SHAPE FROM X

One image:
- **Shading**
- Texture

Two images or more:
- Stereo
- Contours
- Motion

SHAPE FROM X

One image:
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- **Texture**

Two images or more:
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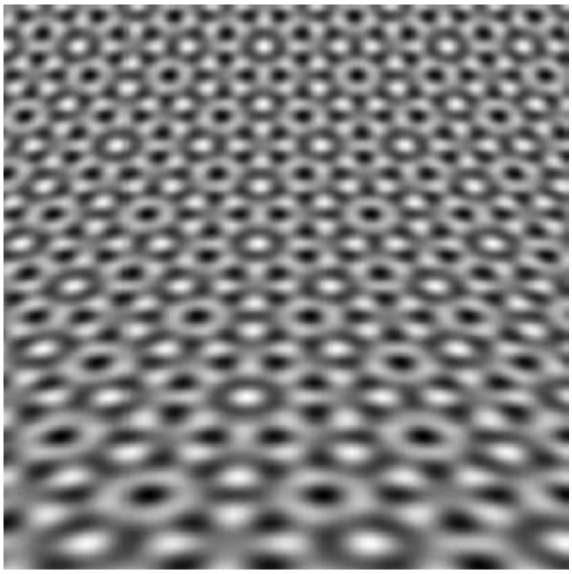

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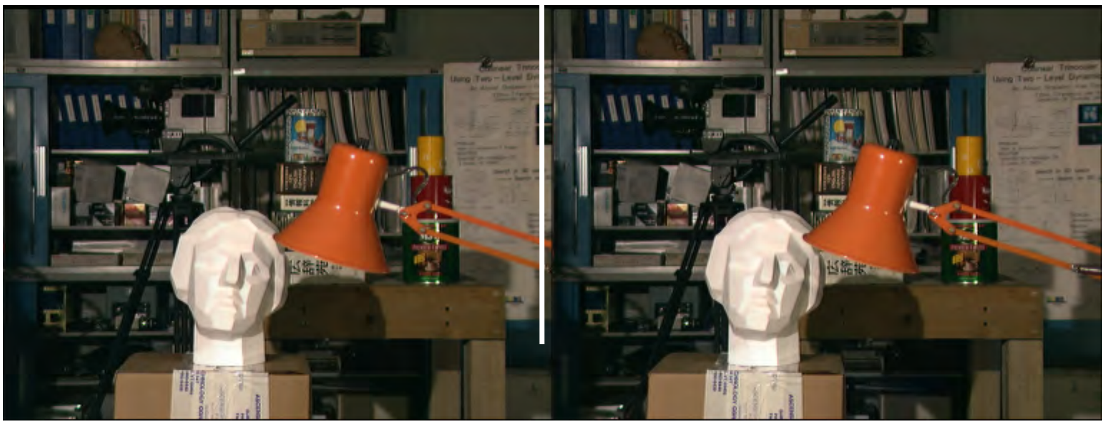

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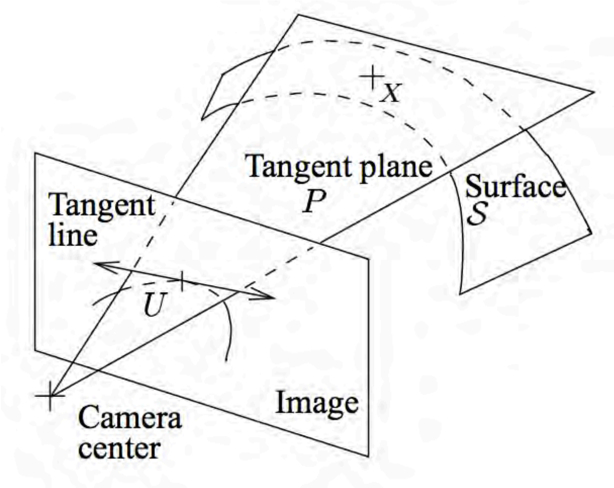

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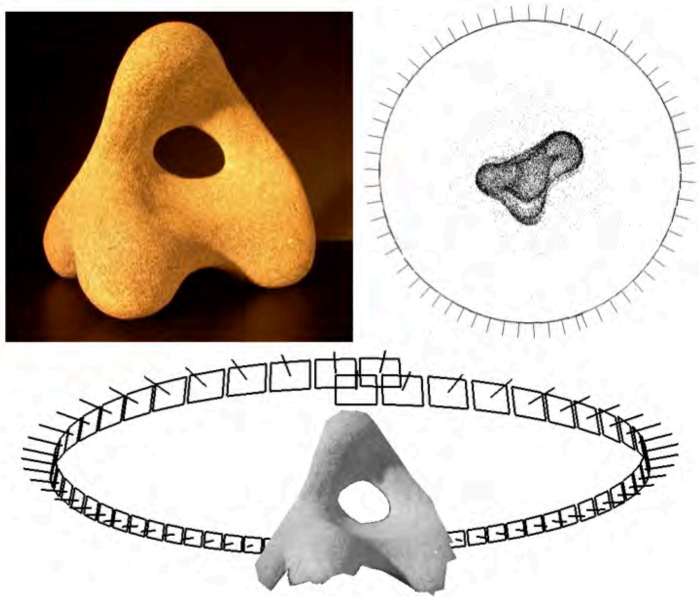
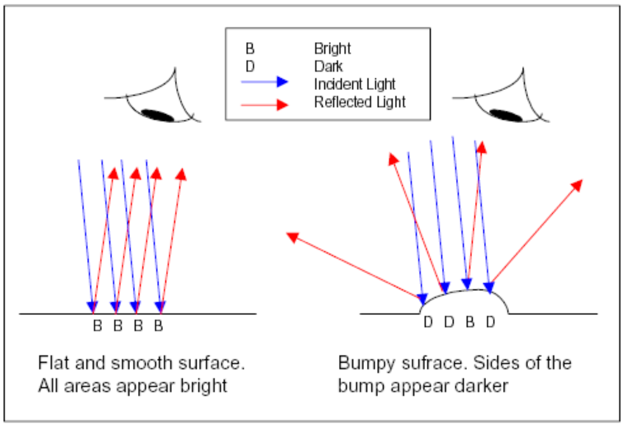
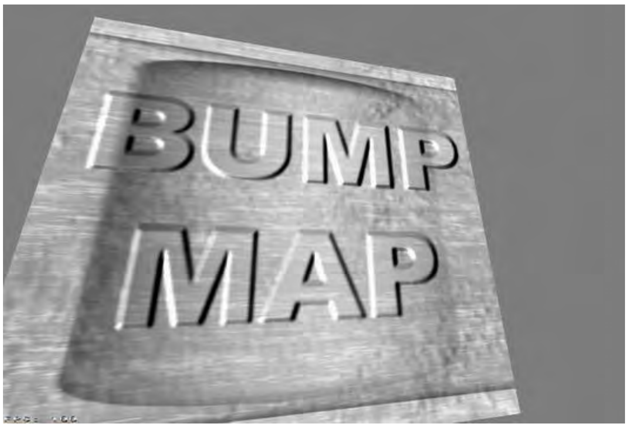


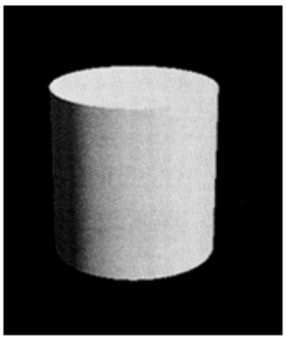
SHADING

- Shading models
- Shape from shading
  - Quantitative
  - Qualitative

BUMP MAPPING

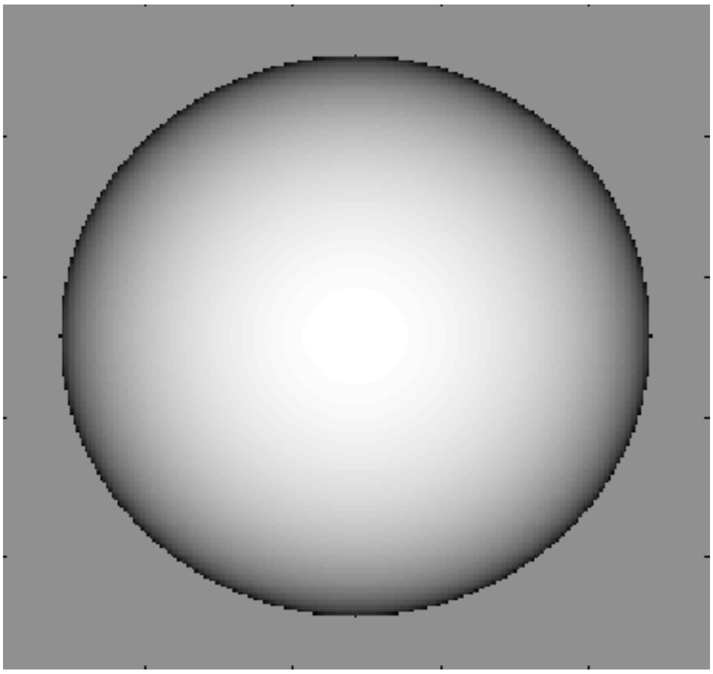
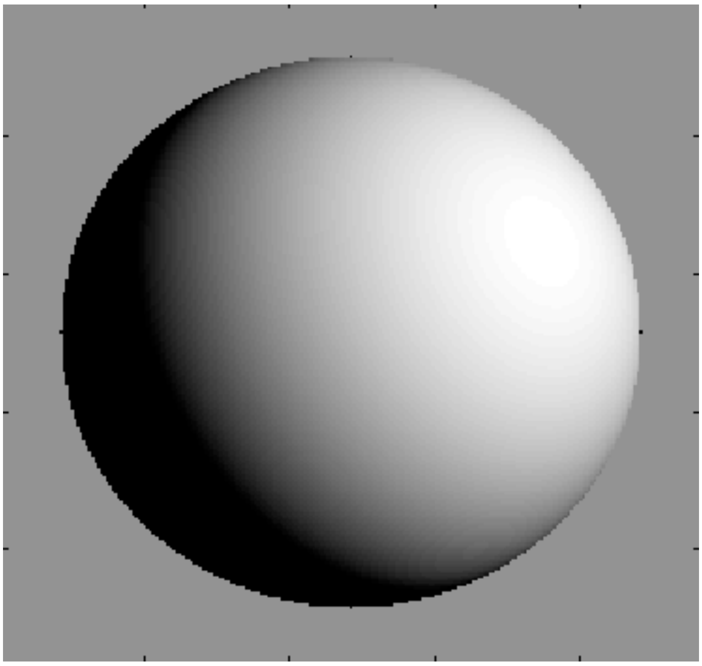






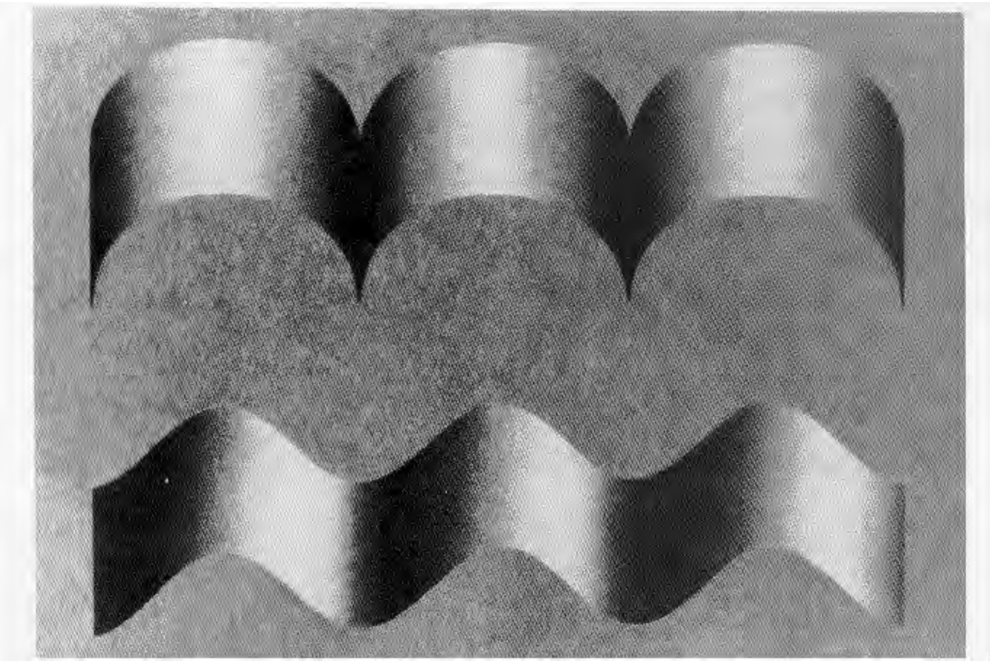
Simple mesh + 2D bump map = Complex looking object

LAMBERTIAN HALF-SPHERE

Gray level changes are interpreted as changes in the direction of the surface normal

BOUNDARY CONDITIONS



Shading gives information about surface normals
→ Boundary conditions required for a complete interpretation.

BEETHOVEN

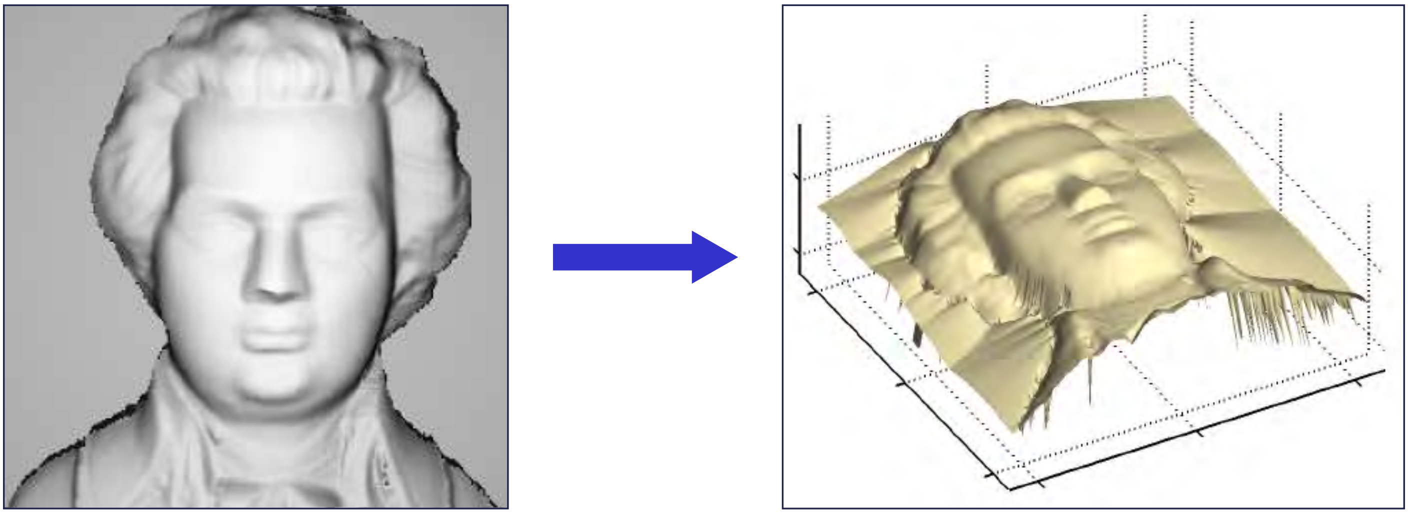
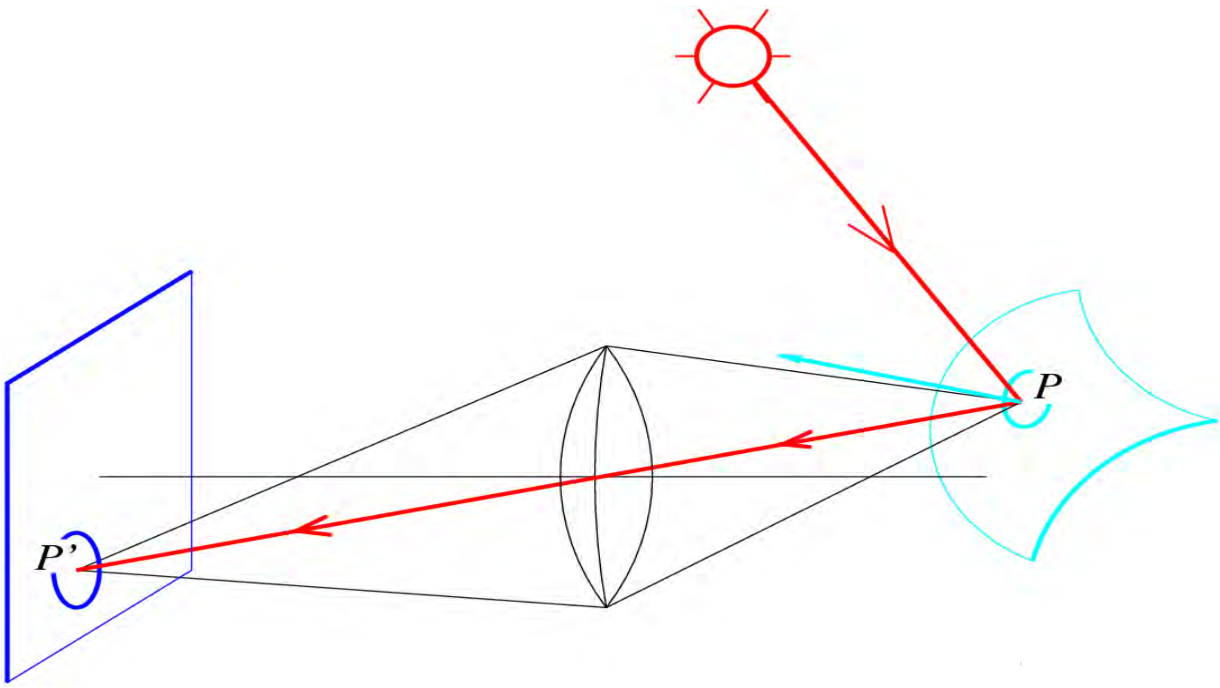
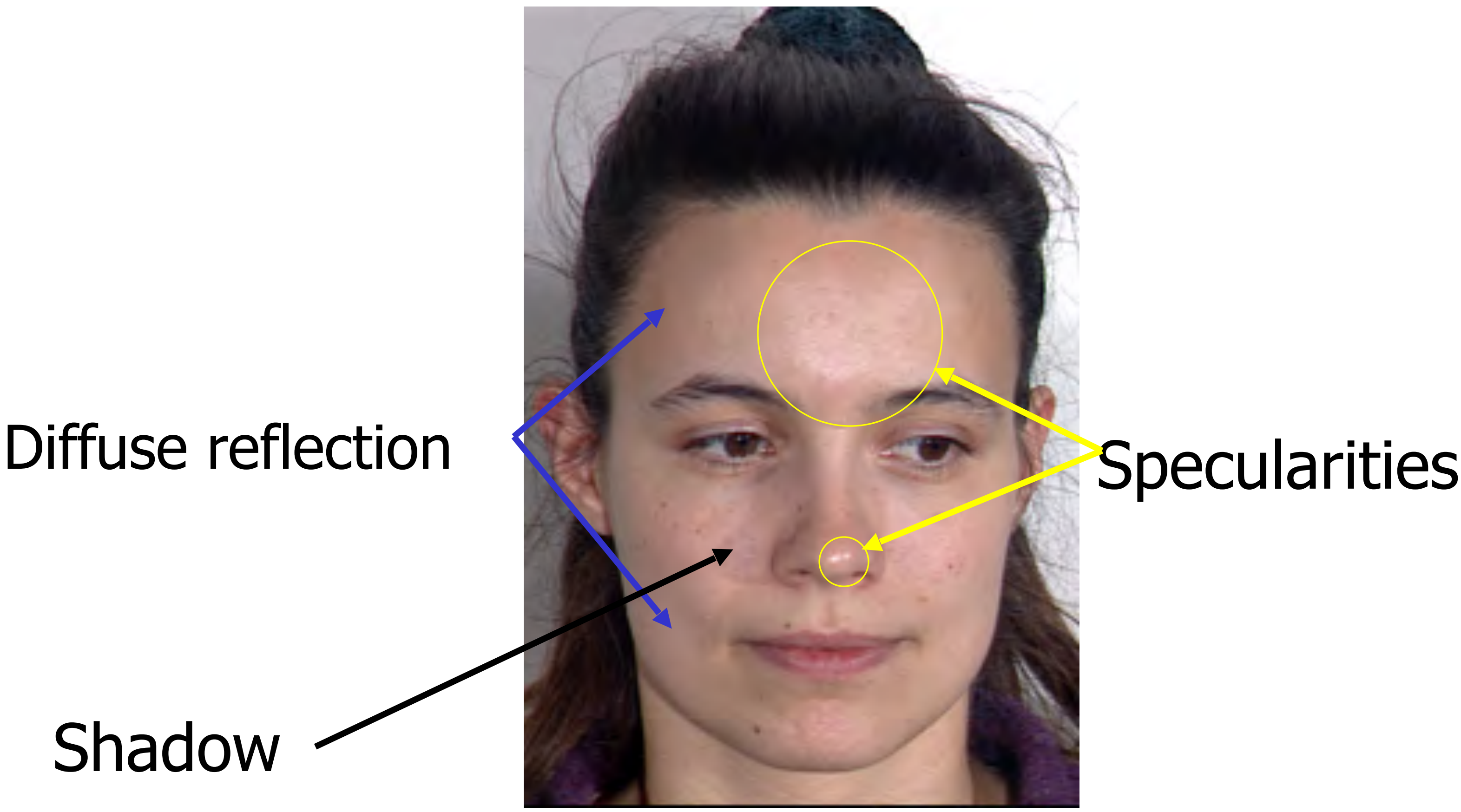


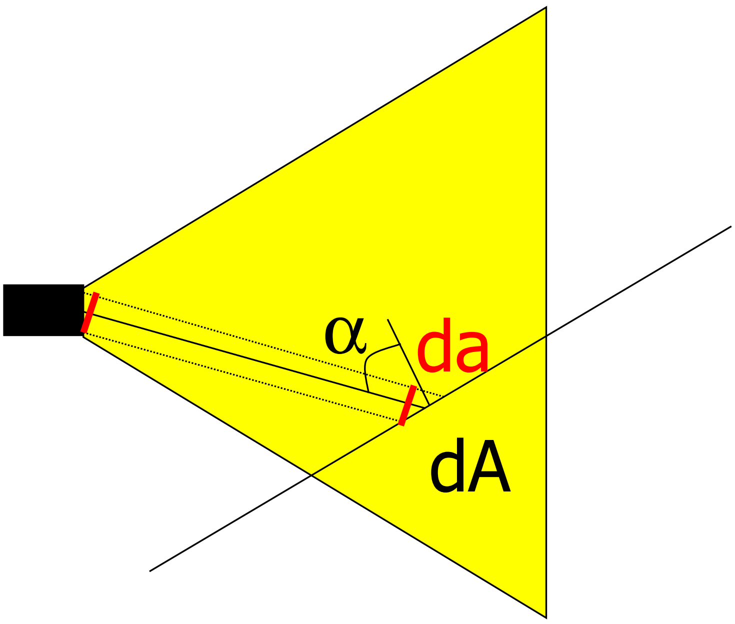
IMAGE FORMATION



DIRECT LIGHTING



LOCAL SHADING MODEL

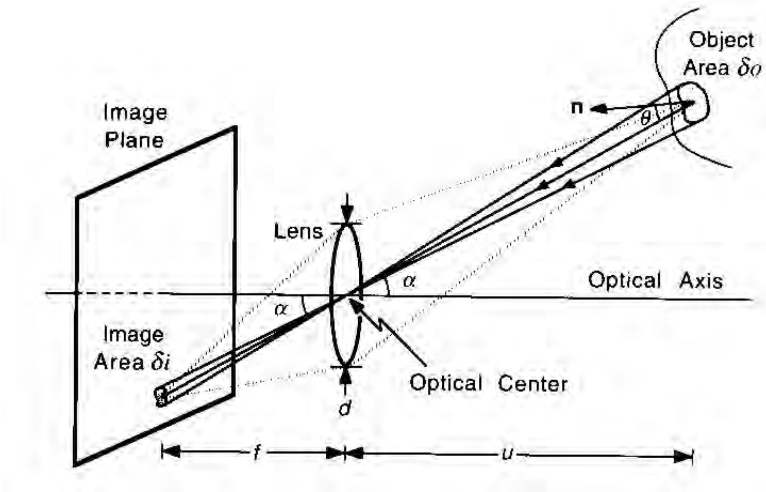


Foreshortening:

$$da = \cos(\alpha) dA$$

The effect of a distant source on a surface patch depends on its **apparent** surface.

FUNDAMENTAL RADIOMETRIC EQUATION



Radiance: amount of light emitted in a given direction

Irradiance: amount of light received on a surface

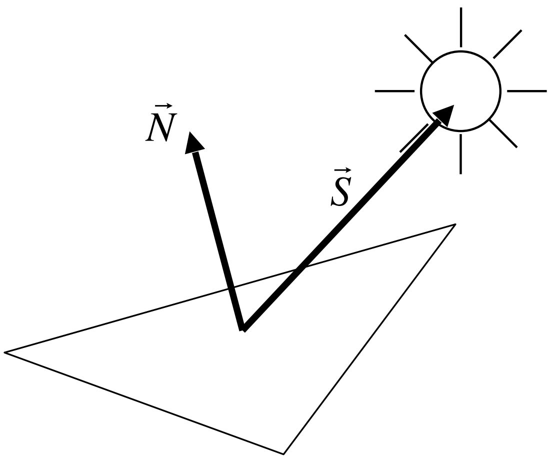
$$Irr = \frac{p}{4}\left(\frac{d}{f}\right)^2 \cos^4(a) Rad$$

Image intensity

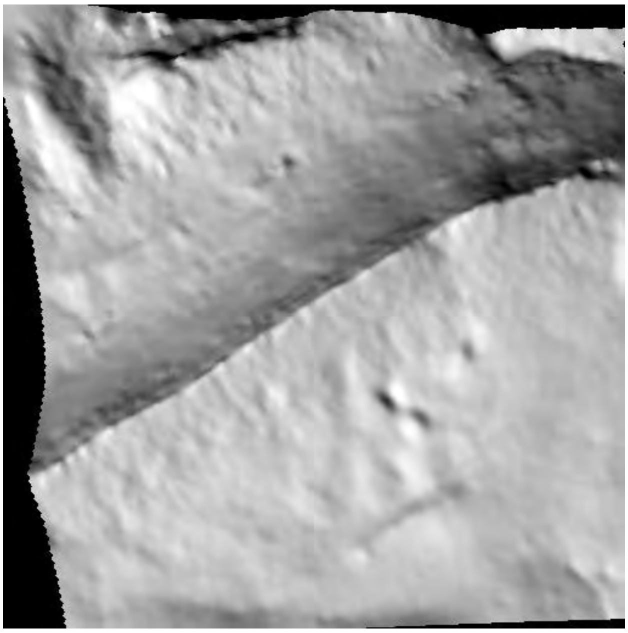
$$\Rightarrow I \propto Irr \propto Rad$$

when the camera is photometrically calibrated.

LAMBERTIAN SURFACE

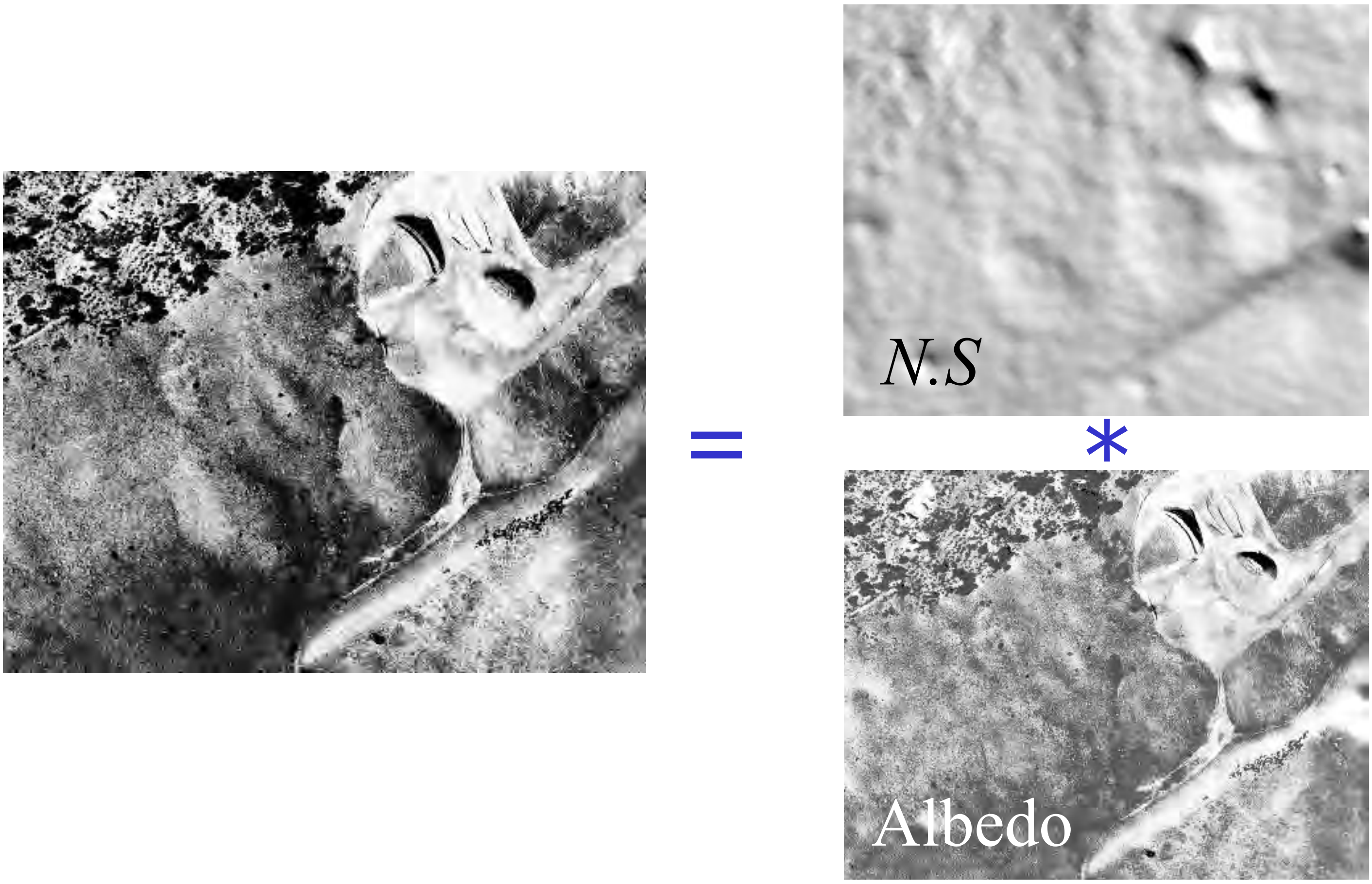


$$I = \max(Albedo \cdot (\mathbf{N} \bullet \mathbf{S}), 0)$$



Perfectly matte surface: The radiance depends only on angle of incidence and not on viewing direction.

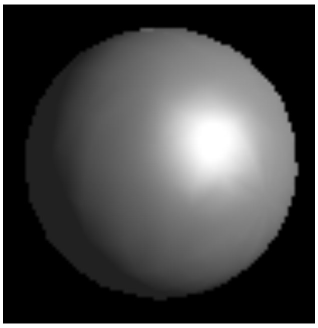
ESTIMATED ALBEDO



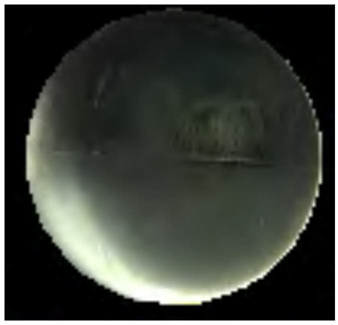
SECONDARY ILLUMINATION

Reflections produce indirect lighting.

Unique light source assumption doe not allow correct albedo recovery.

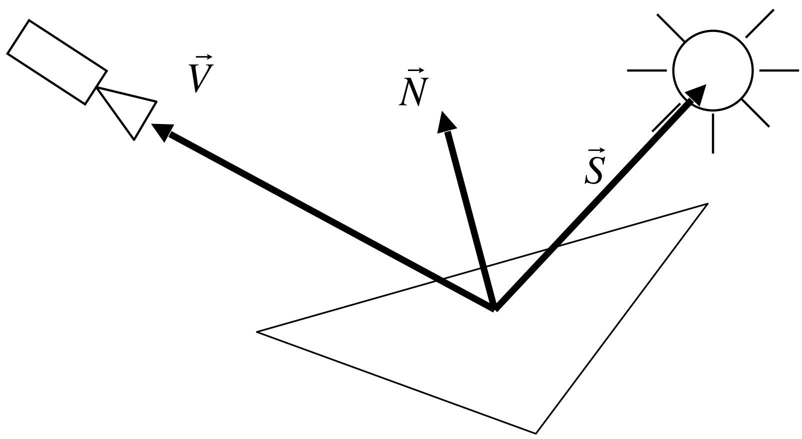


Recovered shading



Recovered albedoes

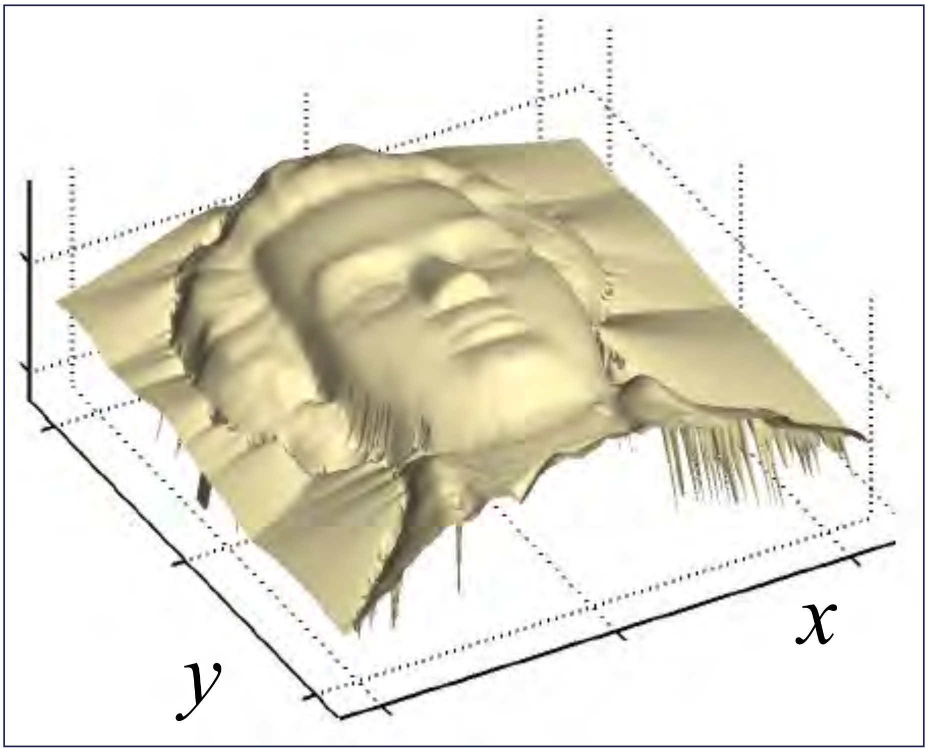
SIMPLIFYING ASSUMPTIONS



- The illumination sources are distant from the imaged surfaces
- Secondary illumination is not significant
- There are no cast shadows

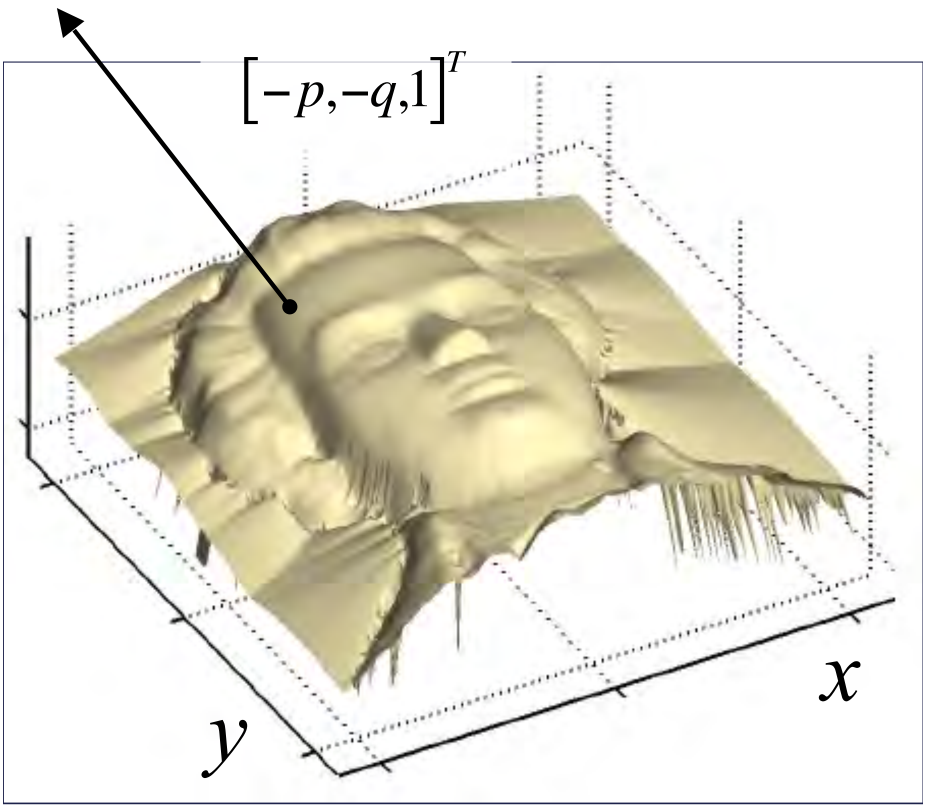
MONGE SURFACE

$$z = f(x, y)$$



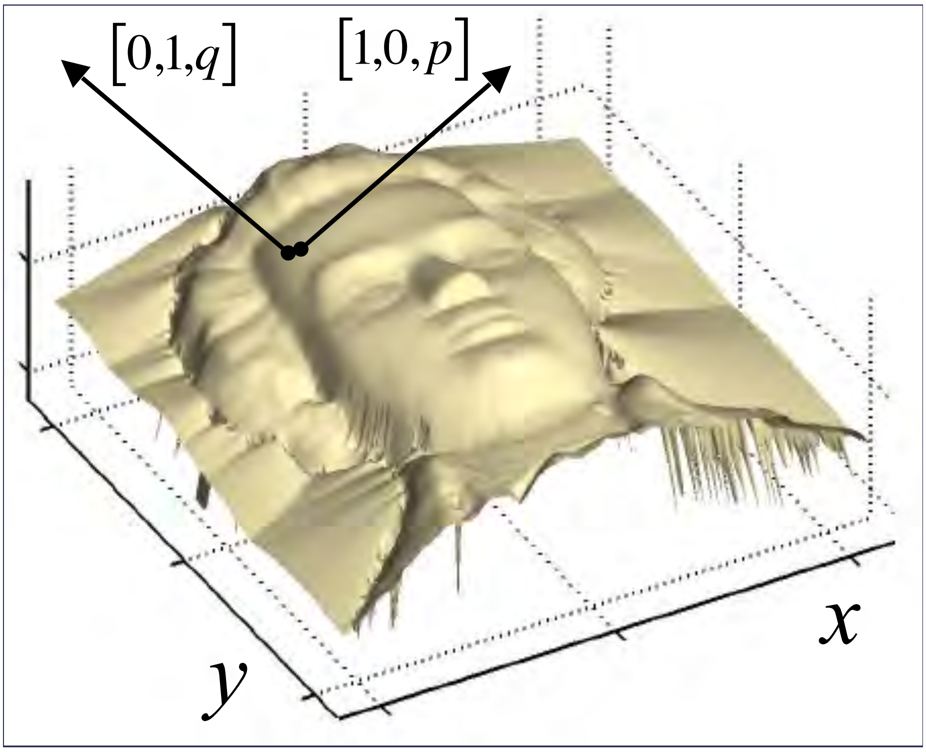
SURFACE NORMALS

$$z = f(x, y)$$
$$p = \frac{\delta z}{\delta x}$$
$$q = \frac{\delta z}{\delta y}$$



TANGENT VECTORS

$$z = f(x, y)$$
$$p = \frac{\delta z}{\delta x}$$
$$q = \frac{\delta z}{\delta y}$$



PROJECTION

Elevation and Normal:

$$z = f(x,y)$$

$$\mathbf{N}(x,y) = \frac{1}{\sqrt{1 + f_x^{\,2} + f_y^{\,2}}} \begin{bmatrix} -f_x \\ -f_y \\ 1 \end{bmatrix}$$

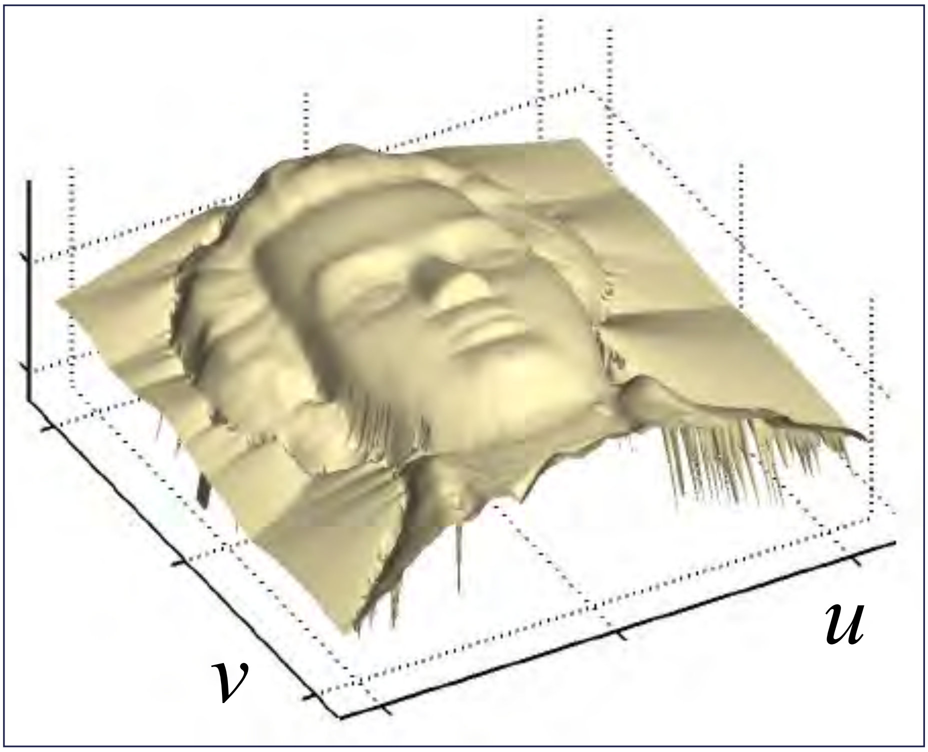
Orthographic Projection:

$$u = sx$$

$$v = sy$$

RE-PARAMETRIZATION

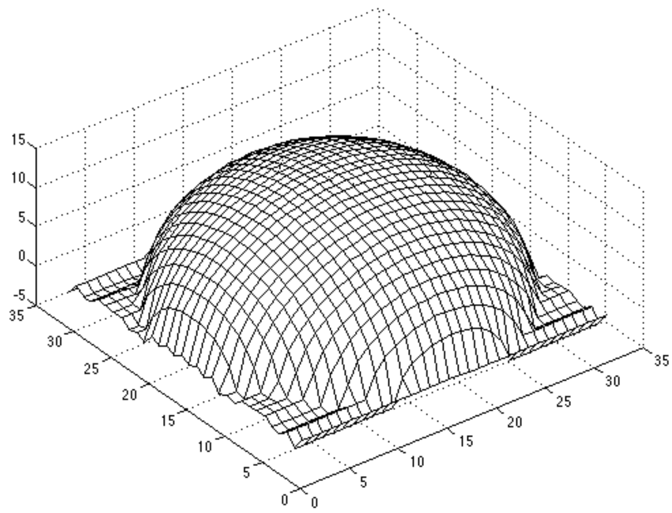
$$z = f(u, v)$$



SHAPE FROM NORMALS

$$
\begin{aligned}
\mathbf{N} &= \frac{1}{\sqrt{1+\frac{\delta z}{\delta x}^2+\frac{\delta z}{\delta y}^2}}\begin{bmatrix}-\frac{\delta z}{\delta x}\\ -\frac{\delta z}{\delta y}\\ 1\end{bmatrix} \propto \begin{bmatrix}-\frac{1}{s}\frac{\delta z}{\delta u}\\ -\frac{1}{s}\frac{\delta z}{\delta v}\\ 1\end{bmatrix}\\
\Rightarrow &\quad \frac{\delta z}{\delta u} = -s\frac{n_x}{n_z} \text{ and } \frac{\delta z}{\delta v} = -s\frac{n_y}{n_z}\\
\Rightarrow &\quad \frac{\delta \bar{z}}{\delta u} = -\frac{n_x}{n_z} = n_1 \text{ and } \frac{\delta \bar{z}}{\delta v} = -\frac{n_y}{n_z} = n_2, \text{ with } \bar{z} = \frac{z}{s}
\end{aligned}
$$

FINITE DIFFERENCES



$\forall u, v$

$\bar{z}(u+1, v) - \bar{z}(u, v) = n_1(u, v)$

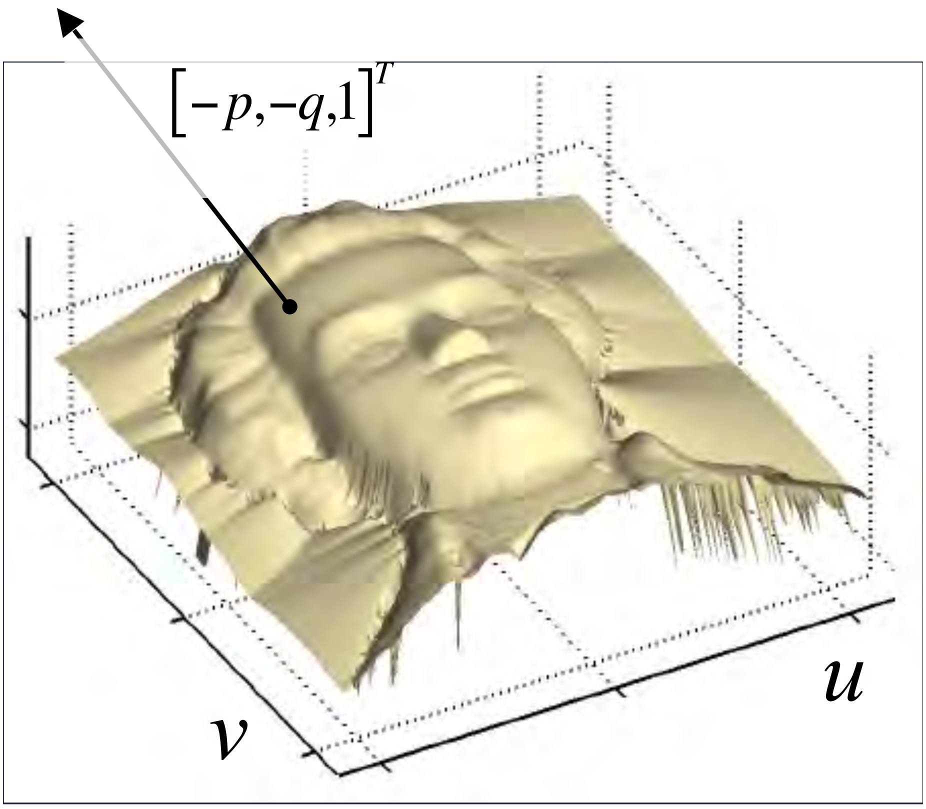
$\bar{z}(u, v+1) - \bar{z}(u, v) = n_2(u, v)$

$\Rightarrow$ Twice as many equations as there are unknown.

$\Rightarrow$ Least square solution up to a scale factor.

GRADIENT SPACE

$$
\begin{aligned}
z &= f(u, v) \\
p &= \frac{\delta z}{\delta u} \\
q &= \frac{\delta z}{\delta v}
\end{aligned}
$$



REFLECTANCE MAP

Reflectance:

Amount of light reflected towards the camera.

Albedo:

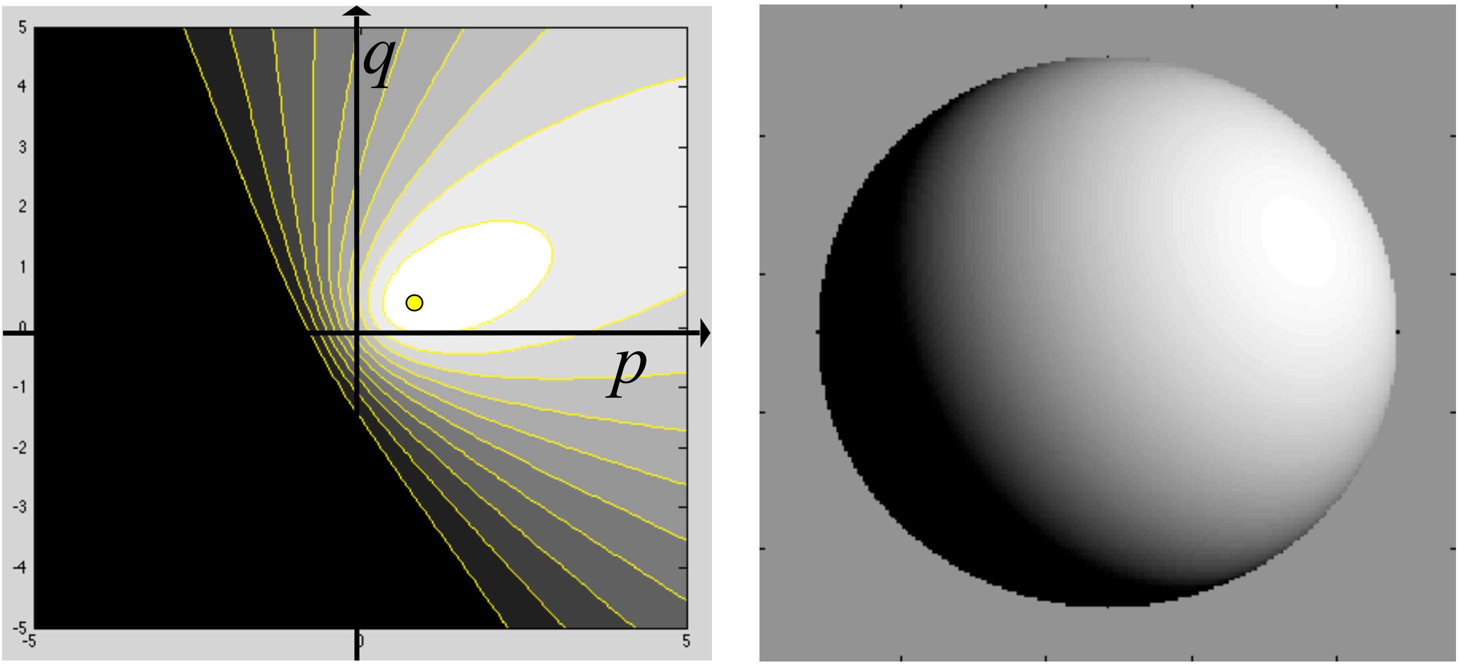
Fraction of the light incident on the surface that is reflected over all directions.

➡ In the Lambertian case and for a constant albedo

$$I(u,v) \propto Ref(p(u,v),q(u,v))$$

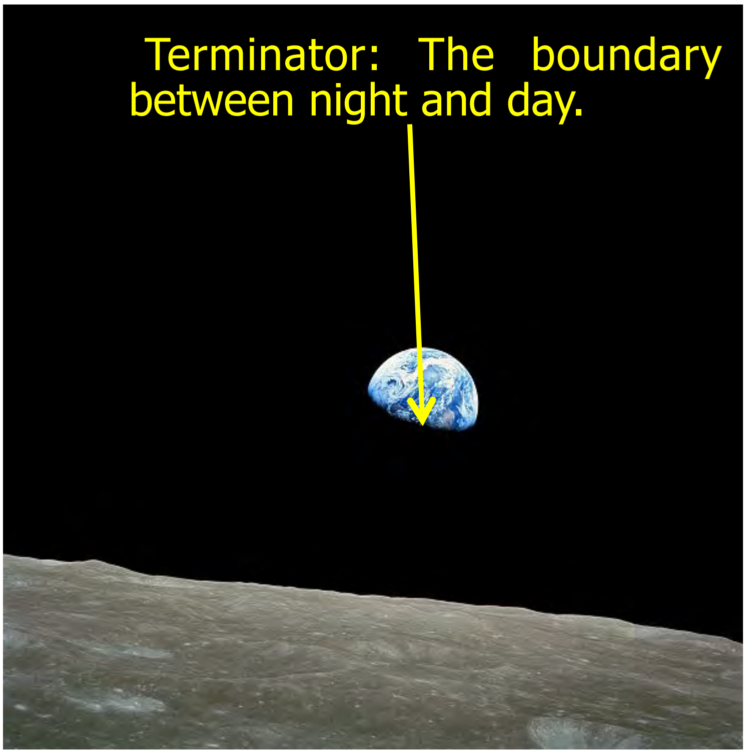
$$\propto \left[p(u,v),q(u,v),-1\right] \bullet \mathbf{S}$$

LAMBERTIAN REFLECTANCE MAP



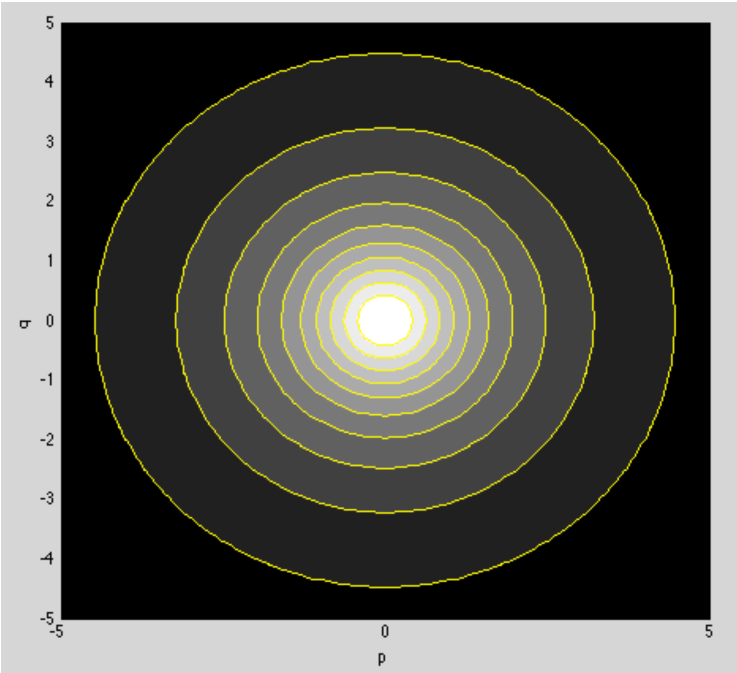
Reflectance map and shaded surface for Lambertian surface illuminated in the direction [-1 -0.5 -1].

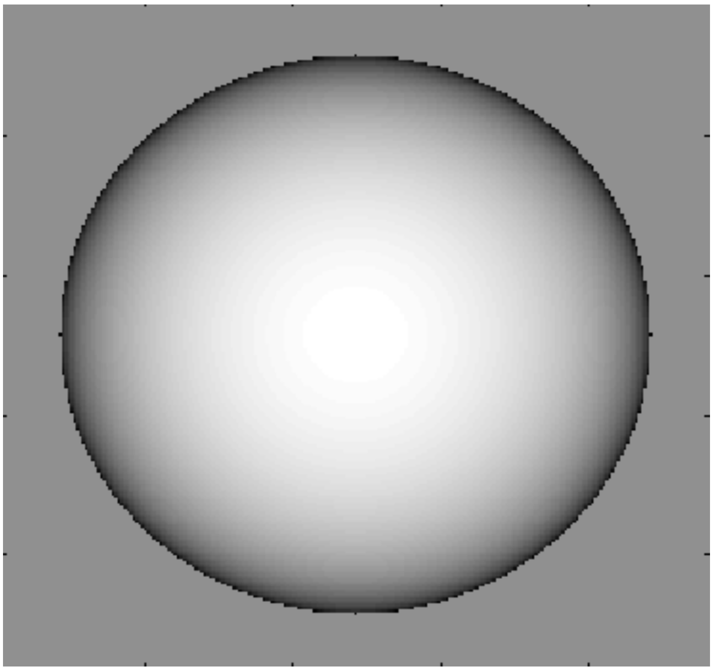
EARTH RISE



The earth seen from the moon. Apollo 8, 1968.

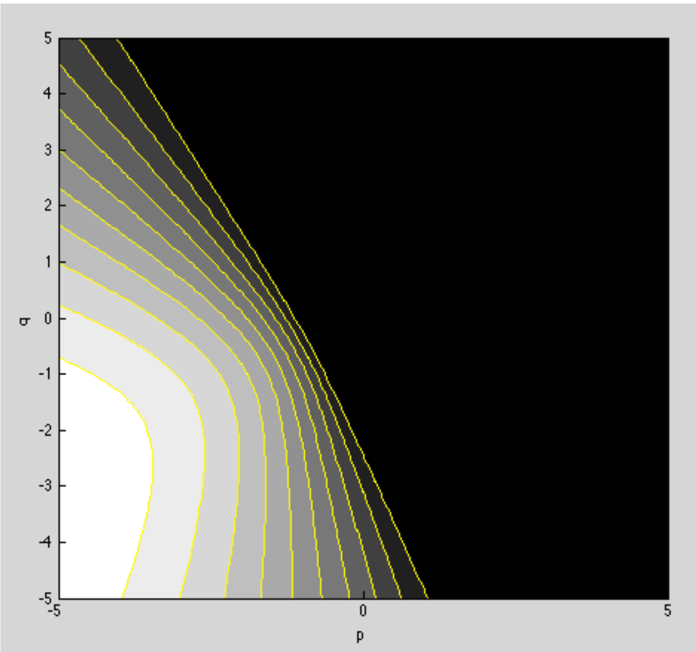
LAMBERTIAN REFLECTANCE MAP

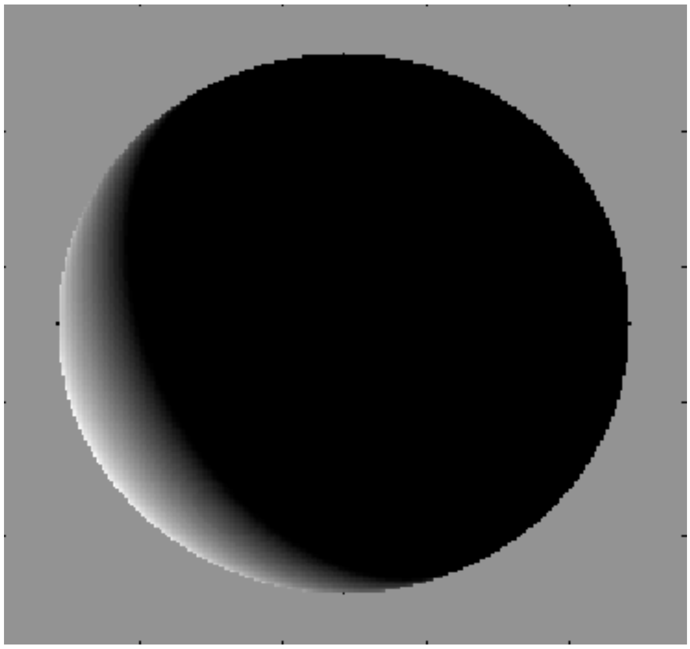




Reflectance map and shaded surface for Lambertian surface illuminated in the direction [0 0 -1].

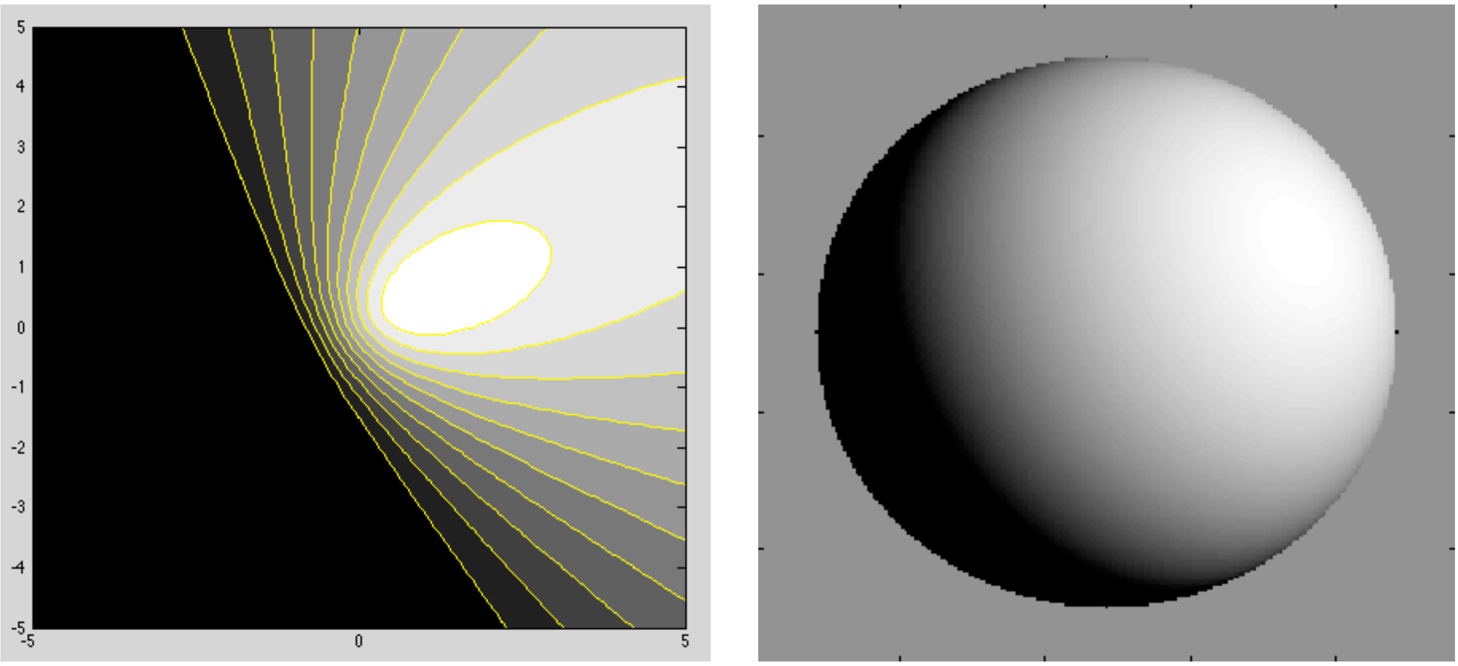
LAMBERTIAN REFLECTANCE MAP





Reflectance map and shaded surface for Lambertian surface illuminated in the direction [1 0.5 -1].

CAN WE DETERMINE (P,Q) UNIQUELY FOR EACH IMAGE POINT INDEPENDENTLY?



NO -> Global optimization required.

VARIATIONAL METHODS

Minimize:

$$\int\int\left(\left[I(u,v)-Ref(\frac{\delta z}{\delta u},\frac{\delta z}{\delta v})\right]^2+\lambda\left[\left(\frac{\delta^2 z}{\delta u^2}\right)^2+\left(\frac{\delta^2 z}{\delta u\delta v}\right)^2+\left(\frac{\delta^2 z}{\delta v^2}\right)^2\right]\right)dudv$$

Brightness constraint

Smoothness term

or:

$$\int\int\left([I(u,v)-Ref(p.q)]^2+\lambda\left[\left(\frac{\delta p}{\delta u}\right)^2+\left(\frac{\delta p}{\delta v}\right)^2+\left(\frac{\delta q}{\delta u}\right)^2+\left(\frac{\delta q}{\delta v}\right)^2\right]+\mu\left[\frac{\delta p}{\delta v}-\frac{\delta q}{\delta u}\right]^2\right)dudv$$

Integrability constraint

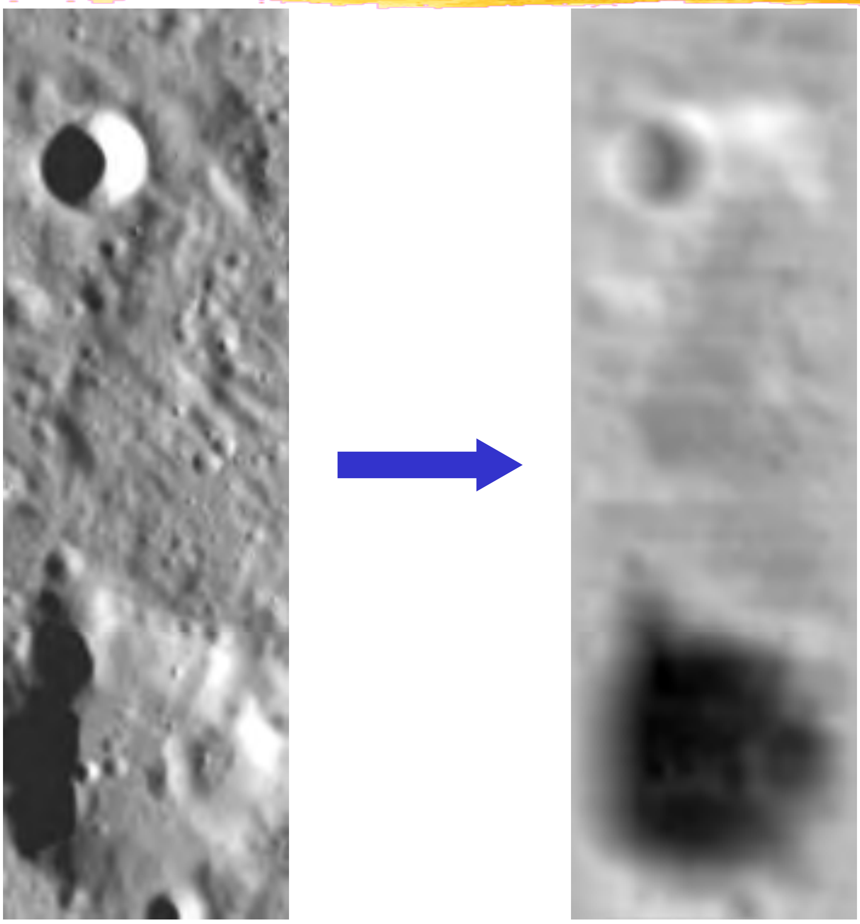
VARIATIONAL METHODS

$$\int\int \left(\left[I(u,v) - Ref(\frac{\delta z}{\delta u}, \frac{\delta z}{\delta v}) \right]^2 + \lambda \left[\left(\frac{\delta^2 z}{\delta u^2} \right)^2 + \left(\frac{\delta^2 z}{\delta u \delta v} \right)^2 + \left(\frac{\delta^2 z}{\delta v^2} \right)^2 \right] \right) dudv$$

Once p and q are estimated, integrate to recover f.

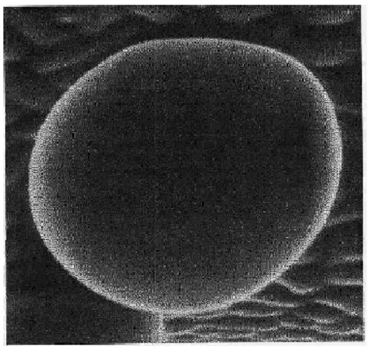
→ Need to know the boundary conditions.

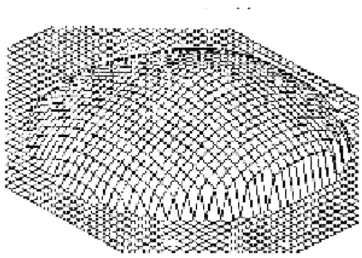
MOONSCAPE



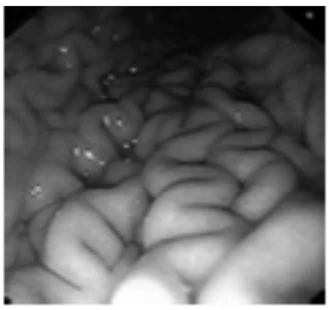
RESULTS IN MEDICAL IMAGING

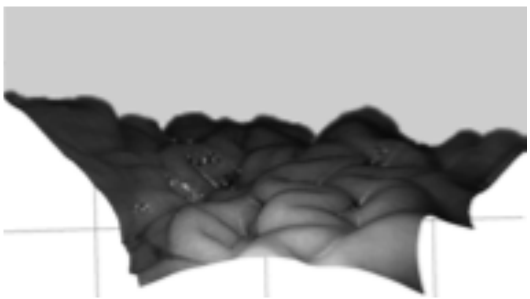
Scanning Electron Microscope (inverse intensity). Ikeuchi and Horn, AI'81:



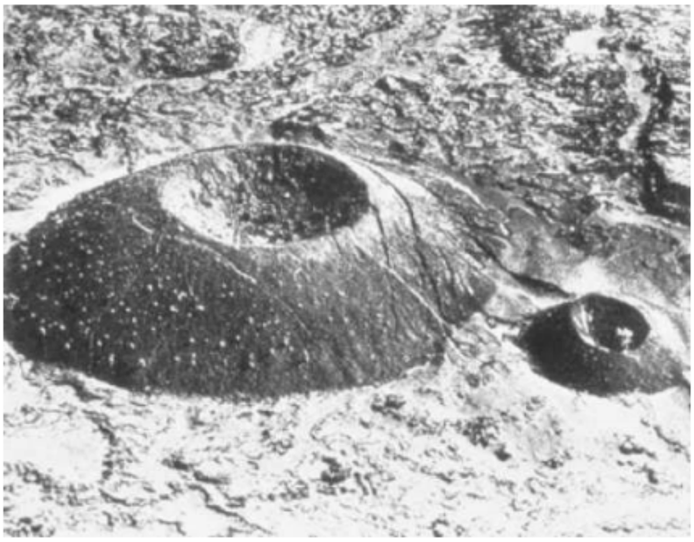


Inside a human stomach: Light source located close to the camera optical center. Prados and Faugeras, ICCV'03:



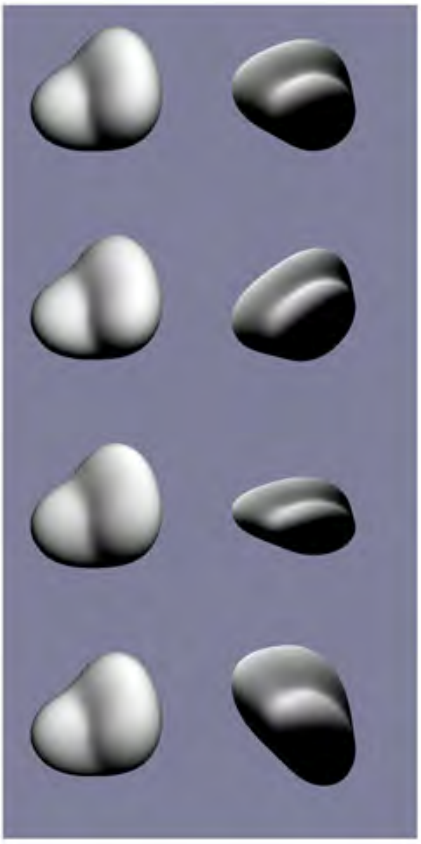


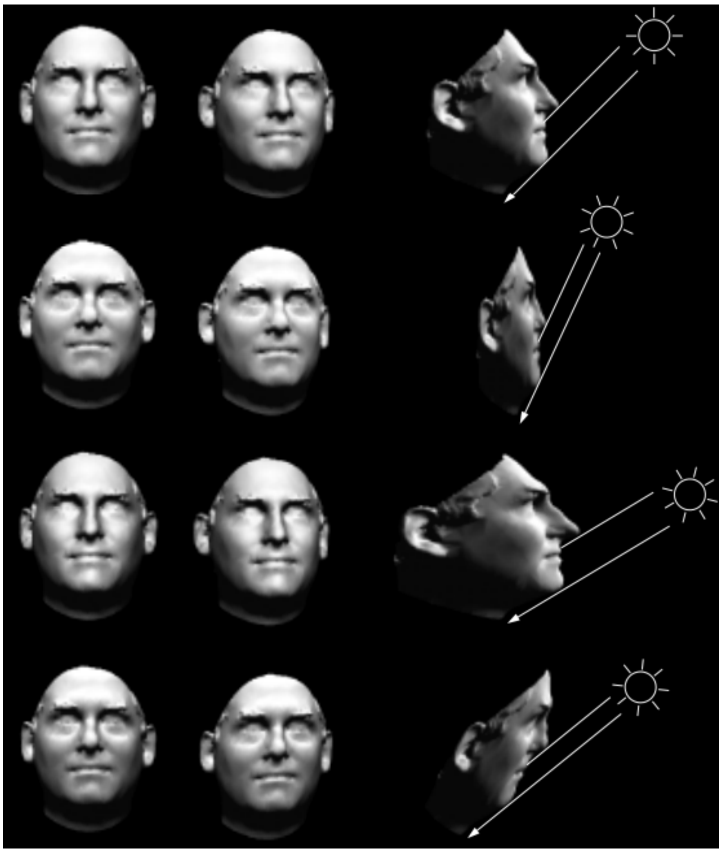
AMBIGUITIES



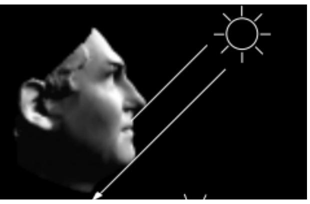
THE BAS-RELIEF AMBIGUITY

MORE GENERALLY



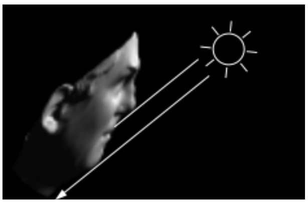


BAS-RELIEF AMBIGUITY



$$Ref = \mathbf{N} \bullet \mathbf{S}$$

For any invertible 3x3 linear transformation A:



$$\mathbf{N} \bullet \mathbf{S} = N^T S = \left(A\mathbf{N}\right)^T A^{-T}\mathbf{S}$$

But for a valid surface z=f(u,v), we should have:

$$\left.\begin{aligned} \frac{\partial f}{\partial u} &= \frac{\mathbf{N}_1^*}{\mathbf{N}_3^*} \\ \frac{\partial f}{\partial v} &= \frac{\mathbf{N}_2^*}{\mathbf{N}_3^*} \end{aligned}\right\} \Rightarrow \frac{\partial \frac{\mathbf{N}_1^*}{\mathbf{N}_3^*}}{\partial v} = \frac{\partial \frac{\mathbf{N}_2^*}{\mathbf{N}_3^*}}{\partial u} \quad \text{with } \mathbf{N}^* = \mathbf{A}\mathbf{N}$$

BAS-RELIEF AMBIGUITY

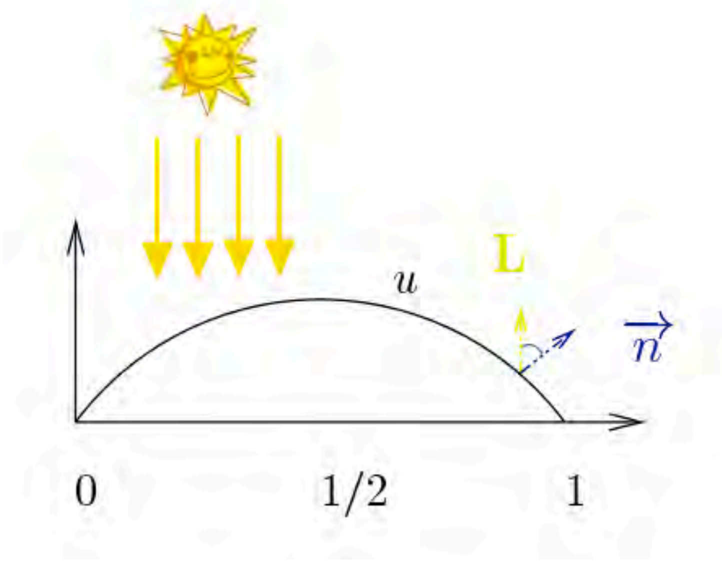
But for a valid surface, we should have:

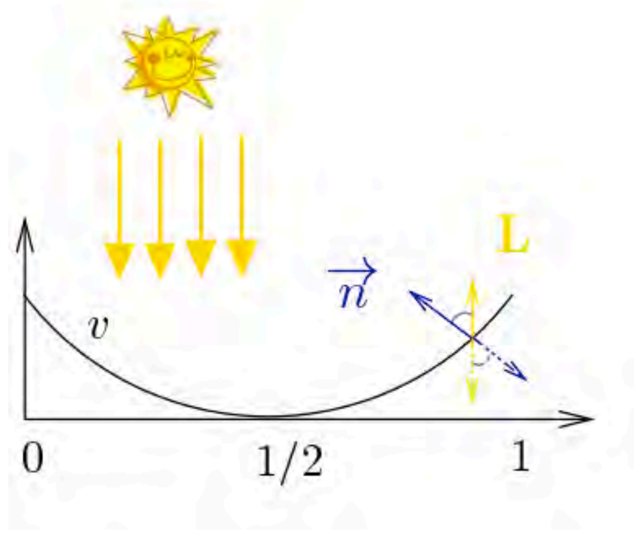
$$\frac{\partial \frac{\mathbf{N}_1^*}{\mathbf{N}_3^*}}{\partial v} = \frac{\partial \frac{\mathbf{N}_2^*}{\mathbf{N}_3^*}}{\partial u} \text{ with } \mathbf{N}^* = \mathbf{AN}$$

$$\Rightarrow \mathbf{A} \text{ restricted to } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m & u & l \end{bmatrix}$$

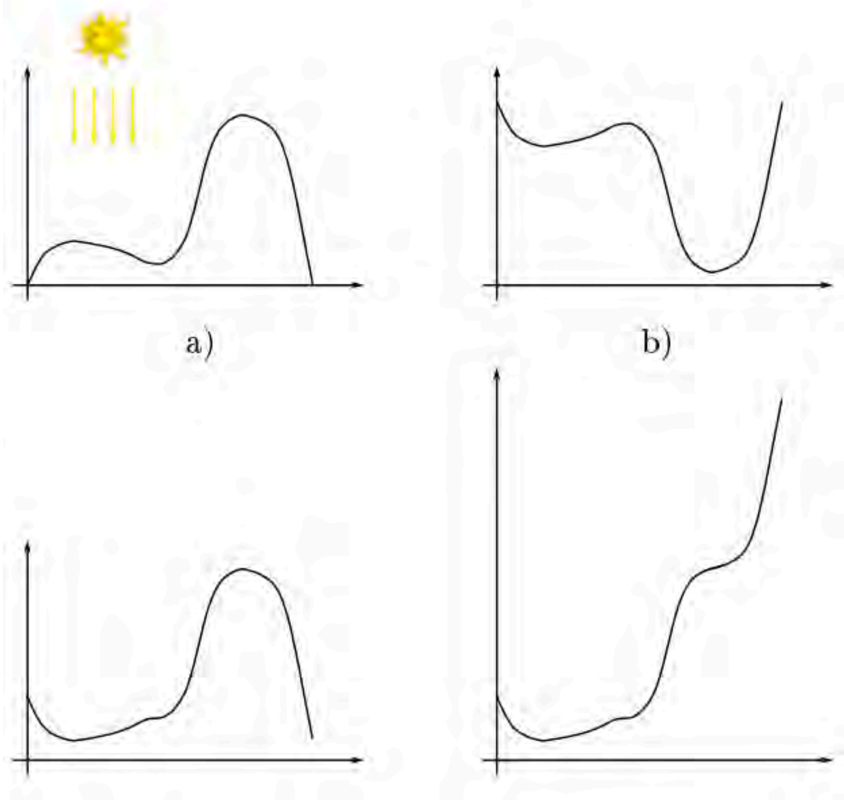
➢The surface f(u, v) can be changed into λf(u,v) + μu + νv and still produce the same image.

EVEN WHEN THE LIGHT SOURCE IS KNOWN...





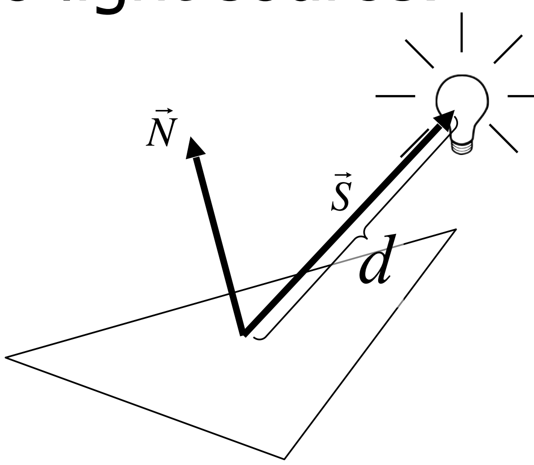
EVEN WHEN THE LIGHT SOURCE IS KNOWN...



MAKING THE PROBLEM WELL-POSED

- Use perspective projection model;
- Radiance depends on distance to light source:

$$I = \frac{Albedo \cdot (\mathbf{N} \bullet \mathbf{S})}{d^2}$$



instead of

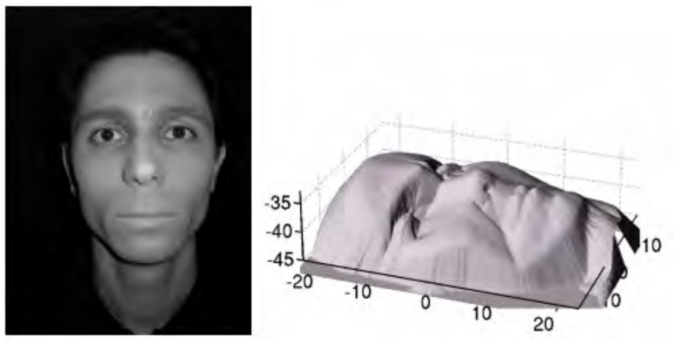
$$I = Albedo \cdot (\mathbf{N} \bullet \mathbf{S})$$

- Light source located at the optical center.

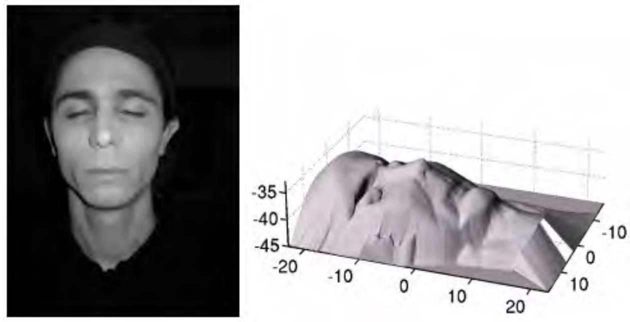
-> Unique solution.

Prados and Faugeras, CVPR'05.

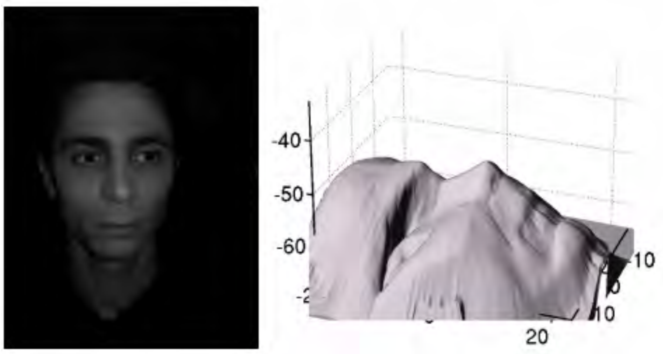
RESULTS



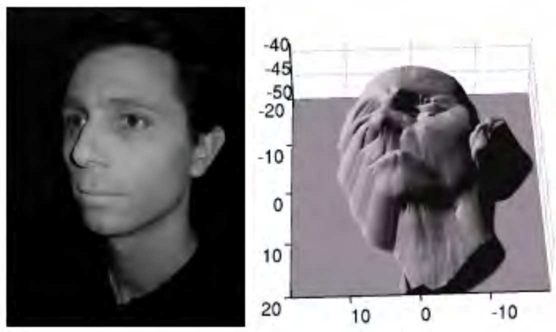
-35
-40
-45
-20
-10
0
10
20
10
0



-35
-40
-45
-20
-10
0
10
20
-10
0
10



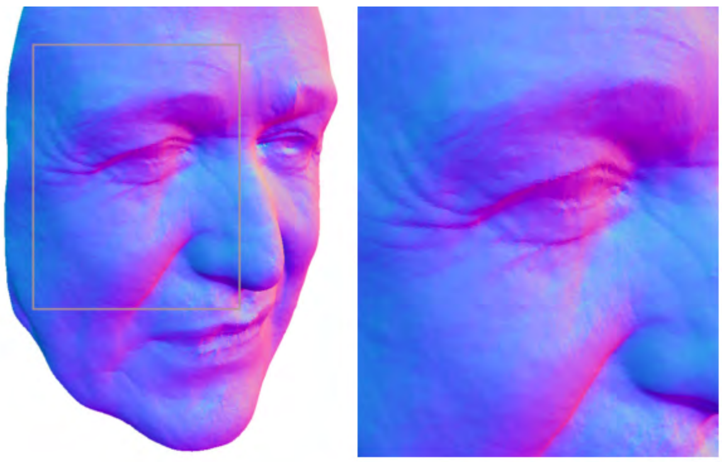
-40
-50
-60
20
10
10



-40
-45
-50
-20
-10
0
10
20
10
0
-10

SHAPE REFINEMENT

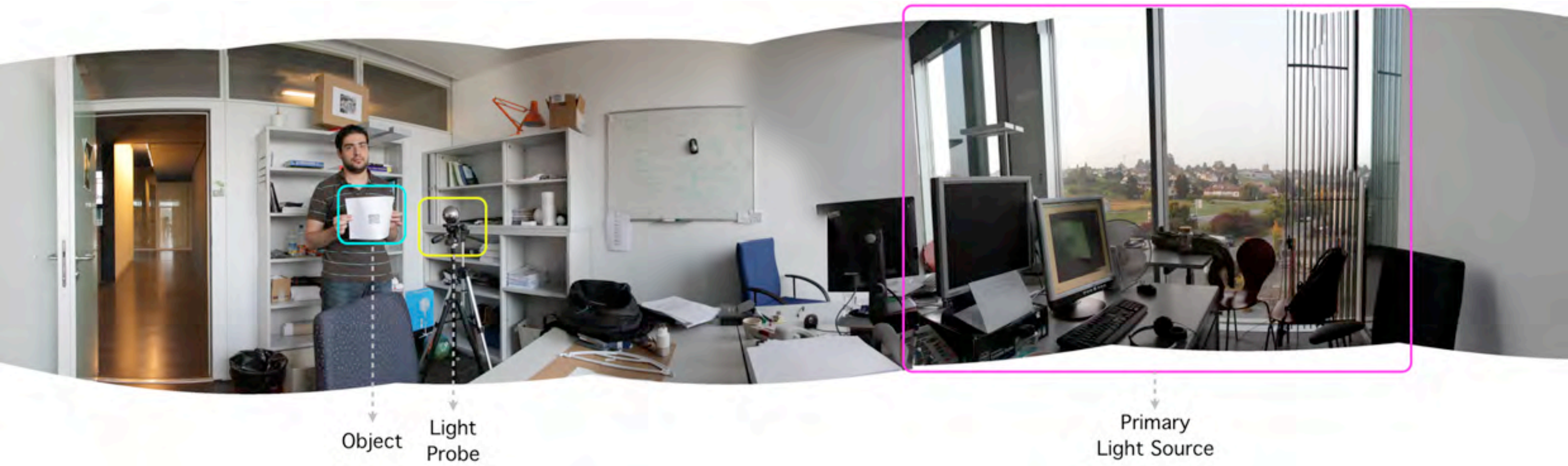
Stereo Only Stereo + Flow



Shape-from-shading is used to refine the shape and provide high-frequency details.

Valgaerts et al. SIGGRAPH Asia'12

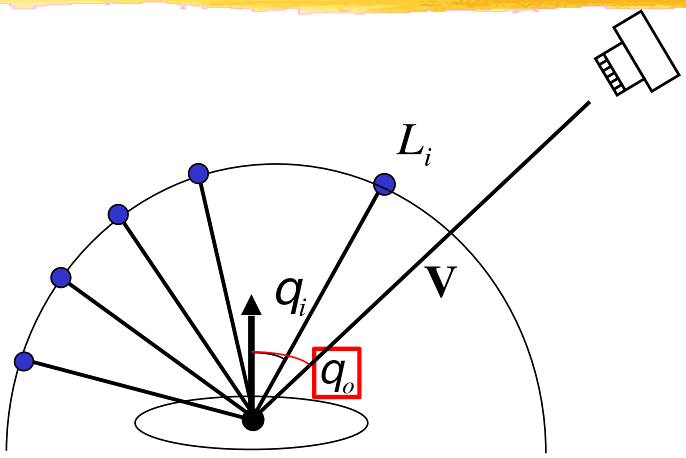
EVERYDAY SETTING



Multiple extended light sources:

- Illumination modeled as a weighted sum of spherical harmonics.
- Illumination parameters estimated using the light probe.

ILLUMINATION HEMISPHERE

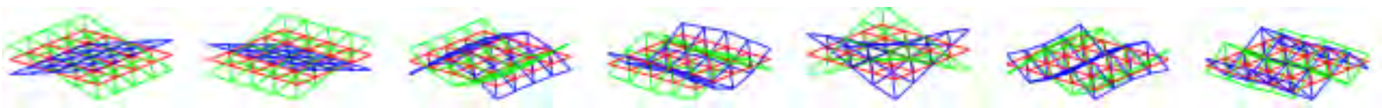


$$
\begin{aligned}
L_o(\mathbf{P},q_o,f_o) &= \int_{\Omega_i} r_{bd}\left(q_o,f_o,q_i,f_i\right) L_i(\mathbf{P},q_i,f_i)\cos\left(q_i\right)dw_i \\
&= \int_{\Omega_i} L_i(q_i,f_i)\, r_{bd}\left(q_o,f_o,q_i,f_i\right)\max(0,\cos\left(q_i\right))\,dw_i \\
&= \int_{\Omega_i} L_i(q_i,f_i)\, r^*\left(q_i,f_i\right)dw_i
\end{aligned}
$$

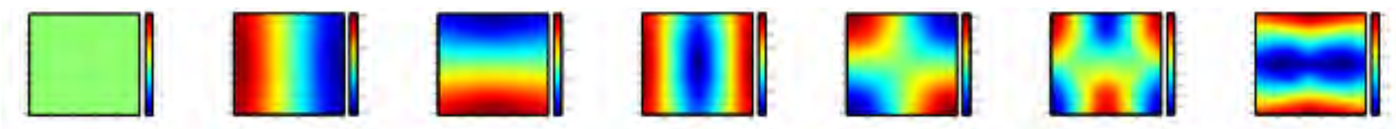
with $r^*\left(q_i,f_i\right) = r_{bd}\left(q_o,f_o,q_i,f_i\right)\max(0,\cos\left(q_i\right))$ is the BRDF product function.

PATCH INTENSITY AND DEFORMATION MODES

Synthesize a training database containing deformed patches and the corresponding intensity patterns. Compute:

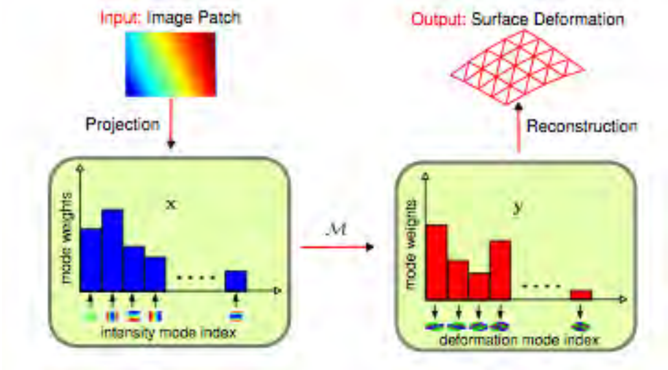


Deformation modes : $D = D_0 + \sum_i y_i D_i$

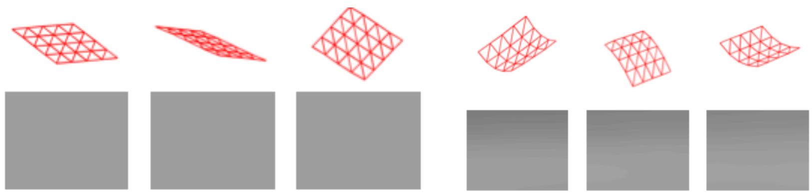


Intensity modes : $I = I_0 + \sum_i x_i I_i$

AMBIGUOUS MAPPING



Mapping from intensity to surface deformation.

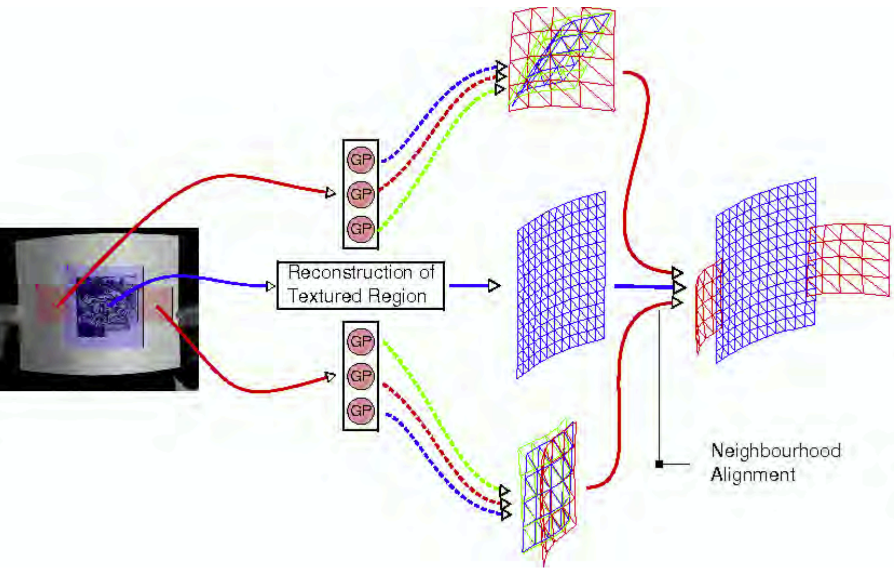


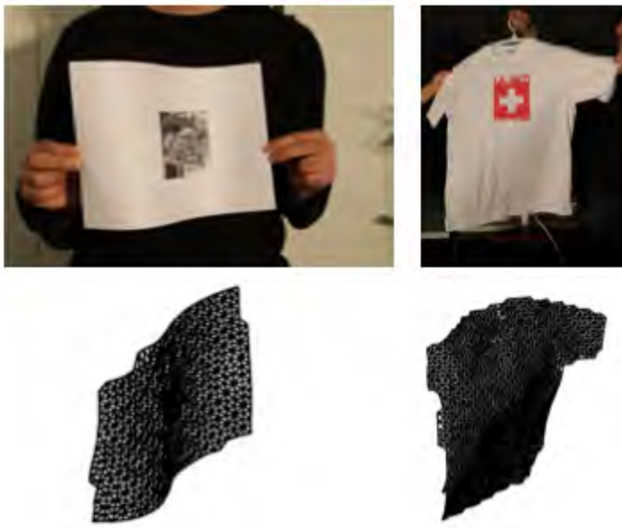
Unfortunately, the mapping is not one to one.

Algorithm:

- Partition the training database according to average normal to learn an unambiguous mapping.
- At run-time, predict several potential shapes for each image-patch and use a Markov-Random field to pick a set of consistent interpretations.

ALGORITHMIC FLOW

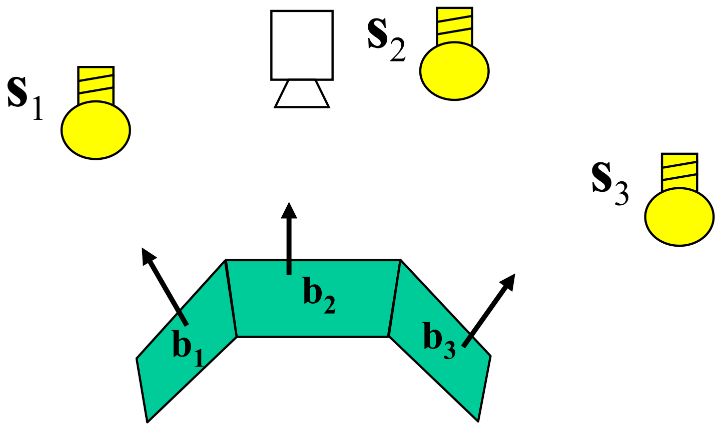




→ The light environment and the camera need to be very carefully calibrated but there is hope …

Varol et al. , PAMI'12

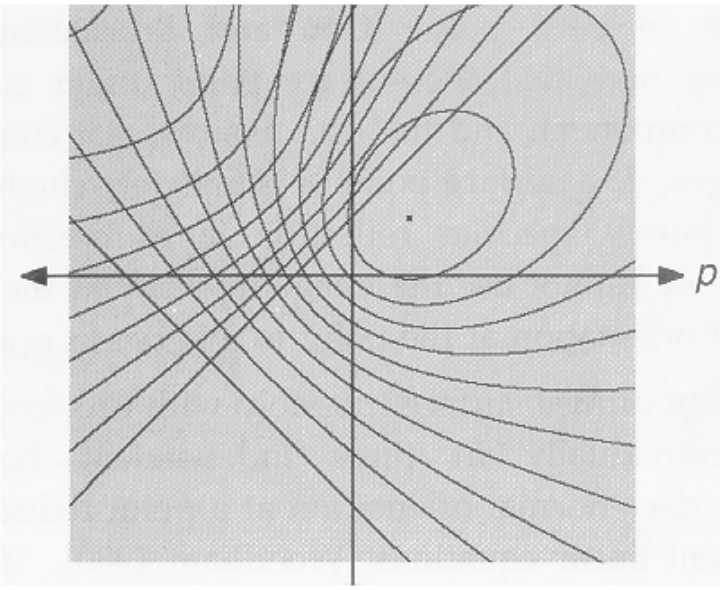
PHOTOMETRIC STEREO

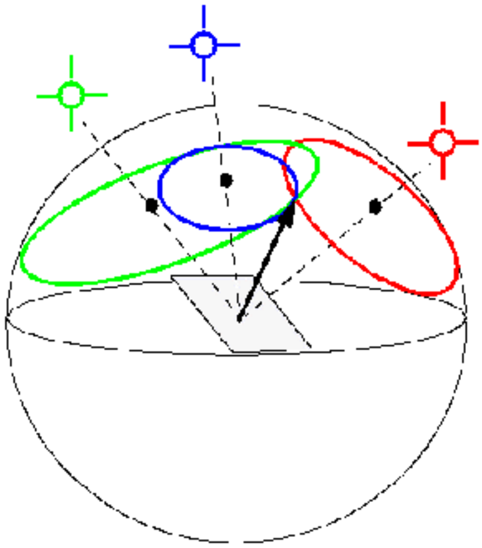


Given multiple images of the same surface under different known lighting conditions, can we recover the surface shape?

Yes! (Woodham, 1978)

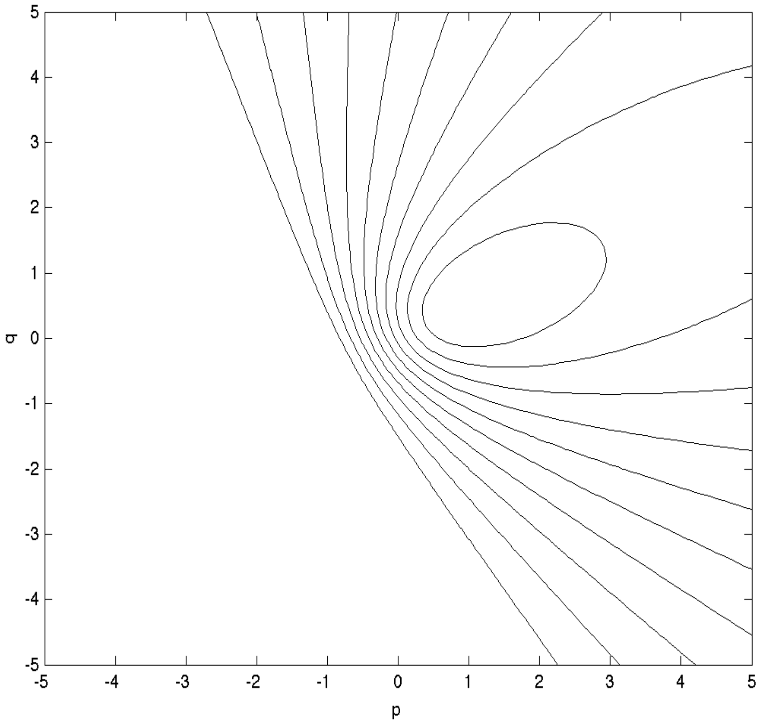
PHOTOMETRIC STEREO

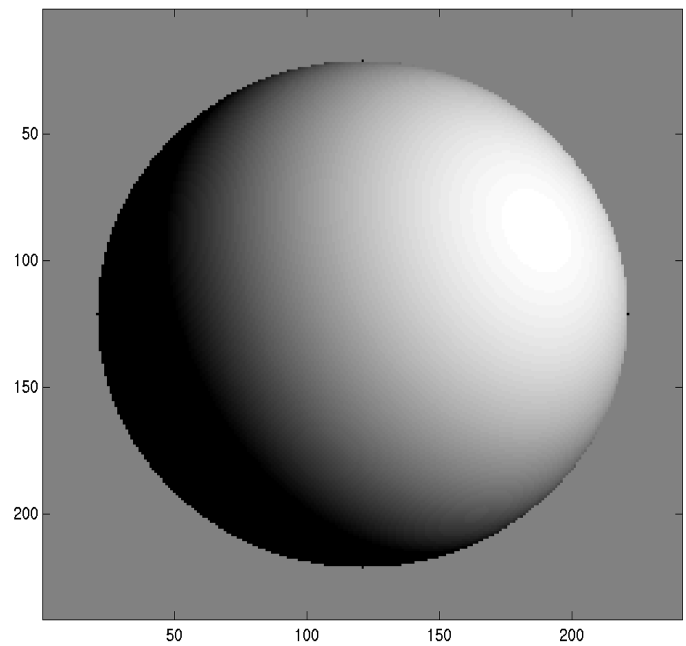




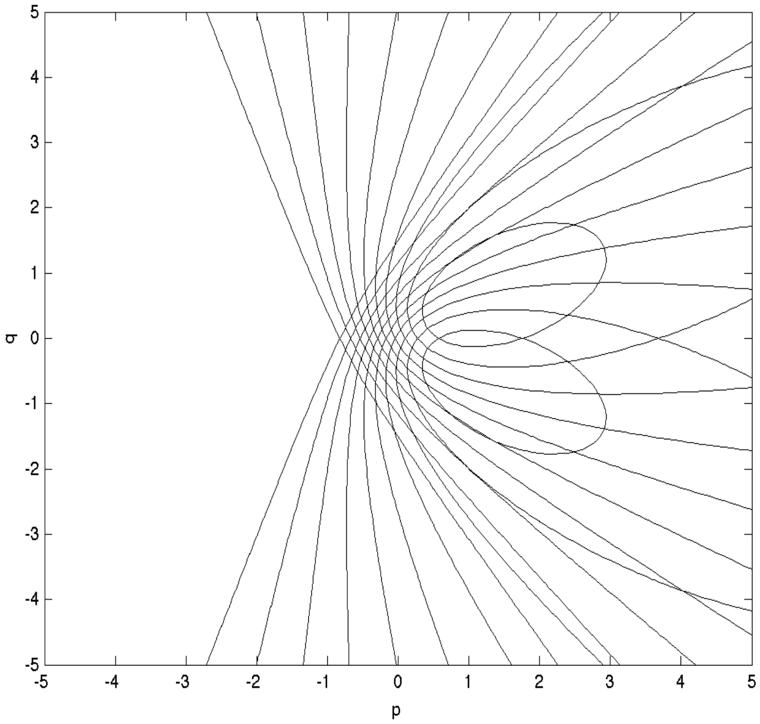
- Take several images under different lighting conditions.
- Infer the orientation from the changes in illumination.

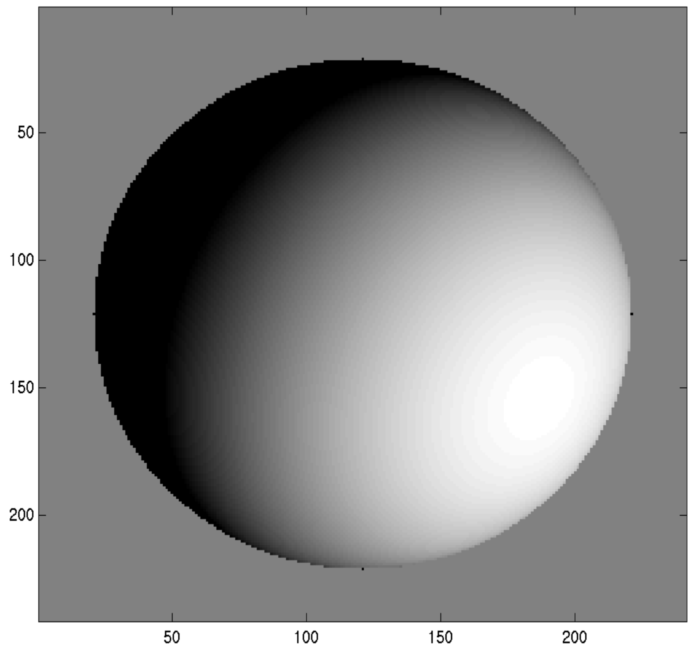
PHOTOMETRIC STEREO



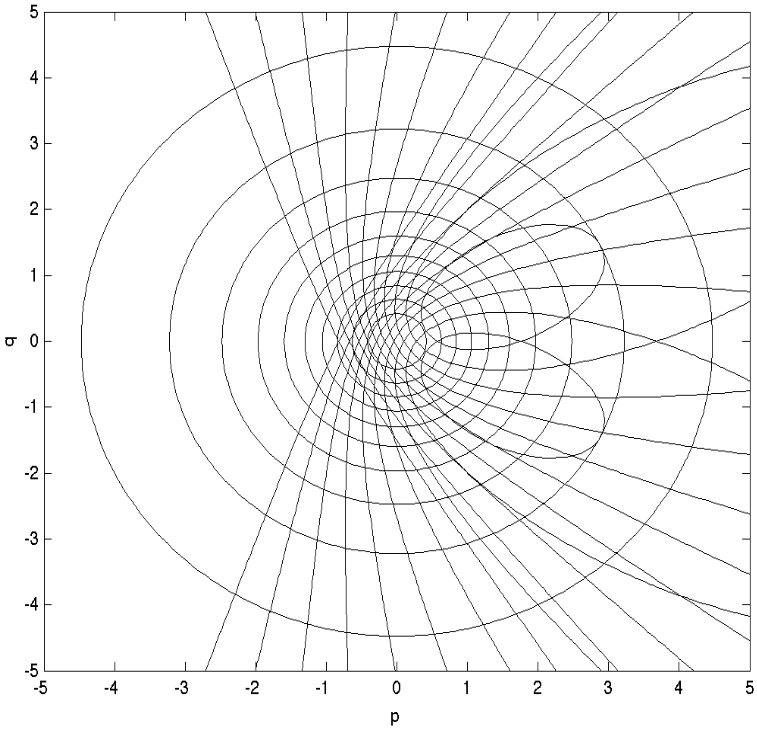


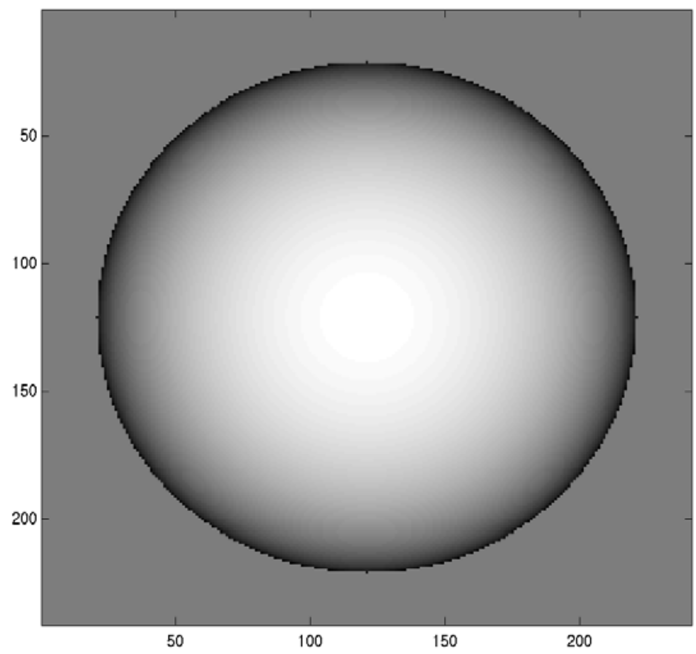
PHOTOMETRIC STEREO





PHOTOMETRIC STEREO





ALGEBRAIC FORMULATION

Lambertian model:

$$I = a(\mathbf{N} \bullet \mathbf{L}) = \mathbf{L} \bullet \mathbf{M}$$

Three light sources:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \mathbf{L}_3 \end{bmatrix} \mathbf{M}$$

$$\mathbf{N} = \frac{\mathbf{M}}{\|\mathbf{M}\|}$$

$$a = \|\mathbf{M}\|$$

ADDITIONAL LIGHTS

Over-constrained problem:

$$\mathbf{I} = \mathbf{L}\mathbf{M}, \text{ with } \mathbf{I} = \begin{bmatrix} I_1 \\ : \\ I_n \end{bmatrix} \text{ and } \mathbf{L} = \begin{bmatrix} \mathbf{L}_1 \\ : \\ \mathbf{L}_n \end{bmatrix}$$

$$\Rightarrow \mathbf{L}^t\mathbf{L}\mathbf{M} = \mathbf{L}^t\mathbf{I} \text{ (Least - squares solution)}$$

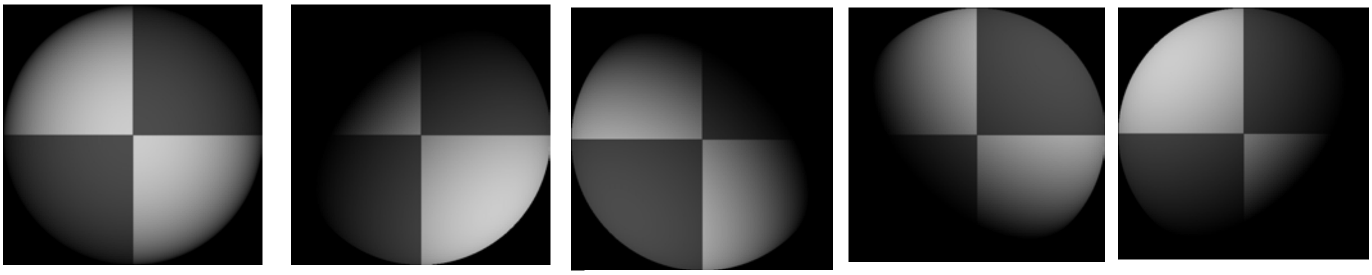
SHADOWS

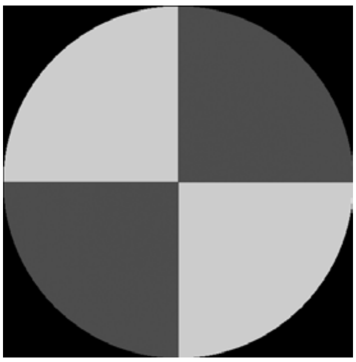
- Shadowed pixels for a given light source position are outliers.
- Premultiplying by a thresholded weight matrix eliminates their contributions.

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \boldsymbol{I} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \boldsymbol{LM}$$

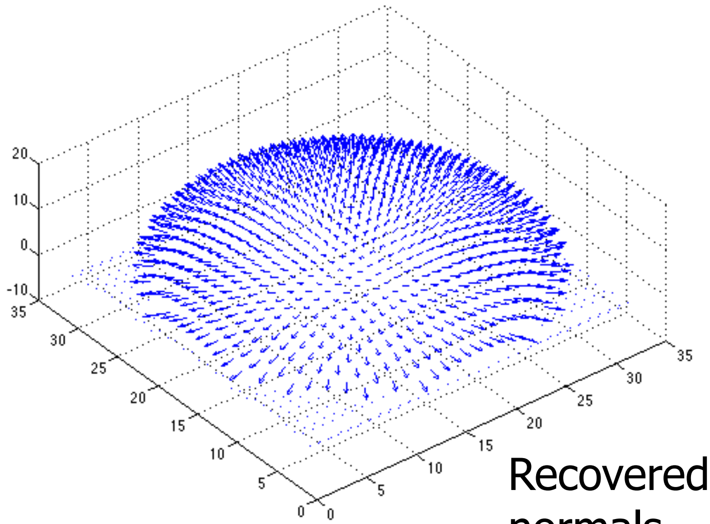
SYNTHETIC SPHERE IMAGES

Five different lighting conditions:





Recovered albedo

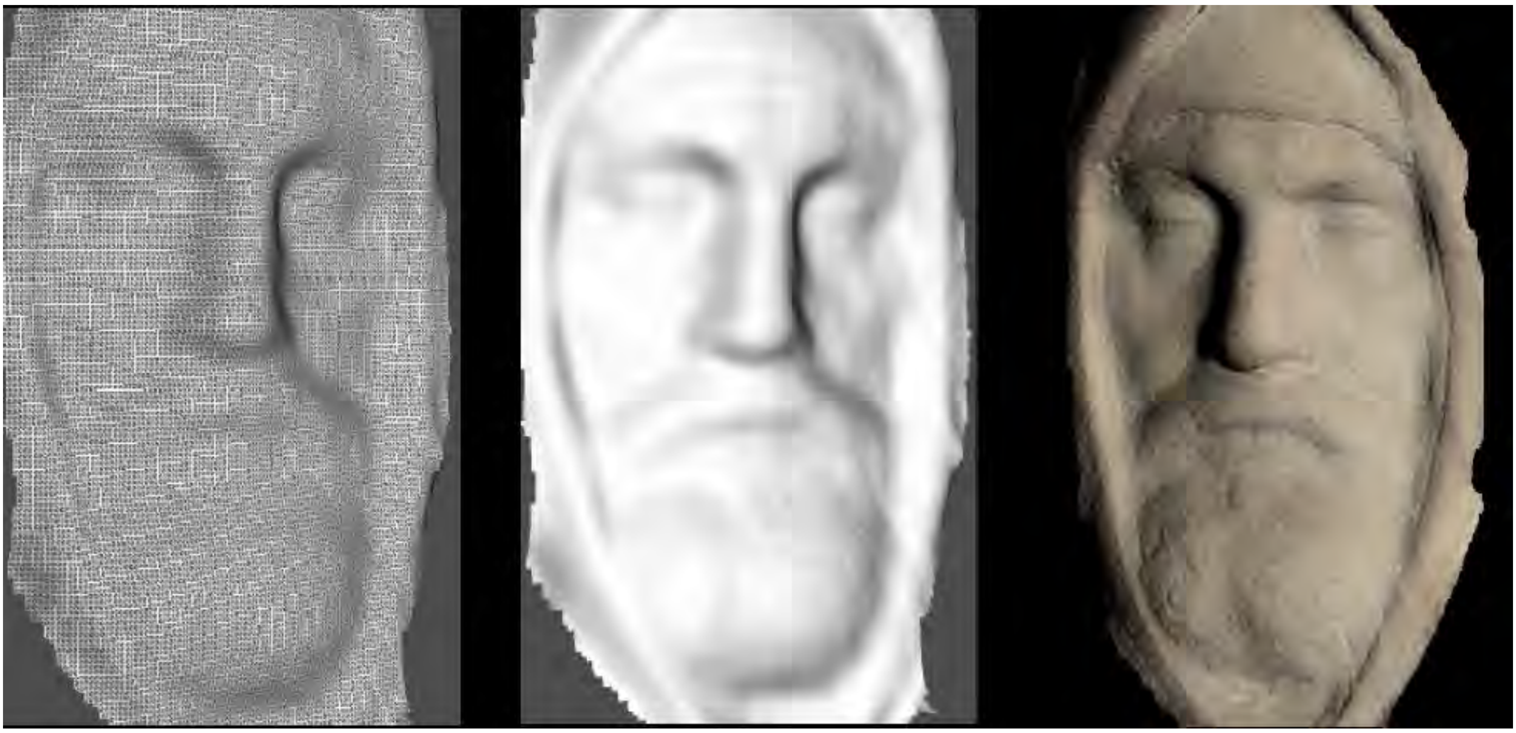


Recovered surface normals

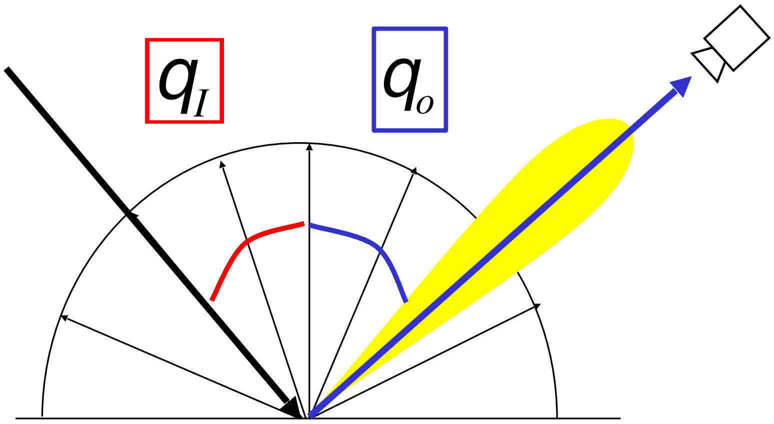
VIRTUOSO

One camera and five light sources

DELIGHTED TEXTURE MAPS

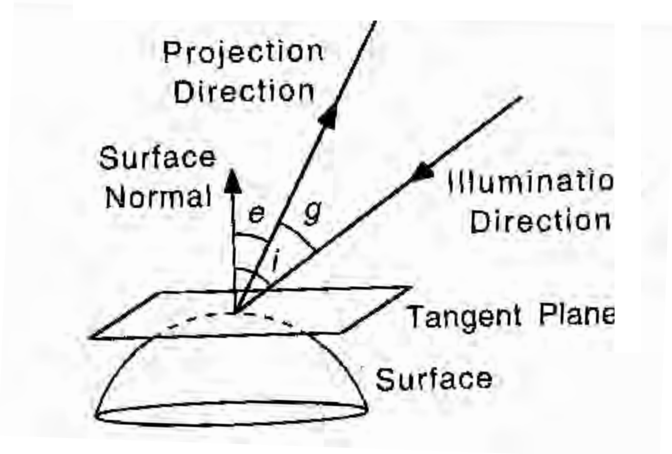


SPECULARITIES

- At specular points Lambertian assumptions are violated.
- However, they can be used to infer normal information.

ANGLES

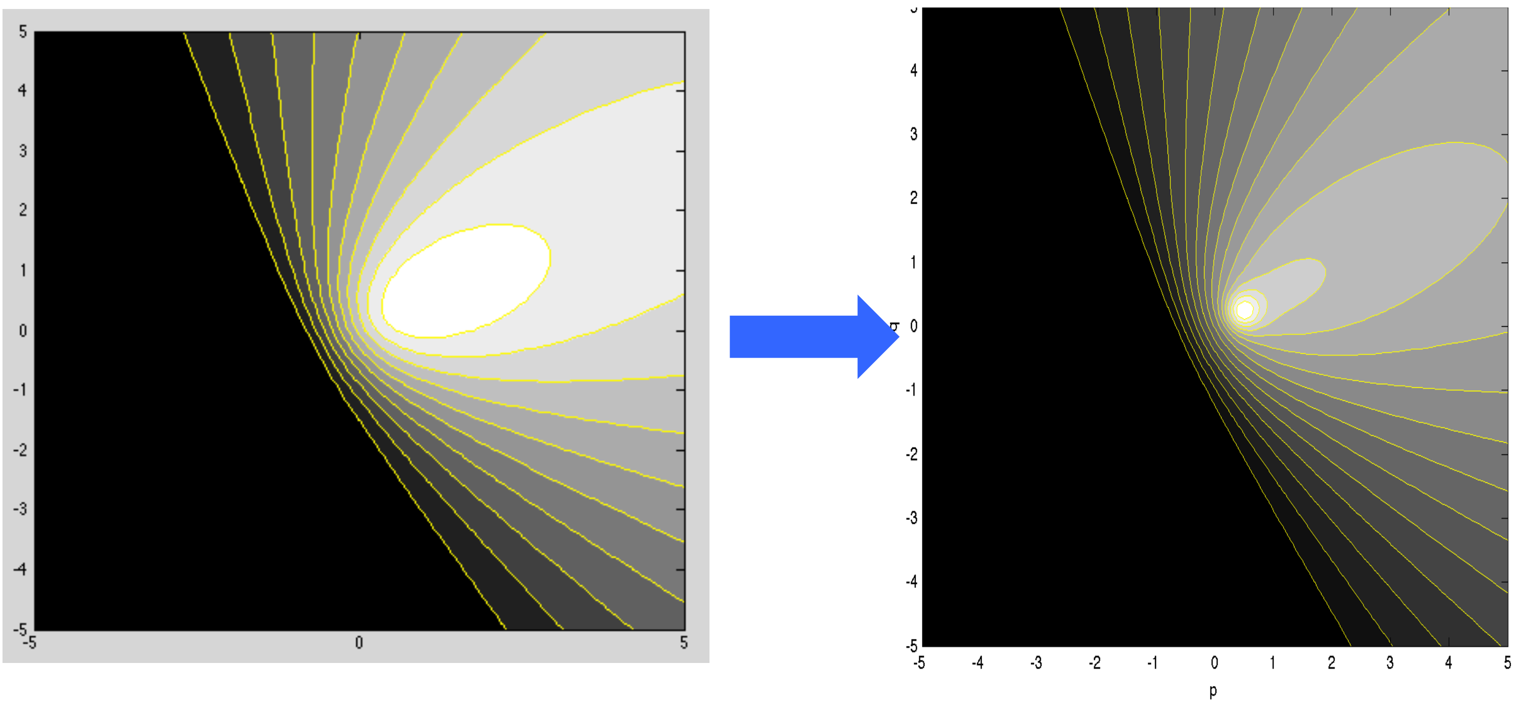


Angle of incidence (i): angle between the surface normal and the direction of the incident light ray.

Angle of emittance (e): angle between emitted light ray and surface normal.

Phase angle (g): angle between incident and emitted light ray.

LAMBERTIAN+SPECULAR REFLECTANCE MAP



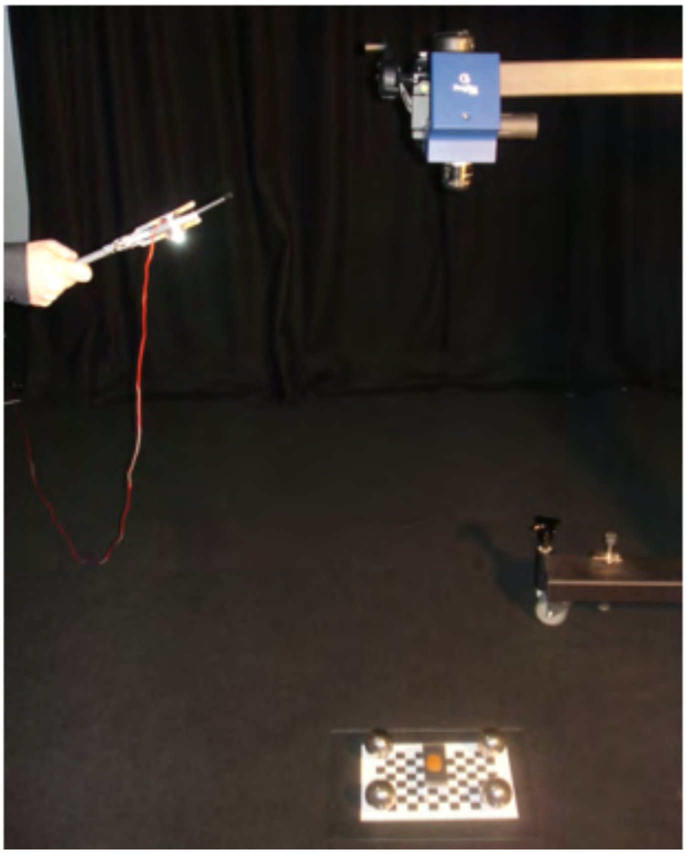
Weighted average of the individual diffuse- and specular-component of glossy white paint.

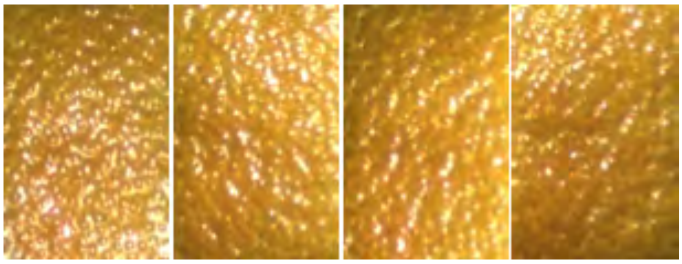
SPECULAR SURFACE

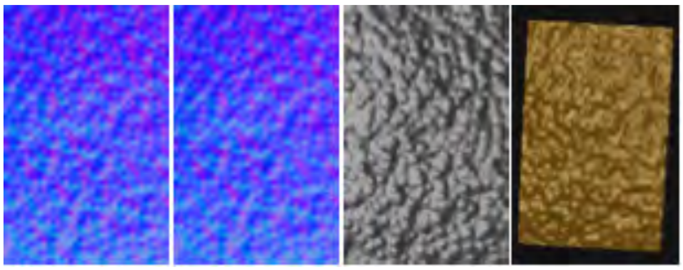
Perfect mirror: Reflects light only when i = e and i + e = g.

→ In practice, most surfaces are a mixture of specular and Lambertian.

SHAPE FROM SPECULARITIES

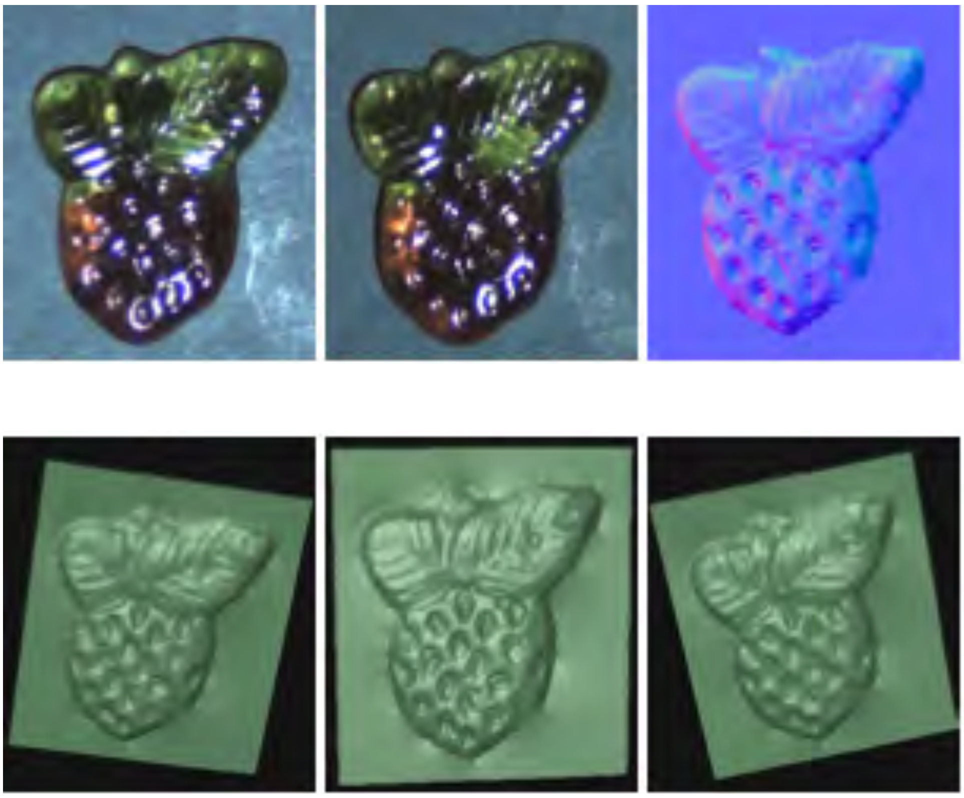






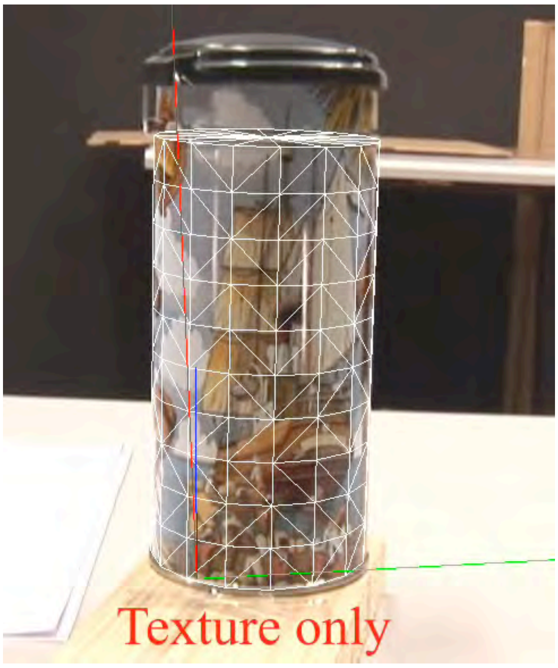
Chen et al., CVPR 2006

SHAPE FROM SPECULARITIES



Chen et al., CVPR 2006

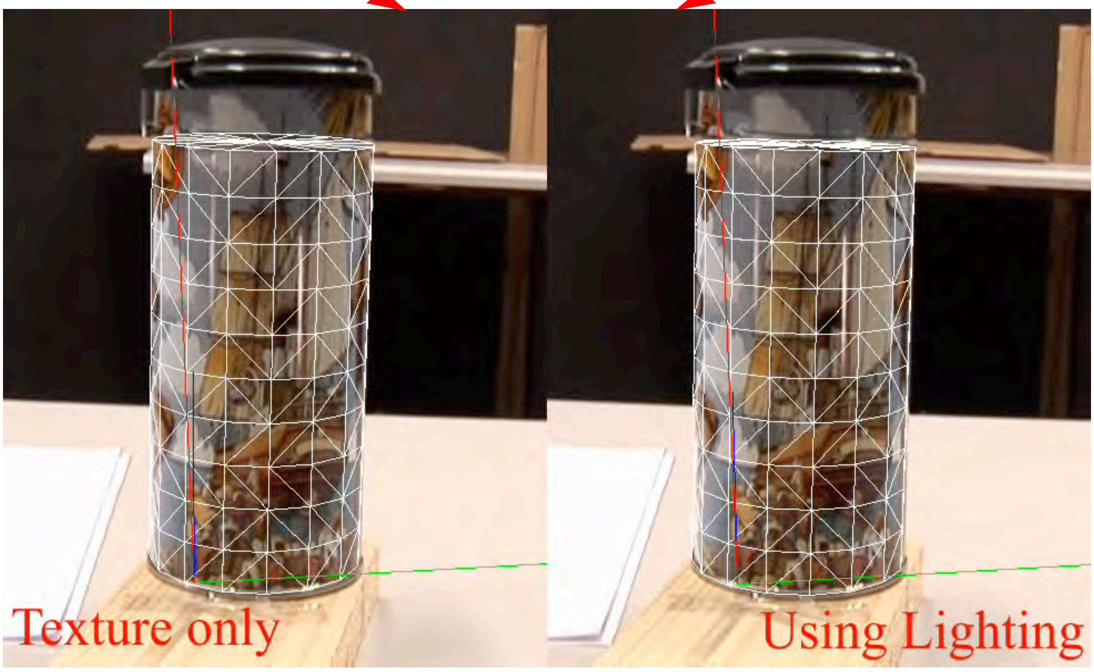
POSE FROM TEXTURE



- Texture tracking yields correct reprojection of frontal facets.
- But the jittering top shows that the tracking is nevertheless inaccurate.

POSE FROM SPECULARITIES

Taking advantage of specularities yields better poses



→ No more jittering top!

TEXTURE AND SPECULARITIES

Specularities are:

- Very sensitive to motion
- Affected differently than texture by the same motion

Specularities are not:

- Capable of constraining all the degrees of freedom

Therefore texture and specularities must be combined

IN SHORT

Traditional Shape-from-Shading requires making strong assumptions:

- Constant or piece-wise constant albedo
- No inter-reflections
- No shadows
- No specularities

→ In a single image context, it is most useful in conjunction with other information sources.