CHAPTER 37

SPECIAL RELATIVITY (SR)

Einstein's Postulates
Relativity of simultaneity
Relativity of time intervals – time dilation
Relativity of length – length contraction
Lorentz transformations
(Doppler effect)
Relativistic momentum and energy

TIMELINE

Classical Mechanics = study of macroscopic world at speeds $\ll c$
Classical Electro-magn.

1642-1727	Newton	light is not a wave; it is made out of particles
1801	Young	double-slit experiment
1861-2	Maxwell	wave theory of el.mg. radiation/light

Quantum Physics = study of microscopic world - not restricted to a type of phenomenon
quantum *- quanta (pl)- elementary amount of quantized physical quantities*

1800s(late)		black-body radiation	
1886-87	Hertz:	discovers photo-electric effect (later Von Lenard does more experiments) experimental proof of el.mg. waves	
1893	Thomson	*electron*	
1895	Roentgen	X-rays	
1900	Planck	mathematical model for BB radiation	$E = n\,h\,f$
1905 *(annus mirabilis)*	Einstein	03/18 radiation/light is quantized in photons & light absorption and emission occur in atoms 05/11 Brownian motion – kinetic theory of gases 06/30 special relativity 09 27 mass-energy equivalence	$E = n\,h\,f$ $E_0 = m\,c^2$
1909	Taylor	double-slit experiment in single-photon version	
1911	Rutherford	*nucleus*	
1916	Einstein	quanta of light have linear momentum	$p = \frac{E}{c} = \frac{h\,f}{c} = \frac{h}{\lambda}$
1917	Rutherford	*proton*	
1923	Schrodinger main QM equation		
	Compton	single-λ X-rays on carbon target scattered	$\Delta\lambda = \frac{h}{mc}(1 - cos\varphi)$
1924	De Broglie wavelength		$\lambda = \frac{h}{p}$
1927	Heisenberg uncertainty principle = basic limitation on our experimental capabilities Davisson-Germer/Thomson experiment		

CH.37 SPECIAL RELATIVITY
INTRODUCTION

Relativity

concepts of space and time and their transformation between different reference frames

Special vs. general relativity

Special – inertial reference frames (IRF) – not undergoing acceleration
General – reference frames can have gravitational acceleration

Low vs. high speeds

Classical/slow relativity is wrong at high speed
SR - is an extension to all physically possible speeds
- includes classical/slow relativity as a limit for $v \ll c$

Space and time

Are entangled (time between events depends on how far apart they are)
and entanglement is $\neq$ for $\neq$ observers
Time does not pass at a fixed rate

Easy and difficult

Easy – math
Difficult – who/what/how/when

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POSTULATES (1905)

1. *Relativity postulate (principle of relativity)*

   Laws of physics are the same in all inertial reference frames IRF
   Note: laws NOT measured values

Consequences – alter the definitions of energy/momentum
- energy/mass equivalence

1887 Michelson-Morley experiment
measured same speed of light in two directions – using interference pattern

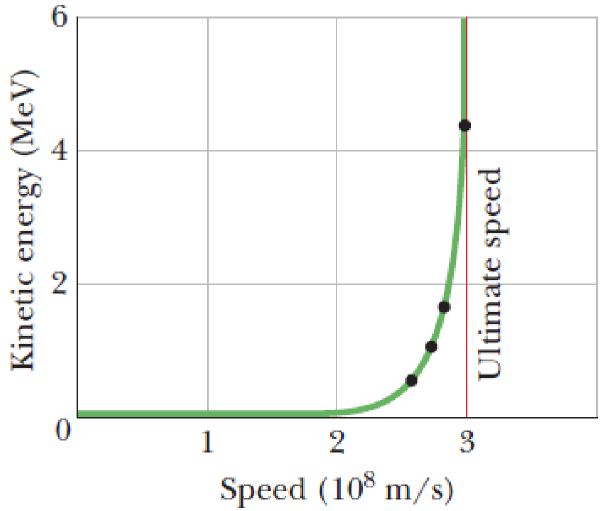
2. *Speed of light postulate*

a. The speed of light in vacuum has the same value c in all directions and in all reference frames, and it is independent of the motion of IRF

OR

b. There is an ultimate speed c, no entity can exceed it, and no particle with mass can reach it.

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299\,792\,458 \frac{m}{s} \cong 3\ 10^8 \frac{m}{s}$$



1964 experiment with accelerated electrons
$v_e = 0.999\,999\,999\,95\ c$
green – special relativity prediction

1964 CERN experiment with $\pi^0 \rightarrow 2\gamma$
$v_\pi = 0.999\,75\ c$
$v_\gamma = c$ (same as for $v_\pi = 0$)

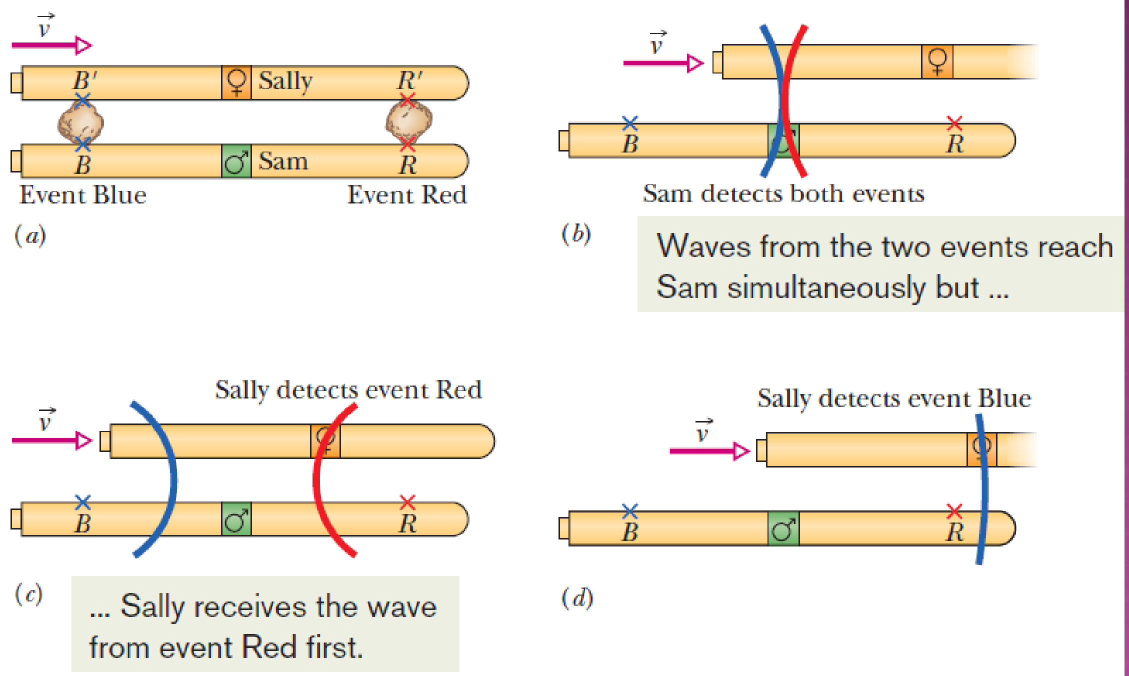
RELATIVITY OF SIMULTANEITY

Thesis: Simultaneity is not an absolute concept, but a relative one, depending on the motion of the observer
OR
Two observers in relative motion might not agree that two events are simultaneous

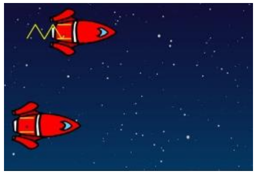
Example:
RF = spaceships or trains
Start at same point
Sally has relative speed v
Observers at midpoint
Events = 2 meteorites (R/B) strike and emit red/blue light

Result:
Sam – simultaneous events
Sally – not simultaneous

Both are right!



RELATIVITY OF TIME



Thesis: The time interval between two events depends on how far apart they occur in both space and time (entangled)

Example: observer on moving train (v) with light source, mirror, clock

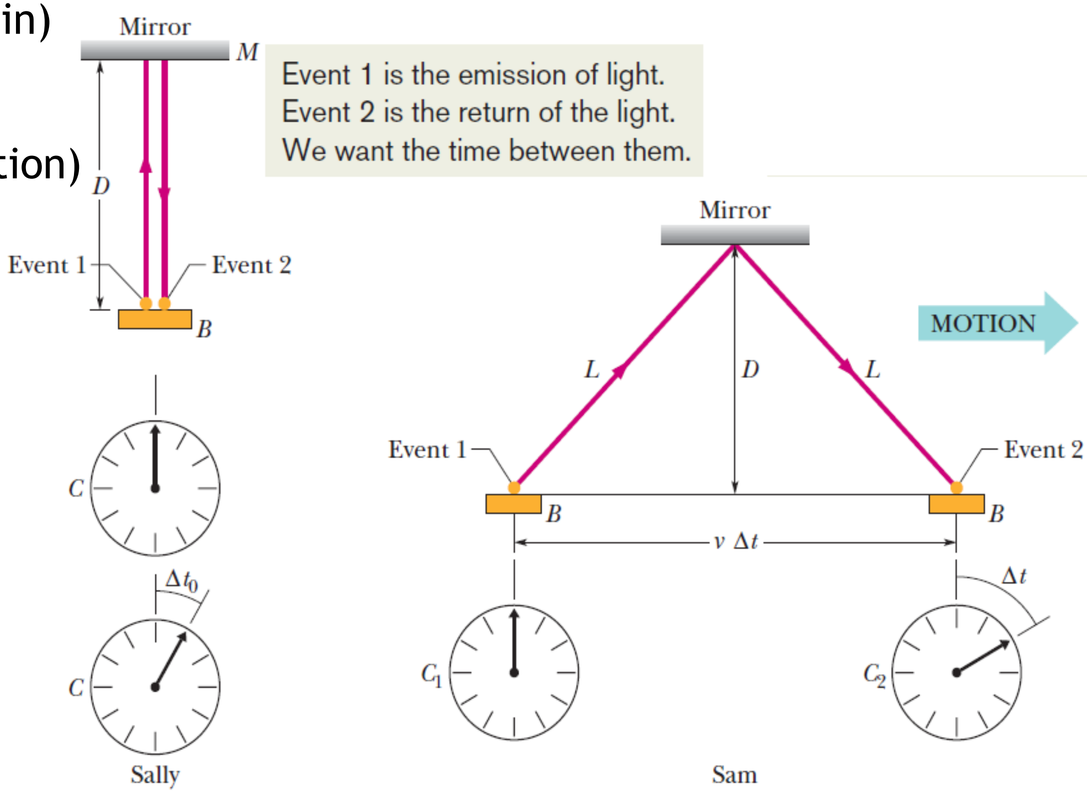
$$\Delta t_0 = \frac{2D}{c}$$ (on train)

$$\Delta t = \frac{2L}{c} = \frac{2}{c}\sqrt{D^2 + (\frac{v\Delta t}{2})^2}$$ (in station)

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} > \Delta t_0$$

→ shorter time in own RF

since $\sqrt{1-\frac{v^2}{c^2}} < 1$



RELATIVITY OF TIME

Previous result $\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}$

Time dilation $\boxed{\Delta t = \gamma\, \Delta t_0 > \Delta t_0}$

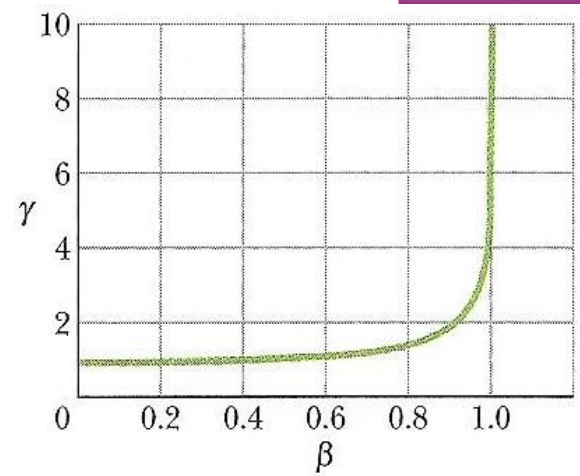
Proper time (interval) Δt_0 time between two events occurring at the <u>same location</u> in an inertial RF
shortest possible measured time interval

Speed parameter $\boxed{\beta = \frac{v}{c} \leq 1}$

Lorentz factor $\boxed{\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} = \frac{1}{\sqrt{1-\beta^2}} > 1}$ for $v \neq 0$

Low-speed approx. $\gamma = 1 + \frac{1}{2}\,\beta^2 \cong 1$ for $v < 0.1c$

NR limit $\gamma = 1.02$ for $v = 0.2c$



TIME DILATION TESTS

1. Microscopic clocks –

   muon (μ meson) – 1936 cosmic rays and 1937 cloud chamber

   $m = 105.66\frac{MeV}{c^2}$; $q = -e$; $s = \frac{1}{2}$

   $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

   at rest $\tau_0 = 2.2\ \mu s$

   at $v = 0.9994c$ $\tau = \gamma\tau_0 = 63.51\ \mu s$

Rest frame = stationary RF at the muon itself

2. Macroscopic clocks –

   1971 Hafele & Keating

   flew 4 atomic clocks around the world once in each direction

   results within 10% of special relativity

   ~1975 UMD – 15 h flight w/ atomic clock over the Chesapeake Bay

   results within 1%

RELATIVITY OF LENGTH

Thesis: The distance between two points depends on RF and is related to relativity of simultaneity, therefore, is relative

Proper (rest) length L_0 -

distance between 2 points measured at the same time by an observer at rest relative to both points

length measured in the RF of the object - where the object is at rest

Length contraction $\boxed{L = \frac{L_0}{\gamma} < L_0}$

Notes:

1. “length” can be the distance between to points/objects
2. length contraction - occurs only along the direction of relative motion
   - is a direct consequence of time dilation

CH.37 SPECIAL RELATIVITY

RELATIVITY OF LENGTH

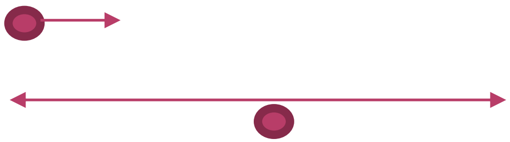
Proper length (rest length) L_0

distance between 2 points measured at the <u>same time</u> by an observer at rest relative to both points

length measured in the RF of the object - where the object is at rest

Example:

1. Measuring the length of the station platform



Sam (at station): L_0 proper length (at rest)

Δt time Sally moves through this length (not proper time – at ≠ places)

$$L_0 = v\,\Delta t$$

Sally (in train): sees the platform moving past her

Δt_0 time it passes through the station (proper time)

$L = v\,\Delta t_0$ not proper length / not in RF of the station

Use time dilation

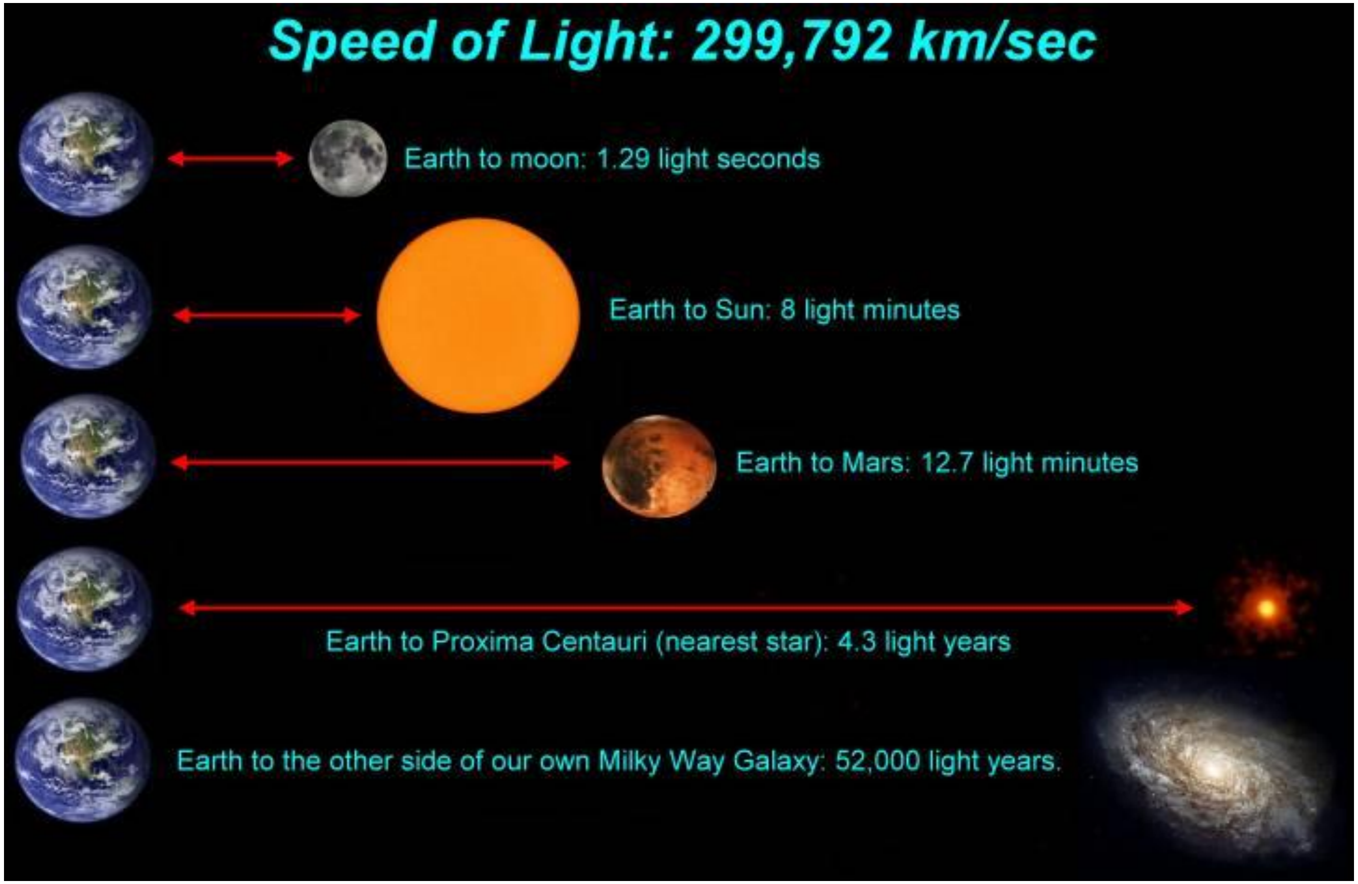
$\Delta t = \gamma\,\Delta t_0$ to obtain length contraction $\qquad \frac{L}{L_0} = \frac{\Delta t_0}{\Delta t} = \frac{1}{\gamma} \rightarrow \boxed{L = \frac{L_0}{\gamma} < L_0}$

2. Ruler w/light source and mirror on train – YF p.1233

RELATIVITY OF LENGTH

Light-year = distance traveled by light in one year

$1\ ly = 9.46\ 10^{15}\ m$



LORENTZ TRANSFORMATIONS

Galilean transformations (NR speed)

$x = x' + u\,t$ | $x' = x - u\,t$

$v = v' + u$ | $v' = v - u$

$t = t'$ | $t' = t$ | time passes at the same rate

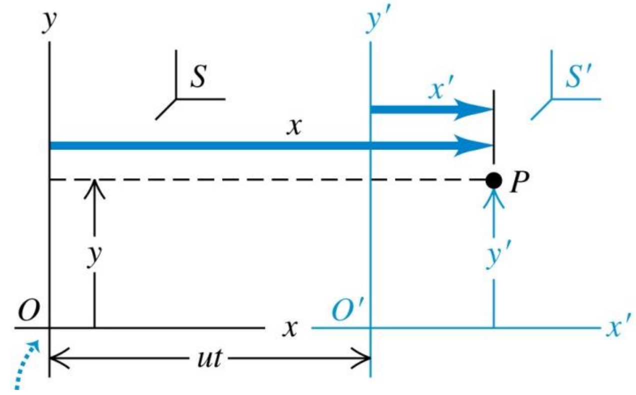
Lorentz transformation (∀ speed) - Lorentz factor $\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$

$x = \gamma(x' + u\,t')$ | $x' = \gamma(x - u\,t)$ | *proper length*

$y = y'$ | $y' = y$

$z = z'$ | $z' = z$

$t = \gamma(t' + \frac{u\,x'}{c^2})$ | $t' = \gamma(t - \frac{u\,x}{c^2})$



Lorentz velocity transformation

$v = \frac{v'+u}{1+\frac{v'u}{c^2}}$ | $v' = \frac{v-u}{1-\frac{v\,u}{c^2}}$

Notes:

Can be derived from the two relativity postulates

Time dilation and length contraction are ensured /confirmed

Classical limit $\gamma \to 1$ leads to Galilean transformation

Simultaneity, time dilation, and length contraction are consequences

RELATIXVISTIC MOMENTUM + ENERGY

Energy and momentum re-defined so that conservation laws hold in all RFs
1905 Einstein – mass is a form of energy = consequence of theory of relativity

Rest energy $\boxed{E_0 = m\,c^2}$ energy associated with the mass of an object

Chemistry – energy and mass are conserved separately ($\Delta m/m$ too small)
Special relativity - unifies conservation of mass and energy
Nuclear reactions – large ΔE & can measure Δm

$\pi^0 \rightarrow 2\gamma$ where $\gamma = photon$ (not Lorentz factor)

${}_1^2D + {}_1^3T \rightarrow {}_2^4He + {}_0^1n\ (+\mathrm{Q})$ fusion $E_i = E_f + Q$ where $Q > 0$

Examples

1 penny ~ $78\ GW\ h$

few 100 kg matter ~ annual US energy production

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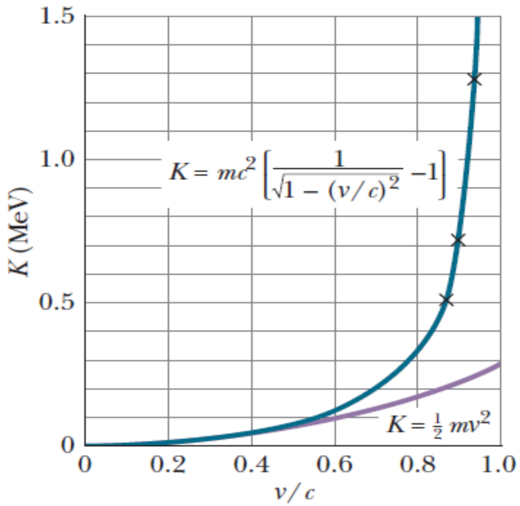
MOMENTUM AND ENERGY

Total energy $\boxed{E = \gamma\, m\, c^2}$ (w/out proof)

$$E = E_0 + K = mc^2 + \gamma mc^2$$

Kinetic energy $\boxed{K = (\gamma - 1)\, mc^2}$

$$K = (\gamma - 1)\, E_0$$



Conservation total energy for Isolated systems

$$\boxed{E_i = E_f}$$

$$\sum(E_0 + K)_i = \sum(E_0 + K)_f$$

Example: Find the minimum kinetic energy of the protons in a head-on collision to produce a pion and a deuteron (proton+neutron). Assume all masses are known.

$$p + p \rightarrow \pi^+ + d$$

MOMENTUM AND ENERGY

Rest energy $E_0 = m\,c^2$

Total energy $E = \gamma\,m\,c^2$

Kinetic energy $K = (\gamma - 1)\,mc^2 = (\gamma - 1)\,E_0$

Relativistic momentum $\boxed{\vec{p} = \gamma\,m\,\vec{v}}$

Note: $\vec{p} = m\,\vec{v}$ NR limit $v \ll c$ i.e. $\gamma \to 1$

Classical $p = mv$

$K = \frac{1}{2}mv^2$ eliminate v → $K = \frac{p^2}{2m}$

CH.37 – SPECIAL RELATIVITY

MOMENTUM AND ENERGY

Relativistic $p = \gamma\, mv$

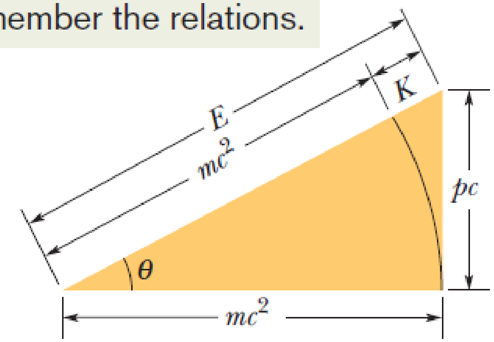
$E = \gamma\, mc^2$ eliminate γ, v^2

Momentum-energy relation

$$\boxed{E^2 = (pc)^2 + (mc^2)^2}$$

$$E^2 = (pc)^2 + (\,E_0)^2$$

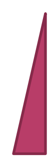
This might help you to remember the relations.



Notes

1. *NR / at rest* $E \rightarrow E_0$

2. *Highly relativistic* $E \rightarrow pc$



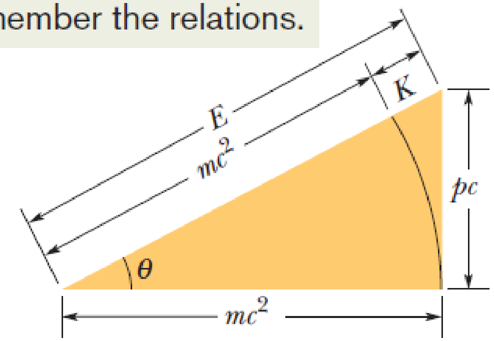
3. $m = 0$ $E = pc$

MOMENTUM AND ENERGY

Momentum-energy relation

$$E^2 = (pc)^2 + (mc^2)^2$$

This might help you to remember the relations.



Units

$[E] = eV$ $\quad 1eV = 1e\ 1V = 1.6\ 10^{-19}\,J$

$1\ keV = \ 10^3 eV$ (atomic scale)

$1\ MeV = \ 10^6 eV$ (nuclear scale)

$[m] = eV/c^2$

OR

$[m] = u = 1.66\ 10^{-27} kg \sim$ mass of nucleon

$[p] = \frac{[E]}{c} = \frac{eV}{c}$