A Closed Form Solution for Frame Slotted ALOHA Utilizing Time and Multiple Collision Recovery Coefficients

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***Abstract*— Minimizing the reading time of large tag populations is a critical issue in Radio Frequency Identification (RFID) systems. The usual approach to reduce the reading time is to select the frame size attaining the highest throughput per frame. Previous studies have focused on conventional frame length calculations. In such systems, only the answer of a single tag is considered as a successful slot. If multiple tags respond simultaneously within a slot a collision occurs. Then, all tags within this slot are discarded. However, modern system have the capability of converting part of the collided slots into successful slots. This is called Collision Recovery. Moreover, modern RFID readers have the ability to identify the type of the slot (successful, collided, or empty). Then, the readers are able to terminate the slot earlier when they recognizes that there is no tag reply. This system is called a time aware system. Recent studies focused on calculating the optimal frame length taking into consideration the time aware and the collision recovery properties. However, these studies have assumed constant collision recovery probability coefficients, i.e. the probability to recover one tag from i collided tags is constant, regardless of the number of collided tags i. Moreover, they proposed only numerical solutions for the optimum frame length. In this paper we propose a novel closed form solution for the optimal Frame Slotted ALOHA (FSA) frame length. The novel solution considers the multiple collision recovery probability coefficients, and the different slot durations. Timing comparisons are presented in the simulation results to show the reading time reduction using the proposed frame length compared to other the state-of-the-art algorithms.**

I. INTRODUCTION

Radio Frequency Identification (RFID) is an identification technology that wirelessly transmits the identity of a tag which may be attached to an object or a person. If tags respond simultaneously to a reader, a collision on the air interface occurs and the information is discarded, leading to reduced throughput. Our research focuses on the improvement of the throughput using the EPCglobal class 1 gen 2 RFID standards [1]. The conventionally used anti-collision algorithm is Framed Slotted ALOHA (FSA), which is a Medium Access Control (MAC) layer protocol. Using this algorithm, only the single tag replies (successful slot) are able to be decoded and then identified. Therefore, the conventional definition of the reading efficiency η_{conv} is equivalent to the probability of success $P(S)$ [2]:

$$\eta_{conv} = P(S) = P(1), \tag{1}$$

where, $P(1) = \frac{n}{L}\left(1 - \frac{1}{L}\right)^{n-1}$, n represents the number of tags in the reading area, and L is the frame length.

The main goal is to find the optimal frame length L, which maximizes the reading efficiency η_{conv}. Based on (1), the reading efficiency η_{conv} is maximized to $\eta_{conv(max)} = 36\%$ when $L = n$ [2]. [3] considered the RFID reader capability of collision resolving. They have used the characteristics of the RFID signals to separate signals from collisions on the physical layer (PHY). They have proposed a new reading efficiency metric that includes the tags which are recovered based on the PHY layer. The authors assumed that the probability to recover a single tag from i collided tags is constant and equals to 100%, independently of the value of i. However in reality, the probability to recover a single tag from i collided tags reduces with the number of collided tags i. Moreover, there are no practical readers that offer a 100% collision recovery probability. Finally, the authors did not consider the effects of the different slot durations. In [4], the authors merged the collision recovery probability with the effect of the different slut durations for a new reading efficiency metric. They have assumed also a constant collision recovery capability, regardless of the number of collided tags. However, this probability decreases with an increasing number of collided tags. Moreover, they proposed a numerical solution for the optimum frame length by searching for the value of the frame length L which maximizes the reading efficiency. Thus, they require a Multi-dimensional look-up table.

In this paper, we propose a novel reading efficiency metric called Time Aware Multiple Collision Recovery Coefficients Reading Efficiency η_{TAMCRC}. The new metric includes different collision recovery coefficients for each number of collided tags. Furthermore, it takes into consideration the different slot durations. Hence, we propose a novel closed form solution for the optimum FSA frame length at RFID systems. The proposed solution gives a direct relation between the optimal frame length and the number of tags n in the reading area, in addition to the collision recovery coefficients and the different slot durations.

This paper organized as follows: Section II presents the system model under variable slot duration and multiple collision recovery coefficients and the proposed corresponding closed form solution for the optimal frame length. Then, section III gives numerical results on the improvements of the new optimization criterion, before we conclude in section IV.

II. PROPOSED SYSTEM MODEL

In this section we present a new FSA reading efficiency metric called Time Aware Multiple Collision Recovery Coefficients Reading Efficiency η_{TAMCRC}. The main contribution in this new efficiency is: It contains a unique collision recovery coefficient α_i for each probability of collision $P(i)$. These new coefficients indicate the ability of the reader to recover one tag from i collided tags, where this ability varies based on the number of collided tags. Moreover, it takes into consideration the different slot durations.

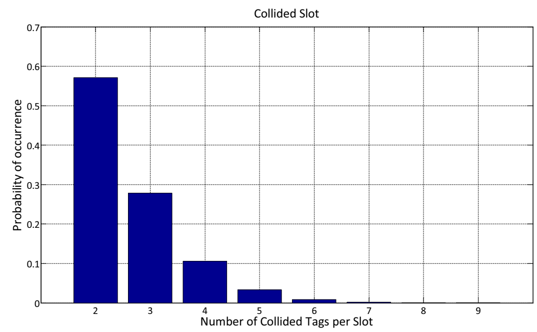

Fig. 1. Collision distribution probability in FSA, under condition of $\frac{n}{2} \leq L \leq 2n$

Figure 1 presents the distribution of the average collision probability in a frame length uniformly distributed within $0.5 \leq \frac{L}{n} \leq 2$, which is the practical range of the RFID frame length. According to figure 1, the probability that a collision results from two or three collided tags is approx. 85%. Moreover, the values of the collision recovery coefficient α_i when $i \geq 4$ (i.e. 4 or more collided tags) will be small. Therefore, we will only consider up to three collided tags. We will now normalize the slot duration t_k of successful and collided tags to unity. We furthermore take the assumption that empty slots are shorter than successful slots (i.e. $t_0 \leq t_k$), which is the case for practical readers. Then, the proposed reading efficiency η_{TAMCRC} can be expressed as:

$$\eta_{TAMCRC} = \frac{P(1) + \alpha_2 P_{col.}(2) + \alpha_3 P_{col.}(3)}{1 + P(0) \cdot (C_t - 1)}, \qquad (2)$$

where $C_t = \frac{t_0}{t_k}$ represents the slots duration constant, and α_2, α_3 are respectively the second, third collision recovery coefficients.

The next step is to derive a closed form for the new optimum frame length L_{TAMCRC} which maximizes η_{TAMCRC}. According to [5], if $L \gg 1$, and $n \gg i$ we get:

$$P(i) \simeq \frac{1}{i!} \cdot \beta^{-i} \cdot e^{-\frac{1}{\beta}}, \qquad (3)$$

where, $\beta = \frac{L}{n}$. After substituting by (3) in (2) we obtain:

$$\eta_{TAMCRC} = \frac{e^{-\frac{1}{\beta}} \cdot \left(\beta^{-1} + \frac{\alpha_2}{2}\beta^{-2} + \frac{\alpha_3}{6}\beta^{-3}\right)}{1 + e^{-\frac{1}{\beta}} \cdot (C_t - 1)} \quad (4)$$

Now we have to find the value of β which maximizes η_{TAMCRC}. This is achieved by differentiating the reading efficiency in (4) with respect to β and equate the result to zero:

$$\frac{\partial \eta_{TAMCRC}}{\partial \beta} = 0 \quad (5)$$

After differentiating and simplifications, the final equation is a fourth order polynomial:

$$a \cdot \beta^4 + b \cdot \beta^3 + c \cdot \beta^2 + d \cdot \beta + e = 0, \quad (6)$$

where: $a = \underbrace{-C_t}_{(-)}$, $b = \underbrace{C_t \cdot (1 - \alpha_2) - 1}_{(-)}$

$$c = \underbrace{\underbrace{2 - C_t - \alpha_2}_{(+)} + \underbrace{\frac{C_t}{2}(\alpha_2 - \alpha_3)}_{(+)}}_{(+)}$$

$$d = \underbrace{\frac{1}{2}(\alpha_2 - \alpha_3) + \frac{1}{2}\alpha_2 \cdot (1 - C_t) + \frac{1}{6}C_t \cdot \alpha_3}_{(+)}$$

$$e = \underbrace{\frac{1}{6}\alpha_3 \cdot (2 - C_t)}_{(+)}$$

As $0 \leq \alpha_i \leq 1$, $0 < C_t \leq 1$, and $\alpha_2 \geq \alpha_3$, equation (6) has four roots [6]:

$$\beta_{1,2} = -\frac{b}{4a} - S \pm 0.5\sqrt{\underbrace{-4S^2 - 2P + \frac{q}{S}}_{X}} \quad (7)$$

$$\beta_{3,4} = -\frac{b}{4a} + S \pm 0.5\sqrt{\underbrace{-4S^2 - 2P - \frac{q}{S}}_{Y}},$$

with $P = \frac{8ac-3b^2}{8a^2}$, $q = \frac{b^3-4abc+8a^2d}{8a^3}$ and $S = 0.5\sqrt{-\frac{2}{3}P + \frac{1}{3a}\left(Q + \frac{\triangle_0}{Q}\right)}$, $Q = \sqrt[3]{\frac{\triangle_1 + \sqrt{\triangle_1^2 - 4\triangle_0^3}}{2}}$

with $\triangle_0 = c^2 - 3bd + 12ae$, $\triangle_1 = 2c^3 - 9bcd + 27ad^2 - 72ace$

According to the practical ranges of the collision recovery coefficients α_i and C_t, we can prove that the signs of the polynomial coefficients are constants and do not change in all ranges of α_i and C_t. Thus their signs will be:
$a = (-)$, $b = (-)$, $c = (+)$, $d = (+)$, and $e = (+)$.
Using Descartes' rules of sign [7] we can count the number of real positive solutions of the polynomial.

Let us assume that the polynomial in (6) is $P(\beta)$, and let ν be the number of variations in the sign of the coefficients a, b, c, d, e, i.e. $\nu = 1$, and let n_p be the number of real positive solutions. According to Descartes' rules of sign [7] we get:

- $n_p \leq \nu$, which means that $n_p = 0$ or 1.
- $\nu - n_p$ is an even integer. Therefore $n_p = 1$.

Consequently, there is only one valid real positive solution, one valid real negative solution, and two complex solutions for our equation. Our target is to identify which solution from the four solutions is the valid one. We have either $\beta_{1,2}$ or $\beta_{3,4}$ real solutions, so S should be a positive real number, and the complex values come only from the square roots $\sqrt{X}$ and $\sqrt{Y}$. According to the coefficient signs: q must be always positive real value. Therefore, in (7) the value of $X > Y$. So $\beta_{3,4}$ have to be the complex roots, with $\beta_1 > \beta_2$. Therefore, β_2 is the negative root and β_1 is the positive real root. Based on the above discussions, the proposed closed form optimum frame length L_{TAMCRC} under time and multiple collision recovery coefficients environment leads to:

$$L_{TAMCRC} = \left(-\frac{b}{4a} - S + 0.5\sqrt{-4S^2 - 2P + \frac{q}{S}}\right) \cdot n \quad (8)$$

The proposed equation gives a linear relation wrt. the number of tags n, and includes the effect of different collision recovery coefficients and the slot duration constant. The values of these coefficients are set based on the RFID reader type as shown in [8]. The value of C_t can be calculated based on the transmission rate as shown in [9]. Based on (8), if the RFID reader has no collision resolving capability ($\alpha_2 = \alpha_3 = 0$) and equal slots durations are used ($C_t = 1$), we get $L_{TAMCRC} = n$. This is identical to the optimum frame length in the conventional case.

III. SIMULATION RESULTS

In this section, we will firstly discuss the accuracy of our closed form, then we will show the gain of using the proposed closed form shown

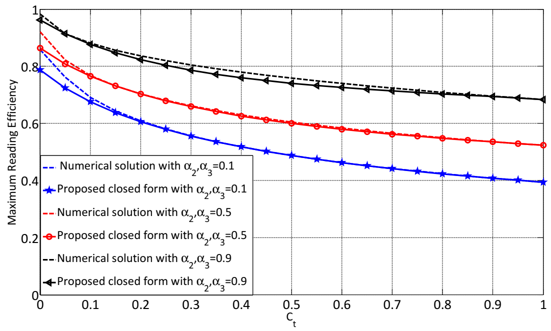

Fig. 2. Maximum reading efficiency $0 \leq C_t \leq 1$

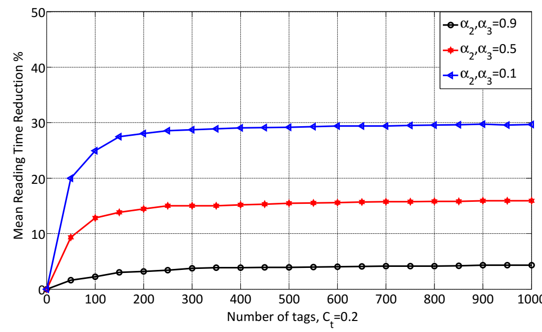

Fig. 3. Mean reading time reduction using the proposed frame length compared to the proposal of [3]

in (8) in terms of mean reading time reduction compared to the literature. Figure 2 presents the maximum efficiency using the proposed closed form compared to the numerical solutions in [4] for different values of collision recovery coefficients α_i in the full range of the slots duration constant C_t. It is clear that the proposed formula almost approaches the numerical solution. There is a small bias between the curves due to the approximation in (3). However, the closed form equation has a direct relation of the optimum frame length function of the number of tags in the reading area, the collision recovery coefficients, and the slots duration constant, so we need neither storage nor searching each reading cycle. However using the numerical methods, we have to store and search in a multidimensional look-up table to get the optimal frame length corresponding to each parameters combination.

Figure 3 presents the mean reading time reduction using the proposed frame length equation compared to the frame length proposed by [3]. The simulation results are based on the slot duration constant $C_t = 0.2$, as it is considered as a practical value used in the EPCglobal class 1 gen 2 standards [1]. The authors in [3] used identical collision recovery coefficients, then based on that has a constant frame lengths. According to figure 3, the mean reduction in reading time increases, when the collision recovery coefficients decreases. In this case, the proposed frame length will adapt for the new optimum size. However, the frame length proposed by [3] is fixed based on the assumption of 100% collision recovery capability.

IV. CONCLUSION

This paper proposes a novel closed form solution for the optimum value of the FSA (Frame Slotted ALOHA) frame length in RFID systems. The proposed equation takes the effect of different collision recovery probabilities into account, as well as the different slot durations. The theoretical derivations lead to a new optimization criterion that can be easily implemented in RFID readers. The closed form solution needs neither storage nor searching like the numerical solutions presented in the literature. Timing comparisons were presented to show the mean reading time reduction using the proposed frame length compared to other proposals.

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