الاسم: حازم مرسي حسن
رقم: 16

Sheet ③ : AI

1) $\Rightarrow P(A,B|C) = P(A|C)\,P(B|C)$ ①

$P(A,B|C) = P(D|C)$ let $(A,B) = (D)$

$= \frac{P(D \cap C)}{P(C)} = \frac{P(A \cap B \cap C)}{P(C)}$ ②

$P(B|C) = \frac{P(B \cap C)}{P(C)}$ ③

by substituting ③, ② in ①:

$\frac{P(A \cap B \cap C)}{P(C)} = P(A|C)\,\frac{P(B \cap C)}{P(C)}$

$\frac{P(A \cap B \cap C)}{P(B \cap C)} = P(A|C)$

$P(A|B,C) = P(A|C) \Rightarrow$ required to prove ①

$P(A|C) = \frac{P(A \cap C)}{P(C)}$ ④

by substituting ④, ② in ①:

$\frac{P(A \cap B \cap C)}{P(C)} = \frac{P(A \cap C)}{P(C)}\,P(B|C)$

$\frac{P(A \cap B \cap C)}{P(A \cap C)} = P(B|C)$

$\frac{P(B \cap A \cap C)}{P(A \cap C)} = P(B|C)$

$P(B|A,C) = P(B|C) \Rightarrow$ required to prove ②

2) $P(\text{Toothache}) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

$P(\text{cavity}) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$

$$P(\text{Toothache} \mid \text{cavity}) = \frac{P(\text{Toothache}, \text{cavity})}{P(\text{cavity})}$$

$$= \frac{0.108 + 0.012}{0.2} = 0.6$$

$$P(\text{cavity} \mid \text{toothache or catch}) = \frac{P(\text{cavity}, \text{toothache or cavity, catch})}{P(\text{toothache or catch})}$$

$$= \frac{0.108 + 0.012 + 0.072}{0.108 + 0.012 + 0.016 + 0.064 + 0.072 + 0.144}$$

$$= 0.4615$$

3) let B : taxi is blue
let SB : taxi seems blue

Then we need $P(B \mid SB)$ which is how likely that the taxi is blue given that it looks blue

by using bayes $P(B \mid SB) = P(SB \mid B)\, P(B)$

$$= 0.75 \underbrace{P(B)}_{\text{unknown}}$$

we can't estimate the probability before knowing $P(B)$ which is probably of blue taxis in city.

$P(B) = \frac{1}{10} = 0.1$ (10% taxis are blue)

$P(B \mid SB) = 0.75 * 0.1 = 0.075$