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Observatinal Astrophysics Laboratory (OAL)

Lab Manual

Lab (3)

Contents

<u>Grading sheet</u>

<u>Student Name</u> --------------------------------

<u>Student ID</u> --------------------------------

Item	*Grade*	*Maximum*
Experiment VI		30
Latex submission		**10**
Total		**40**

TA Signature:

Experiment (VI)

Observation of the the nearst open star cluster, the Hyades

(I) *Objectives:*

The purpose of this experiment is to determine the distance to members of an open stellar cluster by two methods. **The first** is trigonometric parallax, the same method by which the distances to the nearest stars have been measured for more than a century. **The second**, called *convergence-point* or *moving-cluster parallax*, is applicable only to a few clusters: those that occupy a significant fraction of the sky and are close enough to us that the proper motions of the individual stars can be measured with fair accuracy.

(II) *Introduction*

The proper motions of stars that belong to the same constellation are usually so different in magnitude and direction that it is obvious there is no physical connection between them. In certain cases, however, the motions of a group of stars are strikingly similar, and it is apparent that the stars in the group are physically related and move as a unit through space. Usually, the radial velocities of such stars are also nearly the same, and the spectra can show marked similarity. These criteria, similar proper motions, radial velocities and spectra, furnish a sound basis for grouping stars in clusters, and for distinguishing cluster members from other, foreground and background stars along the same line of sight.

If a cluster occupies a significant fraction of the sky, the proper motion of the individual stars, when plotted on a star chart, are found to be not exactly parallel, but instead to converge toward or diverge from a point. The convergence or divergence of the proper motions is an effect of perspective, like the divergence of meteor-shower trails from a radiant point and the convergence of railroad tracks in the distance. When the motions converge, the stars recede from us, and the observed radial velocities are positive (+); when the motions diverge, the radial velocities are negative (-). If the *convergence point*, or point in the sky toward which the cluster stars appear to move, can be found, and if the radial velocity of one of the stars is known, it turns out – by an ingenious bit of geometry which we will describe below – to be possible to determine the distance to this star relatively accurately. If the cluster is much further away than its diameter, this will do as an estimate of the distance to all the stars in the cluster. This procedure is moving-cluster parallax, is based on Zeilik and Gregory's *Introductory Astronomy and Astrophysics* on pages 383-385 and in Shu's *Physical Universe* on pages 169-170. There are only a few open clusters on which the technique can be used, but these clusters provide us with the means to determine the distances of all other open clusters, so the method is of crucial importance in the determination of the cosmic distance scale.

The first example found of the apparent convergence of proper motions of a cluster of stars was the Hyades, in Taurus (Figure1). From the proper motions of 41 of the Hyades and the radial velocities of three, Lewis Boss (1908, *Astron. J.* **26**, 31) determined the distance to the cluster. This is the nearest moderately rich open cluster; nowadays we know of about 400 stars that are virtually certain to be members, and many more that might be. A decade ago, the European Space Agency's astrometric satellite, *Hipparcos*, measured the proper motion *and* the trigonometric parallax of 197 confirmed Hyades for which radial velocities are known from ground-based observations (Perryman *et al.* 1998, *Astron Astrophys.* **331**, 81). These results allow a substantial improvement in the accuracy of distances to the Hyades, and the first extensive comparison of moving-cluster and trigonometric parallax.

Figure 1: Wide-angle photograph of the Hyades (left) and Pleiades (upper right), by Hermann Gumpp. The very bright red star at lower left is Aldebaran (α Tau).

(III) *Moving-cluster parallax*

There are two main steps in the application of moving-cluster parallax: determination of the convergence point of the cluster stars' proper motions, and combination of proper motion and radial velocity observations to obtain the distance and velocity vector of each cluster star.

(III.a) *Determination of the Convergence Point*

In the Table below, are proper motion, radial velocity and trigonometric parallax data for eight of the Hyades in the *Hipparcos* catalogue, chosen to be spread widely across the face of the cluster. Proper motion is there expressed in orthogonal components, $\mu_\alpha \cos\delta$ and μ_δ , of proper motion in right ascension α and declination δ . Note the similarity among the magnitudes of the proper motion and radial velocity among these stars.

In the following you will plot the positions of these stars in at least two epochs, in order to find the direction toward which the motions converge. You may do this by hand on normal graph paper, or better yet by use of the spreadsheet (Origin Lab/Microsoft Excel).

Table 1: Some data on Hyades members (Perryman et al. 1998, ESA 1997).

Hipparcos catalogue number	α, J1991.25 (h m s)	δ, J1991.25 (° ′ ″)	$\mu_\alpha \cos\delta$ (10^{-3} arcsec year^{-1})	μ_δ (10^{-3} arcsec year^{-1})	v_r (km sec^{-1})	p (10^{-3} arcsec)
18170	03 53 09.96	+17 19 37.8	143.97±1.06	-29.93±0.84	35.0±2.5	24.14±0.90
19554	04 11 20.20	+05 31 22.9	146.86±1.00	5.00±0.85	36.6±1.2	25.89±0.95
20261	04 20 36.24	+15 05 43.8	108.79±0.95	-20.67±0.82	36.2±1.2	21.20±0.99
20901	04 28 50.10	+13 02 51.5	105.17±0.84	-15.08±0.63	39.9±4.1	20.33±0.84
21589	04 38 09.40	+12 30 39.1	101.73±0.96	-14.90±0.76	44.7±5.0	21.79±0.79
22157	04 46 01.70	+11 42 20.2	67.48±1.11	-7.09±0.79	43.0±1.0	12.24±0.86
23497	05 03 05.70	+21 35 24.2	68.94±0.75	-40.85±0.52	38.0±1.7	20.01±0.91
24019	05 09 45.06	+28 01 50.2	55.86±1.33	-60.57±0.77	44.9±0.5	18.28±1.30

Procedure:

The following procedure applies in detail to the use of Excel, so it is a good exercise to you map this to OriginLab instructios.

1. Arrange the epoch 1991.25 right ascension and declination of each star at the tops of adjacent columns in a fresh worksheet of your Excel workbook. You may wish to deal with α and δ in degrees and decimal fractions thereof, rather than hours, minutes, seconds, arcminutes and arcseconds.

2. In the next row, enter formulas to calculate the positions 100,000 years later, from the initial α and δ, and the corresponding proper motions (μ_α cosδ and μ_δ). Note that Excel expects the arguments of trigonometric functions to be expressed in radians. If in your formulas you refer to the proper motion components in the original worksheet, make sure you use absolute references (e.g. B5 rather than B5), but use relative references (e.g. A2 rather than A2) for the initial positions from the cells above.

3. Select this new row of formulas and duplicate it for many rows downward in the worksheet, using the Fill Down command (under the Edit menu, or control-d on the keyboard).

4. Each corresponding pair of columns comprises the path of a star through the sky. Plot all of these lines in a single X-Y scatter graph. (Ask Excel to connect the points with lines.) Make sure you have filled down enough rows that all of these lines intersect each other.

5. You will note that the intersections among these lines occurs in a rather small area on the plot; the convergence region is thus defined. Draw an "error box" around the densest concentration of intersections, and from its center and dimensions derive values of right ascension and declination and α_C and δ_C of the convergence point, and uncertainties $\Delta\alpha_C$ and $\Delta\delta_C$ in these values. Compare your result to that obtained by Perryman et al. (1998) from a statistical analysis of all of the Hipparcos Hyades:

$$\alpha_C = 96.6^0$$
$$\delta_C = +5.8^0$$

(III.b) *Determination of the velocity vectors and distances to the Hyades*

Figure 2 represents one of the cluster stars S moving with velocity v with respect to the observer O. We assume that the observer O is at rest with respect to the Sun.

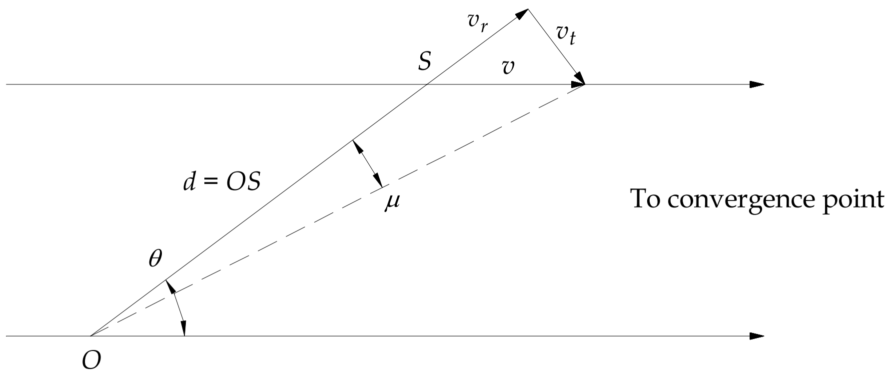

Figure 2: Represents one of the cluster stars S moving with velocity v with respect to the observer O. We assume that the observer O is at rest with respect to the Sun.

With v_t and v_r as the tangential and radial velocity components for the star, then we see in Figure 2 that

$$\tan\theta = \frac{v_t}{v_r}$$

and

$$\mu = v_t / 4.74047 d = \text{total proper motion}$$
$$= \sqrt{(\mu_\alpha \cos\delta)^2 + \mu_\delta^2} \quad ,$$

where the velocities are entered in km/sec, and the distance in parsecs, with the proper motion coming out in arcsec/year (The constant 4.74047 is simply the astronomical unit, expressed in km. year sec^{-1}). θ is the angle, as seen from O, between the directions to the star and to the vertex. From all these quantities, the distance d to the star can be calculated:

$$d = \frac{v_t}{4.74047\mu} = \frac{v_r \tan\theta}{4.74047\mu} \quad ,$$

where once again the distance is in parsecs, the velocities in km/sec and the proper motion in arcsec/year. The angle θ can be found by spherical trigonometry since the convergence-point coordinates are known:

$$\cos\theta = \sin\delta\sin\delta_C + \cos\delta\cos\delta_C\cos(\alpha - \alpha_C) \quad .$$

Procedure:

1. Using the above discussion and your results from section III.a, find θ, and then v and d, for each of the stars in Table 1. Again, you will find it most convenient to carry out these calculations, and those that follow, with formulas in an Excel worksheet.

2. **Error Propagation Formula:**

   For a function f(x,y,z), the error in f is $\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2 + \left(\frac{\partial f}{\partial z}\Delta z\right)^2}$

   where Δx, Δy, and Δz are the errors in x, y, and z, respectively.

   From the uncertainties listed in Table1, and from the size of the "error box" around the convergence point, calculate the uncertainties Δθ (in radians), Δv_r (in km/sec) and Δμ (in arcsec/year) in θ, radial velocity and proper motion for each star. Then calculate the contribution of each to the uncertainty in the distance to each star (in parsecs), from

$$(\Delta d)_v = \left|\frac{\partial d}{\partial v_r}\Delta v_r\right| = \frac{\tan\theta}{4.74047\,\mu}\Delta v_r$$

$$(\Delta d)_\theta = \left|\frac{\partial d}{\partial \theta}\Delta \theta\right| = \frac{v_r}{4.74047\,\mu}\left(1+\tan^2\theta\right)\Delta\theta$$

$$(\Delta d)_\mu = \left|\frac{\partial d}{\partial \mu}\Delta \mu\right| = \frac{v_r\,\tan\theta}{4.74047\,\mu^2}\quad\Delta\mu \quad .$$

   You may assume that uncertainties in orthogonal components of a variable are independent and related to the uncertainty in the variable by the square root of the sum of their squares; for instance,

$$\Delta\theta = \sqrt{\left(\Delta\alpha_C\right)^2 + \left(\Delta\delta_C\right)^2}\ .$$

   Which variable contributes most to the uncertainty in the stellar distances?

3. Estimate crudely the uncertainty in distance to each star, Δd, by taking the square root of the sum of the squares of the quantities calculated with the Equations in step 2. Calculate also the percentage uncertainty in distance, 100 Δd/d, for each star, and the average value of 100 Δd/d for our eight stars.

(III.c) *Comparison of moving-cluster-parallax and trigonometric-parallax distances*

The rightmost column of Table1 contains measurements of the trigonometric parallax *p* for our eight Hyades, from which the distance *d* and uncertainty Δ*d* can be computed directly:

$$d = \frac{1}{p}$$

$$|\Delta d| = \frac{\Delta p}{p^2} = d\frac{\Delta p}{p}$$

Procedure:

1. Calculate d and Δd for each of the stars. Again, you will find it most convenient to do this, and the following, with formulas in an Excel worksheet.

2. Calculate the corresponding percentage uncertainty in distance, 100Δd/d, for each star, and the average value of 100Δd/d for the eight stars.

3. Plot the distance determined from trigonometric parallax against distance determined from moving-cluster parallax, in an X-Y scatter graph. Ask Excel to plot error bars in both variables, using the average percentage uncertainties in the two methods as calculated above, but ask it not to connect the data points with lines. Overlay on this plot a line through the origin with a 45^0 slope.

4. On the basis of this plot, discuss the relative accuracy of the two methods of stellar distance determination, both in terms of the size of the uncertainties and the significance of any systematic departure of the plotted points from the 45^0 line.

5. Under the assumption that our eight stars sample the volume of the cluster uniformly, calculate from the appropriate averages the distance and speed of the center of mass of the Hyades. Compare your result to that derived by Perryman et al. (1998), using all of the Hipparcos Hyades that lie within the central 10 parsecs of the cluster:

$$v = 45.93 \pm 0.23 \text{ km sec}^{-1} \quad ,$$
$$d = 46.34 \pm 0.27 \text{ parsecs} \quad .$$

Comments: