Chapter 15
Waves

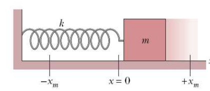
Review Oscillations
Types of waves
- mechanical, el-mg, (sub)atomic/molecular level
- transverse, longitudinal

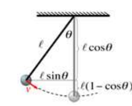
Transverse harmonic wave on a tensioned string
Wave equation
Interference
- Standing waves
- Resonance on string

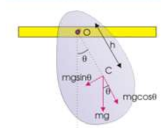
Chapter 14- Oscillations (Review)

Simple Harmonic Motion

Mass on Spring	Simple Pendulum	Physical Pendulum
$\frac{d^2x}{dt^2} + \frac{k}{m}\,x = 0$	$\frac{d^2\theta}{dt^2} + \frac{g}{l}\,\theta = 0$	$\frac{d^2\theta}{dt^2} + \frac{mgh}{I}\,\theta = 0$
$x(t) = x_m\, cos(\omega t + \varphi)$	$\theta(t) = \theta_m \cos(\omega t + \varphi)$	$\theta(t) = \theta_m \cos(\omega t + \varphi)$
$\omega = \sqrt{\frac{k}{m}} = \text{const.}$	$\omega = \sqrt{\frac{g}{l}} = \text{const.}$	$\omega = \sqrt{\frac{mgh}{I}} = const.$
$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$	$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$	$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgh}}$







Chapter 14- Oscillations (Review)

Motion of Mass on Spring (SHM)

$$x(t) = x_m \cos(\omega t + \varphi) = x_m \cos\left(2\pi \frac{t}{T} + \varphi\right)$$

position (displacement)

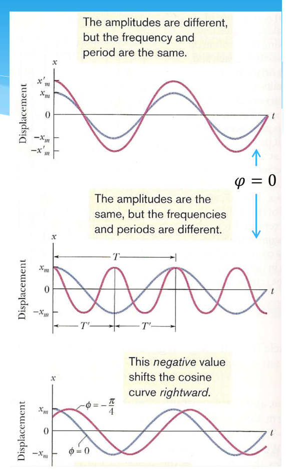
$x_m = const > 0$ — *amplitude (max displ. from eq.)*

$\omega = \sqrt{\frac{k}{m}} = 2\pi f$ — *angular frequency*

$T = \frac{2\pi}{\omega}$ — *period*

$(\omega t + \varphi)$ — *phase of the motion*

φ — *phase angle (phase constant)*



Chapter 14- Oscillations (Review)

Motion of Mass on Spring (SHM)

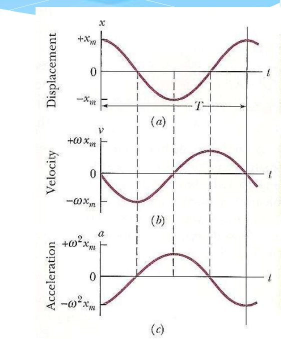
Period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Position $x(t) = x_m \cos(\omega t + \varphi)$

Velocity $v(t) = -\omega\, x_m \sin(\omega t + \varphi)$

Acceleration $a(t) = -\omega^2\, x_m \cos(\omega t + \varphi)$

$\varphi = 0$

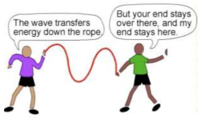


Chapter 15 - Waves

Waves

disturbance traveling:
- through *space*
- as a function of *time*

very different kind of motion
- usually transfers energy and momentum
- without transporting matter
- (mass oscillates around equilibrium)



∞ number of coupled oscillator
local oscillations around (almost) fixed position

Chapter 15
Types of Waves

Mechanical waves
- governed by Newton's laws
- can ∃ only in material medium
  - deformed by perturbation
    - produces elastic restoring force
- *ex:* water, sound, seismic

Electromagnetic waves
- do not require ∃ of material medium
- $c = 3\,10^8\,\frac{m}{s}\quad in\ vacuum$
- *ex:* light (visible, UV), X-rays, radio, TV, microwaves, radar

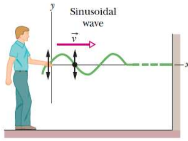
Atomic/molecular (matter) waves
- probability waves associated with $e^-, p^+, atoms, molecules$

Note: ∃ entire areas of physics devoted to a particular kind of waves
- *ex:* optics, acoustics, etc.

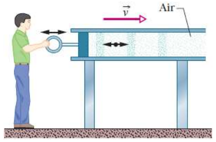
Chapter 15
Types of Waves (2)

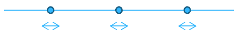
Transverse waves
- travel ⊥ to the direction in which the oscillators move
- *ex:* light, stretched string http://phet.colorado.edu/en/simulations/category/physics/

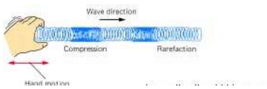
Ideal string
- long
- no friction



Longitudinal waves
- travel along the direction in which the oscillators move
- *ex:* sound



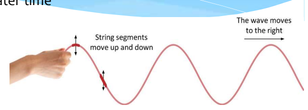




Chapter 15
(Harmonic) Waves in Taut Strings

Taut string = $\sum_1^\infty dm$ (∞ number of <u>*coupled*</u> oscillators)

Wave in the taut string lying on a horizontal frictionless floor
- Disturbance propagates down the string
- All points have the same motion as the LHS end but at a later time
- The string is only the medium for the traveling wave



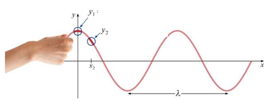
Harmonic wave in taut string

Displacement of one end

$$y_1(t) = A\sin(\omega t)$$

Displacement of element at x_2

$$y_2(t) = A\sin(\omega t - kx)$$



Notes:
- same A, f
- minus sign is for waves moving in $+x$ direction
- k = constant = *wave number*

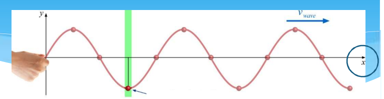
Displacement of transverse harmonic wave on a taut string traveling along the $+x$ direction

$$\boxed{y(t) = A\sin(kx - \omega t)}$$

Chapter 15- Waves

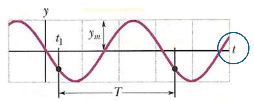
Period and Frequency

Focus on any piece of string



Graph at $\forall x$ position, e.g. at $\boxed{x = 0}$

$y(0,t) = A\sin(-\omega t) = -A\sin\omega t$



Period (T)= time for an element to move through a full oscillation
smallest time before a point repeats its motion

$\omega T = 2\pi \rightarrow \boxed{T = \frac{2\pi}{\omega}}$

Angular frequency $\omega = \frac{2\pi}{T}$

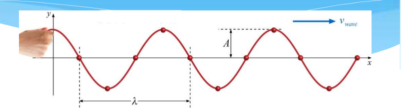
Frequency $\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$ number of oscillations per unit time

Chapter 15- Waves

Wavelength

Freeze at $\forall$ moment

Snapshot at $\forall$ time, e.g. at $\boxed{t = 0}$



$y(x,0) = A\sin kx$

Wavelength (λ)
distance (∥ to direction of travel) between repetitions of wave form
smallest distance between points with identical motion
length of a complete wave pattern
$k\lambda = 2\pi$

Angular wave number $\boxed{k = \frac{2\pi}{\lambda}}$

$[k] = \frac{rad}{m}$

Note : k does not represent the spring constant

Chapter 15- Waves

WaveForm

Wave form (shape of the wave) described by a function $\boxed{y = y(x,t)}$

Sinusoidal wave form

$$\boxed{y(x,t) = A\sin(kx \mp \omega t + \varphi) = A\sin(2\pi\frac{x}{\lambda} \mp 2\pi\frac{t}{T} + \varphi)}$$

$y(x,t)$ *displacement at time* t *and position* x
how far is the particle from equilibrium position

A *amplitude >0*
magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them

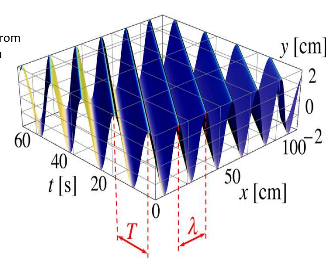
$\sin(kx \mp \omega t + \varphi)$ *oscillating term*

$(kx \mp \omega t + \varphi)$ *phase*
as the wave sweeps through a particular position x, the phase changes linearly with time t.

k *angular wave (phase) number*
related to wavelength λ by $k = \frac{2\pi}{\lambda}$

ω *angular frequency*
related to time period T by $\omega = \frac{2\pi}{T}$

φ *phase constant*
the value of ϕ is determined by the initial conditions (x=0, t=0)



Chapter 15- Waves

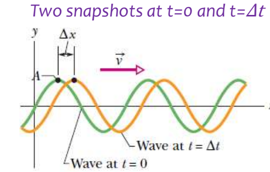
Speed of traveling wave

Focus on $\forall$ piece of the string:
waves moves exactly one wavelength along the string during one period of its oscillation

Wave speed (phase speed)
speed of waveform traveling along x axis

$\boxed{v = \frac{\lambda}{T} = \frac{\omega}{k} = \lambda f}$

Two snapshots at t=0 and t=Δt



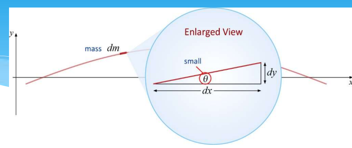
Wave speed along a taut string

$\boxed{v = \sqrt{\frac{F}{\mu}}}$ *where* $F =$ *tension in the string*

$\mu = \frac{M}{L}$ *linear mass density*

Note: speed is set by properties of medium
while frequency is set by the source of oscillation

Chapter 15- Waves
Wave equation



Assume: tension same along the string
$y(x,t) \ll \lambda \rightarrow \sin\theta = tan\theta = \theta$ (SAA)

Net force on $\forall$ dm element from either side

$$\sum F_x = Tcon\theta_2 - Tcos\theta_1 = 0$$
$$\sum F_y = Tsin\theta_2 - Tsin\theta_1 = T(\theta_2 - \theta_1) = T\, d\theta$$

Newton's 2nd law in the y direction

$$T\, d\theta = dm\, a_y = \mu\, dx\, a_y \quad \rightarrow \quad T\frac{d\theta}{dx} = T\frac{d^2y}{dx^2} \quad \text{where} \quad \theta = tan\theta = \frac{dy}{dx}$$

$$a_y = \frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dx^2} = \frac{\mu}{T}\frac{d^2y}{dt^2}$$

Harmonic waveform $y(t) = A\sin(kx - \omega t)$ satisfies this equation if-and-only-if $\frac{k}{\omega} = \sqrt{\frac{\mu}{T}} \equiv \frac{1}{v}$

Wave equation $\boxed{\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}}$

Chapter 15- Waves
Wave equation

$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$ *where* $\boxed{v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f}$

Any function of $(x - vt)$ is a solution of the wave equation IF wave speed $v = constant$

$kx - \omega t = k\left(x - \frac{\omega}{k}t\right) = k(x - vt)$ where $(-)$ wave travels in $+x$ direction
$(+)$ wave travels in $-x$ direction

Examples:
$y(x,t) = A\ln(x + vt)$
$y(x,t) = \sqrt{ax + bt}$
$y(x,t) = \sin(a\,x^2 - b\,t)$

In particular: $y(x,t) = A\,sin(kx - \omega t) = A\,sin\,2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) = A\,sin\,k(x - v\,t)$

Chapter 15- Waves
Speed of traveling wave (2) and Transverse velocity

Transverse velocity of element x –
associated with transverse oscillation on y axis

$$\boxed{v_y(x,t) = \frac{\partial y}{\partial t}} = \omega\,A\cos(kx - \omega t + \varphi)$$

Max transverse velocity $v_{y\,max} = \omega\,A$

Transverse acceleration of element x

$$a_y(x,t) = \frac{\partial v_y}{\partial t} = \frac{\partial^2 y}{\partial t^2} = -\omega^2\,A\sin(kx - \omega t + \varphi)$$

Max transverse acceleration $a_{y\,max} = \omega^2 A$

Chapter 15- Waves
Energy, Power, and Intensity

Waves transmit energy from one place to another
Examples: energy form sun, earthquake.

Energy comes from the work done to produce initial disturbance.
During propagation each portion of the medium exerts a force and does work on the next portion.

$$K_\lambda = \int \frac{1}{2}\,dm\,v_y^2 = \int_0^\lambda \frac{1}{2}\,\mu\,dx\,A^2\omega^2 cos^2(kx - \omega t) = \frac{1}{4}\,\mu\,A^2\omega^2\lambda$$

$$U_\lambda = \int \frac{1}{2}\,k_s\,y^2 = \int \frac{1}{2}\,(\omega^2 dm)y^2 = \int_0^\lambda \frac{1}{2}\omega^2\mu\,dx\,A^2 sin^2(kx - \omega t) = \frac{1}{4}\,\mu\,A^2\omega^2\lambda$$

where $\omega = \sqrt{\frac{k_s}{dm}}$ and k_s=spring constant

$$\boxed{E_\lambda = K_\lambda + U_\lambda = \frac{1}{2}\,\mu\,A^2\omega^2\lambda}$$

Note: $E_\lambda \sim A^2, f^2$

Chapter 15- Waves

Energy, Power, and Intensity

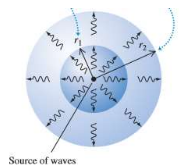
Average power

$$P_{ave} = \frac{E_\lambda}{T} = \frac{\frac{1}{2}\mu A^2\omega^2\lambda}{T} = \frac{1}{2}\,\mu\, v\, \omega^2\, A^2$$

Intensity of the wave – average energy rate transmitted per unit area

$$I = \frac{P}{A_\perp}$$

In 3D $\quad I = \frac{\frac{1}{2}\mu\, v\, \omega^2 A^2}{4\,\pi\, r^2}$ $\quad$ where $\quad A_\perp = 4\,\pi\, r^2$



Chapter 15- Waves

Interference of Waves

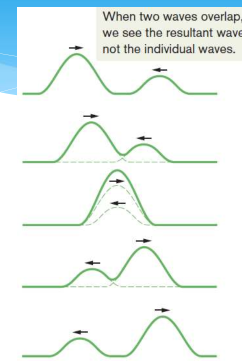
Superposition principle =
when several effects occur simultaneously,
the net effect is the sum of individual effects

IF $y_1(x,t)$ *and* $y_2(x,t)$ are solutions of wave equations
⇒ ∀ linear combination
$y(x,t) = a\, y_1(x,t) + b\, y_2(x,t)$ is a solution

Waves: $\quad y(x,t) = y_1(x,t) + y_2(x,t)$

displacement of string when waves overlap
overlapping algebraically → resultant wave

Notes: sum seems to be standing still
after passing both waves look exactly the same
i.e. overlapping waves do not alter each other's travel (see phone calls)



http://www.acs.psu.edu/drussell/Demos/superposition/superposition.html

Chapter 15- Waves

Interference of Waves

Interference
resultant wave depends on the phase shift φ

$y_1(x,t) = A\sin(kx - \omega t)$
$y_2(x,t) = A\sin(kx - \omega t + \varphi)$

Use: Superposition principle $\quad$ and $\quad \sin a + \sin b = 2\cos\frac{a-b}{2}\sin\frac{a+b}{2}$

$$\boxed{y(x,t) = 2A\cos\tfrac{1}{2}\varphi\ \sin(kx - \omega t + \tfrac{1}{2}\varphi)}$$

amplitude $\quad$ *oscillating term*

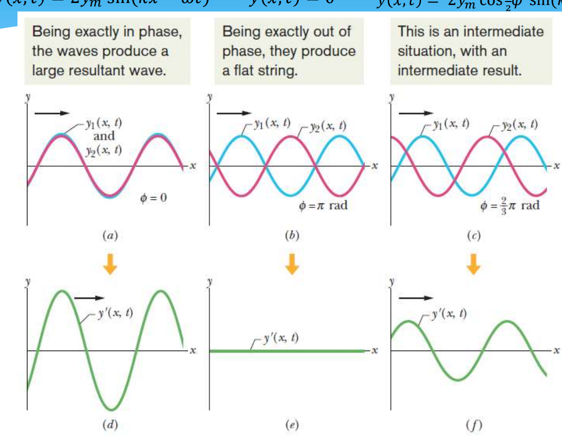
Fully constructive *- when* $\boxed{\varphi = 0}$ i.e. the 2 waves are exactly ***in phase***
$y(x,t) = 2A\sin(kx - \omega t)$

Fully destructive *- when* $\boxed{\varphi = \pi}$ i.e. the 2 waves are exactly ***out of phase***
$y(x,t) = 0$

Chapter 15- Waves

Interference of Waves

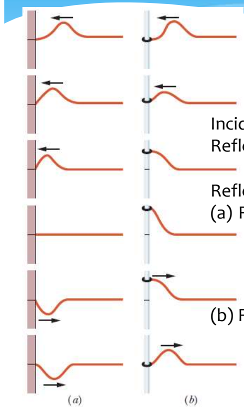
$y(x,t) = 2y_m\sin(kx - \omega t)$ $\qquad y(x,t) = 0$ $\qquad y(x,t) = 2y_m\cos\frac{1}{2}\varphi\ \sin(kx - \omega t + \frac{1}{2}\varphi)$



Start

Chapter 15- Waves

Standing Waves – Reflection at Boundary



Incident wave – before hitting the boundary
Reflected wave – after encountering the boundary

Reflection at boundary
(a) Fixed end – reflected pulse is inverted:
travels in opposite direction and has opposite displacement
due to reaction from wall on string
energy not absorbed in fixed end but remains in string as reflected wave
"hard" reflection
(b) Free end – reflected pulse is not inverted
ring is free to move to max. displacement and back to equilibrium
"soft" reflection

Chapter 15- Waves

Standing (stationary) waves

Standing waves **=** *special case* of interference: Incident + Reflected weave
same $\boldsymbol{A}$ **and** $\boldsymbol{\lambda}$ **but opposite directions**

$\varphi = 0$
$y_1(x,t) = A\,sin(kx - \omega t)$ incident wave traveling in +x direction
$y_2(x,t) = A\,sin(kx + \omega t)$ reflected wave traveling in –x direction

Use superposition principle and $sin\,a + sin\,b = 2\,sin\frac{a+b}{2}\,cos\frac{a-b}{2}$

$\boxed{y(x,t) = 2A\,sin\,kx\ cos\,\omega t}$ ***Standing*** wave equation (NOT traveling)

$A_{sw} = 2A$

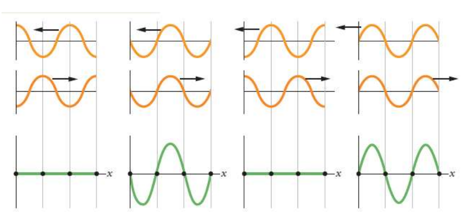
Fixed t: shape of the string is a sin wave, not traveling
Given x: amplitude is oscillating with all points between nodes oscillating in phase

Chapter 15- Waves

Standing (stationary) waves

$y(x,t) = 2A\,sin\,kx\ \ cos\,\omega t$ ***Standing*** wave equation (NOT traveling)

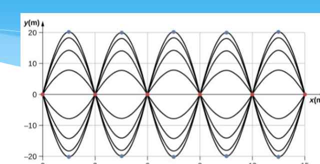
5 snapshots:



$t = \frac{T}{4}$ $t = \frac{T}{2}$ $t = \frac{3T}{4}$ $t = T$

Chapter 15- Waves

Standing Waves (2)



As the waves move through each other:
some points never move (nodes)
others move the most (antinodes)

Nodes: amplitude is 0
$y = 0$
$kx = n\pi$ $\frac{2\pi}{\lambda}x = n\pi$ $\boxed{x = n\frac{\lambda}{2}}$ $n = 0,1,2,\ldots$

Antinodes: amplitude is maximum
$y = 2A$
$kx = \frac{2n+1}{2}\pi$ $\frac{2\pi}{\lambda}x = \frac{2n+1}{2}\pi$ $\boxed{x = \frac{2n+1}{2}\frac{\lambda}{2}}$

Q: What is the separation between 2 nodes or 2 antinodes?

Chapter 15- Waves

Standing Waves – Resonance on string

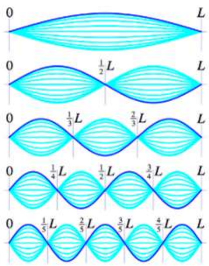
Interference of many reflected waves with both ends fixed = string instruments
For certain frequencies (called resonant frequencies)
interference → standing wave patterns (normal modes)

$\boxed{L = n\,\frac{\lambda}{2}}$ *where* $n = 1, 2, 3, \ldots$

$f_n = \frac{v}{\lambda} = \frac{v}{\frac{2L}{n}} = n\,\frac{v}{2L}$ $\boxed{f_n = n\,\frac{v}{2L} = n\,f_1}$

Note: $\boldsymbol{n}$ is the number of antinodes

$n = 1$ 1^{st} *harmonic* $=$ *fundamental mode*
$n = 2$ 2^{nd} *harmonic OR* 1^{st} *overtone*



String instruments – combination of many wavelengths
longest λ (lowest f) dominates