A Closed Form Solution For ALOHA Frame Length Optimizing Multiple Collision Recovery Coefficients Reading Efficiency

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***Abstract*—Minimizing the reading time of the tag population is a critical issue in RFID systems. The usual approach to reduce the reading time is to select the frame size attaining the highest throughput per frame. Previous studies have focused on the frame length calculations using the conventional Framed Slotted ALOHA (FSA) algorithms. In such systems, only the answer of a single tag is considered as a successful slot, and if multiple tags respond simultaneously, a collision occurs. Then all the replied tags are discarded. However, modern systems have the capability of recovering this collision and convert the collided slot into a successful slot. Recent studies focused on calculating the optimal frame length taking into consideration the collision recovery probability. However, these studies have assumed a constant collision recovery probability coefficient, i.e. the probability to recover one tag from i collided tags per slot is constant regardless the value of i. In this work we propose a novel closed form solution for the optimal FSA frame length which considers the differences in the collision recovery probabilities. The values of the collision recovery coefficients are extracted from the physical layer parameters. Timing comparisons are presented in simulation results to show the mean reduction in reading time using the proposed frame length compared to the other proposals.**

I. Introduction

Radio Frequency Identification (RFID) is an identification technology that wirelessly transmits the identity of a tag that is attached to an object or a person. Recently the number of applications that use RFID technology has increased, and the reading speed became one of the most critical issues in these applications. In RFID systems, the tags typically share a common communication channel. Thus, there is a certain probability of tags collisions, i.e. multiple tags answer simultaneously. This collision probability naturally increases in dense networks. According to EPCglobal class 1 gen 2 standards [1], the reader is responsible of coordinating the network and has to avoid tags collisions using specific anti-collision algorithms. The conventional anti-collision algorithm is the Framed Slotted ALOHA (FSA) algorithm [2], which is only a Medium Access Control (MAC) layer protocol. In this algorithm, only the single tag reply (successful slot) can be decoded and identified. Therefore, the conventional definition of the expected reading efficiency η_{conv} is equivalent to the probability of success $P(1)$, which is expressed as:

$$\eta_{conv} = P(1) = \frac{n}{L}\left(1 - \frac{1}{L}\right)^{n-1}, \tag{1}$$

where n presents the number of tags in the reading area, and L is the frame length.

The main goal is to find the optimal frame length L, which maximizes the reading efficiency η_{conv}. Based on (1), the reading efficiency η_{conv} is maximized to $\eta_{conv(max)} = 36\%$ when $L = n$ as shown in [2].

Some research groups (e.g. [3] and [4]) have concentrated more on the effect of the collision resolving probability and the optimum value of the FSA frame length. They have proposed another equation for the reading efficiency:

$$\eta_{PHY[3]} = P(1) + \alpha \cdot P(c), \tag{2}$$

where $P(c)$ is the probability of collision, and α is the average collision resolving probability coefficient. In this efficiency equation, the RFID reader can convert part of the collided slots into successful slots. Based on this efficiency, the authors have calculated a closed form solution for the optimum frame length:

$$L_{opt[3]} = \alpha + (1 - \alpha) \cdot n. \tag{3}$$

The authors here have assumed unlimited and equal collision resolving probability coefficients. For example, the probability to resolve two collided tags is equal to the probability to resolve ten collided tags. However, this is not an accurate assumption, because the collision recovery probabilities decrease when the number of collided tags increases, as we will see in the next sections. Moreover, there is a practical saturation limit for the collision recovery capability.

Another research group [5] considered the limited RFID reader capability of collision resolving. They have proposed a limited reading efficiency expressed as:

$$\eta_{PHY[5]} = \sum_{i=1}^{M} P(i), \tag{4}$$

where $P(i) = \binom{n}{i}\left(\frac{1}{L}\right)^{i}\left(1 - \frac{1}{L}\right)^{n-i}$, and M represents the number of collided tags that the reader is capable to recover. The authors assumed that the probability to recover one tag from i collided tags is equal to 100% independent of i, which is not a practical assumption. In reality, the probability to recover a single tag from i collided tags varies based on the receiver type, the number of collided tags i, and the Signal to Noise Ratio (SNR). According to the efficiency equation in (4), the authors proposed fixed values for the optimum FSA frame length based on the collision resolving capability of the RFID reader. They calculated these values numerically by searching for the frame length which maximizes their proposed reading efficiency.

In this paper we propose a new reading efficiency metric called Multiple Collision Recovery Coefficients Reading Efficiency η_{MCRC}, which includes a unique collision resolving coefficient for each number of collided tags. Then, we calculate these coefficients based on a simple RFID reader model to show how the proposed system could be applied on real-life applications. Hence, we propose a novel closed form solution for the optimum FSA frame length which maximize the proposed efficiency metric. This paper is organized as follows: Section II presents the system model under multiple collision recovery coefficients in addition to an example to calculate the collision recovery coefficients. Next, section III presents the proposed closed form for the optimal frame length based on the collision recovery coefficients. Finally, section IV gives numerical results on the improvements of the new optimization criterion, before we conclude in section V.

II. Frame Length Optimization Under Multiple Collision Recovery Coefficients

In this section, we will present a new FSA efficiency metric called Multiple Collision Recovery Coefficients Reading Efficiency η_{MCRC}. Afterwards, a closed form solution for the new optimum frame length L_{MCRC} under multiple collision recovery coefficients environment will be presented.

A. Proposed Multiple Collision Recovery Coefficients Reading Efficiency

The main contribution in this efficiency is that it contains a unique collision recovery coefficient α_i for each probability of collision $P_{col.}(i)$. These new coefficients indicate the ability of the reader to recover one tag from i collided tags. The proposed reading efficiency η_{MCRC} is expressed as:

$$\eta_{MCRC} = P(1) + \sum_{i=2}^{n} \alpha_i P_{col.}(i). \tag{5}$$

Figure 1 presents the distribution of the average collision probability in a frame of length $0.5 \cdot n \leq L \leq 2 \cdot n$ uniformly, which is the practical range of the frame length in RFID systems [6]. According to figure 1, the probability that the collided slot comes from two, three, or four collided tags is equal to $P_{col.}(2) + P_{col.}(3) + P_{col.}(4) \simeq 96\%$, and for the remaining tag collisions $\sum_{i=5}^{n} P_{col.}(i) \simeq 4\%$. Moreover, the values of the collision recovery coefficients α_i, $i > 4$ are practically very small.

Therefore, the proposed η_{MCRC} for the practical RFID environment can be expressed as shown:

$$\eta_{MCRC} = P(1) + \alpha_2 P_{col.}(2) + \alpha_3 P_{col.}(3) + \alpha_4 P_{col.}(4), \tag{6}$$

where α_2, α_3, and α_2 are respectively the second, third, and fourth collision recovery coefficients.

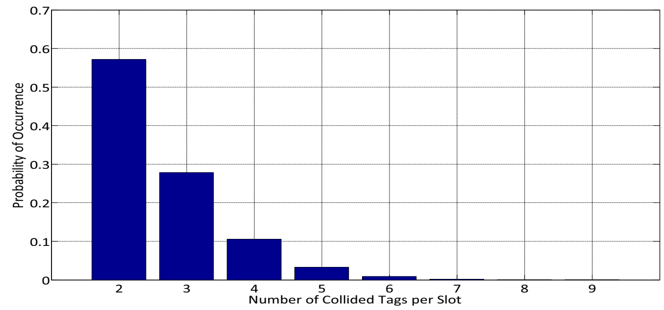


Figure 1: Distribution of collision probability for a collided slot in FSA, under condition of $\frac{n}{2} \leq L \leq 2n$

B. Proposed Closed Form Frame Length

The next step is to derive a closed form solution for the new optimum frame length L_{MCRC} under multiple collision recovery coefficients environment. L_{MCRC} can be optimized by finding the value of L which maximizes η_{MCRC}. According to [7], if $L \gg 1$, and $n \gg i$, we can assume a Poisson distribution:

$$P(i) \simeq \frac{1}{i!} \cdot \beta^{-i} \cdot e^{-\frac{1}{\beta}}, \tag{7}$$

where $\beta = \frac{L}{n}$. After substituting by (7) in (6) we get:

$$\eta_{MCRC} = e^{-\frac{1}{\beta}} \cdot \left(\beta^{-1} + \frac{\alpha_2}{2}\beta^{-2} + \frac{\alpha_3}{6}\beta^{-3} + \frac{\alpha_4}{24}\beta^{-4} \right). \tag{8}$$

Now we have to find the value of β which maximizes η_{MCRC}. This is achieved by differentiating the reading efficiency in (8) with respect to β and equate the result to zero. After differentiating, the equation can be simplified as:

$$-e^{-\frac{1}{\beta}} \cdot \Bigg(\beta^{-2} + \beta^{-3}(\alpha_2 - 1) + \frac{\beta^{-4}}{2}(\alpha_3 - \alpha_2) + \frac{\beta^{-5}}{6}(\alpha_4 - \alpha 3) - \frac{\beta^{-6} . \alpha_4}{24} \Bigg) = 0. \tag{9}$$

After multiplying the equation by $-e^{\frac{1}{\beta}} \cdot \beta^6$, the equation finally results in:

$$\underbrace{1}_{a}\beta^4 + \underbrace{(\alpha_2 - 1)}_{b}\beta^3 + \underbrace{\frac{(\alpha_3 - \alpha_2)}{2}}_{c}\beta^2 + \underbrace{\frac{(\alpha_4 - \alpha_3)}{6}}_{d}\beta - \underbrace{\frac{\alpha_4}{24}}_{e} = 0. \tag{10}$$

Equation (10) has four roots [8]:

$$\beta_{1,2} = -\frac{b}{4a} - S \pm 0.5 \sqrt{\underbrace{-4S^2 - 2P + \frac{q}{S}}_{X}} \tag{11}$$

$$\beta_{3,4} = -\frac{b}{4a} + S \pm 0.5 \sqrt{\underbrace{-4S^2 - 2P - \frac{q}{S}}_{Y}},$$

where $P = \frac{8ac - 3b^2}{8a^2}$, $q = \frac{b^3 - 4abc + 8a^2 d}{8a^3}$

and, $S = 0.5\sqrt{-\frac{2}{3}P + \frac{1}{3a}\left(Q + \frac{\triangle_0}{Q}\right)}$, $\quad Q = \sqrt[3]{\frac{\triangle_1 + \sqrt{\triangle_1^2 - 4\triangle_0^3}}{2}}$.

with, $\triangle_0 = c^2 - 3bd + 12ae$, $\quad \triangle_1 = 2c^3 - 9bcd + 27ad^2 - 72ace$

According to practical ranges for the collision recovery coefficients, α_i , $0 \leq \alpha_i \leq 1$, and $\alpha_2 \geq \alpha_3 \geq \alpha_4$. Therefore, we can proof that the signs of the polynomial coefficients are constant and not changing in all ranges of α_i and can be as follows: $a = +$, $b = -$, $c = -$, $d = -$, and $e = -$.

According to Descartes' rules of sign [9], there is only one valid real positive solution for the equation. Now, we will identify which solution is the valid one. There are two possibilities for the solutions:

1. One positive real solution and the remaining three solutions are negative. In this case, all solutions are real and we need just to identify what is the root which has the largest values from the four solutions. According to (11), the value of the square roots $\sqrt{X}$ and $\sqrt{Y}$ are positive real, because we do not have complex solutions. This means, $\beta_1 > \beta_2$ and also $\beta_3 > \beta_4$. So, the solution will be either β_1 or β_3. Moreover, the value of S should be also positive real, and q has always negative real value. so $\beta_3 > \beta_1$ which means in this case that our solution is β_3.

2. Two complex solutions, one real positive solution, and one negative solution. In this case, we have either $\beta_{1,2}$ or $\beta_{3,4}$ real solutions. S should be positive real number, and the complex value comes only from the square roots $\sqrt{X}$ and $\sqrt{Y}$. Moreover, q has always a negative real value. Therefore, in (11) the value of $X < Y$. So $\beta_{1,2}$ must be the complex roots, and as $\beta_3 > \beta_4$, β_4 is the negative root and β_3 is the positive real root.

Based on the above discussion, the proposed closed form optimum frame length L_{MCRC} under the multiple collision recovery coefficients environment is:

$$L_{MCRC} = \left(-\frac{b}{4a} + S + 0.5\sqrt{-4S^2 - 2P - \frac{q}{S}}\right) \cdot n \tag{12}$$

According to (12), the proposed equation gives a linear relation wrt. the number of tags n, and includes the effect of different collision recovery coefficients. In case that the RFID reader has no collision resolving capability, i.e. $\alpha_2 = \alpha_3 = \alpha_4 = 0$, the proposed formula gives $L_{MCRC} = n$, which is identical to the frame length in the conventional case. When the RFID reader has a full and equal collision resolving capability for the two, three, and four collided tags per slot, i.e. $\alpha_2 = \alpha_3 = \alpha_4 = 1$, the proposed formula gives $L_{MCRC} = 0.452 \cdot n$, which matches the results in [5].

III. Collision Recovery Coefficients Calculations

Now the calculation of the the collision recovery coefficients α_2, α_3 and α_4 will be clarified, which are the main optimization variables in our proposal. The values of these coefficients are strongly depend on the receiver type. Calculations of the collision recovery coefficients are done based on a RFID reader model that utilizes the capture effect. The reader resolves the collision based on the strongest tag reply. Therefore, the collision can be resolved with a certain probability, if the strongest tag reply is stronger than the summation of the other collided tags at the same slot. The main advantage of this reader is that it does not need any channel state information (CSI) to recover the strongest tag. According to EPCglobal C1G2 standard [1], collisions in RFID systems occur only within the 16 bits packet called $RN16$. If any single bit error occurs in this packet, the total packed has to be considered lost. Therefore, the meaning of the collision recovery probability coefficients α_i is the probability that the RFID reader can identify a complete $RN16$ packet from i collided tags. Therefore, the collision recovery coefficient can be expressed as $\alpha_i = (1 - PER_i)$, where PER_i is the Packet Error Rate for i collided tags. In this work, we measure the SNR for each slot, then we calculate the average SNR per frame as $E\left\{\frac{|h|^2 \cdot x^2}{\sigma^2}\right\}$, where σ is the standard deviation of the Additive White Gaussian Noise (AWGN) per slot, and we used normalized signal power, i.e. $E\left\{x^2\right\} = 1$. Based on [10], we assumed that the equivalent channel coefficients h follow Rayleigh fading. The channel coefficients are independent zero mean circularly symmetric complex Gaussian random variables with normalized energy $E\left\{|h|^2\right\} = 1$, and all tags are statistically identical, which means all of them experience the same average path loss. Therefore, the average SNR per frame is $E\left\{\frac{1}{\sigma^2}\right\}$. Figure 2 shows the values of the collision recovery coefficients versus the average SNR per frame using the strongest tag reply receiver. In these simulations, we used a sampling frequency of $f_s = 8\,\text{MHz}$, and tags use FM0 as an encoding scheme. Finally, to clarify the worst case effect of the collision recovery coefficients, we used the highest symbol rate $640\,\text{kHz}$

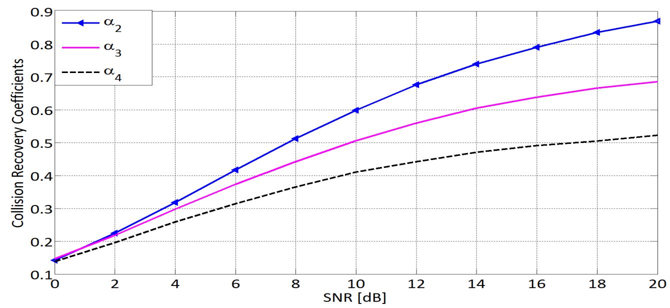


Figure 2: Collision recovery coefficients versus SNR using the strongest tag reply receiver. (example)

IV. Simulation Results

For each RFID reader, each SNR leads to corresponding values for the collision recovery coefficients α_2, α_3 and α_4. Figure 3 shows a comparison frame length comparison between the proposed formula in (12) and the numerical solution which maximizes the reading efficiency in (8) versus the SNR. Both simulations used the same receiver model, which is the strongest tag reply receiver. According to figure 3, the proposed formula gives identical results compared to the numerical solution at the complete SNR range.

According to practical measurements, the typical range of the SNR for successful slots is 4 to $12\,\text{dB}$. Therefore, we calculated the total average number of slots needed to identify a complete bunch of tags using the strongest tag reply receiver. FSA with initial frame length $L_{ini.} = 16$ is used as an anti-collision algorithm. Figure 4 shows a comparison between the proposed formula and the frame length proposed by [3], and [5] for the boundary of the practical range of the SNR. The authors of [3] used the reading efficiency equation in

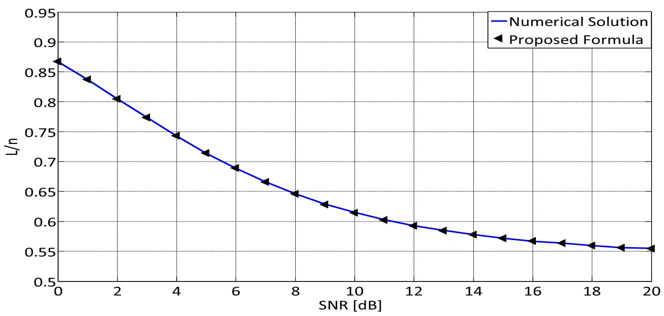


Figure 3: Frame length comparison between the proposed formula and the numerical solution versus the SNR using the strongest tag reply receiver.

(2) assuming constant collision recovery coefficient $\alpha = \alpha_2$ regardless the number of collided slots. In [5], the authors assumed limited collision recovery capability. However, they have assumed a unity collision recovery coefficients $\alpha_2 = \alpha_3 = \alpha_4 = 1$ as shown in (4). Based on figure 4a and 4b, the proposed formula gives better performance than the frame lengths of [3] and [5]. As shown in figure 4a, the performance of [3] is closer than [5] to our proposed performance, because the values of the collision recovery coefficients are small. So the effect of remaining terms in [3] is smaller than the effect of assuming $\alpha_2 = \alpha_3 = \alpha_4 = 1$. However, in case of high SNR $12\,\mathrm{dB}$, the values of the collision recovery coefficients are bigger to make the performance of [5] is closer than [3] to our proposed performance as shown in figure 4b.

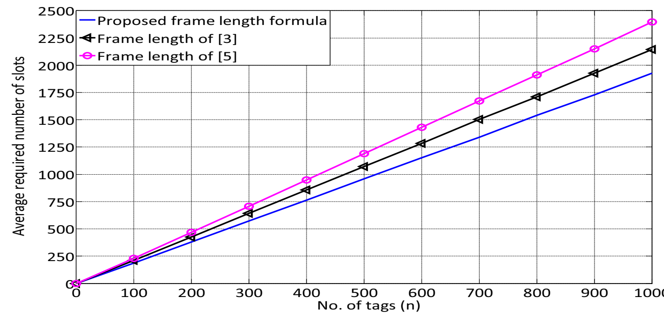
To characterize the performance of the proposed solution, we will define a performance metric called Relative Average Saving Time (RAST):

$$\mathrm{RAST}_{[\mathrm{i}]} = \left(\frac{T_{[\mathrm{i}]} - T_{\mathrm{proposed}}}{T_{[\mathrm{i}]}} \right) \times 100, \qquad (13)$$

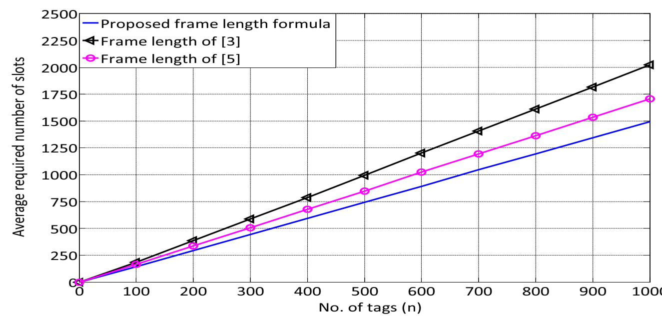
where $T_{[\mathrm{i}]}$ is the average number of slots using [i] algorithm. T_{proposed} is the average number of slots using the proposed formula. According to figure 4, the RAST of [3] and [5] are constants, regardless of the number of tags. At SNR $= 4\,\mathrm{dB}$ $\mathrm{RAST}_{[3]} \simeq 10\%$ and $\mathrm{RAST}_{[5]} \simeq 19\%$. At SNR $= 12\,\mathrm{dB}$, $\mathrm{RAST}_{[3]} \simeq 25\%$ and $\mathrm{RAST}_{[5]} \simeq 12\%$.

V. Conclusion

This paper proposes a novel closed form solution for the optimum value of the FSA frame length in RFID systems. The proposed equation takes into account the effects of the practical limitations and the differences in collision recovery probability coefficients of the RFID readers. The theoretical derivations lead to a new optimization criterion that can be easily implemented in RFID readers. The proposed frame length equation gives the most accurate results in the complete range of the collision recovery coefficients α_i compared to other solutions. Timing comparisons are presented to show the saving in reading time using the proposed frame length compared to the other proposals.



(a) $SNR = 4\,\mathrm{dB}$



(b) $SNR = 12\,\mathrm{dB}$

Figure 4: Average number of slots needed to identify a complete bunch of tags using the proposed formula, frame length proposed by [3], and [5].

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References

[1] "EPC radio-frequency protocols class-1 generation-2 UHF RFID protocol for communications at 860 MHz 960 MHz version 1.1.0 2006."
[2] J. Park, M. Y. Chung, and T.-J. Lee, "Identification of RFID Tags in Framed-Slotted ALOHA with Tag Estimation and Binary Splitting," in *First International Conference on Communications and Electronics, 2006. ICCE '06.*, pp. 368–372, 2006.
[3] B. Li and J. Wang, "Efficient anti-collision algorithm utilizing the capture effect for iso 18000-6c rfid protocol," *Communications Letters, IEEE*, vol. 15, pp. 352–354, March 2011.
[4] X. Yang, H. Wu, Y. Zeng, and F. Gao, "Capture-aware estimation for the number of rfid tags with lower complexity," *Communications Letters, IEEE*, vol. 17, pp. 1873–1876, October 2013.
[5] L. C. V. L. D. De Donno, L. Tarricone and M. M. Tentzeris, "Performance enhancement of the rfid epc gen2 protocol by exploiting collision recovery," *Progress In Electromagnetics Research B,*, vol. 43, 53-72, 2012.
[6] J. Alcaraz, J. Vales-Alonso, E. Egea-Lopez, and J. Garcia-Haro, "A stochastic shortest path model to minimize the reading time in dfsa-based rfid systems," *IEEE Communications Letters*, vol. 17, pp. 341–344, February 2013.
[7] J. Kaitovic, R. Langwieser, and M. Rupp, "A smart collision recovery receiver for rfids," *EURASIP Journal on Embedded Systems*, vol. 2013, no. 1, 2013.
[8] M. Abramowitz and I. A. Stegun, *Solutions of Quartic Equations*. New York: Dover,, 1972.
[9] D. Struik, *A Source Book in Mathematics 1200-1800*. Princeton University Press, 1986.
[10] C. Angerer, R. Langwieser, and M. Rupp, "Rfid reader receivers for physical layer collision recovery," *Communications, IEEE Transactions on*, vol. 58, pp. 3526–3537, December 2010.