SHAPE FROM X

One image:
- **Shading**
- Texture

Two images or more:
- Stereo
- Contours
- Motion

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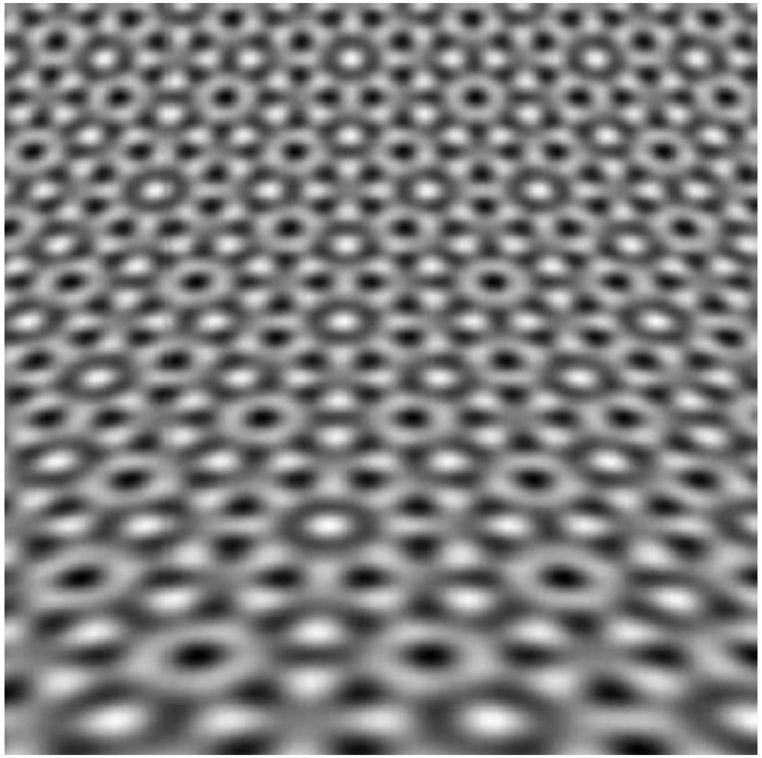

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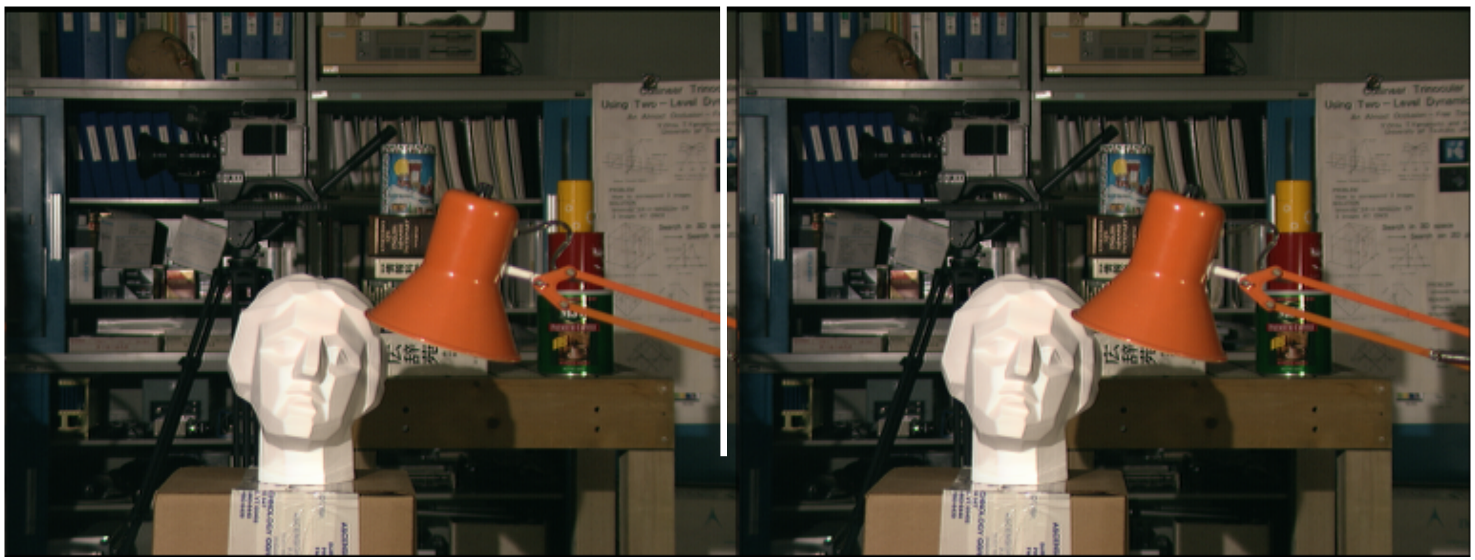

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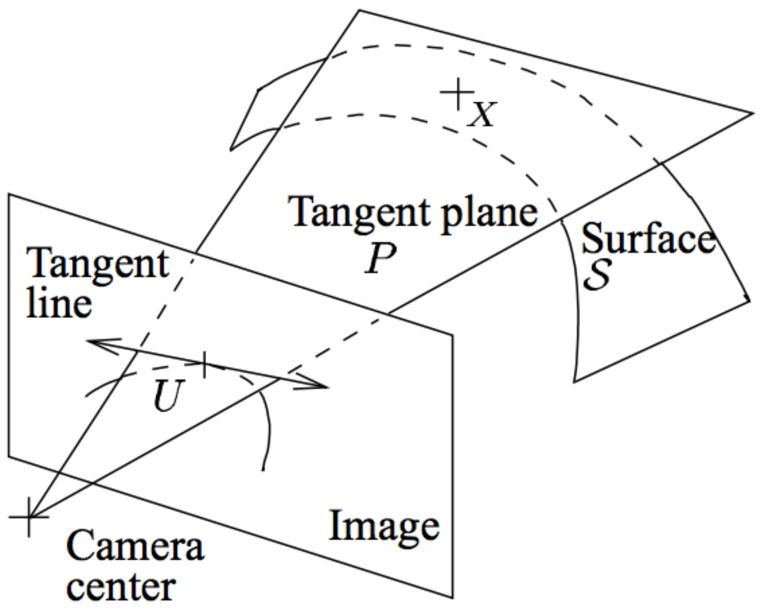

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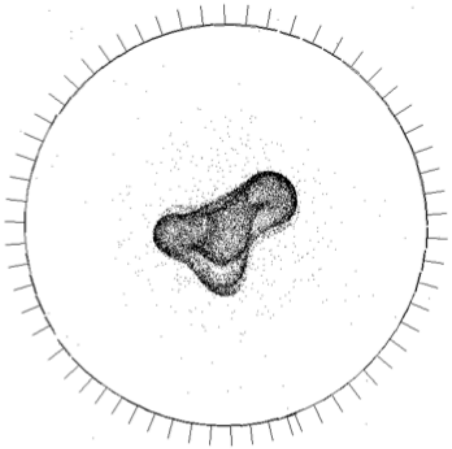
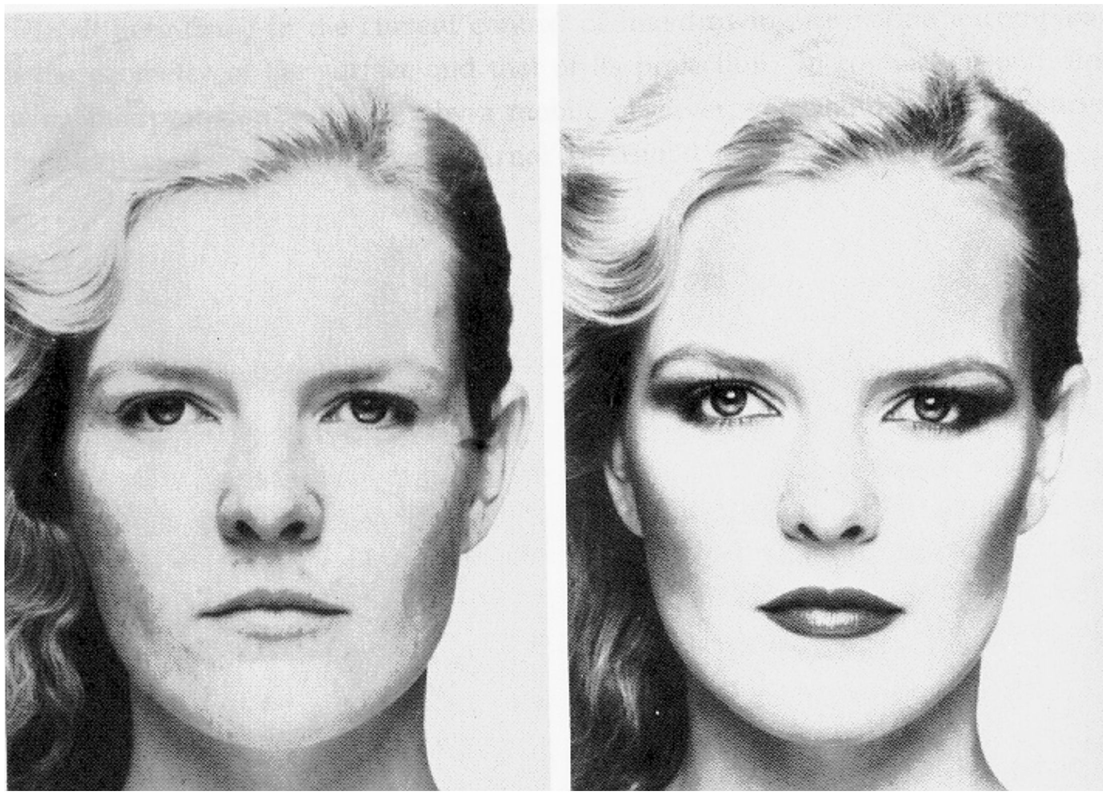


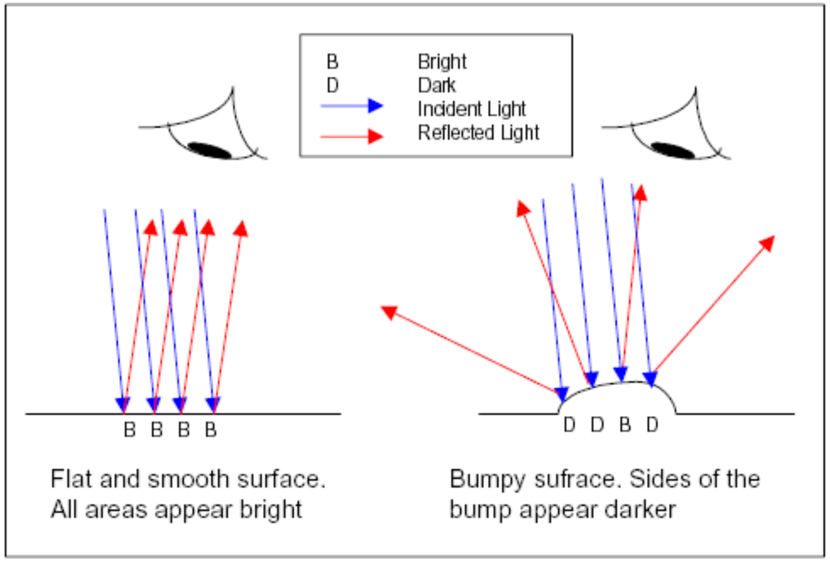
SHADING

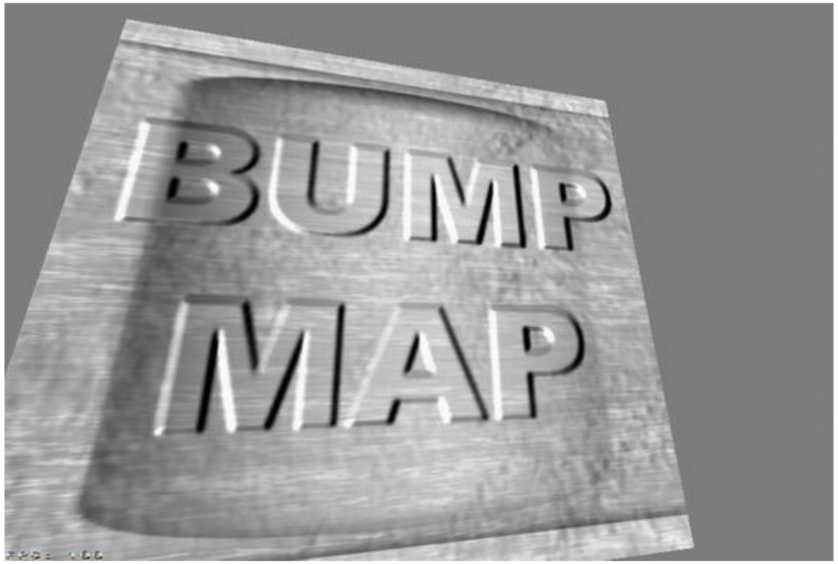
- Shading models
- Shape from shading
  - Variational Methods
  - Photometric Stereo

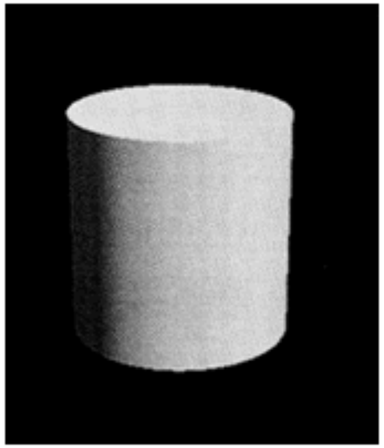


BUMP MAPPING

use shadows / darkness to estimate curvature

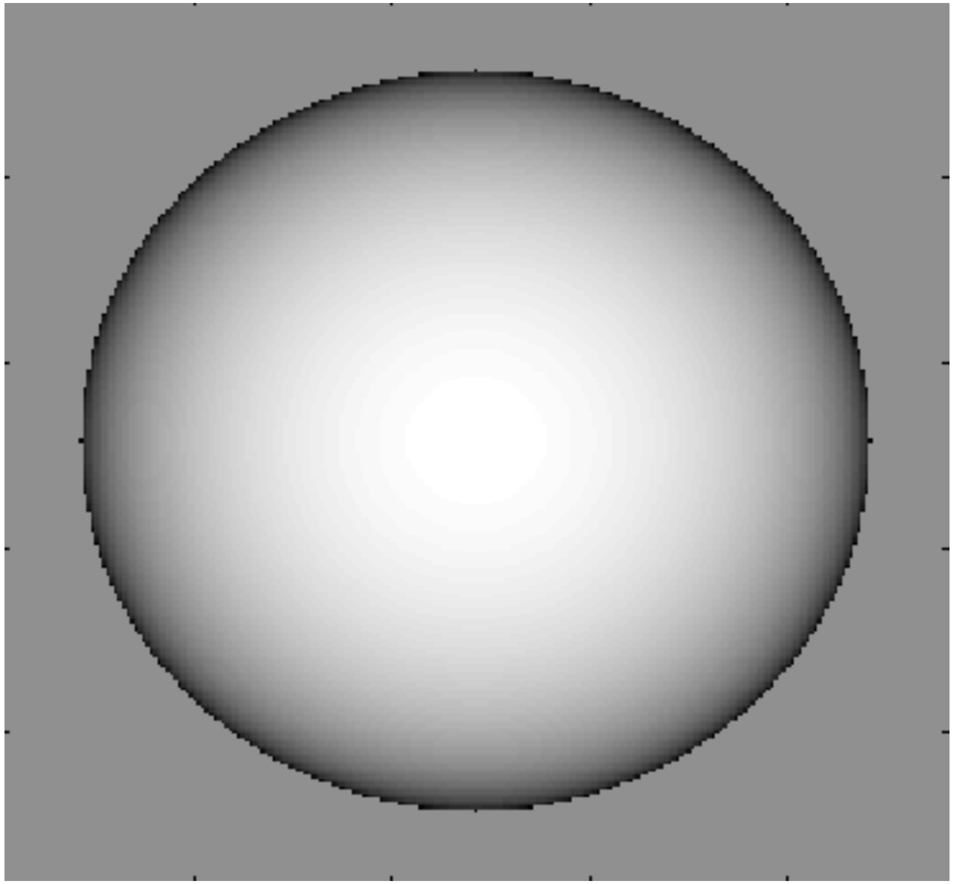


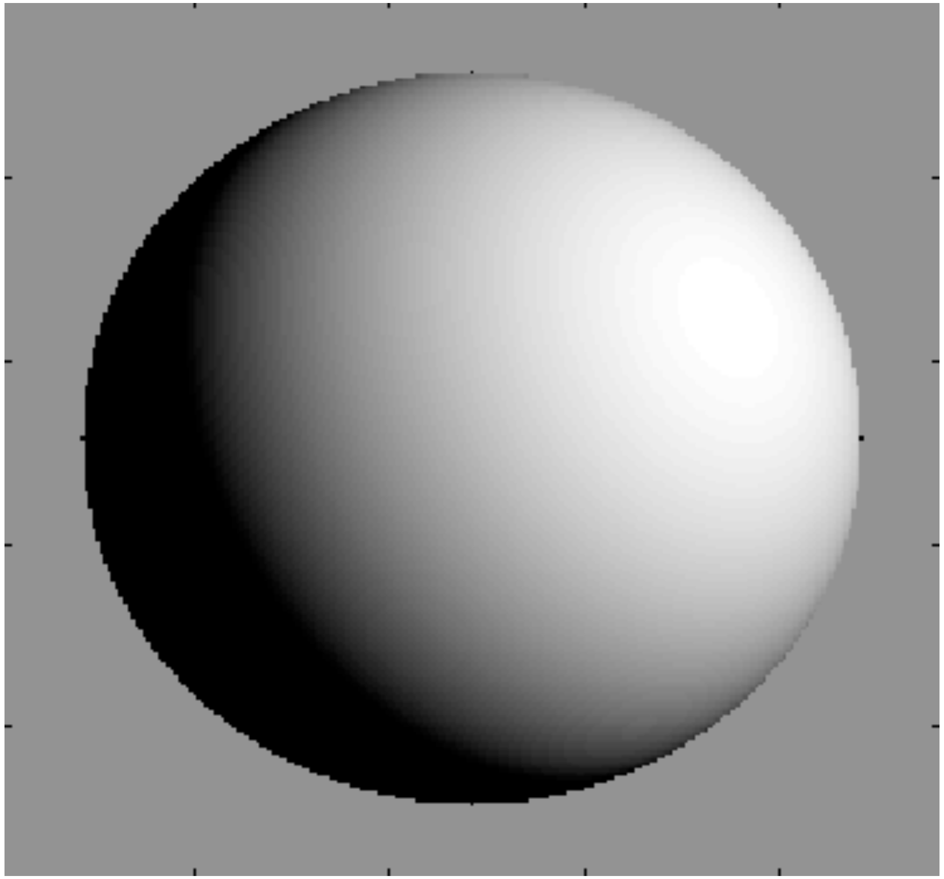




Simple mesh + 2D bump map = Complex looking object

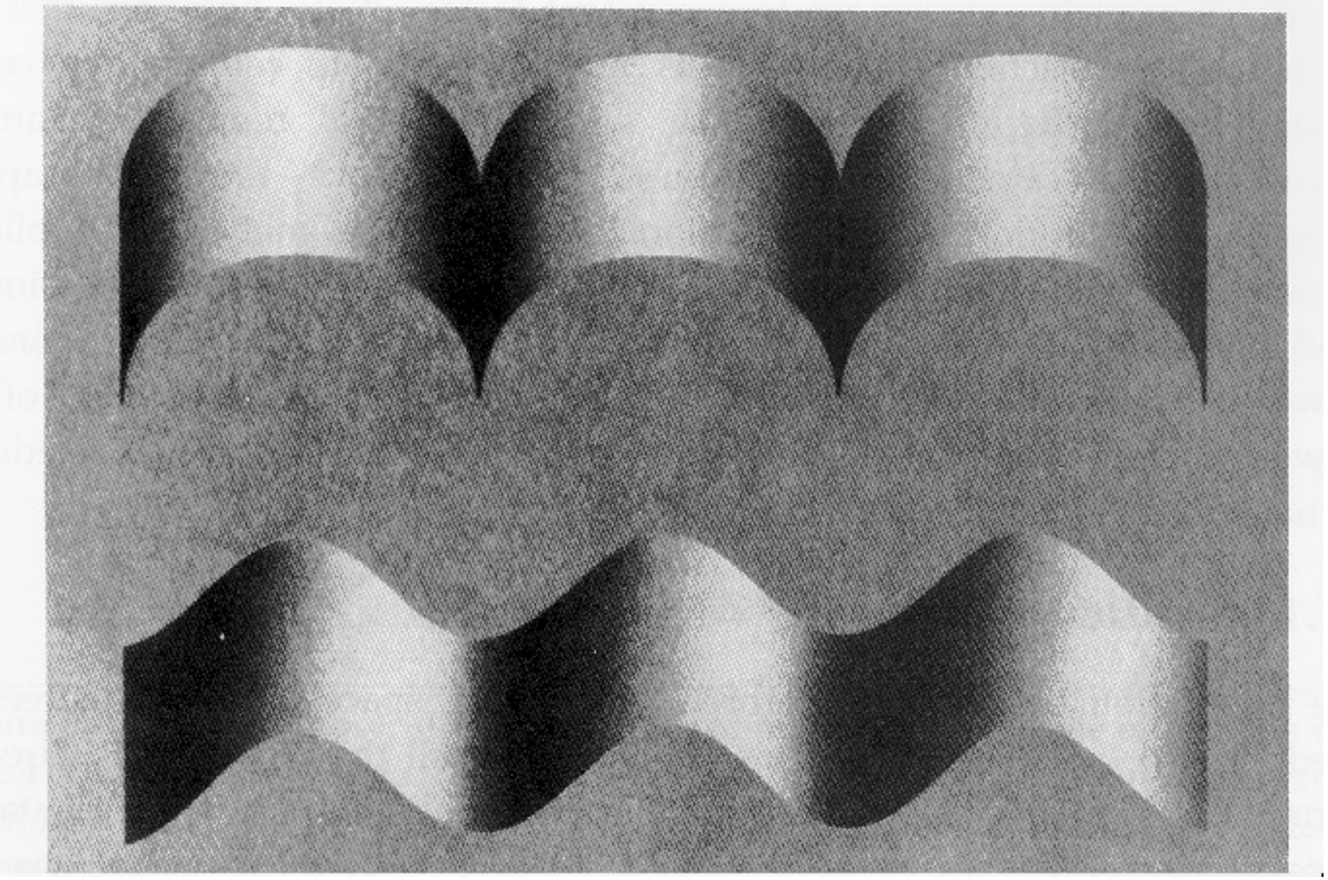
LAMBERTIAN HALF-SPHERE





Gray level changes are interpreted as changes in the direction of the surface normal

BOUNDARY CONDITIONS



todo: what does the boundary condition here mean

Shading gives information about surface normals

→ Boundary conditions required for a complete interpretation.

BEETHOVEN

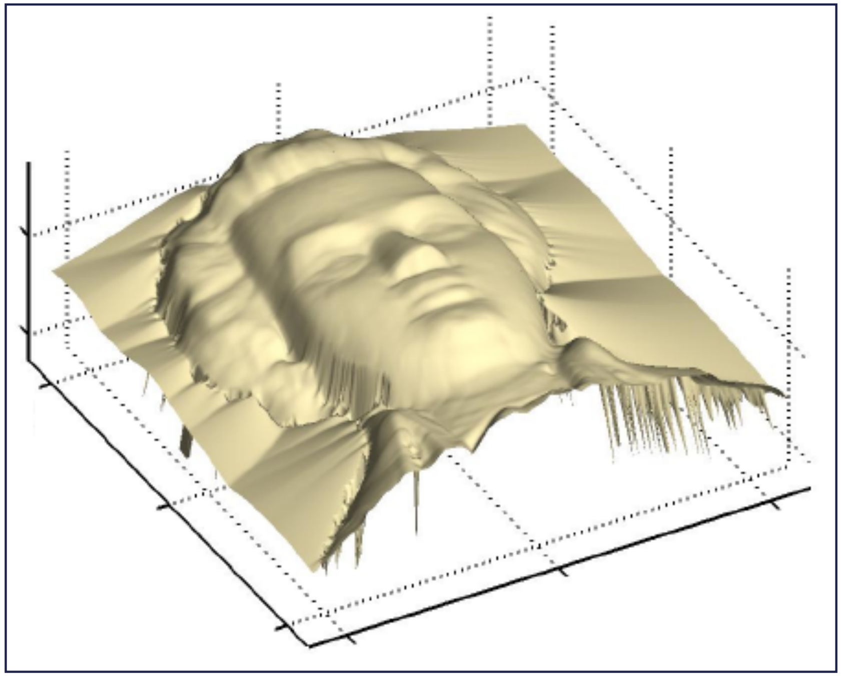
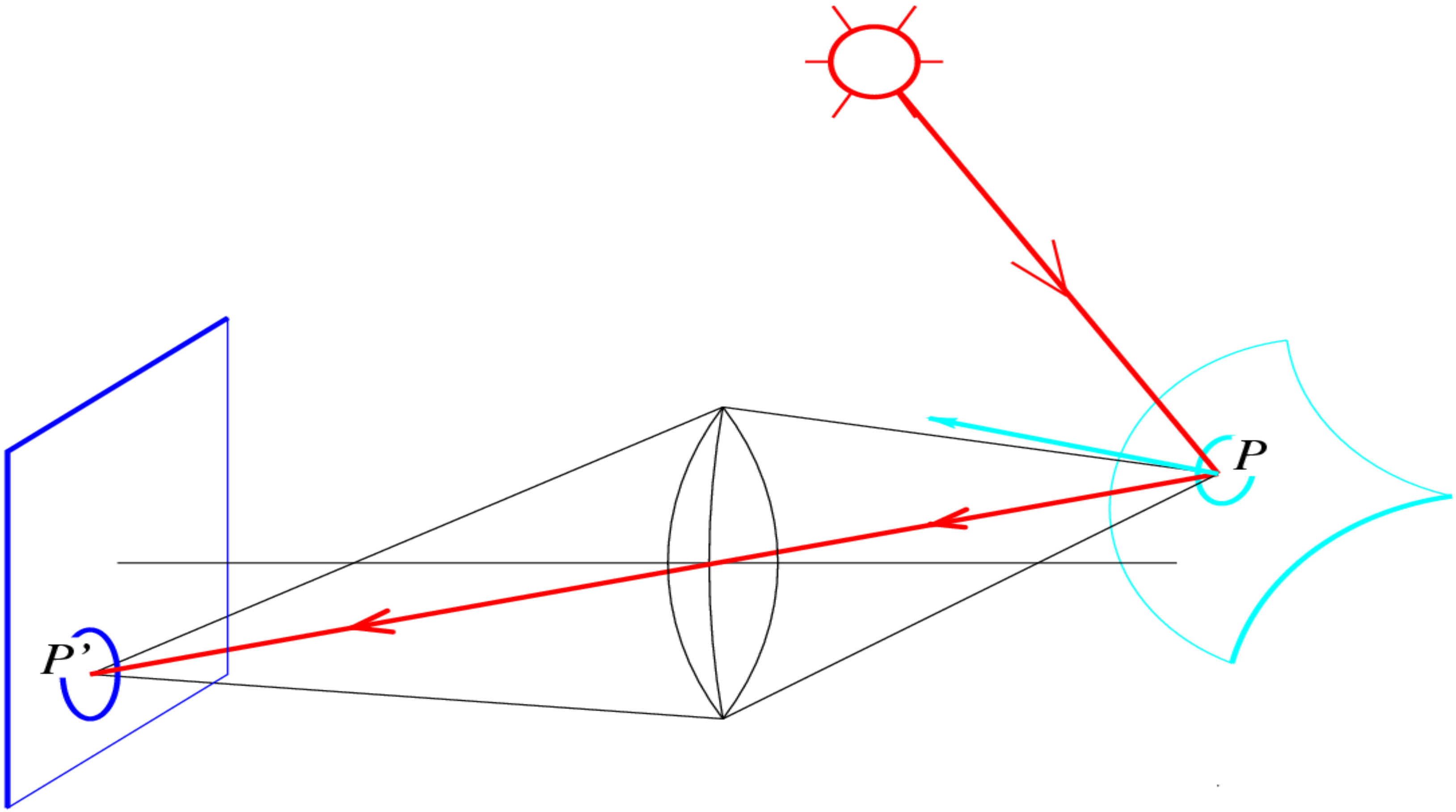
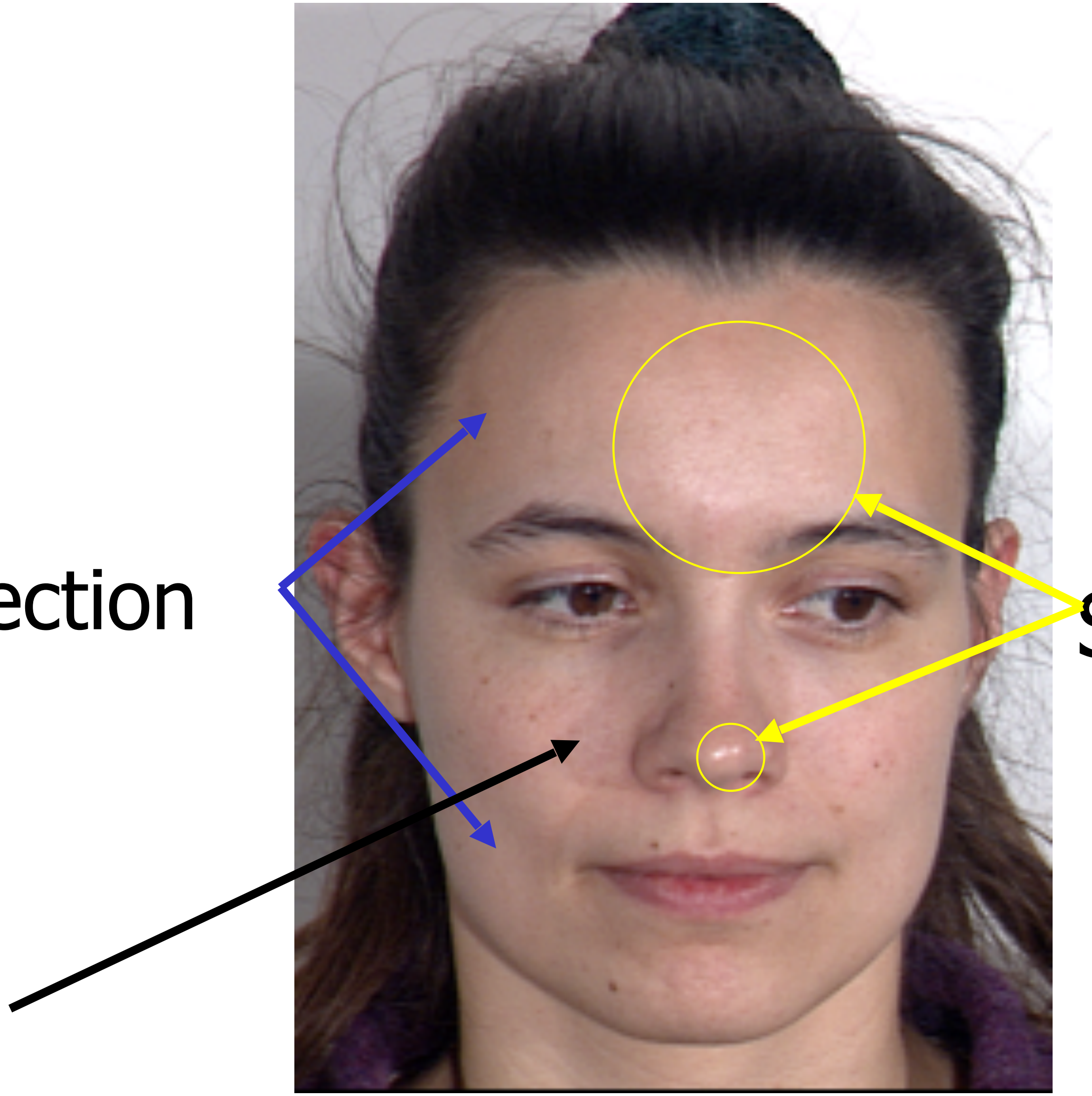


IMAGE FORMATION

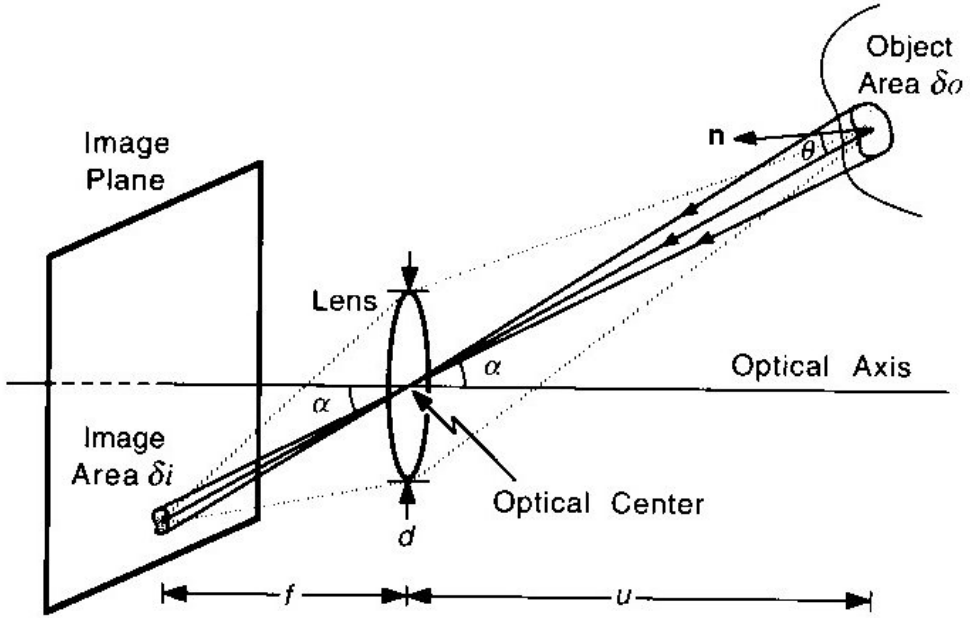


DIRECT LIGHTING



INDIRECT LIGHTING

FUNDAMENTAL RADIOMETRIC EQUATION



Radiance: amount of light emitted in a given direction

Irradiance: amount of light received on a surface

$$Irr = \frac{p}{4}\left(\frac{d}{f}\right)^2 \cos^4(a) Rad$$

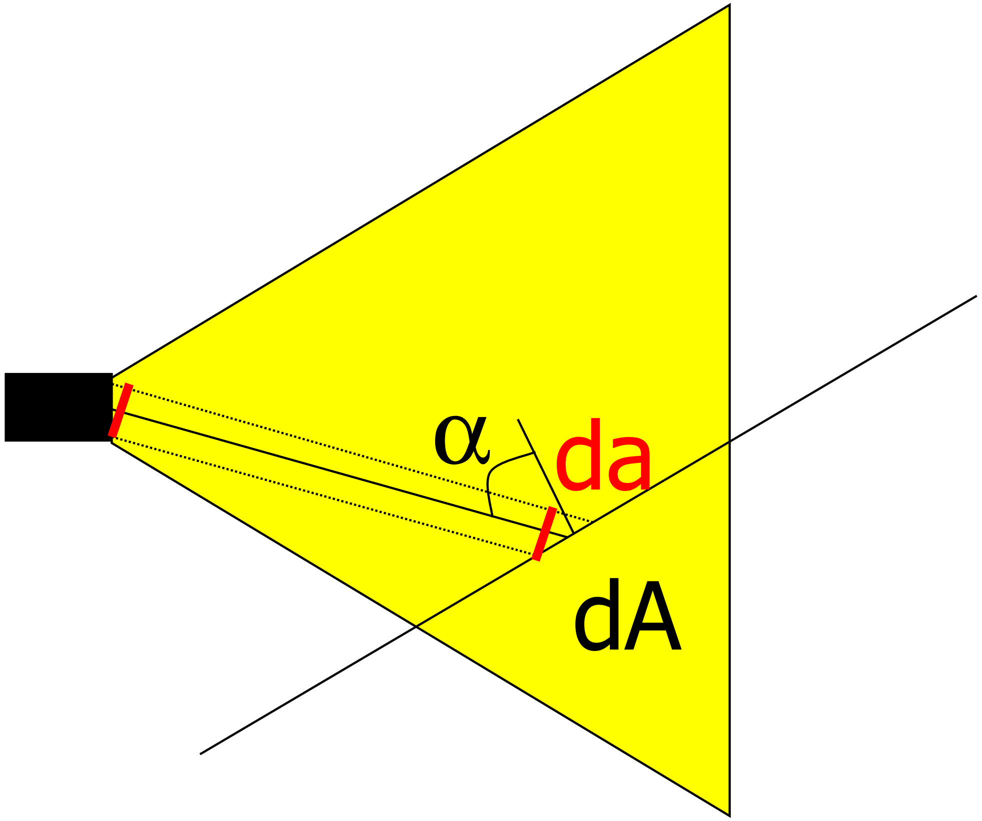
Image intensity

$$\Rightarrow I \propto Irr \propto Rad$$

Image Intensity is simply proportional to radiance and irradiance

when the camera is photometrically calibrated.

LOCAL SHADING MODEL



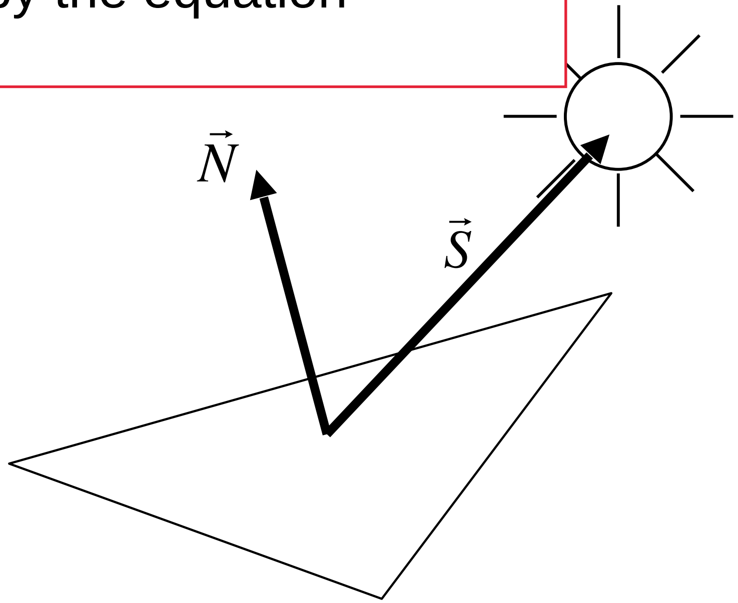
Foreshortening:

$$da = \cos(\alpha) dA$$

The effect of a distant source on a surface patch depends on its **apparent** surface.

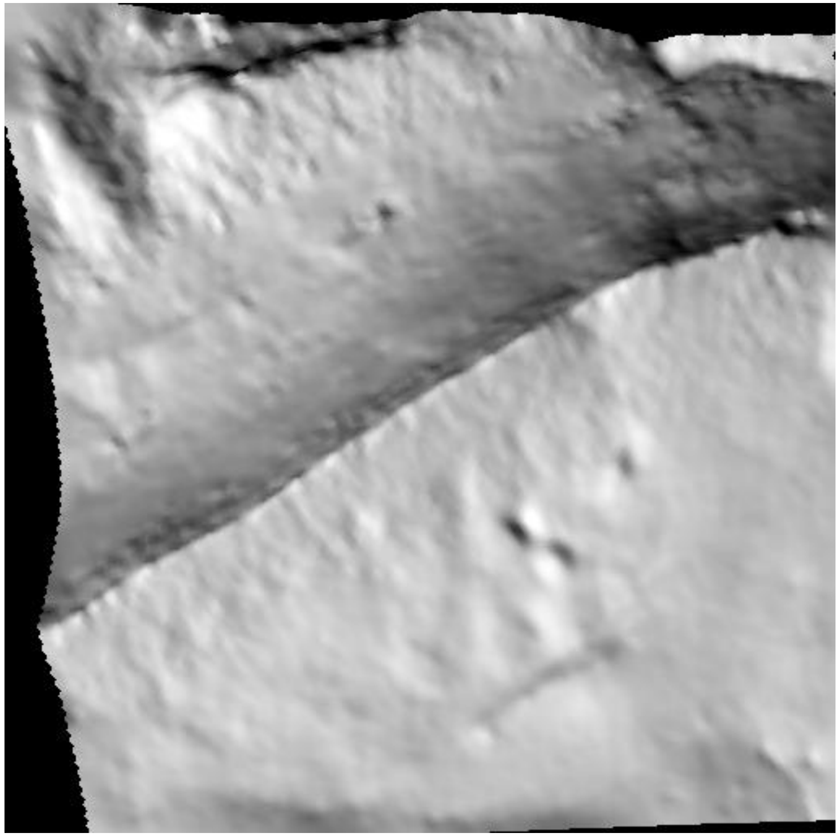
LAMBERTIAN SURFACE

Lambertian object means no specularity i.e. it is completely defined by the equation



$$I = \max(Albedo \cdot (\mathbf{N} \bullet \mathbf{S}), 0)$$

albedo = portion of light being reflected. 0 = black



Perfectly matte surface: The radiance depends only on angle of incidence and not on viewing direction.

ESTIMATED ALBEDO

PER PIXEL PARAMETERS

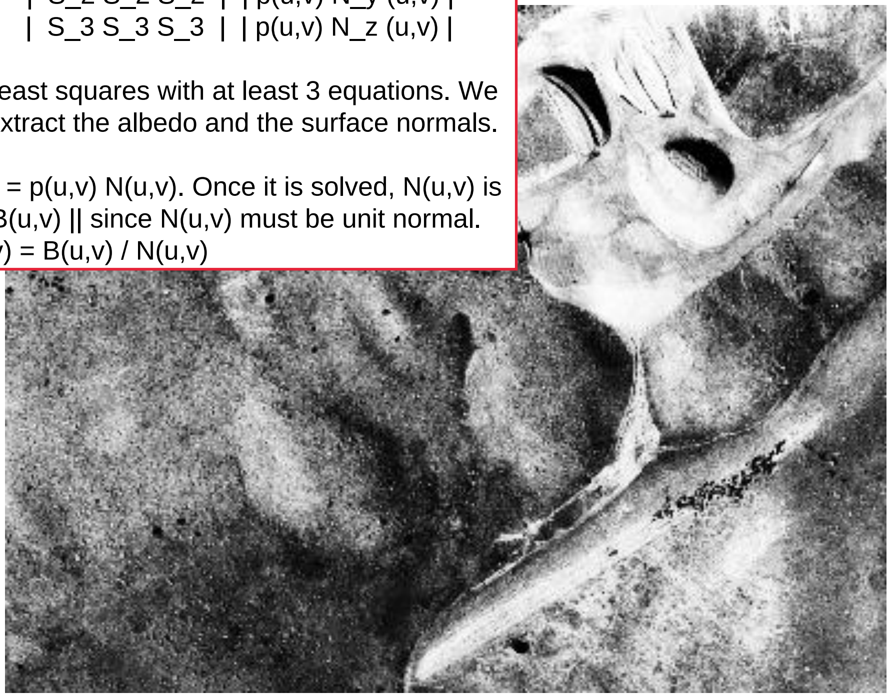
p - Albedo (1 unknown)
N - Surface Normal (2 unknowns since it has 2 DoF)
S_i - Directional Source (known)

I_i = p (N • S_i)
I_i = S_i • (pN)
I_i = S_i • B

I_1 (u,v) | S_1 S_1 S_1 | | p(u,v) N_x (u,v) |
I_2 (u,v) = | S_2 S_2 S_2 | | p(u,v) N_y (u,v) |
I_3 (u,v) | S_3 S_3 S_3 | | p(u,v) N_z (u,v) |

Solve via least squares with at least 3 equations. We can then extract the albedo and the surface normals.

Let B (u,v) = p(u,v) N(u,v). Once it is solved, N(u,v) is B(u,v) / || B(u,v) || since N(u,v) must be unit normal.
Then p(u,v) = B(u,v) / N(u,v)



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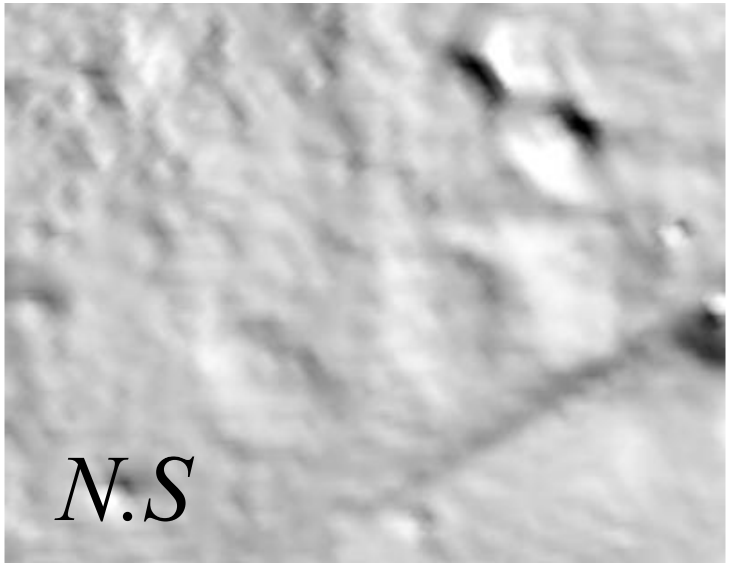
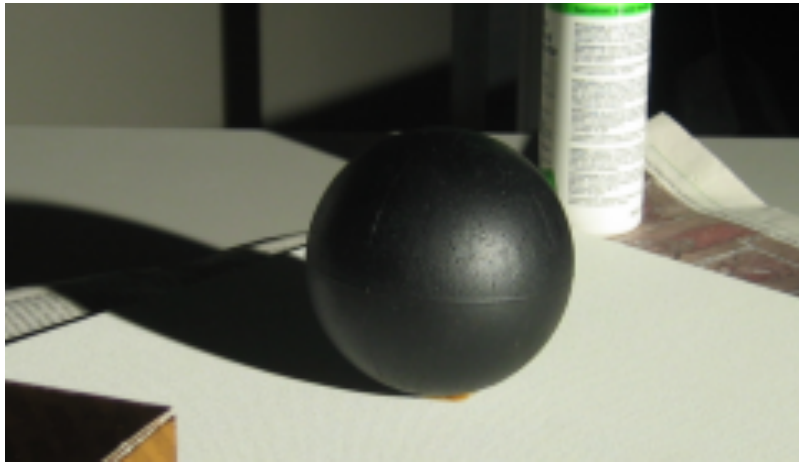


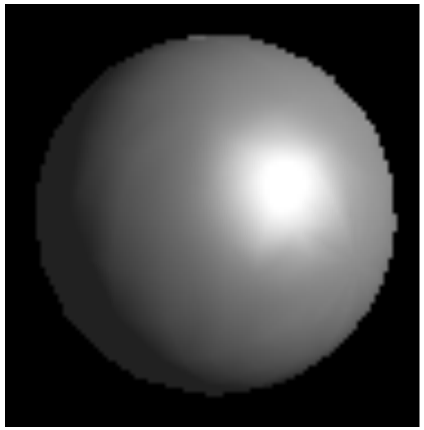
*

SECONDARY ILLUMINATION

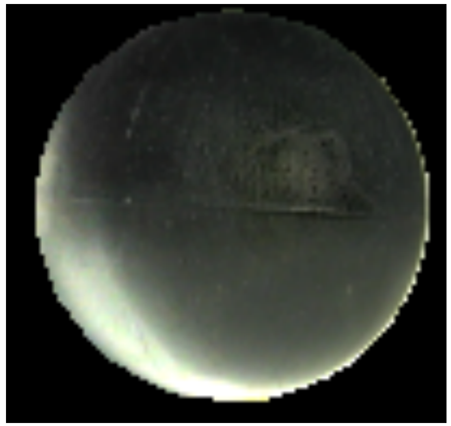
Reflections produce indirect lighting.

Unique light source assumption doe not allow correct albedo recovery.



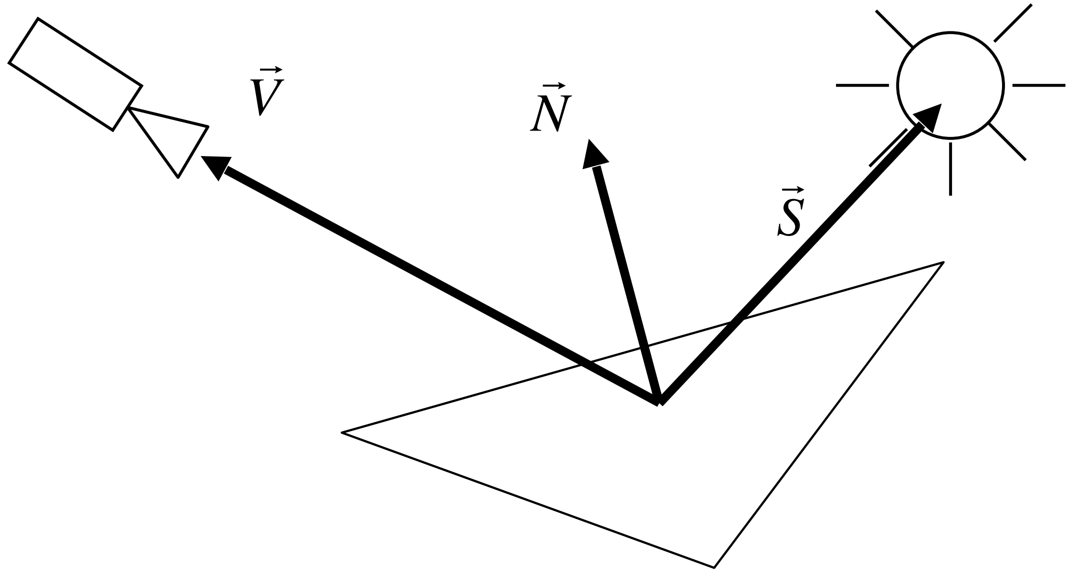


Recovered shading



Recovered albedoes

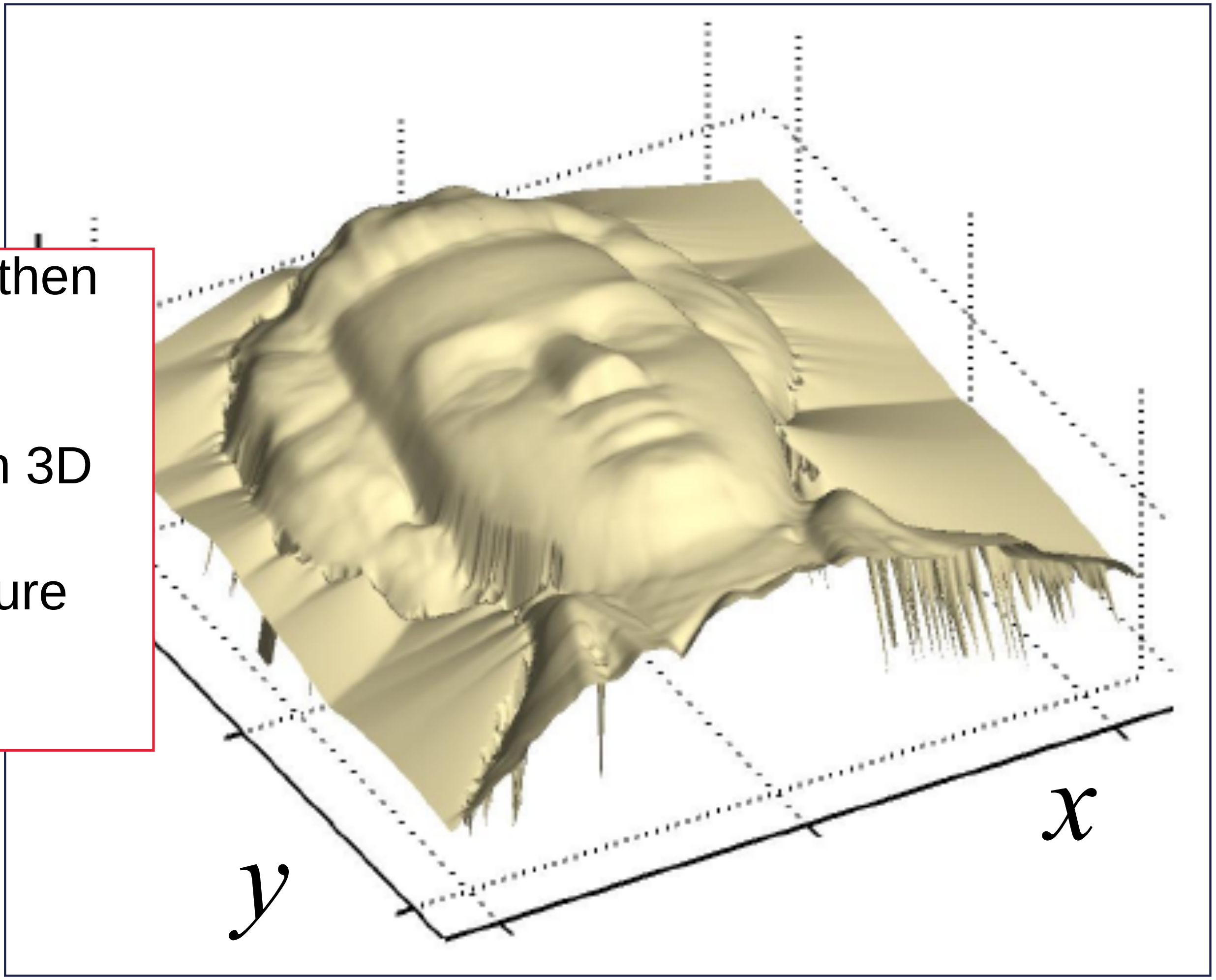
SIMPLIFYING ASSUMPTIONS



- The illumination sources are distant from the imaged surfaces
- Secondary illumination is not significant
- There are no cast shadows

MONGE SURFACE

$$z = f(x, y)$$



if we pretend z is the intensity, then
dz/dx = edge in x
dz/dy = edge in y
which can give us the normal in 3D

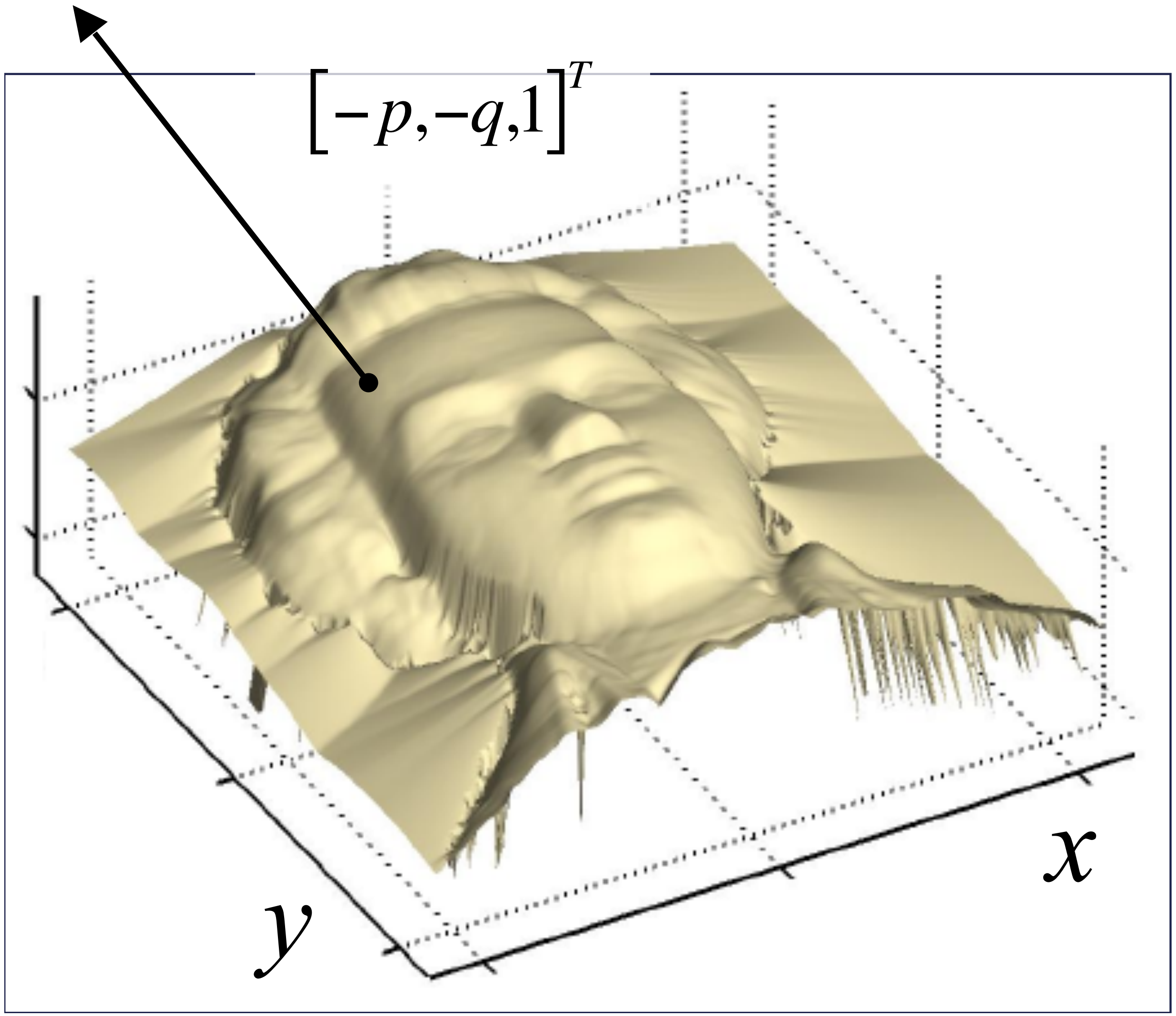
thus intensity = normal / curvature

SURFACE NORMALS

$$z = f(x, y)$$

$$p = \frac{\delta z}{\delta x}$$

$$q = \frac{\delta z}{\delta y}$$

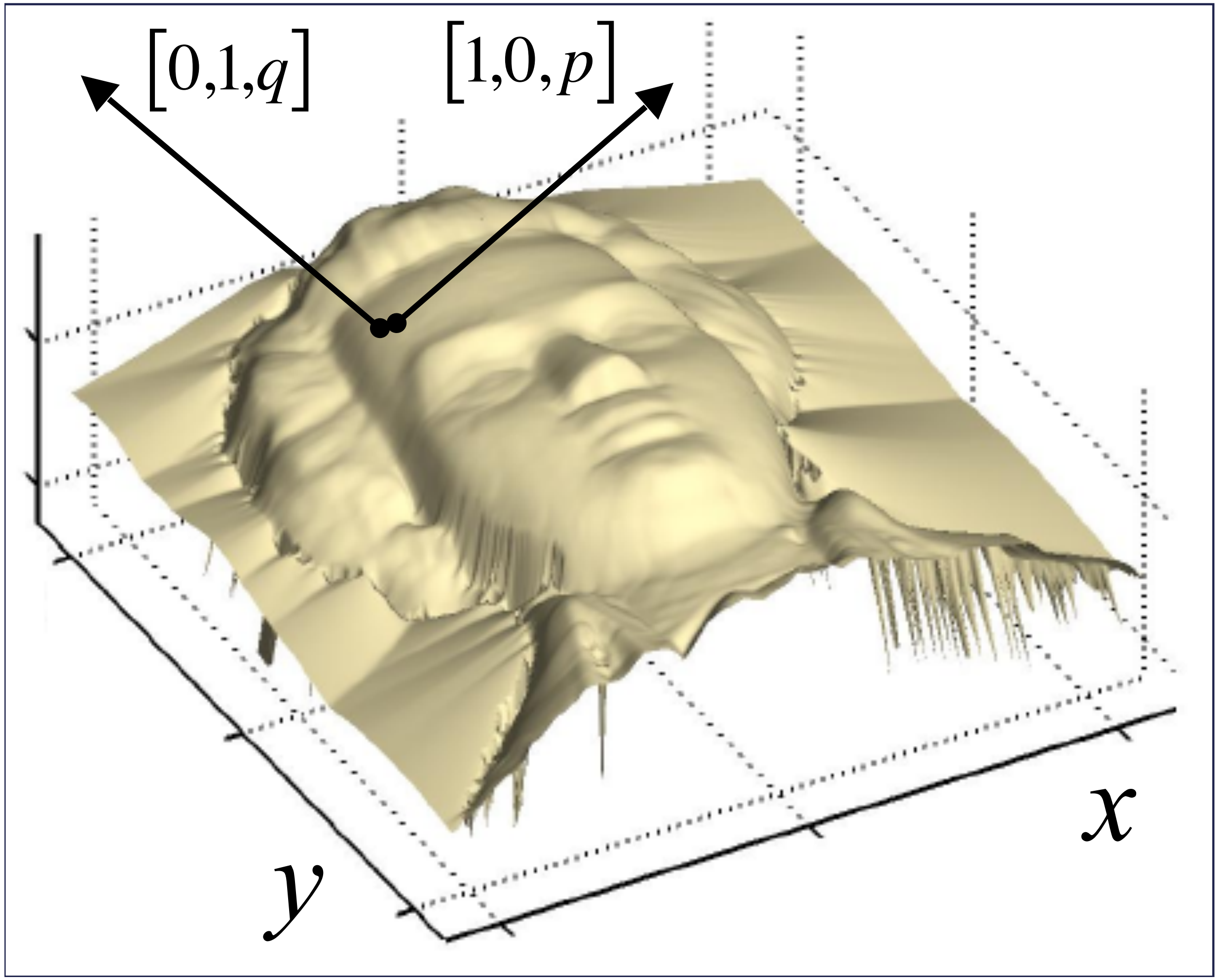


TANGENT VECTORS

$$z = f(x, y)$$

$$p = \frac{\delta z}{\delta x}$$

$$q = \frac{\delta z}{\delta y}$$



PROJECTION

Elevation and Normal:

$$z = f(x,y)$$

Surface normal

$$\mathbf{N}(x,y) = \frac{1}{\sqrt{1 + f_x^{\,2} + f_y^{\,2}}} \begin{bmatrix} -f_x \\ -f_y \\ 1 \end{bmatrix}$$

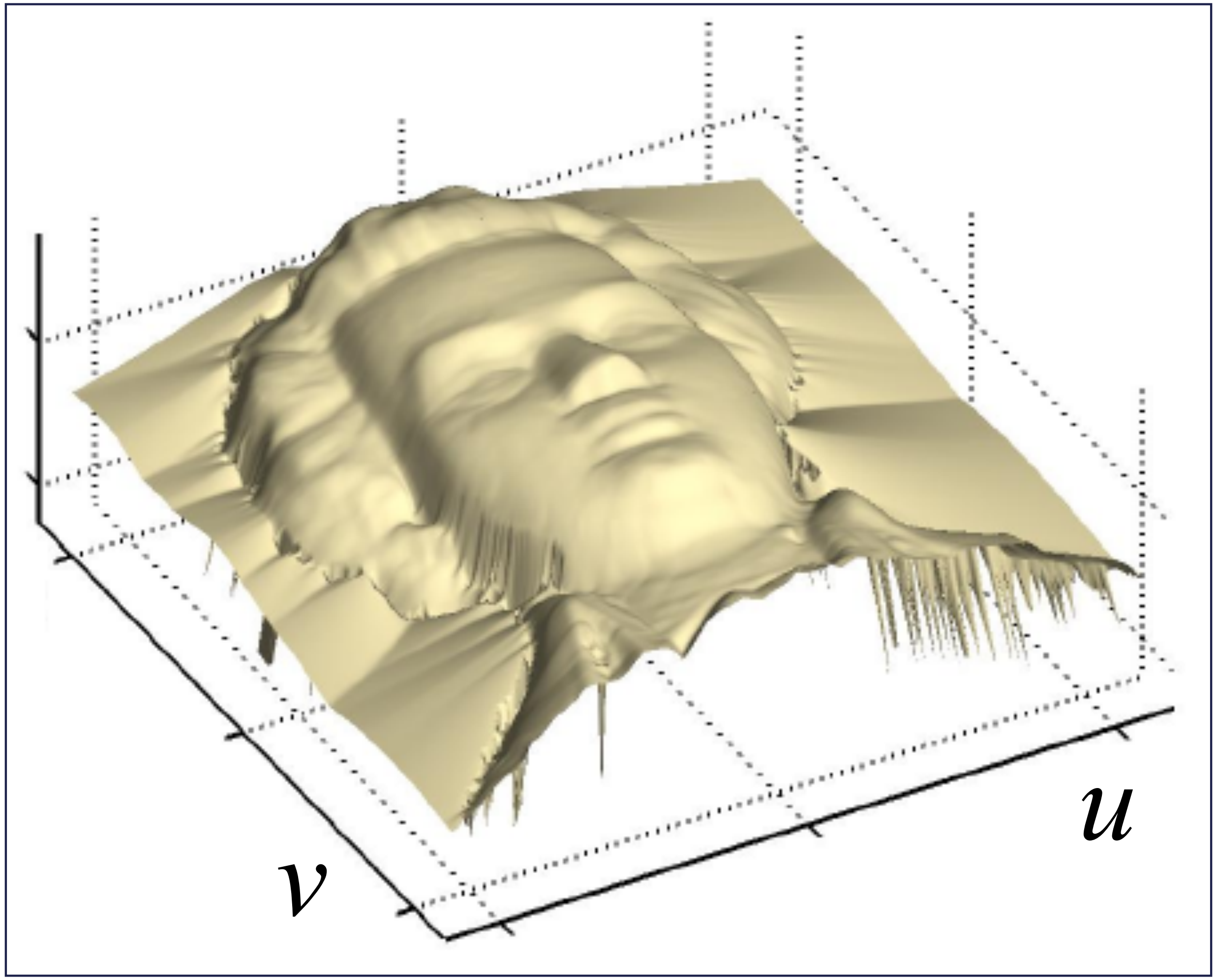
Orthographic Projection:

$$u = sx$$

$$v = sy$$

RE-PARAMETRIZATION

$$z = f(u, v)$$



SHAPE FROM NORMALS

gradient = normal divide by normal

$$\mathbf{N} = \frac{1}{\sqrt{1 + \frac{\delta z}{\delta x}^2 + \frac{\delta z}{\delta y}^2}} \begin{bmatrix} -\frac{\delta z}{\delta x} \\ -\frac{\delta z}{\delta y} \\ 1 \end{bmatrix} \propto \begin{bmatrix} -\frac{1}{s}\frac{\delta z}{\delta u} \\ -\frac{1}{s}\frac{\delta z}{\delta v} \\ 1 \end{bmatrix}$$

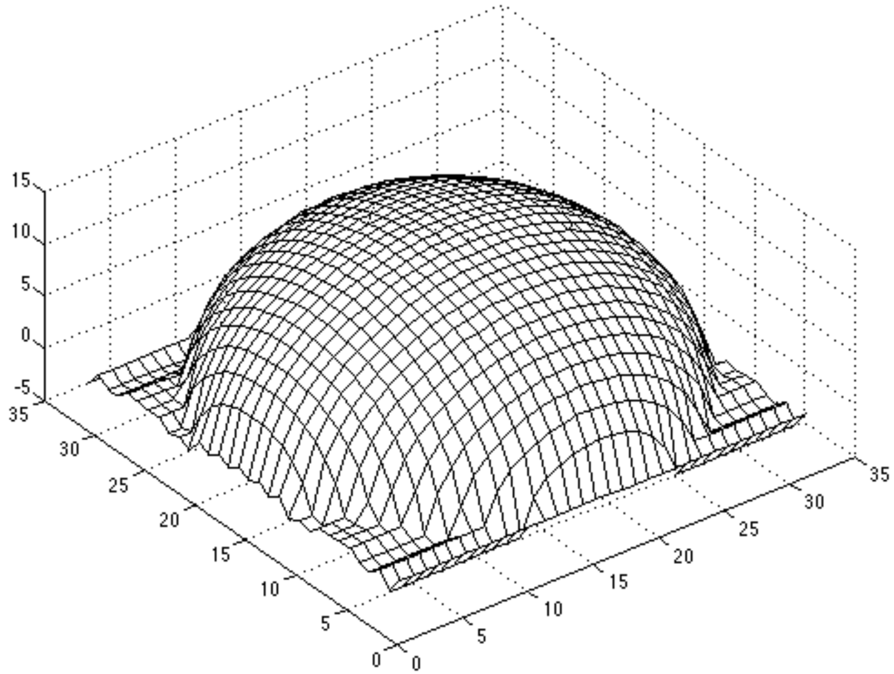
$$\Rightarrow \quad \frac{\delta z}{\delta u} = -s\frac{n_x}{n_z} \text{ and } \frac{\delta z}{\delta v} = -s\frac{n_y}{n_z}$$

$$\Rightarrow \quad \frac{\delta \bar{z}}{\delta u} = -\frac{n_x}{n_z} = n_1 \text{ and } \frac{\delta \bar{z}}{\delta v} = -\frac{n_y}{n_z} = n_2, \text{ with } \bar{z} = \frac{z}{s}$$

FINITE DIFFERENCES

we already have normals.

goal is to estimate z ? depth ?



$$\forall u,v$$

$$\bar{z}(u+1,v) - \bar{z}(u,v) = n_1(u,v)$$

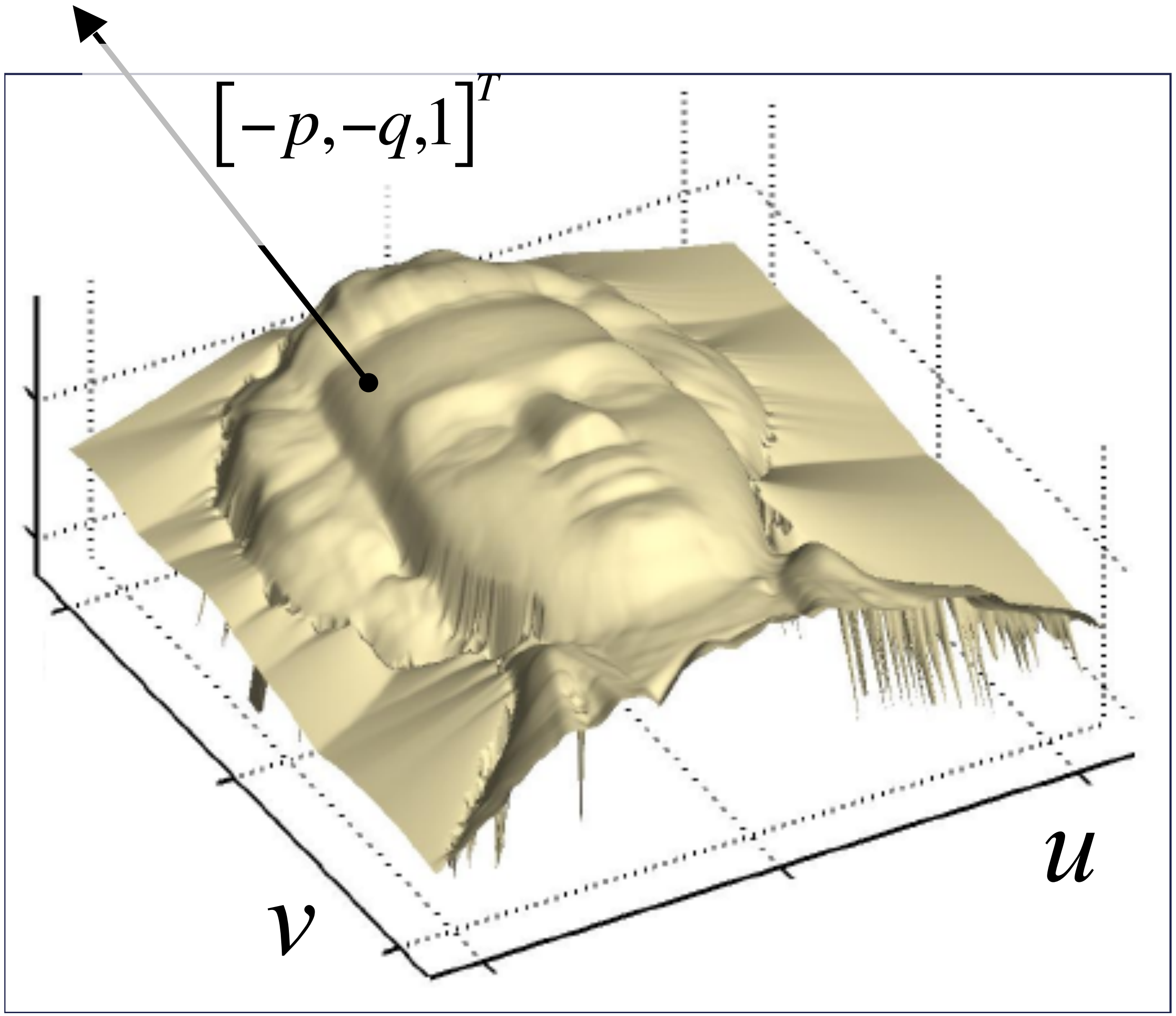
$$\bar{z}(u,v+1) - \bar{z}(u,v) = n_2(u,v)$$

$\Rightarrow$ Twice as many equations as there are unknown.

$\Rightarrow$ Least square solution up to a scale factor.

GRADIENT SPACE

$$z = f(u,v)$$
$$p = \frac{\delta z}{\delta u}$$
$$q = \frac{\delta z}{\delta v}$$



REFLECTANCE MAP

Reflectance:

Amount of light reflected towards the camera.

Reflectance i.e. estimated intensity

Albedo:

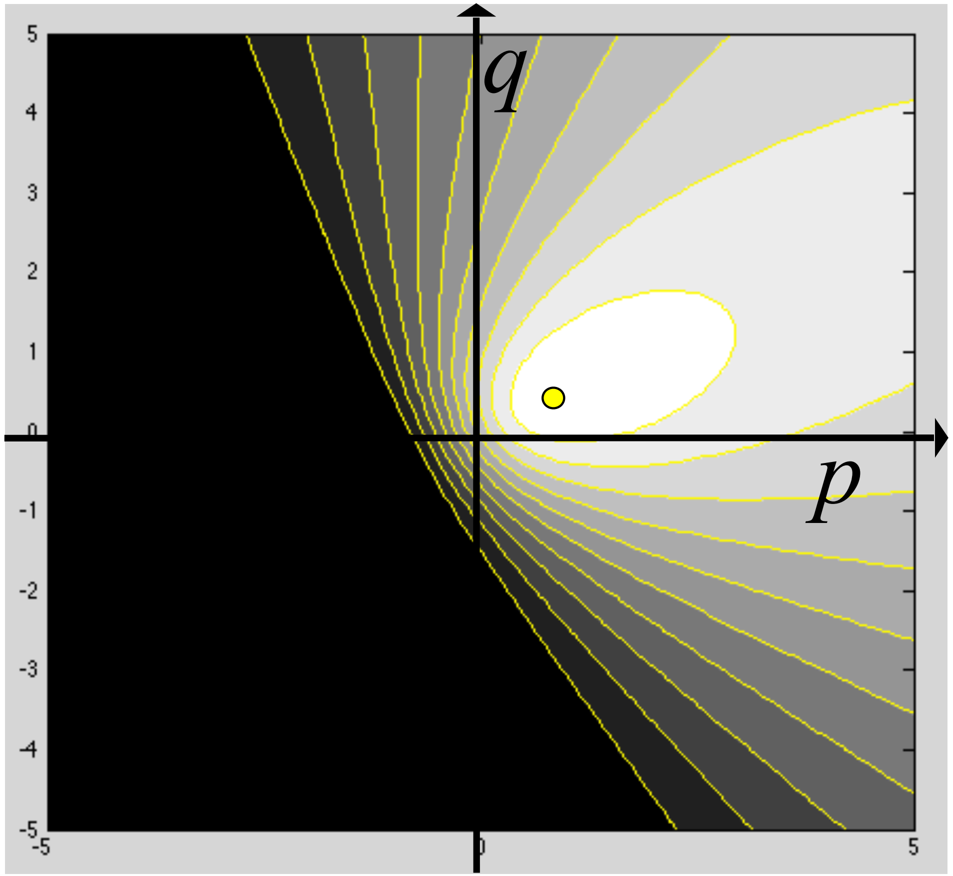
Fraction of the light incident on the surface that is reflected over all directions.

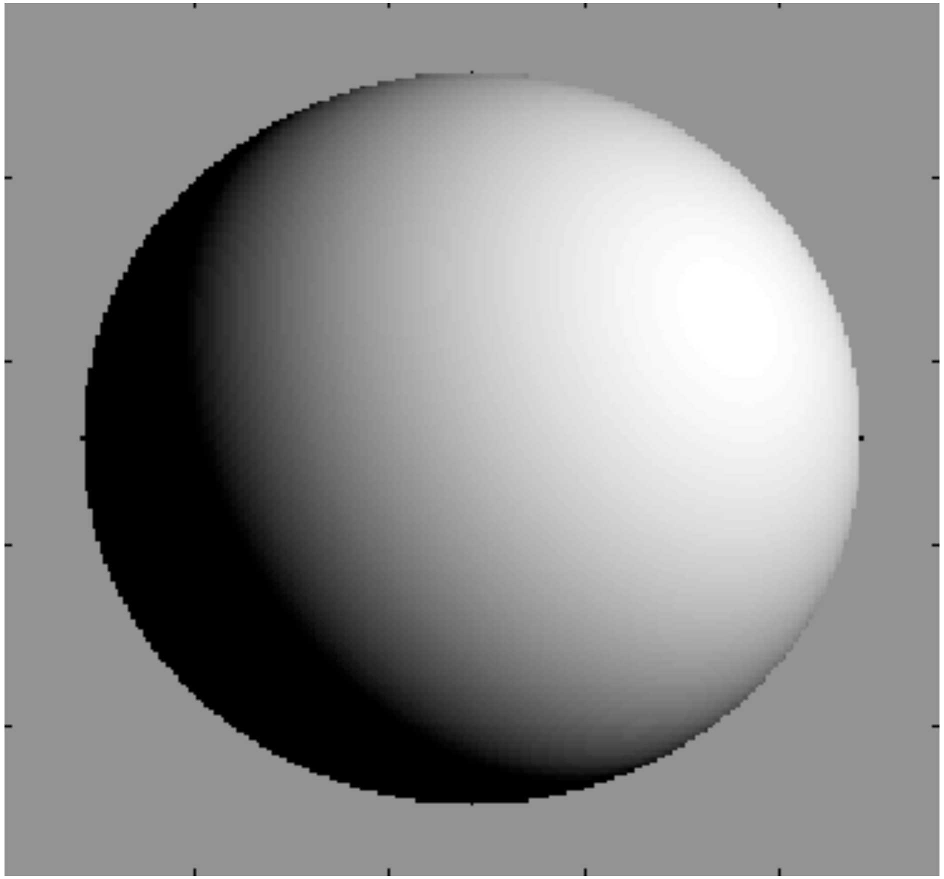
- In the Lambertian case and for a constant albedo

$$I(u,v) \propto Ref(p(u,v),q(u,v))$$

$$\propto \left[p(u,v),q(u,v),-1\right] \bullet \mathbf{S}$$

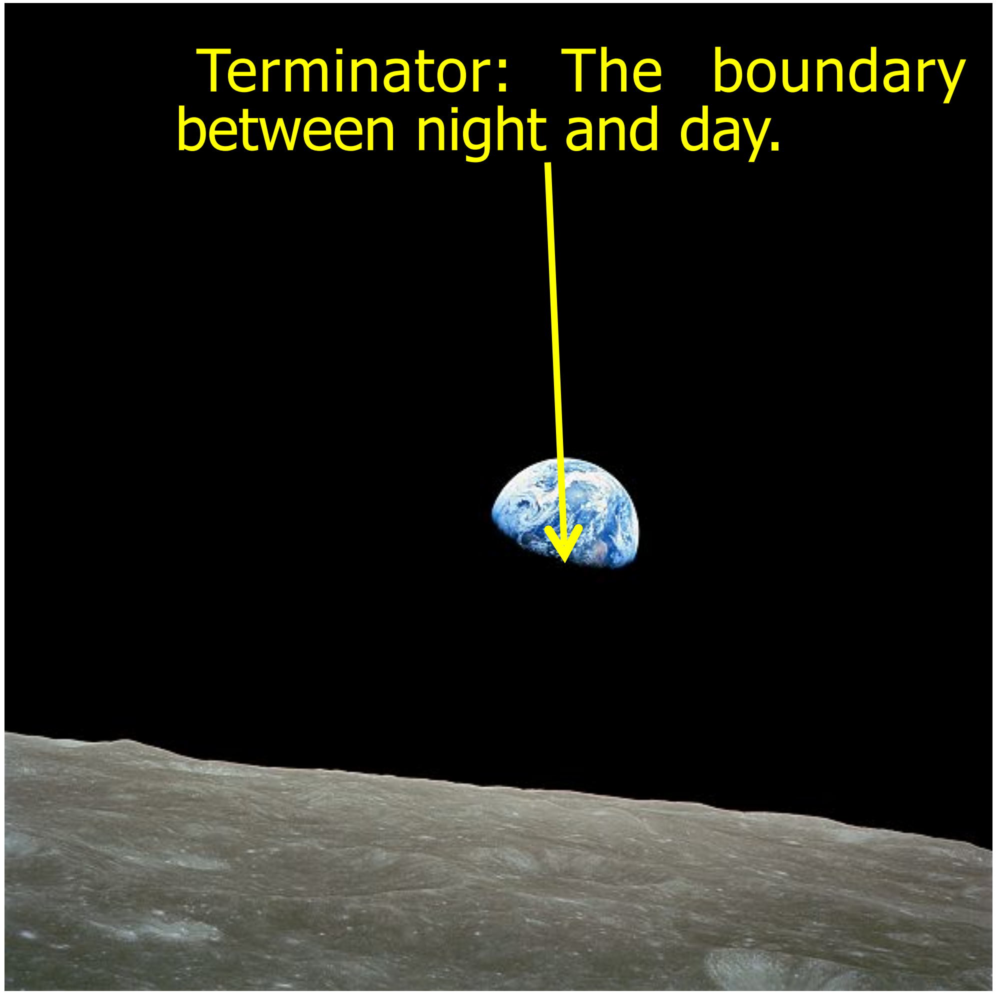
LAMBERTIAN REFLECTANCE MAP





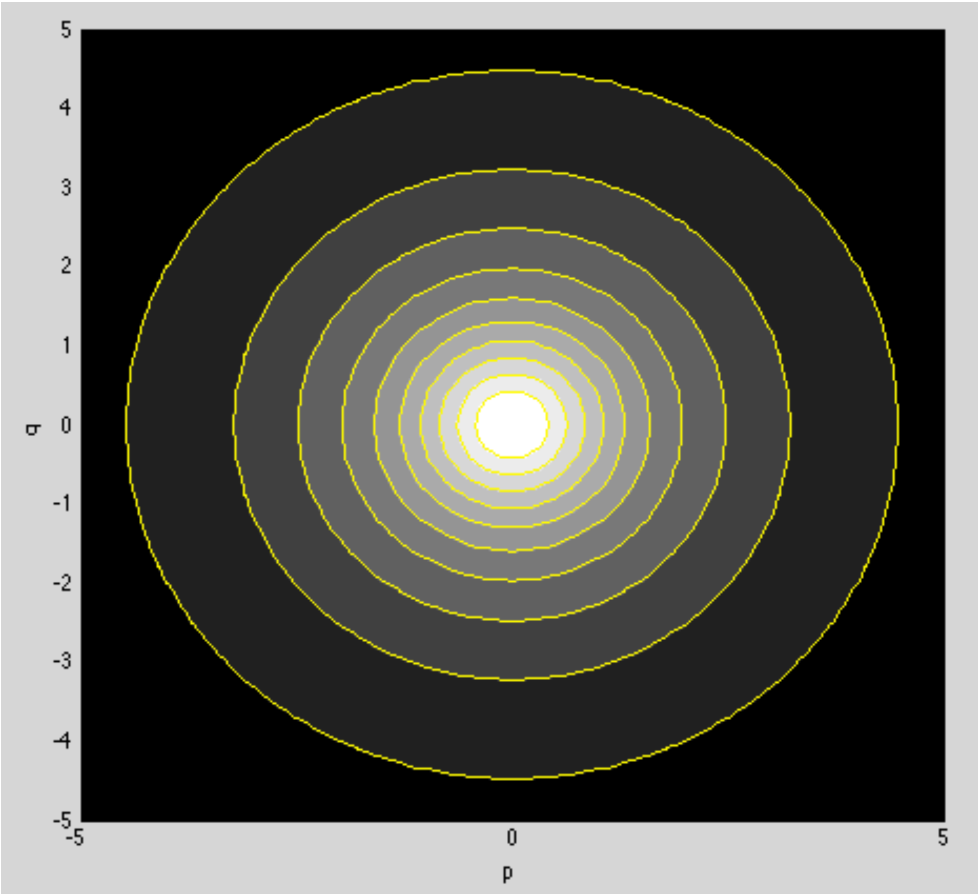
Reflectance map and shaded surface for Lambertian surface illuminated in the direction [-1 -0.5 -1].

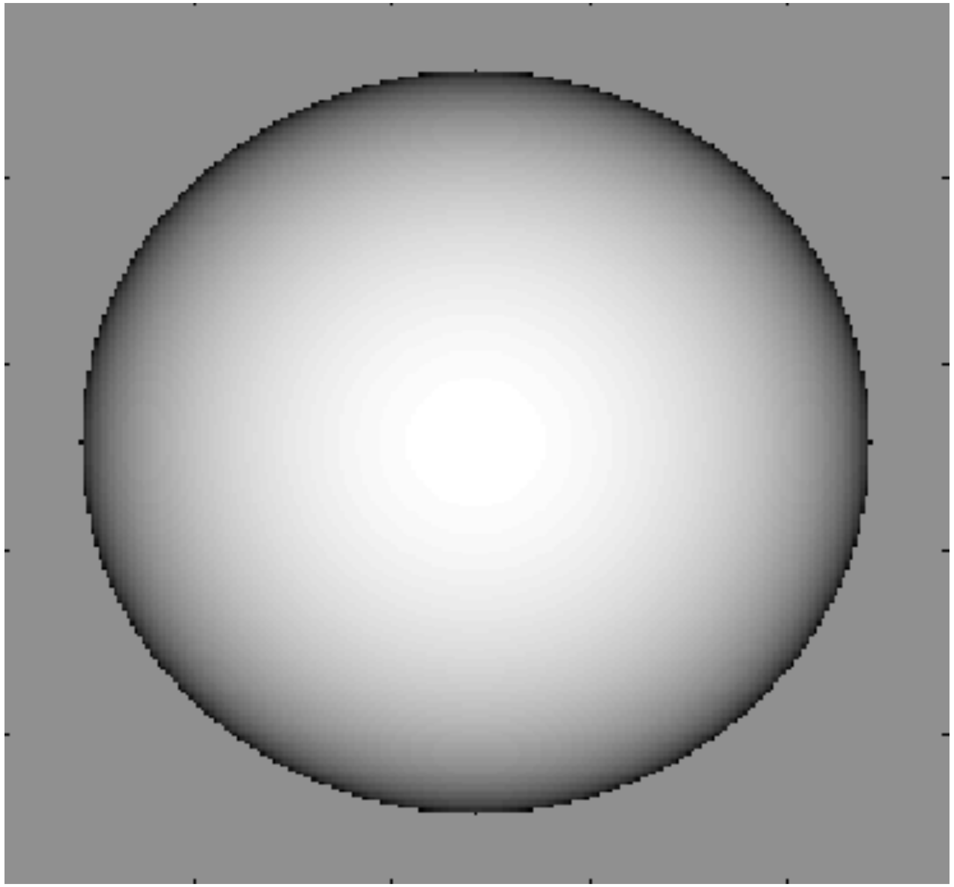
EARTH RISE



The earth seen from the moon. Apollo 8, 1968.

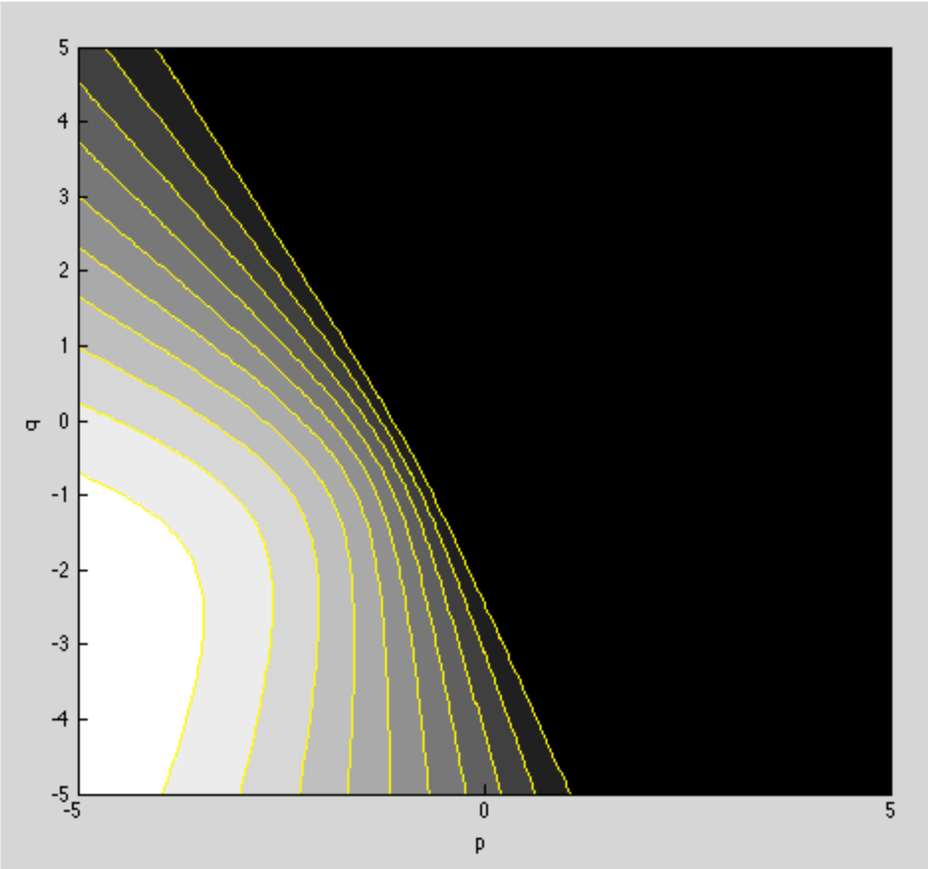
LAMBERTIAN REFLECTANCE MAP

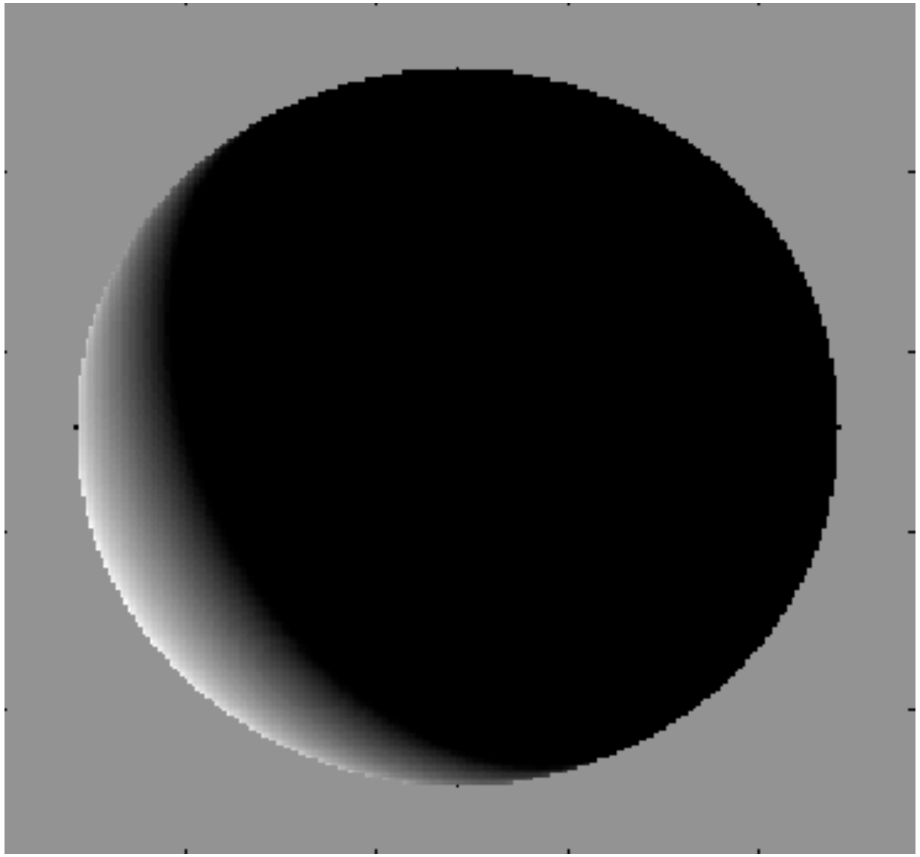




Reflectance map and shaded surface for Lambertian surface illuminated in the direction [0 0 -1].

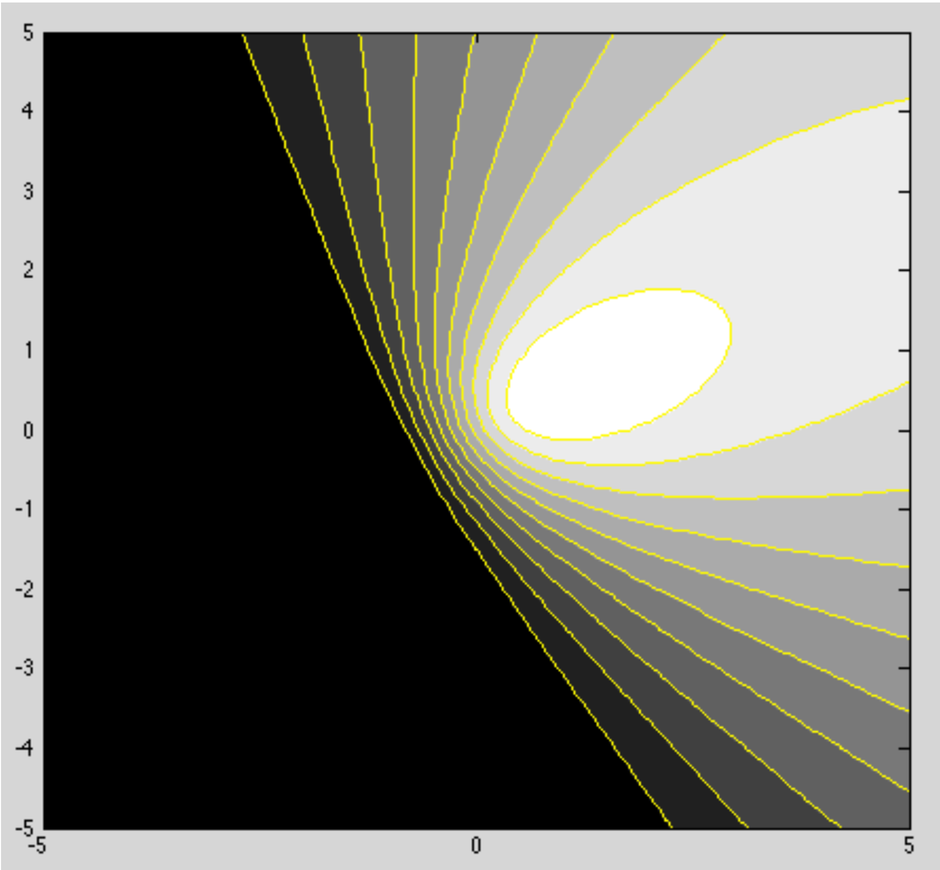
LAMBERTIAN REFLECTANCE MAP

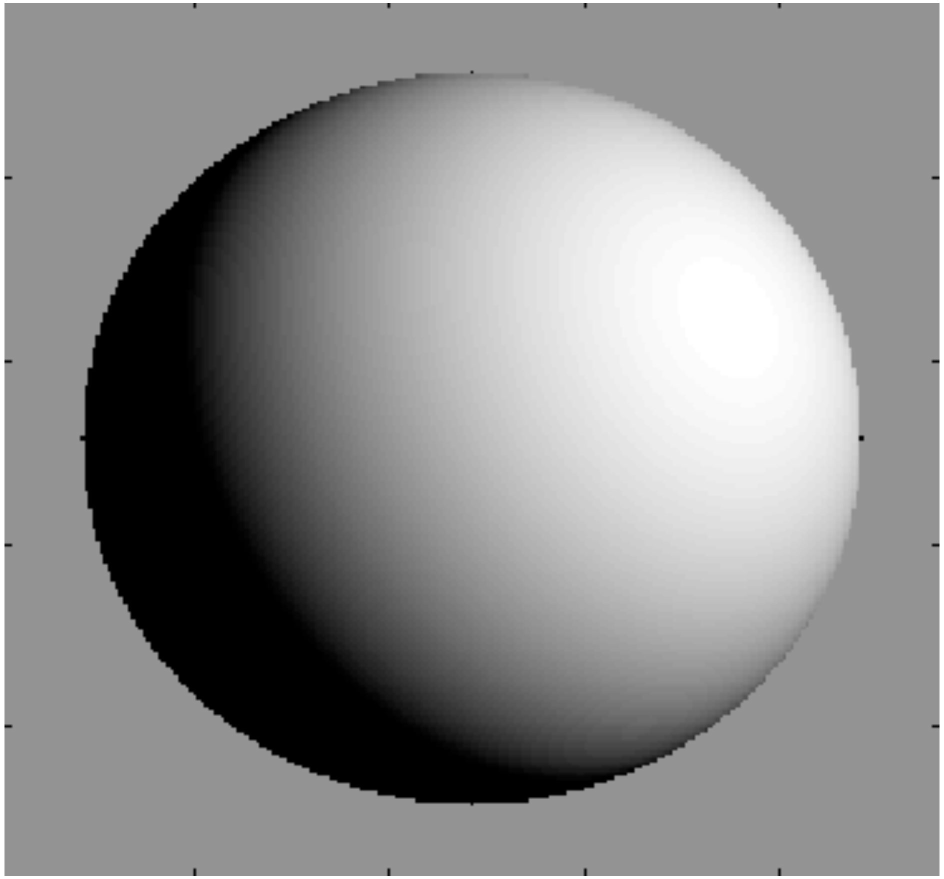




Reflectance map and shaded surface for Lambertian surface illuminated in the direction [1 0.5 -1].

CAN WE DETERMINE (P,Q) UNIQUELY FOR EACH IMAGE POINT INDEPENDENTLY?





NO -> Global optimization required.

VARIATIONAL METHODS

Minimize:

observed intensity

intensity given current estimate of the shape of the surface

u_1
v_1 | z_1 | z_2 |

| z_3 | z_4 |
----

the double integral is actually a summation for the rows and columns. at every u and v we have an unknown z

$$\int\int\left(\left[I(u,v) - Ref(\frac{\delta z}{\delta u}, \frac{\delta z}{\delta v})\right]^2 + \lambda\left[\left(\frac{\delta^2 z}{\delta u^2}\right)^2 + \left(\frac{\delta^2 z}{\delta u \delta v}\right)^2 + \left(\frac{\delta^2 z}{\delta v^2}\right)^2\right]\right) dudv$$

so the surface is smooth. second order penalizes surfaces that are not smooth. minimize curvature

or:

Brightness constraint

Smoothness term

$$\int\int\left([I(u,v) - Ref(p,q)]^2 + \lambda\left[\left(\frac{\delta p}{\delta u}\right)^2 + \left(\frac{\delta p}{\delta v}\right)^2 + \left(\frac{\delta q}{\delta u}\right)^2 + \left(\frac{\delta q}{\delta v}\right)^2\right] + \mu\left[\frac{\delta p}{\delta v} - \frac{\delta q}{\delta u}\right]^2\right) dudv$$

Integrability constraint

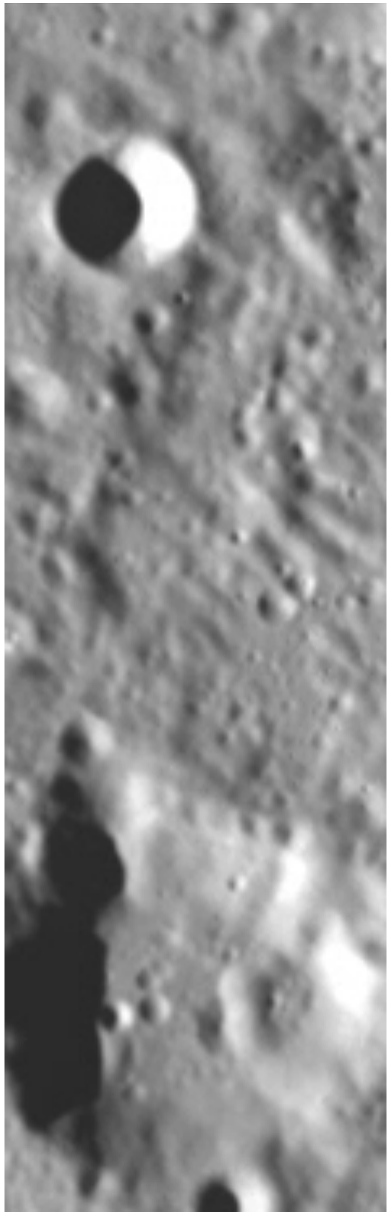
VARIATIONAL METHOD

$$\int\int\left([I(u,v) - Ref(p,q)]^2 + \lambda\left[\left(\frac{\delta p}{\delta u}\right)^2 + \left(\frac{\delta p}{\delta v}\right)^2 + \left(\frac{\delta q}{\delta u}\right)^2 + \left(\frac{\delta q}{\delta v}\right)^2\right] + \mu\left[\frac{\delta p}{\delta v} - \frac{\delta q}{\delta u}\right]^2\right) dudv$$

Once p and q have been estimated, integrate to recover f.

→ Need to know the boundary conditions.

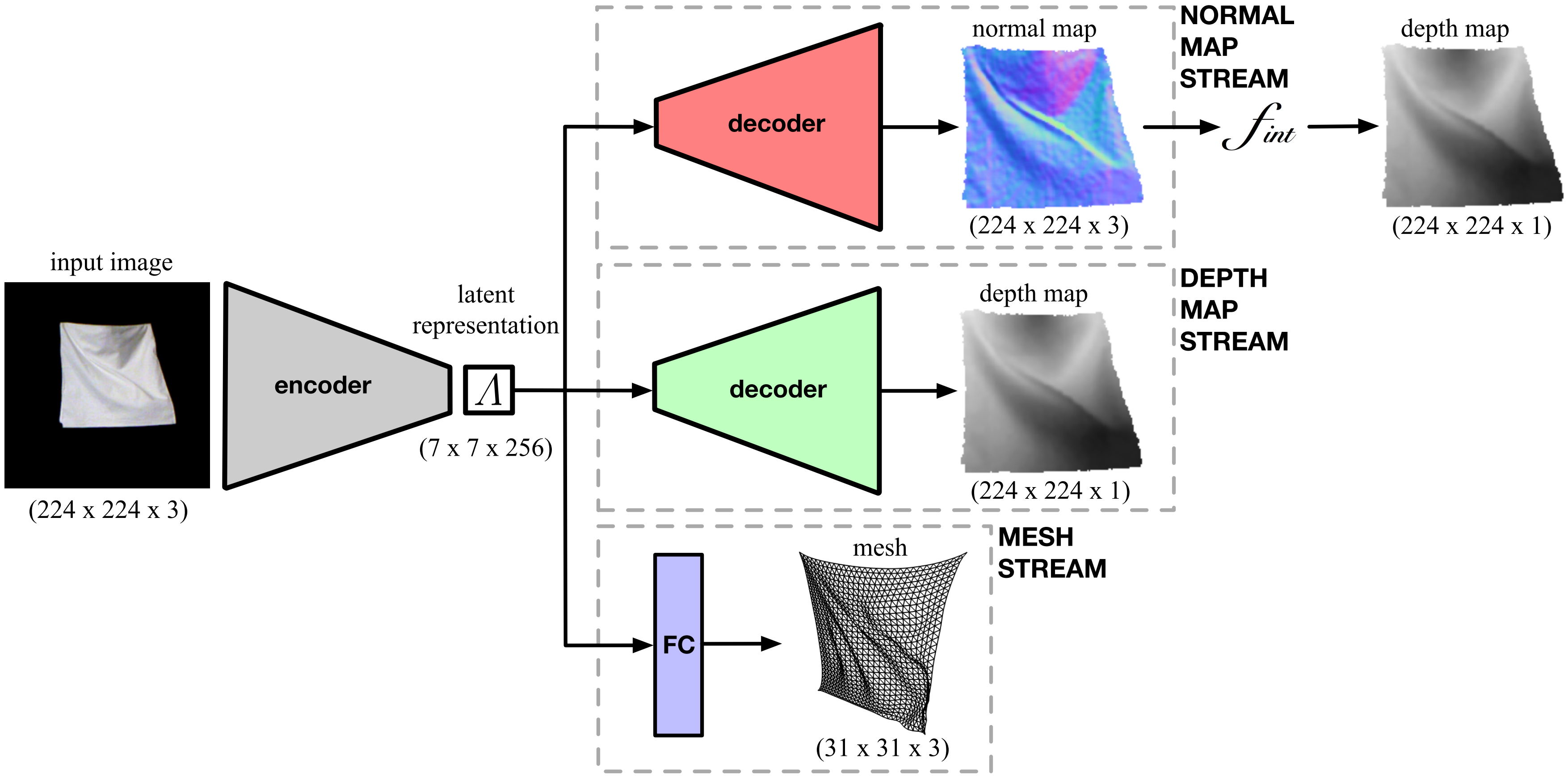
MOONSCAPE



CLOTH

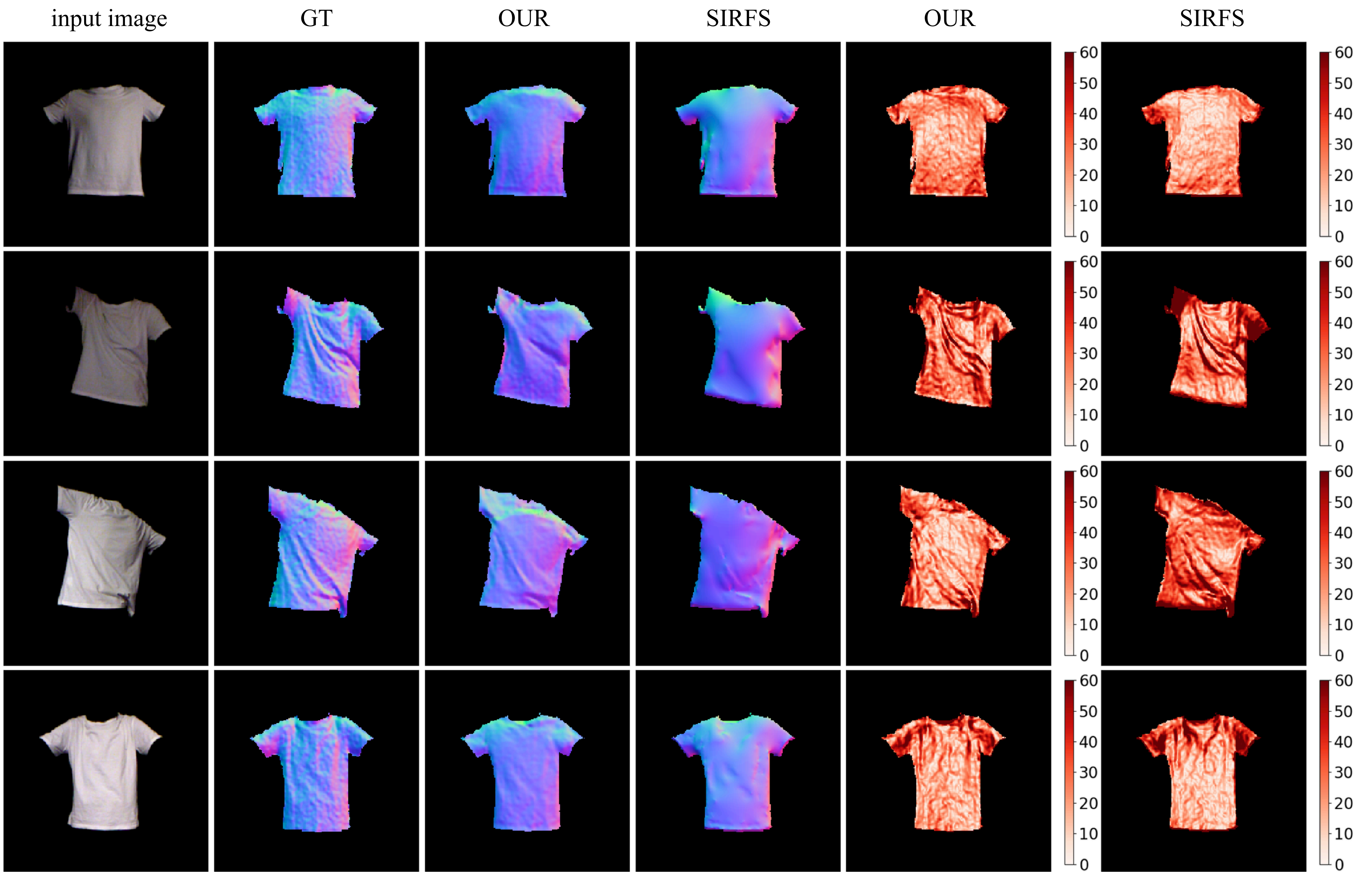
Barron & Malik, PAMI'15

DEEP NETS

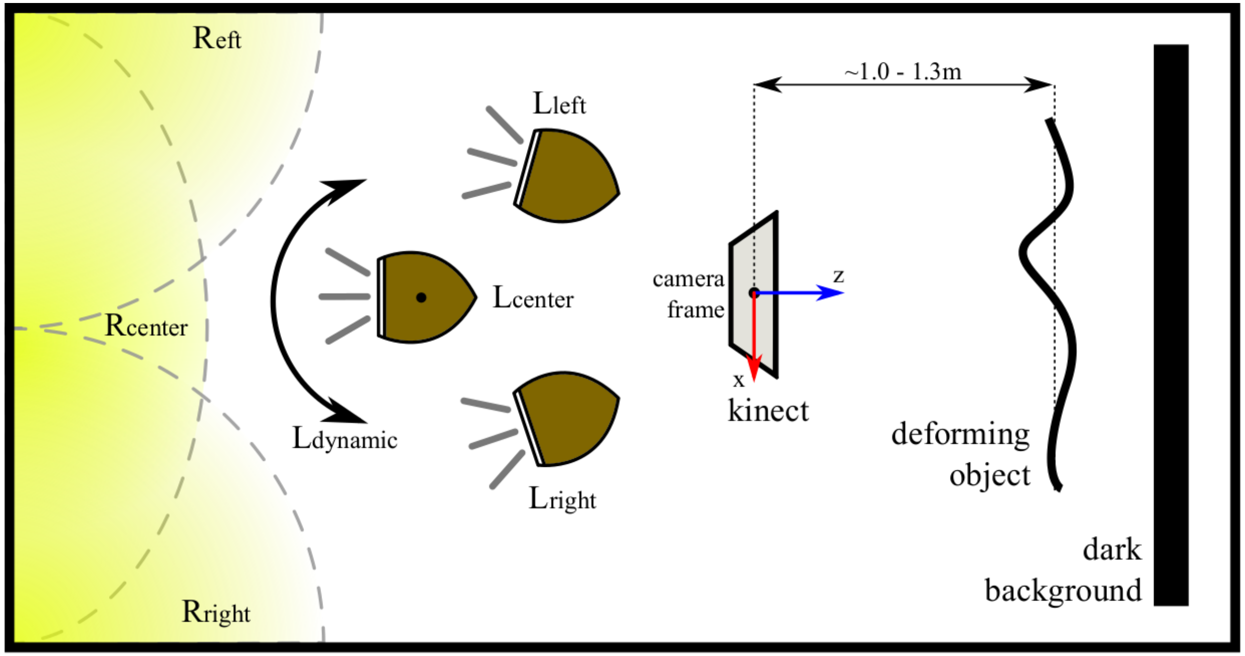


Bednarik et al. , ECCV'18

DEEP NETS



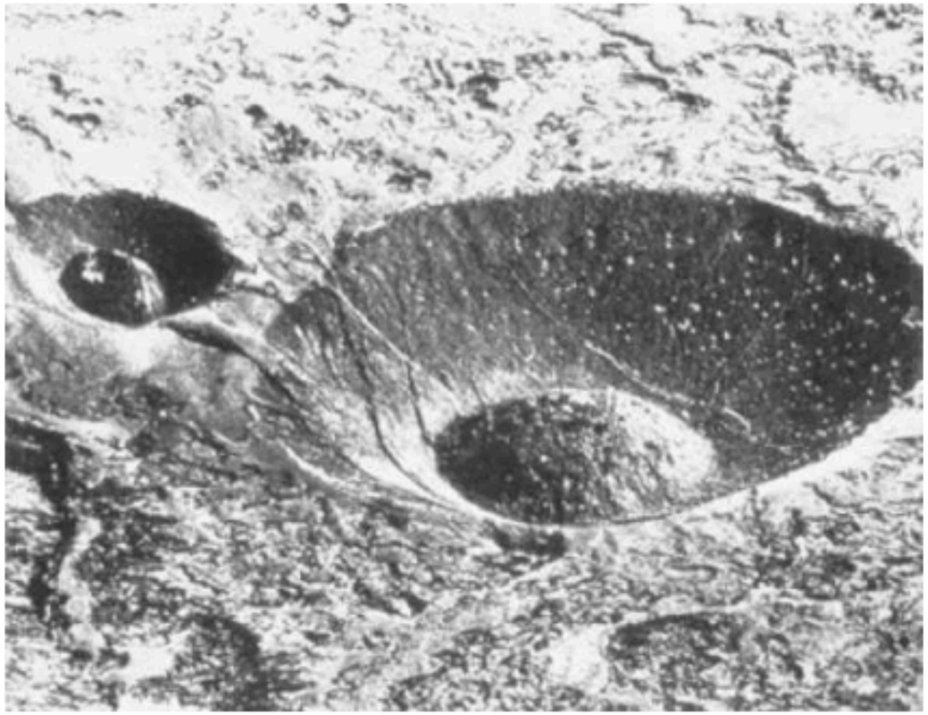
DATA ACQUISITION SETUP

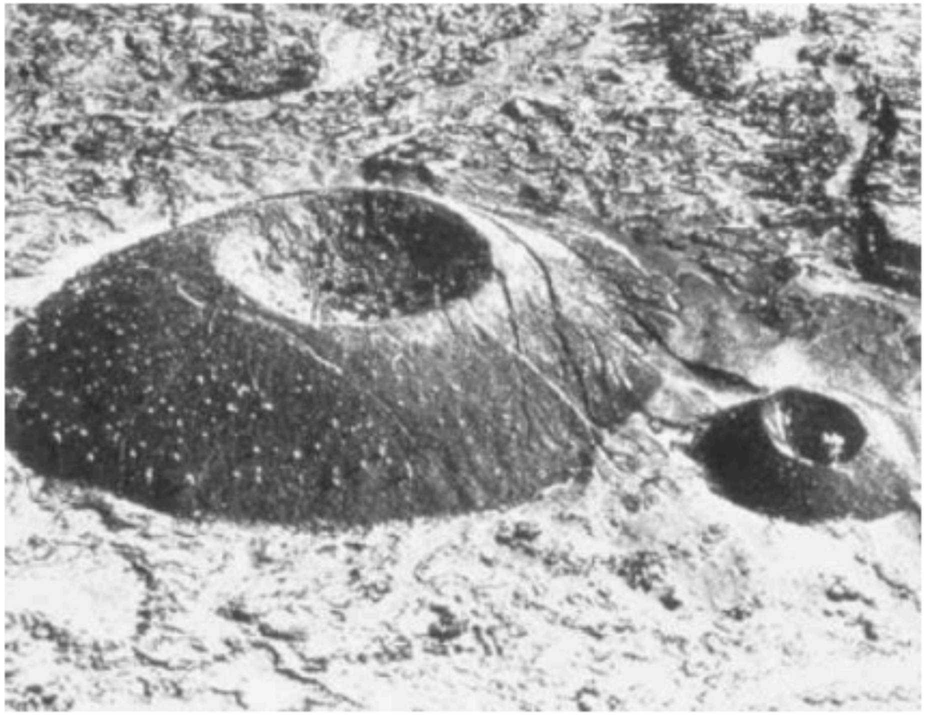


- 3 fixed light sources.
- 1 mobile one.

—> Still a constrained environment.

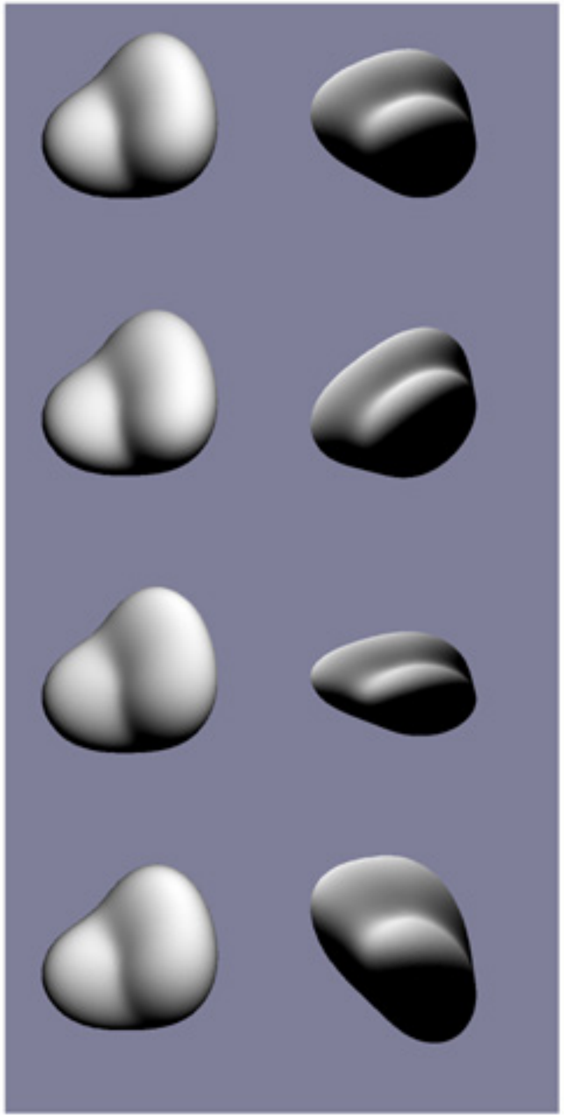
AMBIGUITIES

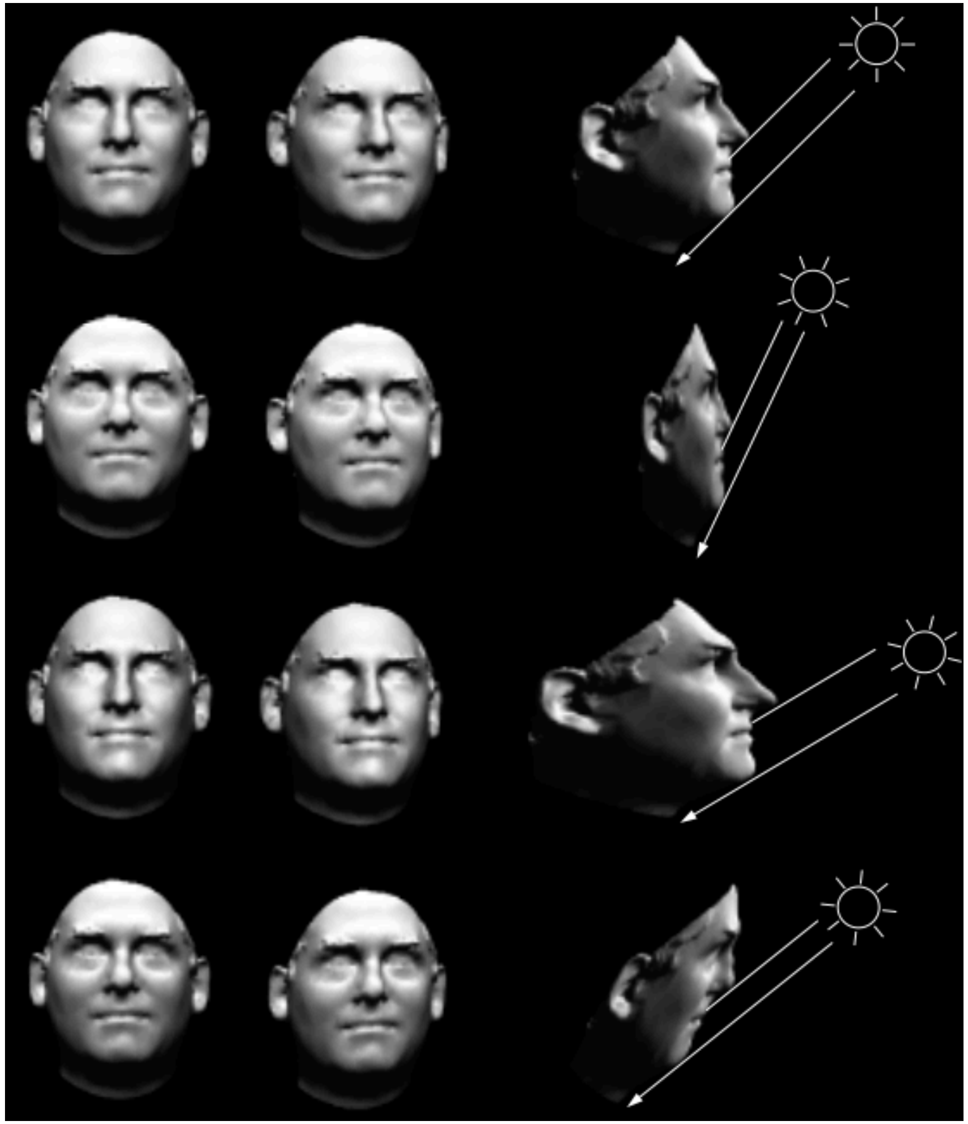




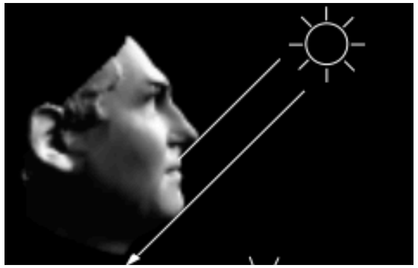
THE BAS-RELIEF AMBIGUITY

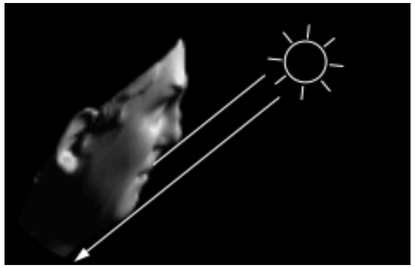
MORE GENERALLY





BAS-RELIEF AMBIGUITY





$$Ref = \mathbf{N} \bullet \mathbf{S}$$

For any invertible 3x3 linear transformation A:

$$\mathbf{N} \bullet \mathbf{S} = N^T S = (A\mathbf{N})^T A^{-T} \mathbf{S}$$

But for a valid surface z=f(u,v), we should have:

$$\left.\begin{array}{rcl} \frac{\delta z}{\delta u} & = & -\frac{n_x^*}{n_z^*} \\ \frac{\delta z}{\delta v} & = & -\frac{n_y^*}{n_z^*} \end{array}\right\} \Rightarrow \frac{\delta \frac{n_x^*}{n_z^*}}{\delta v} = \frac{\delta \frac{n_y^*}{n_z^*}}{\delta u} \text{ with } \begin{bmatrix} n_x^* \\ n_y^* \\ n_z^* \end{bmatrix} = \mathbf{A} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

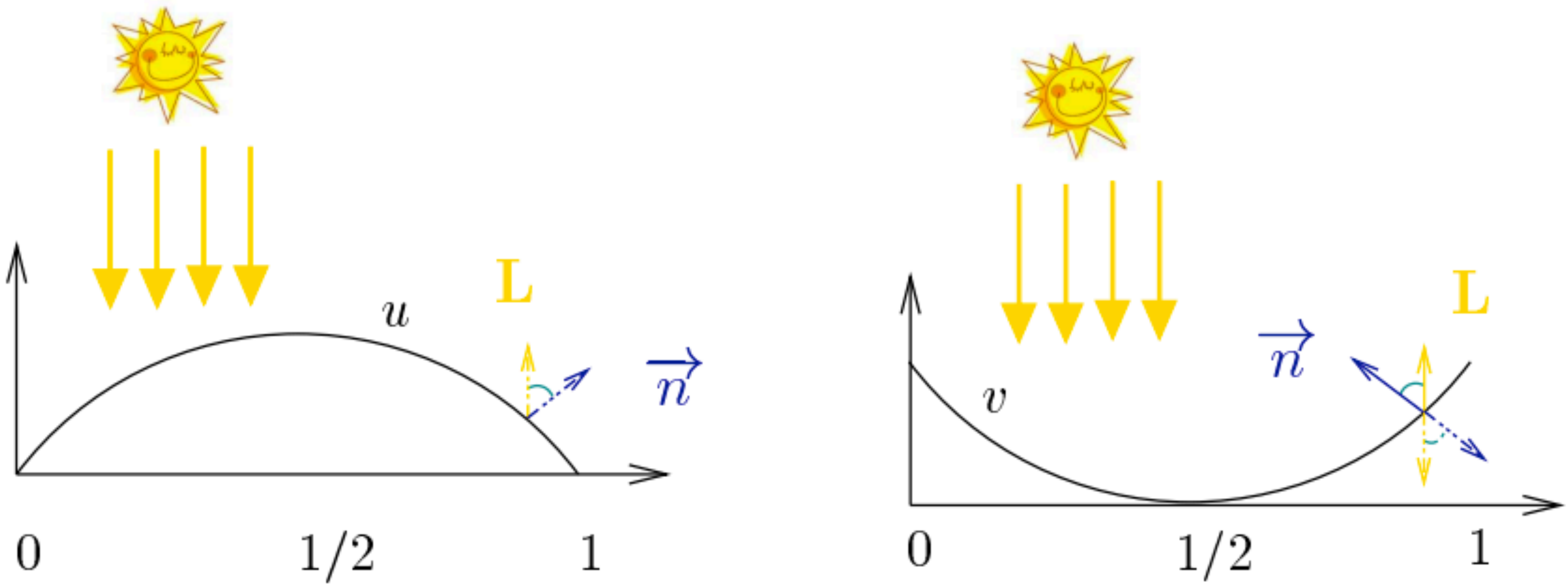
BAS-RELIEF AMBIGUITY

$$\left.\begin{aligned}\frac{\delta z}{\delta u} &= -\frac{n_x^*}{n_z^*}\\ \frac{\delta z}{\delta v} &= -\frac{n_y^*}{n_z^*}\end{aligned}\right\} \Rightarrow \frac{\delta \frac{n_x^*}{n_z^*}}{\delta v} = \frac{\delta \frac{n_y^*}{n_z^*}}{\delta u} \text{ with } \begin{bmatrix} n_x^* \\ n_y^* \\ n_z^* \end{bmatrix} = \mathbf{A} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

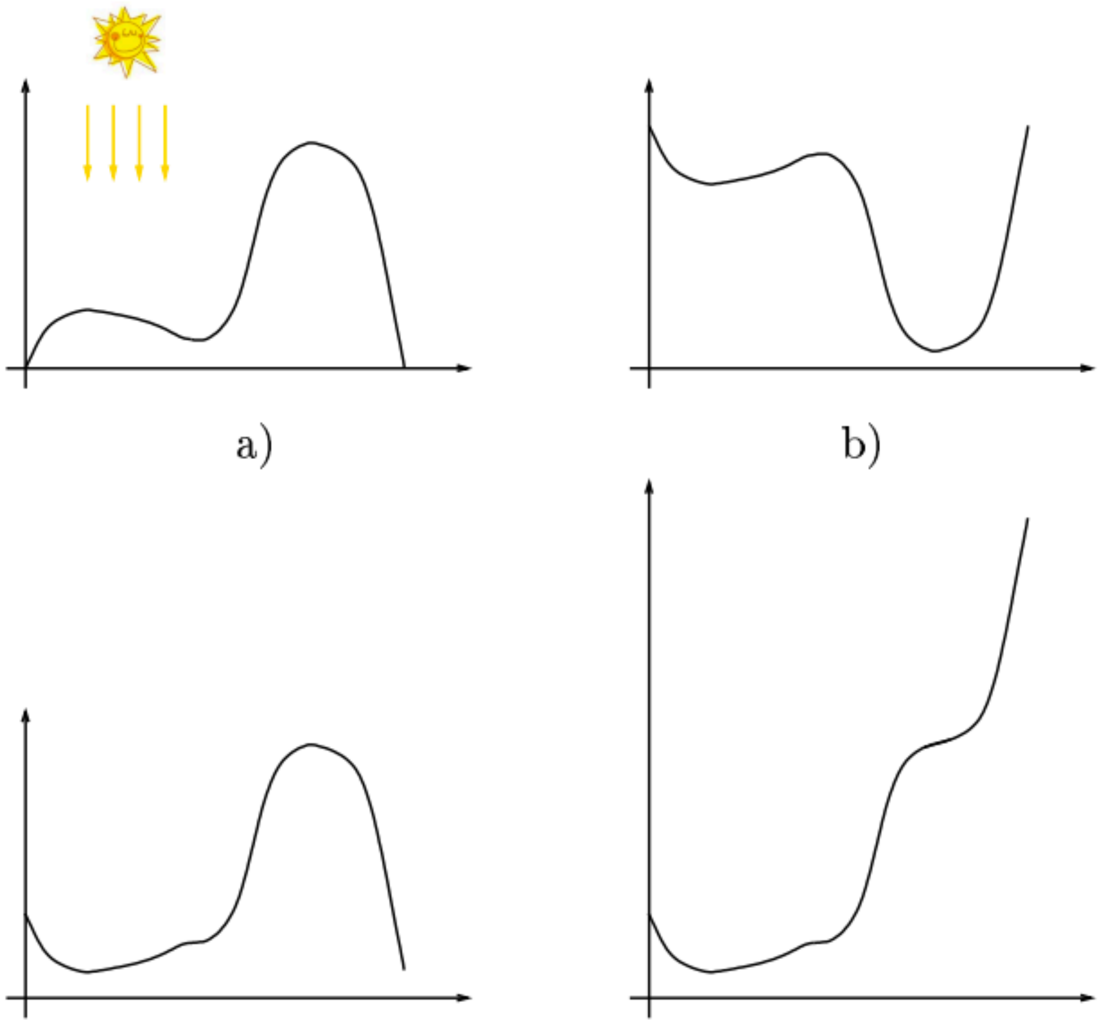
$$\Rightarrow \mathbf{A} \text{ restricted to } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & \nu & 1 \end{bmatrix}$$

- The surface f(u, v) can be changed into λf(u,v) + μu + νv and still produce the same image.

MORE AMBIGUITIES EVEN WHEN THE LIGHT SOURCE IS KNOWN...



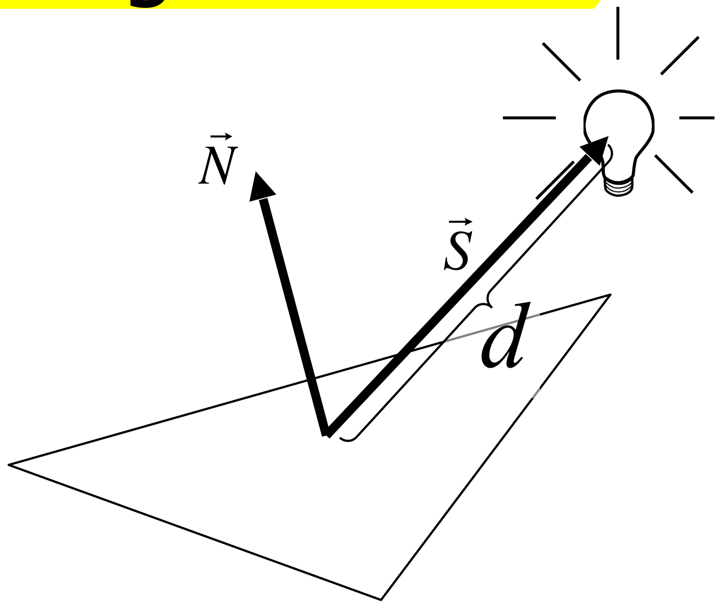
MORE AMBIGUITIES EVEN WHEN THE LIGHT SOURCE IS KNOWN…



MAKING THE PROBLEM WELL-POSED

- Use perspective projection model;
- Radiance depends on distance to light source:

$$I = \frac{Albedo \cdot (\mathbf{N} \bullet \mathbf{S})}{d^2}$$



instead of

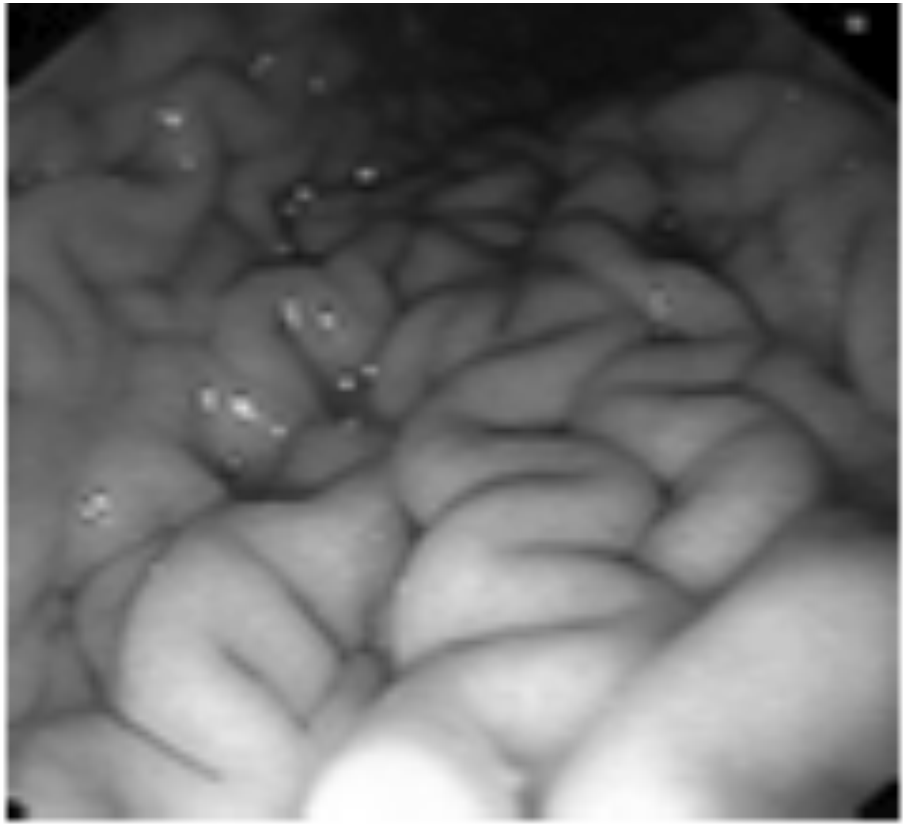
$$I = Albedo \cdot (\mathbf{N} \bullet \mathbf{S})$$

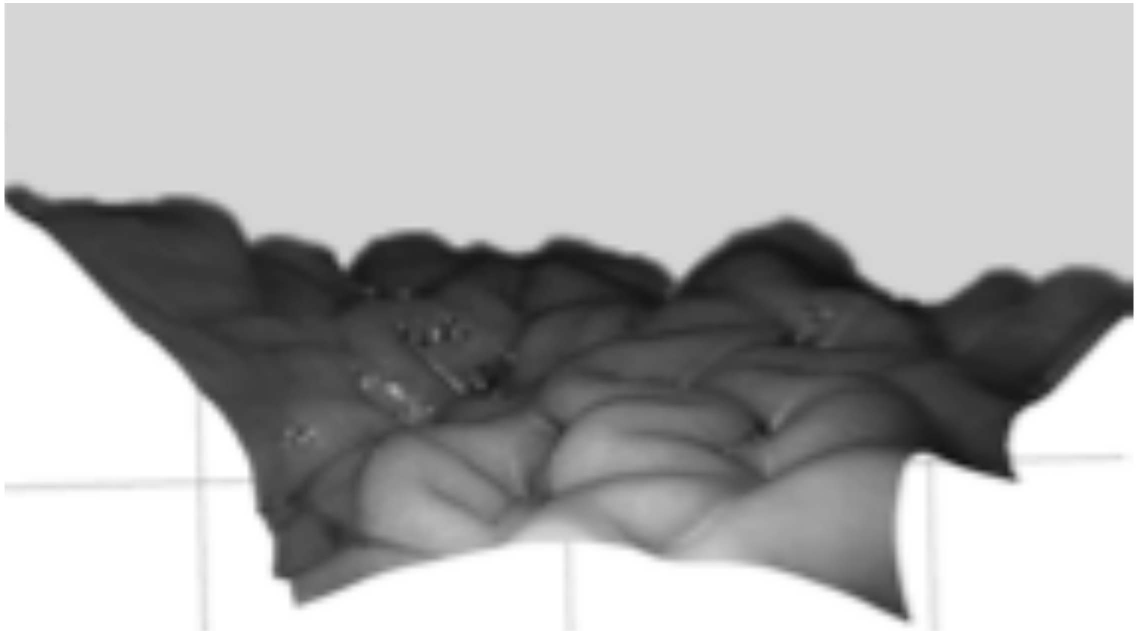
- Light source located at the optical center.

-> Unique solution.

Prados and Faugeras, CVPR'05.

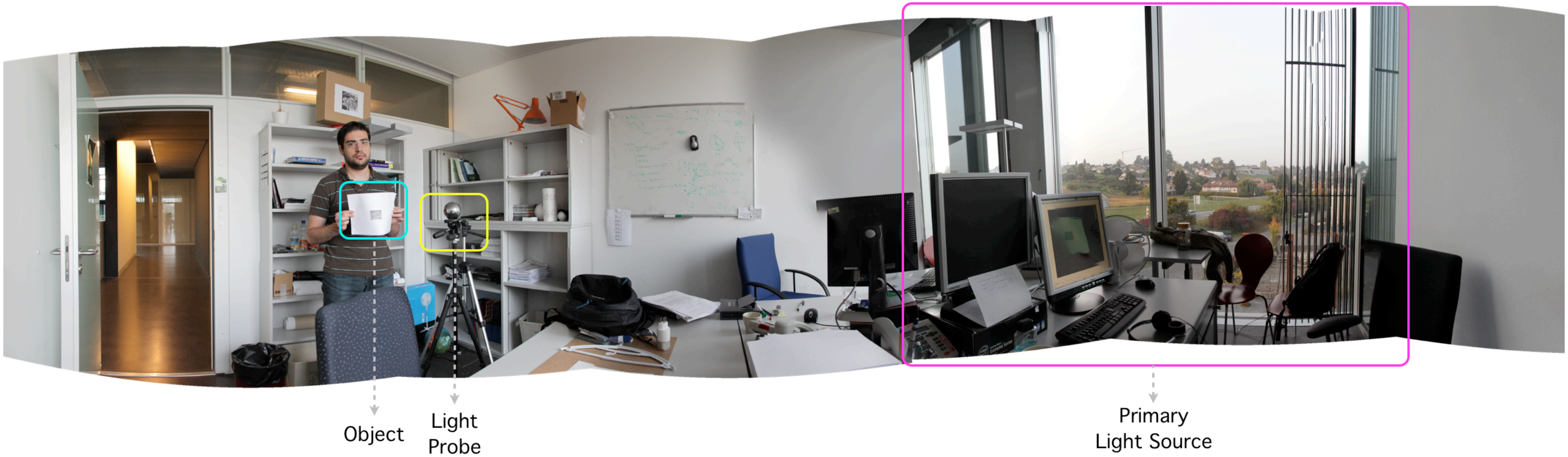
ENDOSCOPY





Prados and Faugeras, CVPR'05.

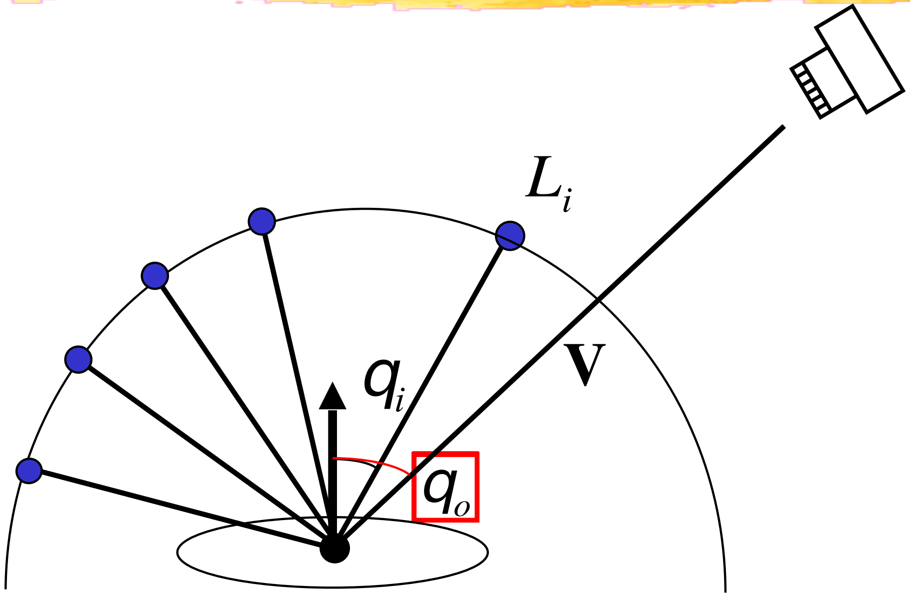
EVERYDAY SETTING



Multiple extended light sources:

- Illumination modeled as a weighted sum of spherical harmonics.
- Illumination parameters estimated using the light probe.

ILLUMINATION HEMISPHERE



$$L_o(\mathbf{P},q_o,f_o) = \int_{\Omega_i} r_{bd}\left(q_o,f_o,q_i,f_i\right) L_i(\mathbf{P},q_i,f_i)\cos\left(q_i\right) dw_i$$

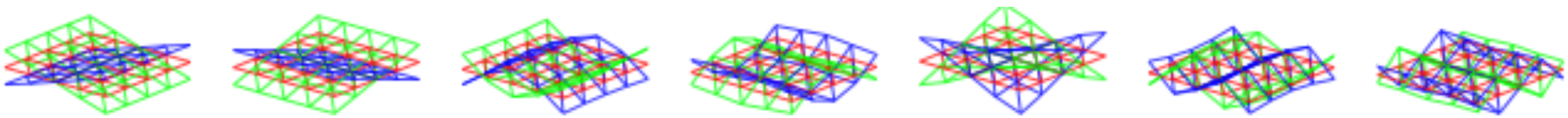
$$= \int_{\Omega_i} L_i(q_i,f_i)\, r_{bd}\left(q_o,f_o,q_i,f_i\right)\max(0,\cos\left(q_i\right)) dw_i$$

$$= \int_{\Omega_i} L_i(q_i,f_i)\, r^*\left(q_i,f_i\right) dw_i$$

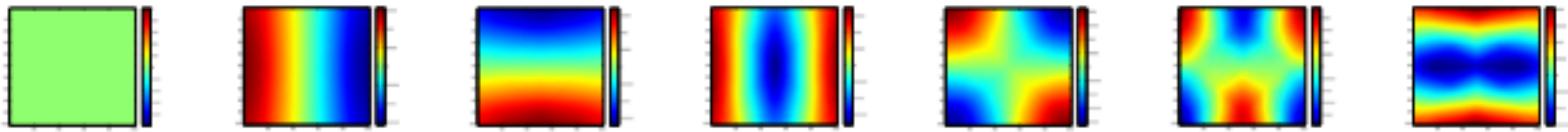
with $r^*\left(q_i,f_i\right) = r_{bd}\left(q_o,f_o,q_i,f_i\right)\max(0,\cos\left(q_i\right))$ is the BRDF product function.

PATCH INTENSITY AND DEFORMATION MODES

Synthesize a training database containing deformed patches and the corresponding intensity patterns. Compute:

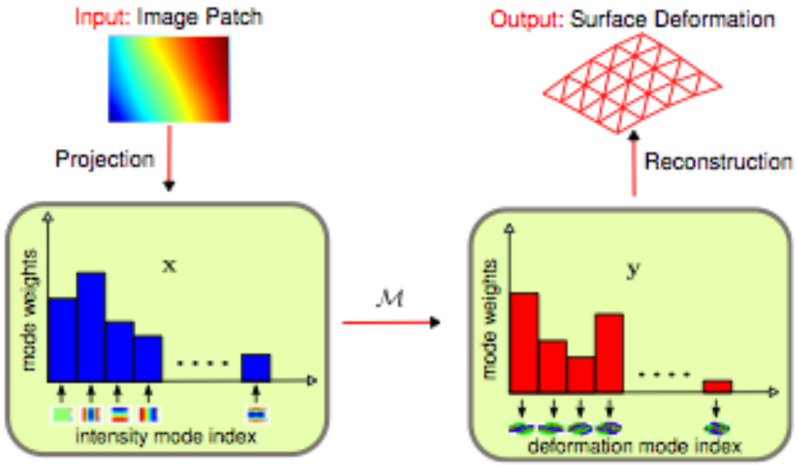


Deformation modes : $D = D_0 + \sum_i y_i D_i$

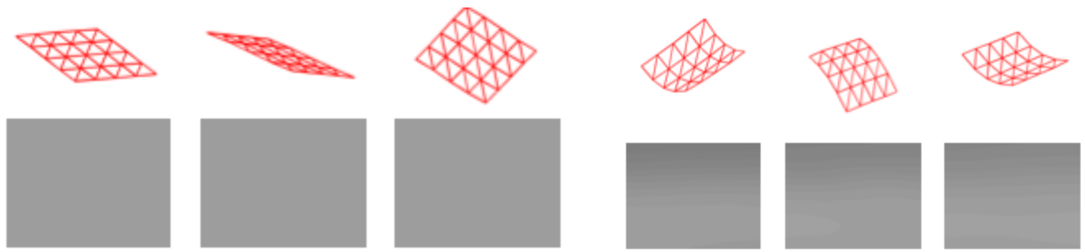


Intensity modes : $I = I_0 + \sum_i x_i I_i$

AMBIGUOUS MAPPING



Mapping from intensity to surface deformation.

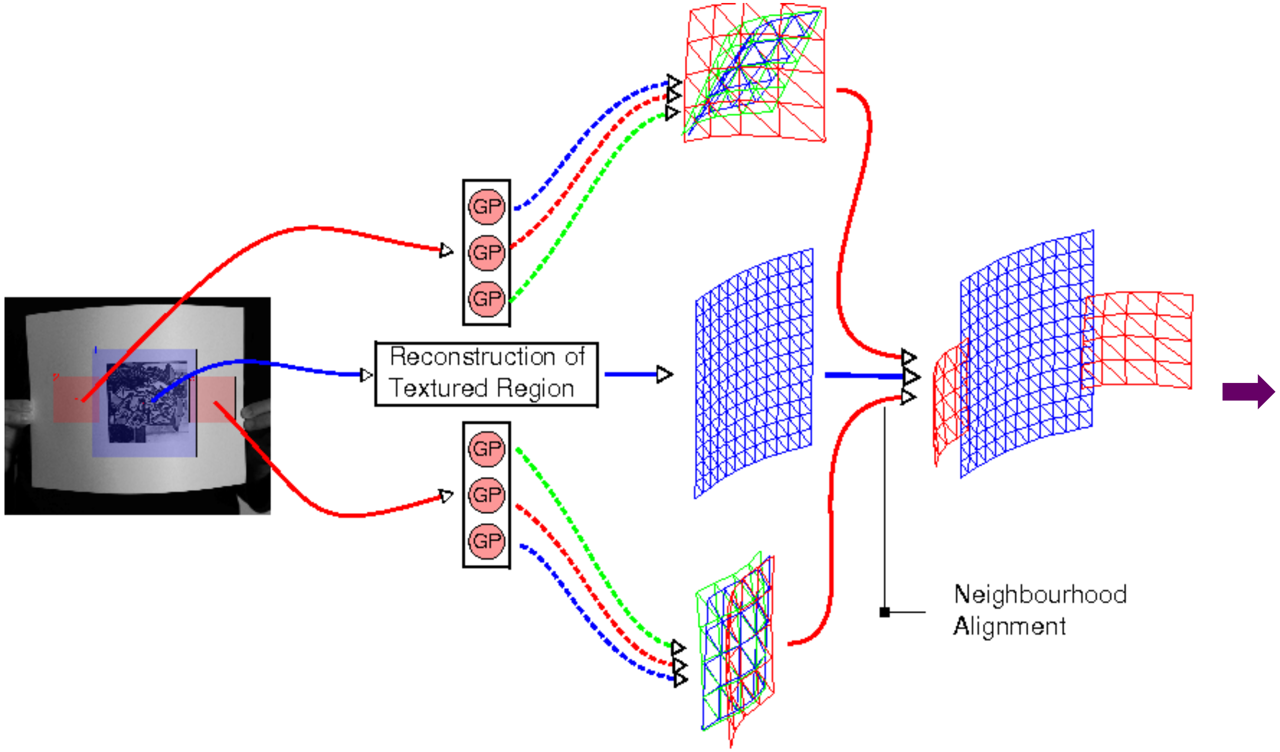


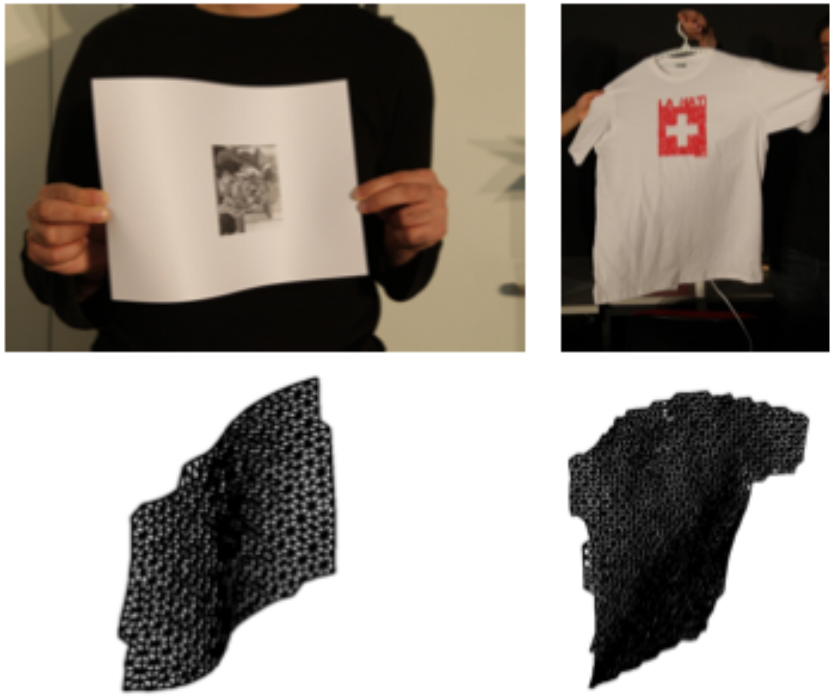
Unfortunately, the mapping is not one to one.

Algorithm:

- Partition the training database according to average normal to learn an unambiguous mapping.
- At run-time, predict several potential shapes for each image-patch and use a Markov-Random field to pick a set of consistent interpretations.

ALGORITHMIC FLOW

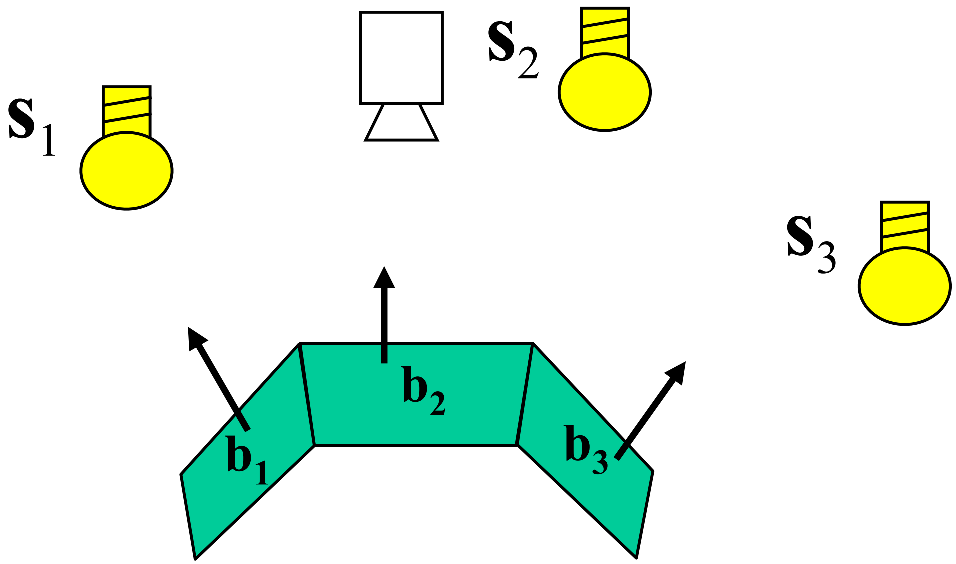




→ The light environment and the camera need to be very carefully calibrated but there is hope …

Varol et al. , PAMI'12

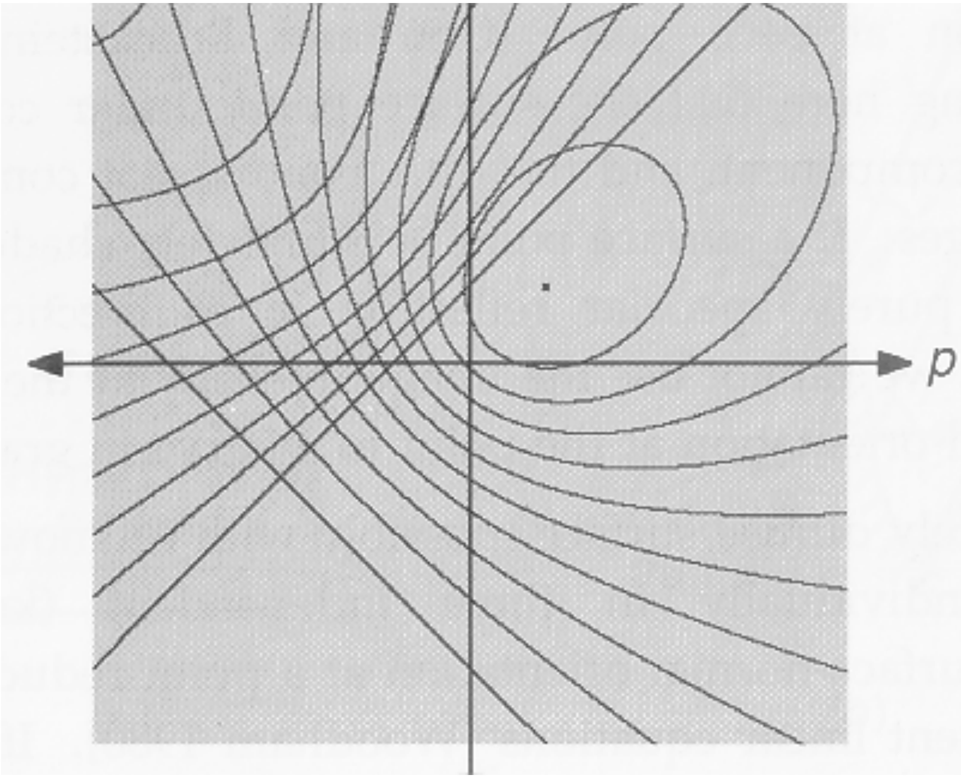
PHOTOMETRIC STEREO

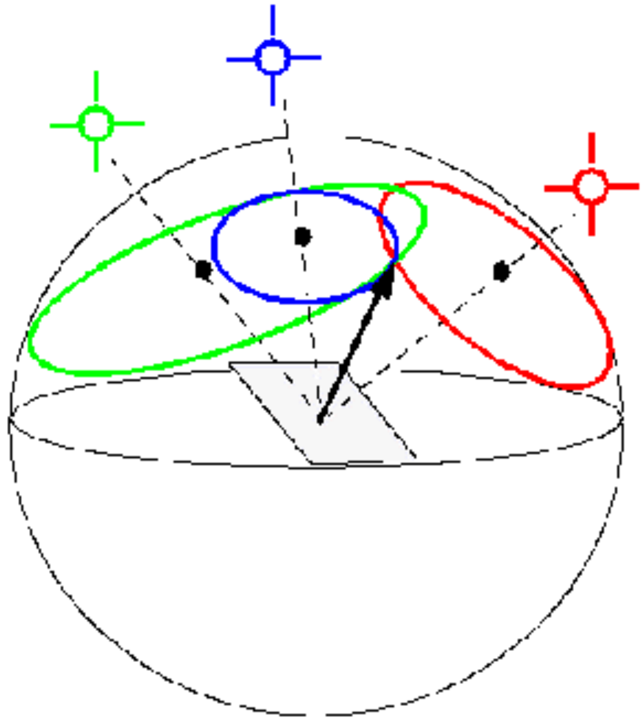


Given multiple images of the same surface under different known lighting conditions, can we recover the surface shape?

Yes! (Woodham, 1978)

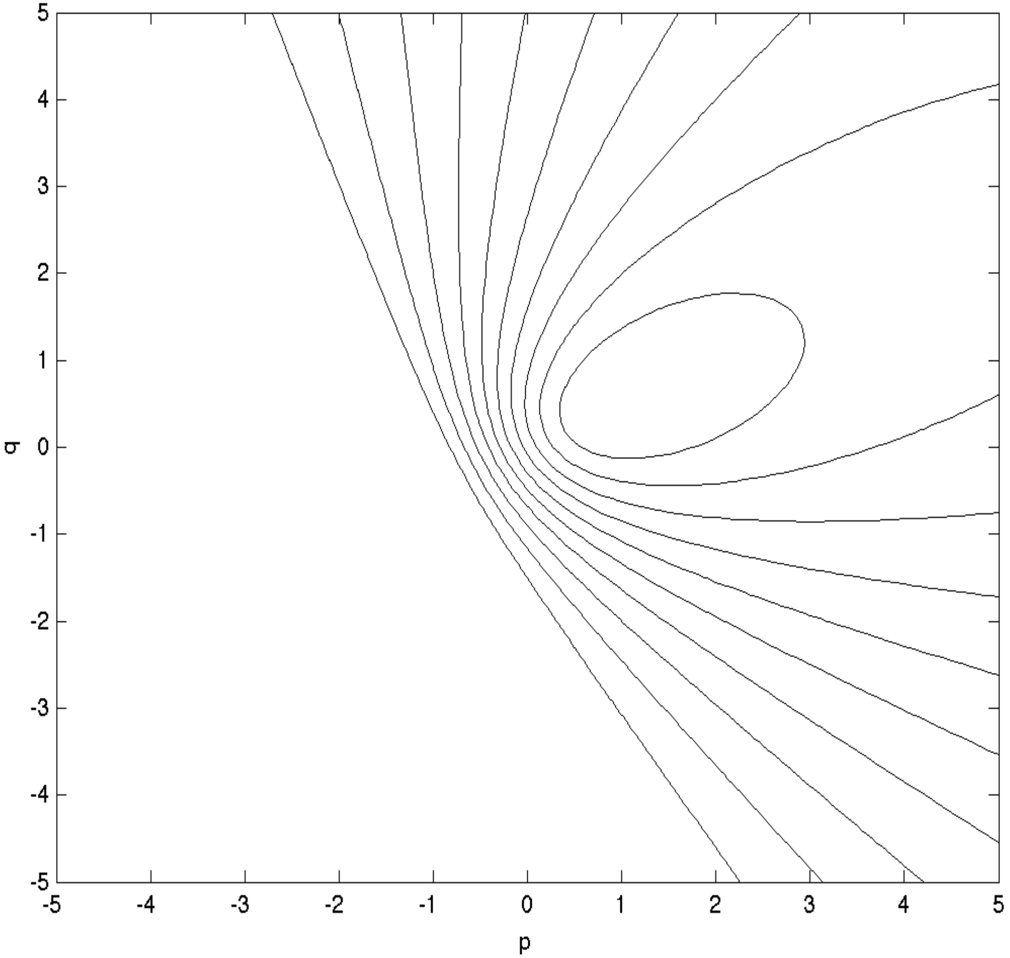
PHOTOMETRIC STEREO

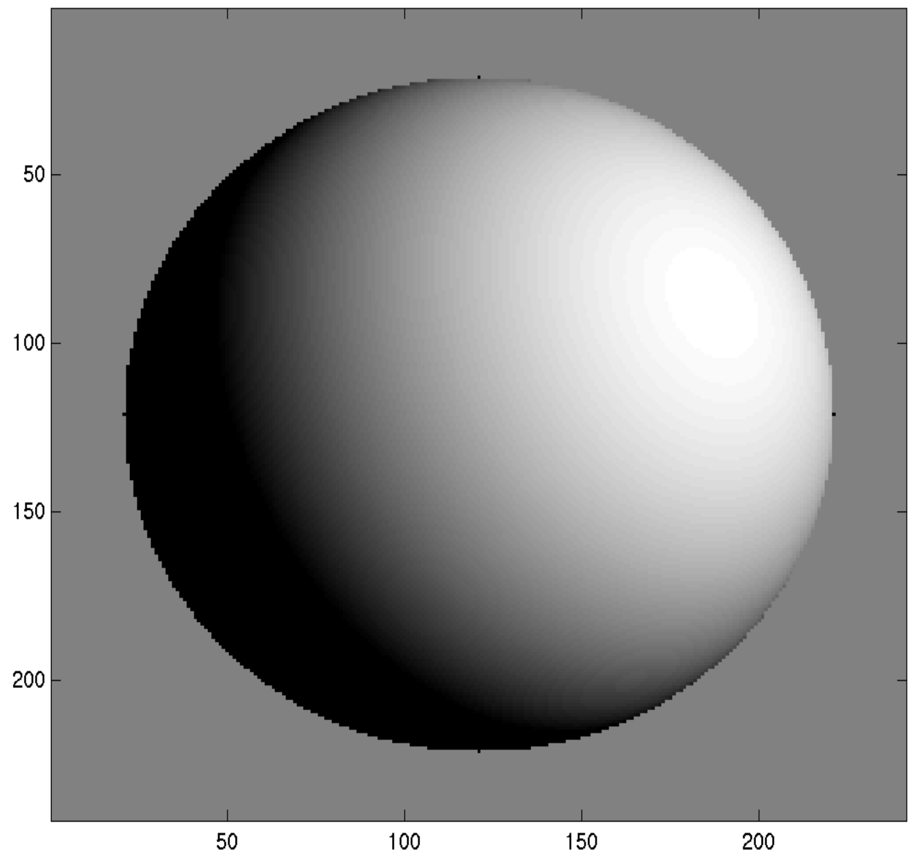




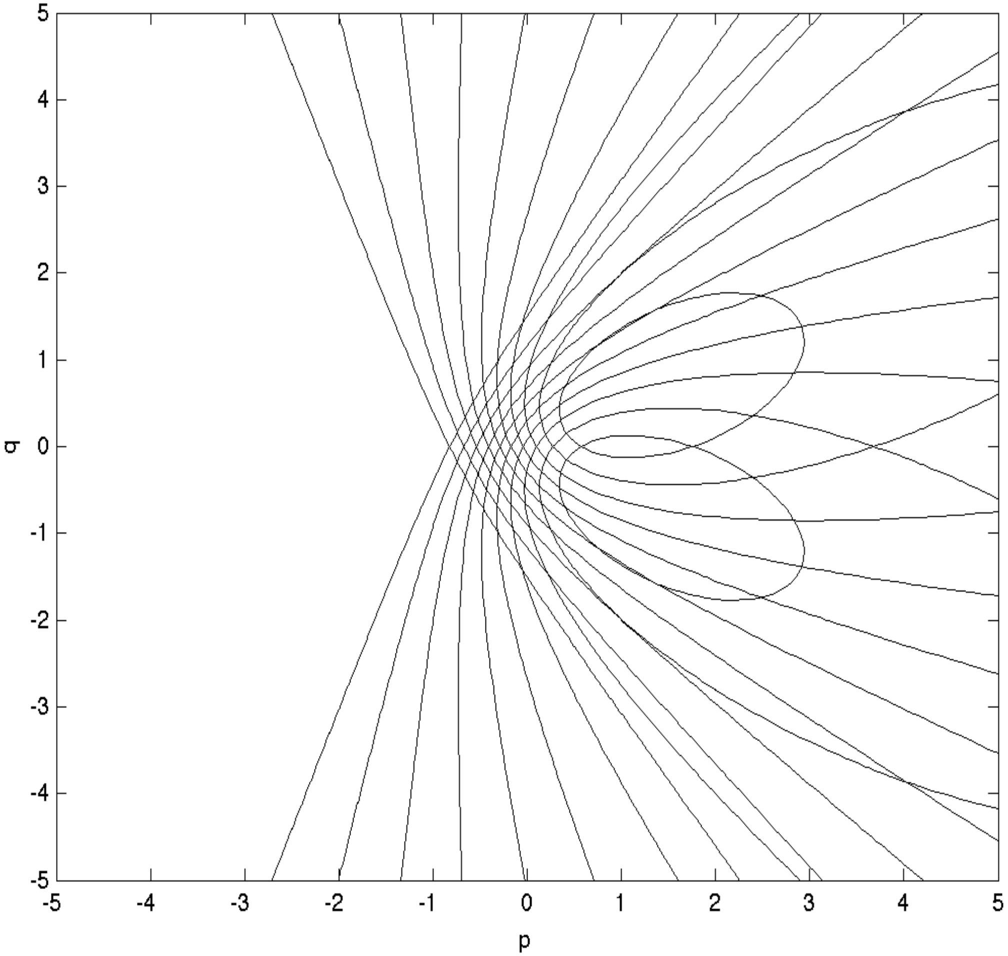
- Take several images under different lighting conditions.
- Infer the orientation from the changes in illumination.

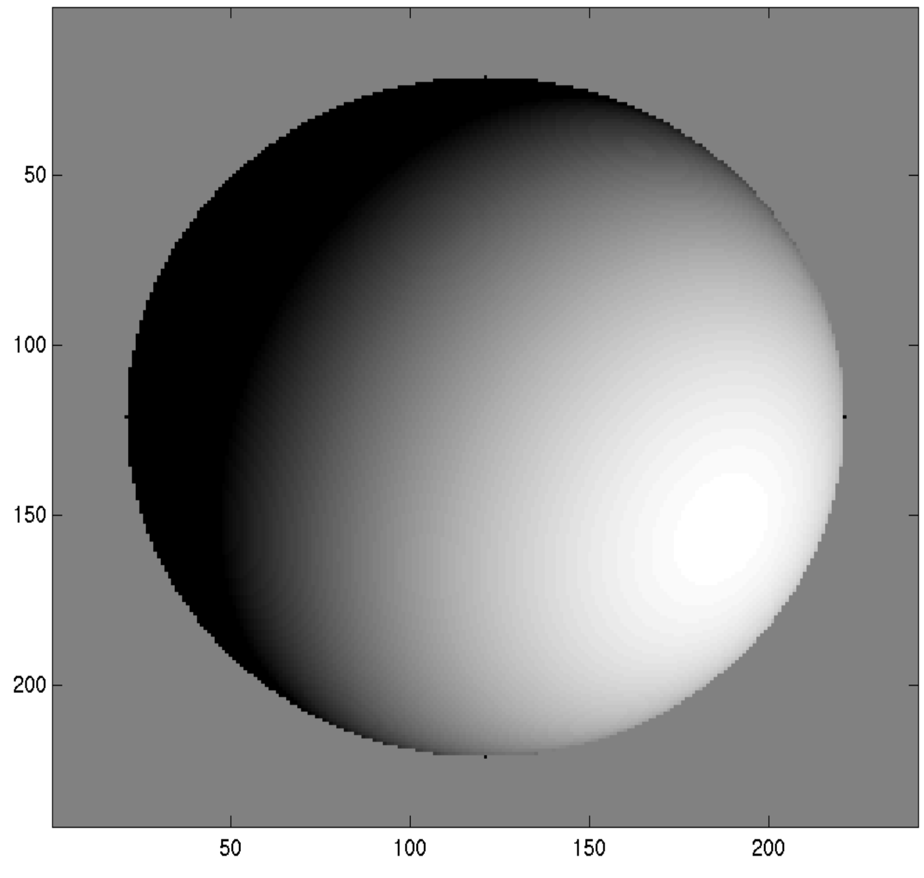
PHOTOMETRIC STEREO



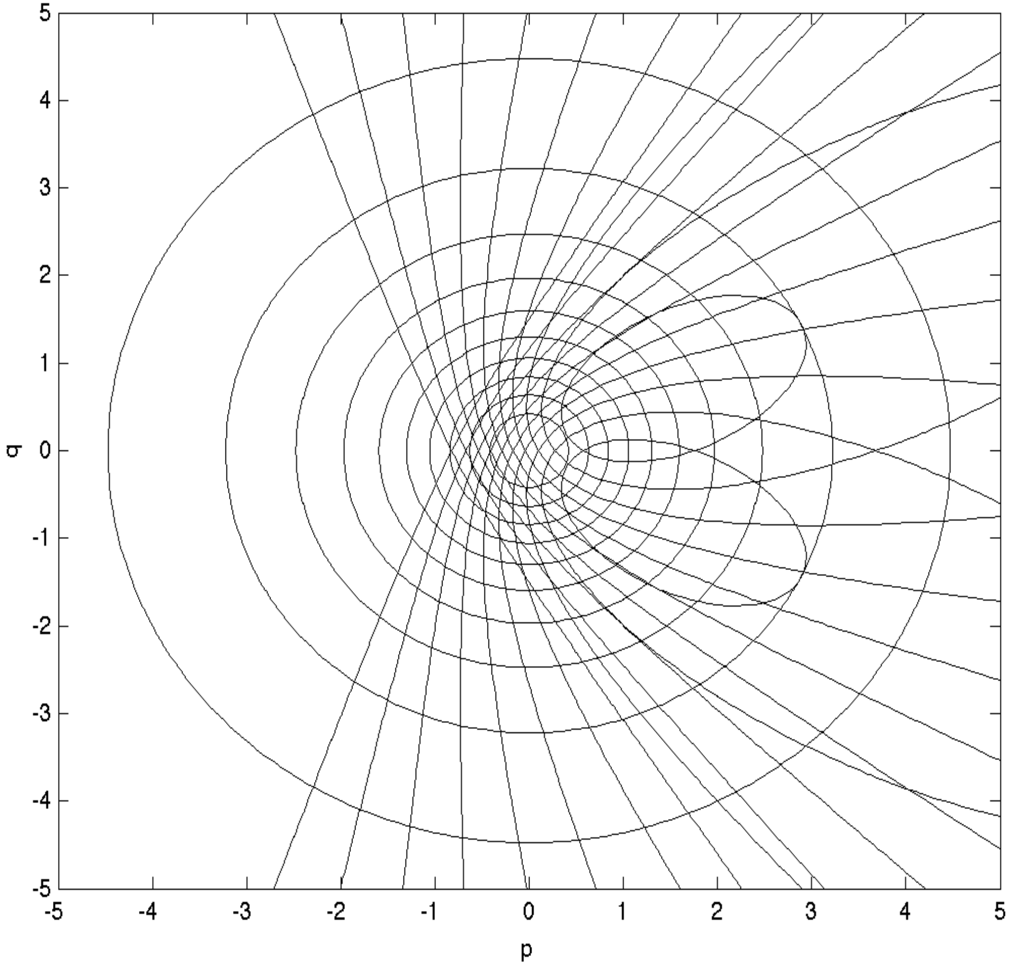


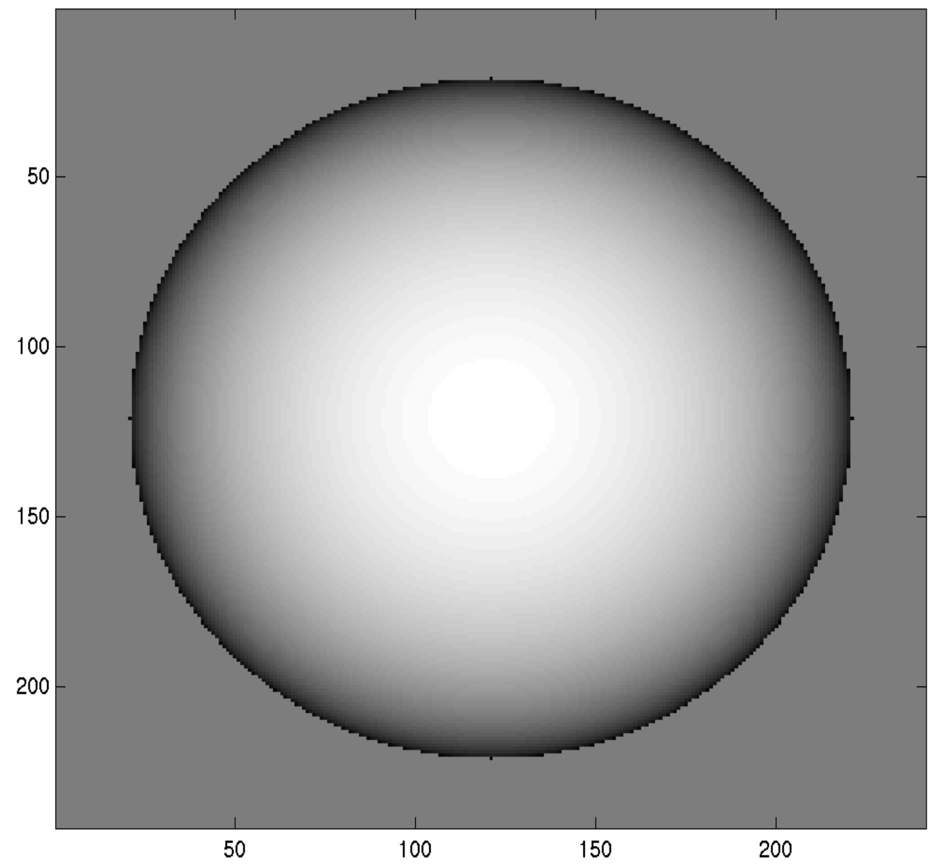
PHOTOMETRIC STEREO





PHOTOMETRIC STEREO





ALGEBRAIC FORMULATION

Lambertian model: $$I = \alpha(\mathbf{L}\cdot\mathbf{N}) = (\mathbf{L}\cdot\mathbf{M})$$

Three light sources:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \mathbf{L}_3 \end{bmatrix} \mathbf{M}$$

$$\mathbf{N} = \frac{\mathbf{M}}{||\mathbf{M}||}$$

$$\alpha = ||\mathbf{M}||$$

ADDITIONAL LIGHTS

Over-constrained problem:

$$\mathbf{I} = \mathbf{LM}, \text{ with } \mathbf{I} = \begin{bmatrix} I_1 \\ : \\ I_n \end{bmatrix} \text{ and } \mathbf{L} = \begin{bmatrix} \mathbf{L}_1 \\ : \\ \mathbf{L}_n \end{bmatrix}$$

$$\Rightarrow \mathbf{L}^t\mathbf{LM} = \mathbf{L}^t\mathbf{I} \text{ (Least - squares solution)}$$

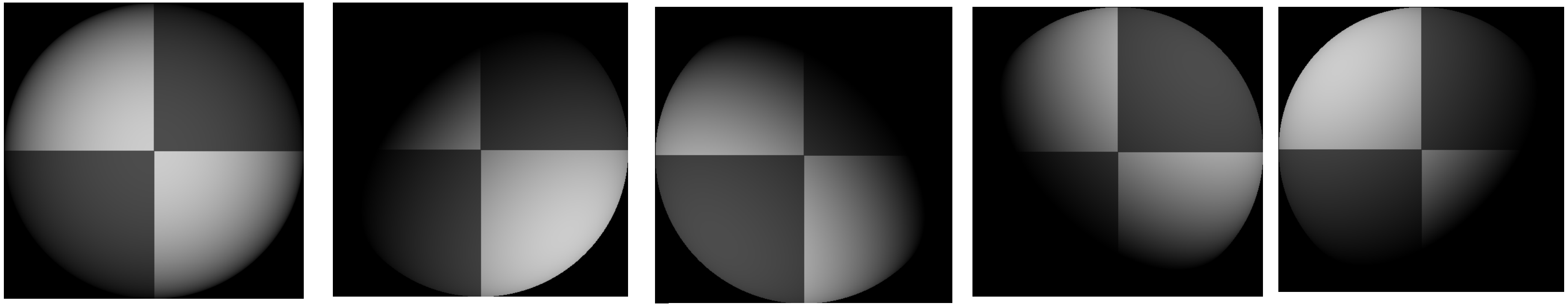
SHADOWS

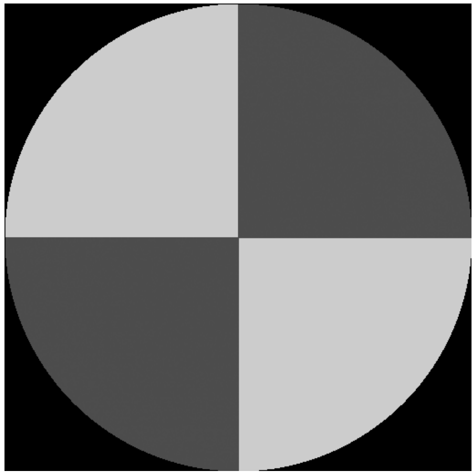
- Shadowed pixels for a given light source position are outliers.
- Premultiplying by a thresholded weight matrix eliminates their contributions.

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \boldsymbol{I} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \boldsymbol{LM}$$

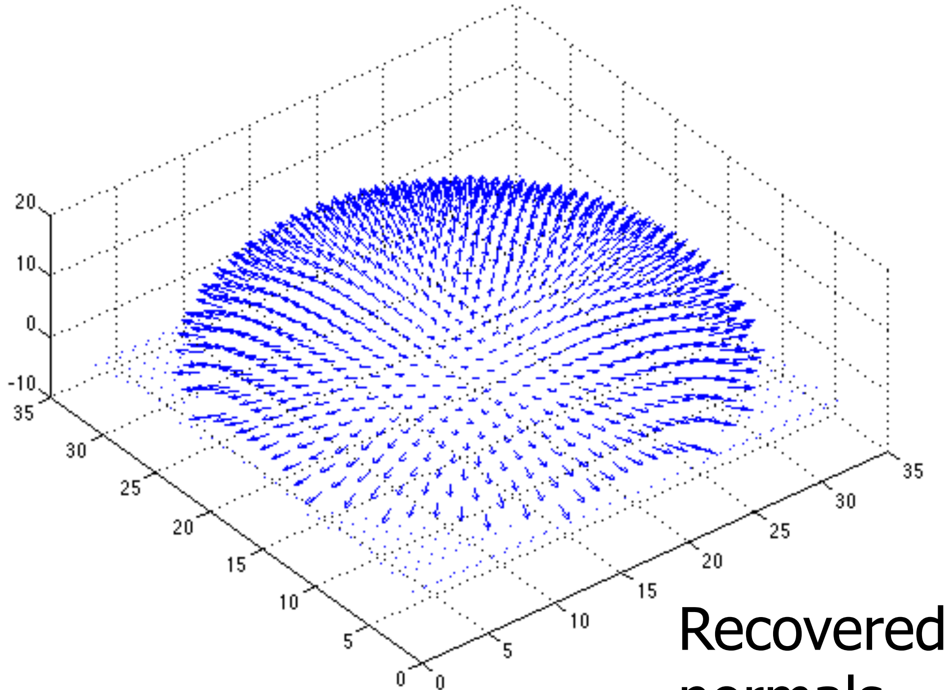
SYNTHETIC SPHERE IMAGES

Five different lighting conditions:





Recovered albedo

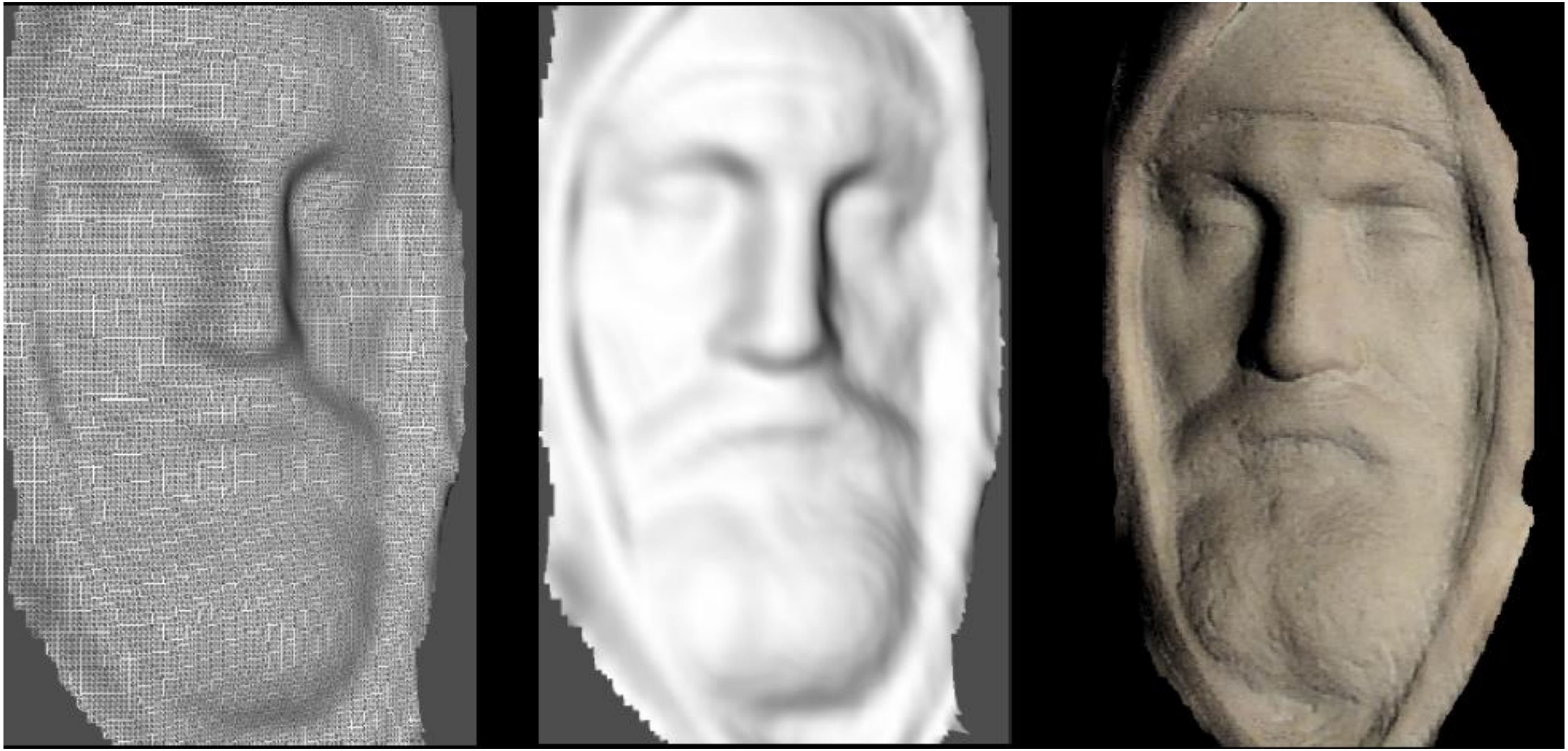


Recovered surface normals

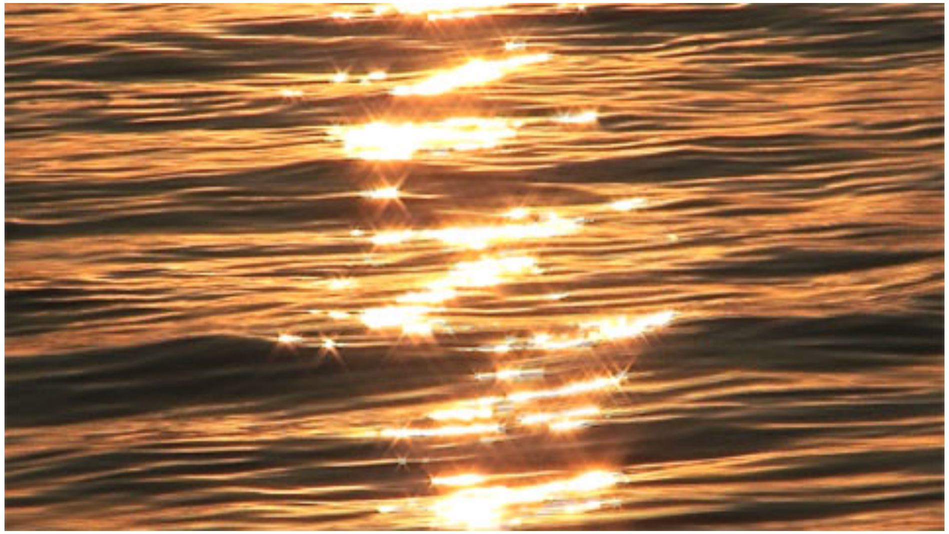
VIRTUOSO

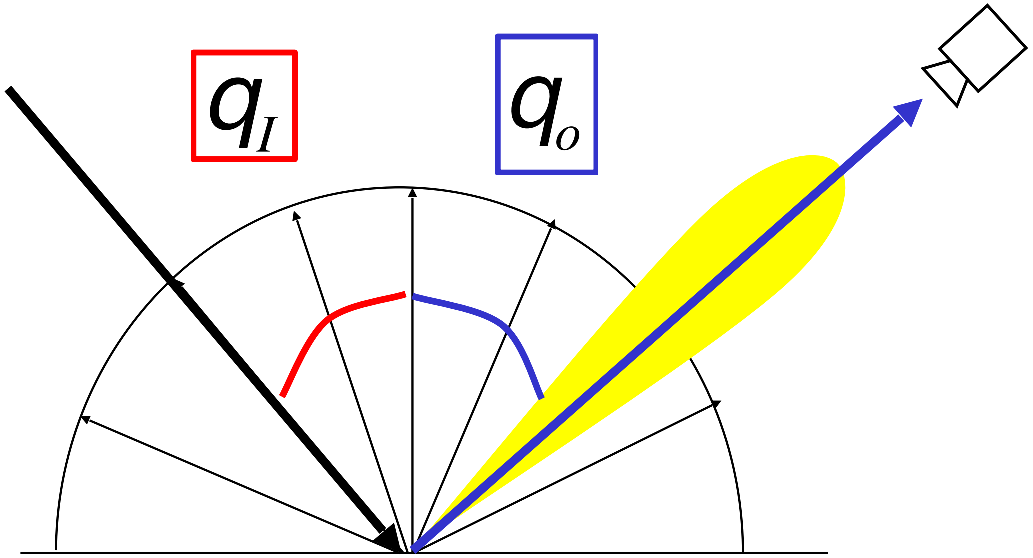
One camera and five light sources

DELIGHTED TEXTURE MAPS



SPECULARITIES

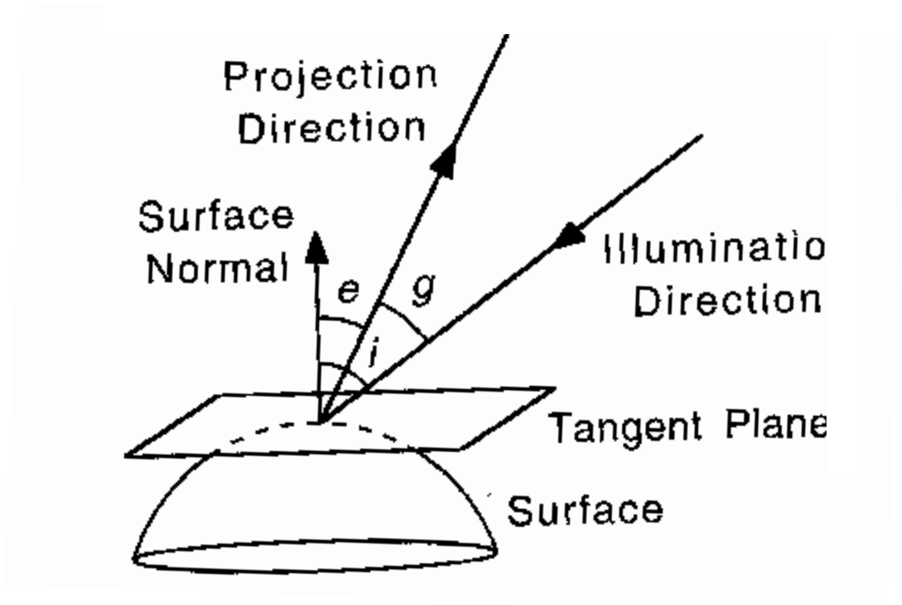




places where the surface behaves like a mirror. normal must be such that reflection ray is symmetric

- At specular points Lambertian assumptions are violated.
- However, they can be used to infer normal information.

ANGLES

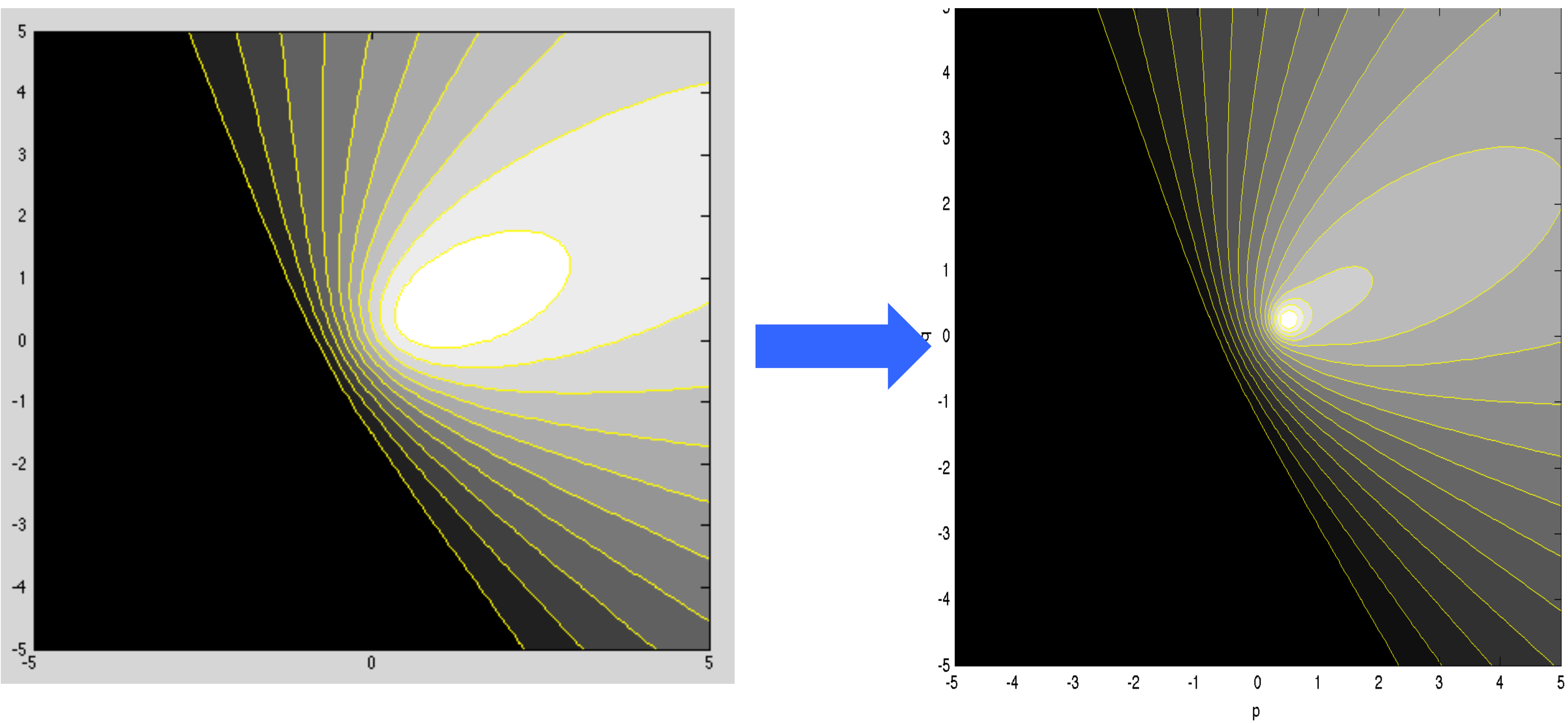


Angle of incidence (i): angle between the surface normal and the direction of the incident light ray.

Angle of emittance (e): angle between emitted light ray and surface normal.

Phase angle (g): angle between incident and emitted light ray.

LAMBERTIAN+SPECULAR REFLECTANCE MAP



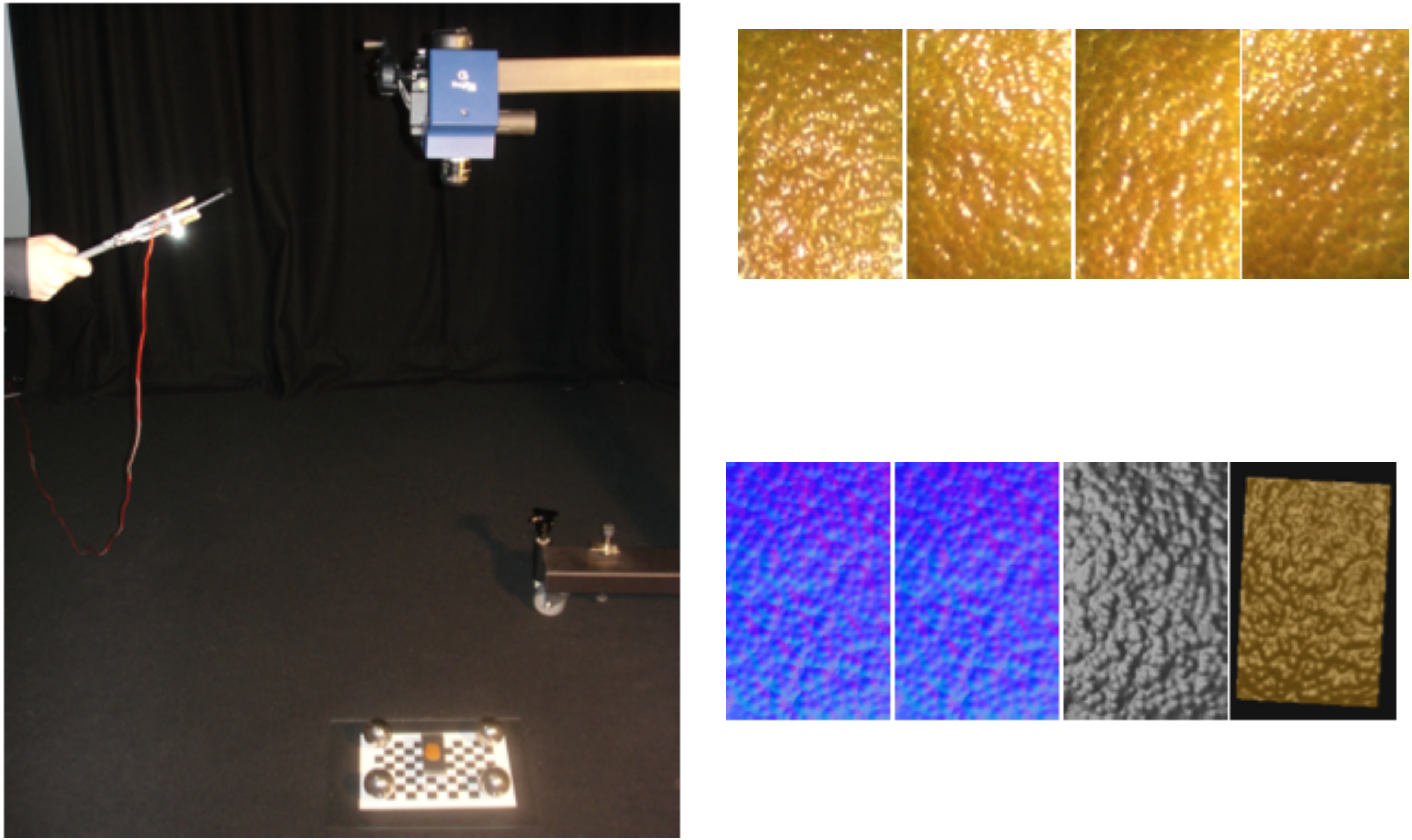
Weighted average of the individual diffuse- and specular-component of glossy white paint.

SPECULAR SURFACE

Perfect mirror: Reflects light only when i = e and i + e = g.

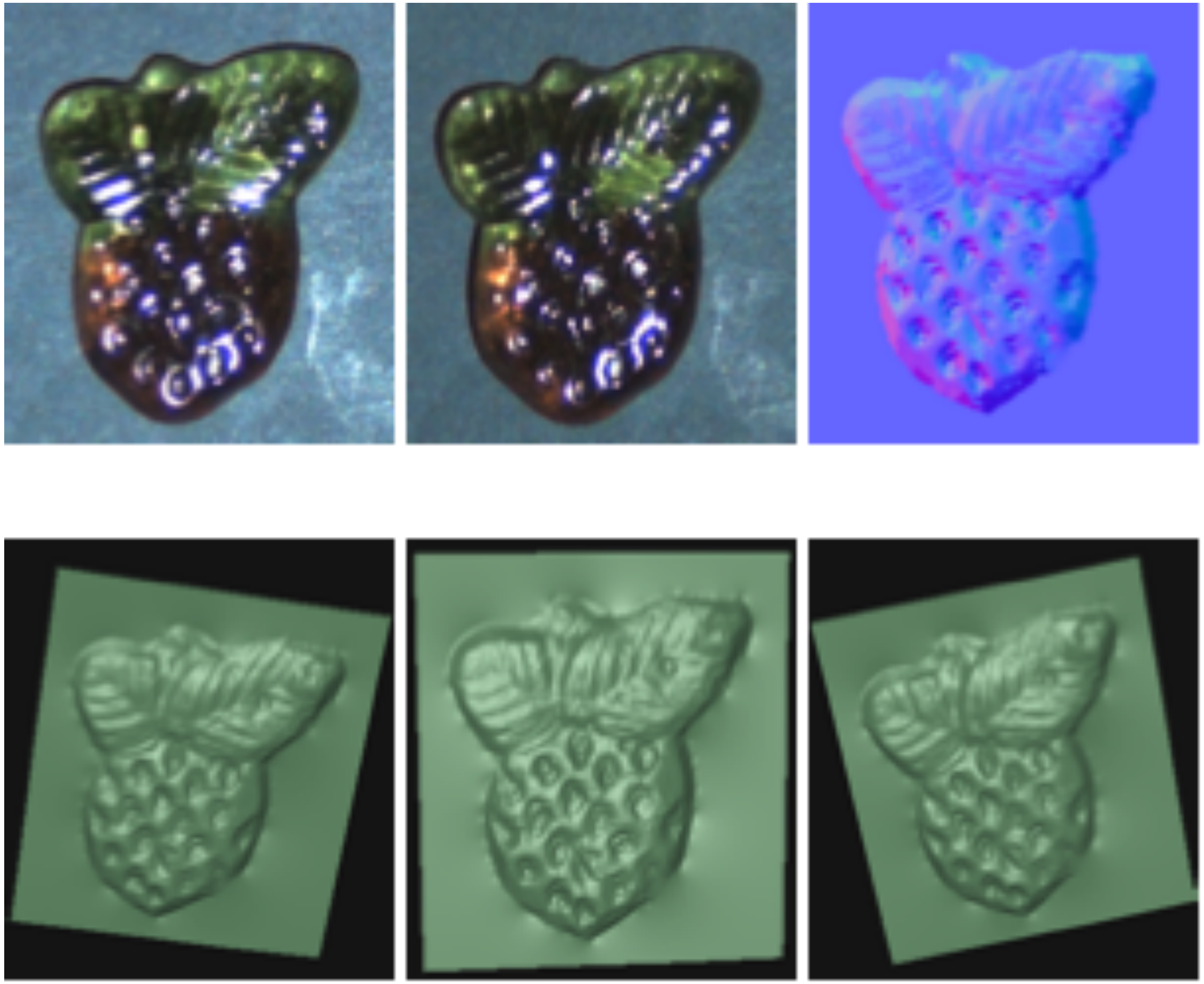
→ In practice, most surfaces are a mixture of specular and Lambertian.

SHAPE FROM SPECULARITIES



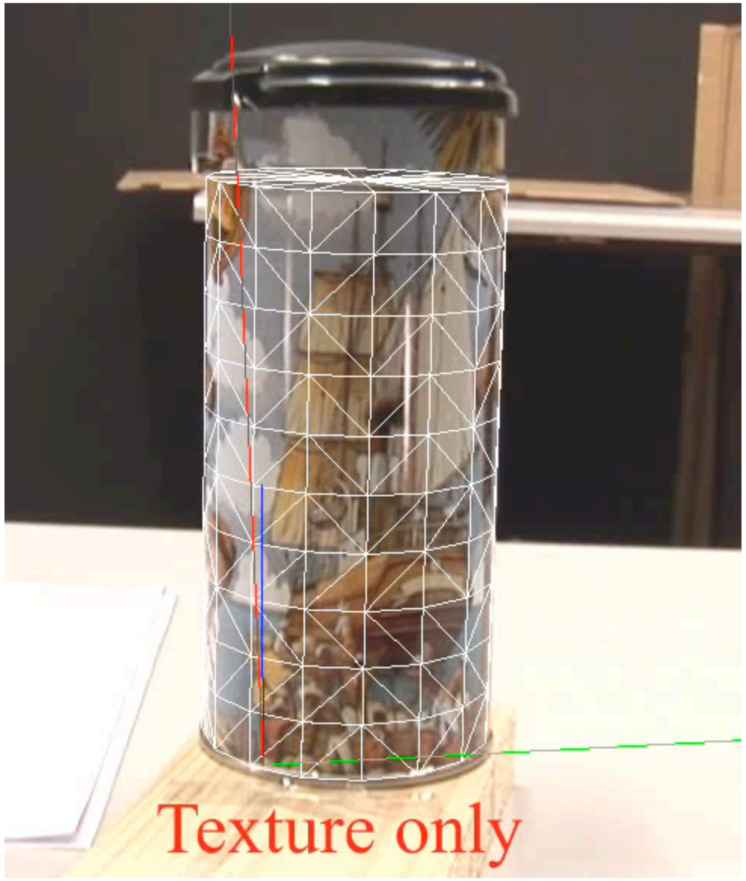
Chen et al., CVPR 2006

SHAPE FROM SPECULARITIES



Chen et al., CVPR 2006

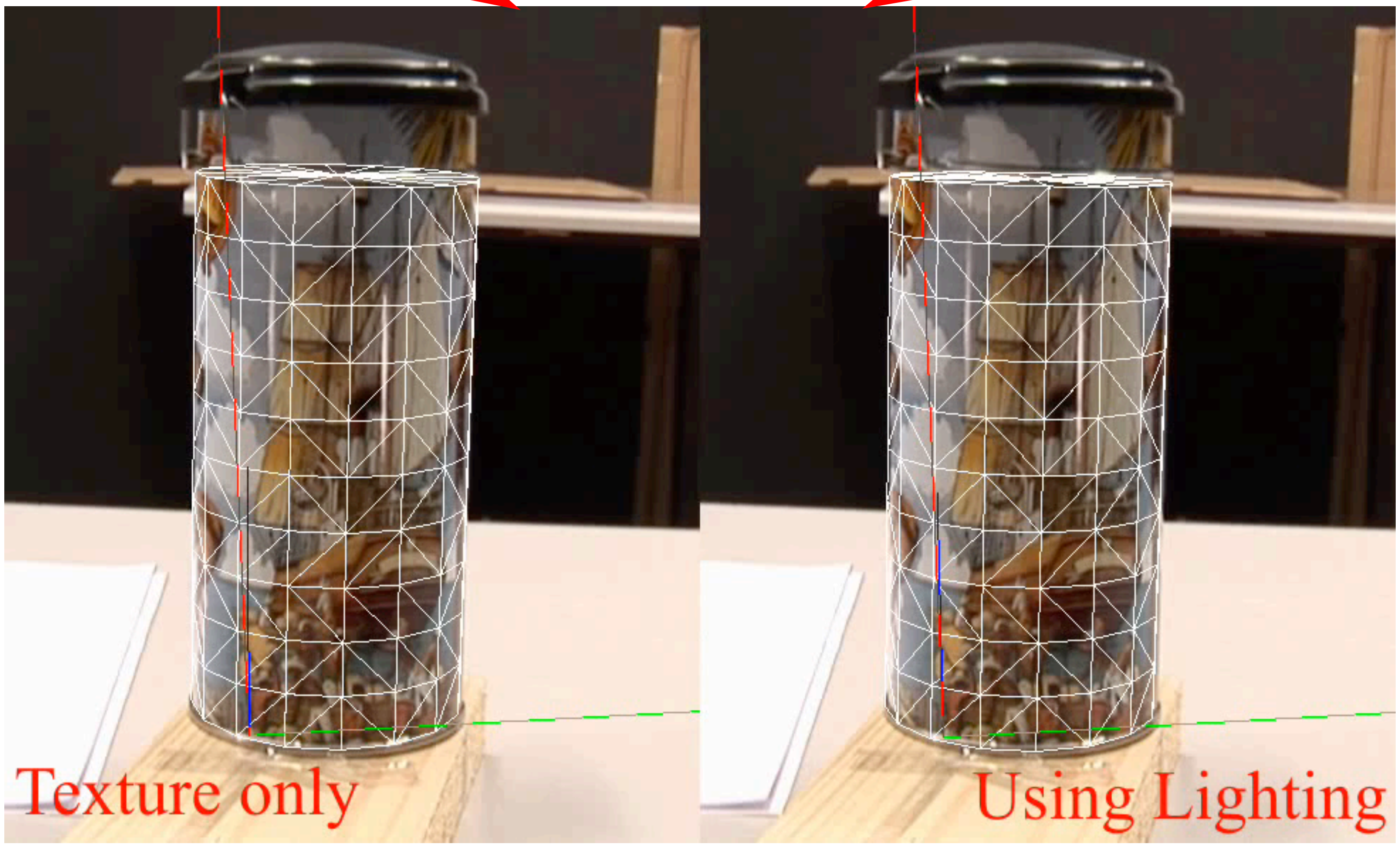
POSE FROM TEXTURE



- Texture tracking yields correct reprojection of frontal facets.
- But the jittering top shows that the tracking is nevertheless inaccurate.

POSE FROM SPECULARITIES

Taking advantage of specularities yields better poses



→ No more jittering top!

TEXTURE AND SPECULARITIES

Specularities are:

- Very sensitive to motion
- Affected differently than texture by the same motion

Specularities are not:

- Capable of constraining all the degrees of freedom

Therefore texture and specularities must be combined

IN SHORT

Traditional Shape-from-Shading requires making strong assumptions:

- Constant or piece-wise constant albedo
- No inter-reflections
- No shadows
- No specularities

stereo works best
when albedo is
changing

→ In a single image context, it is most useful in conjunction with other information sources.