ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Ordinary Level

PURE MATHEMATICS   **4027/2**
PAPER 2

SPECIMEN PAPER   2 hours 30 minutes

Candidates answer on the question paper.
Additional materials: Data booklet
                          Mathematical tables/ electronic calculator

Allow candidates 5 minutes to count pages before the examination.

This booklet should not be punched or stapled and pages should not be removed.

TIME   2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces at the top of this page and your Centre number and Candidate number on the top right corner of every page of this paper.

Answer **all** questions in **Section A** and any **four** from **Section B**.
Check that all the pages are in the booklet and ask the invigilator for a replacement if there are duplicate or missing pages.
Write your answers in the spaces provided on the question paper using **black** or **blue** pens.
If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
Decimal answers which are not exact should be given correct to three significant figures unless stated otherwise. Answers in degrees should be given correct to one decimal place.

INFORMATION FOR CANDIDATES
The number of marks is given in brackets [ ] at the end of each question or part question.
Electronic calculators or Mathematical tables may be used to evaluate explicit numerical expressions.

_____

This specimen paper consists of 18 printed pages and 2 blank pages.

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Section A [52 marks]

*Answer **all** questions in this Section.*

1 **(a)** Functions f and g are defined by

$$f : x \to \frac{1}{3x - 1} \ , \quad x \neq \frac{1}{3}$$

$$g : x \to x^2 - 1.$$

Find an expression for

(i) $fg(x)$,

Answer _____ [2]

(ii) $f^{-1}(x)$,

Answer _____ [2]

(iii) $gg(x)$.

Answer _____ [2]

_____

(b) Solve the inequality;

$$\frac{(x-2)(x+3)}{(x-4)} > 0$$

Answer _____ [3]

2 (a) Solve the simultaneous equations;

$$4x - 3y = 15$$
$$8x^2 - 27y = 45$$

Answer _____ [4]

(b) Prove the identity $\dfrac{(\sin x + \cos x)^2}{\sin x \cos x} \equiv 2 + \sec x \operatorname{cosec} x.$

Answer

[3]

4

(c) Solve the equation $\sin^2\theta + 2\sin\theta\cos\theta = 0$ for $0° \leq \theta \leq 360°$.

Answer _____ [3]

3 **(a)** **(i)** Expand $\left(1 - \dfrac{x}{3}\right)^7$ in ascending powers of x, up to and including

the term in x^3.

Answer _____ [3]

(ii) State the set of values of x for which the expansion is valid.

Answer _____ [2]

(b) Find the range of values of p for which the equation $x^2 - 2px + 3p = 0$ has real roots.

Answer _____ [4]

4   **(a)**   The table shows experimental values of two quantities x and y which are connected by the equation $y = kb^x$.

x	1	2	3	4
y	30	75	190	470

(i)   Draw an approximate straight line graph on the grid on page 6

Answer on graph   [3]

(ii)   Use the graph to estimate the values of k and b.

Answer $k =$ _____

$b =$ _____   [3]

_____

4   **(a)**   **(i)**
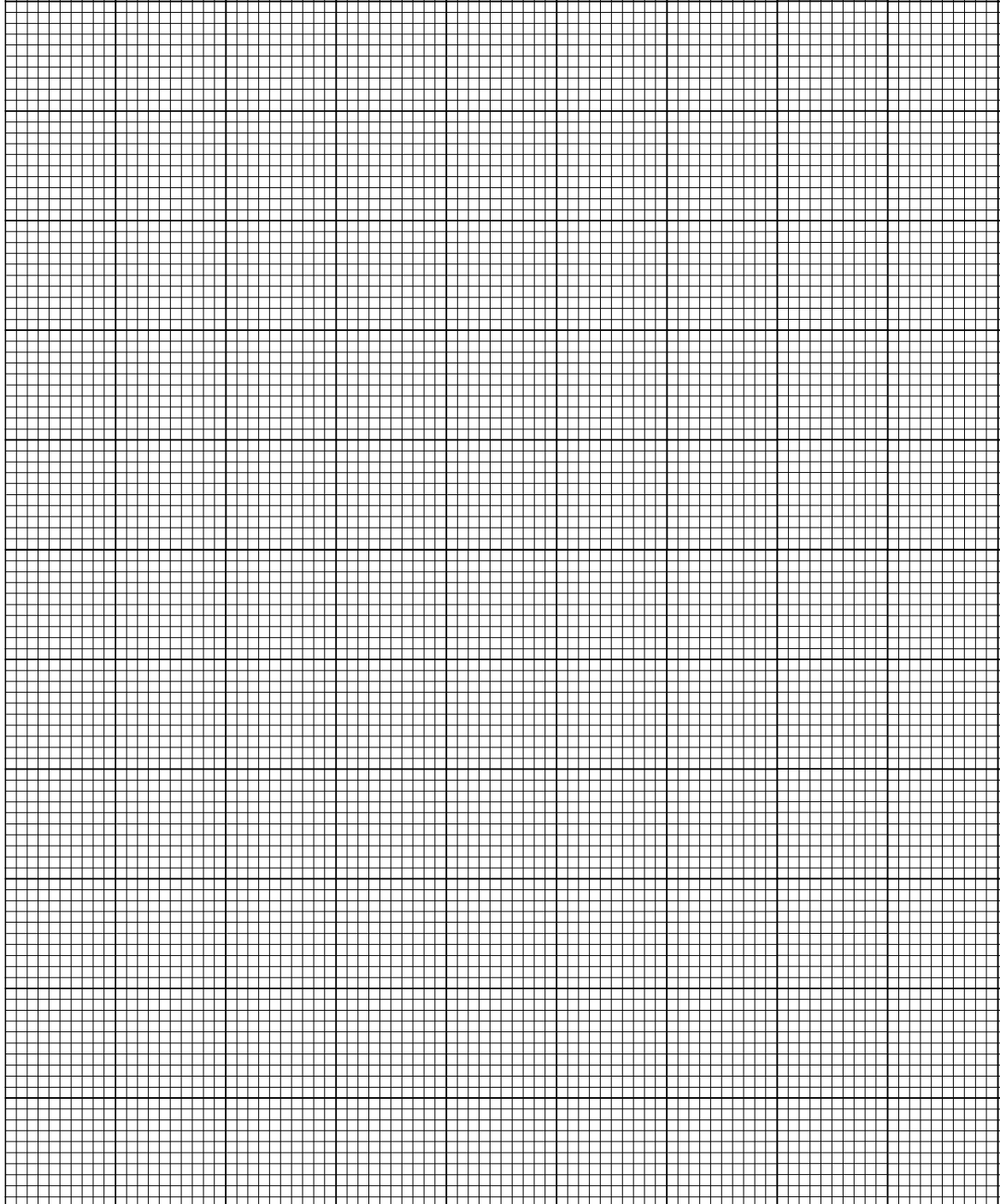
[3]

4 **(b)** Two points **A** and **B** have coordinates $(-2; -3)$ and $(7; 4)$ respectively. The perpendicular bisector of the line joining **A** and **B** meets the y-axis at **C**. Calculate the coordinates of point **C**.

Answer _____ [4]

5 **(a)** A particle P travels in a straight line from a fixed point O. Its velocity, v m/s is given by $v = t^2 - 10t + 24$, where t is the time in seconds after leaving O.

Find the

(i) values of t for which P is instantaneously at rest,

Answer _____ [2]

(ii) distance OP when $t = 3$,

Answer _____ [2]

(iii) range of values of t for which the acceleration is negative.

Answer _____ [2]

[Turn over

8

(b)   **O, P** and **Q** are three points such that $\overrightarrow{OP} = 3i + 4j$ and $\overrightarrow{OQ} = 7i + lj$.

(i)   Find $\overrightarrow{OP} \bullet \overrightarrow{PQ}$ in terms of l.

Answer _____   [2]

(ii)   Given that $O\hat{P}Q = 90^{o}$, calculate the value of l.

Answer _____   [3]

(iii)   Show that triangle **OPQ** is isosceles.

Answer _____   [3]

Section B [48 marks]

*Answer any **four** questions in this section.*

Each question in this section carries 12 marks

6   **(a)**   The remainder when $x^3 - 5x + a$ is divided by $x + 3$ is twice the remainder when it is divided by $x - 2$. Find the value of a.

Answer _____ [4]

(b)   An arithmetic progression has 14 terms. The sum of the odd terms (i.e. 1st , 3rd, 5th etc.) is 40 and the sum of the even terms (i.e. 2nd, 4th, 6th etc.) is 161

Find the

   (i)   first term and the common difference of the progression,

Answer _____ [6]

   (ii) 14th term.

Answer _____ [2]

_____

[Turn over

7   **(a)**   **OAB** is a sector of a circle with centre **O** and radius 6 cm. The mid-point of **OA** and **OB** are **C** and **D** respectively. **CD** is a straight line.
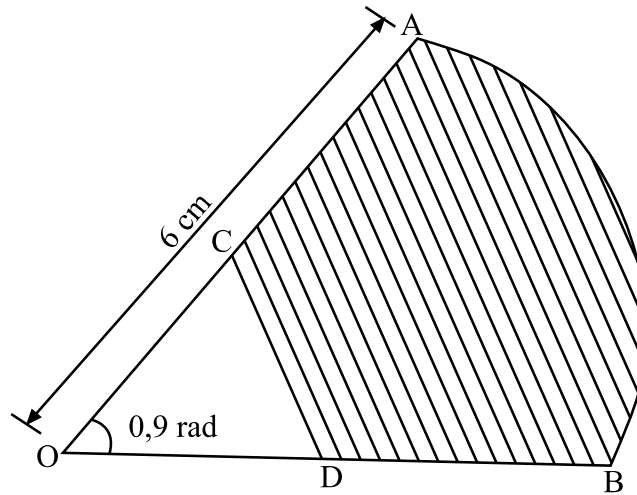
Given that angle **AOB** = 0,9 radians, calculate the:

(i)   length of the arc AB,

Answer _____ [2]

(ii)   area of the shaded region.

Answer _____ [4]

(b) A rectangular block is such that the sides of its base are of length
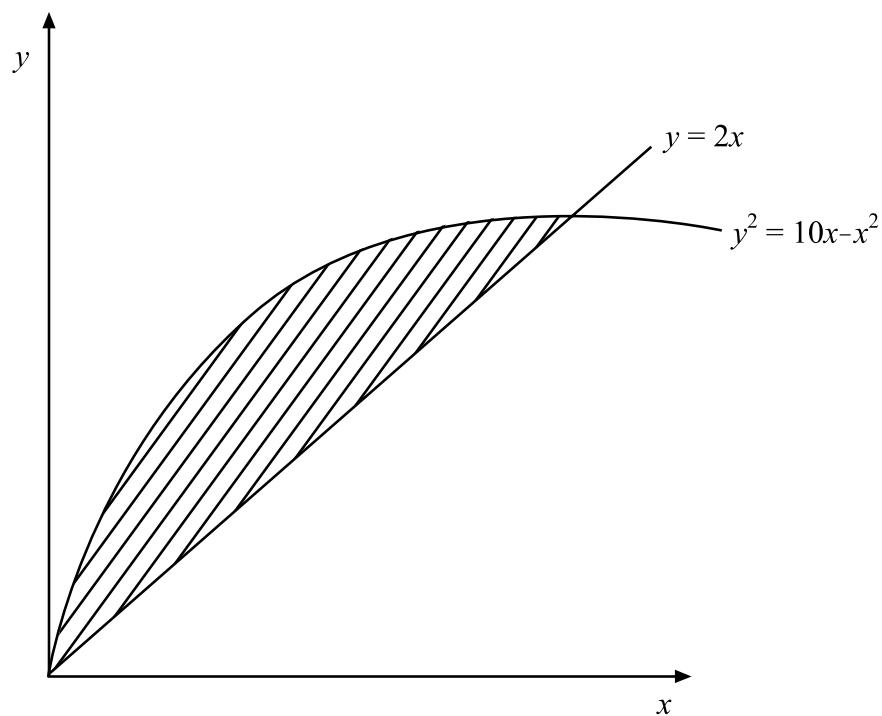x cm and $3x$ cm. The sum of the lengths of all its edges is 200 cm.

Find the

(i) volume, v cm^3 in terms of x,

Answer _____ [3]

(ii) value of x for which v has a maximum value.

Answer _____ [3]

[Turn over

8   **(a)**   The diagram shows part of the curve $y^2 = 10x - x^2$ and the straight
line $y = 2x$.

Calculate the

(i)   area of the shaded part,

Answer _____   [4]

(ii)   volume of the solid generated when the shaded region is rotated through 360° about the *x*-axis.

Answer _____ [4]

(b)   Find the gradient of the curve $x^2 + y^2 + 5x - 4y + 7 = 0$ at the point $(-4; 3)$.

Answer _____ [4]

9   **(i)**   Expand $(9 - 4x)^{-\frac{1}{2}}$ in ascending powers of *x* up to and including the term in x^2.

Answer _____ [4]

_____

(ii)   State the range of values of x for which the expansion is valid.

Answer _____ [2]

(iii)   By using $x = \dfrac{1}{9}$ in the expansion, find an approximation for $\sqrt{77}$

leaving the answer in the form $\dfrac{p}{q}$.

Answer _____ [6]

10   **(a)**   Evaluate $\displaystyle\int_{0}^{3} \dfrac{12}{2x+3}\, dx$

Answer _____ [3]

(b)   A curve is defined parametrically by the equations.

$$x \;=\; t + \frac{1}{t}, \qquad y \;=\; t - \frac{1}{t}$$

Find,

(i) $\dfrac{dy}{dx}$ in terms of t,

Answer _____   [3]

(ii)   the values of t at the points on the curve where $\dfrac{dy}{dx} = 3$,

Answer _____   [2]

(iii)   the equation of the tangent to the curve at the point $\left(\dfrac{5}{2}, \dfrac{3}{2} \right)$.

Answer _____   [4]

_____

[Turn over

11   **(a)**   Solve the equation $4^{x+1} = 16^{2x}$.

Answer _____   [3]

(b)   **(i)**   Express $2 \cos \theta - 5 \sin \theta$ in the form $r\cos(q + a)$.

Answer _____   [3]

(ii)   Hence solve the equation

$2 \cos \theta - 5 \sin \theta = 2$ for $0° < \theta < 360°$.

Answer _____   [3]

_____

(c) Solve the equation $|2x - 3| = 3x$.

Answer _____ [3]

12 **(a)** The time of swing T, of a simple pendulum of length x units is given by $T = k\sqrt{x}$, where k is a constant. Find in terms of dx and x an expression for the approximate value of

(i) dT,

Answer _____ [4]

(ii) $\dfrac{dT}{T}$.

Answer _____ [3]

_____

[Turn over

(b) A geometric progression is such that the 4th term is twice the 3rd term. Given that the 7th term is 512, find

(i) the first term and the common ratio,

Answer _____ [3]

(ii) the sum of the first 10 terms of the progression.

Answer _____ [2]

19
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20
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