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2023-12-27

Report 1: Exponential Distribution Simulation and Central Limit Theorem

This report presents the findings from a simulation exercise involving the exponential distribution and its relation to the Central Limit Theorem. We compare the sample means and variances from the simulation to their theoretical counterparts and demonstrate the normal approximation of the distribution of sample means.

Simulations

```
set.seed(123) # Setting seed for reproducibility
lambda <- 0.2
n <- 40
simulations <- 1000
sample_means <- replicate(simulations, mean(rexp(n, lambda)))
```

The above R code block simulates the process of taking averages of 40 exponential random variables with a rate parameter of 0.2, over 1000 simulations. The resulting distribution of these means will be analyzed to check its accordance with the Central Limit Theorem.

Sample Mean versus Theoretical Mean:

```
theoretical_mean <- 1 / lambda
sample_mean <- mean(sample_means)
```

We calculate the theoretical mean of the exponential distribution to be 5 and compare it to the sample mean obtained from the simulations. A histogram of `sample_means` with a vertical line indicating the `theoretical_mean` will be provided here.

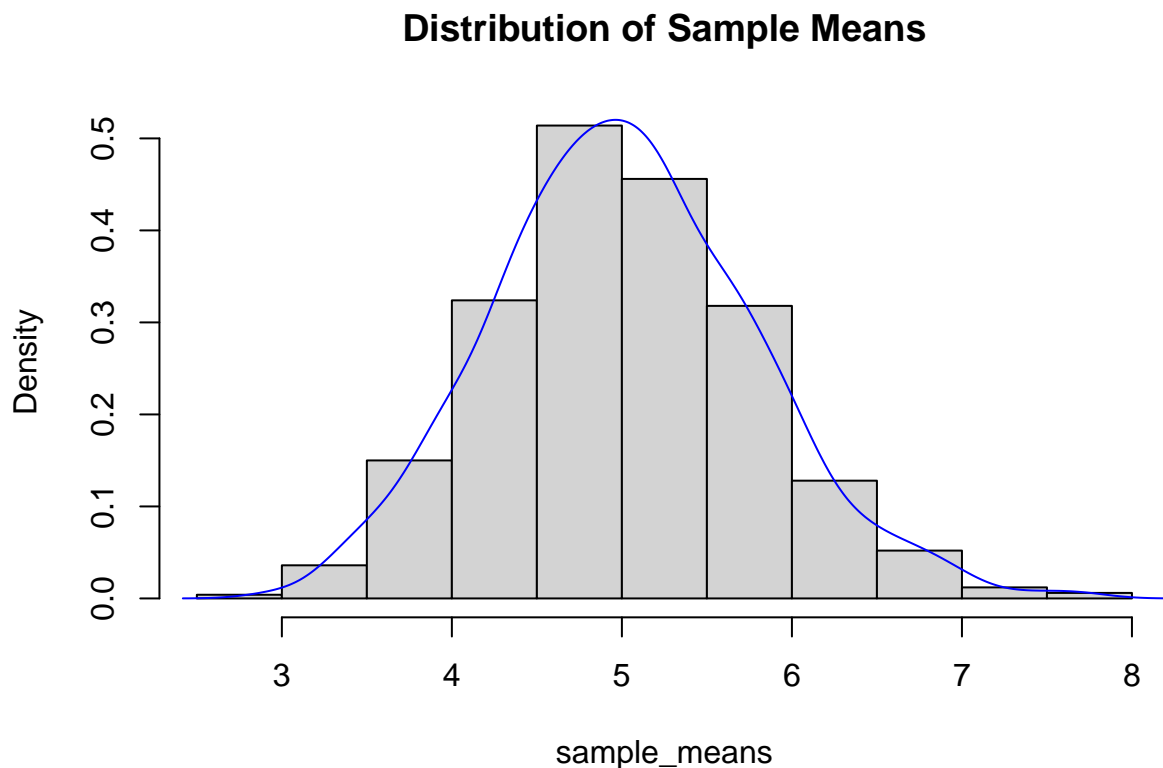
Sample Variance versus Theoretical Variance:

```
theoretical_variance <- (1 / lambda)^2 / n  
sample_variance <- var(sample_means)
```

The theoretical variance for the distribution of the mean of 40 exponentials is calculated and compared to the sample variance obtained from the simulation. A visualization of the distribution of `sample_means` with indicators for both variances will be provided here.

Distribution Normality:

```
# R code to show the distribution is approximately normal  
hist(sample_means, probability = TRUE, main = "Distribution of Sample Means")  
lines(density(sample_means), col = "blue") # Overlaying the density plot
```



The histogram of the `sample_means` overlaid with a density plot demonstrates the approximation to a normal distribution. This fulfills the conditions of the Central Limit Theorem, as the distribution of the sample means is normal, regardless of the shape of the original distribution, given a sufficiently large sample size.