Exercise 5

1. The probability density of the random variable Y is given by

$$f(y) = \begin{cases} \frac{1}{8}(y+1), & \text{for } 2 < y < 4 \\ 0, & \text{elsewhere} \end{cases}$$

   a. Show that the function $f(y)$ represents a density function.

   b. Sketch a graph of this function, and indicate the area associated with the probability that $2.9 < Y < 3.2$.

   c. Find $P(2.9 \leq Y < 3.2)$.

   d. Find the mean of this distribution.

2. The distribution function of the random variable X is given by

$$F(x) = \begin{cases} 0, & \text{for } x < -1 \\ \frac{x+1}{2}, & \text{for } -1 \leq x < 1 \\ 1, & \text{for } x \geq 1 \end{cases}$$

   a. Find $P\left(-\frac{1}{2} < X < \frac{1}{2}\right)$.

   b. Find $P(2 < X < 3)$.

   c. Write down the probability density distribution $f(x)$.

3. The random variable X is normally distributed with mean $\mu = 18$ and standard deviation $\sigma = 7.6$.

   a. Find the probability

      i. $P(0 < X \leq 5)$

      ii. $P(X < 9 \text{ or } X > 27)$

   b. Find the value of x that has 88.3% of the distribution's area to its left.

   c. Find the value of x that has 64.8% of the distribution's area to its right.

4. The amounts of time Facebook users spend on the website each week are normally distributed, with a mean of 6.7 hours and a standard deviation of 1.8 hours.
    a. Find the probability that a Facebook user spends less than four hours on the website in a week.
    b. Out of 800 Facebook users, about how many would you expect to spend between 2 and 3 hours on the website in a week?
    c. What is the lowest amount of time spent on Facebook in a week that would still place a user in the top 15% of times?

5. A recent study of the life span of a wireless sound system found the average to be 3.7 years with a standard deviation of 0.6 year.
    a. A random sample of 16 people who own the wireless sound system is selected.
        i. Find the mean and standard deviation of the sampling distribution of the sample mean.
        ii. In this situation, can the distribution of the sample mean be approximated by a normal distribution using the Central Limit Theorem? Explain why or why not.
    b. If a random sample of 32 people who own the wireless sound system is selected, find the probability that the mean lifetime of the sample will be less than 3.4 years.