Electric and magnetic fields in el.mg. waves

1. YF.32.4. Consider each of the following electric and magnetic field orientations. In each case, draw the axes, the field vectors and how the direction of propagation of the wave.
a. $\vec{E} = E\,\hat{\imath}$ and $\vec{B} = -B\,\hat{\jmath}$
b. $\vec{E} = E\,\hat{\jmath}$ and $\vec{B} = B\,\hat{\imath}$
c. $\vec{E} = -E\,\hat{k}$ and $\vec{B} = -B\,\hat{\imath}$
d. $\vec{E} = E\,\hat{\imath}$ and $\vec{B} = -B\,\hat{k}$
Answer: $-\hat{k}$; $-\hat{k}$; $\hat{\jmath}$; $\hat{\jmath}$

2. YF.32.8. An electromagnetic wave of wavelength $435\ nm$ is traveling in vacuum in the negative z direction. The electric field has amplitude $2.7\ 10^{-3}\,\frac{V}{m}$ and is parallel to the x axis. What are:
a. The frequency of the wave?
b. The magnetic field amplitude?
c. Write the waveforms for the electric and magnetic field vectors.
Answer: $6.9\ 10^{14}\ Hz$; $9\ 10^{-12}\ T$

Wave intensity and Poynting vector

3. HR.33.73/81. The magnetic component of an electro-magnetic field in SI is:
$\vec{B} = 4\ 10^{-6}\sin(1.57\ 10^{7}y + \omega t)\,\hat{\imath}$
a. Find the wavelength and the frequency of the wave and specify in which region of the electromagnetic spectrum is this wave.
b. Write an expression for the electric field component of the wave, including the value for ω and the unit vector specifying its direction.
c. Find the intensity of the wave and write an expression for the Poynting vector, including the value for ω and the unit vector specifying its direction.
Answer: $4\ 10^{-7}m$; $7.5\ 10^{14}\ Hz$; $1.19\ kW/m^2$

4. HR.p899. When you look ay the North Star (Polaris), you intercept light from a star at a distance of $431\ ly$, where $1\ ly = 9.46\ 10^{15}\ m$, and emitting energy at a rate of $2.2\ 10^{3}$ times that of our Sun, where $P_{Sun} = 3.9\ 10^{26}\ W$. Neglecting any atmospheric absorption, find the rms values of the electric and magnetic fields when the starlight reaches you.
Answer: $4.1\ 10^{-12}\ T$

5. BW.31.41. A continuous-wave argon-ion laser beam has an average power of $10\ W$ and a beam diameter of $1\ mm$. Assume the intensity is the same throughout the cross section of the beam.
a. Calculate the intensity of the laser beam and compare with the average intensity of the sunlight at Earth's surface ($1400\,\frac{W}{m^2}$).
b. Find the rms electric field in the electric field.
c. Find the time average of the Poynting vector.
d. If the wavelength of the laser beam is $514.5\ nm$ in vacuum, write an expression for the instantaneous Poynting vector so that it is zero at $t = 0$ and $x = 0$.
Answer: 10^{4} ; $6.93\ 10^{4}\,\frac{V}{m}$; $1.27\ 10^{7}\,\frac{W}{m^2}$

Radiation Pressure

6. BW.31.47. A laser that produces a spot of light that is $1\ mm$ in diameter is shone perpendicular on the center of a thin, perfectly reflecting circular aluminum plate mounted vertically on a flat piece of cork that floats on the surface of the water in a large beaker. The mass of this "sailboat" is $0.1\ g$. The "sailboat" travels $2\ mm$ in $63\ s$. Assuming that the laser power is constant and neglecting air resistance and water viscosity, what is the power of the laser?
Answer: $15\ mW$

7. YF.32.54. A solar sail-craft uses a large, low mass sail and the energy and momentum of sunlight for propulsion.
a. Should the sail be absorbing or reflecting? Why?
b. If the total power output of the Sun is $3.9\ 10^{26}\ W$, how large a sail is necessary to propel a 10,000 kg spacecraft against the gravitational force of the Sun? Express your answer in km^2. Explain why your answer is independent of the distance from the Sun.
Answer: $6.42\ km^2$