\ C \tag{d}$$

$$C\ G\ C \tag{e}$$

$$C\ A^7\ D^7\ G\ C \tag{f}$$

$$C\ A^7\ A\flat^7\ G\ C \tag{g}$$

$$C\ A^7\ F\sharp^0\ G\ C \tag{h}$$

$$(*)\quad C\ A^7\ C\ G\ C \tag{i}$$

$$(*)\quad C\ A^7\ E\flat m\ G\ C \tag{j}$$

The first set of alterations of the base sequence (a) shows that some deletions of elements are acceptable and others not (examples b–e). Whereas A^7 or both $A^7\ Dm$ can be removed without making the sequence unacceptable, Dm or G cannot be removed, since the single A^7 chord or the $A^7\ Dm$ block would be unconnected to either of the preceding C or succeeding G or C chord. This entails that the A^7 chord is best accounted for by being dependent on Dm rather than the preceding C chord, and similarly, Dm and G, grouped with their dependent antecedent chords A^7 and $A^7\ Dm$, are both dependent on their consequent chord as whole blocks. Accordingly, this motivates a representation in which the chords are dependent based on a tree structure (Figure 2).

The second set of examples illustrates the replacement of a chord by functionally equivalent chords. In this case, Dm could be replaced by the major dominant seventh chord D^7, since both fulfil the role of predominant functions. In a Jazz context, it could also be replaced by $A\flat^7$ (g), which constitutes the tritone substitution of the secondary dominant (in a common-practice context, it would constitute a German-augmented sixth chord using different pitch spelling). Similarly, a replacement by $F\sharp^0$ would be possible if the leading note in A^7 is continued downwards. Finally, a replacement by C (i) or $E\flat m$ (j) would be unacceptable, since it would leave the A^7 chord 'hanging' and unconnected to either the preceding or subsequent context, even though the

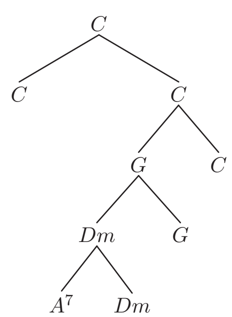


Figure 2. A tree-based representation of the dependency structure of the sequence $C\ A^7\ Dm\ G\ C$.

partial chord progression $C\ G\ C$ would be perfectly fine. Altogether, this example illustrates the organization of chord sequences by recursive dependencies and substitution of functionally equivalent chords and motivates their formalization based on a phrase-structure grammar. The subsequent two paragraphs will explain the two underlying principles.

2.1. *Dependency principle*

This principle states that each element (chord) in a chord sequence is structurally connected to its preceding or succeeding chord or chord group in a dependency relationship. Each group of dependent chords (which may contain more than two elements) recursively distinguishes a head on which the other elements of that group are dependent. The chords in a harmony sequence form recursive dependency relationships until there is only one head for the whole sequence or phrase. This corresponds to the dependencies of syntactic constituents such as '((the man) (took (the (blue hat))))'. Accordingly, the chord sequence '$C\ A^7\ Dm\ G$' would be structured as '$(C\ ((A^7\ Dm)\ G))$' as discussed above. The fact that dependency relationships to non-adjacent groups are disallowed forces the dependency structure to constitute a *planar* tree.

The dependency structure constructed by this principle reflects the way how single chords are justified by their function within the whole overarching context and further recursively defines their (relative) degree of importance. Similarly, the dependency structure, as expressed by the tree, reflects the order by which single chords may be removed from or entered into the sequence.

This principle further entails that harmony sequences may form long-distance dependencies between chords that are separated by other functional chords in between, such as C and G in the example given above. Complementarily, it entails that local adjacencies between structurally unrelated chords may occur when both belong to two different dependency branches and do not share the same parent node. For instance, the progression from C to A^7 in the example given above is not justified by itself, as neither chord could function as their head. The presence of A^7 after C implies that it would be followed by Dm, for instance, so that the A^7 chord would be justified by its consequent chord, which itself would need to be justified within the next context recursively. In this context, in an empirical study, Woolhouse and Rohrmeier [35] found that chord progressions without the same parent node exhibit smaller values of chord attraction than structurally connected surface chords.

2.2. *Functional heads*

This principle states that chords are organized into functional categories which describe their tonal function which may be instantiated or modified by different chords. Following a functional approach [2], the three main tonal functions are *tonic*, *dominant* and *predominant*. In this case, the category of dominants would include chords such as V, VII or $\flat II$ (in the case of Jazz harmony), or predominants would include various chords on $\hat{4}$ or $\hat{2}$.[2]

Such functional categories support the use of abstract category variables for the three main functions instead of the scale degree representation so that the derivation of different chord sequences that are functionally identical on a higher level would reflect this similarity. For instance, Gjerdingen [36] discussed the example of $A\flat\ B\flat\ Cm$, which would reflect a (late-nineteenth-century) progression derived from subdominant–dominant–tonic substitutions. In this context, tonic/dominant/subdominant categories would constitute functional symbols that could be realized by a number of different surface chords. In combination with the first dependency principle it further entails that there may be groups of chords that fulfil (prolongate) tonic/dominant/predominant functions as a whole constituent.

3. Formalization

The proposed formalism is based on functional theories of harmony drawing upon the Riemannian tradition [2,37,38] and employs the two principles described above. The core of the functional approach is rooted in the assumption that chords in a harmonic sequence fulfil functions that derive from three elementary harmonic functions, *predominant*, *dominant* and *tonic*. The formalism distinguishes four levels: a *phrase level*, a *functional level*, a *scale degree level*, and the *surface level*. Accordingly, it employs sets of phrase-level symbols $\mathbb{P} = \{piece, P\}$, key symbols $\mathbb{K} = \{Cmaj, Cmin, C\sharp maj, C\sharp min, D\flat maj, D\flat min, \ldots\}$, functional region symbols $\mathbb{R} = \{TR, SR, DR\}$, functional terms $\mathbb{F} = \{t, s, d, tp, sp, dp, tcp\}$, scale degrees and relative scale degrees $\mathbb{S} = \{I, II, \ldots, VII, V/I, V/II, \ldots, VII/I, VII/II, \ldots\}$, and surface chord symbols $\mathbb{O} = \{Cmaj, Cmin, C^0, C^{\emptyset}, \ldots\}$. The generation begins on the phrase or the functional level and continues recursively until a sequence of surface symbols is generated (or parsed).

3.1. *Phrase level*

On the phrase level, harmony sequences are generated by tonic seeds that constitute the head of single phrases.

$$piece_{key=x\in\mathbb{K}} \longrightarrow P^{+} \tag{1}$$

$$P \longrightarrow TR \tag{2}$$

Rule 1 starts the generative process by the specification of the overarching *key* feature (having values for all different major/minor keys) and defines a piece as a series of co-ordinated parallel phrases. A phrase ending on a perfect cadence is created by rule 2 through the generation of a single tonic seed which will be recursively expanded to a full sequence through subsequent rules on the functional level. A phrase ending on V or a half cadence is generated by rules 2 and 6, which generate a tonic seed (defining tonic and key) and a dominant seed ending the phrase. It is important to note here that a singleton dominant seed (without a preceding or subsequent tonic reference) cannot be generated by this grammar. The plagal cadence is created based on rules 2 and 21. This set of rules is designed in order to match phrase-based forms of harmony, as in Jazz or in pop music, in which phrases may not necessarily be connected through one single overarching tonic prolongation over the level of a whole piece. For harmonic sequences of the common-practice period, one may argue, in analogy to the paradigms by Schenker [13] or Lerdahl and Jackendoff [12], that the generation from a single tonic symbol (or tonic prolongation) would be sufficient to model an entire piece of tonal music recursively.[3] Accordingly, a formalization of the harmonic skeleton of a tonal piece in the full recursive way would only require the following alternative rule:

$$piece \longrightarrow TR_{key=x\in\mathbb{K}} \tag{3}$$

Either initiation of a piece requires the definition of one overarching key property. It is important to note that on a strictly formal level, rules 1 and 2 become obsolete since they can be expressed using rules 3 and 7. However, it was decided to keep the distinction between phrase level and functional level since some analytic applications may want to express differences between phrase structure and functional structure (or add additional phrase definitions), and future computational implementations may add preference constraints (such as metrical or length constraints) specifically at the phrase level.

3.2. *Functional level*

The functional level characterizes harmonic relationships on an abstract level which only concerns relationships between functions and keys and describes different manipulations that may transform functional progressions in an abstract way before they are 'sent off' to a more surface-based representation. The core assumption behind this abstraction is that many chord progressions share the same functional relations even though their scale-degree realizations might be very different and conceal these relationships. Further, this way, the tree reflects and makes explicit all different steps of manipulation by which a chord sequence is derived.

The functional rules characterize the behaviour of tonal chords in four different sets of rules that are applied recursively: a set of expansion rules, according to which core functional sequences may be expanded; a set of substitution rules modelling how functional elements may be substituted by parallels (relatives) and two modulation rules formalising modulation and change of mode.

3.2.1. *Functional expansion rules*

$$TR \longrightarrow DR \quad t \tag{4}$$

$$DR \longrightarrow SR \quad d \tag{5}$$

$$TR \longrightarrow TR \quad DR \tag{6}$$

$$XR \longrightarrow XR \quad XR \qquad \text{for any } XR \in \mathbb{R} \tag{7}$$

$$TR \longrightarrow t \tag{8}$$

$$DR \longrightarrow d \tag{9}$$

$$SR \longrightarrow s \tag{10}$$

These rules characterize the core behaviour of functional regions or units establishing tonic/dominant/ pre-/subdominant functions represented by the symbols $TR/DR/SR$, respectively. They distinguish between progressive and prolongational functional sequences. The expansion rules propose three essential relationships for the progression of tonal functions: dominant regions prepare tonics (rule 4), and predominant regions prepare dominants (rule 5) and any functional region, represented by the variable XR, may expand recursively (rule 7) in a functional prolongation (in which all XR labels are identical). Although this rule induces ambiguities, since, for example, three TR symbols may be parsed in two ways, such ambiguities entail musically meaningful distinctions with respect to the heads and subordination. Rule 7 differs from rule 1 since the latter partitions a piece into different phrases, whereas the former models functional prolongations. The second set of rules describes the generation of elementary functional chord terms from functional regions.

These rules define a general (diatonic) framework around the main tonal functions starting from tonic phrases TR that are defined on a superordinate level. The rules express features of overarching phrase structure as well as cadential contexts. For instance, an antecedent – consequent period can be modelled using rules 2, 4 or 6. All functional rules pass on their *key* property assigned from the superordinate parent nodes to their children.

3.2.2. *Substitution rules*

By a second class of derivations, each functional symbol may be replaced or substituted by their relatives or parallels (in the sense of [2,37]). Rule 11 captures the replacement function of tonic parallels, which is, for instance, important for deceptive cadences. Similarly, rules 13 and 14 characterize subdominant or dominant parallels which could replace subdominants/dominants functionally in a harmonic context (though the use of dp is restricted to minor and rare). Rule 12 defines the rare case of the tonic counter parallel [37] or the Riemannian 'Leittonwechselklang' as, for instance, found at the beginning of Schubert's Lied '*Im Frühling*', D.882.

$$t \longrightarrow tp \tag{11}$$

$$t \longrightarrow tcp \tag{12}$$

$$s \longrightarrow sp \tag{13}$$

$$d \longrightarrow dp \tag{14}$$

3.2.3. *Modulation rule*

One of the most important features of the grammar is modulation, which is formalized in the following way: each functional region may itself become a local tonic during the generation/derivation process. For instance, a d, tp or s node may become the new local tonic dominating all the respective children of that node. The change of key has the effect that new functional symbols that are recursively related to the new local tonic are transposed with respect to the parent tonic. This formalization captures diatonic as well as enharmonic modulation.

$$X_{key=y} \longrightarrow TR_{key=\psi(X,y)} \quad \text{for any } X \in \mathbb{F} \text{ and } y \in \mathbb{K} \tag{15}$$

$$X_{key=y\ maj/min} \longrightarrow X_{key=y\ min/maj} \quad \text{for any } X \in \mathbb{F} \text{ and } y \in \mathbb{K} \tag{16}$$

Rule 15 is the main rule which makes use of the key property of each symbol. Through the generation of the piece and during each rewrite step, each symbol implicitly carries the key property. When it is not notated in the rule, it is assumed that the symbol passes on its key property identically to the rewritten symbols, for instance $TP_{key=y} \rightarrow DP_{key=y}\, TP_{key=y}$. Within the formalism, the variables X and y signify a placeholder for any functional term (like t, tp, s, etc.) or the key feature (like $D\ maj$) (indicated by a lowercase symbol y). The modulation rule 15 specifies that X, representing any functional term except the tonic, may be rewritten as the new (local) tonic which defines the new key according to the respective function and scale degree of X. This constitutes the only way in which a functional term (in $\mathbb{F}$) can reenter the recursive domain of functional regions (in $\mathbb{R}$). The modulation rule involves type casting, since a functional term representing a chord is assigned as the key type. This takes advantage of the fact that both properties of the key type, pitch class and mode, are embodied by functional terms. The root of a functional chord within a particular key defines a unique pitch class and its type defines a unique mode. Hence, the (dominant) diminished *VII* chord in major cannot instantiate a modulation, since it does not define a valid mode property. For instance, a predominant s in G major may be the new tonic, so it would be the tonic in a new key of C major. The rule would be $s_{key=G\ maj} \rightarrow t_{key=\psi(s,G\ maj)=C\ maj}$. The type casting from a function type $f \in \mathbb{F}$ (and a reference key $k \in \mathbb{K}$) to a key type is performed by the function $\psi(f, k) \in \mathbb{K}$. It assigns the resulting key values based on the common definition of functional terms within the diatonic framework (e.g. $\psi(d, B\flat\ maj) = F\ maj$, $\psi(tp, A\flat\ maj) = F\ min$, etc.).[4] When the modulation is initiated through pivot chords in the sequence that may belong to two adjacent keys, the double generation of the pivot chord from two different branches of the parse tree constitutes the preferred form of analysis that captures the double role of the pivot chord (see below). The second rule (16) specifies the change of mode without the change of function. This rule is necessary to capture the phenomena of functional borrowings from the respective complementary modes, such as the use of the major subdominant in minor or the minor subdominant in major.

The grammar incorporates no distinction between modulations, brief tonicizations or changes of local diatonic context. This entails that the difference between these phenomena is gradual and that the stability of a (change of) key is greater, the higher the node is located in the tree and the more children it dominates.

3.3. *Scale degree level*

3.3.1. *Secondary dominant rules*

$$X \longrightarrow D(X)\ X \qquad \text{for any } X \in \mathbb{S} \tag{17}$$

$$X \longrightarrow \Delta(X)\ X \qquad \text{for any } X \in \mathbb{S} \tag{18}$$

$$D(X) \longrightarrow \begin{cases} V/VI/X \mid VII/VI/X & \text{if } X \text{ refers to a diminished triad} \\ V/X \mid VII/X & \text{otherwise} \end{cases} \tag{19}$$

These rules describe two similar yet different phenomena. The first rule specifies that chords (represented by a scale degree) may be preceded by their relative dominants (secondary, tertiary or any dominant) which may be outside the surrounding diatonic context. In contrast, the second rule characterizes diatonic descending fifth sequences. The reason for not describing the chains of secondary dominants through local modulation[5] is that in the case of a chain of two dominants, for example, $A^7\ D^7\ G$, the first dominant could not be explained by modulation and a $d\ t$ progression, since the chord D^7 cannot fulfil a double function as relative tonic and a dominant seventh chord at the same time within this formalism.[6] Moreover, both rules are located on the scale degree level and not on the functional level in order to avoid the reentry of any part of the sequence into the whole recursive generative process and avoid its subsequent elaboration, expansion, tonicization or the like. In analogy to the definition of the functional d term (rule 23), the $D(X)$ function (rule 19) assigns the scale degree of a perfect fifth above X to a given chord on scale degree X and assumes a missing fundamental with respect to the diminished triads (for instance, $F\sharp^0$ would be treated like a D major (seventh) chord). This makes it possible that diminished triads may replace dominant chords within a secondary dominant sequence (cf. Example h in Section 2), but avoids that secondary dominants a fifth above VII or VII/X are produced. The second rule (18) characterizes similar sequences along the diatonic circle of fifths (which may not be dominant seventh chords) expressed by $\Delta(X)$. The $\Delta(X)$ function assigns the scale degree of a fifth above X within the diatonic scale (modulo 7) to a given scale degree X.[7] For instance a diatonic fifth sequence as in the Jazz standards 'Autumn leaves' ($Cm\ F^7\ B\flat^{maj7}\ E\flat^{maj7}\ Am^{ø7}\ D^7\ Gm$, see Figure 4) or 'Fly me to the moon' ($Cm\ Fm^7\ B\flat^7\ E\flat^{maj7}\ A\flat^{maj7}\ D^{ø7}\ G^7\ Cm$) may be modelled this way.

3.3.2. *Function-scale degree interface*

A subsequent set of rules describes the interface between the functional level and the scale degree level, which mostly accord with the common functional theory.

$$t \longrightarrow I \tag{20}$$

$$t \longrightarrow I\ IV\ I \tag{21}$$

$$s \longrightarrow IV \tag{22}$$

$$d \longrightarrow V \mid VII \tag{23}$$

$$tp \longrightarrow \begin{cases} \mathit{VI} & \text{if key is major} \\ \mathit{III} & \text{if key is minor} \end{cases} \tag{24}$$

$$dp \longrightarrow \mathit{VII} \quad \text{if key is minor} \tag{25}$$

$$sp \longrightarrow \begin{cases} \mathit{II} & \text{if key is major} \\ \mathit{VI}, \flat \mathit{II} & \text{if key is minor} \end{cases} \tag{26}$$

$$tcp \longrightarrow \begin{cases} \mathit{III} & \text{if key is major} \\ \mathit{VI} & \text{if key is minor} \end{cases} \tag{27}$$

These rules express the scale degree chord representations of the respective Riemannian functions as commonly known. In this case, the Neapolitan chord *Sn* is subsumed under the subdominant parallels sp, knowing that in the traditional formalism, it constitutes an independent entity; however, in this case, it is more economic (saving two rules) and equivalent for the dependency structure to subsume the Neapolitan chord under sp in the present way. Furthermore, the rule for dominant parallels does not incorporate *III* in the major case since in common-practice or Jazz harmony, *III* cannot typically replace V. The functional reading of *III* as dp which relates to *VI* as tp is reflected in a generalized way in rules (17) and (18). Merely one, rather modal, use of *III* as progressing to *IV* is indirectly modelled by rule 12.[8]

This formalism decouples the scale degree level from the functional level and models sequential dependencies on the functional level for two reasons. First, deep structural relationships for chord sequences that differ on the scale degree level are maintained, and, secondly, different structural implementations within a functional framework such as [39] become possible.

On the scale degree level, a number of *voice-leading rules* may be postulated to cover the various surface structures of chords and dissonance phenomena, that stem from voice leading, for instance the 6/4-suspension V^{6-5}_{4-3} which translates into $V \rightarrow V^{6-5}_{4-3}$. Furthermore, the derivation of altered chords, such as altered sixth chords, would be located on this level since they constitute derivations of core functional chords. The description of the specific details of specialized and stylistic rules will be a matter of fine-grained style-specific rules that do not belong to the general formalism presented here.

3.3.3. *Typing*

During the process of derivation a chord might be derived in a sequence of several different functions by virtue of modulation. In this case, the last function in the chain of derivation defines its surface function and form. For instance, a tonic chord which acts as a higher order dominant will have the surface of a tonic and not a dominant seventh. There are several surface features of chords that are distinctive for their function such as a major triad with a minor seventh, which implies a dominant function, a half-diminished seventh chord, which implies a predominant chord on *II* in minor, or a major chord with a *sixte ajoutée*, which implies a subdominant function in common practice music or also a tonic function in Jazz. Such relationships may be expressed in rules like $d \rightarrow V^7$ or $s \rightarrow IV^6$ in addition to the rules described above. Such constraints are very effective for disambiguation during the parsing process. Since they are dependent on the specific musical style these rules are not discussed in detail here and have to be addressed in future work when modelling a specific style [34].

4. Surface level

The analysis completes with simple transformational *surface level rules* which transform a scale degree into a chord on the surface level, given its key property. These rules follow the standard definition of scale degrees in a straightforward way and are therefore trivial, for instance: $V^7_{key=E\flat\ maj} \rightarrow B\flat^7$.

$$X \longrightarrow X^+ \quad \text{for any } X \in \mathbb{O} \tag{28}$$

This rule makes it possible for any surface chord to be repeated. This rule is located at a lower level than the functional rules, since a functional replication may be itself subject to another recursive transformation, whereas a mere repetition of chords without change of function or scale degree is regarded as a phenomenon located at a surface level which does not enter recursive expansion and may often not even be analysed as a sequence of separate events.

5. Sample analyses

In practice, it turns out that few rules suffice to cover a large number of cases. One example (Figure 3) shows the general application of the rules for the parsing of a phrase from a Bach

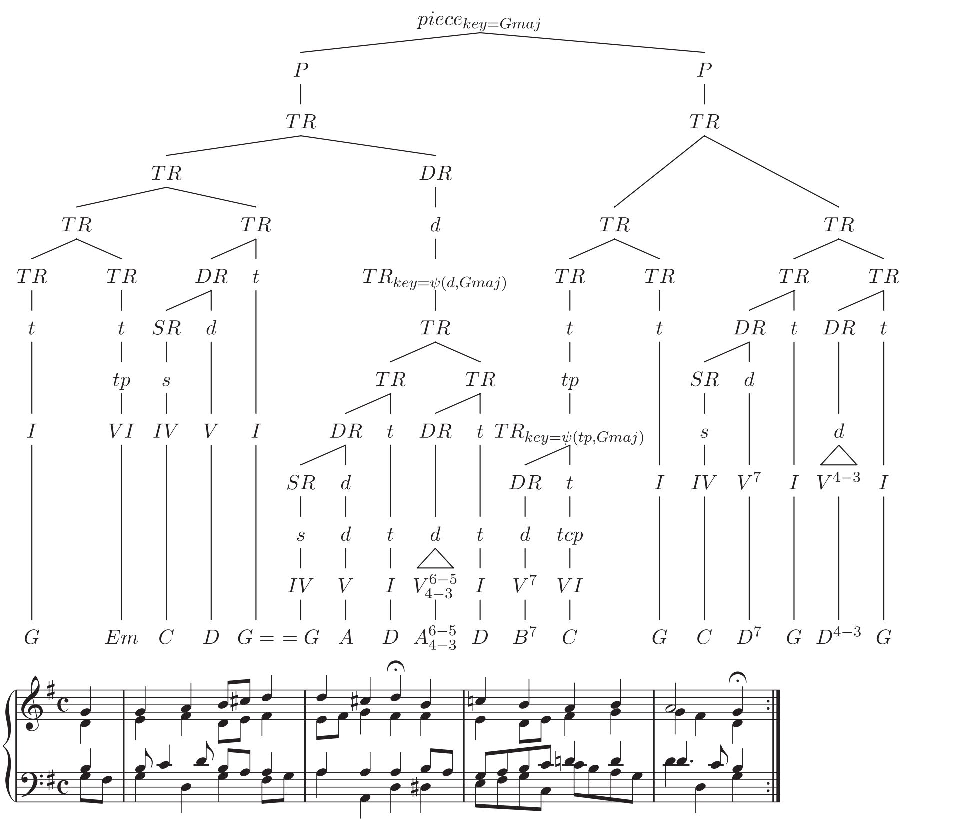
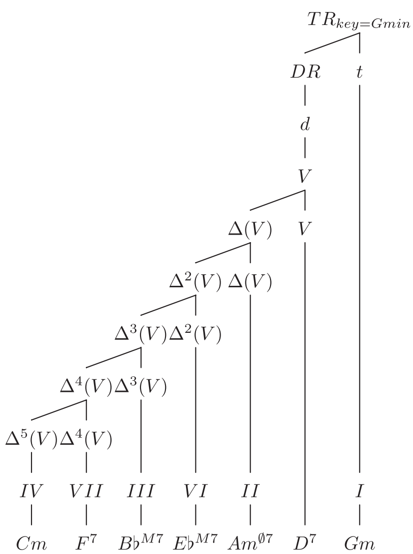


Figure 3. Analysis of the beginning of Bach's chorale 'Ermuntre Dich, mein schwacher Geist', mm.1–4. The '=' signs indicate that both instances of the *G* chord refer to the identical surface pivot chord. The triangle symbol indicates the omission of a self-evident derivation, e.g. $^{6-5}_{4-3}$ movement (in this specific case figured bass notation is used in order to express the surface movement within the respective cadential context).

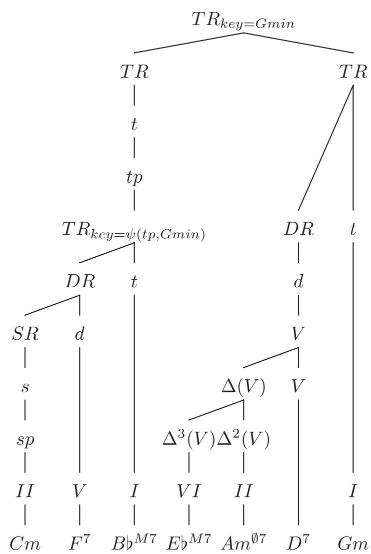
chorale, and illustrates the analysis of modulations. The diagram reads like common linguistic parsing trees (although some of the element relationships expressed in the tree differ from linguistic relationships). It is important to note that the chord symbols in the analysis do not incorporate figured bass notation or chord inversions since such differences as well as resulting features of the underlying bass movement need to be modelled independently. As the example shows, the structuring into different subphrases, simple prolongational and progressive phenomena, as well as basic cadences, modulations and deceptive cadences can be accounted for. The top level of the tree represents the analytic choice to characterize the example through a phrase ending on V and a tonic phrase (rules 6 and 2). Similarly, a different choice may have connected the same dominant expansion to the overarching tonic expansion on its right, applying rule 4, in order to avoid the use of rule 6. The triangles denote the subsumption of the generation of surface voice-leading progressions under a single derivation step. The modulation from G major to D major in the first half phrase involves a case in which the modulation employs a pivot chord (G) which belongs to both keys. Accordingly, the chord is derived twice from the respective adjacent branches of the different parent derivations. Another solution for the parse tree would be to generate the pivot element from both adjacent branches, which, however, would result in the loss of the mathematical tree structure that requires the branches to be disjunct. Moreover, the example illustrates that a phrase that reaches a final V through modulation is modelled in a structurally similar way to a half cadence.

Figure 4 illustrates that the first phrase of the Jazz standard ‘Autumn leaves’ constitutes an example of a descending fifth sequence that can be analysed in two ways. It could either be read as an example of a head-recursive sequence of fifth relationships along the diatonic cycle (in this diatonic context, the tritone progression $E\flat$-A would be accounted under this principle as well). Another analysis would group the phrase into two tonal regions, Gm and $B\flat$, which are both established by cadential/fifth relationships.

A particular feature of the proposed method of analysis is that it further allows to account for more complex adjacencies of structurally/functionally not closely related chords, such as the progressions F-D^7, G-E^7, a-$F\sharp^0$ in Figure 5 or the beginning of Beethoven’s Waldstein sonata (Figure 6). In the first example, the secondary dominant rule (17) accounts for the



(a) Analysis 1



(b) Analysis 2

Figure 4. Two alternative analyses of the first phrase of the Jazz standard ‘Autumn leaves’. $\Delta^x(y)$ refers to the multiple recursive applications of $\Delta(\Delta(\ldots(y)))$.

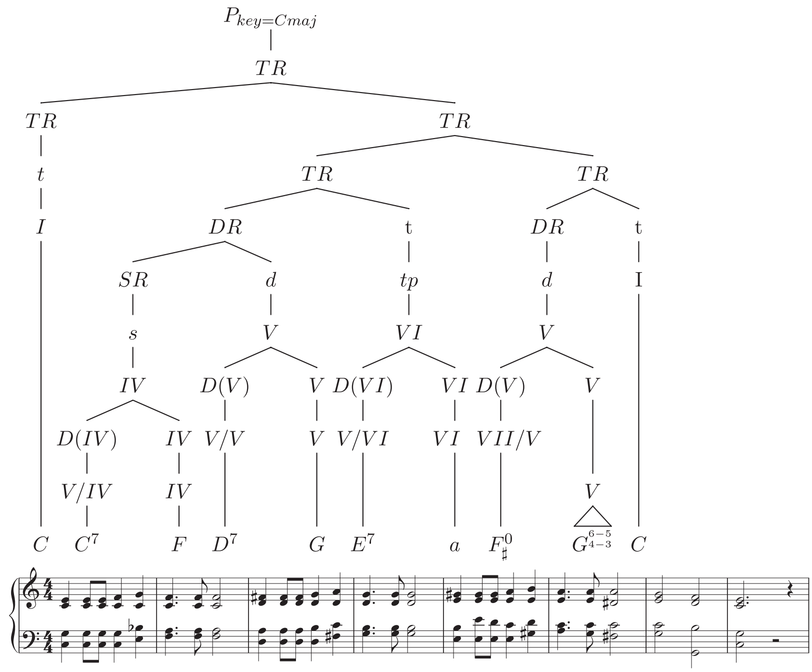


Figure 5. Analysis of a phrase of Bortnianski's piece 'Tebe Poëm'.

derivation/occurrence of the chords C^7, D^7, E^7 and $F\sharp^0$ within this Monte sequence which fall out of the diatonic framework of C major. Moreover, it expresses that the sequential pattern reduces to a well-formed functional harmonic sequence $F\ G\ Am$ at a deeper level. The example further illustrates that the formalism expresses abstract tonal relationships between the harmonic entities. These match the composed structure or the final retrospective cognitive representation after multiple listening rather than the cognitive experience during listening. The latter may require multiple revision processes, for instance, reinterpreting an initial tonic function of the F chord after additional context.

In the case of the Waldstein sonata, the whole sequence characterizes an overarching tonic–dominant progression in C which is elaborated by the change of mode from major to minor and the recursive establishment of the intermediate harmonic goals G and F which are briefly tonicized. The advantage of the proposed form of syntactic analysis is that the parallelism of the two intermediate goals can be captured, and that the local G-$B\flat$ transition, that is comparably rare in (common practice) C major, can be accounted for as adjacent events on locally disjunct subtrees. Note that the application of the changed key and functional properties does not intend to imply that each segment is modulating in the full theoretical sense (the respective tonic goals are unstable since they do not occur in root position). Rather, it signifies the way in which chords from different diatonic contexts (such as D^7 or $B\flat$) are licensed and derived through passing tonicization and the change of the local diatonic framework; otherwise, a chord like $B\flat$ could not be derived within its surrounding context. The second analysis illustrates some of the difficulties of the presented model with respect to some sequential progressions. Once the harmonic progression is understood as an instance of a sequential pattern in which the tonic C progresses/departs to G and, by analogy, $B\flat$ to F, two related problems occur which result in the fact that the subsequence $B\flat\ C\ F$ cannot be connected to the surrounding context and violates the dependency principle. If the sequential parallelism is carried through and the region on top of $B\flat$ is assigned to be head (resulting in a brief tonicization of $B\flat$), the *TR* region cannot be functionally connected/subordinated to the

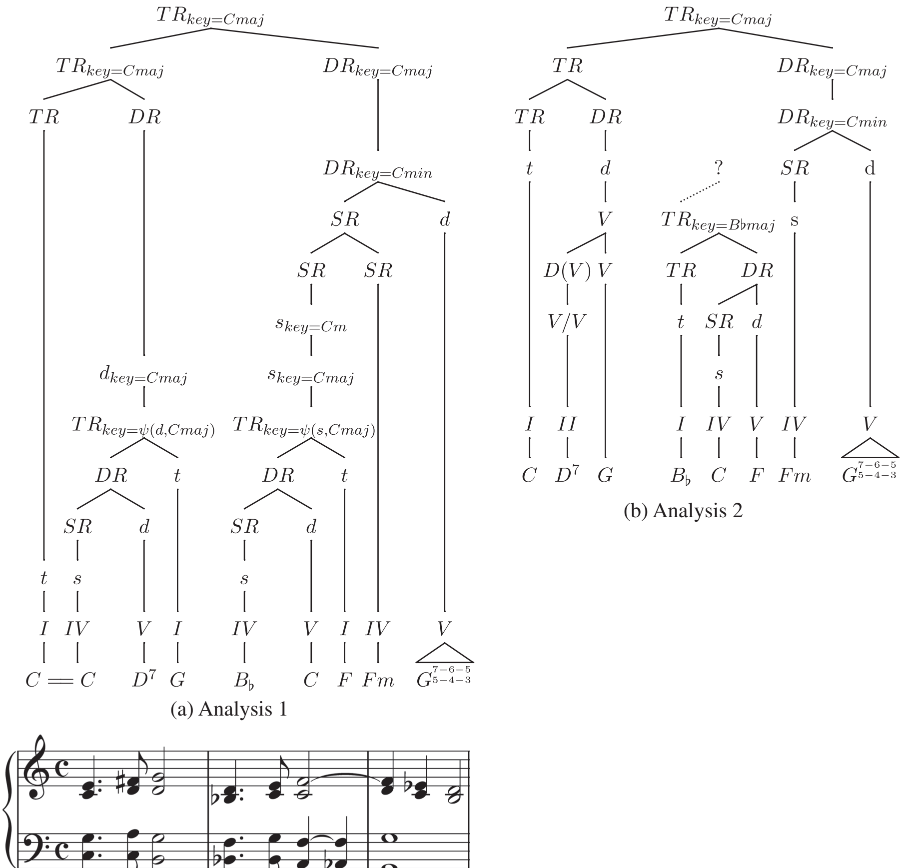


Figure 6. Analysis of the beginning of Beethoven's Waldstein sonata op.53, I, mm.1–13 (reduction). It should be noted here that the analysis does not imply here that there are changes of key to *G* or *F* major in analysis 1. The tree rather details the derivation steps that are necessary in order to derive and license the non-diatonic chords *D* or *B*♭ and indicates the local diatonic context they are derived from. In this respect, the analysis accords with the analysis proposed by Lerdahl [40, p. 222] on the least reduced level. Analysis 2 illustrates the difficulty that arises when aiming to capture the sequential character of the chord progression.

subsequent *SR* branch any more with a simple and straightforward derivation that maintains the sequential character. Similarly, *DR* cannot be the head of the stranded sequence, since the head of a modulating subbranch has to fulfil a tonic function. This example illustrates that abstract sequential relationships, should they be modelled in ways that capture the sequential parallelism between parts of the sequence, may require an additional or independent set of specific, potentially context-sensitive rules that capture features of the governing voice-leading process and override some of the harmonic constraints (such as connectivity or dependency principles).

6. Discussion

Even though some of the rules proposed may be arguable, this paper mainly aims to make a programmatic theoretical contribution by proposing a generative syntax account of tonal harmonic progressions. This contribution is aimed at music theoretical, cognitive and computational perspectives as well as the discussion concerning parallels between music and language.

From a theoretical perspective, this paper aims to reconcile approaches from the Riemannian functional tradition [2,41] with recursive, prolongational approaches. One of the underlying core assumptions is that on an abstract level, harmonic sequences consist of the three main functions (tonic, dominant and pre-dominant) which are categorically distinguished by their role within a tonal context and may have cognitive correlates. They do not necessarily coincide with the surface elements since they may span over ranges beyond the level of single elements (chords). The core set of functional expansion rules expresses recursive prolongational or progressive relationships which combine the Riemannian functional tradition with a recursive, generative approach. The fact that the simple and compact set of rules outlining a general functional context suffices for the explanation of the large range of examples accords with the statements given in [33,42,43] that tonal harmony is fundamentally grounded in elaborations of cadential harmony. While the traditional Riemannian framework models local chord-to-chord progressions, this recursive extension enables it to express complex long-distance hierarchical relationships between musical events. Since abstract functions may be implemented by several different elements (such as *IV*, *II* for pre-dominants, or V, *VII* for dominants), their combined successions (e.g. *IV II*) or even alternative low-level theories or musical domains, the formalism models deep-structure functional relationships independently of the implementation. This also constitutes the reason for tonal functions being chosen as the high-level heads rather than the musical surface elements (chords or pitches) as in [12,40]. In contrast, elaborations such as secondary dominants are assumed to relate to a representation one level below the functional level since they refer to a concrete chord (or scale degree) rather than to an abstract function. This functional approach entails an important implication for the description of third relationships between chords. While some third progressions are licensed through functional substitutions (e.g. $IV - II$ or $V - VII$) or functional progressions (e.g. $VI - IV$ or $II - VII$), other third progressions as in Figures 5 and 6 can only be derived as adjacent events from locally disjunct subtrees. Similar restrictions hold for other scale degree progressions such as ascending or descending seconds. With respect to this feature, the formalism offers an analysis of harmonic sequences that does not require the set of licensed progressions to be based on a descending third sequence (pace [44]).

In distinguishing separate levels of phrase, functional and scale degree structures and assigning rules of harmony to these different levels, this grammar differs from the earlier approaches and extends their scope. For instance, Steedman's grammar [18,19] does not include such distinctions and does not model modulation or change of mode. The proposed grammar further differs from [19] with respect to a number of rules, such as the omission of a rule $X \rightarrow X\ IV(X)$ or $X \rightarrow X\ II(X)\ III(X)$.[9]

Understanding the important role of harmony for the constitution of form is one of the most central music-theoretical challenges in the investigation of common-practice tonal music [45]. Whether the proposed set of recursive rules extends to the level of entire pieces (like the Schenkerian or GTTM analyses) in a meaningful way or whether the rules reflect principles of formal organization shall not be argued on the basis of this paper. Parse trees based on the rules presented here may reflect some formal structures above the phrase level such as the form of entire Jazz standards, or overarching structural relationships of a Baroque suite movement (without employing phrase rules, i.e. based on the seed of rule 3). While it is not the goal of this paper to generate formal analyses, the investigation of structural overlaps or divergences between harmonic and formal analyses has to be regarded as a separate subject of investigation.

It is important to note that the proposed grammar is a model of the subsystem harmony within the tonal language and does not aim to model musical structure outside this domain. Accordingly, processes that involve the interaction with systems of counterpuntal structure or bass motion (such as complex cases of sequences [45,46], fauxbourdon-style parallel triads or extended nineteenth-century harmony) cannot be sufficiently expressed within the proposed framework.[10] However, since many cases of sequential schemata embody harmonic relationships that exceed merely coincidental chord adjacencies, some interaction with the harmonic system may be assumed. The modelling of such complex interactions, which might even require features of context-sensitive languages, remains to be addressed in future work. Similarly, it remains open whether the generative principles presented here result from or constitute epiphenomena of dependencies grounding on voice leading such as the Schenkerian theory or the GTTM or whether they constitute an autonomous subsystem that interacts with voice-leading structure. On the other hand, the rules constitute principles in their own right for specific forms of music that are based on or transmitted by chord/harmony representations such as Jazz, pop music or figured bass. The present model is related to the GTTM, the Tonal Pitch Space Model [40] or dependent approaches [15]. Whereas the latter are, precisely speaking, not generative grammars *per se*, since they do not specify concrete generative production rules, the structure proposed here constitutes an explicit grammar for the subsystem of harmony in its full sense.

The present framework lends itself to comparably simple computational implementation. It further complies with the principles of empirical contestability on the basis of corpus analyses since it specifies a well-defined set of context-free rules that make explicit structural and analytic predictions unlike the GTTM. Note that the grammar is far less complex than formalizations of Schenkerian or GTTM analyses [29–32,47]. This is still true for the machine-readable representation of the grammar in Backus–Naur Form.[11] Such an implementation of the grammar or its stochastic counterpart bears potential in modelling harmonic similarity [34] through partial tree-matching algorithms taking advantage of the fact that the similarity of harmonic sequences (e.g. two versions of a Jazz standard) is reflected in the tree-shaped dependency structure. The success of such similarity measures additionally constitutes a form of empirical underpinning of the structural relevance of the proposed rules.

From another perspective, this text argues that Western tonal music, or at least the harmonic subsystem, exhibits features of recursion, hierarchical organization, and long-distance dependencies in ways that are structurally similar to linguistic syntax [10].[12] However, as some specific features of linguistic syntax, such as overt movement, case assignment, or argument structure, seem not to be required in the musical case, the present findings would motivate further explorations in order to understand deep parallels and non-parallels between music and language [14,48,49] and their origins on a structural and potentially cognitive level. One fundamental difference lies in the fact that all dependency relationships expressed in the tree (except for transformations like $t \rightarrow tp$) are temporal in essence and not as abstract (atemporal) as in linguistic syntax. On the other hand, empirical findings concerning the cognitive overlap between music and language processing or shared neural substrates [50–53] provide converging evidence that links with the assumption of some structural parallels.

The syntactic formalism presents a challenge for the Markovian models [54] of harmony (such as Piston's seminal table of root progressions or empirical versions such as [6,44,55,56]) and invites further reflections about the Markovian models. Markovian approaches by themselves offer no way of deriving harmonic sequences from primitives, explaining them as elaborations of simpler underlying sequences, or discerning differences between, e.g. progressive and prolongational continuations, similarities between chord categories, or expressing similarities between large scale and local harmonic processes, based on the same core set of (functional) rules. The presented syntactic formalism constitutes an analytic and descriptive model in the sense of [57,58], which derives and generates, and explains in part features of deep structure functional progressions or correspondences between abstract and local structure (see above). It therefore offers a theoretical alternative to the Markovian generation processes. Figures 5 and 6 show that the grammar provides the potential to express the parallelism in the examples in some, but not all levels of abstraction. However, it cannot explain (or directly generate) sequential patterns which might require context-sensitive, template- or schema-based approaches [59–61].

Transition matrices or n-gram models may well merely reflect statistical properties of underlying more complex deep structure processes [6] and therefore cannot be easily argued to constitute *'the'* structure-building process (pace [44]). By themselves, they do not embody sufficient complexity to express the formal structure of harmonic tonality, modulation processes or overarching formal processes. Some features of tonal harmony inherently require context-free dependency structures, such as closure in the original key or return to a chord from which an intermitting sequence was departing. A context-free grammar embodies this kind of minimal memory, whereas the left- or right-expanding nature of regular languages embodies no such memory to non-local elements. Hence a Markovian model of chords that is rich enough to model modulation possesses no comparable structural memory and will generate a 'Brownian' harmonic motion that will only randomly (if at all) return to its original or previous key. Similarly, a Markovian process would only randomly comply with the dependency principle described above (if it can be assumed to hold in general). It could happen that a chord departure (e.g. $C\ E^7$ in C major) is not connected to the subsequent events if the Markov process places another remote chord departure to the end of the sequence, for instance $C\ E^7\ A$ to be continued by $C\sharp\ F\sharp$.[13] Such a sequence would lose its binding to the overarching C major context.[14] Moreover, prolongational sequences cannot be captured, depending on the limits of context length and data size. For a prolongation '$A\ x\ y\ A$', a n-gram model would need to observe an exponential mass of data of that structure in order to exhibit behaviour that reflects such a prolongation. Given that n-gram distributions in a corpus obey a Zipf-distribution [6,55], such an n-gram model would require a (even cognitively implausible) massive set of data.

Last but not least, the generative syntax model proposed here specifies a grammar that models structural dependencies rather than a cognitive system. Although the relevance of such structural principles should be reflected in some cognitive processes or mental representations (or vice versa), a simplistic one-to-one mapping of the generative syntax to a cognitive instantiation cannot be assumed. For instance, the objects of analysis on which the grammar is based constitute well-crafted, composed pieces of music that are designed from a bird's-eye perspective and principally independently of an online construction or perception process. Although music history may be compared with a large-scale cognitive experiment, the *cognitive* reality of recursive dependencies on the largest levels (or other mathematical relationships found in *scores*) cannot be taken for granted. Empirical results are undecided about the cognitive reality of musical long-distance relationships [62–64]. However, the grammar makes it possible to generate some explicit (structural) hypotheses about licensed progressions or predictions about expected cognitive parsing and revision processes (cf. [65]) that may be investigated in behavioural or neuroscientific ways. Similarly, questions concerning how such context-free dependencies are acquired implicitly or constitute forms of implicit knowledge [66–69] require further empirical evidence.

Altogether the proposed formalism and its examples may constitute a concrete basis for debate concerning recursion in music and deep structural relationships between music and language syntax and may form a fundament for its computational implementation or empirical exploration.

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Notes

1. It is important to note here that Kostka and Payne's analysis is to some extent inconsequent: the fact that the final V picks up and prolongs its first occurrence at bar 3 suggests that the final V is structurally more important than its earlier occurrence.

2. In an empirical study, Rohrmeier and cross [6] found that some theoretically meaningful categories of music are reflected directly from the antecedent and consequent patterns of chords through hierarchical clustering methods.
3. Accordingly, phrases involving half cadences would need to be licensed by overarching tonic–dominant relationships.
4. Note that in the case of a modulation in minor, the definition of ψ lets *sp* refer to *VI* and not $\flat II$, e.g. $\psi(sp, Amin) = Fmaj$.
5. This does not imply to argue that this is a case of modulation.
6. It could happen, however, that a first dominant seventh chord in a chain of dominant seventh chords implies a tonic chord whose analysis is revised as soon as the chord is heard.
7. It is well noted that some applications of rules 17 and 18 create ambiguities with the cadential rules 4 and 5. However, both sets of rules constitute conceptually different and meaningful parts of the grammar. While the latter models cadential contexts on a functional level (i.e. it models the common feature of progressions, only some of which involve fifth progressions), the former models sequential contexts such as progressions along the cycle of fifths.
8. If one chooses to avoid the rare rule 12, the progression *III IV* can be modelled by a rule $IV \rightarrow III\ IV$. This rule would be intentionally located on a scale degree level rather than on a functional level, since *IV* as *s* could typically not be replaced by *II* in this context.
9. It is important to note, however, that Steedman's article never claimed to model the entirety of tonal or Jazz progressions.
10. Similarly, note that the grammar produces a number of ambiguities. This is important, since many of these ambiguities correspond with small conceptual or structural differences that the theorist might want to capture. Some of these ambiguities cannot be resolved on the level of mere harmonic sequences only (since they are inherently ambiguous without further information (for instance, with respect to the difference between half cadence or an authentic cadence within a well-formed sequence '$X\ \ldots\ VI\ Y\ \ldots$'). The integration of additional information of at least metrical structure, preferred phrase length in terms of preference rules, will be required for a computational implementation.
11. A more comprehensive computational implementation of the present grammar together with appropriate parsing algorithms will, however, require the formulation of a number of low-level style-specific rules, as well as the formalization of constraints with respect to the key finding/preference and key change, and the interaction of metrical and phrase structures. The investigation of parsing techniques, and in particular the choice of musically and cognitively reasonable lookaheads, is subject to ongoing work. This research recourses to the available results from the authors who have contributed significantly to the implementation and evaluation of such parsers [26, 30–32,34,70]. A BNF representation of the rules with respect to its current implementation will be supplied at http://www.mus.cam.ac.uk/CMS/people/martin-rohrmeier/.
12. The formalism presented here is intended to be independent of any particular formalism in linguistics. The core rules are sufficiently simple to be expressed by any of the current main models of linguistic syntax [71–76] without requiring the use of the special features of any of these models.
13. The reason for the secondary dominant rule being located at the scale degree level reflects the avoidance of such a phenomenon, which could occur within the recursive process at the functional level.
14. It remains open to further research, however, on how far such limits of n-gram models could be overcome with multiple-viewpoint approaches [77,78], at least within some cognitively plausible limits.

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