Stochastic Simulation in Engr Applications

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Abstract

Civil engineers consider the dynamic response of structures to exterior inputs, such as earthquakes and winds, to be of great importance. These dynamic responses can create uncomfortable and, more importantly, hazardous conditions for building occupants. The acceleration generated in the floor during such events can cause discomfort for residents, while the lateral displacement in columns can increase drift in each floor. When considering the p-delta effect, this can further increase the damage to the structure. Various methods have been introduced to mitigate the impact of exterior excitations, among which tuned mass dampers (TMDs) are widely utilized. However, TMDs need to be precisely calibrated to the primary structure in order to effectively absorb energy from vibrations and reduce their amplitude. This study employs stochastic simulation methods to account for the uncertainty associated with the optimization parameters of TMDs and their impact on the induced acceleration (or drift) on structures. The study employs several stochastic simulation techniques, including Monte Carlo Simulation (MCS) and Importance Sampling (IS), to predict the optimal parameters for TMDs. The probability of experiencing a high level of acceleration on the floors of the structure is assessed using probability of failure theories with the help of MCS and IS.

Introduction

The rapid growth of urbanization has led to a historic surge in the number of structures and high-rise buildings. External forces, such as earthquakes and wind, are expected to induce excessive vibration in high-rise buildings [1]. This vibration can lead to structural damage and unsatisfactory performance, potentially causing significant inconvenience and even casualties. Consequently, structural designers have long focused on vibration control of structural systems to enhance the safety and functionality of buildings, which is a major technological competitiveness [2]. Various technologies have been developed and adopted to control excessive vibration and mitigate its impact on the structural response, ensuring that it remains within sustainable limits during unpredictable events like earthquakes. Numerous vibration control technologies have been employed to reduce damage and enhance structural performance, including damping, vibration isolation, excitation force control, and vibration absorbers. Each system has its own strengths and limitations, and the selection of a specific control system is typically determined by various factors such as effectiveness, convenience, and life cycle cost. Vibration absorbers, such as Tuned Mass Dampers (TMDs), Active Mass Dampers (AMDs), Semi-active Mass Dampers (SAMDs), and Hybrid Mass Dampers (HMDs), have been widely studied and implemented in high-rise buildings to regulate their behavior under excitations [3]. The Tuned Mass Damper (TMD) is a widely used passive control device that was first developed in 1911 to stabilize ships under wave loads and control mechanical systems under dynamic loads. Over time, it has proven to be an effective solution for controlling vibrations in tall buildings and structures subjected to dynamic wind loads. With significant research, its efficiency has been widely recognized in recent years [4]. The fundamental concept of a classical TMD is illustrated using a single-degree-of-freedom (SDOF) system in Figure 1, where M_s, K_s, and C_s represent the mass, stiffness, and damping coefficient of the structure, respectively. Meanwhile, m_t, k_t, and c_t represent the mass, stiffness, and damping coefficients of the damper, respectively [5].

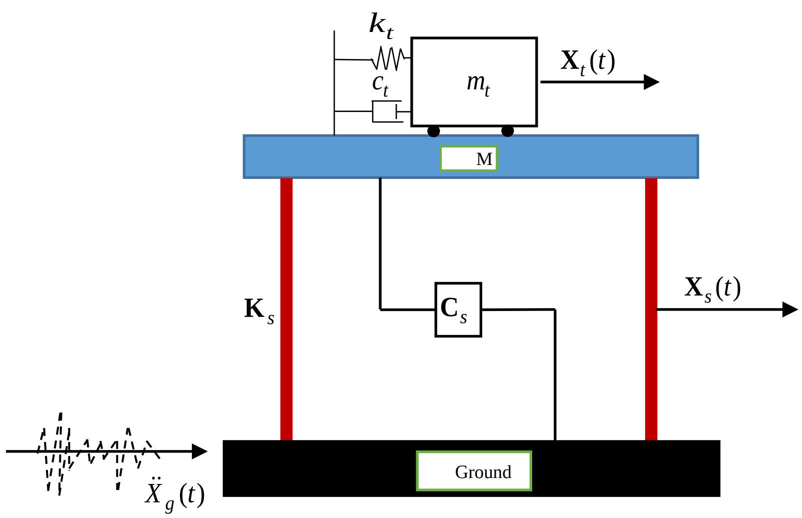


Fig1: Dynamic model of TMD [6].

Numerical, analytical, and experimental studies have sought to optimize the design parameters of the TMD, including mass, frequency, and damping ratios defined by Eq. (1). However, due to construction limitations, the mass ratio (μ) is typically a preselected constant not exceeding 10% of the controlled mode's mass, with the frequency (f) and damping ratio (ζ_t) treated as design variables [7]. Various approaches and closed formulas for the free vibration parameters of TMDs have been proposed in the literature over the past century [8].

$$\begin{aligned} &\omega_s = \sqrt{\frac{k_s}{m_s}} \quad \omega_t = \sqrt{\frac{k_t}{m_t}} \quad f = \frac{\omega_t}{\omega_s} \\ &\zeta_s = \frac{c_s}{2m_s\omega_s} \quad \zeta_t = \frac{c_t}{2m_t\omega_t} \quad \mu = \frac{m_t}{\bar{M}_1} \end{aligned} \tag{1}$$

Classical TMD tuning typically sets the mass ratio below 5% due to limitations, but for seismic applications, larger TMD masses improve control. Researchers have explored unconventional TMDs with masses exceeding 50%, such as using the top floors of buildings as the attached mass. [9]. The concept of the tuned mass damper was first introduced in 1909 by Farham, who created a dynamic vibration absorber to control vibration [10]. Since then, significant research has been conducted in passive control theory and its applications. In the early 1950s, engineers in the former Soviet Union employed impulsive pendulums on steel towers and chimneys to mitigate structural vibrations caused by wind loads. In the 1970s, hundreds of tons of TMD were installed on the 343.5m high John Hancock Tower in Boston and the 292.6m high Citicorp Center in New York City, effectively reducing wind-induced vibrations [11]. Subsequently, TMDs were successfully installed on the Sydney Tower in Australia in 1980 and the Chiba Port Tower and Fund Bridge in Osaka, Japan [12]. The largest TMD in the world, weighing 660 tons and shaped like a sphere, was installed on the Taipei 101 Tower in Taiwan in 2004 [13].

After the vibration control strategy was first proposed by Frahm [10], various TMD configurations have been developed. The first design criterion for TMD optimization was proposed by Ormondroyd and Den Hartog [14]. This criterion aimed to minimize the system response to stationary harmonic excitation at the most critical frequency. Another option is to consider stochastic stationary excitation, such as white noise. Various optimum methods have been developed to design TMD for effective vibration control under different types of excitation sources. The optimization procedures consider different mathematical models for the primary system and external loading. Several types of excitations, including stationary harmonic and white noise, have been studied. Notably, Warburton and Ayorinde [15] obtained the optimal parameters of TMD for harmonic support excitation based on Den Hartog's method, while Grigoriu and Soong [16] proposed solutions for various types of white noise excitation using different minimization criteria. Other researchers have explored unconstrained optimization of single nonlinear [17] and multiple linear [18] TMDs using variance of the system displacement as the objective function. "One widely used method to calculate the optimal values for TMD parameters was introduced by Sadek et al. [19], where the optimum damping (ζ_{topt}) and frequency f_{topt} presented as:

$$\begin{aligned} \zeta_{topt} &= \bar{\varphi}_n \left[\frac{\zeta_s}{1+\bar{\mu}} + \sqrt{\frac{\bar{\mu}}{1+\bar{\mu}}} \right] \\ f_{opt} &= \frac{1}{1+\bar{\mu}\bar{\varphi}_n} \left[1 - \zeta_s \sqrt{\frac{\bar{\mu}\bar{\varphi}_n}{1+\bar{\mu}\bar{\varphi}_n}} \right] \end{aligned} \tag{2}$$

Where,

$$\bar{\boldsymbol{\varphi}} = \frac{\boldsymbol{\varphi}_{\mathbf{1}}{}^{T} \mathbf{m}_{\mathbf{s}} \mathbf{L}}{\boldsymbol{\varphi}_{\mathbf{1}}{}^{T} \mathbf{m}_{\mathbf{s}} \boldsymbol{\varphi}_{\mathbf{1}}} \boldsymbol{\varphi}_{\mathbf{1}} \quad \bar{M}_1 = \bar{\boldsymbol{\varphi}}^T \mathbf{m}_{\mathbf{s}} \bar{\boldsymbol{\varphi}} \tag{3}$$

In this equation $\mathbf{L}$ is $\mathbf{L} = \{1, \ 1, \ \ldots, \ 1\}^T$, $\boldsymbol{\varphi}_{\mathbf{1}}$ is the first mode of the structure and $\bar{\phi}_n$ is the mode shape parameter related to the placement of the TMD. We can obtain the optimum parameters by substituting equation 3 into 1, resulting in:

$$
\begin{aligned}
m_t &= \bar{\mu}\bar{M}_1 \\
k_t &= f_{opt}{}^2\omega_s{}^2 m_t \\
c_t &= 2\zeta_{topt} m_t \omega_t
\end{aligned}
\tag{4}
$$

Uncertainties parameters and Structural models

In this project, we will obtain nominal parameter values from Eq. 4, which will be used for stochastic analysis. The TMD control values, $\bar{\mu}$, f_{opt}, and ζ_{opt}, are calculated from this equation. We will also consider uncertainties for these parameters by assigning probability distributions. We will model a shear building with five stories. Table 1 shows the mass and stiffness for the structure.

Table 1: Data of five and ten floor structure [6].

5-Storey Structure		
$St.No.$	$m_i\,(ton)$	$k_i\,(10^3 \times KN/m)$
1	12	22000
2	12	20000
3	12	17800
4	11	16000
5	10	14300

Based on the equations (2) and (4) and assume $\bar{\mu}$ equal to 10% based on the Sadek et al. [19] paper, we can find the optimum values as below:

Table2: Optimum values for TMD.

5-Storey Structure	
$St.No.$	$m_i\,(ton)$
$\bar{\mu}$	10%
f_{opt}	0.87
ζ_{opt}	0.45
m_t	4.8567 (ton)
k_t	$527.58\times10^3\,(KN/m)$
c_t	4.5655×10^4

We will use two different types of input excitations for the structures in this project. The first is the El Centro earthquake (figure 1), and the second is white Gaussian noise (figure 2), which is used to incorporate input uncertainty.

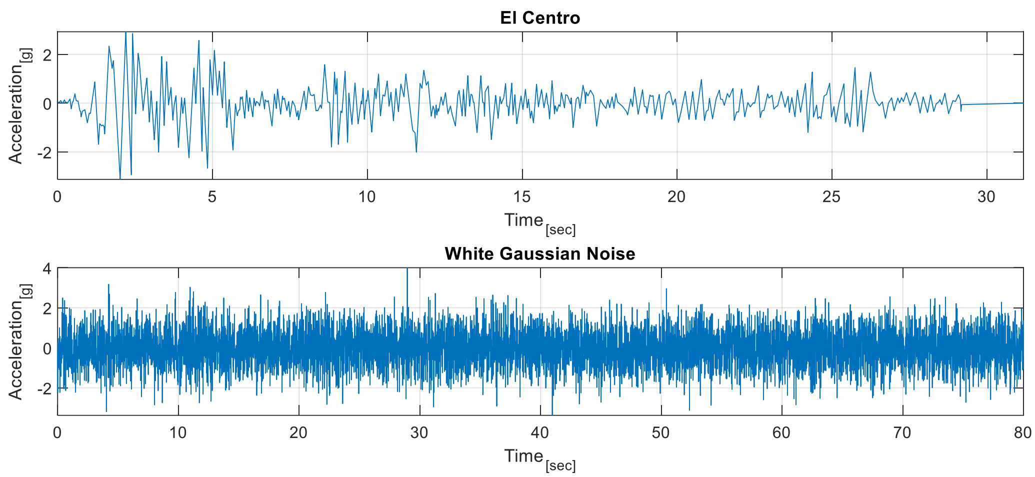
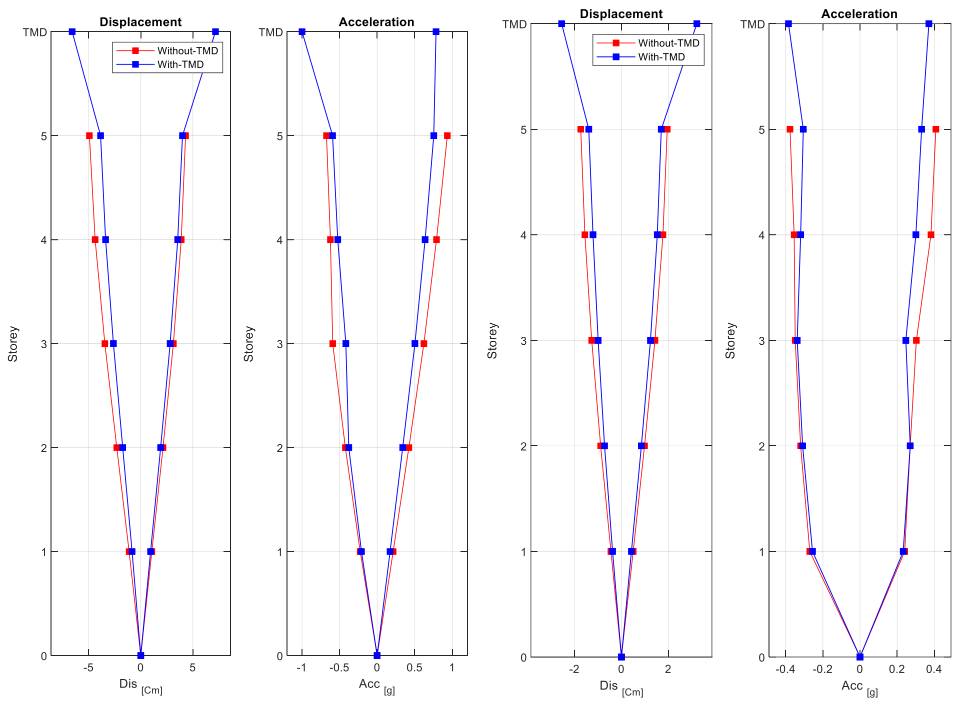


Figure 1: Input excitation for structures.

Based on these excitations and the values from Table 2, we obtain:



(a) (b)

Figure 2: Maximum displacement and acceleration of 5-Story structure under a) El Centro, b) White Gaussian Noise.

As we can see in figure (2) adding the TMD to the structure reduced both maximum acceleration and displacement in the floors. As it mentioned before in this project based on the equation (4) we consider $\bar{\mu}$, f_{opt}, and ζ_{opt} parameters as uncertainty parameters, Table 3 presents the mean and coefficient of variation along with the proposed distribution for the parameters of the model based on previous work by Sadek et al. [19].

Table3: Statistical parameters for TMD width model random variables.

Parameter	Mean	COV	Distribution
$\bar{\mu}$	0.1	0.02	Log-Normal
f_{opt}	1	0.05	Log-Normal
ζ_{opt}	0.05	0.01	Log-Normal
Excitation	El Centro, WGN		
Structure	5 Floor		

Although there are different parameters that can be considered to assess the probability of structure failure, in this project we will focus on the peak floor acceleration as the failure parameter. The peak floor acceleration is directly related to the comfort level of the building occupants. Based on Table (4), we will consider two cases: one with a high probability of failure and another with a low probability of failure, corresponding to rare events.

Table 4: Limit state based on the drift and peak floor acceleration.

Limit State	θ (%) Steel	θ (%) Composite	a_{floor} (g)
(I) - None	$\theta \leq 0.3$	$\theta \leq 0.1$	$a_{floor} \leq 0.04$
(II) - Slight	$0.3 < \theta \leq 0.35$	$0.1 < \theta \leq 0.2$	$0.04 < a_{floor} \leq 0.08$
(III) - Light	$0.35 < \theta \leq 0.56$	$0.2 < \theta \leq 0.4$	$0.08 < a_{floor} \leq 0.16$
(IV) - Moderate	$0.56 < \theta \leq 1.2$	$0.4 < \theta \leq 1.2$	$0.16 < a_{floor} \leq 0.63$
(V) - Heavy	$1.2 < \theta \leq 3.0$	$1.2 < \theta \leq 3.0$	$0.63 < a_{floor} \leq 1.24$
(VI) - Major	$3.0 < \theta \leq 8.0$	$3.0 < \theta \leq 8.0$	$1.24 < a_{floor} \leq 2.21$
(VII) - Collapsed	$\theta > 8.0$	$\theta > 8.0$	$a_{floor} > 2.21$

Based on table 4, we consider Moderate (IV) limit state as high probability failure parameter and Heavy as low probability failure parameter.

$$\max\left(\max\left(a_t^{floor}\right)\right) \leq a_{Lim}$$
$$For \rightarrow \begin{cases} low\ probability & a_{Lim} = 0.63g \\ high\ probability & a_{Lim} = 1.24g \end{cases} \tag{5}$$

METHODOLOGY

The methods utilized in this paper involve stochastic simulation methods. To predict the peak floor acceleration of the structure presented in this study, we considered it as a shear building and used MATLAB software. Monte Carlo Simulation (MCS) and Importance Sampling (IS) methods were employed for the prediction. We then calculated the probability of failure associated with different limit states. The calculations involved the use of commonly used stochastic integrals, as shown in equation (6) [20].

$$H \stackrel{\Delta}{=} E_f\left[g(\xi)\right] = \int_{\Xi} g(\xi)\, f(\xi)\, d\xi = \int_{\Xi} k(\xi)\, d\xi \tag{6}$$

In this case, the PDF of the estimated parameter is represented by $f(\xi)$, and $g(\xi)$ is a scalar function of the random variable. The coefficient of variation (CV) and predicted values of both methods will be compared. The uncertainty associated with each method will be referenced using the CV. The failure probability will be calculated using Equation (5) and will be estimated using both MCS and IS methods. The indicator function used to describe the failure probability is given by Equation (7).

$$P_f = E_f\left[I_F(\xi)\right] = \int_{\Xi} I_F(\xi)\, f(\xi)\, d\xi \tag{7}$$

DIRECT MONTE CARLO SIMULATION

MCS is a stochastic simulation method that is based on the Central Limit Theorem. The method involves generating random samples for each parameter and estimating the integral of the function $g(\xi)$, which in this paper represents the crack width (W) [20]. The evaluation of the integral H_k is computed as the average of the integral samples, divided by the number of simulations (K), as shown in equation (8) [21].

$$\hat{H}_{MCS,K} = \hat{H}_K = \frac{1}{K}\sum_{i=1}^{K} g(\xi_i) \tag{8}$$

The coefficient of variation (CV) for MCS can be calculated using the following equation (9).

$$CV = \frac{1}{\sqrt{K}} \frac{\sqrt{\underset{\{\xi_k, k=1,\ldots,K\}}{\text{variance}}\left(g(\xi)\right)}}{\underset{\{\xi_k, k=1,\ldots,K\}}{\text{mean}}\left(g(\xi)\right)} \tag{9}$$

The peak floor accelerations predicted by MCS is compared to the threshold value to determine if failure has occurred. An indicator function is used for each simulation to count the number of failures in the entire sample vector, which is used to calculate the probability of failure with an associated CV. [21]

IMPORTANCE SAMPLING

Importance sampling is a powerful technique used to estimate the properties of a particular distribution when the generated samples are different from the distribution of interest. This sampling process has a high potential to achieve the optimum integral prediction. In general, when comparing the coefficient of variation associated with Importance Sampling (IS) versus Monte Carlo Simulation (MCS), the IS CV is expected to be lower than the CV associated with MCS [21]. For Importance Sampling, equation (6) can be used, as shown in equation (10). Using equation (11), an optimal sampling density $q^*(\xi)$ can be calculated.

$$q^*(\xi) = \frac{|g(\xi)|\, f(\xi)}{\int |g(\xi)|\, f(\xi)\, d\xi} \propto |g(\xi)|\, f(\xi) \tag{10}$$

$$H \overset{\Delta}{=} E_f\left[g(\boldsymbol{\xi})\right] = \int_{\Xi} g(\boldsymbol{\xi}) \frac{f(\boldsymbol{\xi})}{q(\boldsymbol{\xi})} q(\boldsymbol{\xi})\, d\boldsymbol{\xi} = E_q\left[g(\boldsymbol{\xi}) \frac{f(\boldsymbol{\xi})}{q(\boldsymbol{\xi})} \right] \tag{11}$$

The following equation represents the coefficient of variation.

$$\delta_{IS,K} \approx \frac{1}{\sqrt{K}} \frac{\sqrt{\underset{\{\xi_k, k=1,\ldots,K\}}{\text{variance}} \left(\dfrac{g(\xi)\, f(\xi)}{q(\xi)} \right)}}{\underset{\{\xi_k, k=1,\ldots,K\}}{\text{mean}} \left(\dfrac{g(\xi)\, f(\xi)}{q(\xi)} \right)} \tag{12}$$

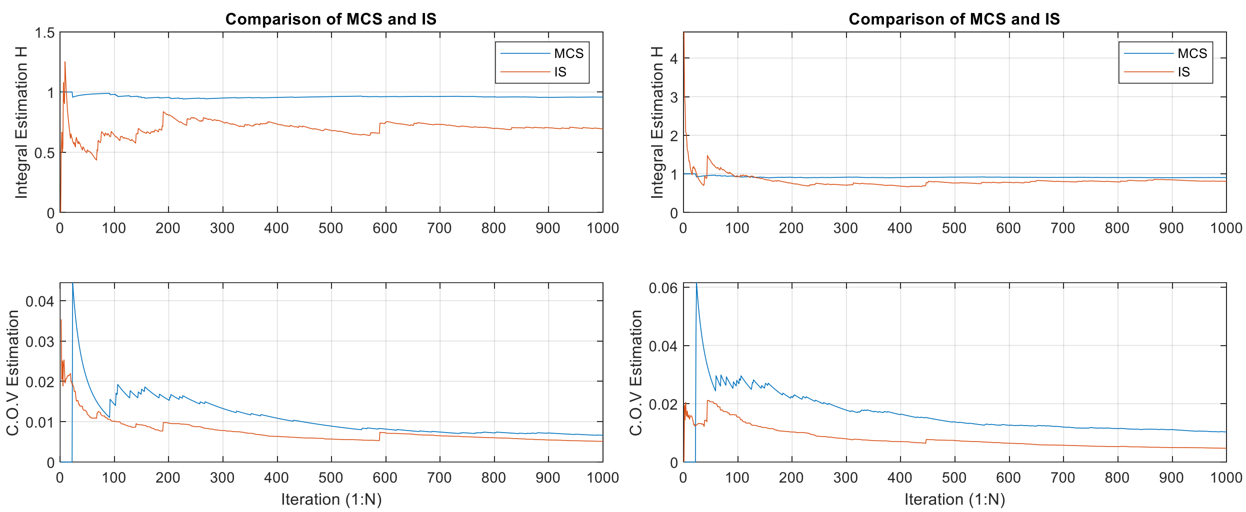
RESULTS AND DISCUSSION

In this section the results of the analysis will be presented. The predictions of the peak floor acceleration for the 5 and 10 floor structures under the El Centro earthquake and WGN excitation will be discussed and compared using different stochastic simulations techniques. The probability of failure for different TMD parameters will be discussed using MCS and IS methods.

We will start with the high probability failure parameter, for the 5-story structure under El Centro and WGN excitation, using 1000 simulations for direct MCS, Then, using samples from standard Gaussian distribution will be used, at the end the estimations of the integrals and coefficient of variation for 1000 iterations using IS and MCS is presented in Figure 3 and 4 and we can see the results for mean, COV and probability of failure in table 5.

Table 5: Results for direct and standard gaussian distribution MCS and IS method for High probability failure.

Structure	Excitation	Value	MCS	Standard Gaussian MCS	IS
5- Story	El Centro	Mean floor peak acceleration	1.1043 (g)	1.1 (g)	0.709 (g)
5- Story	El Centro	COV	0.938 %	0.9065 %	0.906 %
5- Story	El Centro	P_f	94.3%	95.8%	69.395 %
5- Story	El Centro	COV (P_f)	0.778%	0.6624%	0.51 %
5- Story	WGN	Mean floor peak acceleration	0.8648 (g)	0.8627 (g)	0.752 (g)
5- Story	WGN	COV	0.726 %	0.724 %	0.724 %
5- Story	WGN	P_f	89.2 %	90.4 %	89.55%
5- Story	WGN	COV (P_f)	1.1 %	1.03 %	0.09438 %

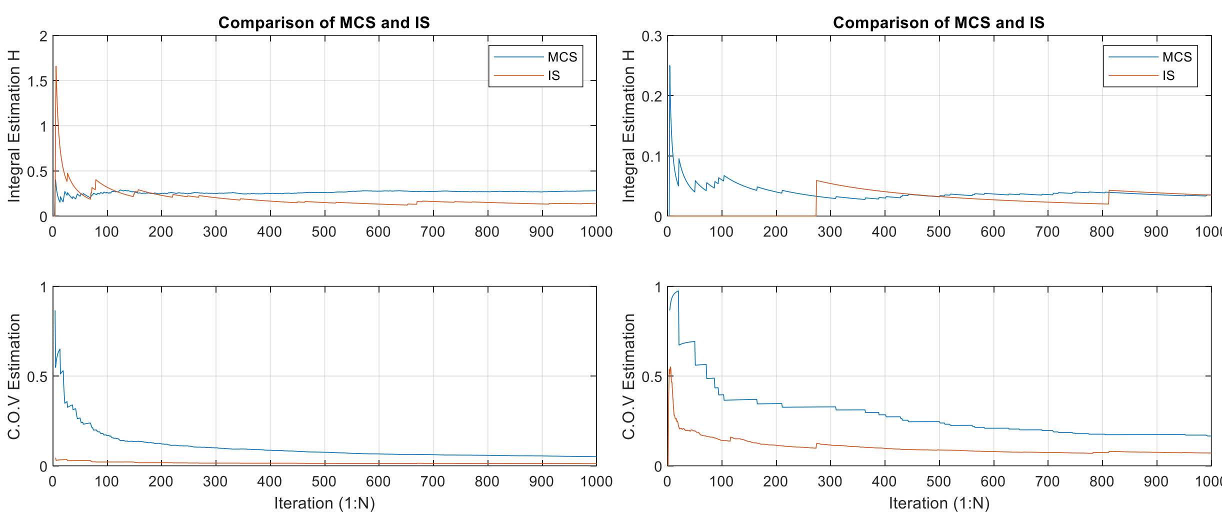


(a) (b)

Figure 3: High probability of failure, estimate of integral and COV vs number of iterations using MCS and IS, (a) El Centro, (b) WGN

Table 6: Results for direct and standard gaussian distribution MCS and IS method for Low probability failure.

Structure	Excitation	Value	MCS	Standard Gaussian MCS	IS
5- Story	El Centro	Mean floor peak acceleration	1.1043 (g)	1.1 (g)	0.844 (g)
5- Story	El Centro	COV	0.938 %	0.9065 %	0.9065 %
5- Story	El Centro	P_f	28.2 %	27.9 %	15.19 %
5- Story	El Centro	COV (P_f)	5.04 %	5.08 %	2.05 %
5- Story	WGN	Mean floor peak acceleration	0.865 (g)	0.8627 (g)	0.7489 (g)
5- Story	WGN	COV	7.267 %	0.724 %	0.724 %
5- Story	WGN	P_f	5.2 %	3.5 %	4.54%
5- Story	WGN	COV (P_f)	13.5 %	16.61 %	7.58 %



(a) (b)

Figure 4: Low probability of failure, estimate of integral and COV vs number of iterations using MCS and IS, (a) El Centro, (b) WGN

After performing 1000 simulations using direct MCS, the peak acceleration was estimated to be 1.1043 g with a coefficient of variation of 0.938% for both failure probabilities. Using samples from a standard Gaussian distribution, the peak acceleration was estimated to be 1.1 g with a coefficient of variation of 0.9065%, which is very close to the first approach. Importance sampling was also used, and the parameter values were optimized using the function fminunc. The updated parameter values are shown in Tables 5 and 6. The peak acceleration estimated using importance sampling was 0.709 g with a coefficient of variation of 906%. The estimates of the integrals and coefficients of variation for 1000 iterations using IS and MCS are presented in Figure 3 for high failure probability and Figure 4 for low failure probability. The TMD parameters found using fminunc showed that $\bar{\mu}$ always go to the highest possible value and the ξ_{opt} didn't experience a lot of change how ever frequency f_{opt} has change and effect among the results, althoug should be noted that to limit the high increase in the $\bar{\mu}$ parameter, a penalty function was created in the optimization function. At follow we just consider this parameter as uncertainty and repeat the above simulation (just for El Centro) again to see the effect of the f_{opt}, and we consider $\bar{\mu} = 0.15$ since if we consider it equal to 0.1 as before for high failure the probability of failure became 1.

Table 7: Results for direct and standard gaussian distribution MCS and IS method for high probability failure for f_{opt}.

Structure	Excitation	Value	MCS	Standard Gaussian MCS	IS
5- Story	El Centro	Mean floor peak acceleration	0.719 (g)	0.72 (g)	0.642 (g)
5- Story	El Centro	COV	0.3 %	0.3 %	0.3 %
5- Story	El Centro	P_f	90.1%	90.6%	78.78 %
5- Story	El Centro	COV (P_f)	1.048%	1.01%	0.99 %

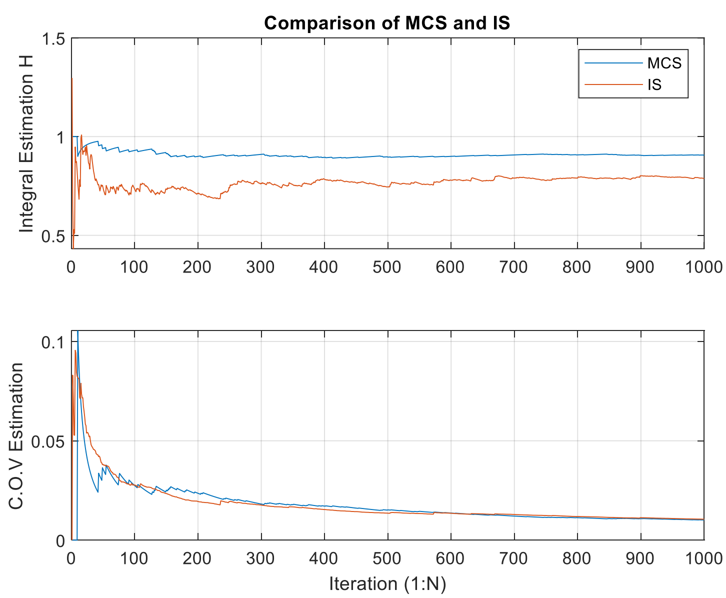


Figure 5: High probability of failure, estimate of integral and COV vs number of iterations using MCS and IS, El Centro

Table 8: Results for direct and standard gaussian distribution MCS and IS method for Low probability failure for f_{opt} .

Structure	Excitation	Value	MCS	Standard Gaussian MCS	IS
5- Story	El Centro	Mean floor peak acceleration	0.719 (g)	0.72 (g)	0.642 (g)
5- Story	El Centro	COV	0.3 %	0.3 %	0.3 %
5- Story	El Centro	P_f	0	0	0
5- Story	El Centro	COV (P_f)	--	--	--

As we can see in Table 7, when we consider the $\bar{\mu} = 0.15$ the P_f and peck acceleration reduced since this parameter has a high effect on the structure response, int this part we just consider the f_{opt} as uncertainty parameter and as we can see for Low probability of failure we have no failure although for high probability we have 90.6% for MCS and 78.78% for IS failure probability,

Conclusion

The peck floor acceleration of a structure is a crucial factor that affects the safety and comfort of its occupants. Therefore, designing an optimized TMD that can effectively reduce the acceleration is of paramount importance. However, finding the optimal design of TMD can be challenging due to the uncertainty in the design parameters. In this study, we aim to investigate the uncertainty associated with the TMD design parameters based on the formulation by Sadek et al. To address this issue, we used the Monte Carlo Simulation (MCS) and Importance Sampling (IS) methods to predict the peck floor acceleration of the presented structure. We first used 1000 simulations for direct MCS and obtained a peck acceleration of 1.1043 g with a coefficient of variation of 0.938% for both failure probabilities. Then, using samples from a standard Gaussian distribution, we obtained a peck acceleration of 1.1 g with a coefficient of variation of 0.9065%, which is very close to the first approach. Next, we optimized the values of the TMD parameters using the function fminunc and updated the parameter values accordingly. The peck acceleration obtained from Importance Sampling was found to be 0.709 g with a coefficient of variation of 906%. We also calculated the probability of failure associated with different limit states using both MCS and IS methods. The results revealed that the IS method provided more accurate estimates than the MCS method, with a lower coefficient of variation. Furthermore, we considered only one of the parameters as the uncertainty parameter and performed the simulation again. The results showed that the probability of failure became zero for the low situation, and the high peak floor acceleration was reduced to 0.642 (g). Our findings demonstrate the significance of considering uncertainty in the TMD design parameters and highlight the effectiveness of Importance Sampling method in providing accurate estimates with fewer simulations.

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