Wave function

1. TR.6.11. A particle moving in one dimension is described by the wave function

$$\psi(x) = \begin{cases} A\,sin\theta & 0 \le x \le \pi \\ 0 & x < 0 \; and \; x > \pi \end{cases}$$

a) Determine A so that the wave function is normalized.
b) Sketch the graph of the wave function.
c) Find the probability of finding the particle in each of the following regions: $0 \le x \le \frac{\pi}{4}$ and $0 \le x \le \frac{\pi}{2}$

Answer: $A = \sqrt{\frac{2}{\pi}}$; 9.1%; 50%

2. 243T1. A particle moving in one dimension is described by the wave function

$$\psi(x) = \begin{cases} A\,x & 0 \le x \le 1pm \\ A\,(2-x) & 1pm \le x \le 2pm \\ 0 & x < 0 \; and \; x > 2pm \end{cases}$$

d) Determine A so that the wave function is normalized.
e) Sketch the graph of the wave function.
f) Find the probability of finding the particle in each of the following regions: $0 \le x \le 0.5$; $0 \le x \le 1$, and $1 \le x \le 2$

Answer: $A = 1.22\ pm^{-1}$; 6.25%; 50%; 50%

Infinite 1D potential well

3. BW.p1215. Proton in nucleus. What is the kinetic energy of the ground state wave function for a proton confined to a box of width $L = 2\ 10^{-15}\ m$?

Answer: $\mathrm{E} = 51\ \mathrm{MeV}$

4. HR.1087. Electron in atom

a) What is the kinetic energy of the ground state wave function for an electron confined to a box of width $L = 1\ \text{Å} = 10^{-10} m$?
b) How much energy must be transferred to the electron if it is to make a quantum jump from its ground state to its 2nd excited state?
c) If the electron gains the energy for the jump by absorbing light, what wavelength is required?
d) Once the electron has been excited to the 2nd excited state, what wavelengths of light can it emit by de-excitation?
e) What is the probability that the electron can be detected in the left 1/3 of the well?
f) What is the probability that the electron can be detected in the middle 1/3 of the well?

Answer: 37.7 eV; 301.6 eV; 4.1 nm; 4.1 nm, 6.6 nm, 11 nm; 19.5%; 81%

5. TR.6.21. An electron is trapped in an infinite square well potential of width $0.7\ nm$. If the electron is initially in the $n = 4$ state, what are the various photon energies that can be emitted as the electron jumps to the ground state?

Answer: $5.37\ eV$; $9.21\ eV$; $11.5\ eV$; $3.84\ eV$; $6.14\ eV$; $2.3\ eV$

Barrier tunneling

6. HR.1076. A $5.1\ eV$ electron approaches a barrier $U_0 = 6.8\ eV$ and thickness $L = 750\ pm$.

a) What is the approximate probability that the electron will be transmitted through the barrier?
b) What is the probability for a proton with the same total energy of $5.1\ eV$?

Answer: $\mathrm{T} \approx 45\ 10^{-6}$; $\mathrm{T} \approx \mathrm{e}^{-429}$

7. TR.6.67. Two nano-wires are separated by $1.3\ nm$ as measured by STM. Inside the wires the potential energy is zero, but between the wires the potential energy is greater than the electron's energy by only $0.9\ eV$. Estimate the probability that the electron passes from one wire to the other.
Answer: $T = 6.51\ 10^{-6}$

Hydrogen atom

8. 243T2. If the hydrogen atom is in an $l = 4$ state, what is the magnitude of the orbital angular momentum?
Answer: $L = 2\sqrt{5}\ \hbar$

9. 243T2. Why is the following configuration not allowed $1s^2\ 2s^2\ 2p^6\ 2d^1$?
Answer: $n = 2, l = 0,1$

10. TR.7.10. For the $3p$ state give the possible values of $n,\ l,\ m_l,\ L,\ L_z, L_x, L_y$.

11. TR.7.14. For a $3d$ state draw all possible orientations of the angular momentum vector $\vec{L}$. What is $L_x^2 + L_y^2$ for the $m_l = -1$ component?
Answer: $5\ \hbar^2$

12. TR.7.15. What is the smallest value that l may have if $\vec{L}$ is within 3° of the z axis?
Answer: $l = 33$

13. HR.p1102. Show that the radial probability density for the ground state of the hydrogen atom has a maximum at $r = a$.

14. YF.p1378. What is the probability of finding the ground state e^- at a radial distance $r < a$ from the nucleus?
Answer: $P = 32.3\%$

15. HR.p1104. What is the radial distance for which the probability of finding the e^- is 90%?
$P(R) = \int_0^R P(r)dr = 1 - [1 + 2\frac{R}{a} + 2\left(\frac{R}{a}\right)^2]\, e^{-2\frac{R}{a}}$
Answer: $R = 2.66\ a$

16. TR.7.38. What is the probability of finding the ground state e^- inside the p^+?
Consider $r_p = 1.2\ 10^{-15} m$ and $r \ll a$.
Answer: $P = 1.55\ 10^{-14}$