ALOHA Frame Length Optimization Using Multiple Collision Recovery Coefficients

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***Abstract*—Calculating the optimal frame length for Frame Slotted ALOHA in RFID systems is a critical issue as it highly affects the reading efficiency, and hence the reading time. Previous studies have focused on the frame length calculations using the conventional Framed Slotted ALOHA (FSA) algorithms. In such a systems, only answers of a single tag is considered as a successful slot, and if multiple tags respond simultaneously, a collision occurs, then all the replied tags are discarded. However, the modern system have the capability of recovering this collision and convert the collided slot into a successful slot. Other studies focused on calculating the optimal frame length taking into consideration the collision recovery probability. However, these studies have assumed a constant collision recovery probability coefficients, i.e. the probability to recover one tag from i-tags is constant regardless the value of i. In this work we propose a novel closed form solution for the optimal FSA frame length which considers the differences in the collision recovery probabilities. Timing comparisons were presented in the simulation results to show the saving in reading time using the proposed frame length compared to the other proposals.**

I. INTRODUCTION

In the recent years, the number of applications that use Radio Frequency Identification Systems (RFID) has increased, and the reading speed became one of the most critical issues in these applications. Such RFID networks consist of: 1) Readers (Interrogators), which are responsible of scanning the interrogation area and identifying the tags. 2) Tags (Transponders), which store the data to be read by the readers. In RFID systems, the tags typically share a common communications channel. Thus, there is a certain probability of tag-collisions, i.e. multiple tags answer simultaneously. This collision probability naturally increases in dense networks with many tags. Since passive tags are the most practical tags in the market, because of their low price and simple design, they cannot sense the channel or communicate with the other tags. A a result, the reader is responsible of coordinating the network and has to avoid tags collisions using specific anti-collision algorithms. This work focuses mainly on Ultra High Frequency tags which follow epcglobal class 1 gen 2 standards [1].

According to epcglobal class 1 gen 2 standards [1], the conventional anti-collision algorithm is Framed Slotted ALOHA (FSA) algorithm, which is only Medium Access Control (MAC) layer protocol. In this algorithm, only the single tag reply (successful slot) are able to be decoded and then identified. Therefore The conventional definition of the expected reading efficiency η_{conv} is equivalent to the probability of success $P(S)$ [2]:

$$\eta_{conv} = P(S) = P(1), \tag{1}$$

where, $P(1) = \frac{n}{L}\left(1 - \frac{1}{L}\right)^{n-1}$, n presents the number of tags in the reading area, and L is the frame length.

The main goal is to find the optimal frame length L, which maximizes the reading efficiency η_{conv}. Based on (1), the reading efficiency η_{conv} is maximized to be $\eta_{conv(max)} = 36\%$ when $L = n$ as shown in [2]. Some research groups: Bo Li [3], and Xi Yang [4] have concetrated more on the effect of the collision resolving probability on the optimum value of the FSA frame length. They have proposed another equation for the reading efficiency:

$$\eta_{PHY2} = P(1) + \alpha \cdot P(c), \tag{2}$$

where, $P(c)$ is the probability of collision, α is the average collision resolving probability coefficient. In this efficiency equation, the RFID reader can convert some of the collided slots in to successful based on this new efficiency, the authors then have calculated a closed form for the optimum frame length:

$$L_{opt} = \alpha + (1 - \alpha) \cdot n \tag{3}$$

The authors here have assumed that the value of α could be equal to the coefficient of the collision resolving probability for only two tags collided. However this is not an accurate assumption, because the collision recovery probability should not be constant value, it should decrease with increasing the number of collided tags. Therefore they assume that the reader have constant collision resolving capability regardless the number of collided tags per slot. Other research groups: Christoph Angerer [5], Kaitovic [6], and D. De Donno [7] considerd the limited RFID readers capability of collision resolving: They have used the characteristics of the RFID signals to separate signals from collisions at the physical layer. They have proposed a new reading efficiency metric which includes the tags which are recovered based on the PHY layer work. This reading efficiency can be expressed as:

Table I
OPTIMAL FRAME LENGTH AND EXPECTED THROUGHPUT FOR DIFFERENT COLLISION RESOLVING CAPABILITIES

M	L/n	Expected reading efficiency
1	1	0.368
2	0.707	0.587
3	0.55	0.726
4	0.452	0.817

$$\eta_{PHY} = \sum_{i=1}^{M} P(i), \tag{4}$$

where $P(i) = \binom{n}{i}\left(\frac{1}{L}\right)^{i}\left(1-\frac{1}{L}\right)^{n-i}$, and M presents the number of collided tags that the receiver is capable of recovering one tag from them. The authors assumed that the probability to recover one tag from i collided tags is equal to 100% independent on the value of i. However in reality, the probability to recover a single tag from i collided tags is inverse proportional with the number of collided tags i. Moreover, there is no practical readers have 100% collision resolving probability. According to the efficiency equation in (4), the authors proposed fixed values of the optimum FSA frame length based on the collision resolving capability of the RFID reader, as shown in table I . they calculated these values numerically by searching for the frame length which maximize their proposed reading efficiency. In this paper we propose a new reading efficiency metric called Multiple Collision Recovery Coefficients Reading Efficiency η_{MCRC} includes a different collision resolving coefficient for each number of collided tags, hence we propose a novel closed form solution for the optimum FSA frame length at RFID systems. The proposed solution gives a direct relation between the frame length and the number of tags n in the reading area in addition to the collision recovery coefficients.

II. SYSTEM MODEL UNDER MULTIPLE COLLISION RECOVERY COEFFICIENTS

In this section, we present a new FSA reading efficiency called Multiple Coefficients Collision Recovery Reading Efficiency η_{MCRC}. The main contribution in this proposed efficiency is: It contains a unique collision recovery coefficient α_i for each probability of collision $P_{col.}(i)$. These new coefficients indicates the ability of the reader to recover i collided tags, where this ability is different based on the number of collided tags. The proposed reading efficiency η_{MCCR} can be expressed as:

$$\eta_{MCRC} = P(1) + \sum_{i=2}^{n} \alpha_i P_{col.}(i) \tag{5}$$

where,

$$0 \le \alpha_i \le 1 \tag{6}$$

.

Figure 1 presents the distribution of the average collision probability in a frame length $0.5 \le \frac{L}{n} \le 2$ uniformly, which is the practical range of the frame length. According to figure 1, the probability that the collided slot is two, three, or four collided tags is equal to $P_{col.}(2) + P_{col.}(3) + P_{col.}(4) \simeq 96\%$, and the remaining tag collisions $\sum_{i=5}^{n} P_{col.}(i) \simeq 4\%$. Moreover, the values of the collision recovery coefficient α_i decrease with increasing of the number of collided tags i.

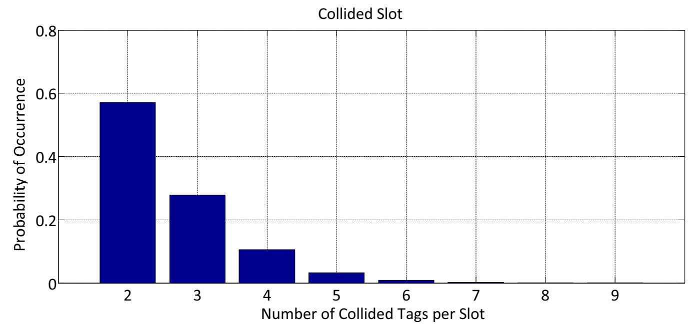

Figure 1. Collision distribution probability in FSA, under condition of $\frac{n}{2} \le L \le 2n$

Therefore, the proposed η_{MCRC} for the practical RFID environment can be expressed as shown:

$$\eta_{MCRC} = P(1) + \alpha_2 P_{col.}(2) + \alpha_3 P_{col.}(3) + \alpha_4 P_{col.}(4), \tag{7}$$

where, α_2,α_3, and α_2 are respectively the second, third, and fourth collision recovery coefficients.

$$\alpha_2 \ge \alpha_3 \ge \alpha_4 \tag{8}$$

III. PROPOSED CLOSED FORM FRAME LENGTH

The next step is to derive a closed form for the new optimum frame length $L_{opt-MCRC}$ under the multiple collision recovery coefficients environment. $L_{opt-MCRC}$ can be optimized by finding the value of L which maximizes η_{MCRC}. According to [8], if $L \gg 1$, and $n \gg i$:

$$P(i) \simeq \frac{1}{i!} \cdot \beta^{-i} \cdot e^{-\frac{1}{\beta}}, \tag{9}$$

where, $\beta = \frac{L}{n}$. After substituting by (9) in (7) we get:

$$\eta_{MCRC} = e^{-\frac{1}{\beta}} \cdot \left(\beta^{-1} + \frac{\alpha_2}{2}\beta^{-2} + \frac{\alpha_3}{6}\beta^{-3} + \frac{\alpha_4}{24}\beta^{-4}\right) \tag{10}$$

Now we have to find the value of β which maximizes η_{MCRC}. This is achieved by differentiating the reading efficiency in (10) with respect to the frame length β and equate the result to zero:

$$\frac{\partial \eta_{MCRC}}{\partial \beta} = 0 \tag{11}$$

After differentiating, the equation can be simplified as:

$$-e^{-\frac{1}{\beta}} \cdot \Bigg(\beta^{-2} + \beta^{-3}(\alpha_2 - 1) + \frac{\beta^{-4}}{2}(\alpha_3 - \alpha_2) + \frac{\beta^{-5}}{6}(\alpha_4 - \alpha 3) + \frac{\beta^{-6}.\alpha_4}{24}\Bigg) = 0 \tag{12}$$

After multiplying the equation by $-e^{\frac{1}{\beta}} \cdot \beta^{6}$, the equation finally results in:

$$\underbrace{1}_{a}\beta^4 + \underbrace{(\alpha_2 - 1)}_{b}\beta^3 + \underbrace{\frac{(\alpha_3 - \alpha_2)}{2}}_{c}\beta^2 + \underbrace{\frac{(\alpha_4 - \alpha_3)}{6}}_{d}\beta - \underbrace{\frac{\alpha_4}{24}}_{e} = 0 \quad (13)$$

Equation (13) has four roots [9]:

$$\beta_{1,2} = -\frac{b}{4a} - S \pm 0.5\sqrt{\underbrace{-4S^2 - 2P + \frac{q}{S}}_{X}} \quad (14)$$

$$\beta_{3,4} = -\frac{b}{4a} + S \pm 0.5\sqrt{\underbrace{-4S^2 - 2P - \frac{q}{S}}_{Y}},$$

where

$$P = \frac{8ac-3b^2}{8a^2},\ q = \frac{b^3-4abc+8a^2d}{8a^3}$$

and where

$$S = 0.5\sqrt{-\frac{2}{3}P + \frac{1}{3a}\left(Q + \frac{\triangle_0}{Q}\right)}, \quad Q = \sqrt[3]{\frac{\triangle_1 + \sqrt{\triangle_1^2 - 4\triangle_0^3}}{2}}$$

with

$$\triangle_0 = c^2 - 3bd + 12ae, \qquad \triangle_1 = 2c^3 - 9bcd + 27ad^2 - 72ace$$

According to practical ranges for the collision recovery coefficients $\alpha_i' s$ in (6) and (8), we can proof that the signs of the polynomial coefficients are constants and not changing in all ranges of $\alpha_i' s$ and can be as follows:
$a = +ve$, $b = -ve$, $c = -ve$, $d = -ve$, and $e = -ve$.
Using Descartes' rules of sign [10], we can count the number of real positive solutions that a polynomial has.

Assume that the polynomial in (13) is $P(\beta)$, let ν be the number of variations in the sign of the coefficients a, b, c, d, e. So $\nu = 1$. Let n_p be the number of real positive solutions. According to Descartes' rules of sign [10]:

- $n_p \leq \nu$ which means that $n_p = 0\, or\, 1$.
- $\nu - n_p$ is an even integer. Therefore $n_p = 1$.

So there is only one valid real positive solution for the equation, and our target to identify which solution from the four solutions is the valid one.

According to that we have one real solution, there are two possibilities for the solutions:

1) One positive real solution and the remaining three solutions are negative. In this case, all the solutions are real and we need just to identify what is the root which has the largest values from the four solutions. According to (14), it is clear that the value of the square roots $\sqrt{X}$ and $\sqrt{Y}$ are positive real, therefore $\beta_1 > \beta_2$ and also $\beta_3 > \beta_4$. and the value of S should be also positive real, so $\beta_3 > \beta_1$ which means in this case that our solution is β_3.
2) Two complex solutions, one real positive solution, and one negative solution. In this case, we have either $\beta_{1,2}$ or $\beta_{3,4}$ real solutions so S should be positive real number and the complex value comes only from the square roots $\sqrt{X}$ and $\sqrt{Y}$. According to the coefficients signs: q must be always negative real value. Therefore, in (14) the value of $X < Y$. So $\beta_{1,2}$ must be the complex roots, and as mentioned before that $\beta_3 > \beta_4$, so β_4 is the negative root and β_3 is the positive real root.

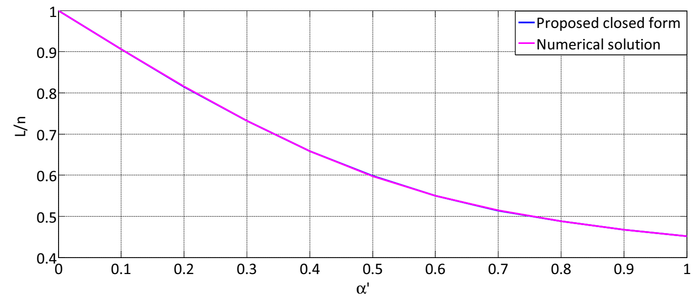

Figure 2. Frame length comparision between the closed form solution and the numerical solution

Based on the above discussion, the proposed closed form optimum frame length $L_{opt-MCRC}$ under the multiple collision recovery coefficients environment:

$$L_{opt-MCRC} = \left(-\frac{b}{4a} + S + 0.5\sqrt{-4S^2 - 2P - \frac{q}{S}}\right) \cdot n \quad (15)$$

According to (15), the proposed equation gives a linear relation wrt. the number of tags n, and includes the effect of different collision recovery coefficients. The values of these coefficients are set based on the powerful of the physical layer part of the RFID receiver as shown in [11]. Based on (15), if the RFID reader has no collision resolving capability ($\alpha_2 = \alpha_3 = \alpha_4 = 0$). In this case $L_{opt-MCRC} = n$ which is equal the frame length in the conventional case. If the RFID reader has a full collision resolving capability for the two, three, and four collided tags per slot ($\alpha_2 = \alpha_3 = \alpha_4 = 1$). In this case $L_{opt-MCRC} = 0.452 \cdot n$ which match the results in [5]. Figure 2 shows a frame length comparision between the proposed closed form solution and the numerical solution. We have assumed that $\alpha_2 = \alpha_3 = \alpha_4 = \alpha'$ to be able to scann the full ranges of $\alpha' s$.

IV. SIMULATION RESULTS

In this section, we compare the performance of the proposed frame length compared to the main two groups who proposed an optimum frame lengths for FSA under collision recovery environment. In our comparisons we have assumed that $\alpha_2 = \alpha_3 = \alpha_4$ with different values scanning the complete range. However the proposed solutions allow them to take different values:

- First [3], the authors used the reading efficiency equation in (2) assuming constant collision recovery coefficient $\alpha = \alpha_2$ regardless the number of collided slots. According to this assumption the author reached to the closed for solution for the optimum frame length in (3). Figure 3 shows the total number of slots required using the

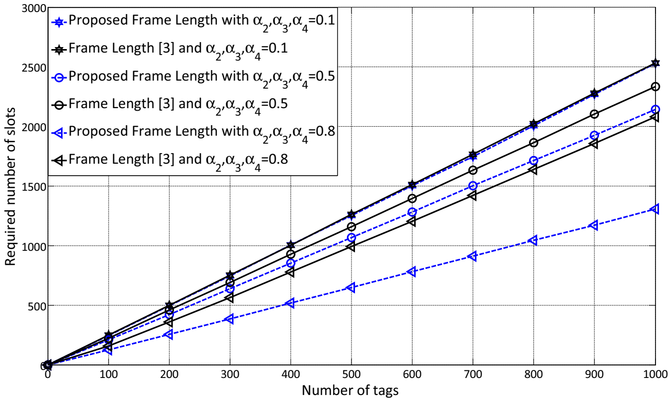

Figure 3. Time slots between the proposed and [3] frame length solutions using different values of collision recovery coefficients

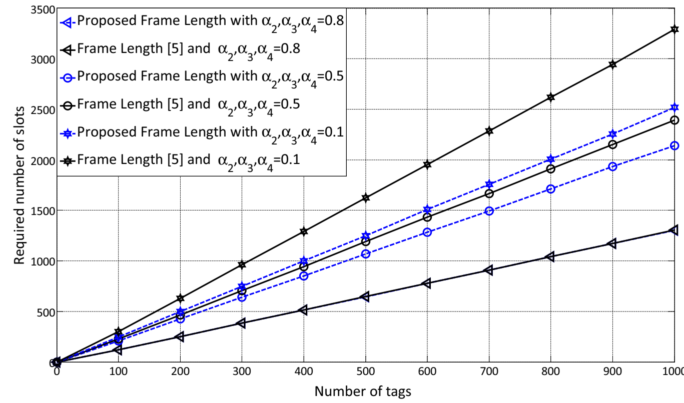

Figure 4. Time slots between the proposed and [5] frame length solutions using different values of collision recovery coefficients

proposed frame length compared to the frame length in [3] to identify bunch of tags. According to figure 3, for low performance readers, the collision recovery probability coefficients are very low ($\alpha_2 = \alpha_3 = \alpha_4 = 0.1$). In this case, the proposed frame length has the same performance of the frame length proposed in [3], as the value remaining terms of $\alpha \cdot \sum_{i=5}^{n} P_{col.}(i)$ can be neglected. However, as soon as the collision recovery coefficients are more than 0.5, the term of $\alpha \cdot \sum_{i=5}^{n} P_{col.}(i)$ will affect the value of the optimal frame length, therefore the proposed system which consider this practical limitations gives better performance when $\alpha's > 0.5$.

- Second [5], the authors assume limited collision recovery capability. However, they have assumed a unity collision recovery coefficients $\alpha's = 1$ as shown in (4). Based on this efficiency equation, the authors proposed numerical results for the optimum frame length based on the collision recovery capability of the reader. Figure 4 presents the total number of slots required using the proposed frame length compared to the frame length in [5] to identify bunch of tags. According to figure 4 In case of high performance readers ($\alpha_2 = \alpha_3 = \alpha_4 \geq 0.8$), the the proposed frame length has the same performance of the frame length proposed in [5]. However in case of ($\alpha's < 0.6$), the proposed frame length gives better performance.

According to the above comparison, the proposed frame length gives an accurate results in the full range of the collision recovery probability coefficients. The proposed system can get the values of the collision recovery coefficients per frame from the physical layer part as shown in [11].

V. Conclusion

This paper proposes a novel closed form solution for the optimum value of FSA frame length in RFID systems. The proposed equation takes the effect of the practical limitations and the differences in collision recovery probability coefficients of the RFID readers. The theoretical derivations lead to a new optimization criterion that can be easily implemented in RFID readers. The proposed frame length equation gives the most accurate results in the complete range of the collision recovery coefficients $\alpha's$ compared to the other solutions. Timing comparisons were presented to shows the saving in reading time using the proposed frame length compared to the other proposals.

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