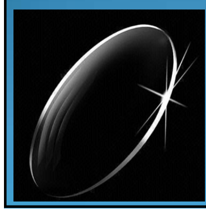
Chapter 34-Geometric Optics
Mirrors & Lenses

Mirrors - Plane and Spherical
Thins Lenses – Converging and Diverging
Human eye
Optical instruments – Magnifier, Telescope, Microscope



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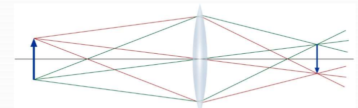
Images

Image = reproduction of an object derived from light rays spreading from the object
(self-luminous or reflecting light from a source)
from mirrors, through lenses – size and distance might be different

Assume light propagates as straight-line rays, i.e. scales $\gg \lambda$
Use ray tracing techniques
Images form at the intersection of light rays



Real image: can be captured on screen
intersection of real light rays

Virtual image exist only in the brain, e.g. back of the mirror
intersection of extension of light rays

Notation O = object; I = image
d_o, p, s = object distance; d_i, i, s' = image distance
h_o, h = object size; h_i, h' = image size

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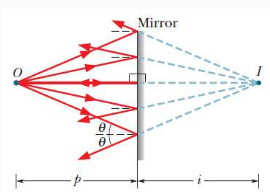
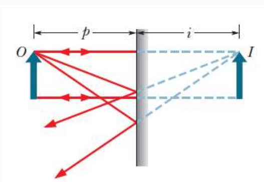
Reflection in Plane Mirrors



d_o d_i

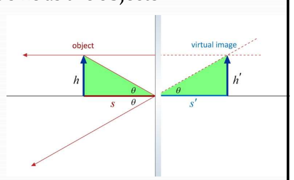


O = object point
is the source of the light rays
I = image point
is the apparent source of the reflected light rays

In plane mirrors images are
as far from the mirror as the object
the same size as the objects

$\boxed{d_o = d_i}$

$\boxed{m = \frac{h_i}{h_o} = 1}$



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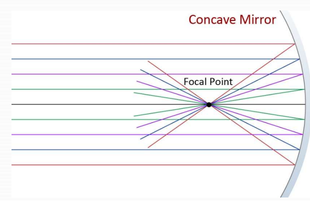
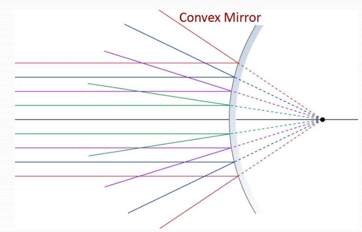
Reflection in Spherical Mirrors





$f > 0$ $f < 0$

$Focal\ point$ or $focus\ (F)$ – image of an ∞ distant object – all rays converge
$Focal\ length/distance\ (f)$ – distance from focal point to mirror

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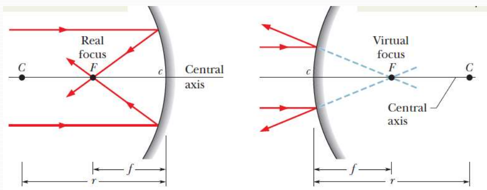
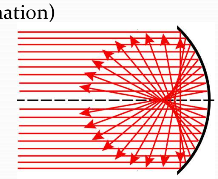
Reflection in Spherical Mirrors

Center of curvature (C) – center of sphere of which the mirror is part
Central axis – from center of curvature to center of mirror



$\boxed{f = \frac{R}{2}}$ (small angle approximation)

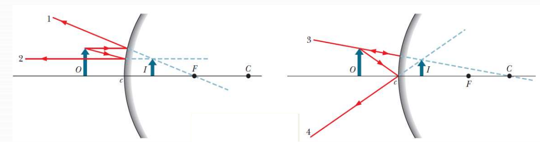
Spherical aberration – rays far from optical axis not focused
Spherical mirrors replaced w/parabolic ones



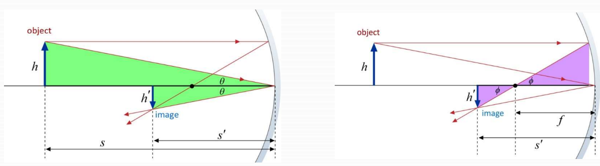
Images in Spherical Mirrors

Rays for forming an image in spherical mirrors:

1. A ray that is initially parallel to the central axis reflects through the focal point F
2. A ray that reflects from the mirror after passing through the focal point emerges parallel to the central axis
3. A ray that reflects from the mirror after passing through the center of curvature C returns along itself
4. A ray that reflects from the mirror at point c is reflected symmetrically about that axis



Spherical Mirror Equation and Magnification



Similar triangles $\frac{|h'|}{h} = \frac{s'}{s}$ $\frac{|h'|}{h} = \frac{s'-f}{f}$

Mirror equation $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

Magnification $m = \frac{h'}{h} = -\frac{s'}{s}$

Example $h > 0,\ h' < 0 \rightarrow m < 0$
$s > 0,\ s' > 0 \rightarrow m < 0$

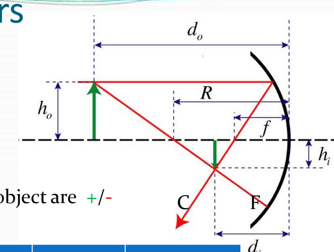
Convention: distances on same/opposite side as object are +/-

Images in Spherical Mirrors

Mirror equation $\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = \frac{2}{R}}$

Magnification $\boxed{m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}}$

Convention: distances on same/opposite side as object are +/-



Mirror		Object	Image				
	f	Location	Type	d_i	↑/↓	Size	m
Plane	∞	$\forall$	virtual	$-$	↑	same	1
Convex	$-$	$\forall$	virtual	$-$	↑	reduced	$0 < m < 1$
Concave	$+$	$d_o > 2f$	real	$+$	↓	reduced	$-1 < m < 0$
		$d_o = 2f$	real	$+$	↓	same	-1
		$f < d_o < 2f$	real	$+$	↓	enlarged	$m < -1$
		$d_o < f$	virtual	$-$	↑	enlarged	$m > 1$

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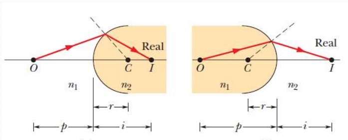
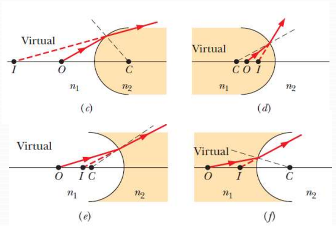
Refraction on Spherical Surface





Single spherical refracting surface $\frac{n_2-n_1}{R}=\frac{n_1}{d_o}+\frac{n_2}{d_i}$ (NOT thin lens)

Lateral magnification $m=\frac{h_i}{h_o}=-\frac{n_1}{n_2}\frac{d_i}{d_o}$

Real images
- refracted ray towards central axis
- object farther away
- opposite side as object → major ≠

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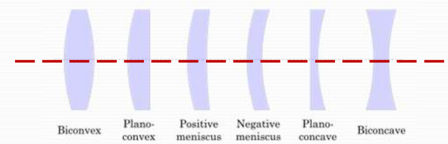
Lenses

Lens = transparent object w/ 2 *refracting surfaces whose central axes coincide*
OR 2 spherically curved boundaries between 2 optically transparent media
constructed so that parallel rays refract toward a *focal point (focus)*



Note: light is refracted (i.e. image is formed) only if lens is in a medium with $\neq n$

Thin lens = thickness t can be neglected because $t \ll d_o, d_i, R_1, R_2$

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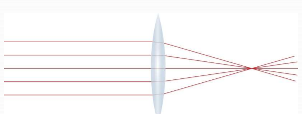
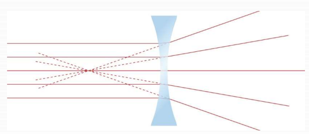
Lenses

Converging lens

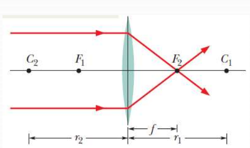


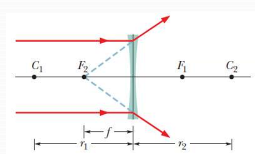
Diverging lens



$f>0$

$f<0$





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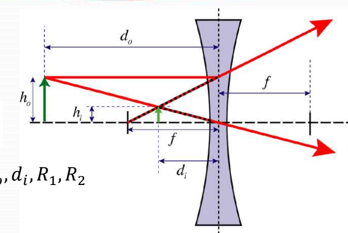
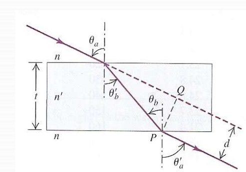
Thin Lenses



Thin lens
thickness t can be neglected because $t \ll d_o, d_i, R_1, R_2$

Rays used to build image
1. Parallel to optical axis – refracts through focus
2. Through center of lens -

At the center $d=t\,\frac{\sin(\theta_a-\theta_b')}{\cos(\theta_b')}\sim t$



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Lenses

Lens		Object	Image				
	f	Location	Type	d_i	↑/↓	Size	m
Diverging	–	∀	virtual	–	↑	reduced	$0 < m < 1$
Converging	+	$2f < d_o$	real	+	↓	reduced	$-1 < m < 0$
		$d_o = 2f$	real	+	↓	same	$m = -1$
		$f < d_o < 2f$	real	+	↓	enlarged	$m < -1$
		$d_o < f$	virtual	–	↑	enlarged	$m > 1$

Conventions for distances and heights
Object distance $d_o > 0$
Image type real $d_i > 0$ image on opposite side from object
virtual $d_i < 0$ image on same side as object
Image orientation upright $h_i > 0$
inverted $h_i < 0$

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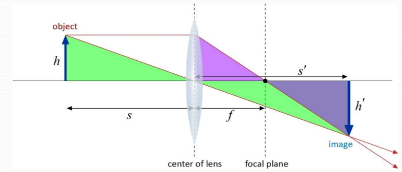
The Lens Equation and Magnification

Similar triangles $\frac{|h'|}{h} = \frac{s'}{s} = \frac{s'-f}{f}$

Mirror equation $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

Magnification $m = \frac{h'}{h} = -\frac{s'}{s}$



Example $h > 0,\ h' < 0 \rightarrow m < 0$
$s > 0,\ s' > 0 \rightarrow m < 0$

Convention: distances on same/opposite side as object are -/+

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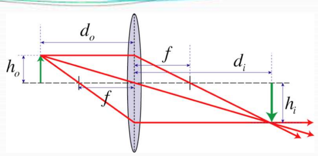
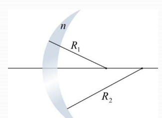
Lenses

Thin lens formula $\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$

Magnification $\boxed{m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}}$



Lens-maker's formula $\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$ in air



$n \rightarrow \frac{n}{n_{med}}$ in other media

Special case $\boxed{\frac{1}{f} = (n-1)\frac{2}{R}}$ $R_1 = -R_2 = R$

Aberrations – at large angles – use parabolic lenses
- chromatic - $n(\lambda)$
- astigmatism – non symmetric curvatures in transverse direction

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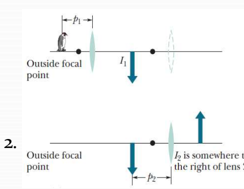
Two-Lens Systems

Simple two-step solution
Step 1 :
Neglecting lens 2, build image I_1 produced by lens 1.
Determine whether the image it is real or virtual, and whether it has the same orientation as the object.
Step 2
Neglecting lens 1, treat I_1 as though it is the object for lens 2.
Locate the image I_2 produced by the second lens.



Total magnification $\boxed{M = m_1 m_2}$

Effective focal length $f = x - \frac{1}{\frac{1}{x-f_1} - \frac{1}{f_2}} = x + \frac{f_2(x-f_1)}{x-(f_1+f_2)}$

where x = distance between lenses
Note: $f = f(x)$ can change f by changing x

$x = 0 \rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

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Human Eye

Most of the refraction occurs at the air/*cornea boundary.*
Lens changes shape → focus changes

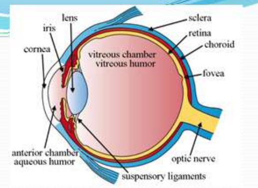
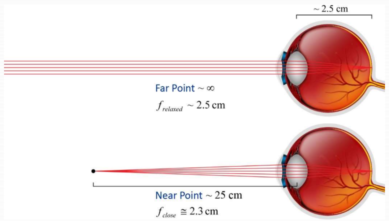


Image must be formed at location of retina
Distinct vision between

Far point (FP) = maximum distance that a relaxed eye can focus on the retina
$few\ m \sim 100\ f_{relaxed}$ → 1% $\neq \sim 1/4\ mm$
Near point (NP) = closest distance that can be focused on the retina
$\sim 7cm$ (child) – $40\ cm$ (50s)



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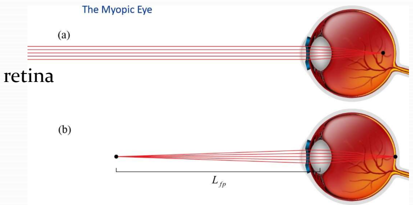
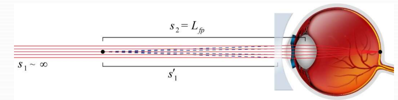
Myopic Eye

Parallel rays are focused in front of the retina
(model eye as elongated)

Far point is at finite distance L_{fp}



Divergent lens produces a virtual image at the far point = object for the eye



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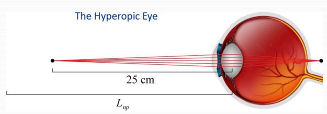
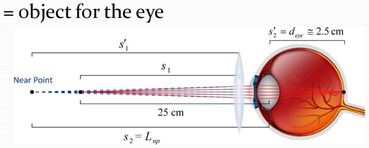
Hyperopic Eye

Rays from the normal near point focused behind the retina
(model eye as shortened)

Near point is at $L_{np} > 25\ cm$



Converging lens $f > 25\ cm$
produces a virtual image at the near point = object for the eye



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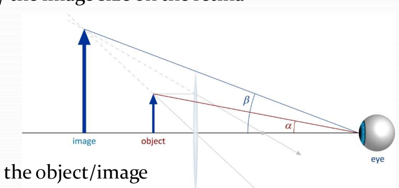
Angular Magnification and Magnifier

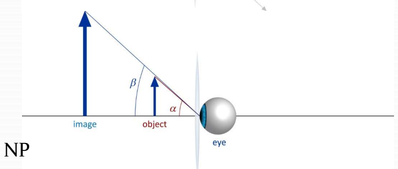
Perceived size of objects is determined by the image size on the retina

Place object inside focal distance
→ enlarged virtual image

Angular magnification
$m_\theta = \frac{\beta}{\alpha}$
where α, β angles subtended by the object/image



Bring eye close to lens
$m_\theta \to 1$



Short f magnifier
→ image at NP of object inside NP
$m_\theta = \frac{\beta\ (image\ at\ NP)}{\alpha\ (object\ if\ at\ NP)}$

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Refracting Telescope

Purpose = gather light from distant objects / produce magnified image
Objective produces real image at focal point → viewed/recorded through eyepiece

Simplest telescope = objective lens + eyepiece

$d_{i,1} \approx f_1$ $\quad$ $(d_{o,1} \to \infty)$

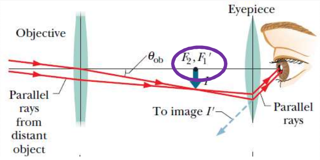
$d_{o,2} \approx f_2$ $\quad$ (set eyepiece)

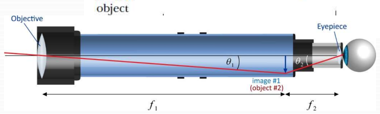
Angular magnification - defined by

θ_1 observed in eyepiece

θ_2 sub-tended by object

$$\boxed{m_\theta = -\frac{\theta_2}{\theta_1} \approx -\frac{f_1}{f_2}}$$





Powerful telescope is long (f_1) and has strong eyepiece (short f_2)

Problem – objective lens is glass
large ~ 1 m
heavy ~ 500 kg
thick → absorbent
chromatic aberration

Galileo - 5.1 cm/2.6 cm/1.3 m long

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Microscope

Purpose = magnified image of small object at short distance
Simplest microscope = objective lens (converging, short f)
eyepiece (converging, longer f)

Object to be magnified is placed just outside the focal length of the objective lens

$d_{o,1} \approx f_1$ $\quad$ image is real, ↓, enlarged

First image is placed just inside the focal length of the eyepiece

$d_{o,2} \approx f_2$ $\quad$ image is virtual, ↑, enlarged

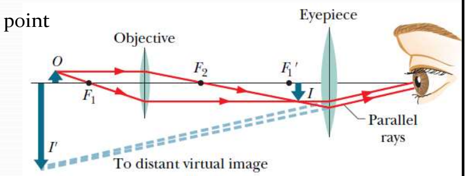
$L \gg f_1, f_2$ → $d_{i,1} \approx L$ $\quad$ where L = distance (objective, eyepiece)

Final image produced around near point

$d_{i,2} \approx 0.25\ m$

Total magnification

$$m = \frac{d_{i,1}}{d_{o,1}}\frac{d_{i,2}}{d_{o,2}} \approx \frac{0.25\ L}{f_1\ f_2}$$



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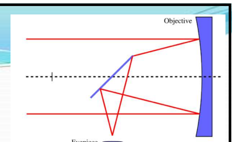
Telescopes



Replace objective lens with mirror
1670 Newton / James Gregory

1990 Hubble Space Telescope (2.4 m D) + COSTAR package
2021 James Webb Space Telescope – 6.5 m D.
(stopped after $3 billion /75% done; restarted)

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