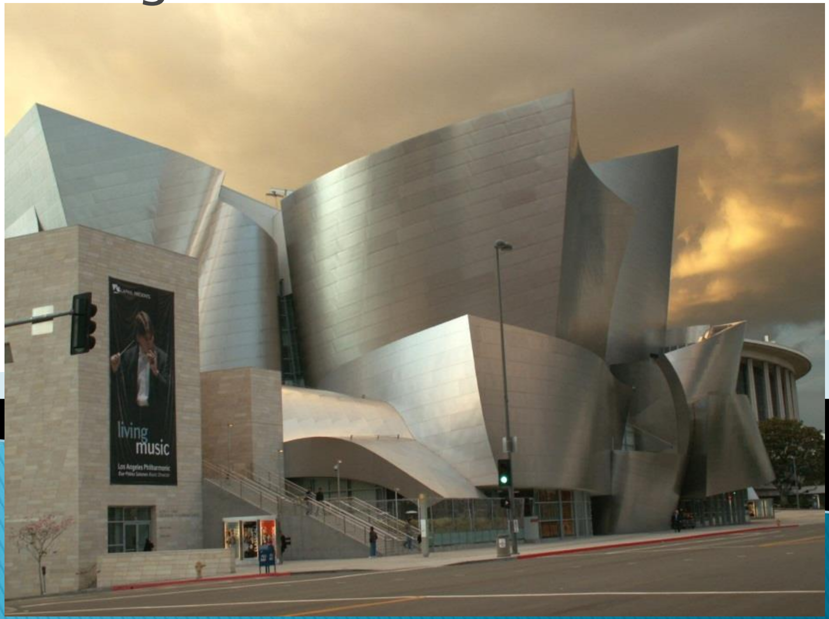
Chapter 16
Sound and Science

Acoustic engineering
    concert halls, speaker systems, freeway noise
Aviation engineering
    shock waves, airport noise
Physiology
    speech, speech impairment, snoring
Medical research
    use heart and lung noise
Military , paleontology, music, etc…



Walt Disney Concert Hall in LA was designed by Frank Gehry to be one of the most acoustically accurate concert halls in the world.

Chapter 16
Sound

Summary

Sound as longitudinal waves

Speed of sound, Reflection

Sound Intensity

Dynamic range

Sound Interference

Doppler Effect and Applications

Sound Resonance

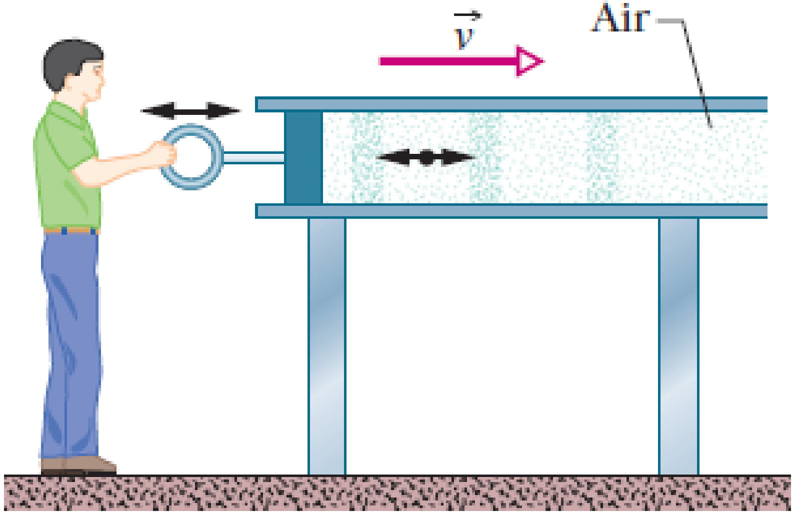
Musical instruments

Sound as Longitudinal Waves

Sound = pressure variation that propagates through a medium

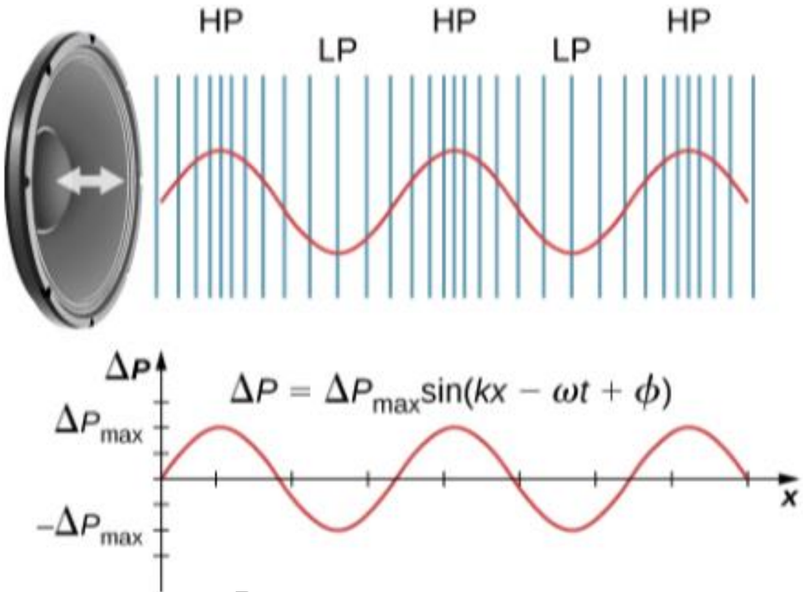
- source = emitter, transmitter, etc. (loudspeaker, etc.)
- medium to propagate
  air, liquids, solids (train track)
- detector, receiver (eardrum, etc.) is set in vibration

Longitudinal waves (in fluids):

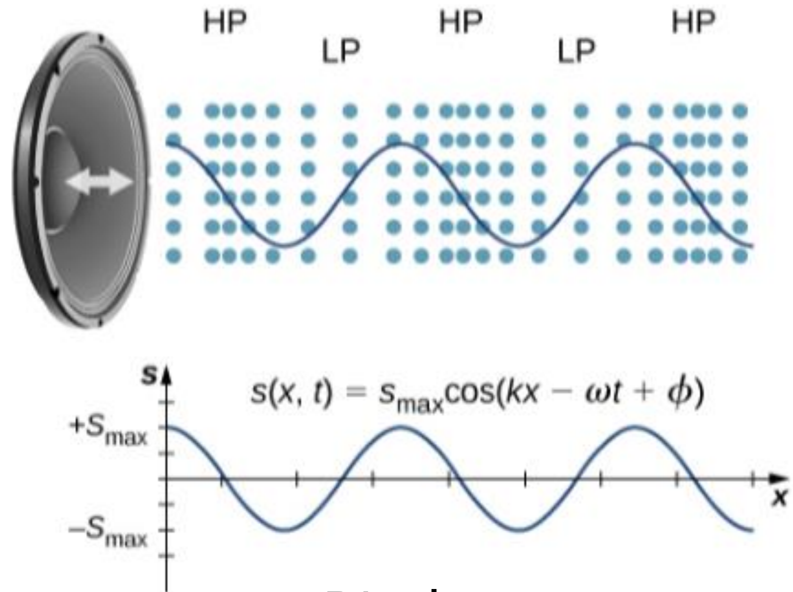


Chapter 16 – Sound

Sound as Longitudinal Waves



Pressure



Displacement

Notes:

- Pressure and displacement are $\pi/2$ out of phase:

  $\Delta p(x,t) = -B \frac{\partial s(x,t)}{\partial x}$

  largest displacement at lowest pressure fluctuation

  $s(x,t) = 0$ for molecules in equilibrium position – at highest compression and rarefaction

- Intensity /amplitude decrease with distance from source

Chapter 16 – Sound
Speed of Sound

Any wave

$$v = \lambda f$$

Waves on String

$v = \sqrt{\frac{F_T}{\mu}}$ where $\mu = linear\ mass\ density$ – inertial property

$F_T = tension\ in\ string$ – elastic property

Sound in Solids

$v = \sqrt{\frac{Y}{\rho}}$ where $\rho = density\ of\ medium$

$Y = Young's\ modulus, modulus\ of\ elesticity$

$(elongation\ due\ to\ external\ pressure)$

Sound in Fluids (liquid, gas)

$v = \sqrt{\frac{B}{\rho}}$ where $\rho = density\ of\ medium$

$B = bulk\ modulus$

$(volume\ change\ due\ to\ external\ pressure)$

Compare

$v_{air} < v_l < v_s$ since $\rho_g < \rho_l < \rho_s$

$B_{fl} \ll Y_s$

Chapter 16 – Sound
Speed of Sound (2)

Speed of sound in air

$v = \sqrt{\frac{\gamma R T}{M}}$ where γ =adiabatic index, R =ideal gas ct., T = absolute temperature, M = molecular mass

$v_{air} = 343 \frac{m}{s}$ at $p = 1\, atm; t = 20°\text{C}$ (standard conditions)

Compare $v_{diamond} = 12{,}000 \frac{m}{s}$

Temperature dependence

$v(T) = 331 + 0.6\, T(°\text{C})$

Lightning-and-thunder rules

metric: $3s$ rule → $d = 1\, km$

British: $5s$ rule → $d = 1\, mile = 1.6\, km$

Sound reflection – determine dist. by measuring time

Applications–ultrasound imaging $\nu = 2 - 15\, MHz$

dolphin, rat navigation $\nu = 14 - 100\, kHz$

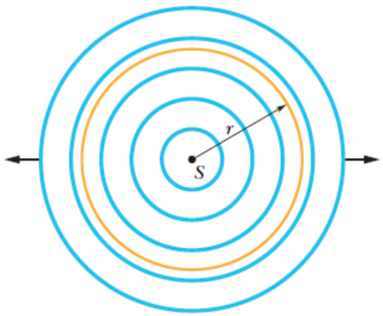
Chapter 16 – Sound
Sound Intensity

Wave intensity

$$I = \frac{power}{area} = \frac{emergy/time}{area}$$ (rate of energy transferred by wave per unit area)

Spherical wave transmitted isotropically

$$I = \frac{P}{4\,\pi\,r^2} \sim \frac{A^2}{r^2}$$



Perceived as $Loudness$

Human sensitivity

$$I_0 = 10^{-12}\,\frac{W}{m^2} \rightarrow whisper$$

$$I\ = 10\,\frac{W}{m^2} \quad \rightarrow\ pain\ threshold$$

$\longrightarrow$ $14\ orders\ magnitude$

Use logarithmic scale

$$y\ = \log x$$

$$y' = \log x' = \log(10x) = log10 + \log x = 1 + y$$

$$concl.:\quad 10x \rightarrow y + 1$$

Chapter 16 – Sound

Sound Intensity (2)

Decibel scale (dB) $I \rightarrow \beta$

Sound level on decibel scale $\boxed{\beta = 10\ log\frac{I}{I_0}}$ where $I_0 = 10^{-12}\frac{W}{m^2}$ $(whisper)$

Relative intensity $\Delta\beta = \beta_2 - \beta_1 = 10\log\frac{I_2}{I_1}$

Dynamic range – measures relative level loudest to quietest source, loudspeakers, etc.

Human sensitivity /hearing threshold depends on

a. frequency (*pitch*)

hearing range $20 - 20k\ Hz$

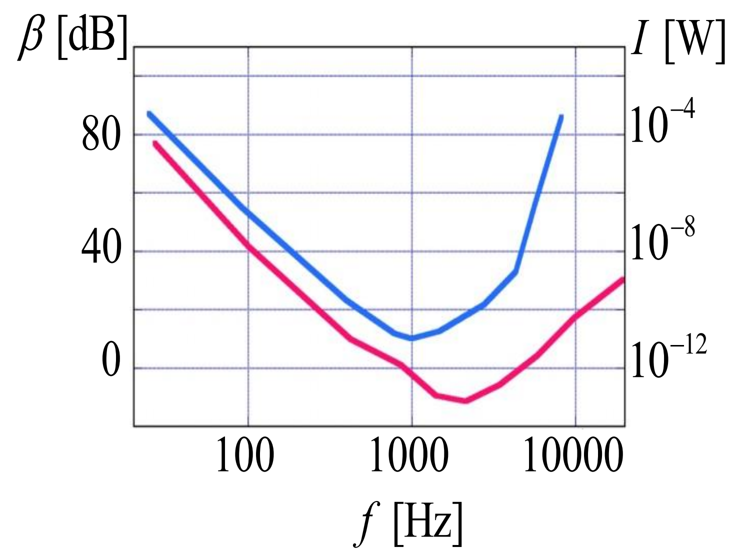
best hearing $\sim 1k\ Hz$

b. age

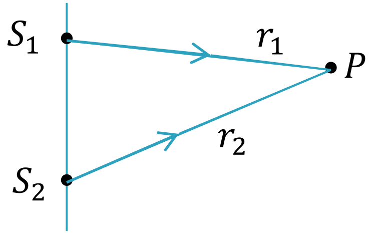
red – teens

blue – 60s

c. long-term exposure over 120 dB



Sound Interference in Space



Wave interference – depends on phase difference

$$y(x,t) = 2A\cos\tfrac{1}{2}\varphi\ \sin(kx - \omega t + \tfrac{1}{2}\varphi)$$

Coherent sources – same frequency (ν) and in phase

Phase difference (φ) at P depends on *path length difference* (Δr)

Relate $\Phi, \Delta r$

$$\frac{\varphi}{2\pi} = \frac{\Delta r}{\lambda} \longrightarrow \Delta r = \frac{\varphi}{2\pi}\lambda$$

Fully constructive $\quad \varphi = 2\pi n \longrightarrow \Delta r = n\lambda$

Fully destructive $\quad \varphi = (2n+1)\pi \longrightarrow \Delta r = (2n+1)\dfrac{\lambda}{2}$

$n = 0, \pm 1, \pm 2, \ldots$

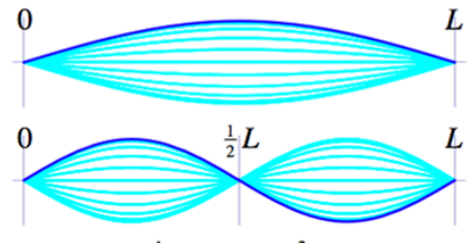
Intermediate $\quad \varphi = \forall$

Chapter 16 – Sound

Sound Resonance – Musical Instruments

String instruments

$$L = n\,\frac{\lambda}{2} \qquad where \quad n = 1, 2, 3, \ldots$$



$$f_n = \frac{v}{\lambda} = n\,\frac{v}{2L} \quad \text{where} \quad v = \sqrt{\frac{T}{\mu}}$$

Tuning (adjust F_T, i.e. change v) vs. *Playing* (change L)

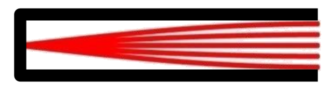
Sound Resonance – Musical Instruments

Wind instruments

similar to string: large, sustained amplitude (IF f (or λ) is matched to L)

unlike string (reflection on fixed ends)

half-open (clarinet, trumpet)



open-open (flute)

closed end → node (no vibration) and highest pressure change $\Delta\varphi = 0$
open end → anti-node (free vibration) and lowest pressure change $\Delta\varphi = \pi$

Playing: change L by – covering holes (flute)
piston-like (trombone)

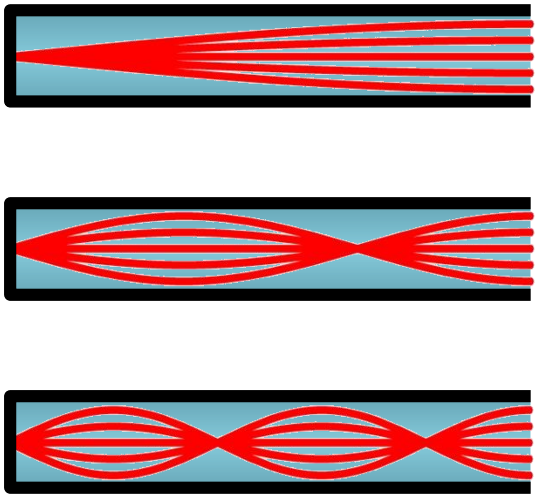
Chapter 16 – Sound
Sound Resonance – Music (2)
Half-open pipes

Resonance condition

$$L = (2n+1)\frac{\lambda}{4} \quad where \quad n = 0, 1, 2, \dots \quad (1)$$

OR

$$\boxed{L = (2n-1)\frac{\lambda}{4}} \quad where \quad n = 1, 2, 3, \dots \quad (2)$$



Note: harmonic number n in (2) corresponds to the number of *nodes*

Resonance wavelength $\boxed{\lambda_n = \frac{4L}{2n-1}}$

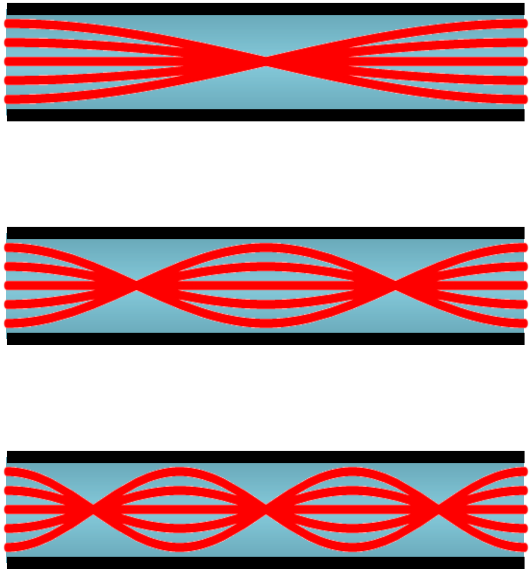
Resonance frequency $\boxed{f_n = \frac{v}{\lambda_n} = (2n-1)\frac{v}{4L}}$

Fundamental mode (1st harmonic)

$$n = 1$$
$$\lambda_1 = 4L$$
$$f_1 = \frac{v}{4L} \quad \rightarrow \quad \boxed{f_n = (2\mathrm{n}-1)\ f_1}$$

Note: in both cases the range of frequency is determined by length of instrument

Sound Resonance – Music (2)
Open pipes



Resonance condition

$$\boxed{L = n\frac{\lambda}{2}} \quad where \quad n = 1, 2, 3, \ldots$$

Note: n corresponds to the number of *nodes*

Resonance wavelength $\boxed{\lambda_n = \frac{2L}{n}}$

Resonance frequency $\boxed{f_n = \frac{v}{\lambda_n} = n\,\frac{v}{2L}}$

Fundamental mode
(1st harmonic)

$$n = 1$$
$$\lambda_1 = 2L$$
$$f_1 = \frac{v}{2L} \quad \rightarrow \quad \boxed{f_n = n\, f_1}$$

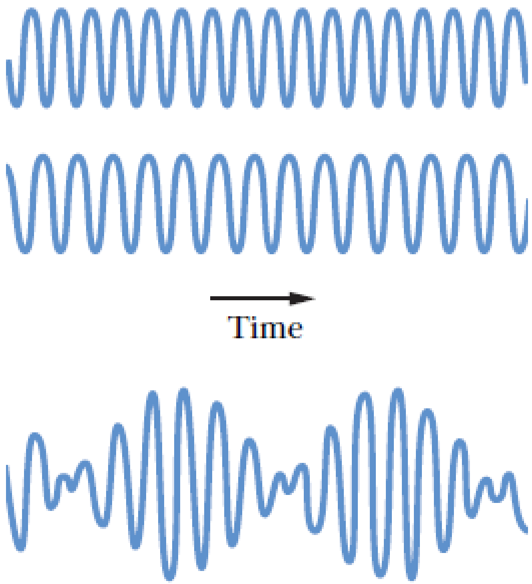
Demo: http://ocw.mit.edu/courses/physics/8-03-physics-iii-vibrations-and-waves-fall-2004/video-lectures/lecture-8/ (min 34)

Chapter 16 – Sound

Sound Interference in Time – Beat Phenomenon

Slightly different frequencies f_1, f_2
At point $x_0 = 0$

$$y_1 = A\,cos(k_1 x_0 - \omega_1 t) \rightarrow \quad A\,cos\,\omega_1 t$$
$$y_2 = A\,cos(k_2 x_0 - \omega_2 t) \rightarrow \quad A\,cos\omega_2 t$$



$$y(x_0, \mathrm{t}) = y_1 + y_2 \;\; = A\,cos\omega_1 t + A\,cos\omega_2 t =$$
$$= 2A\,cos\tfrac{1}{2}(\,\omega_1 - \omega_2)t\;\; cos\tfrac{1}{2}(\,\omega_1 + \omega_2)t$$

With $\omega = 2\pi f$ *and* $(\omega_1 - \omega_2)$ small

$f_b = \;|f_1 - f_2|$ *beat frequency*

$\bar{f} = \frac{1}{2}(f_1 + f_2)$ *average frequency*

$$y(x_0, \mathrm{t}) = 2A\cos\pi\, f_b t\;\; cos\,2\pi\,\bar{f}\,t$$

slowly varying (modulated) amplitude

rapidly changing function

Used in tuning instruments (12Hz)

Chapter 16 – Sound
Doppler Shift

General Doppler Effect

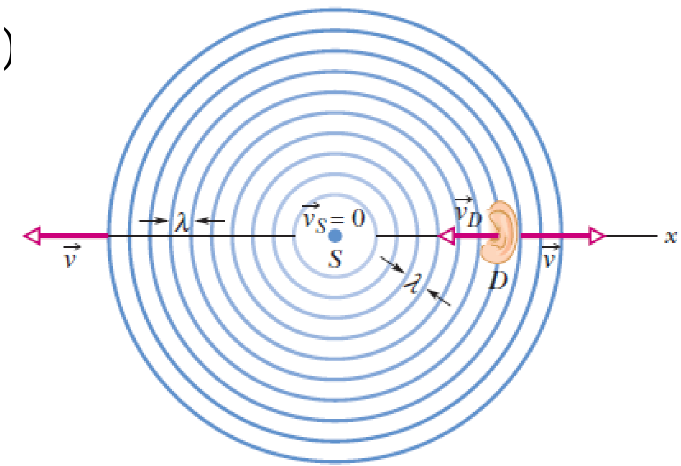
Frequency changes due to motion source and/or detector

Doppler shift

$$\Delta f = |\, f(source) - f'(detected)| \quad IF \quad v_S \neq 0 \; \frac{and}{or} \; v_D \neq 0$$

1842 proposed J.C. Doppler (Austrian)

1845 tested Buys Ballot (Dutch)



Stationary source and detector ($v_S = 0, v_D = 0$)

Know: $v = \lambda f = \frac{\lambda}{T}$ $\quad$ *where* $\quad v = speed\ of\ sound$

$f = \frac{1}{T}$ *frequency from source*

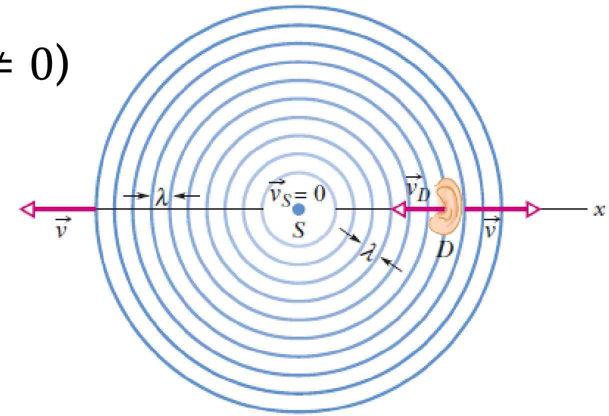
Chapter 16 – Sound
Doppler Shift (2)

Stationary source ($v_S = 0$), moving detector ($v_D \neq 0$)

Detector towards source:

$$f' = \frac{v + v_D}{\lambda} = f\,\frac{v + v_D}{v} > f$$

$$\boxed{f' = f\,\frac{v \pm v_D}{v}}$$



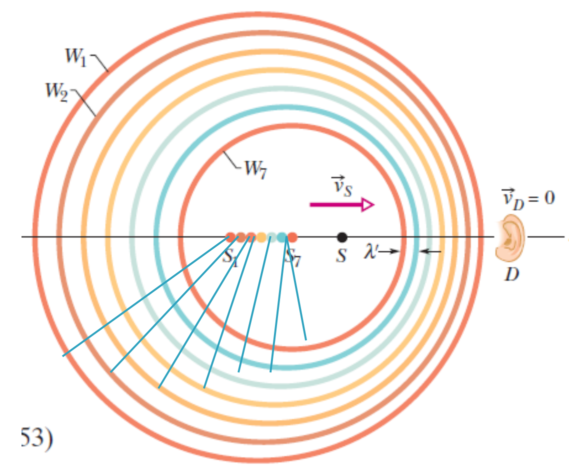
Moving source ($v_S \neq 0$), stationary detector ($v_D = 0$)

(1) $v_S < v$

Source toward detector:

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - v_S T} = \frac{v}{\frac{v}{f} - \frac{v_S}{f}} = f\,\frac{v}{v - v_S} > f$$

$$\boxed{f' = f\,\frac{v}{v \pm v_S}} \qquad for \qquad v_S < v$$



Doppler Shift (3)

General Doppler Effect

$$f' = f\,\frac{v \pm v_D}{v \pm v_S}$$ *where* f = source frequency

v = speed of sound, D = detector, S = source

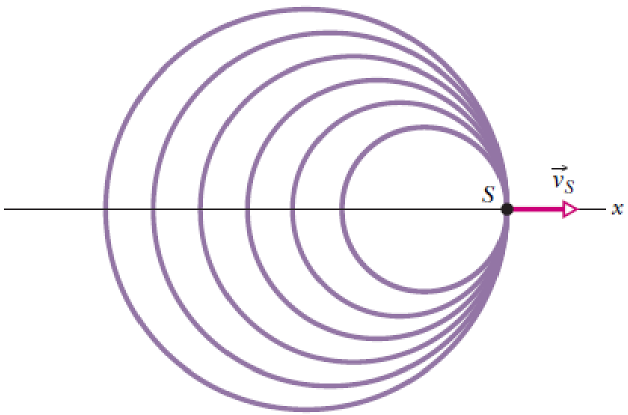
Choice of sign such that

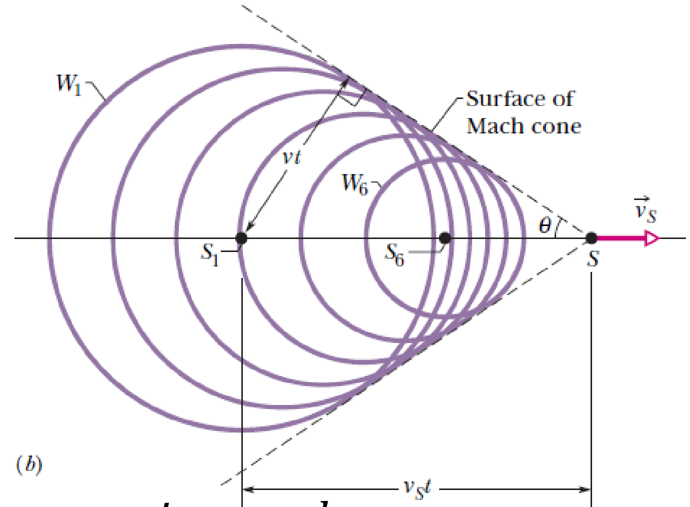
towards ⇔ shift UP $f' > f$

away ⇔ shift DOWN $f' < f$

Detector moves away from source: $-$
Detector moves toward source: $+$
Source moves away from observer: $+$
Source moves toward observer: $-$

Doppler Effect (4) – Shock Waves





Source moving
(2) $v_S = v$

$f' \rightarrow \infty$

(3) $v_S > v$ *supersonic speed*

Mach cone –
constructive interference of waves arriving simultaneously
= shock wave

Mach Angle $\sin\theta_M = \frac{v}{v_S} = \frac{1}{M}$

Mach number $\mathrm{M} = \frac{v_S}{v}$

E.g.: aircraft, bullets, whip crack (sound)
Cherenkov radiation (el.mg.)