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Date

No

1st m. 20210269: ID

Algorithm-HW(1)

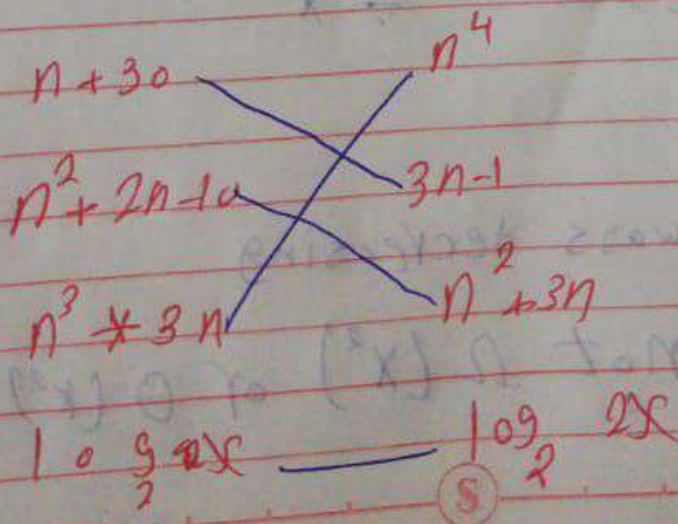
1) Which kind of growth best characterizes these functions?

- 1) $3n$ → Linear
- 2) $3n^2$ → Polynomial
- 3) 2^n → Exponential
- 4) $(3/2)^n$ → Exponential
- 5) 1000 → Constant
- 6) 1 → Constant
- 7) $(3/2)n$ → Linear
- 8) $8n^3$ → Polynomial

2) Rank from slowest → Fastest in growing

$n^2, 2^n, n, n^3, (3/2), 1$ → $1, n, n^2, n^3, (3/2)^n, 2^n$

3) Match each function with equivalent function



20210269

(2)

if $n \rightarrow \infty$ then P/L

5. Functions $\log n, \log_8 n$

a) $\log n$ is $O(\log_8 n)$

$$\log n \leq C \cdot \log_8 n$$

if

$$C=2, n=3$$

b) $\log n$ is $\Omega(\log_8 n)$

$$\log n \geq C \cdot \log_8 n \rightarrow \text{Cont exists}$$

both C are wrong

6. Determine whether each of these functions is $O(x)$, $\Omega(x^2)$ or $\Theta(x^2)$

a) $f(x) = 10$

$$10 \leq C \cdot x \rightarrow C=10, x \geq 1$$

$f(x)$ is $O(x)$

$$10 \geq C \cdot x^2 \div x^2$$

$$\frac{10}{x^2} \leq C$$

\rightarrow always decreasing

$f(x)$ is not $\Omega(x^2)$ or $\Theta(x^2)$

20210269

(3)

if $n \rightarrow \infty$ then

b) $f(x) = 3x + 7$

$$3x + 7 \leq C \cdot x \quad \div x$$

$$3 + \frac{7}{x} \leq C \rightarrow C = 10, x = 1$$

 $f(x)$ is $O(x)$

$$3x + 7 \not\geq C \cdot x^2 \quad \div x^2$$

$$\cancel{3} \rightarrow \boxed{\frac{3}{x} + \frac{7}{x^2}} \geq C$$

\rightarrow always decreasing
 $f(x)$ is not $\Omega(x^2)$ or $\Theta(x^2)$

c) $f(x) = 5 \log x$

$$5 \log x \leq C \cdot x \quad \div x$$

$$\cancel{5} \rightarrow \frac{5 \log x}{x} \leq C \rightarrow C = 1$$

$f(x)$ is $O(x)$ and not $\Omega(x^2)$

20210269

(4)

let $n \rightarrow \infty$ \Rightarrow LE P/LP

d) $F(x) = x^2 + x + 1$

$$x^2 + x + 1 \leq C \cdot x \quad \div x$$

$$x + 1 + \frac{1}{x} \leq C \rightarrow \text{Not true}$$

 $F(x)$ is not $O(x)$

$$x^2 + x + 1 \geq C \cdot x^2 \quad \div x^2$$

$$\cancel{1 + \frac{1}{x} + \frac{1}{x^2} \geq C} \quad \hookrightarrow C \leq 1, x \leq 2$$

$$x^2 + x + 1 \leq C \cdot x^2$$

$$C = 2, x = 3$$

 $F(x)$ is $\Theta(x^2)$

e) $F(x) = 17x + 11$

$$17x + 11 \leq C \cdot x \rightarrow C = 28, x = 1$$

 $F(x)$ is $O(x)$

$$17x + 11 \geq C \cdot x^2 \quad \div x^2$$

$$\frac{17}{x} + \frac{11}{x^2} \geq C$$

 \hookrightarrow always decreasing

(5)

20210269

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~~P111~~ f) $F(x) \leq x^2 + 1000$

$$x^2 + 1000 \leq C \cdot x \quad \div x$$

$$x + \frac{1000}{x} \leq C \quad C \leq 1001, x \leq 1$$

$F(x)$ is $O(x)$

$$x^2 + 1000 \leq C \cdot x^2 \quad \div x^2$$

$$1 + \frac{1000}{x^2} \leq C$$

\downarrow always decreasing

g) $F(x) \leq x \log x$

$$x \log x \leq C \cdot x \quad \div x$$

$$\log x \leq C \quad C \leq 2, x \geq 2$$

$F(x)$ is $O(x)$

$$x \log x \geq C \cdot x^2 \quad \div x^2$$

$$\frac{\log x}{x} \geq C$$

\downarrow always decreasing

2021-2022

(6)

let $n \rightarrow \infty$ and $n \in \mathbb{N}$

$$h) f(x) = 2^x \Rightarrow$$

$$2^x \leq c \cdot x \rightarrow \text{const exist}$$

$$2^x \gg c \cdot x^2 \rightarrow c = 1$$

$$2^x \not\ll c \cdot x^2 \rightarrow c = 2$$

$$f(x) \text{ is } \Theta(x)^2$$

$$7) a) n \log(n^2+1) + n^2 \log n$$

$$\downarrow \quad \downarrow$$

$$O(n \log n^2) + O(n^2 \log n)$$

$$\downarrow$$

$$O(2n \log n^2) + O(n^2 \log n)$$

$$\text{is } O(n^2 \log n)$$

$$b) (n \log n + 1)^2 + (\log n + 1)(n^2 + 1)$$

$$(n \log n)^2 + 2n \log n + n^2 \log n + \log n + 1 + 2$$

$$\downarrow$$

$$O((n \log n)^2) + O(n^2 \log n)$$

$$\hookrightarrow O((n \log n)^2)$$

(7)

202/0264

2 + 1 = 3

$$c) (n^3 + n^2 \log n)(\log n + 1) + (17 \log n + 19)(n^3 + 2)$$

$$= [n^3 \log n + n^3 + n^2 (\log n)^2 + n^2 \log n] + [17n^3 \log n + 34 \log n + 19n^3 + 38]$$

$$O(n^3 \log n) + O(17n^3 \log n)$$

$$\hookrightarrow O(17n^3 \log n)$$

$$d) (2^n + n^2)(n^3 + 3^n)$$

$$= 2^n n^3 + 6^n + n^5 + n^2 3^n$$

$$= 2^n n^3 + n^2 3^n + 6^n + n^5$$

$$= 32 \quad 36 \quad 36 \quad 32$$

when $n \leq 2$

$$= 216 \quad 242 \quad 216 \quad 243$$

when $n \leq 3$

$$= 1024 \quad 1296 \quad 1296 \quad 1024$$

when $n \leq 4$

$$3^n n^2 \geq 2^n n^3, \quad \cancel{6^n} \geq n^5$$

$$= 3^n n^2 \geq 6^n$$

$$\text{is } O(3^n n^2)$$

(8)

20210269

time complexity

8)

a) $n(n+1)$ and $2000n^2$

\downarrow
 n^2+1 $2000n^2 \rightarrow$ same

b) $100n^2$ and $0.01n^3 \rightarrow$ lower

c) $\log_2 n$ and $\ln n \rightarrow$ same

d) $\log_2^2 n$ and $\log_2 n^2$

\downarrow
 $2 \log_2 n \rightarrow$ higher

e) 2^{n-1} and $2^n \rightarrow$ same

f) $(n-1)!$ and $n! \rightarrow$ ~~same~~ lower

Date 20/02/20

Let $a \leq b$ & $c \leq d$

Q1)

~~Let~~

For $i \leftarrow 0$ to $M-1$ do
For $j \leftarrow 0$ to $N-1$ do

if $(a[i] == b[j])$ {
Print $a[i]$;}

10) Find asc (31415, 14142)

1 \rightarrow (14142, 3131)

2 \rightarrow (3131, 1618)

3 \rightarrow (1618, 1513)

4 \rightarrow (1513, 105)

5 \rightarrow (105, 43)

6 \rightarrow (43, 19)

7 \rightarrow (19, 5)

8 \rightarrow (5, 4)

9 \rightarrow (4, 1)

10 \rightarrow (1, 0) = 1

20210269

(10)

11)

Left side > Right side

1. Take The goat and drop at the other side
2. Come back and take the Cabbage.
3. Drop it and take the goat back
4. take the wolf and drop the goat
5. Drop the ~~goat~~ wolf with the Cabbage
6. go back take the goat Cabbage

12) as $VP(P-a)(P-b)(P-c)$, where $P = (a+b+c)/2$

14)

for $i \leftarrow 0$ to $n-1$ do

for $j \leftarrow 1$ to $n-1$ do

if $|a[i] - a[j]| < \delta_{min}$

$\delta_{min} \leftarrow |a[i] - a[j]|$

return δ_{min}