3. Write a program that solves a given LP using simplex algorithm.

Recall that a typical LP can be given as (or transformed into):

$$\min \sum_{j=1}^{n} c_j x_j$$
$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \qquad i = 1, \dots, m$$
$$x_j \geq 0, \qquad j = 1, \dots, n$$

Or equivalently,

$$\begin{aligned} &\min c^T x \\ &\text{s.t.} \quad Ax \leq b \\ &\qquad x \geq 0 \end{aligned}$$

You should write a computer program in C, C++, R or Python which correctly implements simplex algorithm using the matrix form, i.e. you should carry out a number of matrix multiplications in each iteration. You can refer back to the formulations we have derived during the lecture.

Assume that the objective is minimization, variables are non-negative, constraints are less than or equal to type with non-negative right-hand sides and simplex algorithm converges to a solution.

You need to test your code with three problem instances attached. The first number in .txt files gives the number of constraints m and second number gives the number of variables n. Then, there is a $1 \times n$ matrix which gives the cost vector c. Finally, there is an $m \times (n+1)$ matrix which represents the augmented matrix $[A|b]$. You can safely assume that the data is given correctly. Do not change the input format.

You are encouraged not to use any linear algebra libraries available! If you use, you will be graded partially. You may use the inversion operator that you have already developed in the second assignment.

To the report, add your description of the method and output of the problem each instance (you may include screenshots in your report or write your output in a text file.) Check below for output format.

Output formats

"Optimal variable vector: "
$[x_1, x_2, \dots]$

"Optimal result: "
$z = c^T x$