

221 me 2021 P/LD F/VI
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Algorithm #HW 3

1)

Function Find_largest (arr, start, end)

if start == end

return start

else

mid = (start + end) / 2

left_index = Find_largest (arr, start, mid)

right_index = Find_largest (arr, mid + 1, end)

if arr[left_index] > arr[right_index]

return left_index

else

return right_index

$$T(n) = T(n/2) + C$$

$$T(n) = O(n^0 \cdot \log(n)) = O(\log n)$$

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(2) ~~min~~ min ~~max~~ max pile

2)

Function min_max(arr, start, end)

if start == end
return(arr[start], arr[end])

elseif end - start == 1
if arr[start] < arr[end]
return(arr[start], arr[end])

else
return(arr[end], arr[start])

else

mid = (start + end) / 2

left_min, left_max = min_max(arr, start, mid)

right_min, right_max = min_max(arr, mid + 1, end)

return(min(left_min, right_min), max(left_max, right_max))

$$T(n) = 2T(n/2) + c$$

$$T(n) = O(n^{\log_b a} \cdot \log(n)) = O(n \log n)$$

(2)

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4) Yes it's a stable sorting algorithm. A sorting algorithm is said to be stable if it maintains the relative order of equal elements in the input array.

6) Solve the Following

a) $x(n) \leq x(n-1) + 5$ for $n > 1$, $x(1) = 0$

$$x(n-1) \leq x(n-2) + 5$$

$$x(n) \leq x(n-2) + 10$$

$$x(n-2) \leq x(n-3) + 5$$

$$x(n) \leq x(n-3) + 15$$

$$x(n) \leq x(n-k) + 5k \quad n-1 = k$$

$$= x(1) + 5(n-1)$$

$$\leq 5n - 5 \Rightarrow O(n)$$

b) $x(n) \leq 3x(n-1)$ for $n > 1$, $x(1) = 4$

$$x(n) \leq 3^2 x(n-2)$$

$$x(n) \leq 3^3 x(n-3)$$

$$x(n) \leq 3^k x(n-k)$$

$$n-k \leq 4$$

$$n-4 \leq k$$

$$x(n) \leq 3^{n-4} x(1) = 4 \cdot 3^n \Rightarrow O(3^n)$$

(4) Time Complexity

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c) $x(n) = x(n-1) + n$ for $n > 0$, $x(0) = 0$

$$x(n) = x(n-1) + n$$

$$x(n) = x(n-2) + (n-1) + n$$

$$x(n) = x(n-3) + (n-2) + (n-1) + n$$

$$x(n) = n(n+1)/2 \Rightarrow \Theta(n^2)$$

d) $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$

$$n = 2^k$$

$$x(2^k) = x(2^{k-1}) + 2^k$$

$$\text{let } y(k) = x(2^k)$$

$$y(k) = y(k-1) + 2^k$$

$$= y(k-2) + 2^{k-1} + 2^k$$

$$= 2^{k-1} - 2$$

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$$a) T(n) = T(n-1) + 1$$

$$T(n-1) = T(n-2) + 1$$

$$T(n) = T(n-2) + 2$$

$$= T(1) + (n-1)$$

$$= n \Rightarrow \Theta(n)$$

B) Both have the same complexity

Q) A) It returns the smallest value in an array

$$B) T(n) = T(n-1) + 2$$

$$T(n-1) = T(n-2) + 2$$

$$T(n) = T(n-2) + 4$$

$$K = n-1$$

$$T(n) = T(n-K) + 2K$$

$$T(n) = T(1) + 2(n-1) = 2n-1 \Rightarrow \Theta(n)$$