A Stochastic Shortest Path Model to Minimize the Reading Time in DFSA-Based RFID Systems

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***Abstract*—RFID systems implementing Dynamic Frame Slotted Aloha (DFSA) can adjust the number of identification rounds (slots) within an inventory cycle (frame). The usual approach to reduce the identification time of the tag population is to select the frame size attaining the highest throughput in the frame. However, it is more accurate to minimize the identification time of all the tags considering an indefinite long decision horizon. This is done in this paper by means of a Stochastic Shortest Path (SSP) formulation that incorporates capture effect and differentiation among slot durations. Our results show that the optimal policy is even faster than previous approaches.**

***Index Terms*—Radiofrequency identification, framed slotted Aloha, stochastic shortest path, probability analysis.**

I. Introduction

In Passive Radio Identification (RFID) systems, Frame Slotted Aloha (FSA) is one of the most prevalent anti-collision protocols in the UHF band. In FSA, inventory cycles (frames) start with a *Query* packet. These frames consist of l interrogation rounds (slots) delimited by *Query Rep* packets. EPCglobal UHF Class 1 Generation 2 standard [1] specifies that the number of slots in a frame (frame size) must be in the set $\mathbb{L} = \{l = 2^Q : Q \in [0, 1, \ldots, 15]\}$. The value of the parameter Q is sent in the *Query* packets. Each unidentified tag within the interrogation region selects randomly (with uniform distribution) one slot to perform its identification. Correctly identified tags withdraw from the identification process. Colliding tags will try to identify again in the following frame. Fig. 1 shows the duration of different command sequences corresponding to successful, idle and collision slots respectively. The detailed configuration of the timing parameters is provided in [2]. Note that different configurations would result with different command durations.

Depending on the number of unidentified tags, long frames may be inefficient, requiring an unnecessary long time to identify all the tags. On the other hand, short frames may experience excessive collisions which also increases the identification time. For this reason RFID standards currently include Dynamic FSA (DFSA), capable of adjusting the frame size dynamically. Two operation modes are possible: slot-by-slot, in which the system can interrupt the ongoing frame at any slot and start a new frame with a new *Query* (containing an updated Q value); and frame-by-frame, in which the Q value is only applied when the current frame finishes. This paper focuses on the latter mode.

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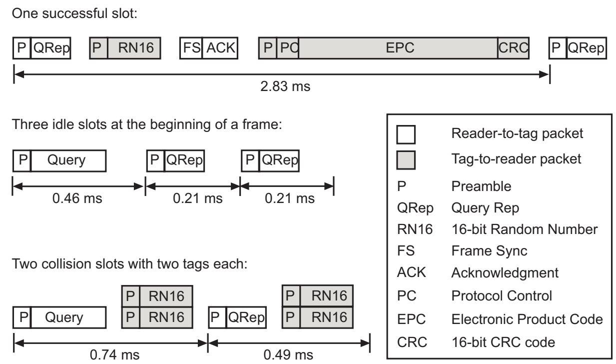

Fig. 1. Durations of different command sequences in an ISO 18000-6C frame computed from the timing parameters specified in [2].

In this paper we consider a reader covering a known number N of unidentified tags. Tags do not move (*static* situation) and only leave the reader's identification region once all of them are identified. Therefore, at every cycle, the system knows the number of unidentified tags i under coverage. The objective is to select, at the beginning of each cycle, the frame size l attaining the minimum expected identification time for the whole tag population. The mapping between i and the selected l is referred to as the *policy*. The static situation described is usual in inventory processes where readers have to identify long sequences of pallets carrying a fixed number of tags.

Our main contribution is a Stochastic Shortest Path (SSP) formulation for the problem described that, in contrast to previous approaches, comprises capture effect and does not assume equal duration for the time-slots of the frame. Our results show that an SSP-based policy identifies the tags faster than a policy maximizing the expected number of identified tags per frame (throughput). The rest of the paper is organized as follows: After discussing related work in Section II we explain, in Section III, how to compute the transition probabilities of the Markov model under capture effect. The SSP formulation and the numerical results are presented in Sections IV and V respectively, and finally Section VI summarizes the conclusions.

II. Related Work

Previous works on DFSA have mainly focused on developing accurate estimators $\hat{N}$ for the tag population N (see [3], [4] and the references therein), and then selecting the frame length l according to $\hat{N}$. The right choice for l is widely assumed to be the one maximizing the throughput per frame. While most works use the frame length closest to N, [4] showed that this strategy is suboptimal when $l \in \mathbb{L}$ and explained how to compute the optimal Q value. In this paper we focus on deriving the best policy for selecting l given N, and the main difference is that the SSP optimizes the duration of the whole identification process, instead of the duration of an isolated frame or the number of slots. In Section V we compare our approach to the policies aimed at throughput maximization. Note that we assume a perfect knowledge of N and therefore the issue of robustness against estimation inaccuracies is outside the scope of this work.

In [5] a different approach was presented, based on optimizing the total identification time by means of a Markov Decision Process which, in contrast to ours, did not consider capture effect and assumed equal duration for collision and idle slots. In a capture effect environment, [6] derived an approximated expression for the frame length attaining the maximum throughput per frame, that we use also later in numerical comparisons.

III. System Model

The system described in the introduction can be modeled by a discrete time Markov chain (DTMC) where the state of the system is determined by the number of unidentified tags under reader's coverage. The Q parameter selection problem is formulated as a Markov Decision Process (MDP), which implies the transition probabilities of the DTMC to be defined in terms of the controlled parameter, which in our case is the frame size $l = 2^Q$. Therefore, our objective is to find the expression for $p_{xy}(l)$ where x is the initial state and y is the destination state for the nontrivial cases where $y \leq x$ ($p_{xy}(l) = 0$ for $y > x$).

Let us define the function $F(k, s, x, l)$ providing the number of ways that a frame of size l can accommodate s successful slots and k collision slots when x tags respond. $F(k, s, x, l)$ is basically the multiplication of two combinatorial numbers: 1) the number of ways that s tags out of x can be successfully identified in a frame with l slots, and 2) the number of ways that the unidentified tags can be allocated into k slots.

Let us address the first combinatorial number. The number of ways that s single responses can be distributed into l slots is given by the binomial coefficient[1] $C(l, s) = \binom{l}{s}$. Since any of the x tags in the system can choose each of the s successful slots, we have to multiply previous quantity by the number of different permutations of s elements taken from a set of x elements. This number is given by the well known k-permutation formula $P(x, s)$=$(x)_s$=$x\,(x-1)\ldots(x-s+1)$. Then, the first quantity is given by $C(l, s)P(x, s)$.

To obtain the second number, let us consider the k collision slots, which are spread over the remaining $l-s$ slots. Similarly as before, there are $C(l-s, k) = \binom{l-s}{k}$ ways of selecting k elements from a set of $l-s$ elements. We must multiply this amount by the number of possible ways of allocating the $j = x - s$ colliding tags into the k collision slots. Because a collision implies that at least two tags respond on the same slot, we can only consider allocations where the number of tags per slot is 2 or higher. The amount of possible allocations is given by the following expression: $S_2\,(j, k)\,k!$ where $S_2\,(j, k)$ is the 2-associated Stirling number of the second kind, which provides the number of ways to partition a set of j objects into k subsets with 2 or more elements. The multiplication by $k!$ is to account for the possible permutations of the k slots (urns or subsets in combinatorial terms). Therefore the function $F(k, s, x, l)$ has the following form

$$F(k, s, x, l) = C(l, s)P(x, s)C(l-s, k)S_2\,(j, k)\,k! \quad (1)$$

Given j responses colliding in k time-slots, let $\Phi(j, k)$ denote the set of possible arrangements for these responses. An arrangement $\boldsymbol{\phi} \in \Phi(j, k)$ is a vector of integers $\boldsymbol{\phi} = (\phi_2, \ldots, \phi_j)$ where $\phi_i \in [0, \ldots, k]$ indicates the amount of slots with i responses. For each $\boldsymbol{\phi}$, we define the sets $\Gamma\,(\boldsymbol{\phi}, c)$ = $\{(c_2, \ldots, c_j) : c_i \leq \phi_i, \sum c_i = c\}$ where c_i represents the number of captures out of the ϕ_i slots with i responses. Considering that p_i denotes the capture probability in a slot with i responses, the conditional probability of capturing c responses for a given $\boldsymbol{\phi}$ is

$$p(c|\boldsymbol{\phi}) = \sum_{\mathbf{c} \in \Gamma(\boldsymbol{\phi}, c)} \prod_{i=2}^{j} \binom{\phi_i}{c_i} p_i^{c_i} (1-p_i)^{(\phi_i - c_i)} \quad (2)$$

Additionally, the probability of a particular arrangement $\boldsymbol{\phi}$ is

$$p(\boldsymbol{\phi}|j, k) = \frac{\binom{j}{2I_{\phi_2}, \ldots, jI_{\phi_j}} \binom{k}{\phi_2, \ldots, \phi_j}}{S_2\,(j, k)\,k!} \quad (3)$$

where $I_{\phi_i} = 0$ if $\phi_i = 0$, and $I_{\phi_i} = 1$ otherwise. With (2) and (3) we can obtain the conditional probability

$$p(c|j, k) = \sum_{\boldsymbol{\phi} \in \Phi(j,k)} p(c|\boldsymbol{\phi})p(\boldsymbol{\phi}|j, k) \quad (4)$$

The number of identified tags is determined by the number of captures c and the number of slots with a single response s. The transition probability $p_{xy}(l)$ is then given by

$$p_{xy}(l) = \sum_{s,c:s+c=x-y} \sum_{k=1}^{k_M} \frac{F(k, s, x, l)}{x^l} p(c|j, k) \quad (5)$$

where k_M = $\min\left(\lfloor j/2 \rfloor, l-s\right)$ denotes the maximum possible number of collision slots given s, j and l. Note that the terms added in the double summation of (5) correspond to $p(c, s, k|l)$, the joint probability of c captures, s slots with a single response, and k collision slots in an l-slot frame. Equation (5) simplifies to the no-capture case by considering $x - y = s$ and $p(c|j, k) = 1$ when $c = 0$, and $p(c|j, k) = 0$ otherwise.

IV. Stochastic Shortest Path Formulation

As explained in the introduction, the reader selects l within the set $\mathbb{L}$. Since the transition probabilities $p_{xy}(l)$ of the DTMC depend on the frame size, the system is, in fact, a controlled DTMC. The objective is to select, for each state $x = 1, \ldots, N$, the frame size $l \in \mathbb{L}$ so that the expected time (in seconds, not in frames) to reach state $x = 0$ is minimum. Our formulation considers the different durations of idle, successful and collision slots.

[1] We consider $C(l, s) = 0$ when l and/or s are negative.

The problem described lies in the category of Stochastic Shortest Path (SSP), a special case of dynamic programming problems. The main elements required to formulate an SSP problem are:

- A state space $S = \{N, \ldots, 1, 0\}$ where 0 is a special termination (final) state.
- A finite control set $\mathbb{L}$=$\{l = 2^Q : Q \in [0, 1, \ldots, 15]\}$.
- A controlled DTMC, where the transition probabilities $p_{xy}(l)$ denote the probability of going from state x to state y when the control applied is $l \in \mathbb{L}$, for $x, y \in S$. Furthermore the termination state 0 is absorbing, *i.e.* for all $l \in \mathbb{L}$, $p_{00}(l) = 1$.
- A cost $g(x, l)$ incurred when control $l \in \mathbb{L}$ is selected at state $x \in S$. The termination state must be *cost-free*, *i.e.* $g(0, l) = 0$ for all $l \in \mathbb{L}$.

The objective in an SSP problem is to find the policy μ minimizing the expected total cost over an infinite number of stages. If $x_t \in S$ denotes the state of the system at the t-th stage, this expected cost is given by

$$J_\mu(x) = \lim_{T\to\infty} E\left\{\sum_{t=1}^{T-1} g(x_t, \mu(x_t)) \,\middle|\, x_0 = x\right\} \tag{6}$$

where E is the expectation operator. Applying the transition probabilities defined in (5), we can write

$$J_\mu(x) = \lim_{T\to\infty} \sum_{t=1}^{T-1} p_{xy}(\mu(x_t))\, g(x_t, \mu(x_t)) \tag{7}$$

Let us see how to obtain the cost $g(x, l)$. When the system is at state x, the cost of selecting a frame size $l \in \mathbb{L}$ depends on how the l slots are distributed among successful, collision and idle slots. Let $\tilde{g}(x, l, y)$ denote the cost of applying l at state x and moving to state y, then

$$g(x, l) = \sum_{y=0,1,\ldots,N} p_{xy}(l)\tilde{g}(x, l, y)\,. \tag{8}$$

The transition characterized by $\tilde{g}(x, l, y)$ implies that $x-y$ tags are successfully identified, including non-colliding responses s and captured ones c. The $j = x - s$ collided responses may collide in k = 1,. . .,$\min(\lfloor j/2\rfloor, l - s)$ slots, each k having a specific conditional probability $p(k|c, s, l)$, that we can obtain from the joint probability $p(c, s, k|l)$.

Let T_s, T_c and T_i denote the duration of a successful, collision and idle slot respectively. For a given combination of s, c and k, we have $x - y$ successful slots, $k - c$ collision slots without capture and $l - k - s$ idle slots. Then, $\tilde{g}(i, l, j)$ is given by the following expectation

$$\begin{aligned}\tilde{g}(x, l, y) = \sum_{s,c:s+c=x-y} \sum_{k=1}^{k_M} p(k|s, c, l)\big[(x - y)T_s +\\ + (k - c)T_c + (l - k - s)T_i + T_g\big]\end{aligned} \tag{9}$$

where T_g accounts for the interframe gap. Using (8) and (9) we can compute the per-stage cost $g(x, l)$.

Let us again turn our attention to the expected cost defined in (7). Clearly, any stationary policy μ for which $l > 1$ when $x > 1$ has a positive probability that the termination state will be reached after a finite number of stages. Such a policy is said to be *proper* in SSP terminology. With an *improper* policy (e.g. $l = 1$ for at least one $x > 1$ such that $p_x = 0$) the termination state is never reached in some cases (e.g. if state x is visited). Because a single-slot frame has a non-zero duration, the expected cost for this state diverges to ∞ when $T \to \infty$. Under these conditions, the optimal costs satisfy Bellman's equation [7],

$$J^*(x) = \min_{l\in\mathbb{L}}\left[g(x, l) + \sum_{y=1}^{N} p_{xy}(l) J^*(y)\right] \quad x = 1, \ldots, N \tag{10}$$

which can be solved by the policy iteration algorithm.

TABLE I
VALUES OF Q ATTAINING THE MAXIMUM THROUGHPUT, Q_{opt}; OPTIMAL POLICY WITHOUT CAPTURE EFFECT, μ^*; AND OPTIMAL POLICY WITH CAPTURE, μ_C^* (CAPTURE RATIO $C = 10\ dB$).

Q_{opt} (no capture) [4]		μ^* (no capture)		μ_C^* (capture)	
Q	N **range**	Q	N **range**	Q	N **range**
0	$N = 1$	0	$N = 1$	0	$N = 1$
1	$1 < N \leq 3$	2	$1 < N \leq 3$	1	$1 < N \leq 2$
2	$3 < N \leq 6$	3	$3 < N \leq 7$	2	$2 < N \leq 4$
3	$6 < N \leq 11$	4	$7 < N \leq 16$	3	$4 < N \leq 10$
4	$11 < N \leq 22$	5	$16 < N \leq 33$	4	$10 < N \leq 21$
5	$22 < N \leq 44$	6	$33 < N \leq 67$	5	$21 < N \leq 43$
6	$44 < N \leq 89$	7	$67 < N \leq 100$	6	$43 < N \leq 88$
7	$89 < N \leq 100$			7	$88 < N \leq 100$

V. NUMERICAL RESULTS

It is reasonable to believe that a Q value maximizing the expected number of identified tags per frame (throughput) would also minimize the expected identification time. For a frame with l slots and N tags, the expected throughput is $N/l\,(1 - 1/l)^{N-1}$ which reaches its maximum (e^{-1}) at $l = N$ (see e.g. [3]). Because the frame length must be chosen from the set $\mathbb{L}$, it is reasonable to select the value in $\mathbb{L}$ nearest to N, i.e. $l = 2^{[log_2(N)]}$, where $[x]$ denotes the nearest integer to x. However, this frame size is not optimum for every N. We explained in [4] how to accurately compute the optimal Q values (Q_{opt}) shown in Table I.

Let us first assume that the capture effect is not present in our system. For this scenario, we compare the policies described above, to the optimal one, μ^*, obtained by solving the SSP problem. We also include two static frame size policies $Q = 4$ and $Q = 5$, to assess the benefits of DFSA versus static FSA. Fig. 2 shows the difference between the identification time obtained by each policy ($l = 2^{[log_2(N)]}$, $l = 2^{Q_{opt}}$, $l = 2^4$ and $l = 2^5$) and the minimum identification time J^*, obtained with μ^*.

It is clear that policies aimed at maximizing the throughput do not minimize identification time. What we achieve by increasing throughput is reducing the expected number of slots of the identification process. However, since each type of slot has a different duration, this does not necessarily implies reducing the identification time. The SSP approach evaluates the decisions made at each stage considering not only their effects on the current stage but on future stages as well. In consequence, the optimal policy tends to use longer and less efficient frames (in terms of throughput) because it reduces the expected number of collision slots (lasting twice the idle

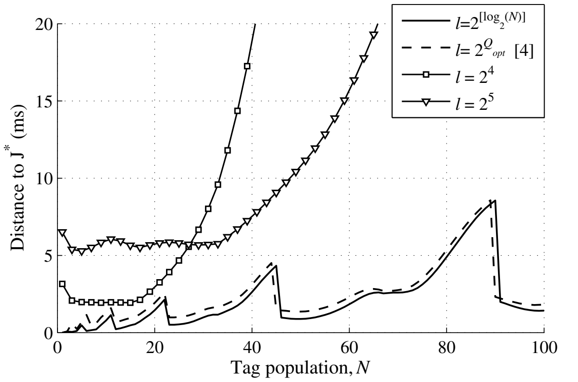

Fig. 2. Differences between the expected identification time of several policies respect to the optimum performance (J^*) in an ideal (no capture effect) system.

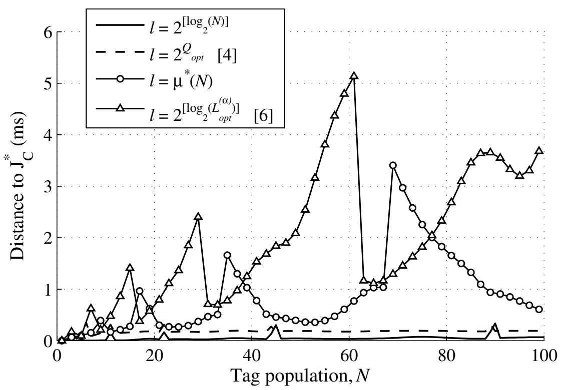

Fig. 3. Differences between the expected identification time of several policies respect to the optimum performance (J_C^*) considering capture effect.

ones). Moreover, identifying more tags at the present stage, reduces the number and duration of the frames required to identify the remaining tags.

Let us consider now the capture effect. The capture probabilities p_i have been obtained by means of Monte-Carlo simulations. We simulated a monostatic system (i.e. a single antenna is used both for transmission and reception) using the pathloss model and parameter configuration described in [8] (frequency: 915 MHz; transmission power: 800 mW; reader's antenna gain: $g_r = 5$; tag's antenna gain: $g_t = 1.6$; tag to reader distance: 2 m). The fading of the backscattered signal is modeled using a product-Rician distribution with a Rician factor $K = 3$ dB, a standard deviation $\sigma = 3$ and a correlation factor $\rho = 0.9$ between the forward and the backscatter link. The capture ratio is set to $C = 10$ dB as in [2]. The probabilities p_i are associated to the slots in which $\frac{P_b^{(1)}}{\sum_{k=2}^{i} P_b^{(k)}} \geq 10^{C/10}$, where $P_b^{(k)}$ is the power of the k-th backscattered signal.

In this case we compare the optimal performance J_C^*, obtained with μ_C^* (shown in Table I) to the following policies: i) The two reference policies ($l = 2^{\lceil log_2(N) \rceil}$, $l = 2^{Q_{opt}}$) used in previous example; ii) the optimal policy for the non-capture assumption, μ^*; iii) a policy selecting the value in $\mathbb{L}$ closest to $L_{opt}^{(\alpha)} = \alpha + (1-\alpha)N$, where α is the average capture probability in a collision slot. The expression for $L_{opt}^{(\alpha)}$ was derived, using approximations, by Li and Wang in [6] as the frame length maximizing the throughput in a capture environment. Figure 3 presents the comparison results.

As in previous case, a policy aimed at maximizing the throughput ($L_{opt}^{(\alpha)}$) obtains a suboptimal performance in terms of identification time. On the other hand, the suboptimal performance of μ^* illustrates the consequences of not considering capture effect in the SSP formulation. Finally, we see that the policies aimed at maximizing the throughput in the non-capture environment obtain a performance very close to J_C^*. The reason is that capture effect increases the number of identifications for a given frame length (capture effect reduces the identification time by 5%), and therefore the SSP tends to choose shorter frames. With a smaller capture ratio, the SSP would choose even shorter frames and we would see more diverging performances.

VI. Conclusions

We have revisited the problem of determining the Q value attaining the minimum expected identification time for a given population of unidentified tags in DFSA-based RFID systems. We formulate the problem as an SSP, for which we have derived the transition probabilities involving capture effect. Our model includes the relevant fact that identification, idle and collision slots have different durations in practical systems. It was generally believed that selecting the Q value that maximizes the throughput also minimizes the identification time. However, our results show that the optimal policy outperforms that approach.

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