A Closed Form Solution For Collision Recovery Aware Maximum Likelihood Tag Estimation

***Abstract*—Radio Frequency Identification (RFID) is a wireless technology allowing for automatic identification of tags (transponders). In case of large tag populations, the estimation algorithm has to be fast and accurate. Previous studies depend on the Medium Access (MAC) Layer information, i.e, the working frame length, number of collided, successful, and empty slots per frame. However, modern systems have a collision recovery capability. They have the ability to convert part of collided slots into successful slots. In such systems, the number of collided and successful slots at the MAC layer are not accurate information about the real number of tags at the reading area. In this work, we propose a novel tag estimation method taking into consideration the collision recovery capability of the system. The main advantage of the proposed method is that it gives a novel closed form solution for the estimated number of tags considering the collision recovery probability of the used system. Simulation results indicate that the proposed solution gives more accurate results compared to the literature without any need for Multi-dimensional look-up tables. Timing comparisons are presented in simulation results to show the mean reduction in identification delay using the proposed estimation method compared to other proposals.**

I. Introduction

Recently the number of applications that use RFID technology have increased, and the reading speed became one of the most critical issues in these applications. In RFID systems, the tags typically share a common communications channel. Thus, there is a certain probability of tag-collisions, i.e. multiple tags answer simultaneously. This collision probability naturally increases in dense networks with many tags. Since passive tags are the most practical tags in the market, because of their low price and simple design, they cannot sense the channel or communicate with the other tags. As a result, the reader is responsible for coordinating the network and has to avoid tags collisions using specific anti-collision algorithms.

The conventional anti-collision algorithm is the Framed Slotted ALOHA (FSA) algorithm [1], which is only a Medium Access Control (MAC) layer protocol. In such systems, only the answer of a single tag is considered as a successful slot, and if multiple tags respond simultaneously, a collision occurs. Then all the replied tags are discarded. The performance of FSA-based protocols is maximized by adapting the frame length L to the number of tags n. However in practical applications, the number of tags n in the interrogation region is unknown. Furthermore, the number of tags may even vary, e.g. when the tags are mounted on moving goods. Therefore, Dynamic Framed Slotted ALOHA (DFSA) [2] is commonly used. DFSA first estimates the number of tags in the interrogation area, and then calculates the optimal frame size L for the next reading cycle. Therefore, the system performance is controlled by how precise and fast we estimate the number of tags in the interrogation area.

A simple estimation methods have been proposed by Vogt and Schoute. The lower bound estimation method proposed by Vogt [2] states that the remaining number of tags is double the number of collided slots in the previous frame. Schoute [3] proposed a posterior expected factor of 2.39 to estimate the number of tags in the interrogation area. However, these methods depend only on a single information which is the number of collided slots. Therefore, these methods lead to increase the tags estimation error in dense networks [4]. In [1], The author proposed more complex estimation method minimizing the distance between the observed empty E_{obs}, successful S_{obs}, collided C_{obs} slots and the expected values E, S, C for a given frame length L. i.e., $\varepsilon_{conv}(L, S_{obs}, C_{obs}, E_{obs}) = \underset{n}{min}\{|E - E_{obs}| + |S - S_{obs}| + |C - C_{obs}|\}$. However, This method requires numerical searching to find the optimum value of n. Moreover, the author assumed that tags are identically distributed in slots, which is generally not accurate assumption. Another approach proposed by [5]. The author assumed that tags in the frame are distributed using the binomial model. Once the E, S, C are obtained for given L, a posteriori distribution is calculated. Afterwards, the author searches for number of tags n which maximizes the given a posteriori probability. An improved version of [5] includes the mutual dependence of different slot types is presented in [6]. However, this method is more complex and needs more iterations to find the optimum value of n. Moreover, it does not improve the performance of the FSA compared to the proposal in [5]. [4] used the same approach in [5] but using Poisson model instead of the binomial model to reach to a less complex equation and decrease the searching complexity. However, it is still needs iterations of searching to obtain the optimum value of n.

In [7], [8], these proposals included a closed form solutions for estimated number of tags n without need for searching iterations. However, both methods use numerical interpolations to reach to the optimum value of n. Therefore, both result equations can not be used utilizing further parameters like the collision recovery probability which will be discussed later. Modern systems have the capability to convert part of collided slots into successful slots. In such systems, the number of collided and successful slots which delivered to the MAC layer are not accurate information about the real number of tags at the reading area. Therefore we should take into consideration the collision recovery probability α. [9] used the estimation approach of [1] taking into consideration the collision recovery probability. However this method leads to have multi-dimensional searching, which time consuming and high complexity.

In this paper, we propose a novel closed form solution for the estimated number of tags n taking into consideration the collision recovery probability of the system. Then we show how to calculate the collision recovery probability from the physical layer parameters. The proposed solution gives a direct relation between the estimated number of tags n and the frame length L , successful and collided number of slots S, C, and the collision recovery probability α. This paper is organized as follows: Section II presents the system model under collision recovery probability in addition to an example to calculate the collision recovery coefficients. Next, section III presents the derivation of the proposed closed form solution for the estimated number of tags n. Finally, section IV gives numerical results on the improvements of the new optimization criterion, before we conclude in section V.

II. Proposed System Model Under Collision Recovery Probability

In this section, we will describe a novel collision recovery aware number of tags estimation method. The proposed method is based on the classical Maximum Likelihood (ML) estimation in [5]. According to the classical ML estimation method, it searches for the optimum value of n_{est} which maximizes the conditional probability of observing vector $v = \langle C,\, S,\, E\rangle$ given that n tags transmit at a frame length L.

$$P(n/L, S, C, E) = \frac{L!}{E!S!C!} P_e^E P_s^S P_c^C, \tag{1}$$

where C, S, E are successively the number of collided, successful, and empty slots per a frame length L, and P_e, P_s, P_c are respectively the probabilities of empty, successful and collided transmission per slot. However, modern RFID readers have a collision recovery capability. Thus, the physical layer convert part of collided slots into successful slots based on the following relation:

$$E = E_b,\, S = S_b + \alpha \cdot C_b,\, C = C_b - \alpha \cdot C_b, \tag{2}$$

where C_b, S_b, E_b are successively the number of collided, successful, and empty slots before collision recovery of the system, and C, S, E are respectively the number of collided, successful, and empty slots after collision recovery of the system. α is the collision recovery probability.

In MAC layer, only the values of C, S, E after collision recovery are known, and there is no information about these values before collision recovery. Thus, the conventional estimation systems including the classical ML number of tags estimation in (1) use the values of C, S, E after collision recovery in their calculations . However, these values are not accurate indicator about the actual number of tags in the reading area. In the proposed system, we estimate the value of the current average collision recovery probability α as shown in [10]. Afterwards, we calculate the corresponding values of C_b, S_b, E_b as:

$$E_b = E,\, C_b = \frac{C}{1-\alpha},\, S_b = S - \frac{\alpha}{1-\alpha} C \tag{3}$$

Under condition:

$$L = E_b + S_b + C_b \tag{4}$$

Thus, $C_{b(max)} = L - E_b$ and $S_{b(min)} = 0$. Therefor, the proposed collision recovery aware ML conditional probability can be formalized as:

$$P(n/L, S, C, E, \alpha) = \frac{L!}{E_b!S_b!C_b!} P_e^{E_b} P_s^{S_b} P_c^{C_b} \tag{5}$$

In this work, we are interested in the dense RFID network so we can use the approximation suggested in [4] for the tag probability of transmission per slot, which are considered as independent Poisson random variables with unknown mean $\gamma = \frac{n_{est}}{L}$ to have:

$$P_e = e^{-\gamma},\, P_s = \gamma \cdot e^{-\gamma},\, P_c = 1 - e^{-\gamma} - \gamma \cdot e^{-\gamma} \tag{6}$$

After substituting by (6) in (5) and normalizing the result equation from the constant $\frac{L!}{E_b!S_b!C_b!}$, the result proposed conditional probability will be:

$$P(n/L, S, C, E, \alpha) = \gamma^{S_b} \cdot e^{-\gamma \cdot L} \cdot \left(e^{-\gamma} - 1 - \gamma\right)^{C_b} \tag{7}$$

III. Derivation of The Proposed Closed Form Solution for The Estimated Number of Tags

The computation of (7) is done numerically to obtain the optimum value of n_{est} which maximizes (7). Thus, the calculation of (7) may lead to numerical instability problems using low-complexity devices. Therefore, in this section, we propose a closed form solution for the collision recovery aware estimation. This is achieved by differentiating (7) with respect to γ and equate the results to zero. After differentiating, the equation can be simplified as:

$$e^{-\gamma}\left(1 + \frac{\gamma\,(\gamma \cdot L - S_b)}{(\gamma \cdot L - S_b - \gamma \cdot C_b)}\right) - 1 = 0 \tag{8}$$

The analysis of (8) indicates that the relevant values for γ are in the region close to one [10]. Hence, we can develop a Taylor series for $e^{-\gamma}$ around one which leads to:

$$e^{-\gamma} \simeq 1 - \gamma + \frac{1}{2}\gamma^2 - \frac{1}{6}\gamma^3. \tag{9}$$

After substituting (8) and some additional simplifications, the final equation is a fourth order polynomial:

$$\underbrace{\frac{1}{120}(L - C_b)}_{(a)}\gamma^4 + \underbrace{\frac{1}{24}\left(L - C_b - \frac{S_b}{5}\right)}_{(b)}\gamma^3 + \underbrace{\frac{1}{6}\left(L - C_b - \frac{S_b}{4}\right)}_{(c)}\gamma^2 + \underbrace{\frac{1}{2}\left(L - C_b - \frac{S_b}{3}\right)}_{(d)}\gamma - \underbrace{\left(C_b + \frac{S_b}{2}\right)}_{(e)} = 0 \tag{10}$$

Equation (10) has four roots [11]:

$$\gamma_{1,2} = -\frac{b}{4a} - S \pm 0.5\sqrt{\underbrace{-4S^2 - 2P + \frac{q}{S}}_{X}} \tag{11}$$

$$\gamma_{3,4} = -\frac{b}{4a} + S \pm 0.5\sqrt{\underbrace{-4S^2 - 2P - \frac{q}{S}}_{Y}},$$

where $P = \frac{8ac-3b^2}{8a^2}$, $q = \frac{b^3-4abc+8a^2d}{8a^3}$

and, $S = 0.5\sqrt{-\frac{2}{3}P + \frac{1}{3a}\left(Q + \frac{\triangle_0}{Q}\right)}$, $Q = \sqrt[3]{\frac{\triangle_1 + \sqrt{\triangle_1^2 - 4\triangle_0^3}}{2}}$.

with, $\triangle_0 = c^2 - 3bd + 12ae$, $\triangle_1 = 2c^3 - 9bcd + 27ad^2 - 72ace$

According to equation (4), we can proof that the signs of the polynomial coefficients are constant and can be as follows: $a = +$, $b = +$, $c = +$, $d = +$, and $e = -$.

Using Descartes' rules of sign [11], we can count the number of real positive solutions that a polynomial has. Assume that the polynomial in (10) is $P(\gamma)$, and let ν be the number of variations in the sign of the coefficients a, b, c, d, e, so $\nu = 1$. Let n_p be the number of real positive solutions. According to Descartes' rules of sign [11]:

- $n_p \leq \nu$ which means that $n_p = 0\, or\, 1$.
- $\nu - n_p$ must be an even integer. Therefore, $n_p = 1$.

According to the above two Descartes' rules of sign, there is only one valid real positive solution for the equation. Now, we will identify which solution is the valid one. There are two possibilities for the solutions:

1. One positive real solution and the remaining three solutions are negative. In this case, all solutions are real and we need just to identify what is the root which has the largest values from the four solutions. According to (11), the value of the square roots $\sqrt{X}$ and $\sqrt{Y}$ are positive real, because we do not have complex solutions. This means, $\gamma_1 > \gamma_2$ and also $\gamma_3 > \gamma_4$. So, the solution will be either γ_1 or γ_3. Moreover, the value of S should be also positive real, and q has always negative real value. so $\gamma_3 > \gamma_1$ which means in this case that our solution is γ_3.

2. Two complex solutions, one real positive solution, and one negative solution. In this case, we have either $\gamma_{1,2}$ or $\gamma_{3,4}$ real solutions. S should be positive real number, and the complex value comes only from the square roots $\sqrt{X}$ and $\sqrt{Y}$. Moreover, q has always negative real value. Therefore, in (11) the value of $X < Y$. So $\gamma_{1,2}$ must be the complex roots, and as mentioned before that $\gamma_3 > \gamma_4$, so γ_4 is the negative root and γ_3 is the positive real root.

Based on the above discussion, the proposed closed form solution for the collision recovery aware tag estimation is:

$$n_{est} = \left(-\frac{b}{4a} + S + 0.5\sqrt{-4S^2 - 2P - \frac{q}{S}} \right) \cdot L \qquad (12)$$

IV. Collision Recovery Probability Calculation

In this part, we will give an example about how we calculate the collision recovery probability α. The capture probability varies in the range of $0 \leq \alpha \leq 1$. Its value depends on the Signal to Noise Ratio (SNR). In this work, we measure the SNR for each slot. Then, we calculate the average SNR per frame. In [12], the authors proposed a method to capture the strongest tag reply based the physical layer properties. They have proposed a Bit Error Rate (BER) curve versus the SNR. We want to calculate the capture probability for a complete collided RN16 packet, which includes 16 random successive bits. The BER is mapped to Packet Error Rate (PER) by simulation as the channel is not Binary Symmetric Channel (BSC). The capture probability can be expressed as: $\alpha = (1 - PER)$. Figure 1 presents the values of the capture probabilities versus the average signal to noise ratio per frame.

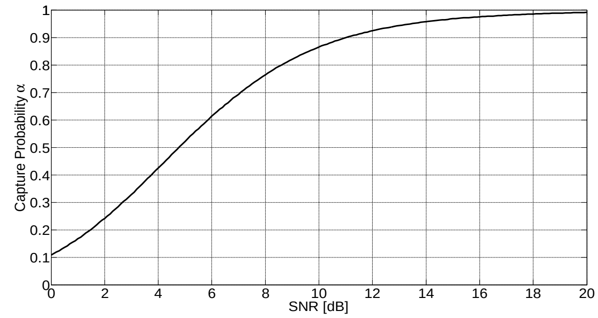
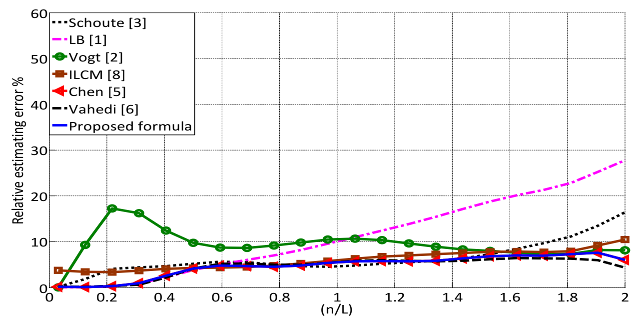
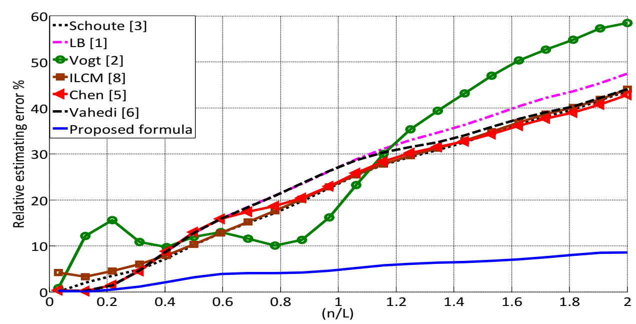


Figure 1: Capture probability versus the signal to noise ratio



(a) System has no collision recovery capability ($\alpha = 0$)



(b) System has collision recovery capability ($\alpha = 0.7$)

Figure 2: Relative estimation error ϵ vs normalized number of tags $^{n}/_{L}$

In this work, we calculate the average capture probability from the corresponding average SNR at the current frame.

V. Simulation Results

Firstly, we will define a performance metric called relative estimation error ϵ. It can be defined as

$$\epsilon = \left| \frac{n_{est} - n}{n} \right| \times 100\,\% \qquad (13)$$

Figure 2 shows the percentage of the relative estimation error for the proposed system compared to the literature versus the normalized number of tags $^{n}/_{L}$. Figure 2a shows system which has no collision recovery capability ($\alpha = 0$). According to figure 2a, the proposed system gives identical relative estimation error compared to [5]. However the proposed system gives a closed form solution but the solution of [5] is based on numerical searching. [6] which included the mutual independence of slot types has almost the same results compared to the proposed results. However, it includes a very complex searching algorithms compared to the proposed closed

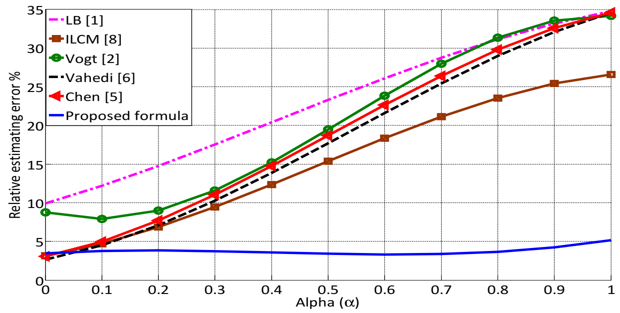
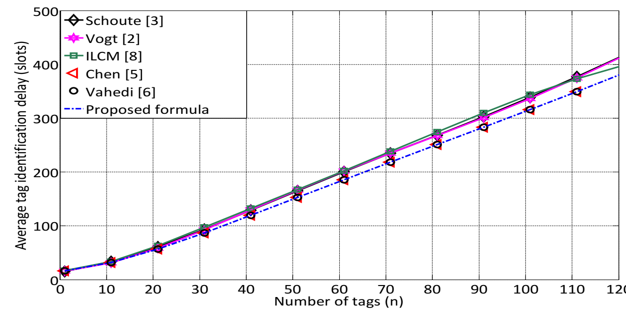


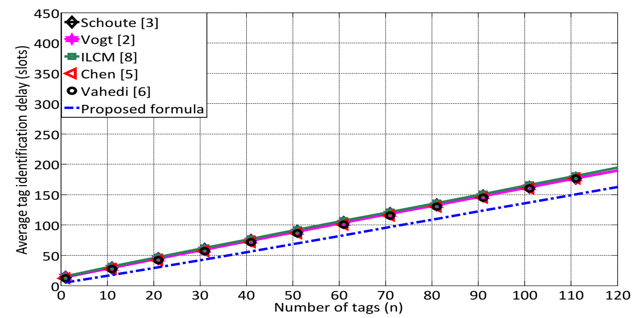
Figure 3: Relative estimation error vs. collision recovery probability α , where, $L = n$

form solution. Figure 2b shows an example for modern systems, which have collision recovery capability. We used collision recovery probability $\alpha = 0.7$. According to figure 2b, The proposed curve has more accurate estimation performance compared to all the literature. Figure 3 shows the relative estimation error versus the collision recovery probability α assuming that the number if tags in the reading area is equal to the frame length i.e. $n = L$. Based on figure 3, when the value of the collision recovery probability increases, the performance of all other proposals decreases, except the proposed method has almost constant performance independent on the value of the collision recovery probability.

Figure 4 shows the average identification delay for a bunch of tags. Figure 4a shows the identification time for systems with no collision recovery capability ($\alpha = 0$). In these simulations we have assumed that the optimum frame length is the nearest quantized 2^Q for $L = n$. According to 4a, the proposed system gives identical results compared to [5] and [6] better than the other literature. Figure 4b, shows the average identification delay for systems has a collision recovery probability $\alpha = 0.7$. According to figure 4b, the average identification delay has decreased for all the systems due to the collision recovery capability. However the proposed system saves the total identification time with almost 10 % compared to the others due to the performance of estimation only.



(a) System has no collision recovery capability ($\alpha = 0$)



(b) System has collision recovery capability ($\alpha = 0.7$)

Figure 4: Average identification delay

VI. Conclusion

This paper proposes a novel closed form solution for the estimated number of tags taking into consideration the collision recovery probability of the system. The theoretical derivations lead to a new analytical number of tags estimation equation that can be easily implemented in RFID readers. Using the proposed formula, we need neither look-up tables nor searching. Furthermore, our analytical solution can be used for further optimizations of RFID systems. Simulation results prove the correctness of our analytical solution.

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