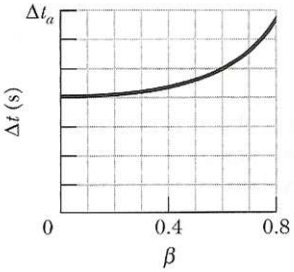
Time dilation

1. HR. p1030. A spaceship passes Earth with a relative speed of $0.999c$. After traveling $10\ y$ it stops at lookout post LP13, turns, and travels back to Earth with the same relative speed. The trip back takes another$10\ y$. How long does the trip take according to measurements made on Earth?
Answer: $t = 448\ y$

2. HR.37.6. Reference frame S' is to pass frame S at speed v along the common x direction. The figure shows the time interval Δt measured in the S frame, versus the speed parameter β. What is the interval Δt if $v = 0.98\ c$? *Hint:* You can use the figure to find the proper time.
Answer: $\Delta t = 40\ s$



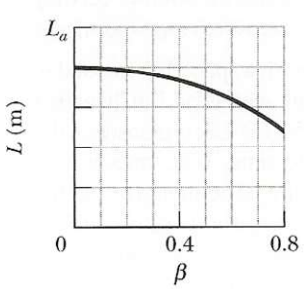
Relativistic particles

3. HR.p1030. A positive kaon (K-meson) has an average lifetime $\tau_0 = 0.1237\ \mu s$ when stationary, i.e. τ_0 is measured in the rest frame of the kaon.
If a positive kaon has a speed of $0.99\ c$ relative to the laboratory RF, how far can it travel in that RF during its lifetime: a) according to classical physics? and b) according to special relativity?
Answer: $36.7\ m;\ 260\ m$

4. (can skip) TR.2.21. A muon (μ-meson) has a mean lifetime of $2.2\ \mu s$. The track left in a particle detector before decaying into an electron and two neutrinos is $9.5\ cm$. What is the speed of the muon? *Hint:* Solve for the speed parameter β, then use the approximation for low relativistic speeds.
Answer: $v = 1.44\ 10^{-4}c$

Length contraction

5. HR.37.14. A rod is to move at constant speed v along the x axis of reference frame S, with the rod's length parallel to that axis. An observer in frame S is to measure the length L of the rod. The figure shows the length L for a range of speed parameters β. What is L if $v = 0.95c$? *Hint:* You can use the figure to find the proper length.
Answer: $L = 0.25m$



Time dilation and Length contraction

6. TR.2.98N. A spaceship travels at a speed $0.995\ c$ to Barnard's star which is $5.98\ ly$ away (the 2nd nearest star to our solar system after Alpha Centauri). The spaceship travels slowly around the star for 3 years, doing research before returning to Earth.
a) What is the Lorentz factor for the spaceship?
b) How much time does the journey take?
c) How much older is the twin brother of the astronaut when the spaceship returns?
Answer: $10.01;\ 4.2\ y;\ 10.8\ y$

Velocity transformation

7. TR.2.31. A spaceship is moving at a speed $0.84\ c$ away from the Earth. A boy in the spaceship shoots a proton gun with the protons having a speed $0.62\ c$.
What is the speed of the protons measured on Earth when the gun is shot: a) away and b) toward the Earth.
Answer: $0.96\ c;\ 0.46\ c$

8. 243T1.5. Object 1 is moving at a speed $0.9\ c$ to the right with respect to the Earth. Object 2 is moving to the left at a speed of $0.7\ c$ with respect to object 1. How fast is object 2 moving with respect to the Earth?
Answer: $0.54\ c$

9. 243.T1.6. A beam of light is moving in the same direction as an object is traveling. If the object is moving at a speed of $0.7\ c$, how fast is the light moving as it passes the object?
Answer: c

Relativistic energy and momentum
The rest energies and lifetimes of particles involved in the next problems are given in the table below.

10. HR.p1046. a) What is the total energy of a $2.53\ MeV$ electron?
b) What is the magnitude of electron's momentum in the unit MeV/c?
c) What is its speed in units of c?
Answer: $3.04\ MeV;\ 3\frac{MeV}{c};\ 0.987\ c$

11. GR.Ex.26.6. Carbon dating is based on the radioactive decay of the ^{14}C nucleus into ^{14}N.The reaction is written as: $^{14}C \rightarrow {}^{14}N + e^- + \bar{\nu}$ and releases $156\ keV$ of energy. If all the energy released appears as the kinetic energy of the electron, how fast is the electron moving? How does it compare with the non-relativistic result?
Answer: $0.643\ c$

12. HR.37.51. What must be the momentum of a particle with mass m so that the total energy of the particle is 3 times its rest energy?
Answer: $2.83\ mc$

13. HR.37.54. What is the speed parameter β for a particle with a) $K = 2\ E_0$ and b) $E = 2\ E_0$?
Answer: $0.943;\ 0.866$

14. HR.37.55. A certain particle of mass m has momentum of magnitude mc. What are the speed parameter β, the Lorentz factor γ, and the ratio K/E_0 ?
Answer: $0.707;\ 1.41;\ 0.414$

15. The most energetic protons ever detected in the cosmic rays coming to Earth from space had an astounding kinetic energy of $3\ 10^{20}\ eV$, which is enough energy to warm a teaspoon of water by a few degrees.
a) Find Lorentz factor. *Note:* This translates into a speed parameter of 0.999 999 999 999 999 999 999 995.
b) Suppose one of these protons travels along a diameter of the Milky Way galaxy, $9.8\ 10^4\ ly$. Approximately how long does it take as measured from the common reference frame of the Earth and the Galaxy?
c) How long does the trip take as measured in the reference frame of the proton?
Answer: $3.198\ 10^{11};\ 9.8\ 10^4\ ly;\ 9.7\ s$

Particle	$m\ (\frac{MeV}{c^2})$	q	s	$\tau\ (s)$	Decay channels	Discovered
e^-	0.511	$-e$	$1/2$		stable	1897
p^+	938.27	e	$1/2$		stable	1919
μ^-	105.66	$-e$	$1/2$	$2.2\ \mu s$	$e^- + \bar{\nu}_e + \nu_\mu$	1936
K^+	493.67	e	0	$0.1237\ \mu s$	$\mu^- + \nu_\mu$; etc.	1947