Chapter 32
Electro-magnetic Waves

Maxwell Equations

Electro-magnetic spectrum

Speed of Light

Energy and Momentum – Poynting Vector and Radiation Pressure

Standing waves

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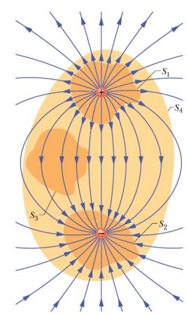
Chapter 32 – El-mg Waves

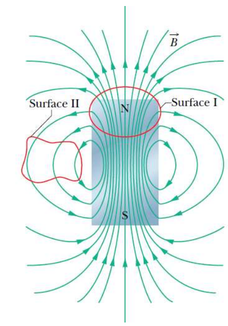
Maxwell's Equation – Gauss' Laws

Gauss' law for ELECTRIC field



$$\oiint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} q_{enc}$$

Gauss' law for MAGNETIC field



$$\oiint \vec{B}\cdot d\vec{A} = 0$$

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Chapter 32 – El-mg Waves

Maxwell's Equation – Integral form

Law	Equation	Relates …
Gauss' law for electric field	$\oiint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} q_{enc}$	net electric flux to net enclosed electric charge
Gauss' law for magnetic field	$\oiint \vec{B}\cdot d\vec{A} = 0$	net magnetic flux to net enclosed magnetic charge
Faraday/Lenz law of induction	$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	induced electric field to change in magnetic flux
Maxwell-Ampere law	$\oint \vec{B}\cdot d\vec{s} = \mu_0\, i_{enc} + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt}$	induced magnetic field to change in electric flux and to current

Conclusions:

Electric and magnetic fields can be separated for electrostatic charges and/or steady currents.

Time-varying fields are no longer independent.

Every accelerating charge radiates el.mg. energy, in particular a charge in SHM – see Fig 32.3 page 1053.

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Chapter 32 – El-mg Waves

Maxwell's Equation – Integral and Differential Forms

Law	Equation	Relates …
Gauss' law for electric field	$\oiint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} q_{enc}$	net electric flux to net enclosed electric charge
	$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho_{enc}$	
Gauss' law for magnetic field	$\oiint \vec{B}\cdot d\vec{A} = 0$	net magnetic flux to net enclosed magnetic charge
	$\vec{\nabla}\cdot\vec{B} = 0$	
Faraday/Lenz law of induction	$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	induced electric field to change in magnetic flux
	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$	
Maxwell-Ampere law	$\oint \vec{B}\cdot d\vec{s} = \varepsilon_0\mu_0\frac{d\Phi_E}{dt} + \mu_0\, i_{enc}$	induced magnetic field to change in electric flux and to current
	$\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} + \mu_0\vec{J}$	

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Chapter 15- Waves

Mathematical Description - Review

Sinusoidal wave form (shape of the wave) described by a function

$$y(x,t) = A\sin(kx - \omega t + \varphi) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) = A\sin(2\pi k(x - vt) + \varphi)$$

Wave equation $\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$ where $\boxed{v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f}$

$y(x,t)$ *displacement at time t and position x*

A *amplitude >0*

magnitude of the maximum displacement of the elements from their eq

$\sin(kx - \omega t)$ *oscillating term*

$(kx - \omega t)$ *phase*

as the wave sweeps through a particular position x,
the phase changes linearly with time t.

k *angular wave (phase) number*

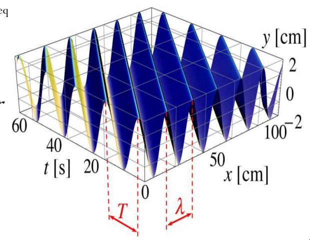
related to wavelength λ by $k = \frac{2\pi}{\lambda}$

ω *angular frequency*

related to time period T by $\omega = \frac{2\pi}{T} = 2\pi f$

φ *phase constant*

the value of ϕ is determined by the initial conditions (x=0, t=0)



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Chapter 32 – El-mg Waves

Electro-magnetic Waves

El-mg waves = changing electric & magnetic fields traveling through vacuum and obeying Maxwell equations (w/out involving moving charges or currents)

Maxwell (1850s) showed this for light (el.mg waves traveling at the c) and predicted radio waves
Heinrich Hertz (1888) - Helmholtz's student – experimental demo for radio waves (& sparks)

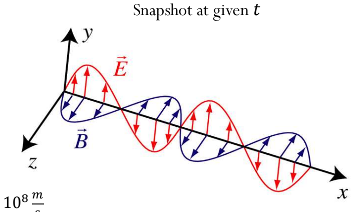
Postulate field configurations with wave-like behavior traveling in the $+x$ direction in phase

$$\vec{E} = E_{max}\sin(\kappa x - \omega t)\ \hat{\jmath}$$

$$\vec{B} = B_{max}\sin(\kappa x - \omega t)\ \hat{k}$$

E,B travel as package in x direction
with $\lambda = \frac{2\pi}{k}$ and $T = \frac{2\pi}{\omega}$

Snapshot at given t



Test if they satisfy Maxwell's equations
Show they propagate at the speed of light $c = 3\ 10^8\ \frac{m}{s}$
See YF derivation page 1055-1057

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Chapter 32 – El-mg Waves

Electro-magnetic Waves

Faraday's Law

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\oiint \vec{B}\cdot d\vec{A}$$

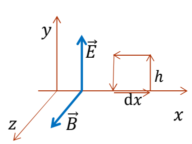
Ampere's law

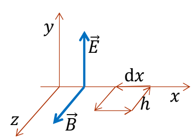
$$\oint \vec{B}\cdot d\vec{s} = \varepsilon_0\mu_0\frac{d}{dt}\oiint \vec{E}\cdot d\vec{A}$$

Harmonic wave solutions

$\vec{E} = E_{max}\cos(\kappa x - \omega t)\ \hat{\jmath}$

$\vec{B} = B_{max}\cos(\kappa x - \omega t)\ \hat{k}$





$((E + dE) - E)\ h = -\frac{d}{dt}\ B\ dx\ h$ $\quad /: dx$

$dB\ h = -\varepsilon_0\mu_0\frac{d}{dt}\ E\ dx\ h$ $\quad /: dx$

$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ $\quad /: \frac{\partial}{\partial x}$

$\frac{\partial B}{\partial x} = -\varepsilon_0\mu_0\frac{\partial E}{\partial t}$ $\quad /: \frac{\partial}{\partial t}$

$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}\frac{\partial B}{\partial x}$

$\frac{\partial}{\partial t}\frac{\partial B}{\partial x} = -\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}$

$\frac{\partial^2 E}{\partial x^2} = \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}$ where $\varepsilon_0\mu_0 = \frac{1}{v^2}$

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Chapter 32 – El-mg Waves

Speed of Light

Speed of the electromagnetic waves

$$\boxed{c = \lambda f = \frac{\omega}{\kappa} = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3\ 10^8\ \frac{m}{s}}$$

Compare to $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$ SHM of mass on spring

$v = \sqrt{\frac{F_T}{\mu}}$ Mechanical wave on taut string

Relationship between electric and magnetic field

$$\boxed{\frac{E(t)}{B(t)} = \frac{E_{max}}{B_{max}} = c}$$

Fields are in phase

Proof: $E = E_{max}\sin(\kappa x - \omega t)$ $\quad /: \frac{\partial}{\partial x}$

$\frac{\partial E}{\partial x} = \kappa E_{max}\cos(\kappa x - \omega t)$

Use $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ from Faraday's law $\quad /\cdot\ dt$

$\int \kappa E_{max}\cos(\kappa x - \omega t)\ dt = \int -\frac{\partial B}{\partial t}\ dt$ $\quad /\int \ldots dt$

$\frac{k}{\omega}E_{max}\sin(\kappa x - \omega t) = \frac{E_{max}}{c}\sin(\kappa x - \omega t) = \frac{E}{c} = B$

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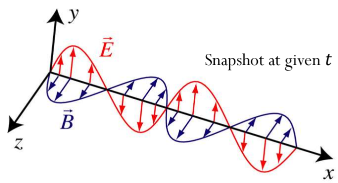
Chapter 32 – El-mg Waves

Key Properties of electro-magnetic Waves

$\vec{E}(\vec{r},t) = E_{max}\cos(\kappa x - \omega t)\ \hat{j}$
$\vec{B}(\vec{r},t) = B_{max}\cos(\kappa x - \omega t)\ \hat{k}$

E,B travel as package in x direction
with $\lambda = \frac{2\pi}{k}$ and $T = \frac{2\pi}{\omega}$



Properties of ALL el.mg. waves

1. Transverse wave $\vec{E}\perp\hat{\imath}$; $\vec{B}\perp\hat{\imath}$
2. Fields are perpendicular to each other $\vec{E}\perp\vec{B}$
   Fields are perpendicular to direction of propagation $\vec{E}\times\vec{B}\sim\hat{\imath}$
3. Fields are in phase
4. Satisfy Maxwell equations under two conditions
   a) field amplitudes/magnitudes are related $E = c\,B$
   b) wave speed in vacuum $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3\ 10^8\,\frac{m}{s}$
5. No medium is required for propagation
   In dielectric medium $v = \frac{1}{\sqrt{\varepsilon\,\mu}} < c$

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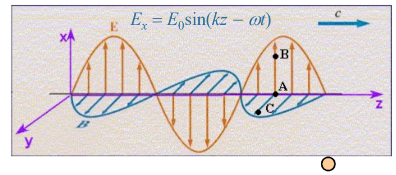
Chapter 32 – El-mg Waves

Key Properties of electro-magnetic Waves



An electro-magnetic wave is traveling in the z direction.

1. The points A,B, C have the same z coordinate. Compare the magnitude of the electric field at A and B:
   A) $E_A > E_B$
   B) $E_A = E_B$
   C) $E_A < E_B$

2. Consider a point (x,y,z) at time t when E_x is negative and has its maximum value.
What is B_y?
   A) B_y is positive and has its maximum value
   B) B_y is negative and has its maximum value
   C) B_y is zero
   D) We do not have enough information

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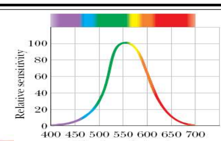
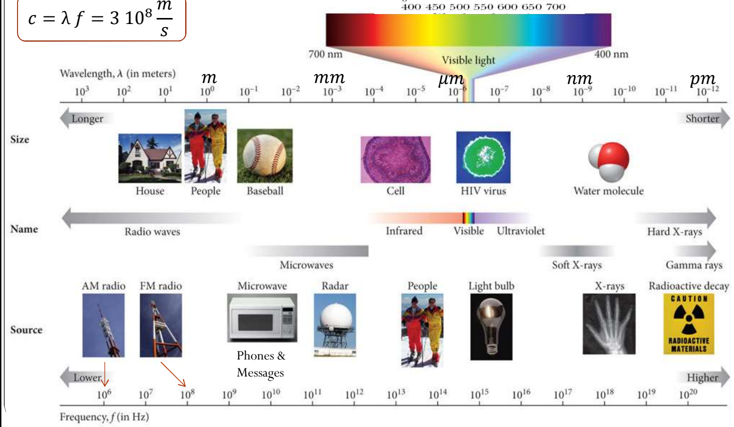
Chapter 32 – El-mg Waves

Electromagnetic Spectrum
Maxwell's Rainbow

$\boxed{c = \lambda\,f = 3\ 10^8\,\frac{m}{s}}$





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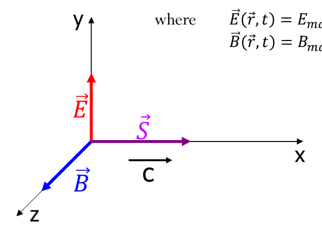
Chapter 32 – El-mg Waves

Energy Transport

El-mg waves carry energy and its rate of transport per unit area is described by:

Poynting vector $\boxed{\vec{S} = \frac{1}{\mu_0}\,\vec{E}\times\vec{B}}$ John Henry Poynting 1884

where $\vec{E}(\vec{r},t) = E_{max}\cos(\kappa x - \omega t)\ \hat{j}$
$\vec{B}(\vec{r},t) = B_{max}\cos(\kappa x - \omega t)\ \hat{k}$



Direction: direction of traveling wave and energy transport

Magnitude: $S(t) = \frac{1}{\mu_0}E\,B = \frac{1}{\mu_0}E\,\frac{E}{c} = \frac{E^2}{c\mu_0} = c\,\varepsilon_0\,E^2$

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Chapter 32 – El-mg Waves

Energy Transport

Relate magnitude of $\vec{S}$ with energy contained in the electric and magnetic field of the el.mg. wave

Energies densities are the same

$u_E = u_B$ using $\boxed{c\,\varepsilon_0 = {}^{1}/_{c\,\mu_0}}$

Prove $u_E = \frac{1}{2}\varepsilon_0\, E^2 \frac{c}{c} = \frac{1}{2c}\,\frac{1}{c\,\mu_0} E^2 = \frac{1}{2\,\mu_0}\left(\frac{E}{c}\right)^2 = \frac{1}{2\,\mu_0} B^2 = u_B$

Total energy density

$u = u_E + u_B = \varepsilon_0\, E^2$

$S = c\, u$

Take averages

$S_{avg} = c\, u_{ave} = c\,\varepsilon_0 (E^2)_{avg} = c\,\varepsilon_0 \frac{1}{2} E_m^2 = c\,\varepsilon_0\, E^2{}_{rms}$

Intensity of the wave

$I = \frac{P_{ave}}{A} = \frac{1}{A}\frac{u_{ave}\, vol}{t} = \frac{1}{A}\frac{u_{ave}(c\,t)\,A}{t} = c\, u_{ave}$

Conclusion

$\boxed{I = S_{avg} = \frac{1}{c\,\mu_0} E_{rms}{}^2}$

Unit: $[S] = \frac{W}{m^2}$

Example: average intensity of sunlight on Earth $\sim 1400\,\frac{W}{m^2} \sim 140\,\frac{mW}{cm^2}$

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Chapter 32 – El-mg Waves

Radiation Pressure

Radiation pressure of el-mag waves (p_r)

Assume el-mg wave incident on object and totally absorbed over Δt

U = energy carried by wave

$\Delta p = \frac{\Delta U}{c}$ (Maxwell) change in linear momentum of an object after absorbing ΔU

$p_r = \frac{F}{A} = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{1}{A}\frac{\frac{\Delta U}{c}}{\Delta t} = \frac{1}{c}\frac{1}{A}\frac{\Delta U}{\Delta t} = \frac{I}{c}$

Total absorption: $\boxed{p_r = \frac{I}{c}}$ since $\Delta p = p_i - 0 = p_i$

Assume el-mg wave incident on object and totally reflected over Δt

$\Delta p = \frac{2\,\Delta U}{c}$ (Maxwell) change in linear momentum of an object after reflecting ΔU

Perfect reflection $\boxed{p_r = \frac{2I}{c}}$ since $\Delta p = p_i - (-p_i) = 2p_i$

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Chapter 32 – El-mg Waves

Radiation Pressure

Examples:

Atmospheric pressure $p_0 = 101\,kPa \cong 10^5 Pa$

Pressure from sun $p_r = \frac{I}{c} = \frac{1400\,W/m^2}{3\,10^8\,m/s} = 4.7\,\mu Pa = 4.6\,10^{-1}\;\; p_0$

Laser pointer $p_r = \frac{2I}{c} = \frac{2\frac{P}{\pi r^2}}{c} = \frac{2 10^{-3}}{\pi\, 10^{-6} 3\, 10^8} = 2.1\,\mu Pa$

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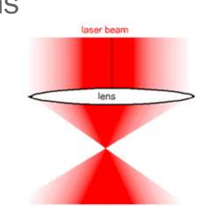
Chapter 32 – El-mg Waves

Radiation Pressure : Applications



Optical (laser) tweezers

intense laser focused on small area

1987 Arthur Ashkin - used in cell biology

2018 ½ Nobel Prize

Solar sail

IKARUS – Japan, 2010-2013

(Interplanetary Kite-craft Accelerated by Radiation Of the Sun)

https://www.youtube.com/watch?v=7Mb47w0vB04

$7.5\,\mu m$ over $200\,m^2$ ($14\,m$ square);); $2\,kg$

reflectance-adjustable LCD panel

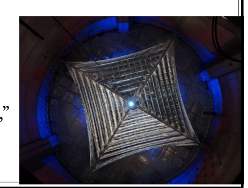
Sunjammer – NASA, Jan 2015 (planned)

$1200\,m^2$ ($35\,m$ square); $32\,kg$

cancelled >4y, >$21M

"It looks like it just wasn't big enough for us to afford it,"

Dana Rohrabacher (R-Calif.)



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