A Time and Capture Probability Aware Closed Form Frame Slotted ALOHA Frame Length Optimization

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***Abstract*—Minimizing the reading time in dense Radio-Frequency Identification (RFID) networks is a critical issue. Commonly used RFID systems are based on Frame Slotted ALOHA (FSA) for tag anti-collision management. The usual approach for improved reading times with large tag populations is the optimization of the number of slots per frame. In real RFID systems, the slot duration depends on the slot type (i.e. idle, successful, or collided). In addition, collided slots might be converted to successful slots by capturing the strongest transponder, i.e. the so-called capture effect. Recent publications have proposed numerical solutions for obtaining the optimum frame length under these assumptions. The authors employ numerical solutions that require Multi-dimensional look-up tables for obtaining the optimum frame length. In this paper we propose a closed form solution for the analytical calculation of the optimum frame length. The proposed solution gives a novel closed form equation for the frame length considering the different slot durations and the capture effect. Moreover, the paper presents a new method to calculate the capture probability per frame. Simulations indicate that the proposed solution gives accurate results for all relevant parameter configurations without any need for Multi-dimensional look-up tables.**

I. INTRODUCTION

OVER the recent years, the number of applications that use Radio-Frequency Identification (RFID) has increased, and that number will further grow in the near future. One main application is the area of logistics, where e.g. hundreds of tags (transponders) may be closely placed on pallets. This naturally requires fast RFID readers in order to not slow down the delivery process of the actual goods. Commonly used RFID standards in the area of logistics (e.g. ISO 18000-6C [1]) base on TDMA (time division multiple access). In commercial RFID systems, the tags have to be of low price and simple design. Hence, they neither can sense the channel nor communicate with the other tags. Thus, there is a certain probability of tag-collisions, i.e. multiple tags answer simultaneously. This collision probability increases in dense networks with many tags. Hence, the readers are responsible for efficiently coordinating the network, and for the avoidance of collisions using anti-collision algorithms.

The EPCglobal C1 G2 standard [1] uses an anti-collision algorithm based on Framed Slotted ALOHA (FSA) [2]. The reader signals the frame length (i.e. the number of available slots in the frame) to the tags. Each tag then randomly assigns itself to one of the available slots and replies to the reader within this slot. Using this algorithm, the reader is able to successfully decode the data in a specific slot if only one tag replies within this slot. We call this slot type a successful slot. Furthermore, we have empty slots (i.e. no tag replies) and collided slots (i.e. multiple tags reply). Due to collisions the reader is not able to identify all tags in one frame. Hence, it has to use multiple frames to read all tags. As a tag is deactivated after its successful reading, the number of tags will decrease from frame to frame and all parameters have to be adjusted.

The conventional definition of the reading efficiency η_{conv} for FSA w.r.t. a single frame is equivalent to the expected number of successful slots S divided by the summation of the expected number of empty E, successful S, and collided C slots.

For a given number of tags n and a frame length of L slots, the main goal is finding the optimal frame length L, which maximizes the reading efficiency $\eta_{_{conv}}$ [2]. The reading efficiency $\eta_{_{conv}}$ is maximized to $\eta_{_{conv}} = 36\%$ for $L = n$ [2]. However, this assumption is not precise, as it does not take into consideration the near-far problem [3], which is very common in passive RFID systems. A tag that is close to the reader would reply with a significantly higher level than other tags that are more distant. In many cases, the reader would be able to decode the closest tag, even in case of a collided slot. This effect is the so-called capture effect. Another important aspect that has to be taken into consideration is the duration of the slots. The conventional FSA anti-collision algorithms assumes identical durations of empty, successful, and collided slots. However, in actual RFID systems these durations differ significantly between the different slot types [4].

According to the literature we can divide the previous research for the optimization of the frame length into three different groups:

The first group considered only the capture probability for the frame length optimizations, such as [5]. The authors proposed a closed form equation for the optimum frame length maximizing the reading throughput per frame.

The second group considered only the different slot durations, such as [4,6]. In [6] the authors have optimized the total latency to read a bunch of tags. However, no closed form solution for the optimum frame length was derived. They calculate the optimum frame length numerically. Only [4] proposed a closed form equation for the optimum frame length taking into consideration the different slot durations. However, the authors did not take into their consideration the effect of the capture probability. Additionally, the equation uses the Lambert-W function that does not exist in an explicit form.

Finally, the third group considered the different slot durations and the capture effect, such as [7]–[9]. Both [7,8] gave numerical solutions for the optimum frame length versus the capture probability and the number of tags. The main problems of their numerical solutions occur when taking into account the multiple degrees of freedom: the solutions require a complex multidimensional look-up table that has to consider all possible degrees of freedom. [9] uses curve fitting to find a closed form solution for the optimum frame length at a specific capture probability and timing. However, this solution can not be generalized for all values of slot's timing and capture probabilities.

In this work, we will derive a closed form solution for the optimum frame length by optimizing the reading throughput per frame that does not require any look-up table [7,8]. In addition, we take into consideration the capture effect and the different slot durations. The capture probability takes a value between 0 and 1, and the ratio between the empty and collided slot varies between 0.18 and 0.7 based on the derived formula in section IV, so two dimensional search is needed to calculate the frame length numerically which is time consuming in a critical time application like RFID. Moreover, we will clarify how we calculate the capture probability based on the physical layer. Afterwards, we will clarify the effect on the optimum frame length calculations due to the estimation error of the capture probability. This paper is organized as follows: Section II presents our system model and considers the different slot durations and the capture effect. In section III, we analytically derive a new closed form equation for the optimal frame length based on the Time and Capture Probability Aware FSA. Finally, we show how we estimate the capture probability and calculate the value of the slot duration constant before proving our derivations by comparing our analytical equations to numerical results.

II. SYSTEM MODEL WITH DIFFERENT SLOT DURATIONS CONSIDERING THE CAPTURE EFFECT

In this section we introduce the Time and Collision Recovery Aware reading efficiency η_{TCA}. The main properties for this efficiency are the consideration of the different slot durations and the capture probability. Firstly, we consider the different slot durations. The new time aware reading efficiency is then given by:

$$\eta_{TA} = \frac{S \cdot t_s}{E \cdot t_e + S \cdot t_s + C \cdot t_c}, \tag{1}$$

where t_e, t_s and t_c are the durations of the empty, the successful, and the collided slots, respectively. In addition, $E = L\left(1 - \frac{1}{L}\right)^n$, $S = n\left(1 - \frac{1}{L}\right)^{n-1}$, and $C = L - E - S$.

Adding the capture effect assuming a capture probability α, the actual values of E, S, and C have to be converted to E_c, S_c and C_c, respectively [5]. Their relation is given by:

$$E_c = E,\, S_c = S + \alpha \cdot C,\, C_c = (1-\alpha) \cdot C. \tag{2}$$

Basically, collided slots are converted to successful slots with probability α. From (2), the time and capture probability aware efficiency [7] can then be written as:

$$\eta_{TCA} = \frac{t_s \cdot S_c}{t_e \cdot E_c + t_s \cdot S_c + t_c \cdot C_c}. \tag{3}$$

The goal is now the maximization of η_{TCA} for obtaining the maximum reading efficiency.

III. DERIVATION OF THE PROPOSED CLOSED FORM SOLUTION FOR THE OPTIMAL FRAME LENGTH

In this section we derive the proposed optimal time and capture-aware frame size L_{TCA} that takes into consideration the different slot durations and the capture effect. This is achieved by differentiating the reading efficiency η_{TCA} of (3) with respect to the frame length L and equate the result to zero.

Clearly, the frame length L is an integer value. Therefore, differentiating the equation is not fully correct. However, we will later show that the resulting error is negligible. According to (2), E_c, S_c, and C_c are a function of L. However, t_e, t_s, t_c are constants for a given system configuration. Thus, the equation can be simplified to:

$$\frac{(t_eE_c + t_sS_c + t_cC_c)t_s\frac{d}{dL}(S_c) - t_sS_c\frac{d}{dL}(t_eE_c + t_sS_c + t_cC_c)}{(t_eE_c + t_sS_c + t_cC_c)^2} = 0. \tag{4}$$

After multiplying both sides by the denominator and dividing by t_s (non-zero constant), and subtracting the term, which is multiplied by t_s, the equation results in:

$$(t_eE_c + t_cC_c)\frac{d}{dL}(S_c) = S_c\frac{d}{dL}(t_eE_c + t_cC_c). \tag{5}$$

Then, the substitution by the values of E_c, S_c, and C_c from (2) leads to the final exact equation for the proposed time and capture probability aware frame length:

$$\begin{aligned}(1-\alpha)\cdot(1-\frac{n}{L_{TCA}}) + \alpha\cdot C_t\cdot\left(1-\frac{1}{L_{TCA}}\right)&\\ +(C_t-1)\cdot(1-\alpha)\cdot\left(1-\frac{1}{L_{TCA}}\right)^n &= 0,\end{aligned} \tag{6}$$

where n is the number of tags, and C_t is the slot duration constant defined as $C_t = \frac{t_e}{t_c}$. (6) shows the exact relation between the proposed capture and time-aware optimum frame length L_{CTA} and the number of tags n. This equation takes into consideration the capture effect and the different slot durations. However, this solution depends on a recursive equation. For reaching an explicit equation, (6) can be expressed as:

$$\begin{aligned}(1-\alpha)\cdot(1-\frac{1}{\beta}) + \alpha\cdot C_t\cdot\left(1-\frac{1}{\beta n}\right)&\\ +(C_t-1)\cdot(1-\alpha)\cdot\left(1-\frac{1}{\beta n}\right)^n &= 0,\end{aligned} \tag{7}$$

where $\beta = \frac{L_{TCA}}{n}$. As we are focusing on systems with a large number of tags n, we can use the approximation

$$\lim_{n\to\infty}\left(1-\frac{1}{\beta n}\right)^n = e^{-\frac{1}{\beta}}, \tag{8}$$

which simplifies (7) to:

$$\begin{aligned}(1-\alpha)\cdot\left(1-\frac{1}{\beta}\right) + \alpha\cdot C_t\cdot\left(1-\frac{1}{\beta n}\right)&\\ +(1-\alpha)\cdot(C_t-1)\,e^{-\frac{1}{\beta}} &= 0.\end{aligned} \tag{9}$$

The analysis of (6) indicates that the relevant values for $\beta = \frac{L_{TCA}}{n}$ are in the region close to one [8]. Hence, we can develop a Taylor series for $e^{-\frac{1}{\beta}}$ around one which leads to:

$$e^{-\frac{1}{\beta}} \simeq 1 - \frac{1}{\beta} + \frac{1}{2\beta^2}. \tag{10}$$

After substituting (9) and some additional simplifications, we get:

$$\beta^2 C_t - \beta C_t (1 + \frac{\alpha}{n}) - 0.5\,(1-\alpha) \cdot (1 - C_t) = 0. \tag{11}$$

As $\frac{\alpha}{n} \ll 1$, (11) can be expressed as:

$$\beta^2 C_t - \beta C_t - 0.5\,(1-\alpha) \cdot (1 - C_t) = 0. \tag{12}$$

By solving (12) and rejecting the negative solution we finally reach to the final solution:

$$L_{TCA} = \frac{n}{2}\left((1-\alpha) + \sqrt{(1-\alpha)^2 + \frac{2}{C_t}\,(1-\alpha)\cdot(1-C_t)}\right). \tag{13}$$

As the optimization of (3) is a convex optimization problem, and there exits only one non-negative solution of (12), (13) leads to the global optimum. According to the literature, [7] is the only work who used the same performance metric η_{TCA} to get the optimum frame length. However, in this work the authors did not propose any closed form solution and they have to rely on Multi-dimensional look-up tables. According to the EPCglobal C1 G2 standard [1], the proposed frame length in equation (13) takes only discrete values of power 2. Therefore, the possible quantized values of the optimum frame length can be expressed as:

$$L_{QTCA} = 2^{\text{round}(\log_2 L_{TCA})}. \tag{14}$$

We will proof in the simulation results that L_{QTCA} leads to the optimum solution.

IV. CAPTURE PROBABILITY AND SLOT DURATION CONSTANT CALCULATIONS

In this section we will discuss in details how can we calculate the slot duration constant C_t and the capture probability from the physical layer:

A. *Slots Duration Constant calculation*

Based on the EPCglobal C1 G2 standard [1], the slot duration constant C_t is $C_t = \frac{t_e}{t_c}$, where t_e is given by:

$$t_e = T_{QRep} + T_1 + T_3. \tag{15}$$

Here, T_{QRep} is the query repeat command time, $T_{QRep} = 3.5 \cdot DR \cdot T_{pri}$, $T_{pri} = \frac{1}{BLF}$, BLF is the backscatter link frequency of the tag reply, and DR is the so-called divide ratio constant that can take the two values $DR = 8$ or $\frac{64}{3}$. Next, T_1 is the time from the reader transmission to the tag response with $T_1 = \max\{DR \cdot T_{pri},\, 10 \cdot T_{pri}\}$. Finally,$T_3$ is the time that the reader waits after T_1 before issuing another command. As it has no constrains it can be assumed to be zero.

Next, t_c is given by:

$$t_c = T_{QRep} + T_1 + T_2 + T_{RN16}, \tag{16}$$

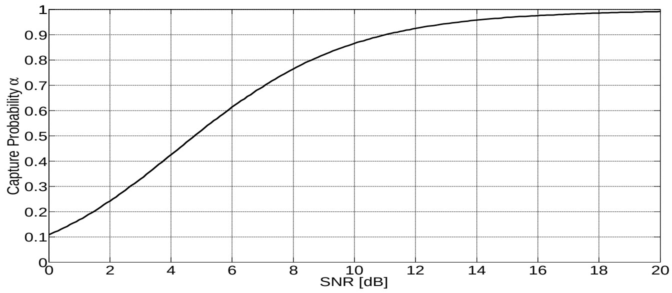


Figure 1: Capture probability versus the signal to noise ratio

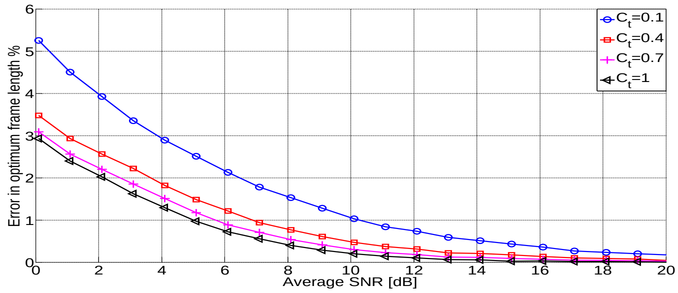


Figure 2: Effect of the SNR measurement error on the proposed formula using sampling rate $f_s = 8\,\text{MHz}$, and symbol rate $640\,\text{kHz}$

where T_2 is the reader response time starting from the end of the tag response, $T_2 = 6 \cdot T_{pri}$, and T_{RN16} is the duration of the 16 bits temporary identifier reply of the tag. It includes 16 bits data, 6 bits preamble, n_p pilot tones, i.e. $T_{RN16} = (22 + n_p) \cdot T_{pri}$. After substituting (15) and (16) in C_t equation, the the final expression of C_t is:

$$C_t = \frac{3.5 \cdot DR + \max\{DR,\, 10\}}{3.5 \cdot DR + \max\{DR,\, 10\} + 6 + 22 \cdot M + n_p \cdot M}. \tag{17}$$

where M equals to 1, 2, 4, or 8 which represents the modulation types FM0, Miller 2, 4, or 8, respectively.

B. *Capture Probability Calculation*

The capture probability varies in the range of $0 \leq \alpha \leq 1$. Its value depends on the Signal to Noise Ratio (SNR). In this work, we measure the SNR for each slot. Then, we calculate the average SNR per frame. In [10], the authors proposed a method to capture the strongest tag reply based the physical layer properties. They have proposed a Bit Error Rate (BER) curve versus the SNR. We want to calculate the capture probability for a complete collided RN16 packet, which includes 16 random successive bits. The BER is mapped to Packet Error Rate (PER) by simulation as the channel is not Binary Symmetric Channel (BSC). The capture probability can be expressed as: $\alpha = (1 - PER)$. Figure 1 presents the values of the capture probabilities versus the average signal to noise ratio per frame. In this work, we calculate the average capture probability from the corresponding average SNR at the current frame.

V. SIMULATIONS RESULTS AND COMPARISONS

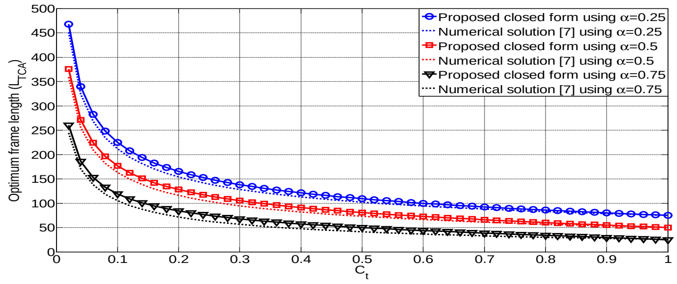
Figure 2 shows the relative error of the proposed formula (13) due to the average SNR measurement error. According to the

European standard [1], the bandwidth of the UHF RFID systems is 4 MHz, so we used a sampling frequency of $f_s = 8\,\text{MHz}$. To clarify the worst case effect of the estimation error, we used the highest symbol rate 640 kHz in this simulations. The SNR per slot is measured at the first part of each slot T_1. The average SNR per frame is calculated as $E\left\{\frac{|h|^2 \cdot x^2}{\sigma^2}\right\}$, where σ is the standard deviation of the Additive White Gaussian Noise (AWGN) per slot, and we used normalized signal power i.e. $E\left\{x^2\right\} = 1$. Based on [10], we assumed that the equivalent channel coefficients h follow Rayleigh fading. The single Rayleigh channel coefficients are independent zero mean circularly symmetric complex Gaussian random variables with normalized energy $E\left\{|h|^2\right\} = 1$, and all tags are statistically identical, which means all of them experience the same path loss. Therefore, the average SNR per frame is $E\left\{\frac{1}{\sigma^2}\right\}$. Afterwards, we get the error in the measured SNR by comparing the measured value by the nominal one. According to the relation between the capture probability α and the SNR (shown in figure 1), and the relation between the proposed optimum frame length and the capture probability in (13), the estimation error of the measured SNR is mapped to an error of the proposed frame length versus the SNR. Based on Figure 2, for a constant value of the slot duration constant C_t, the error in the frame length decreases when the SNR increases. Moreover, when the slots duration constant C_t decreases, the error in the proposed formula increases up to 5% at zero dB SNR, which is negligible.

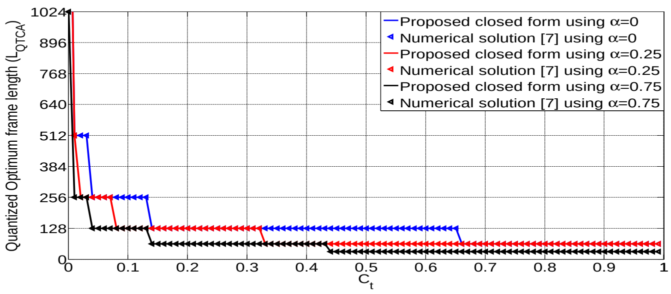
Now we will compare the proposed closed form equation in (13) with the numerical results in [7]. Figure 3a shows the non-quantized optimal frame length of both algorithms for the full range of the slot duration constant C_t. Assuming a fixed number of $n = 100$ tags, figure 3a indicates that the proposed closed form approaches the numerical solution for different α in the full range of C_t. Due to the proposed approximations, there are slight differences between the proposed closed form and the numerical solution, where the numerical is the exact solution. According to the EPCglobal C1 G2 standard [1] the frame length is allowed to take only quantized values (power of 2). Figure 3b shows that the proposed frame length fully match the numerical solution in the full range of C_t.

VI. Conclusions

This paper proposes a novel closed form solution for analytically calculating the optimum frame length for Frame Slotted ALOHA. We especially consider the effects of unequal slot durations and the capture probability, which have to be taken into account for optimizing the reading speed in real RFID systems. The theoretical derivations lead to a new analytical optimum frame length equation that can be easily implemented in RFID readers. The optimum frame length for different slot durations in addition to the capture effect has been previously addressed in literature. However, the authors did not derive any any closed form solution for this problem. Our proposed frame equation gives a direct linear relation with the number of tags in the reading area instead of using Multi-dimensional look-up tables. Furthermore, our analytical solution can be used for further optimizations of RFID systems. Simulation results prove the correctness of our analytical solution.



(a) Non-quantized Frame Length



(b) Quantized Frame Length

Figure 3: Optimum frame length L_{TCA} as a function of the slot duration constant C_t ($n = 100$ tags) for the capture probabilities $\alpha = 0.25,\ 0.5,\ 0.75$.

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