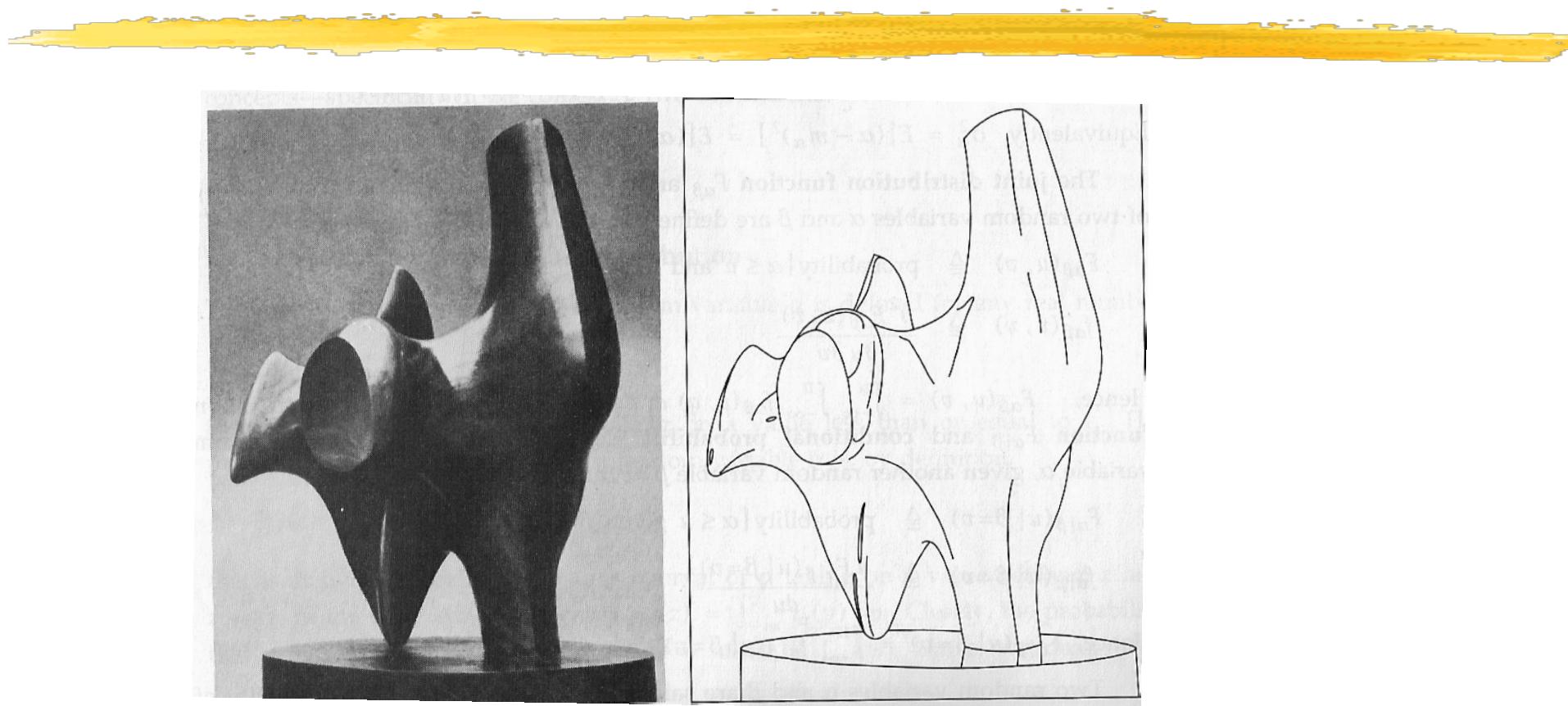
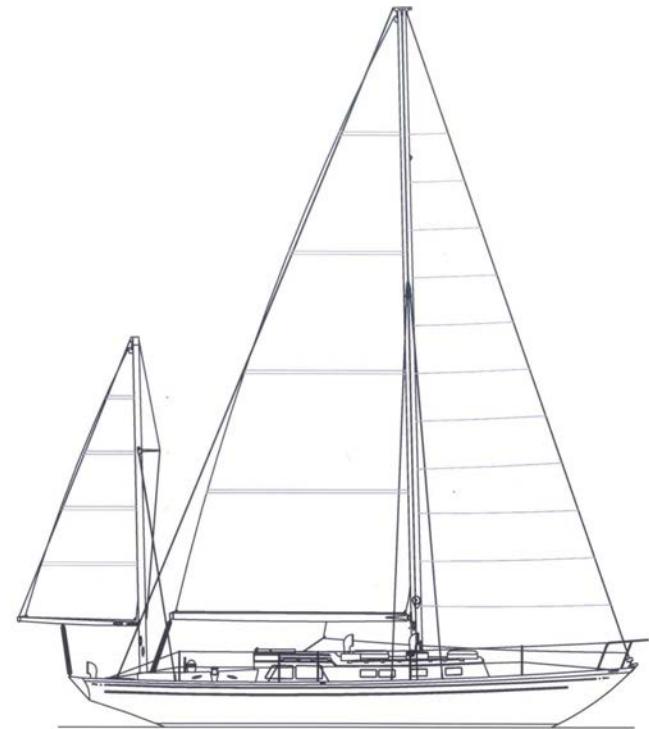
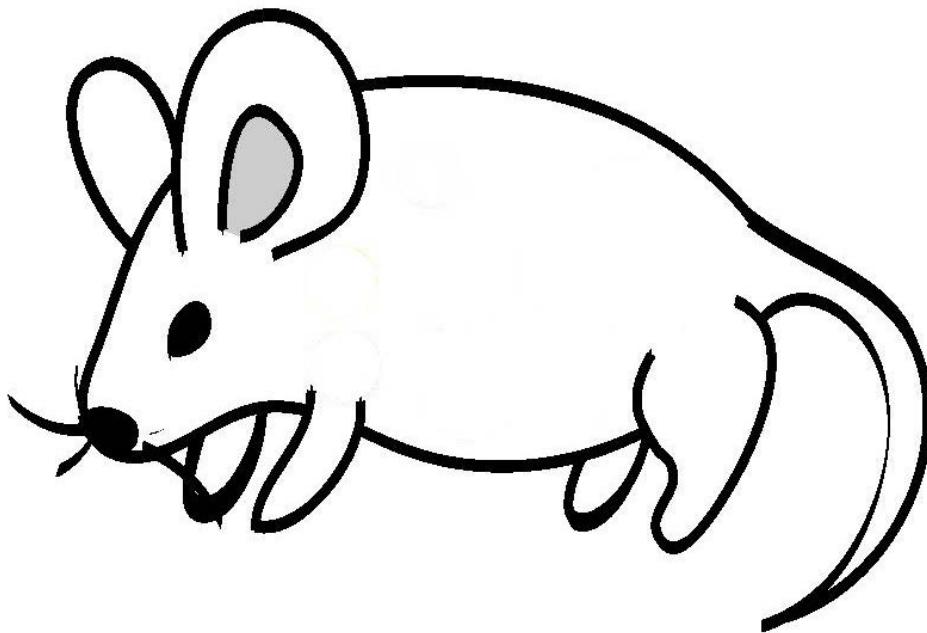


EDGE DETECTION



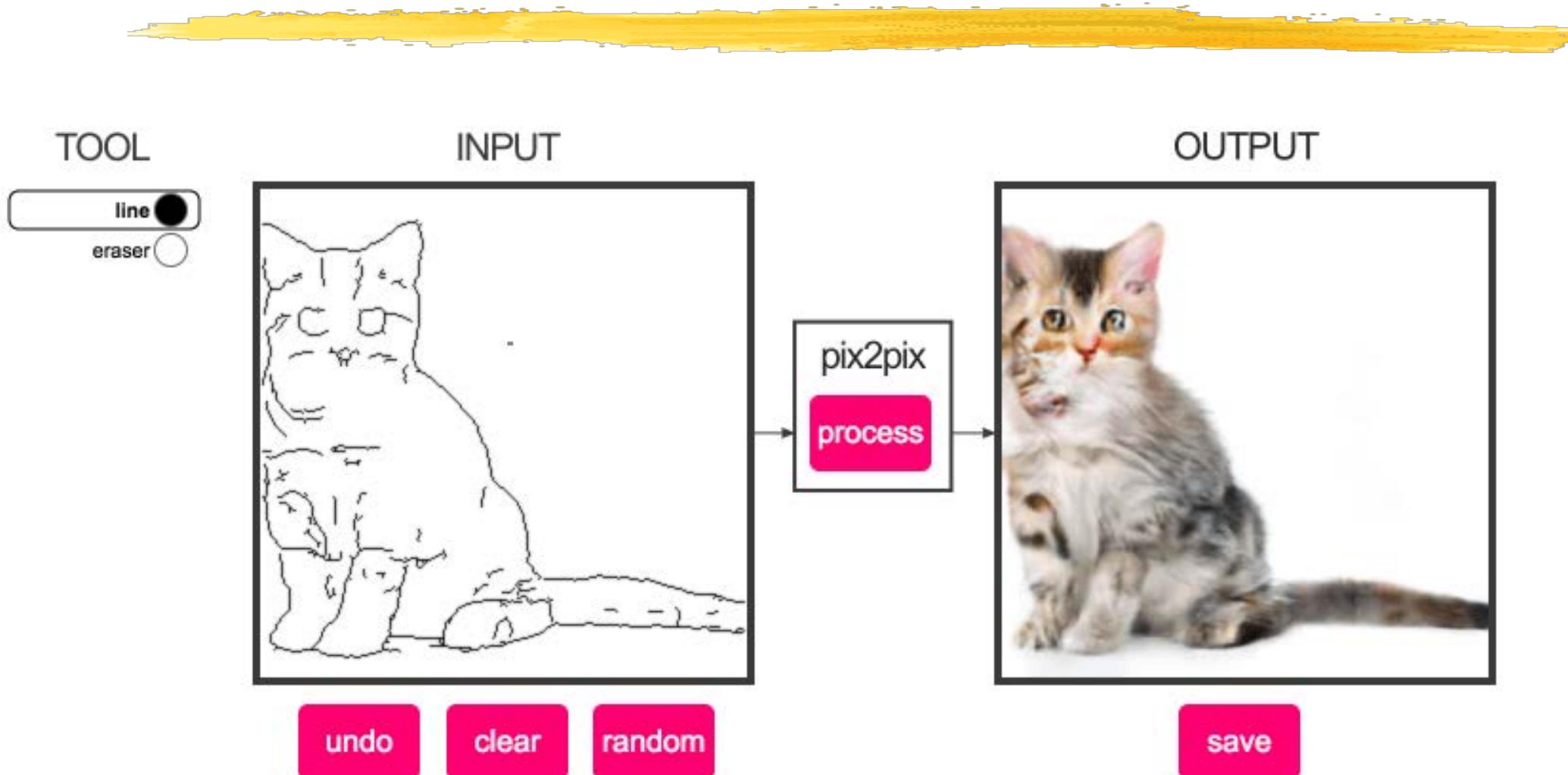
- What's an edge
- Image gradients
- Edge operators

LINE DRAWINGS



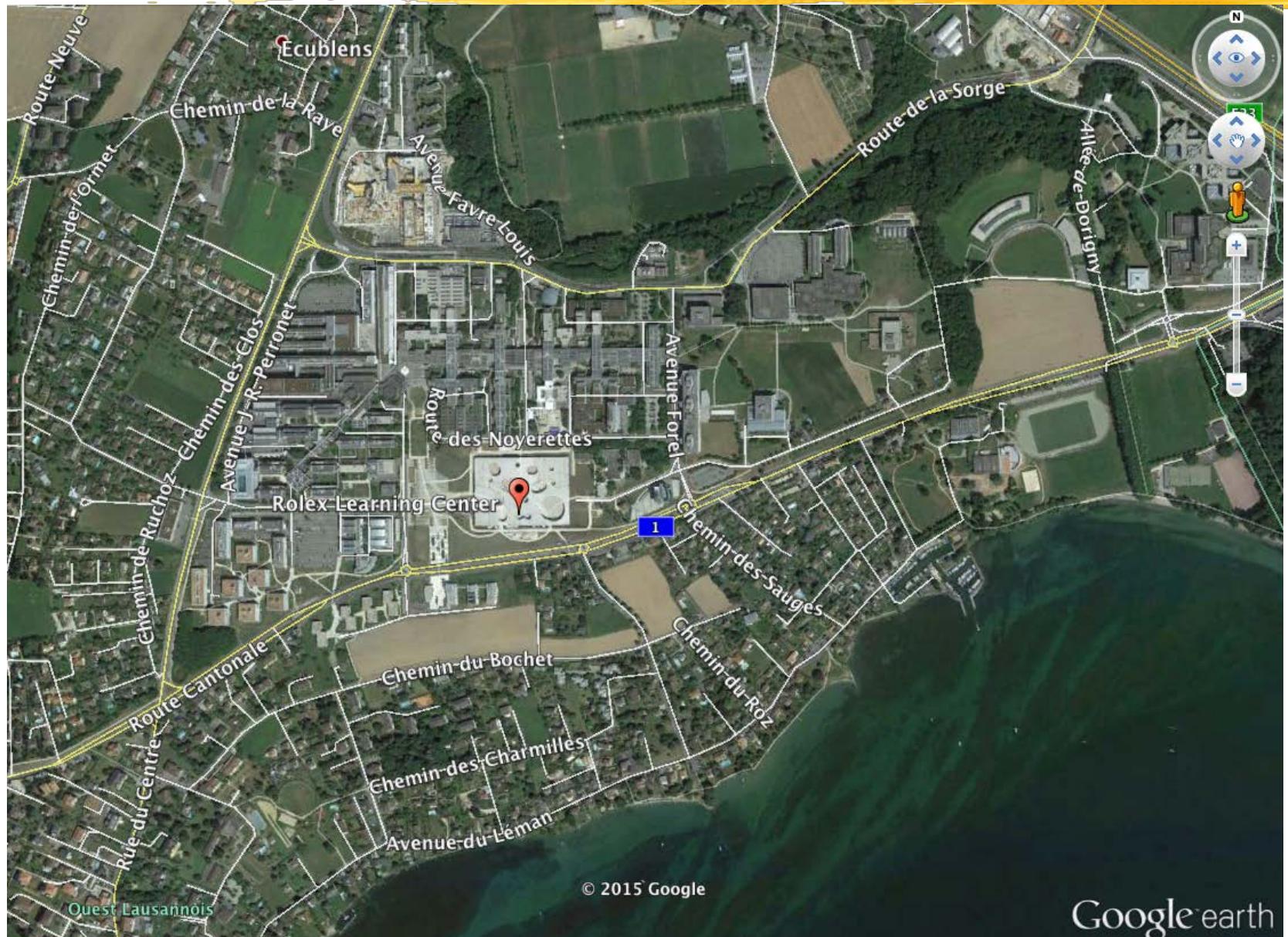
- Edges seem fundamental to human perception.
- They form a compressed version of the image.

FROM EDGES TO CATS



<https://affinelayer.com/pixsrv/>

MAPS AND OVERLAYS



CORRIDOR

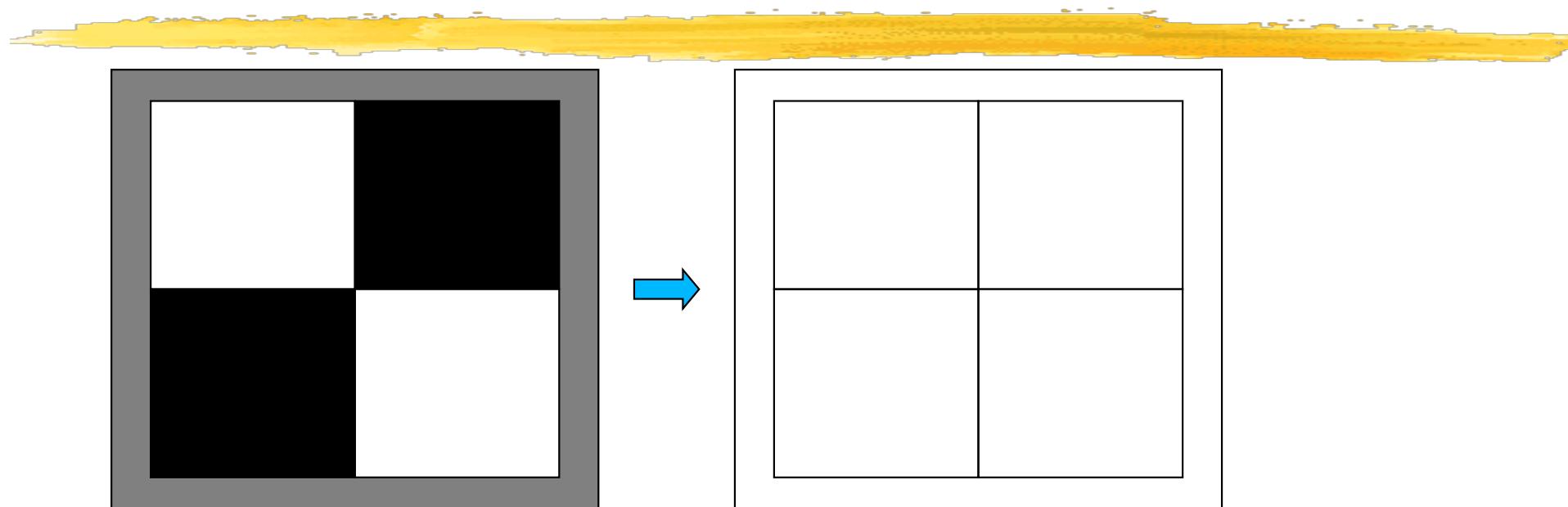


Lab
exlab.epti.ch

CORRIDOR



EDGES AND REGIONS



Edges:

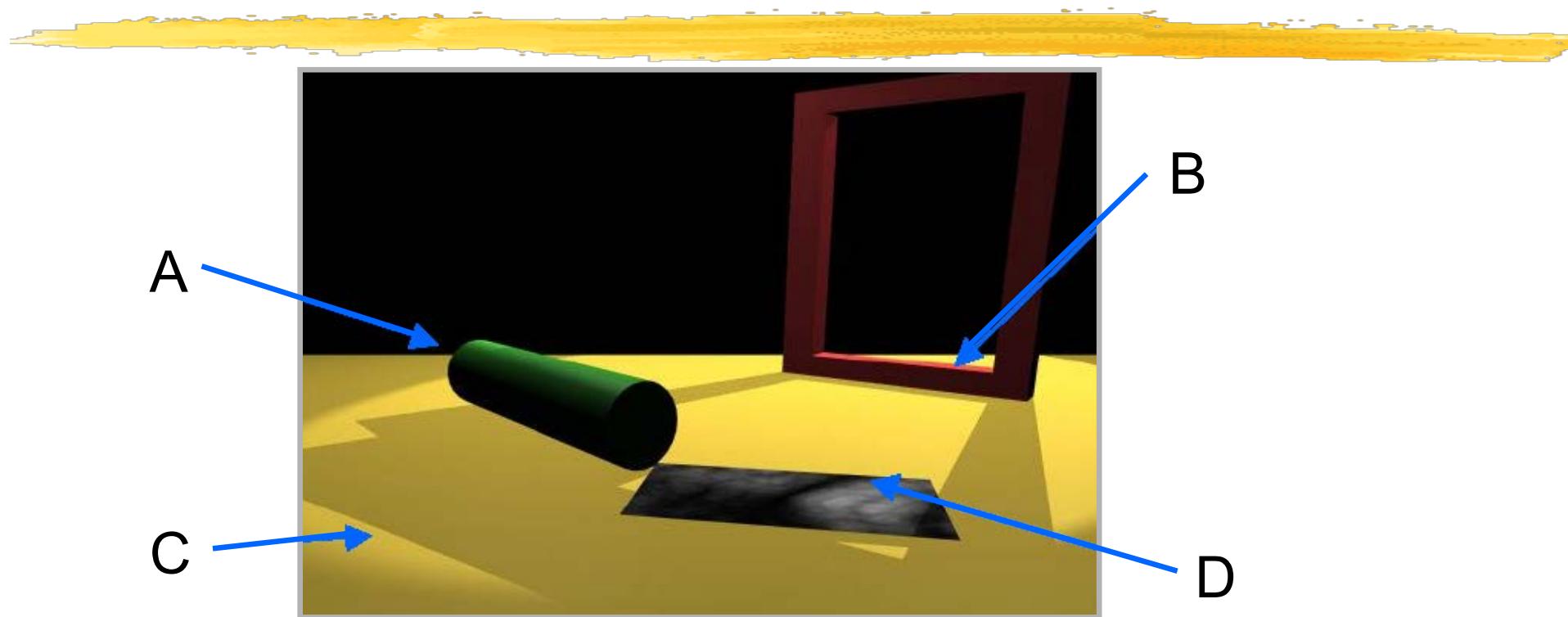
Boundary between bland image regions.

Regions:

Homogenous areas between edges.

→ Duality Edge/Regions.

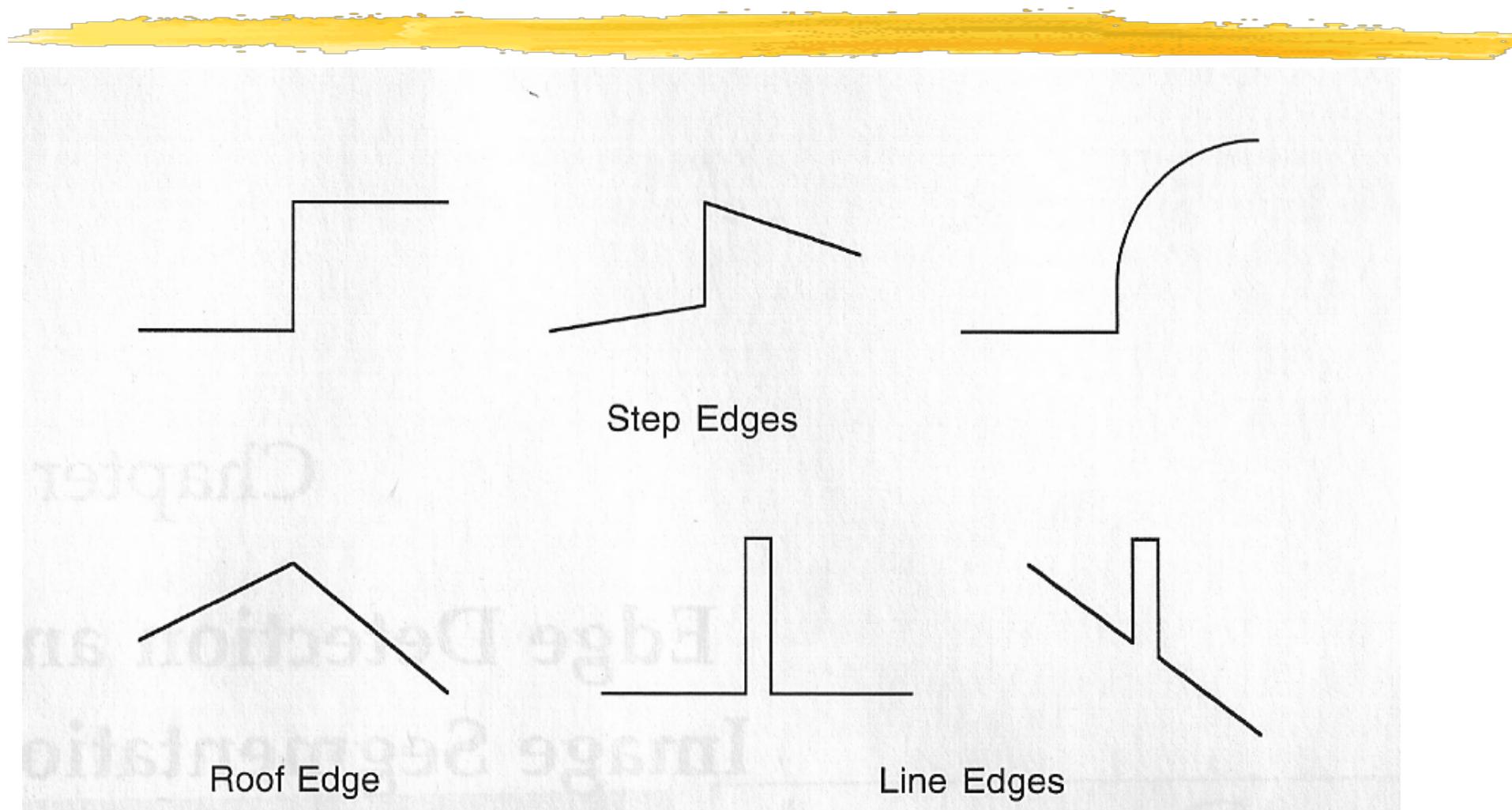
DISCONTINUITIES



- A. Depth discontinuity: Abrupt depth change in the world
- B. Surface normal discontinuity: Change in surface orientation
- C. Illumination discontinuity: Shadows, lighting changes
- D. Reflectance discontinuity: Surface properties, markings

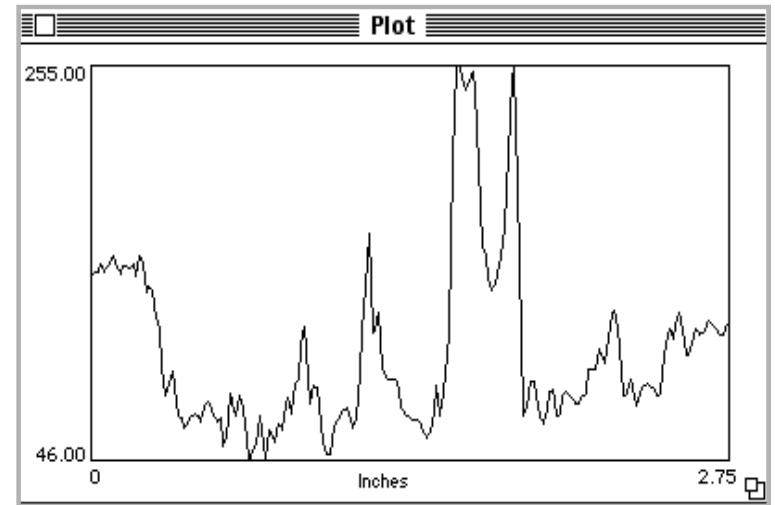
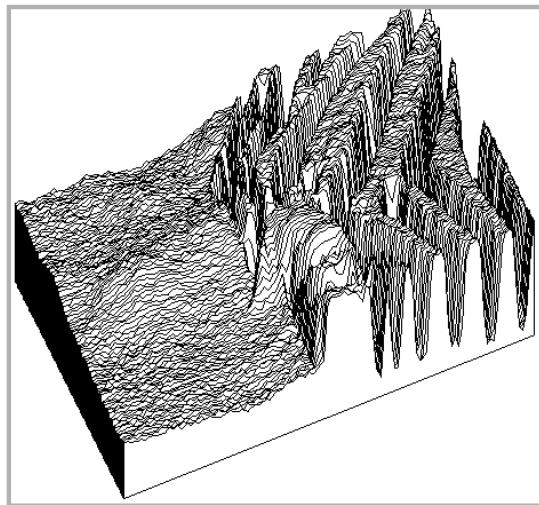
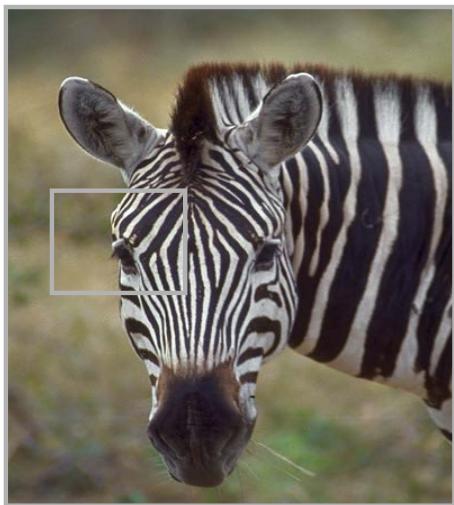
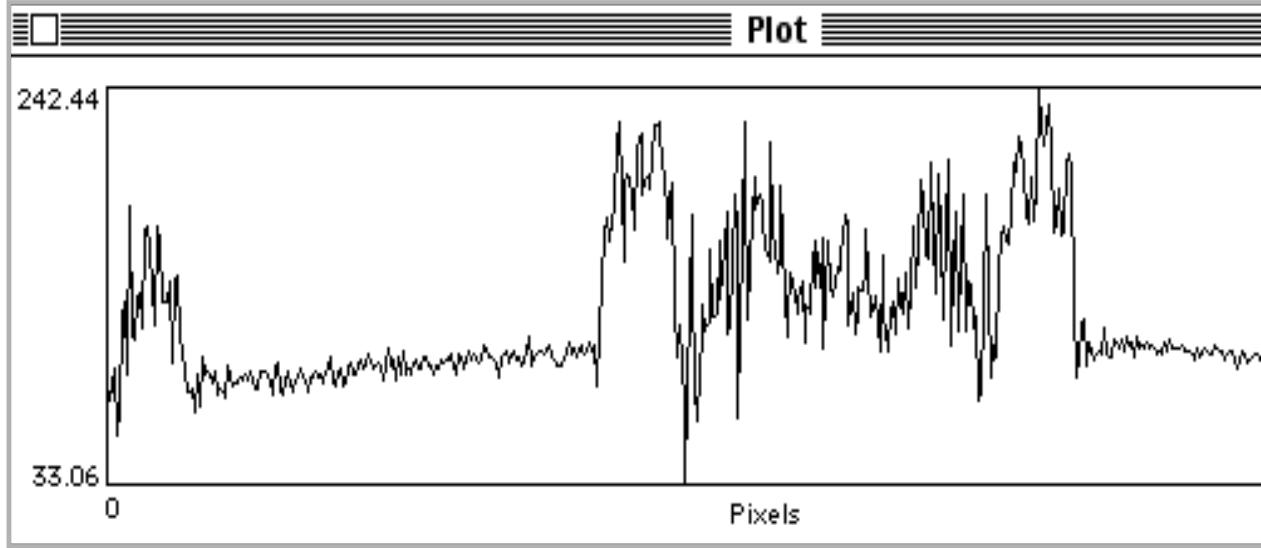
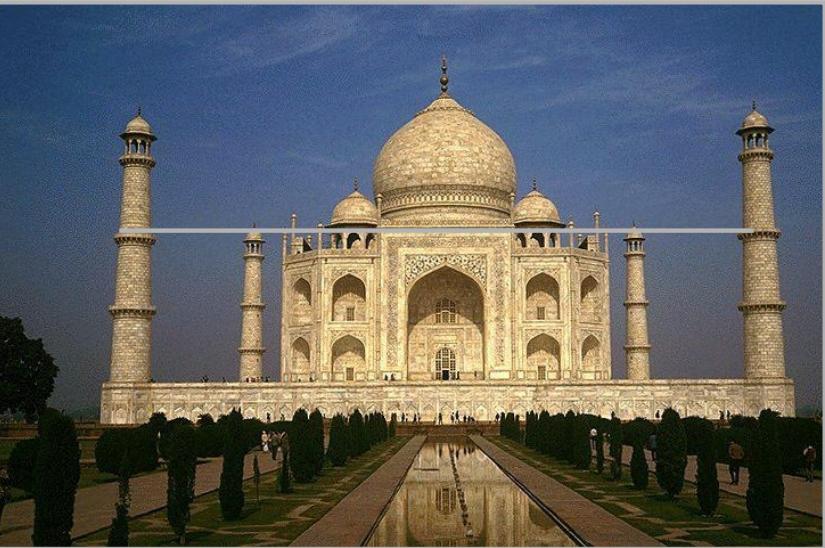
→ **Contrast:** Gray levels different on both sides

EDGE PROFILES

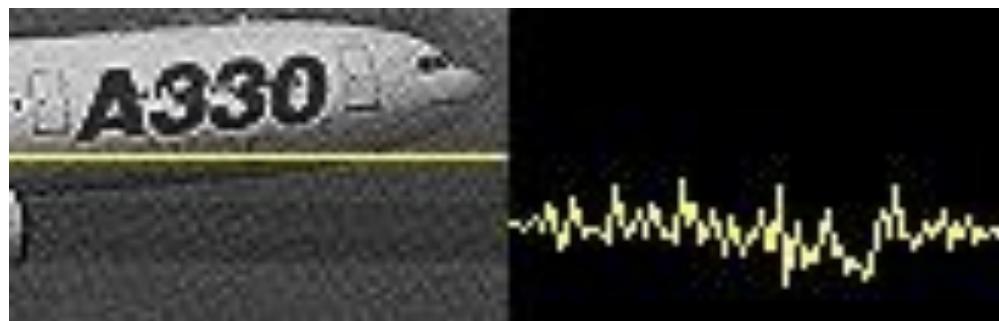
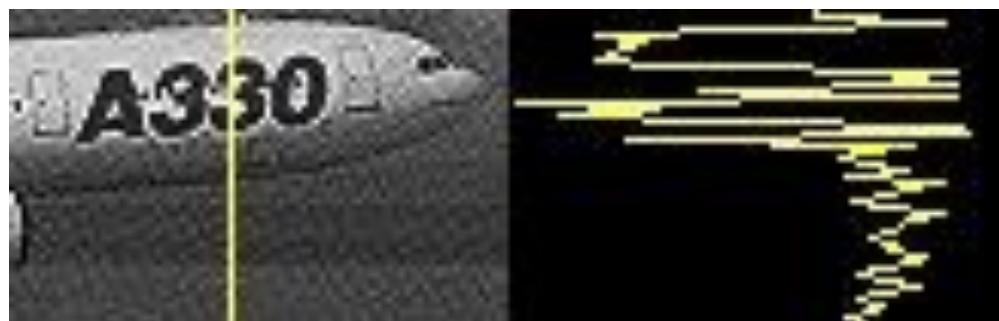


Edges are where a change occurs

REALITY



MORE REALITY



Very noisy signals
→ Much knowledge is required!!

ILLUSORY CONTOURS

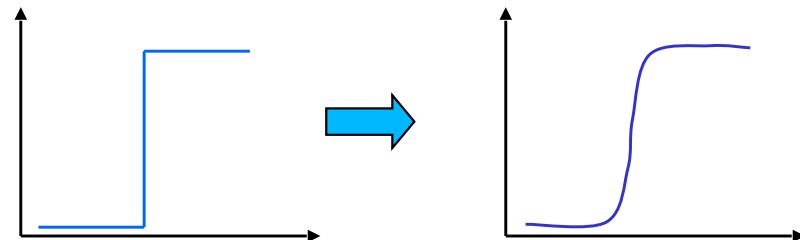


No closed contour, but

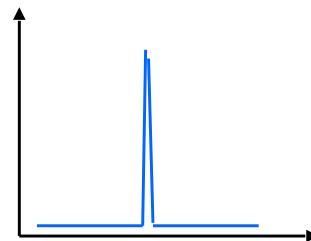
IDEAL STEP EDGE



$f(x) = \text{step edge}$

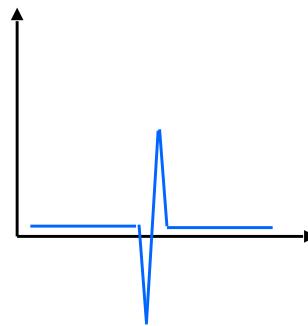


1st Derivative $f'(x)$



maximum

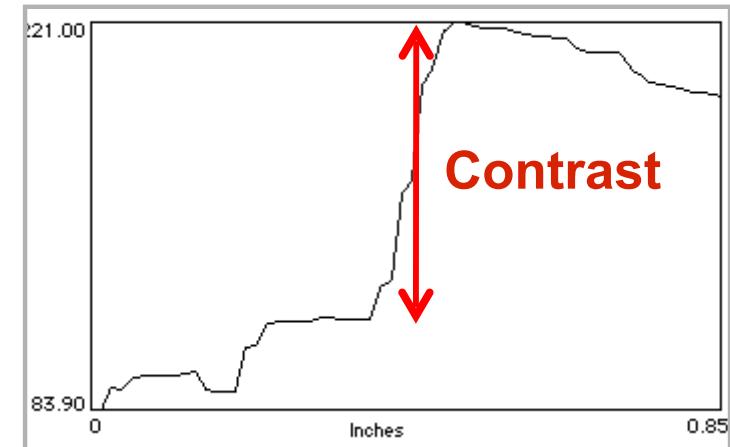
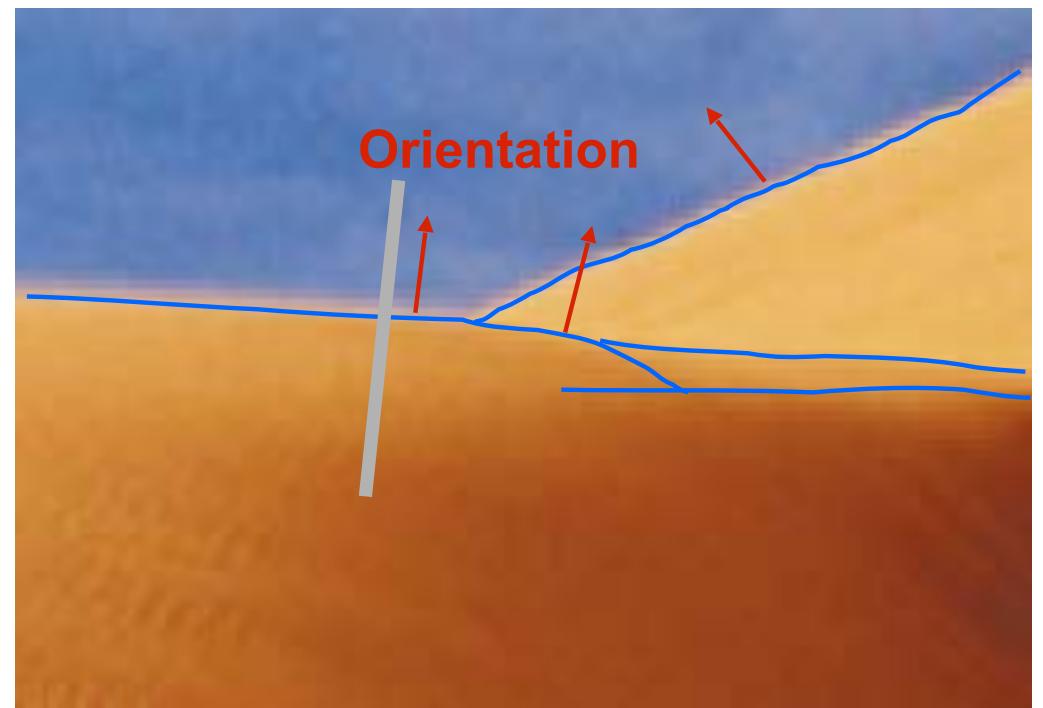
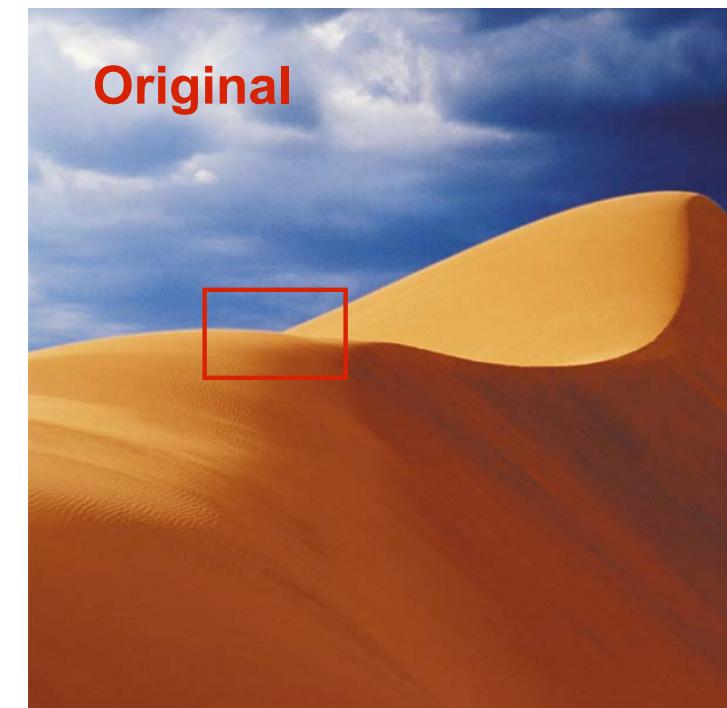
2nd Derivative $f''(x)$



zero crossing

Rapid change in image => High local gradient

EDGE PROPERTIES



EDGE DESCRIPTORS

Edge Normal:

- Unit vector in the direction of maximum intensity change

Edge Direction:

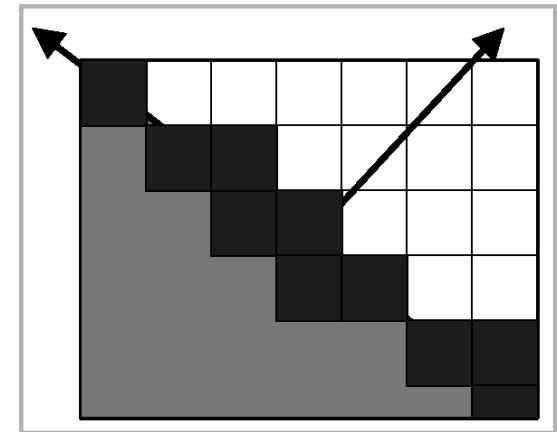
- Unit vector perpendicular to the edge normal

Edge position or center

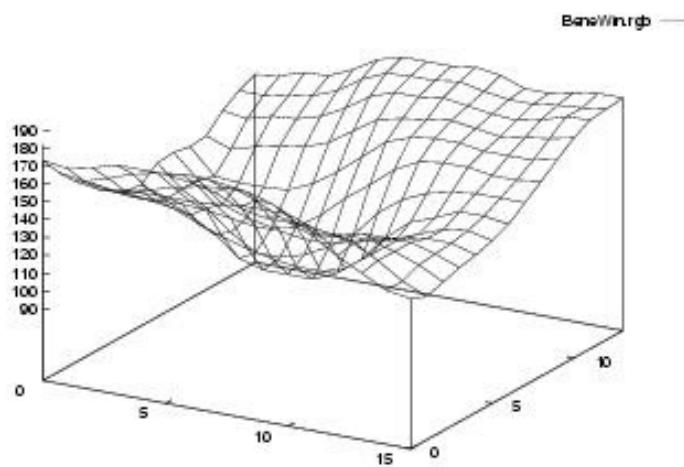
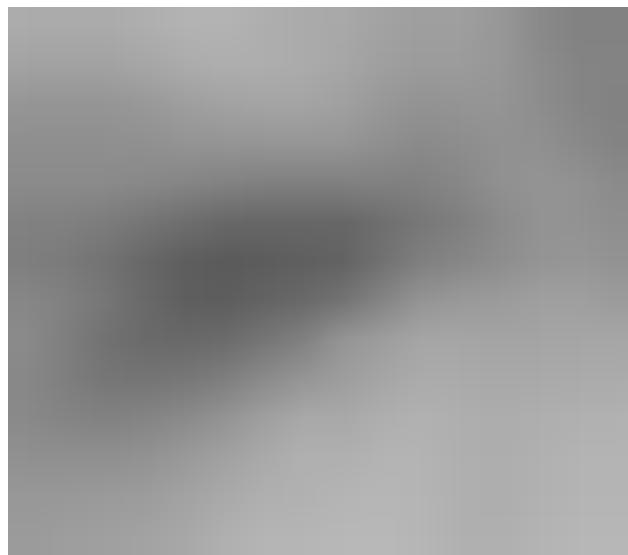
- Image location at which edge is located

Edge Strength

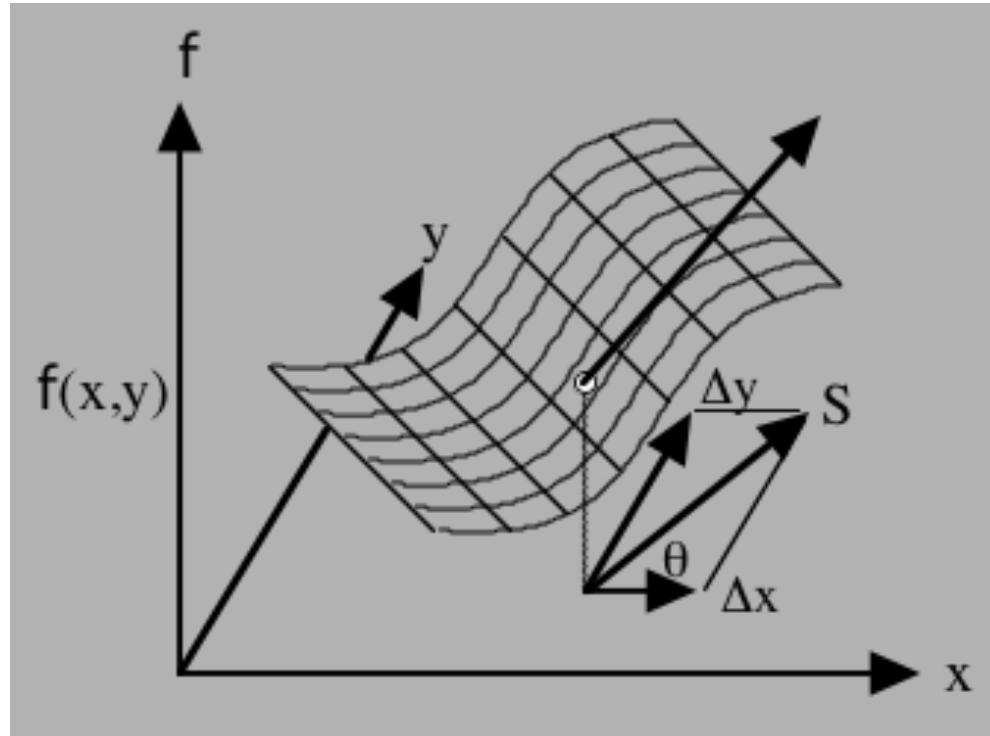
- Speed of intensity variation across the edge.



IMAGES AS 3-D SURFACES



GEOMETRIC INTERPRETATION



Since $I(x,y)$ is not a continuous function:

1. Locally fit a smooth surface.
2. Compute its derivatives.

IMAGE GRADIENT

The gradient of an image

$$\nabla I = \left[\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y} \right]$$

points in the direction of most rapid change in intensity.



$$\nabla I = \left[\frac{\delta I}{\delta x}, 0 \right]$$

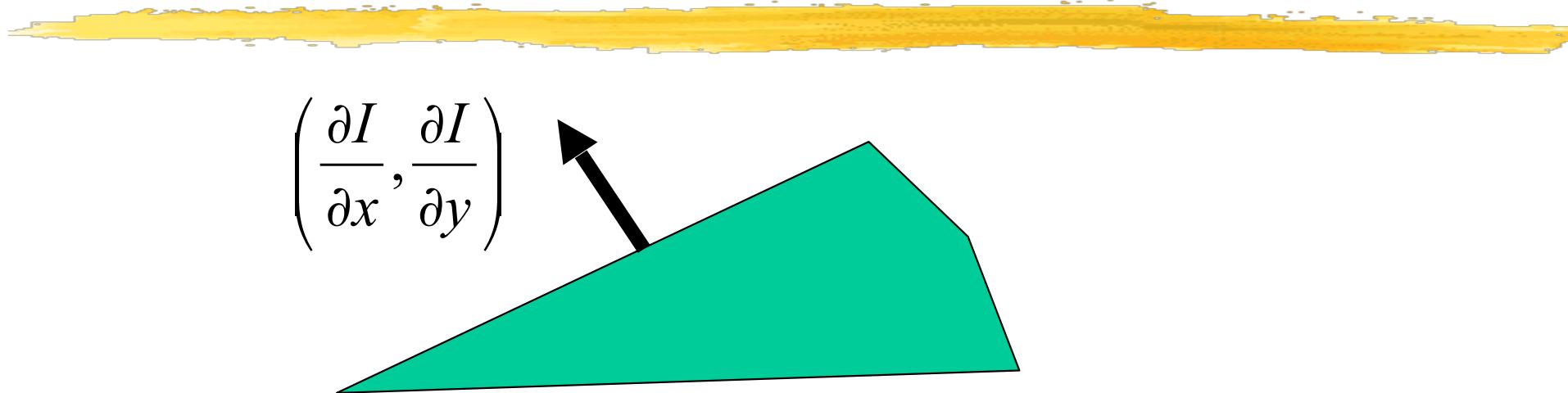


$$\nabla I = \left[0, \frac{\delta I}{\delta y} \right]$$



$$\nabla I = \left[\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y} \right]$$

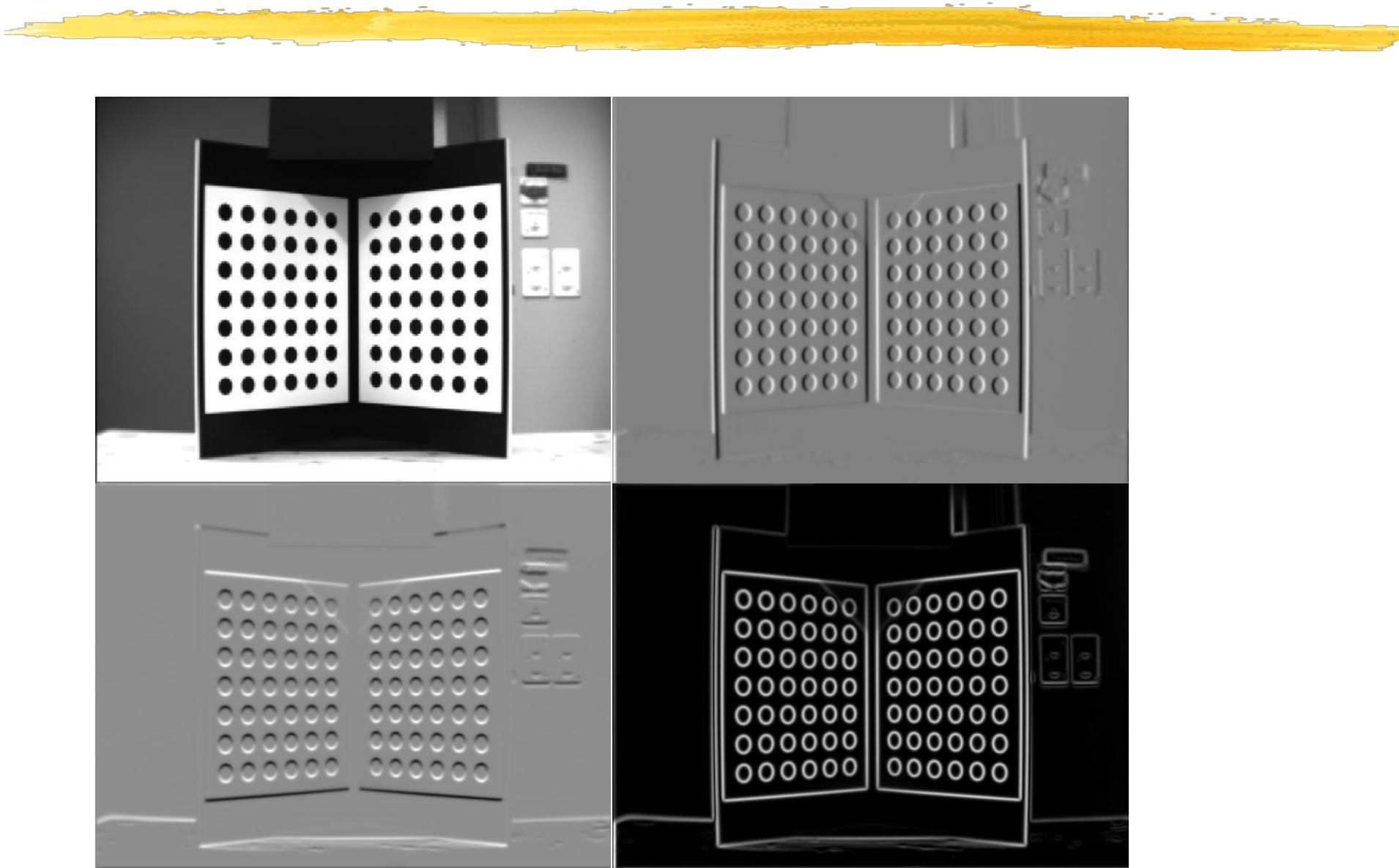
MAGNITUDE AND ORIENTATION



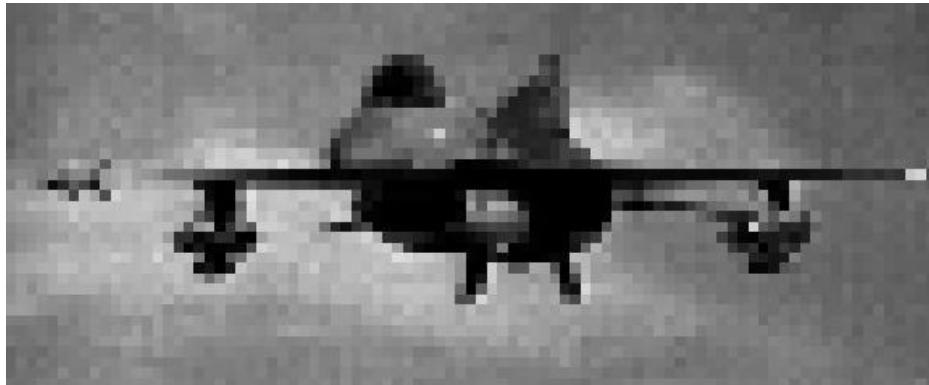
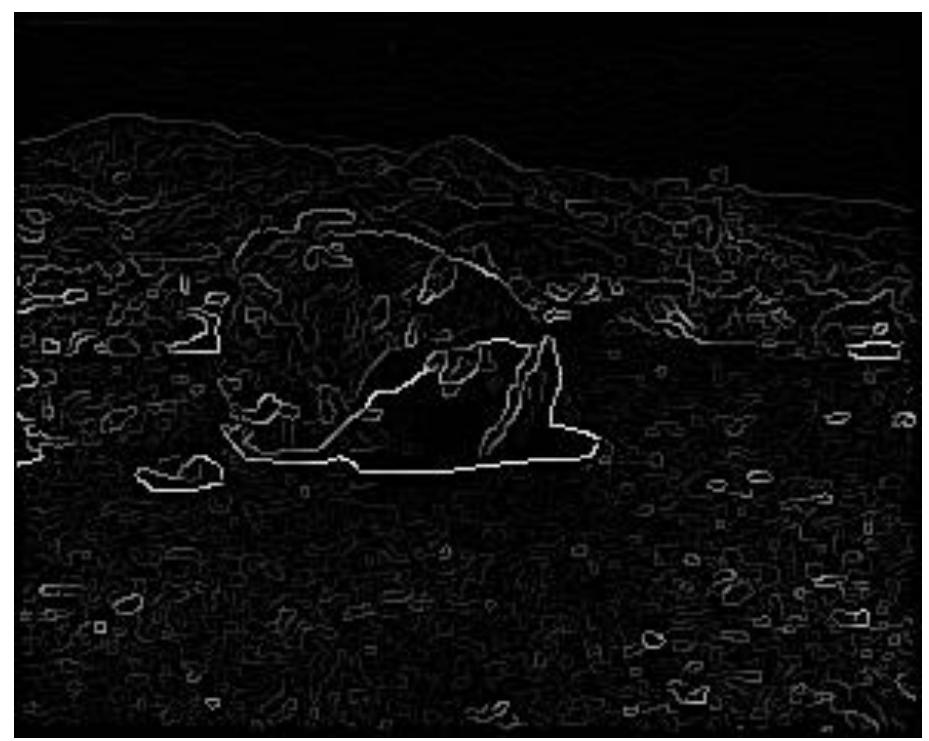
Measure of contrast : $G = \sqrt{\frac{\partial I^2}{\partial x} + \frac{\partial I^2}{\partial y}}$

Edge orientation : $\theta = \arctan\left(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x}\right)$

GRADIENT IMAGES



REAL IMAGES

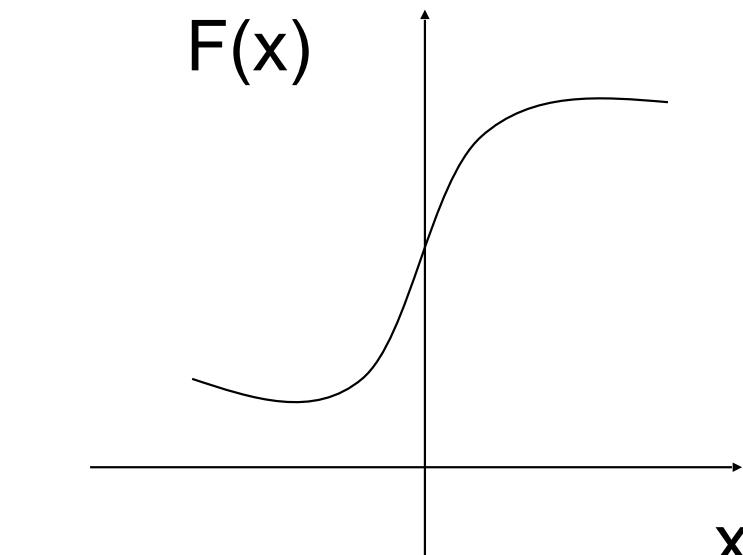


EDGE OPERATORS

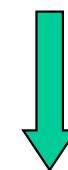
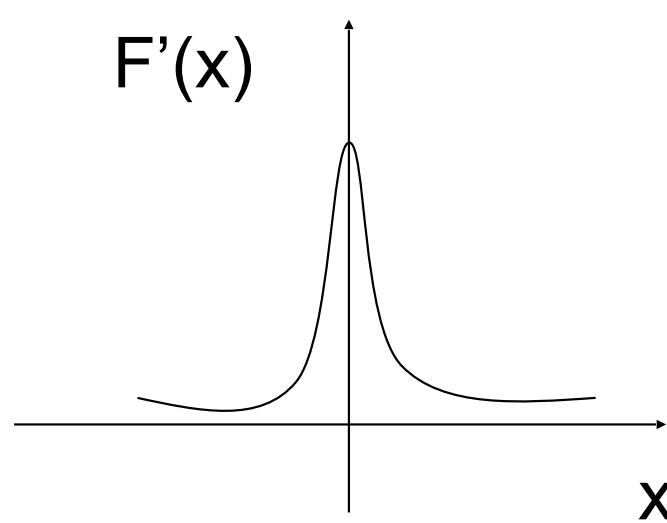


- Difference Operators
- Convolution Operators
- Parametric Matchers
- Trained Detectors

GRADIENT METHODS



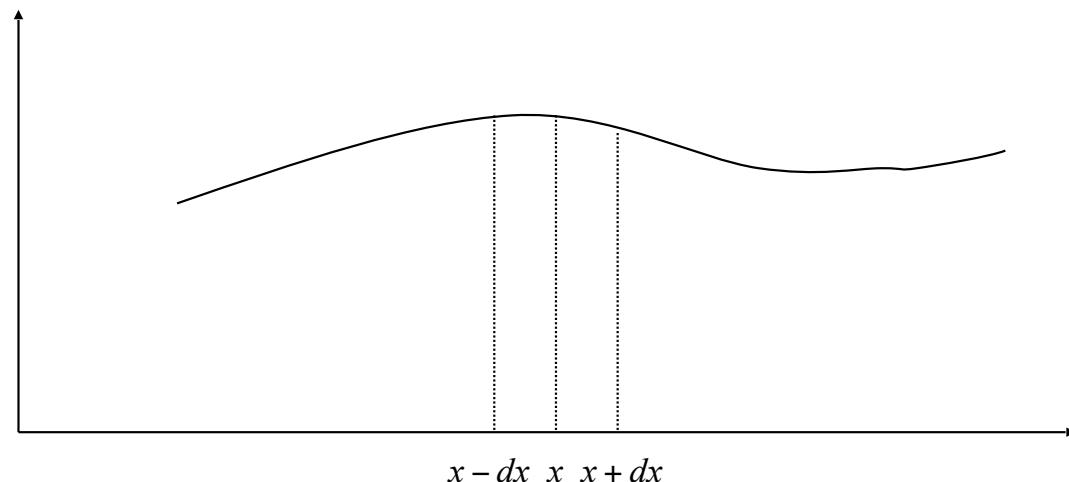
Edge = Sharp variation



Large first derivative

1D FINITE DIFFERENCES

In one dimension:



$$\frac{df}{dx} \approx \frac{f(x+dx) - f(x)}{dx} \approx \frac{f(x+dx) - f(x-dx)}{2dx}$$

$$\frac{d^2f}{dx^2} \approx \frac{f(x+dx) - 2f(x) + f(x-dx)}{dx^2}$$

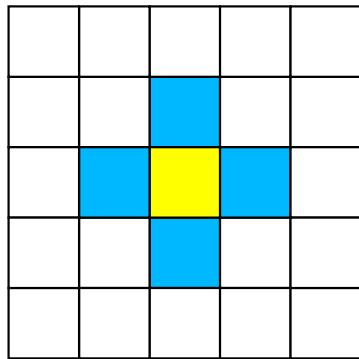
1D FINITE DIFFERENCES IN C



Line stored as an array:

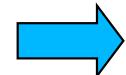
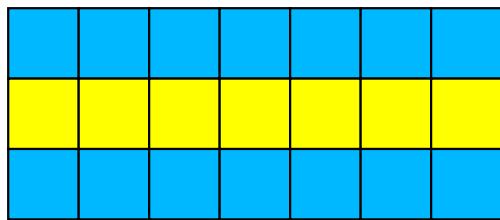
```
{  
    int i;  
    for(i=0;i<n;i++){  
        q[i]=p[i+1]-p[i];  
    }  
}
```

2D FINITE DIFFERENCES



$$\frac{\partial f}{\partial x} \approx \frac{f(x + dx, y) - f(x, y)}{dx} \approx \frac{f(x + dx, y) - f(x - dx, y)}{2dx}$$
$$\frac{\partial f}{\partial y} \approx \frac{f(x, y + dy) - f(x, y)}{dy} \approx \frac{f(x, y + dy) - f(x, y - dy)}{2dy}$$

2D FINITE DIFFERENCES IN C



p

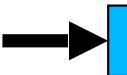


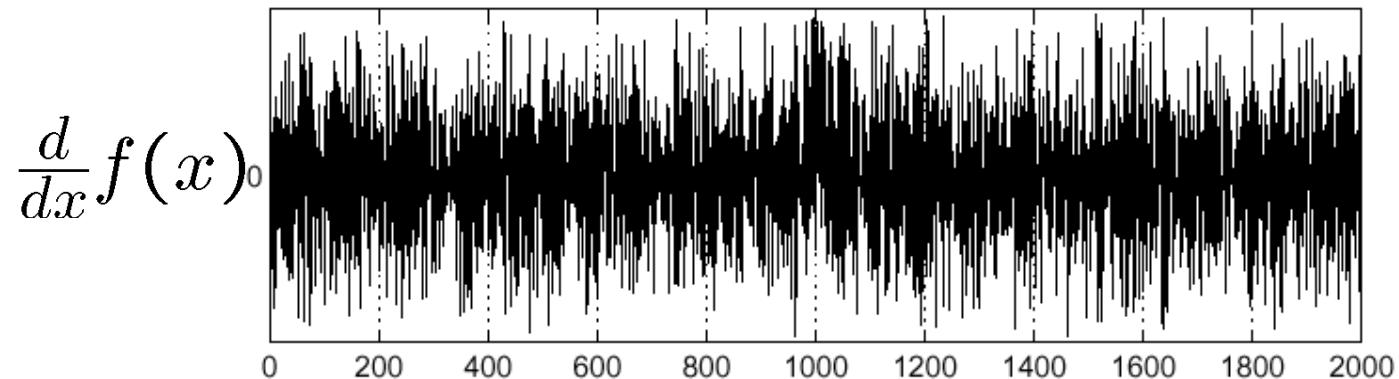
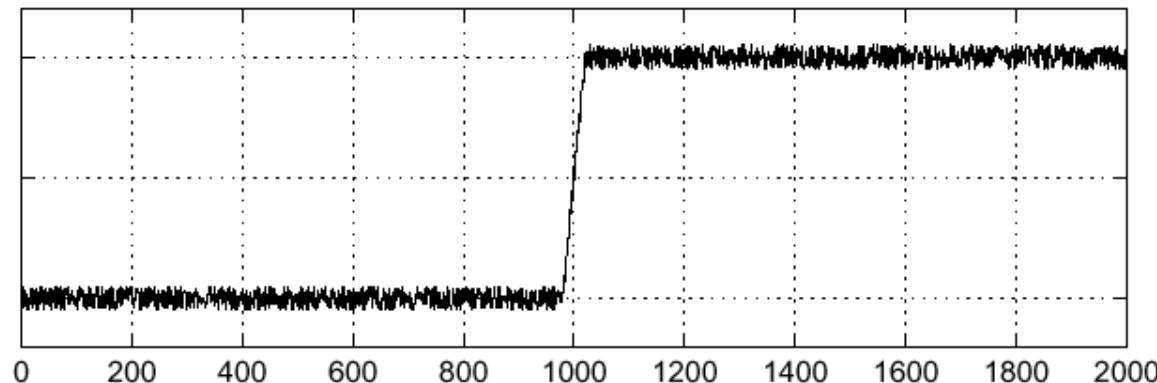
Image stored in raster format:

```
{  
    int i;  
    for(i=0;i<xdim;i++){  
        dx[i] = p[i+1]-p[i];  
        dy[i] = p[i+xdim]-p[i];  
    }  
}
```

NOISE IN 1D



Consider a single row or column of the image:



$$\frac{d}{dx} f(x)$$

FOURIER INTERPRETATION

Function	Fourier Transform
$\frac{\partial f}{\partial x}(x, y)$	$u\mathbf{F}(f)(u, v)$
$\frac{\partial f}{\partial y}(x, y)$	$v\mathbf{F}(f)(u, v)$

→ Differentiating emphasizes noise!

MOVING AVERAGE



Problem:

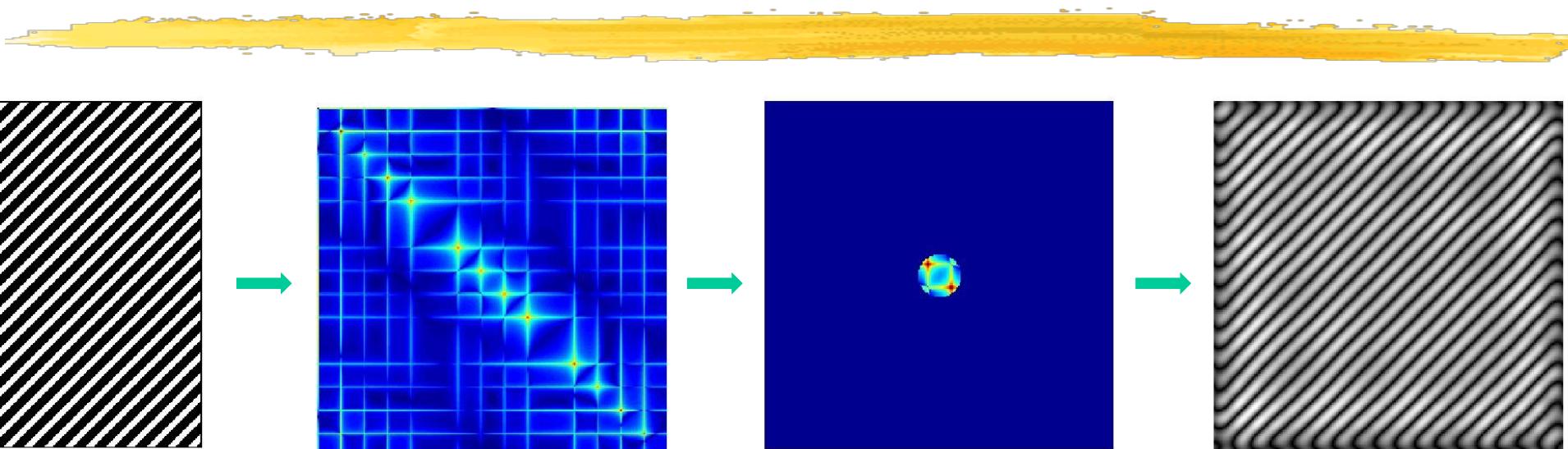
High frequencies lead to trouble with derivation.

Solution:

Suppress high frequencies by

- multiplying DFT of the signal with something that suppresses high frequencies.

DIAGONAL STRUCTURES



Rotated stripes:

- Dominant diagonal structures
- Discretization produces additional harmonics

Removing higher frequencies and reconstructing:

- Smoothed image

REMOVING NOISE



Problem:

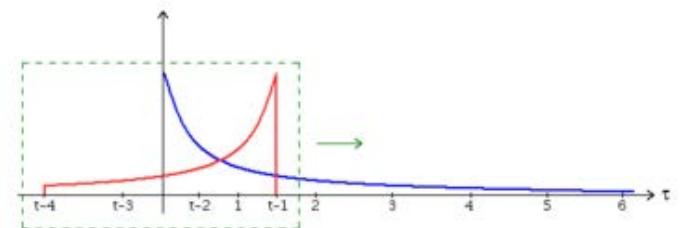
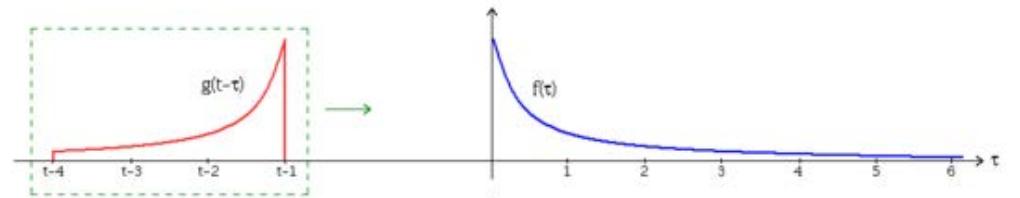
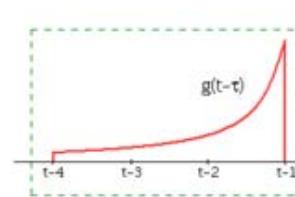
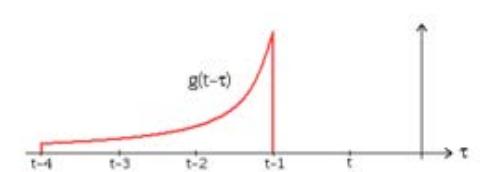
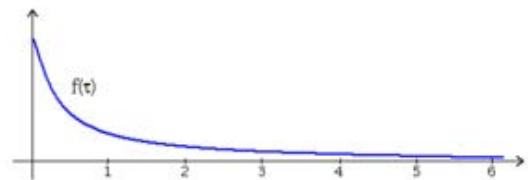
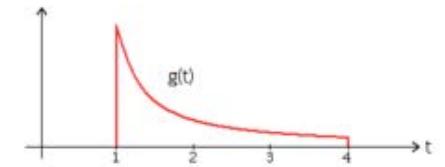
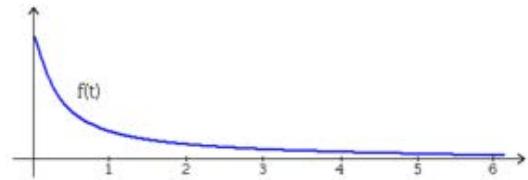
High frequencies lead to trouble with derivation.

Solution:

Suppress high frequencies by

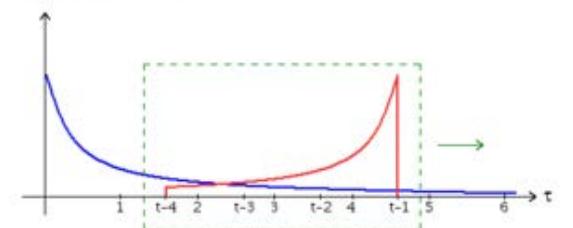
- multiplying DFT of the signal with something that suppresses high frequencies;
- convolving with a low-pass filter.

1D CONVOLUTION

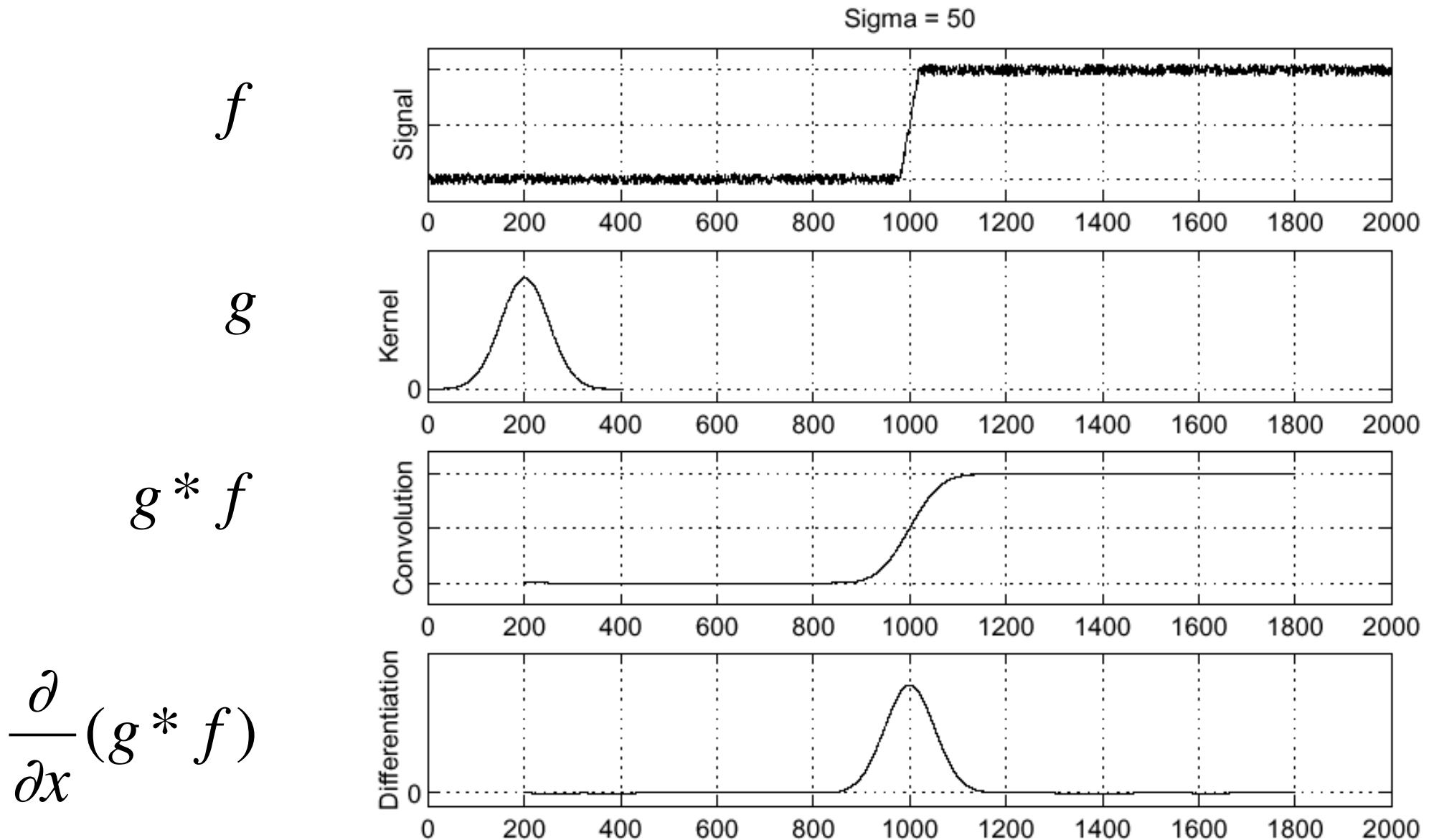


$$(f \star g)(t) = \int_{\tau} f(\tau)g(t - \tau)d\tau$$

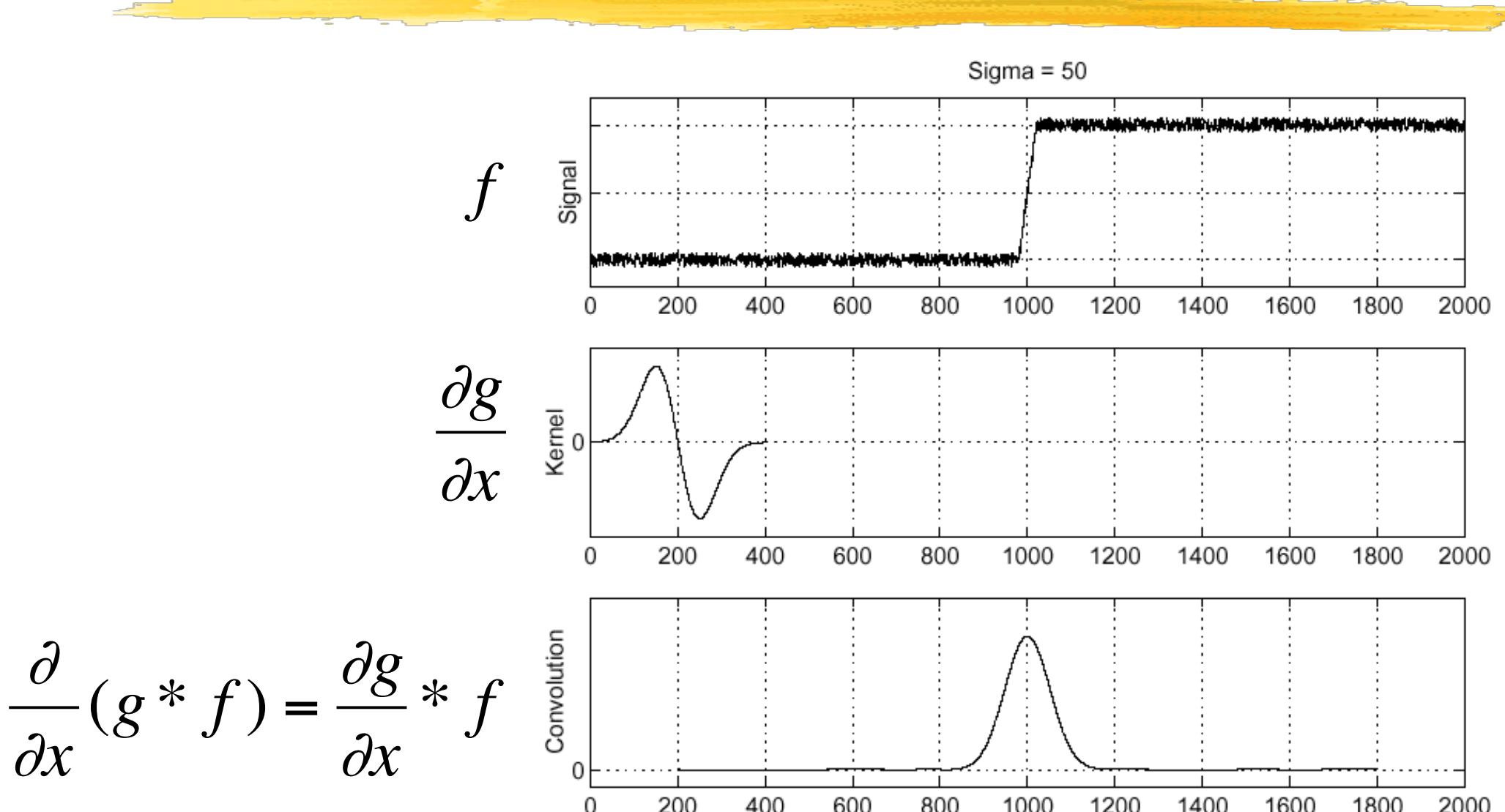
$$(f \star g)(m) = \sum_n f(n)g(m - n)$$



SOLUTION: SMOOTH FIRST



DERIVATIVE THEOREM OF CONVOLUTION



--> Faster because dg/dx can be precomputed.

1D AND 2D CONVOLUTION



Continuous case:

$$m \bullet f(x) = \int_u m(u) f(x-u) du$$

$$m \bullet f(x, y) = \iint_{u v} m(u, v) f(x-u, y-v) dudv$$

Discrete case:

$$m \bullet f(x) = \sum_{i=-w}^w m(i) f(x-i)$$

$$m \bullet f(x, y) = \sum_{i=-w}^w \sum_{j=-h}^h m(i, j) f(x-i, y-j)$$

CONVOLUTION IN C

Naive implementation:

```
static double g[][]={{{-1.0,-2.0,-1.0},{0.0,0.0,0.0},{1.0,2.0,1.0}};  
{  
    for(i=i0;i<N;i++)  
        for(j=j0;j<N;j++){  
            q[i][j]=0;  
            for(a=a0;a<W;a++)  
                for(b=b0;b<W,b++)  
                    q[i][j]+=g[a][b]*p[i-a][j-b];  
    }  
}
```

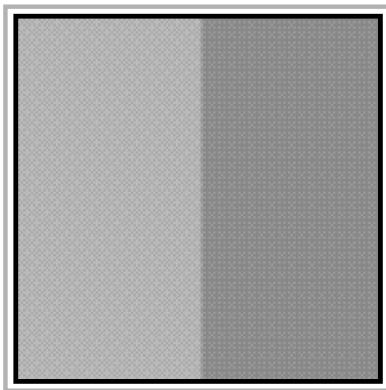
Computational complexity:

- N^2W^2 multiplications for a $N \times N$ image and a $W \times W$ mask.
 - Lots of memory access
- Slow, but can be sped up when the filters are separable.

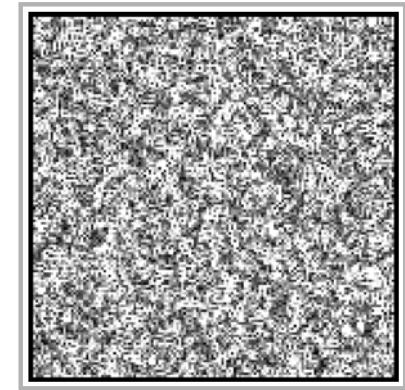
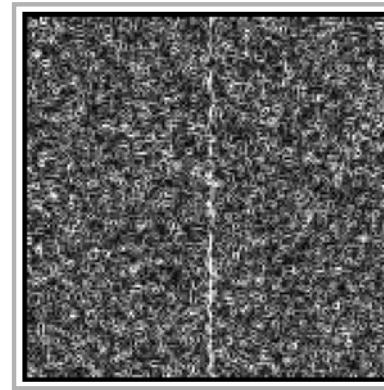
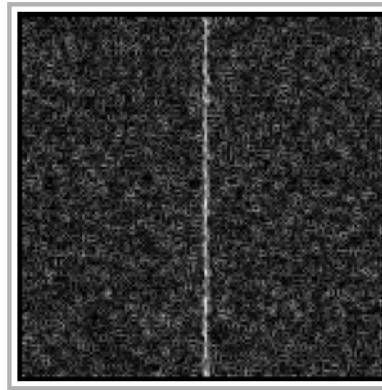
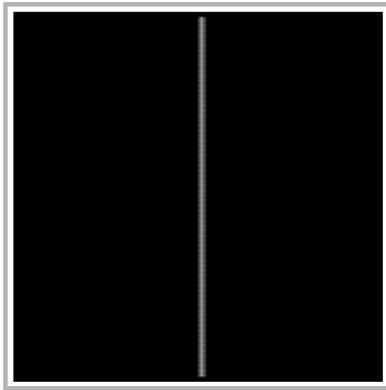
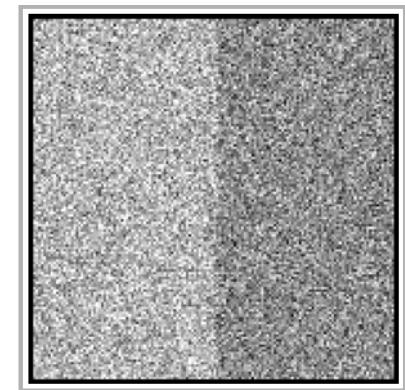
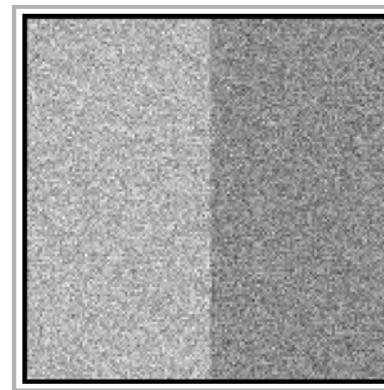
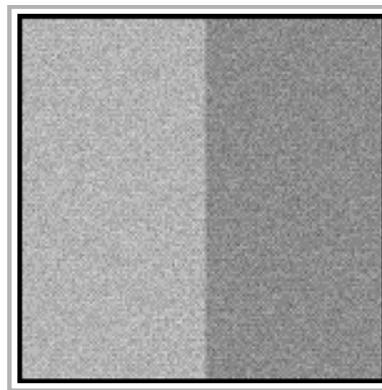
NOISE IN 2D



Ideal step edge



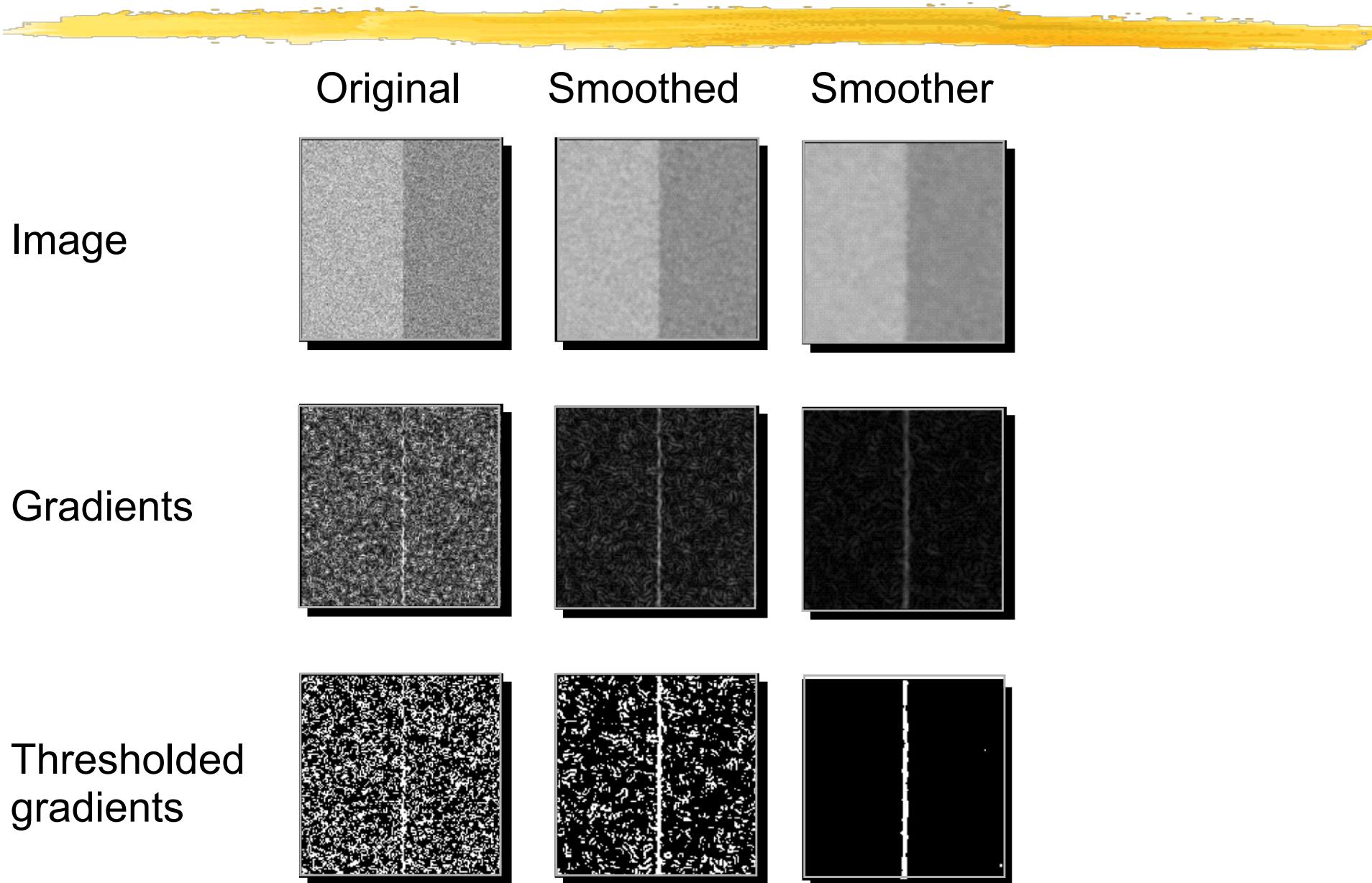
Step edge + noise



Increasing noise



BENEFITS OF SMOOTHING



DIFFERENTIATION AS CONVOLUTION

$$[-1,1] \rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+dx, y) - f(x, y)}{dx}$$

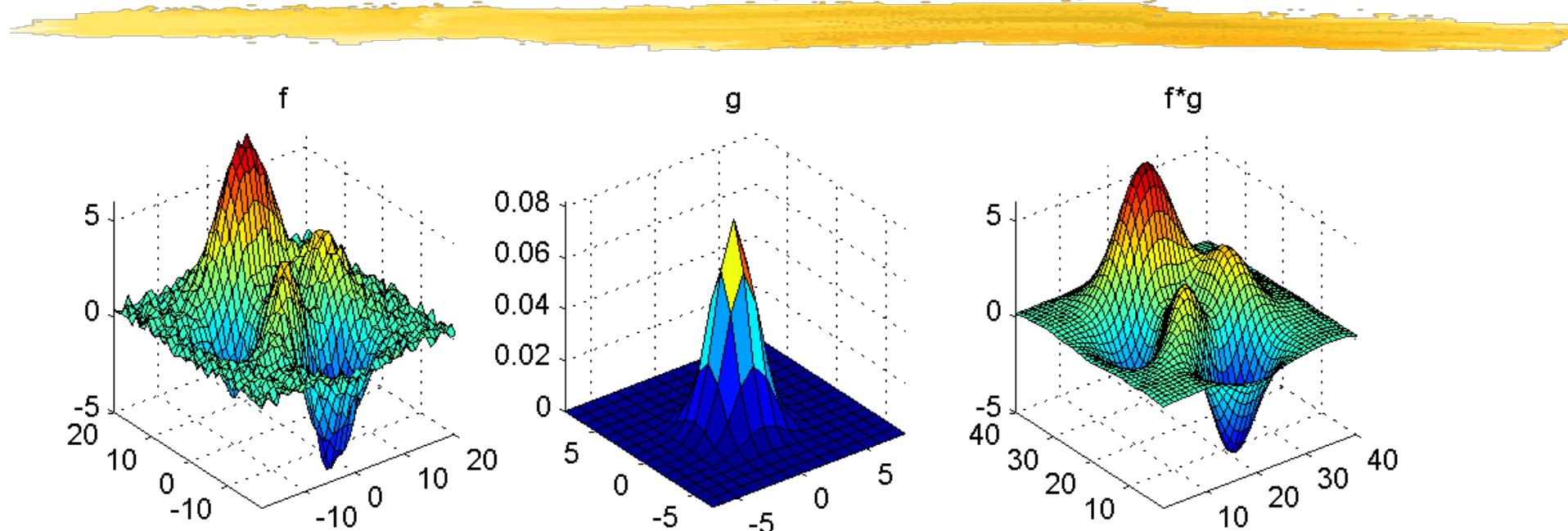
$$[-0.5, 0, 0.5] \rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+dx, y) - f(x-dx, y)}{2dx}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow \frac{\partial f}{\partial y} \approx \frac{f(x, y+dy) - f(x, y)}{dy}$$

$$\begin{bmatrix} -0.5 \\ 0 \\ 0.5 \end{bmatrix} \rightarrow \frac{\partial f}{\partial y} \approx \frac{f(x, y+dy) - f(x, y-dy)}{2dy}$$

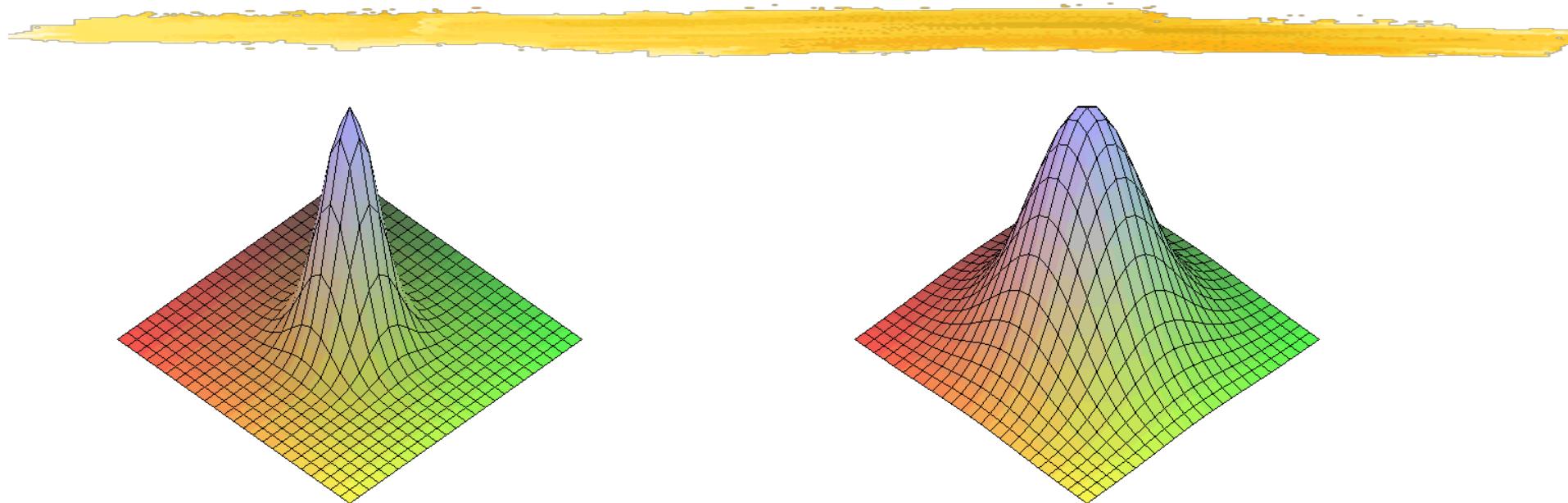
→ Use wider masks to add an element of smoothing

GAUSSIAN SMOOTHING



- Eliminates high frequency noise.
- Is fast because the kernel is
 - small,
 - separable.

GAUSSIAN MASKS

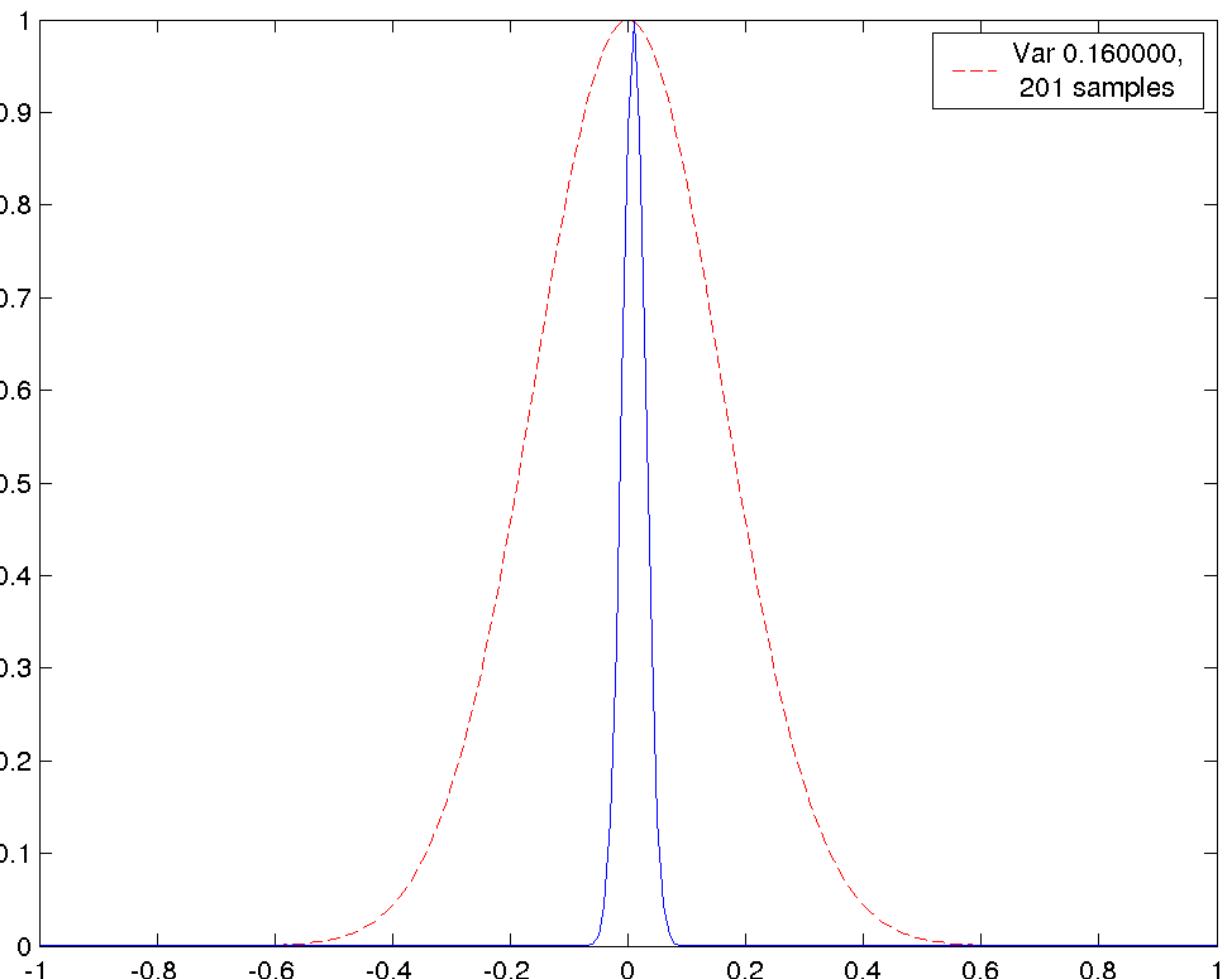


$$\sigma = 1$$

$$\sigma = 2$$

$$g_2(x, y) = \frac{1}{2\pi\sigma^2} \exp(-(x^2 + y^2)/2\sigma^2)$$

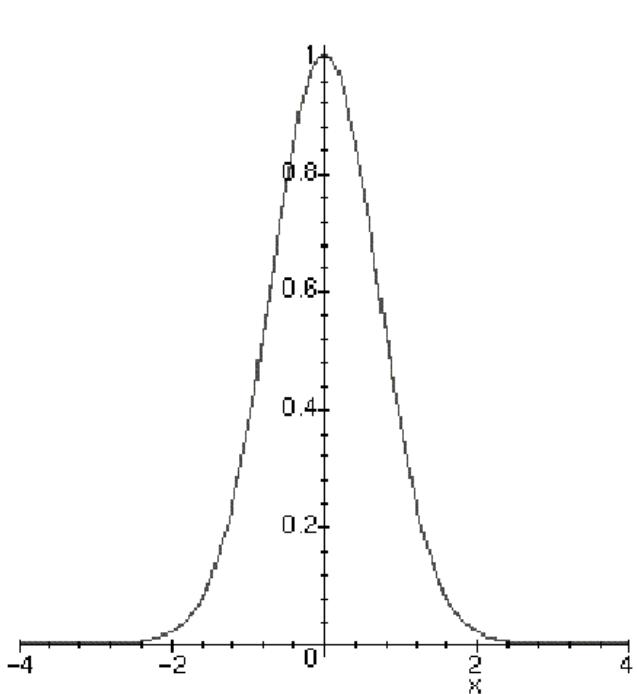
FOURIER TRANSFORM



- The DFT of a Gaussian is a Gaussian.
- It has finite support.
- Its width is inversely proportional to that of the original Gaussian.

→ Convolving with a Gaussian suppresses the high frequencies.

SEPARABILITY

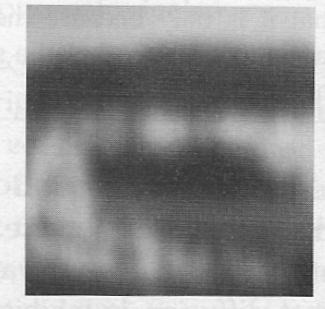
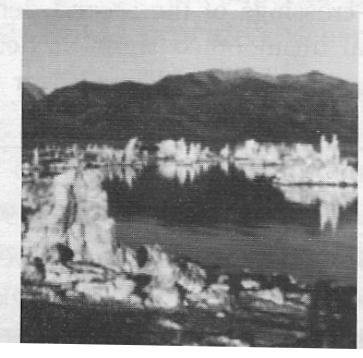
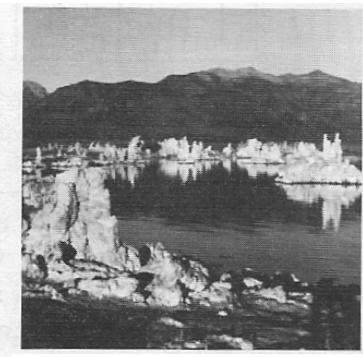
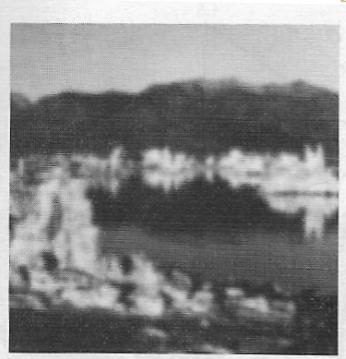
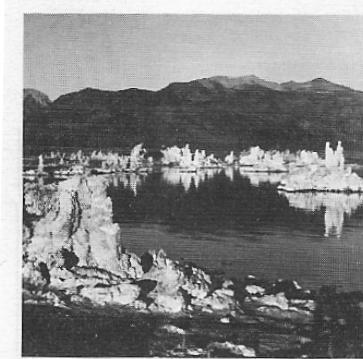


$$g_1(x) = \frac{1}{\sqrt{\pi}\sigma} \exp(-x^2/\sigma^2)$$

$$g_2(x, y) = g_1(x)g_1(y)$$

$$\begin{aligned} \iint_{u,v} g_2(u, v) f(x-u, y-v) du dv &= \int_u g_1(u) \left(\int_v g_1(v) f(x-u, y-v) dv \right) du \\ &= \int_v g_1(v) \left(\int_u g_1(u) f(x-u, y-v) du \right) dv \end{aligned}$$

SMOOTHED IMAGES



GAUSSIAN DERIVATIVES

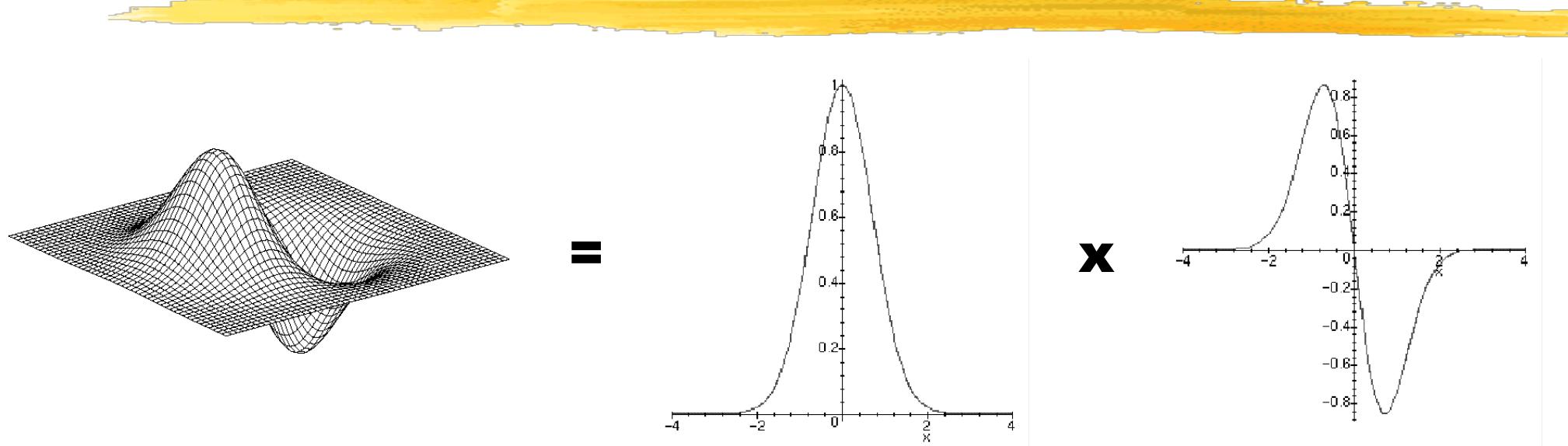


Image derivatives computed by convolving with the derivative of a Gaussian:

$$\frac{\partial}{\partial x} \iint g_2(u, v) f(x-u, y-v) du dv = \int_u g'_1(u) \left(\int_v g_1(v) f(x-u, y-v) dv \right) du$$

$$\frac{\partial}{\partial y} \iint g_2(u, v) f(x-u, y-v) du dv = \int_v g'_1(v) \left(\int_u g_1(u) f(x-u, y-v) du \right) dv$$

GAUSSIAN MASKS



Sigma=1:

g : 0.000070 0.010332 0.207532 0.564131 0.207532 0.010332 0.000070

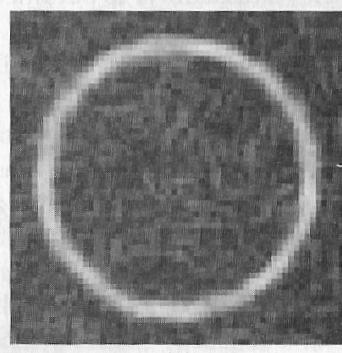
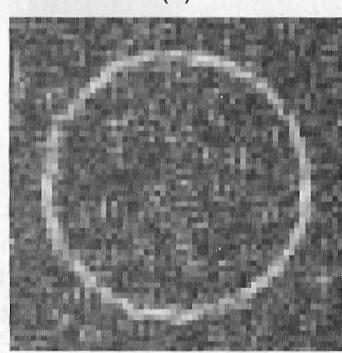
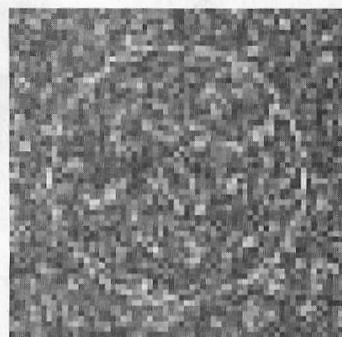
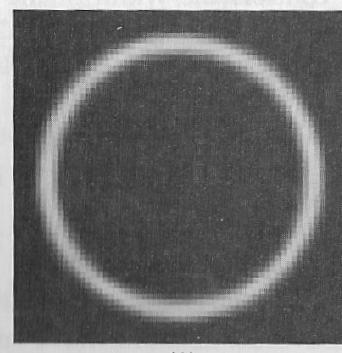
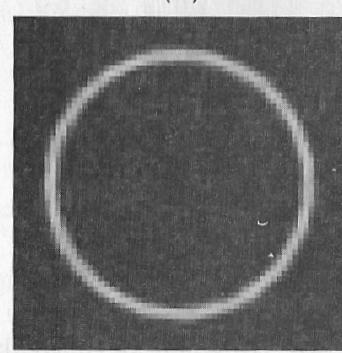
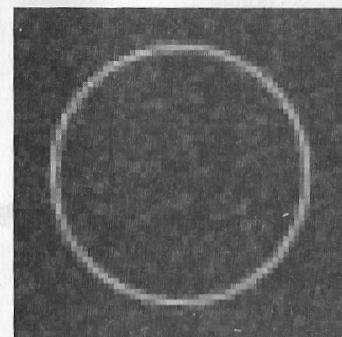
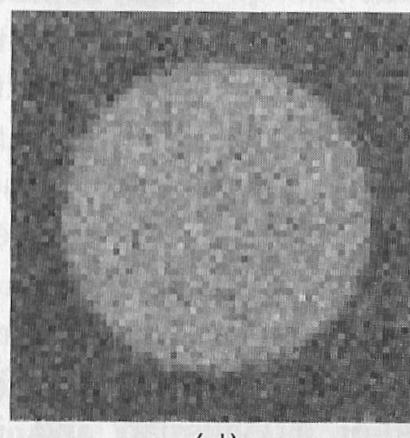
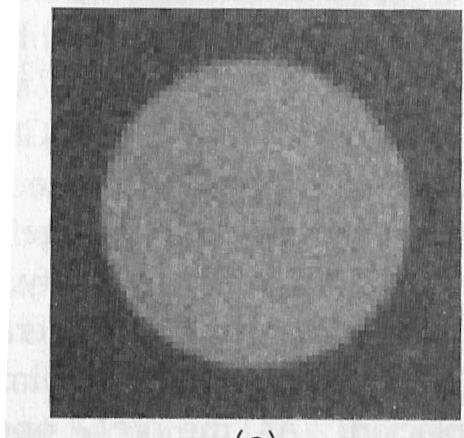
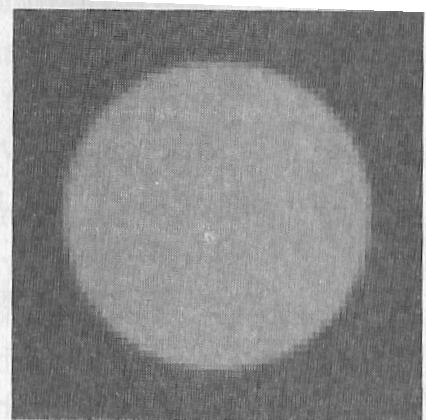
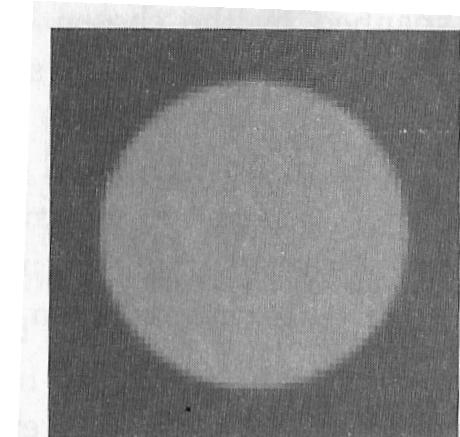
g' : 0.000418 0.041330 0.415065 0.000000 -0.415065 -0.041330 -0.000418

Sigma=2:

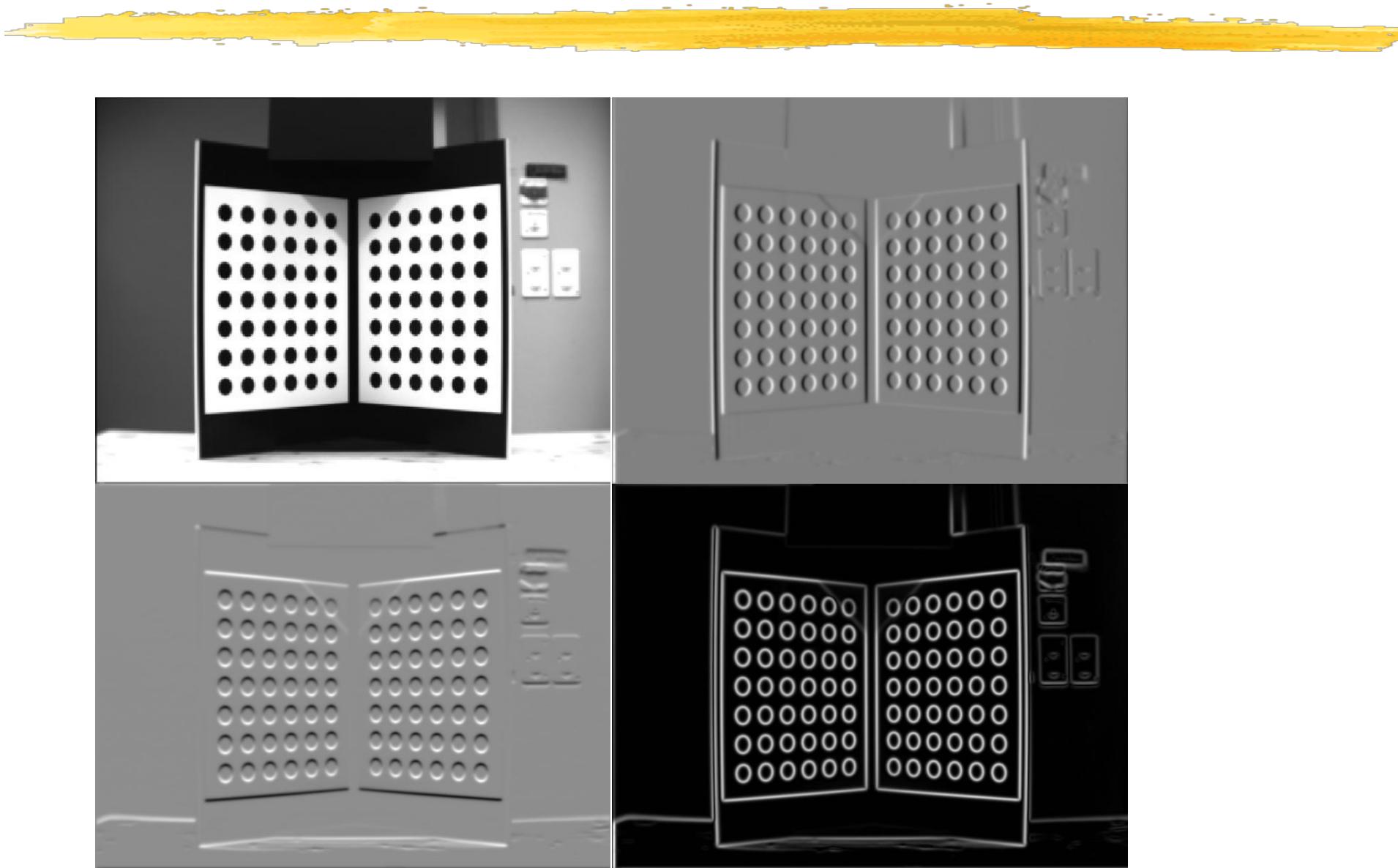
g : 0.005167 0.029735 0.103784 0.219712 0.282115 0.219712 0.103784 0.029735 0.005167

g' : 0.010334 0.044602 0.103784 0.109856 0.000000 -0.109856 -0.103784 -0.044602 -0.010334

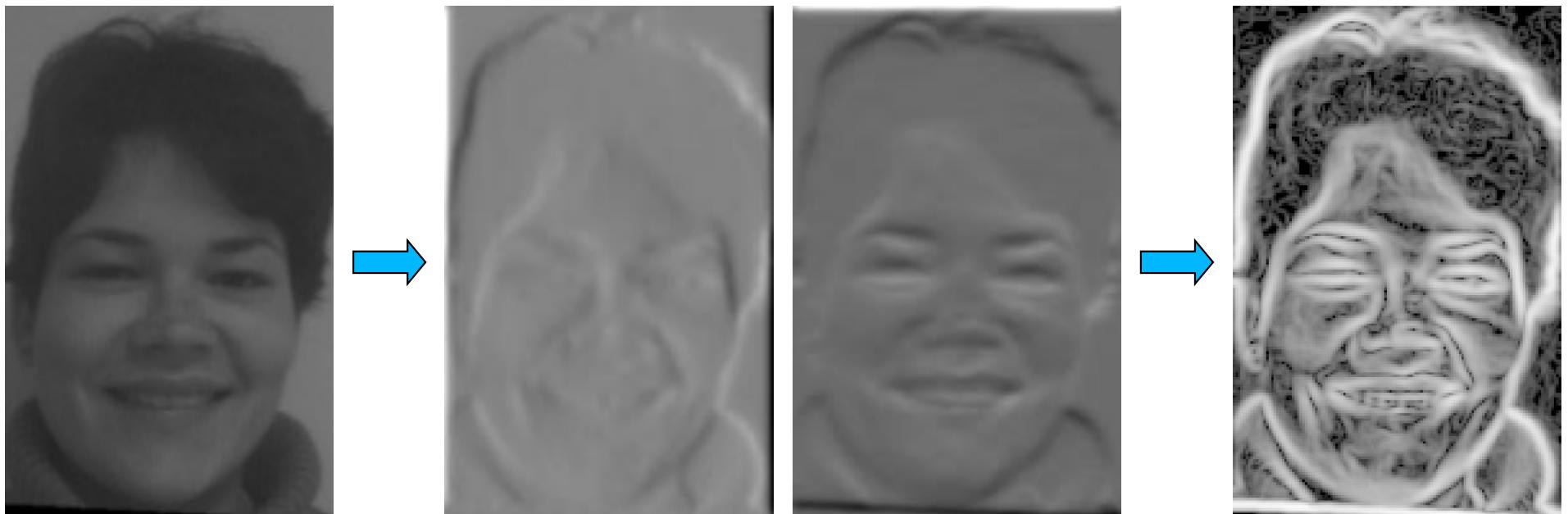
MASK SIZE



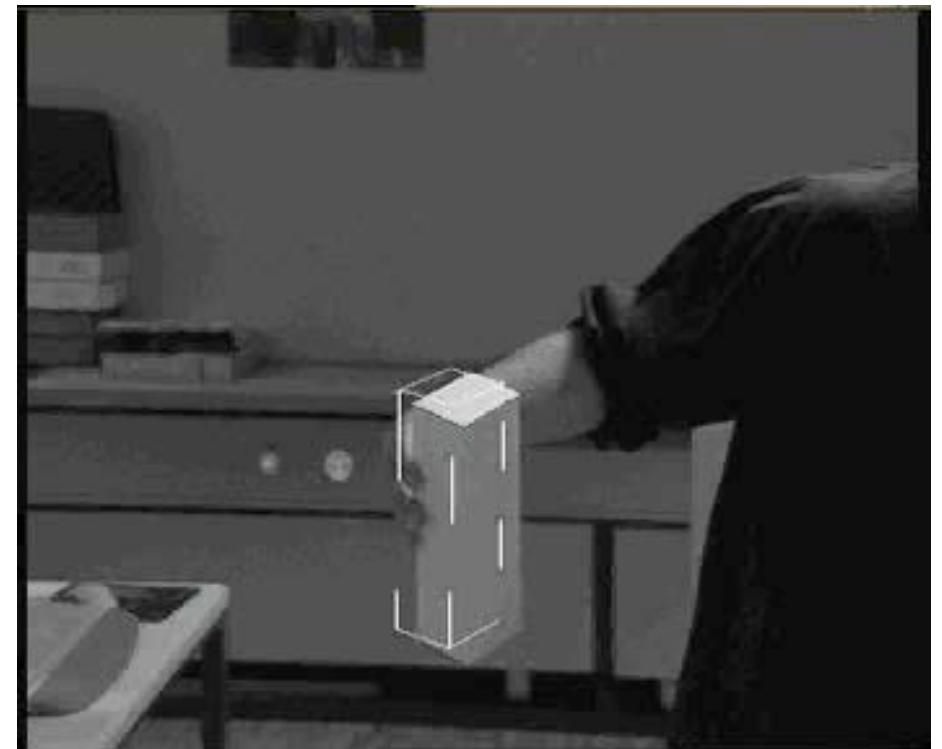
DERIVATIVE IMAGES



DERIVATIVE IMAGES

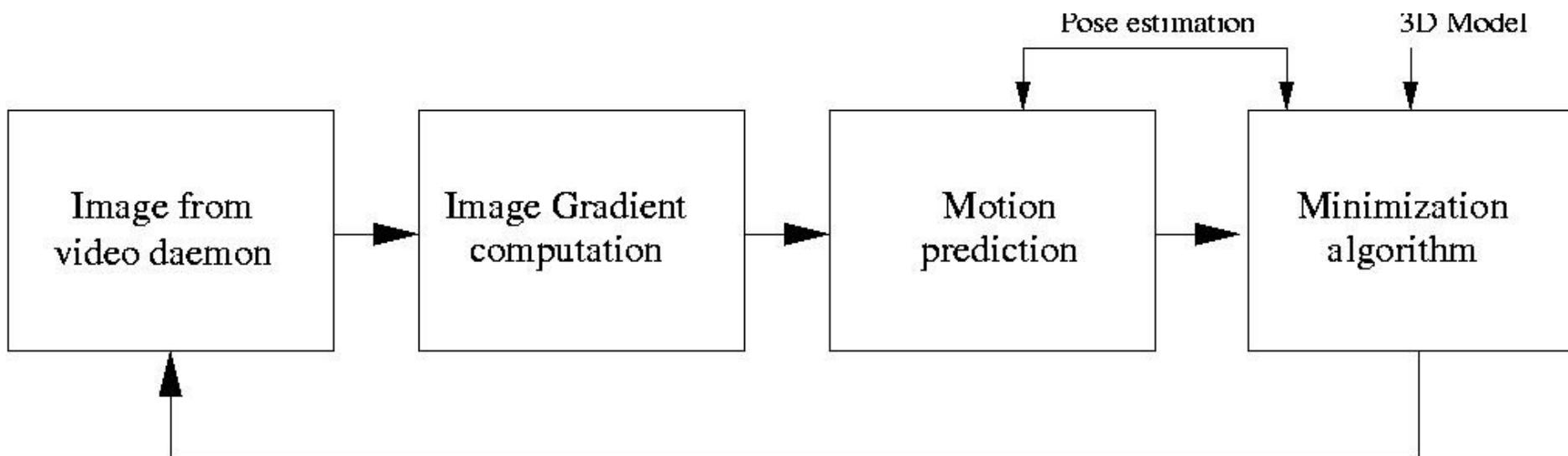
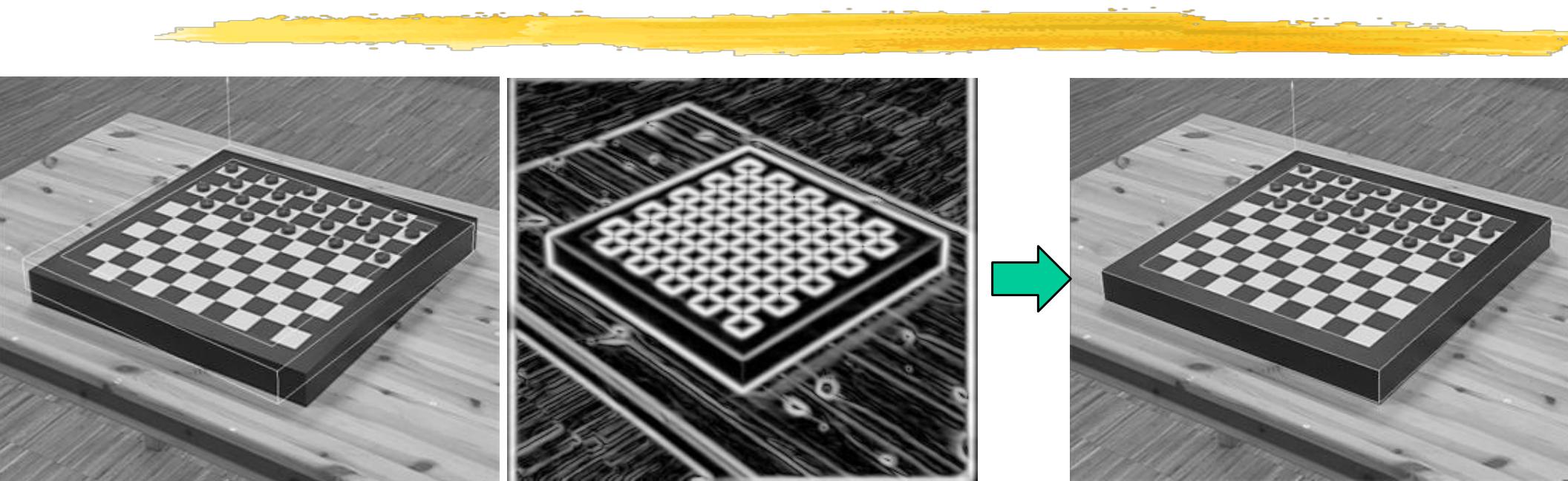


GRADIENT-BASED TRACKING



Maximize edge-strength along projection of the 3-D wireframe.

GRADIENT MAXIMIZATION

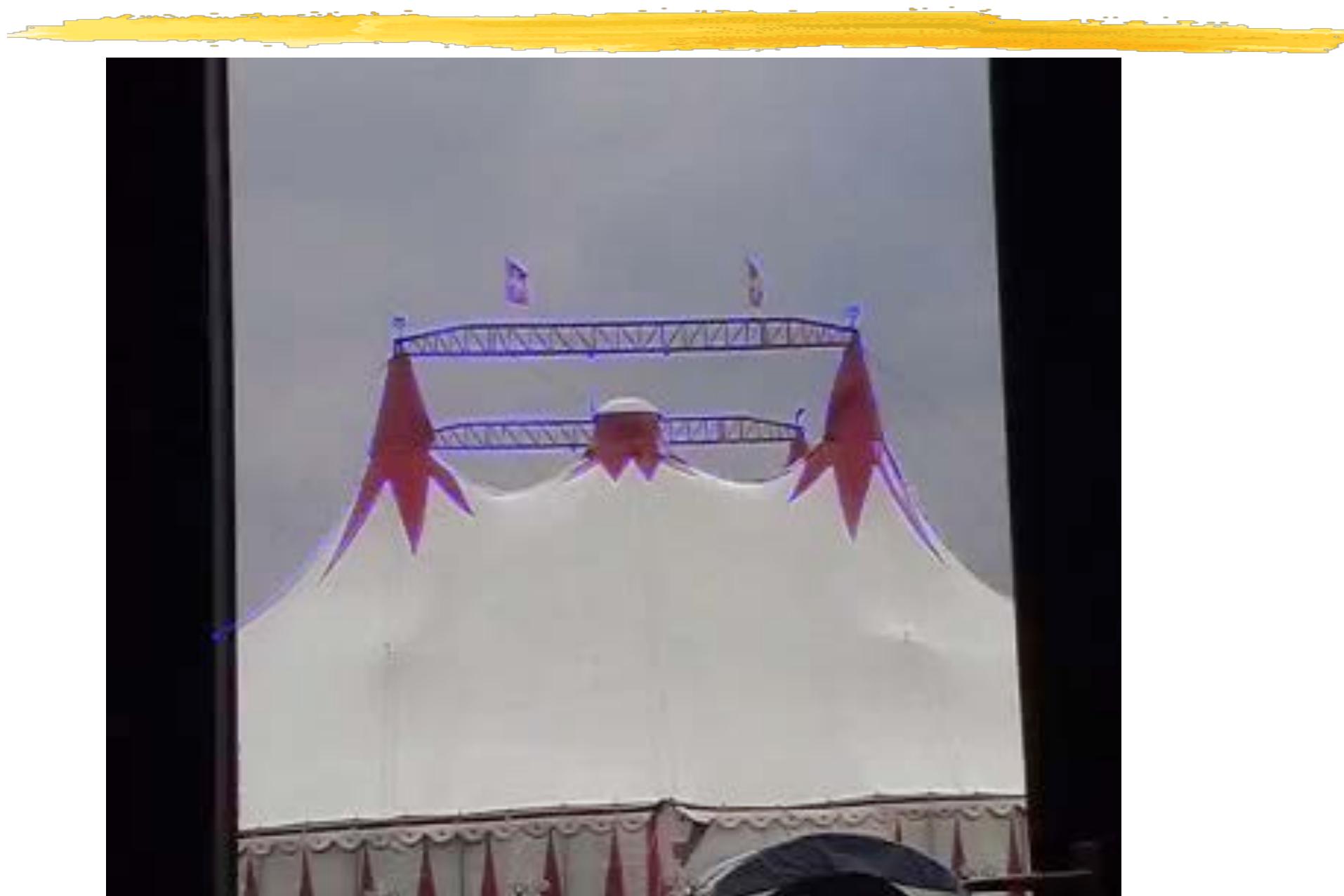


WING DEFORMATION

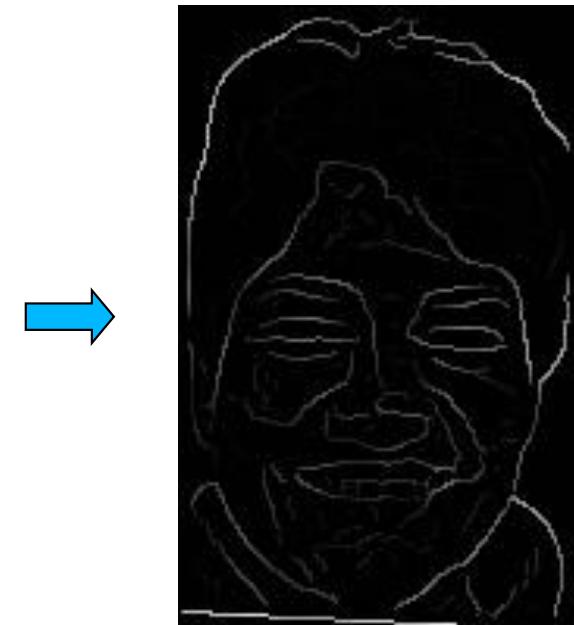
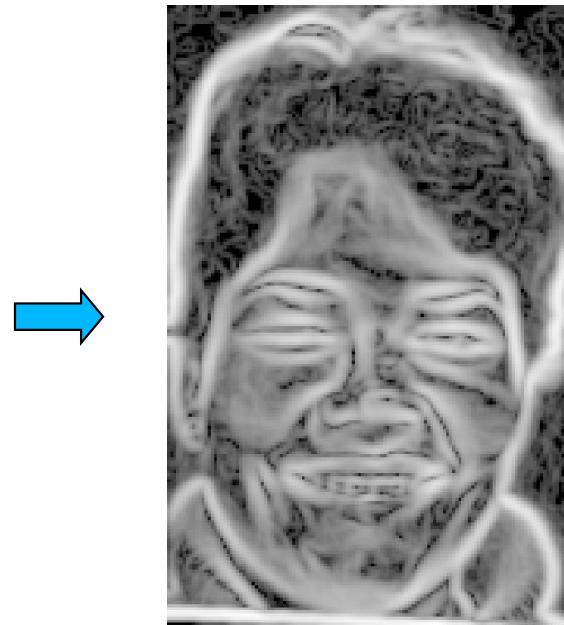
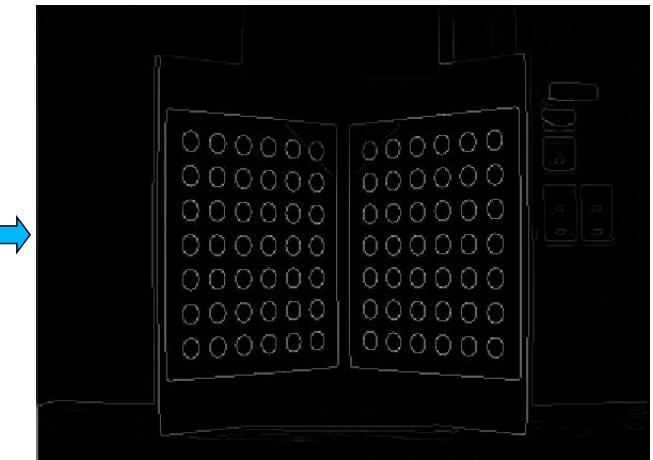
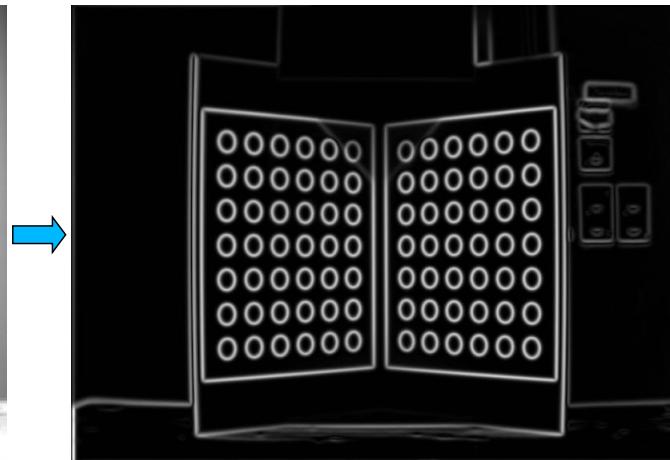
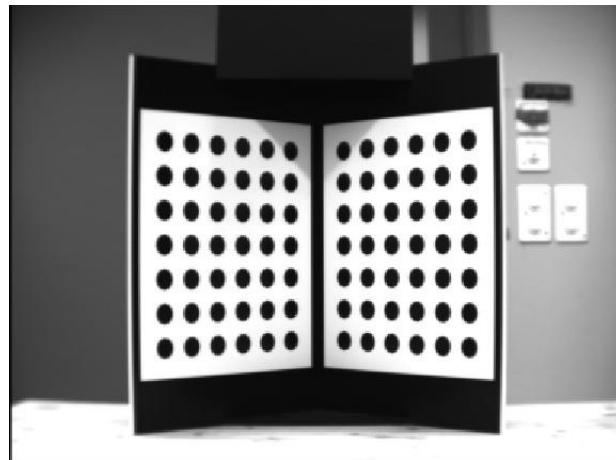


- Measure true behavior in flight.
- Validate computational models.

REAL-TIME TRACKING



CANNY EDGE DETECTOR



CANNY EDGE DETECTOR



Convolution

- Gradient strength
- Gradient direction

Thresholding

Non Maxima Suppression

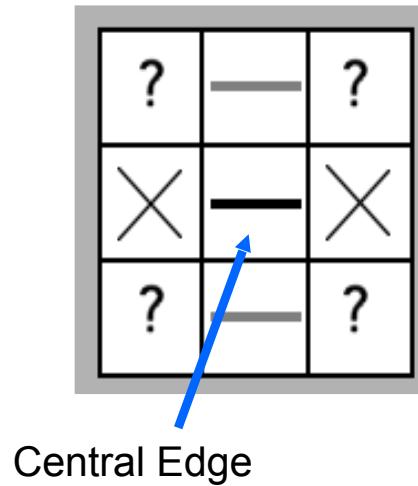
Hysteresis Thresholding

NON-MAXIMA SUPPRESSION



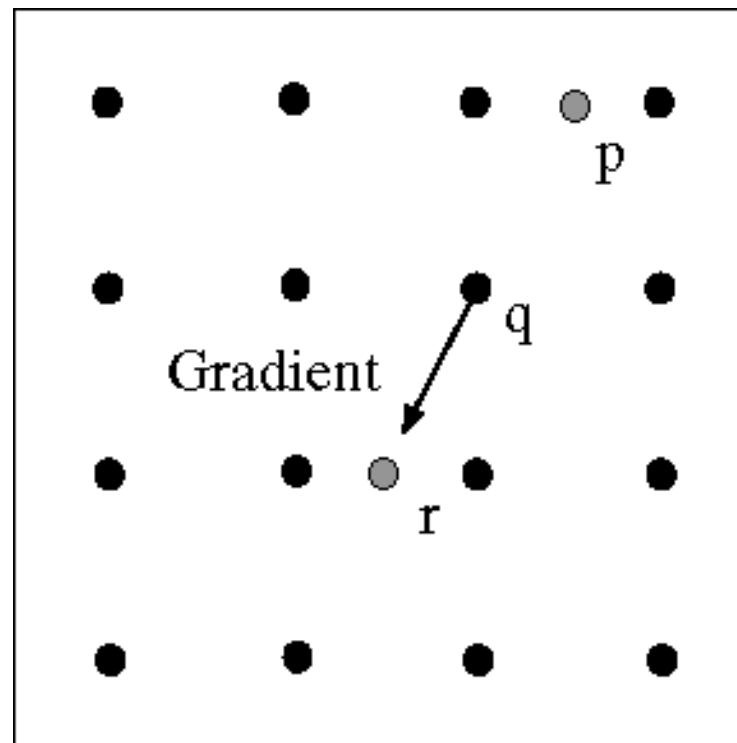
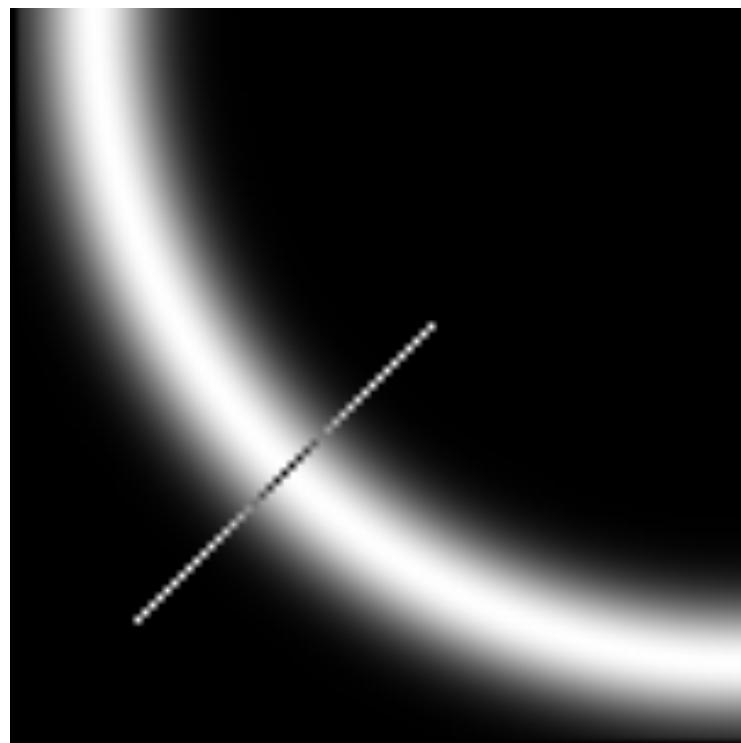
In parallel, at each pixel in edge image, use window to select as a function of edge orientation:

Window W



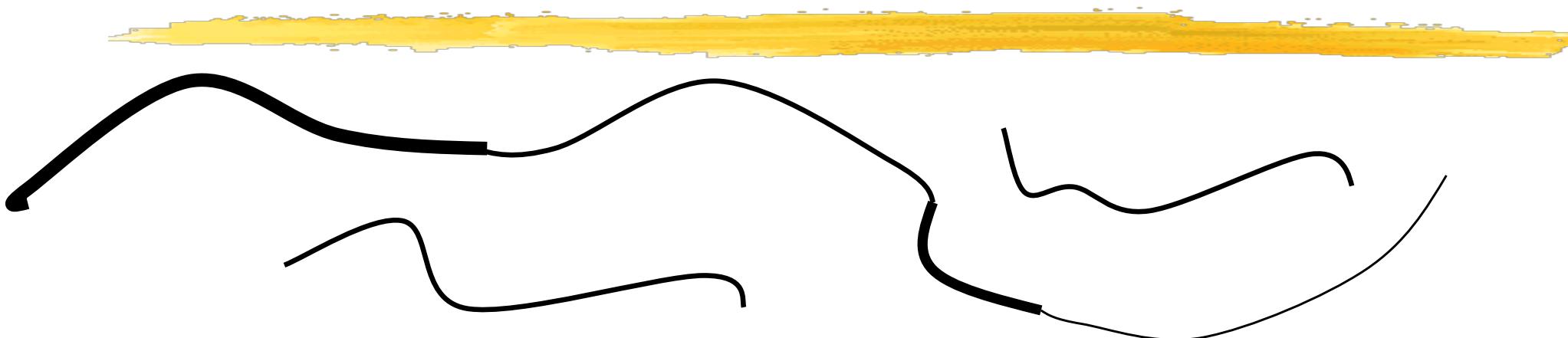
- Definitely consider these edges
- X Do not consider these edges
- ? Maybe consider them, depending on algorithm

NON-MAXIMA SUPPRESSION



Check if pixel is local maximum along gradient direction,
which requires checking interpolated pixels p and r .

HYSTERESIS THRESHOLDING



Algorithm takes two thresholds: high & low

- A pixel with edge strength above high threshold is an edge.
- Any pixel with edge strength below low threshold is not.
- Any pixel above the low threshold and next to an edge is an edge.

Iteratively label edges

- Edges grow out from 'strong edges'
- Iterate until no change in image.

CANNY RESULTS



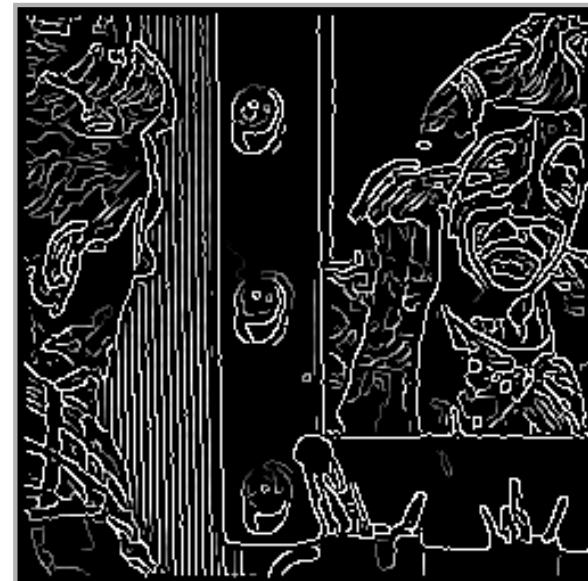
$\sigma=1$, $T_2=255$, $T_1=1$

'Y' or 'T' junction
problem with
Canny operator

CANNY RESULTS



$\sigma=1$, $T_2=255$, $T_1=220$



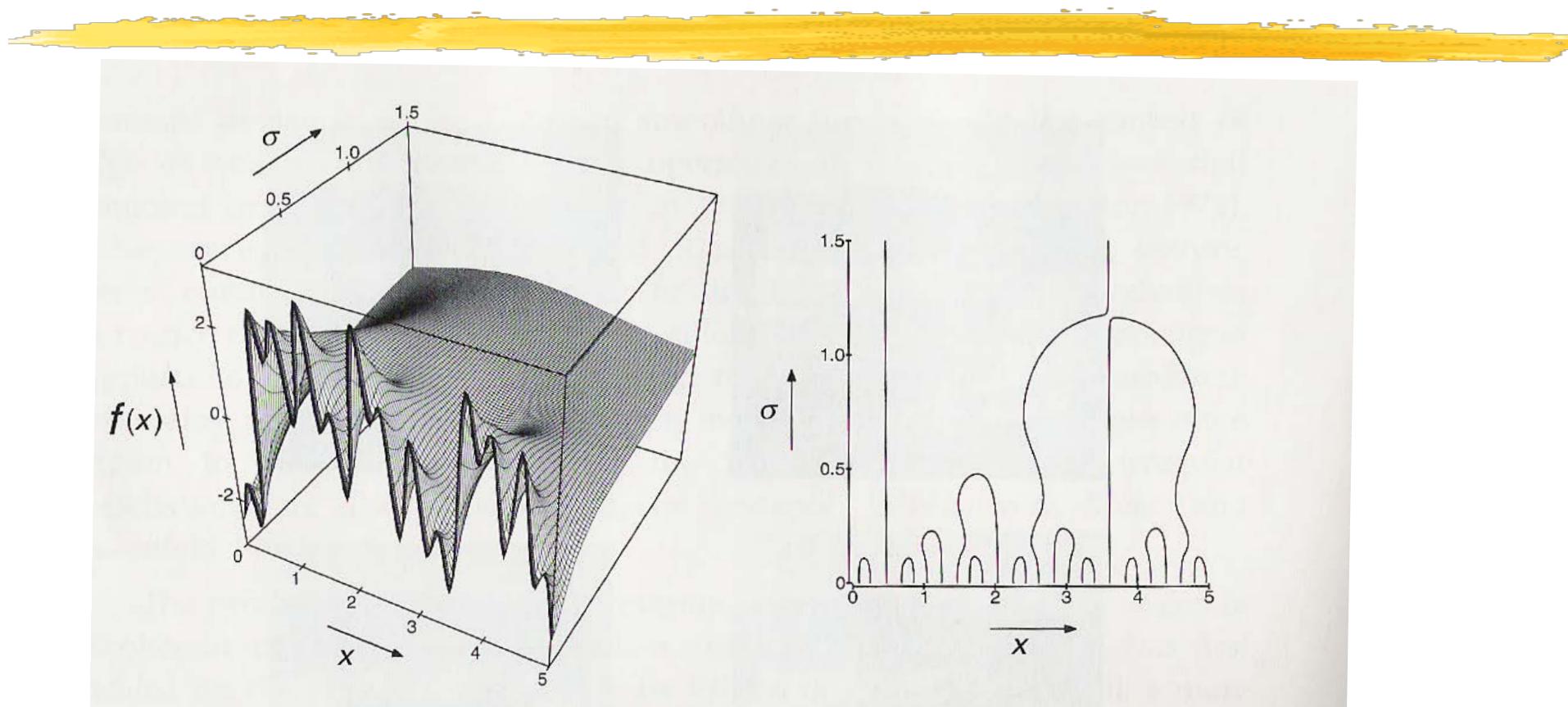
$\sigma=1$, $T_2=128$, $T_1=1$



$\sigma=2$, $T_2=128$, $T_1=1$

M. Heath, S. Sarkar, T. Sanocki, and K.W. Bowyer, "A Robust Visual Method for Assessing the Relative Performance of Edge-Detection Algorithms" IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 19, No. 12, December 1997, pp. 1338-1359.

SCALE SPACE



Increasing scale (σ) removes details but never adds new ones:

- Edge position may shift.
- Two edges may merge.
- An edge may **not** split into two.

MULTIPLE SCALES



$$\sigma = 1$$



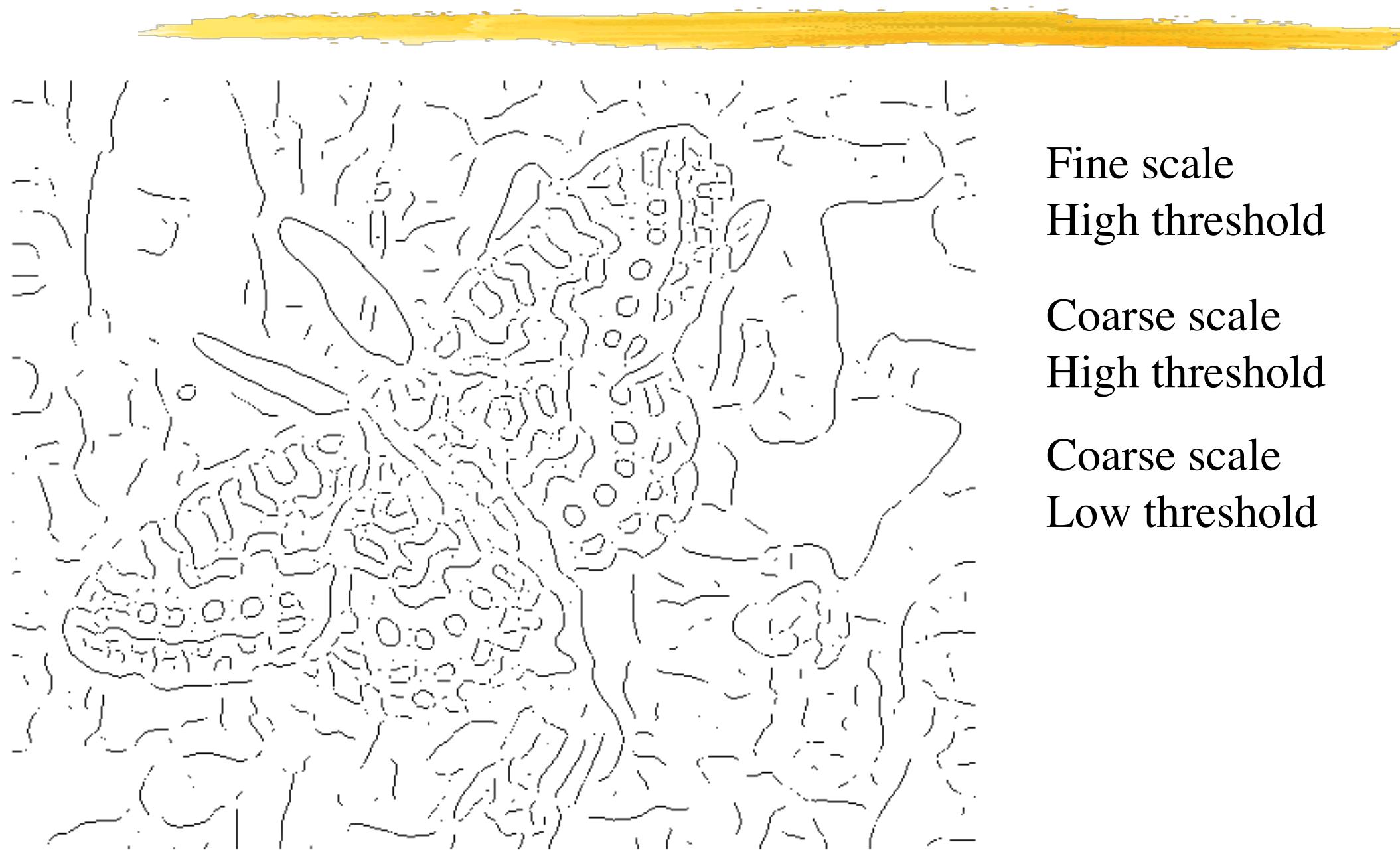
$$\sigma = 2$$



$$\sigma = 4$$

→ Choosing the right scale is a difficult semantic problem.

SCALE vs THRESHOLD

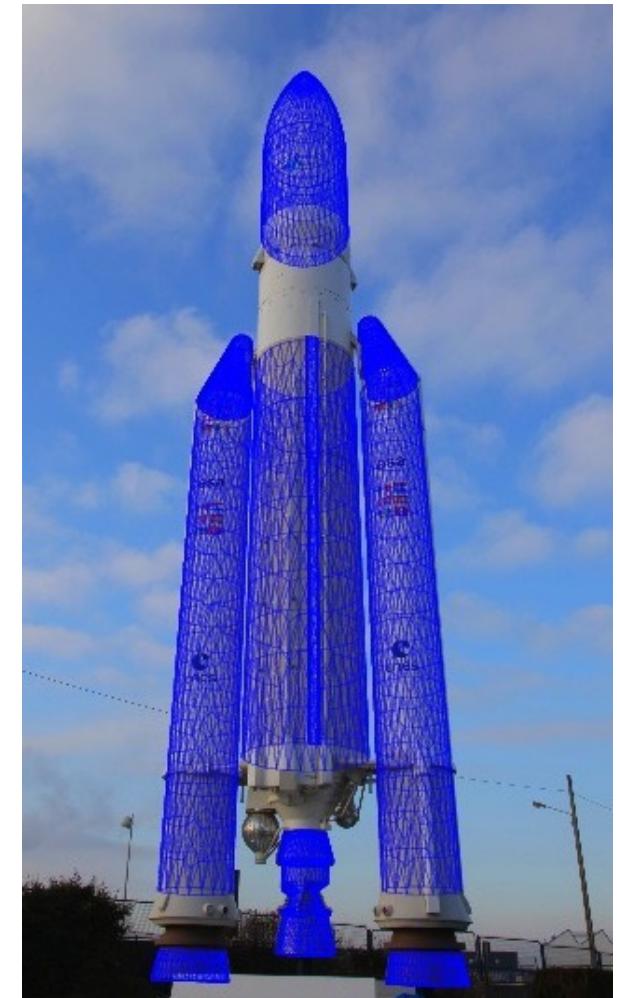
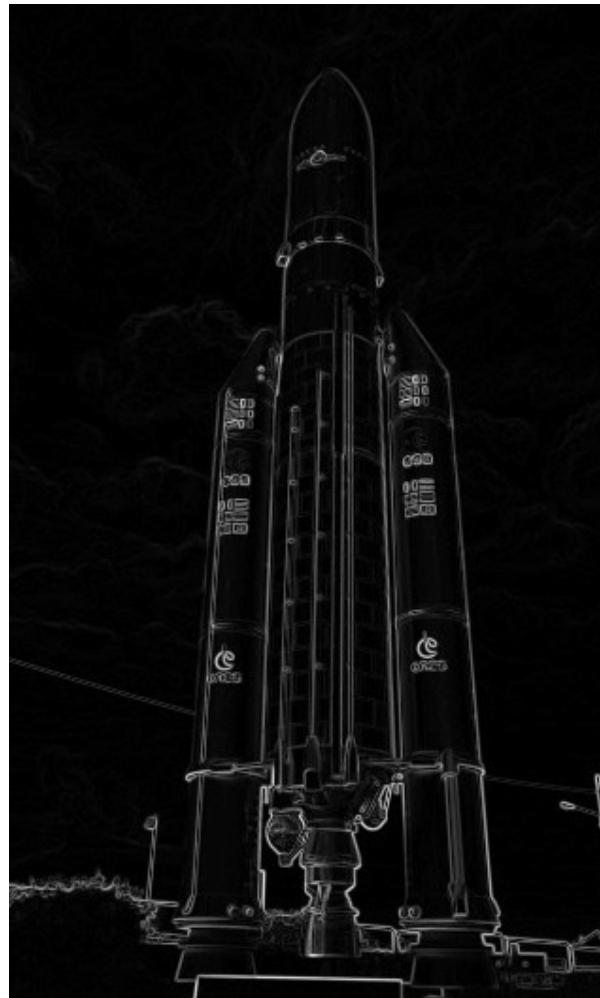


Fine scale
High threshold

Coarse scale
High threshold

Coarse scale
Low threshold

TRACKING ARIANE



RAPID TRACKER

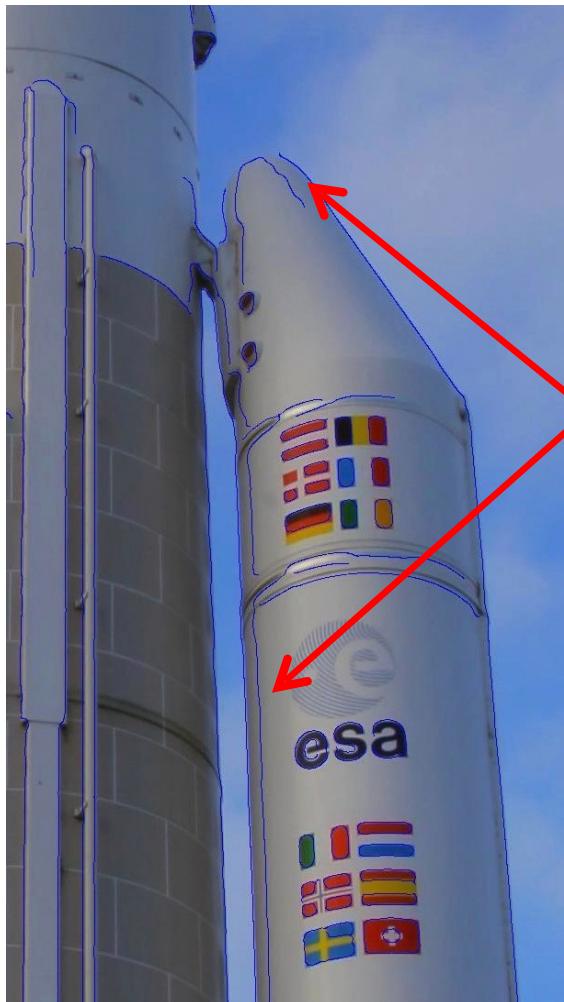


Given an initial pose estimate:



- Estimate the occluding contours.
- Find closest edge points in the normal direction.
- Re-estimate pose to minimize distances in the least squares sense.
- Iterate until convergence.

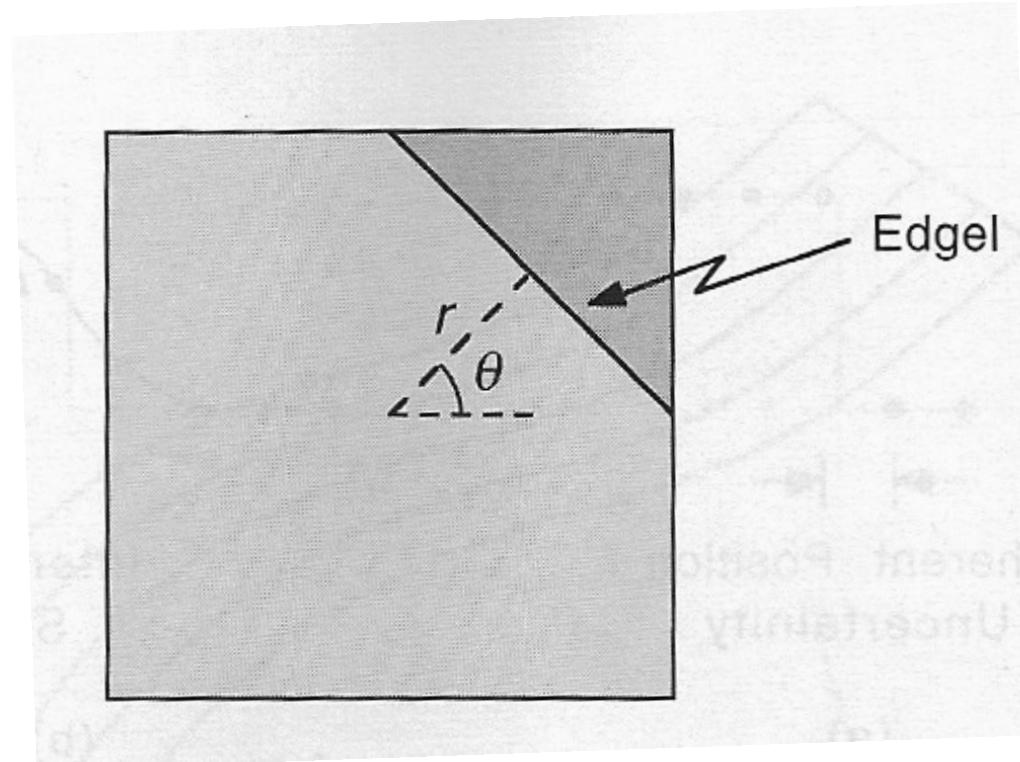
STEEP SMOOTH SHADING



- Rapidly varying gray levels.
- Large gradients.

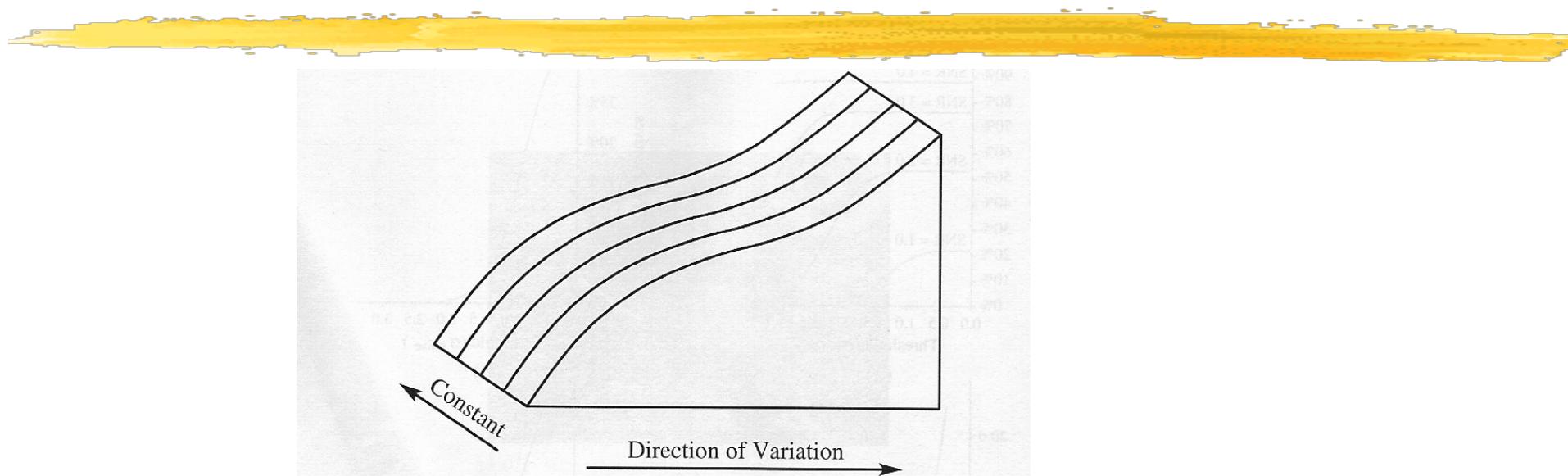
→ Shading can produce spurious edges.

PARAMETRIC MODEL MATCHERS



→ 4 parameters model to be fit in the least squares sense.

SURFACE FITTING



1. Estimate the edgel direction by fitting a cubic surface.
2. Fit a 1-D surface in the direction of the edgel
 - Step shaped surface,
 - Quadratic polynomial.
3. Declare an edge if step shape better than quadratic.

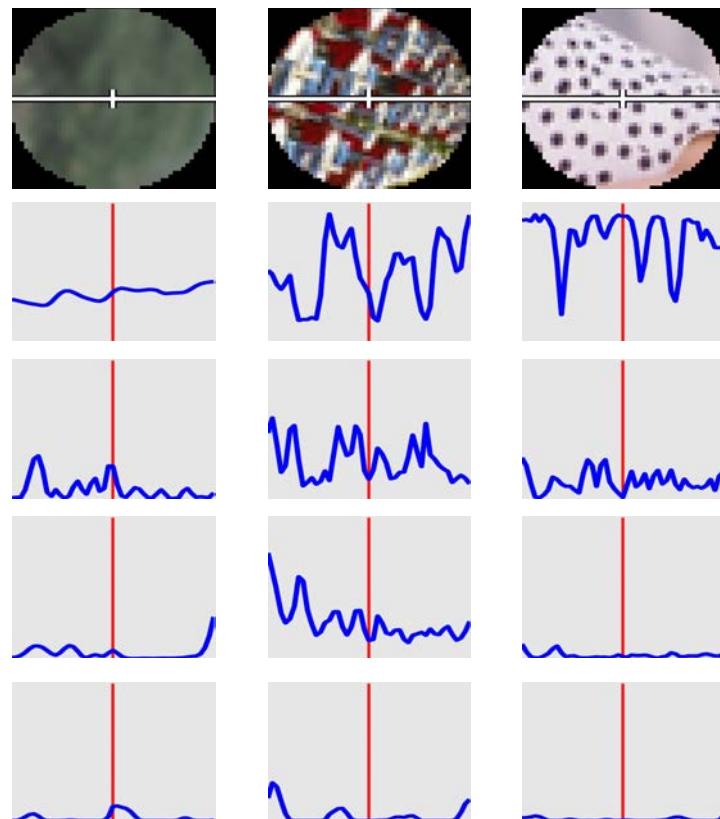
TEXTURE BOUNDARIES



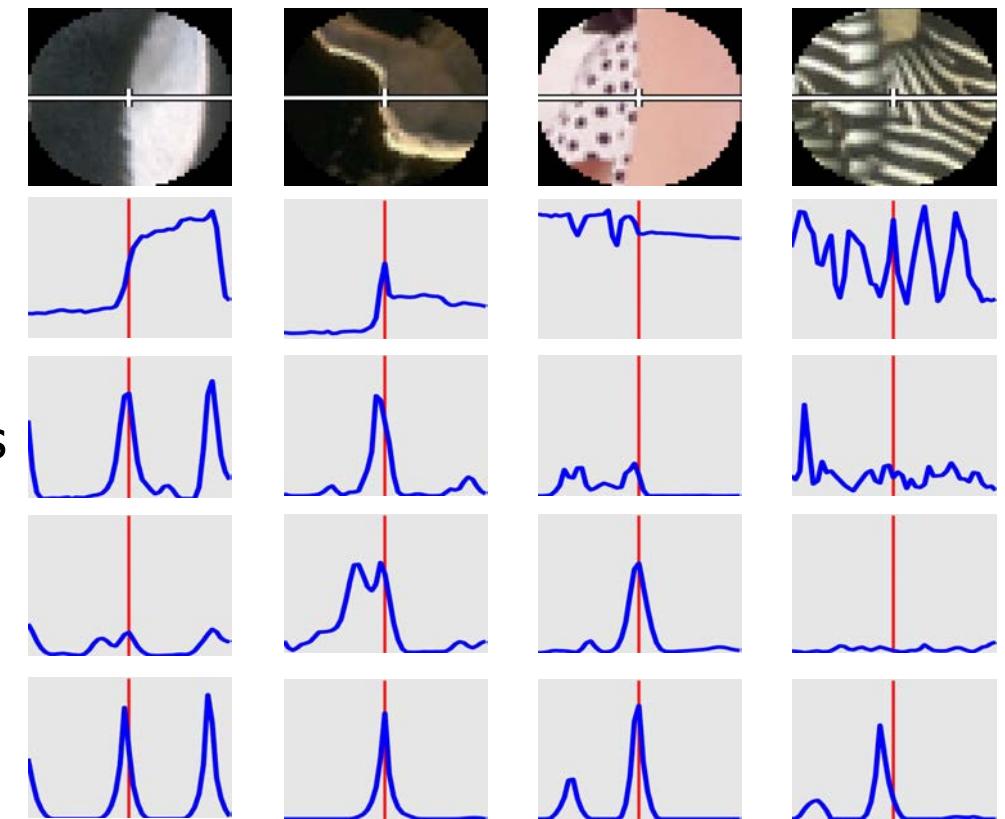
- Not all image contours are characterized by strong contrast.
- Sometimes, textural changes are just as significant.

DIFFERENT BOUNDARY TYPES

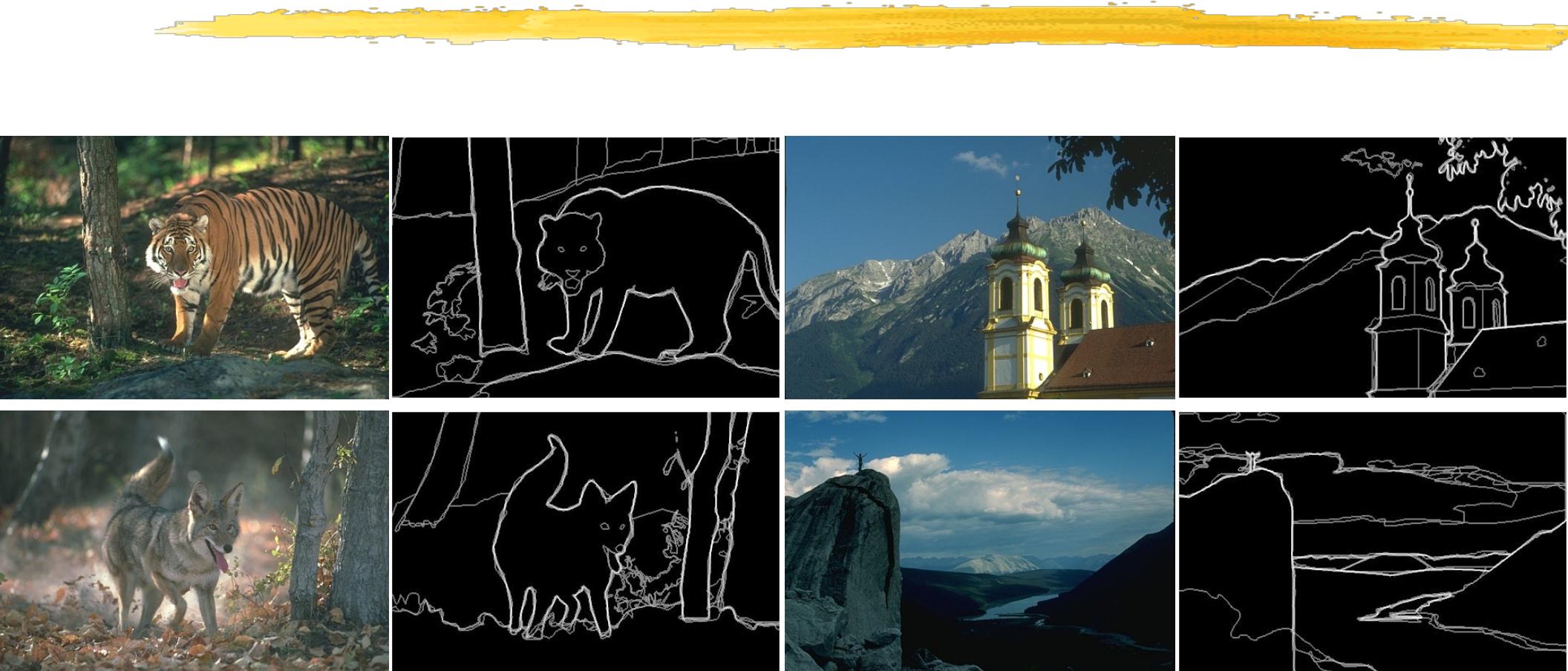
Non-boundaries



Boundaries

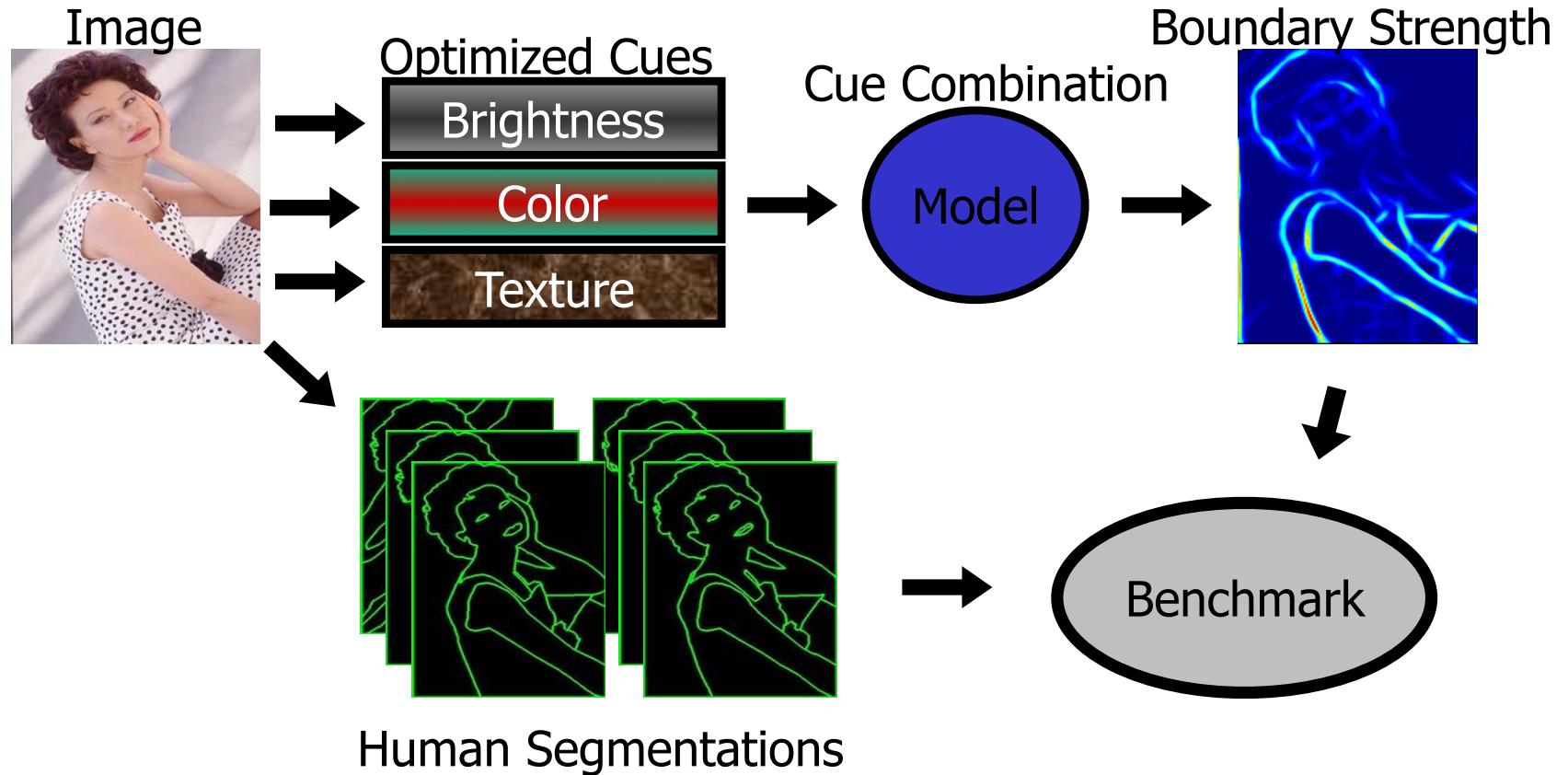


TRAINING DATABASE



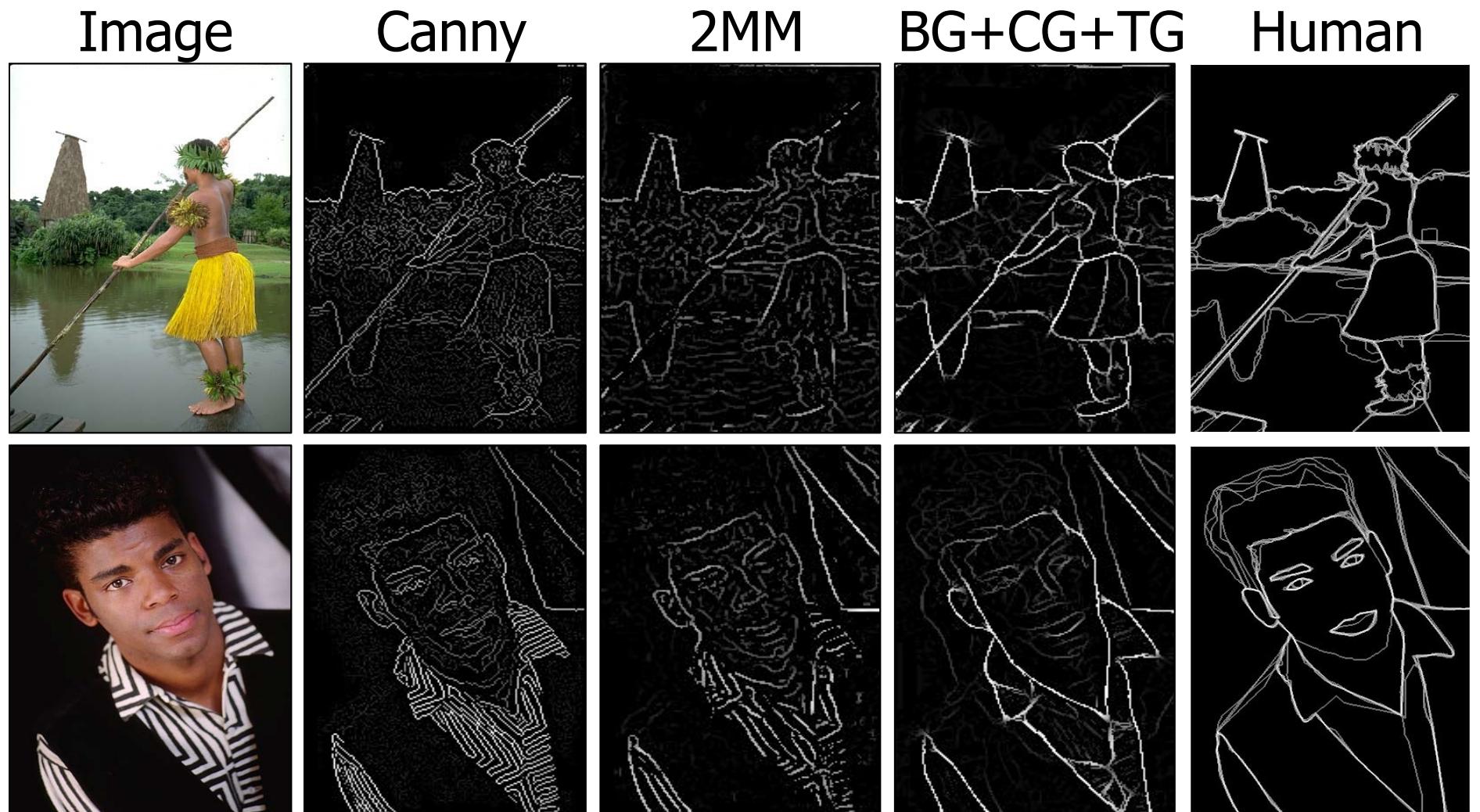
1000 images with 5 to 10 segmentations each.

MACHINE LEARNING

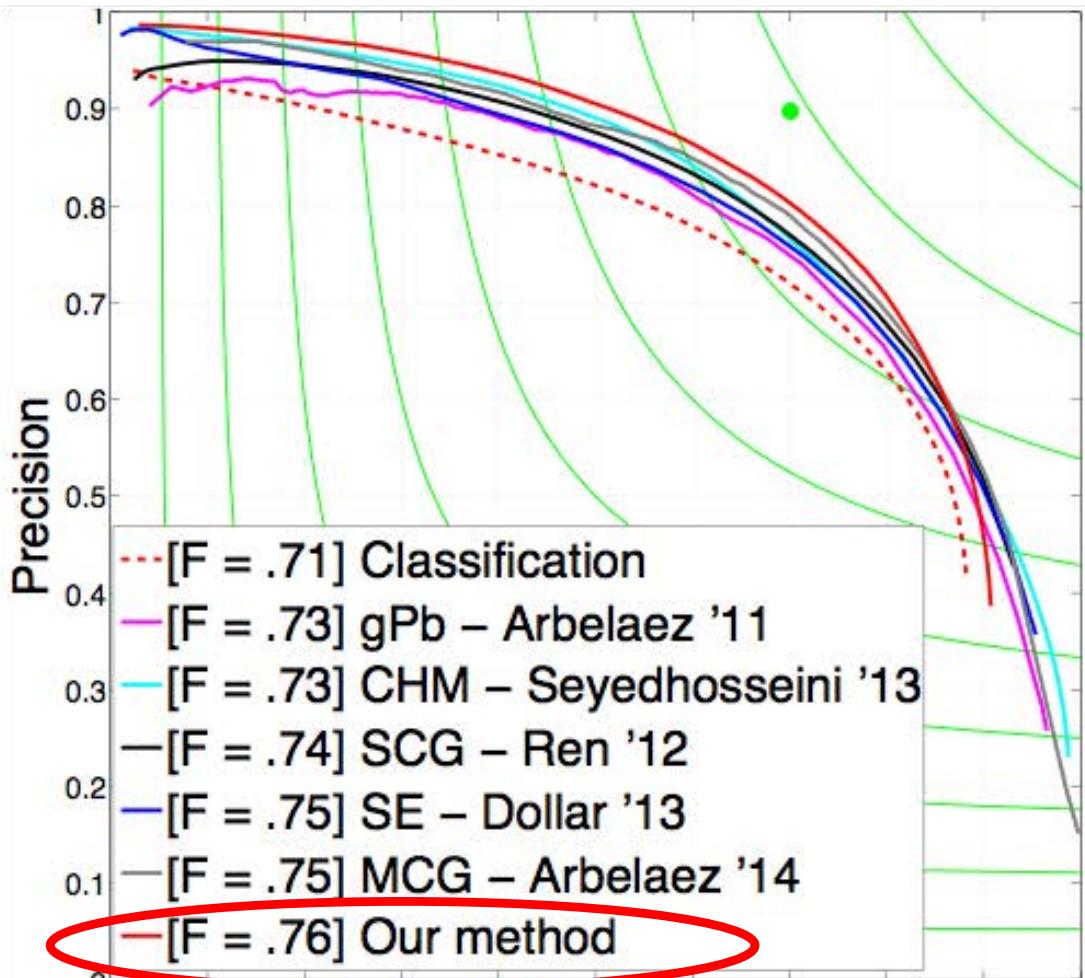


Learn the probability of being a boundary pixel on the basis of a set of features.

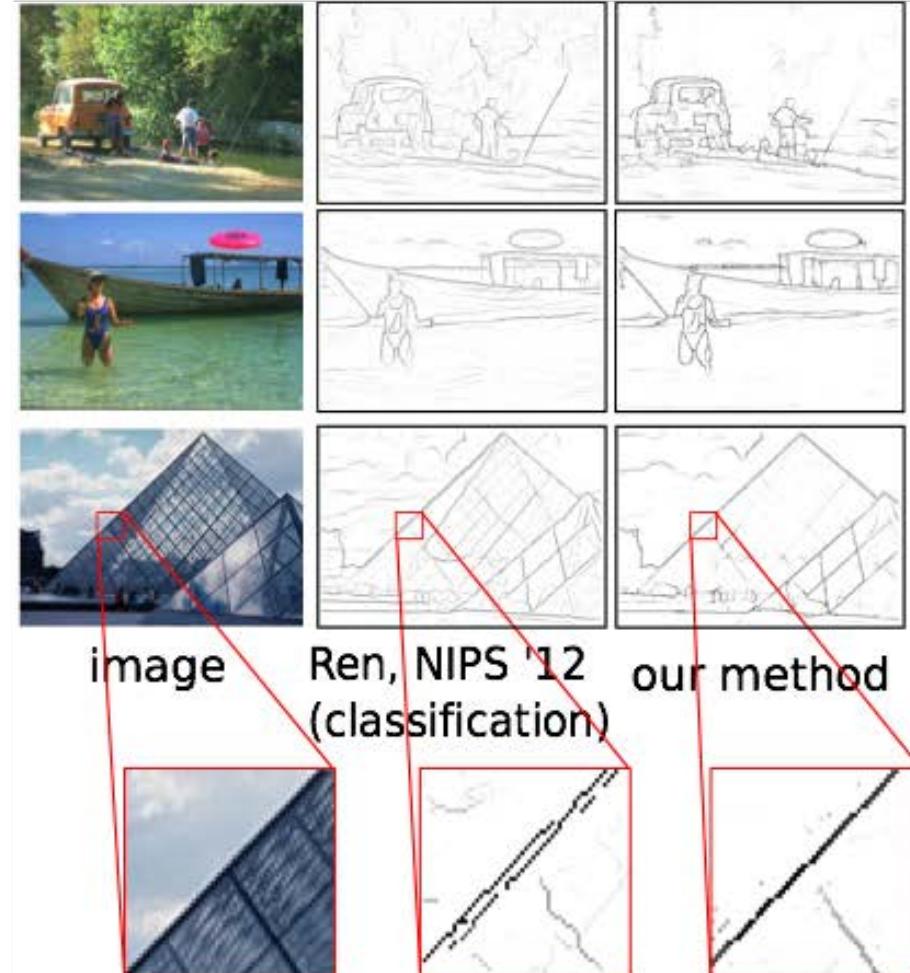
RESULTS



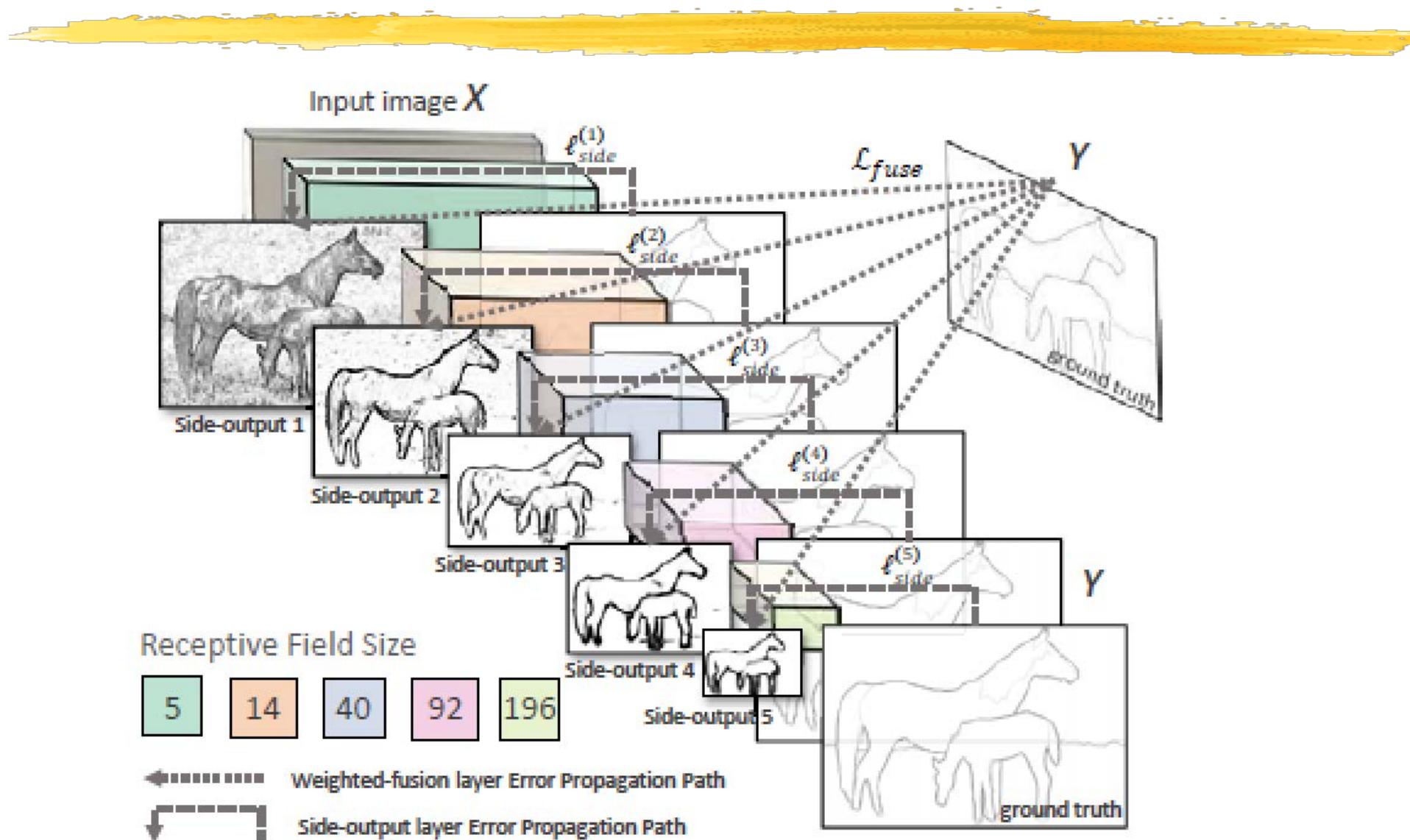
CLASSIFICATION vs REGRESSION



Yes!



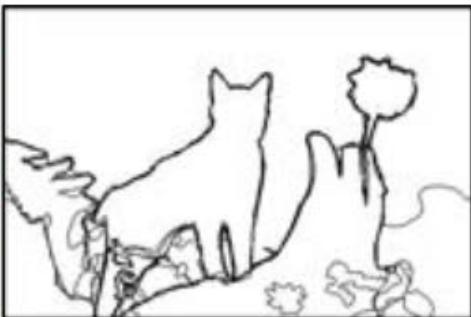
DEEP LEARNING



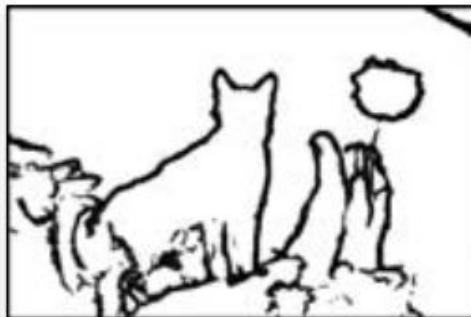
DEEP LEARNING VS CANNY



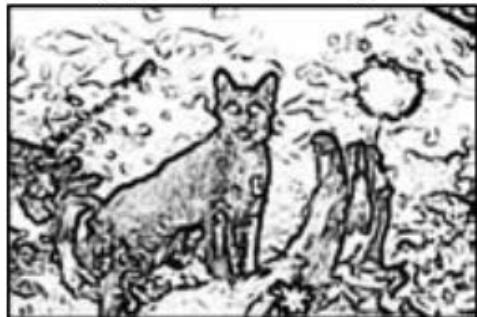
(a) original image



(b) ground truth



(c) HED: output



(d) HED: side output 2



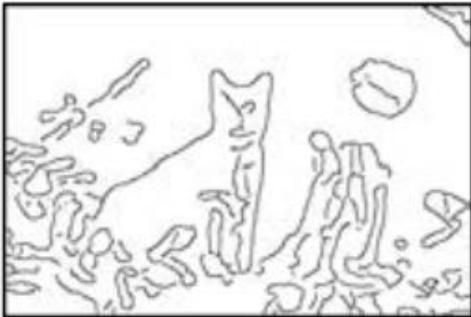
(e) HED: side output 3



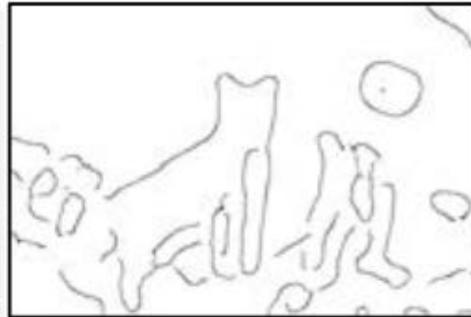
(f) HED: side output 4



(g) Canny: $\sigma = 2$

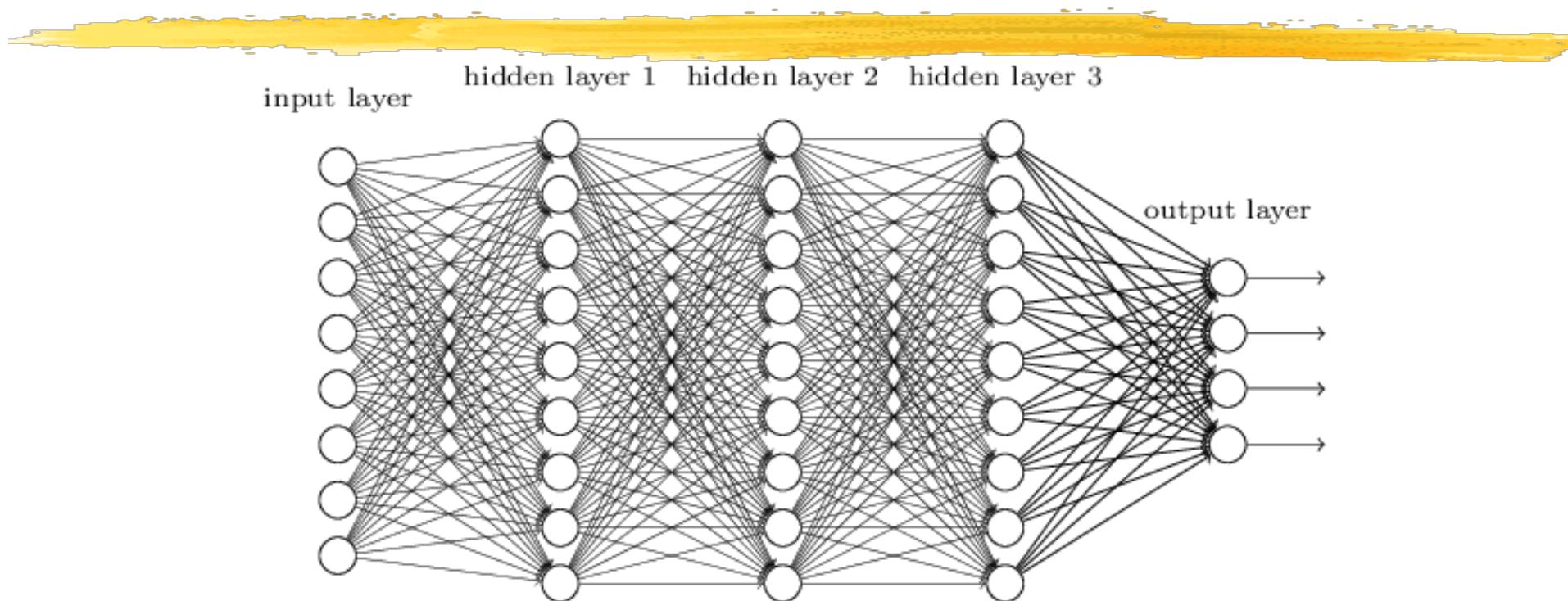


(h) Canny: $\sigma = 4$



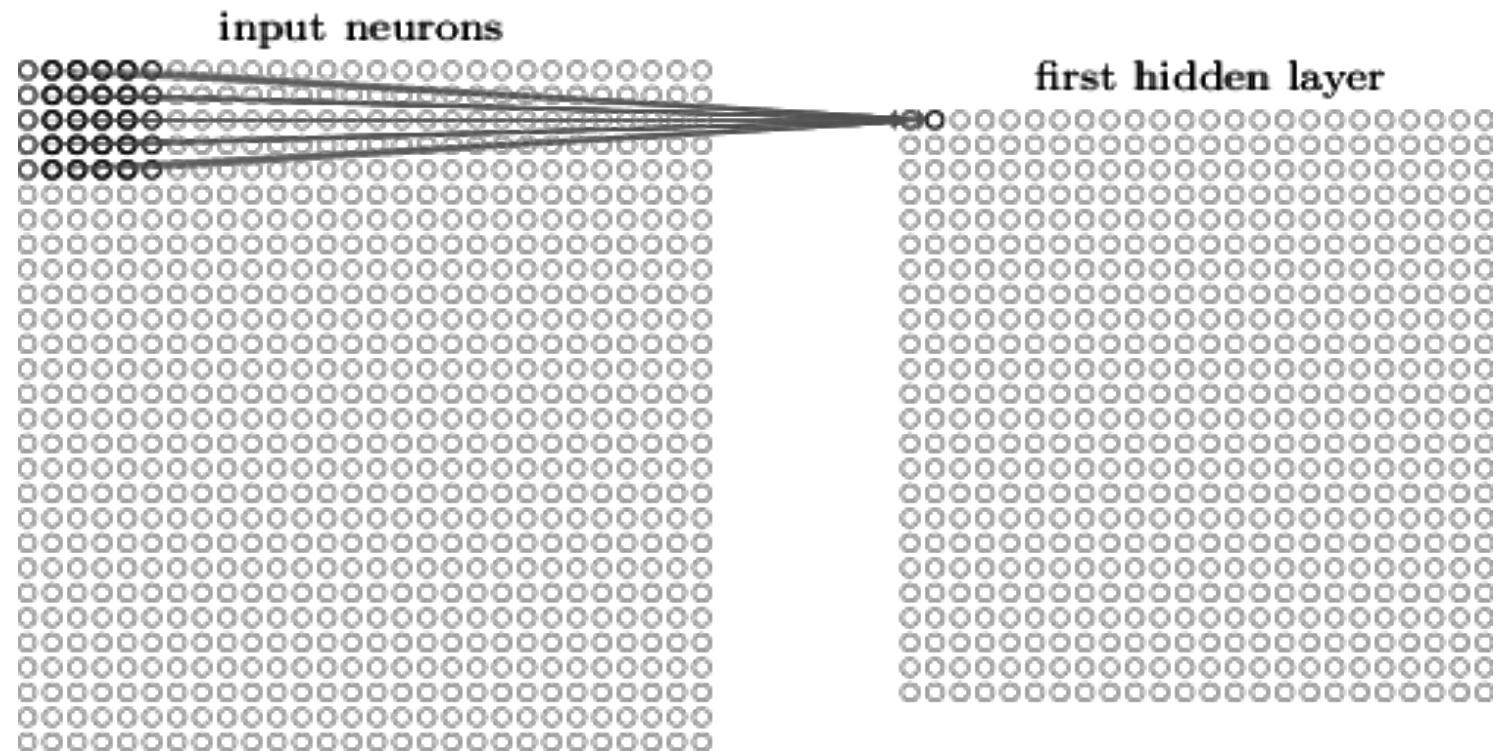
(i) Canny: $\sigma = 8$

DEEP LEARNING



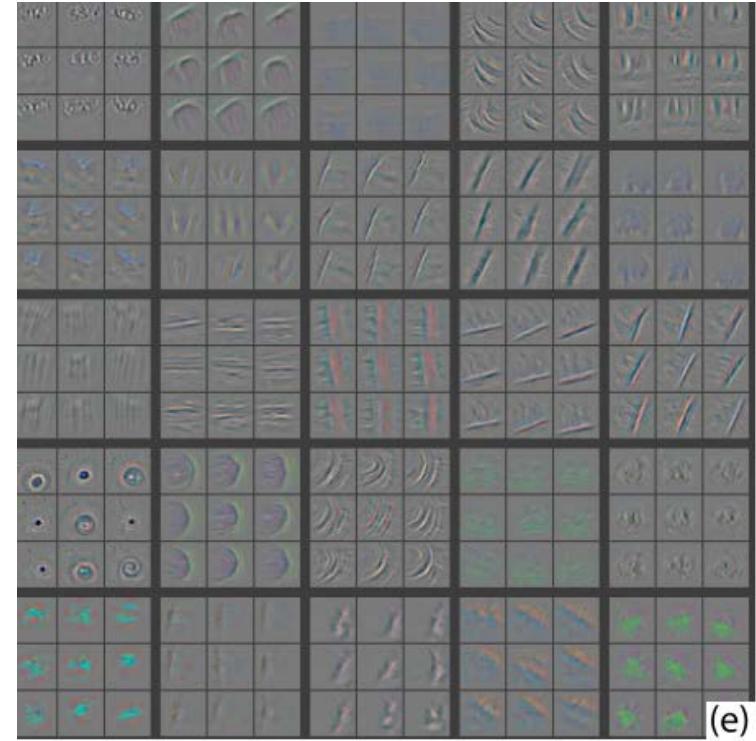
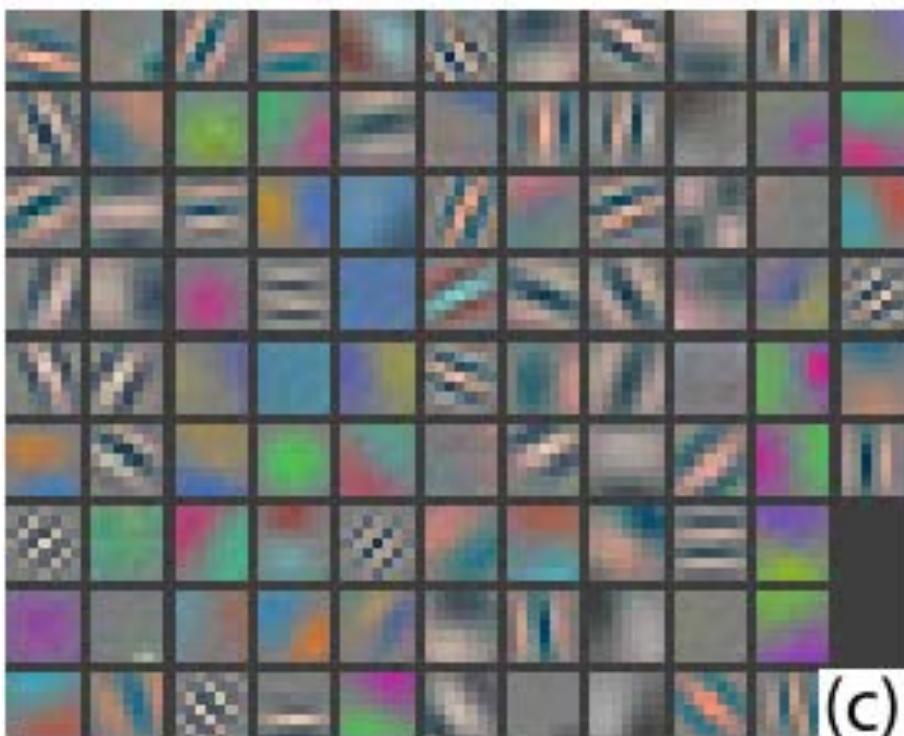
- Store in each node a function of the linear combination of the activations of all nodes in the previous layers.
- The network can be trained to produce a desired output given a specific input by learning the linear combination weights.

CONVOLUTIONAL LAYER



$$\sigma \left(b + \sum_{x=0}^{n_x} \sum_{y=0}^{n_y} w_{i,j} a_{i+x,j+y} \right)$$

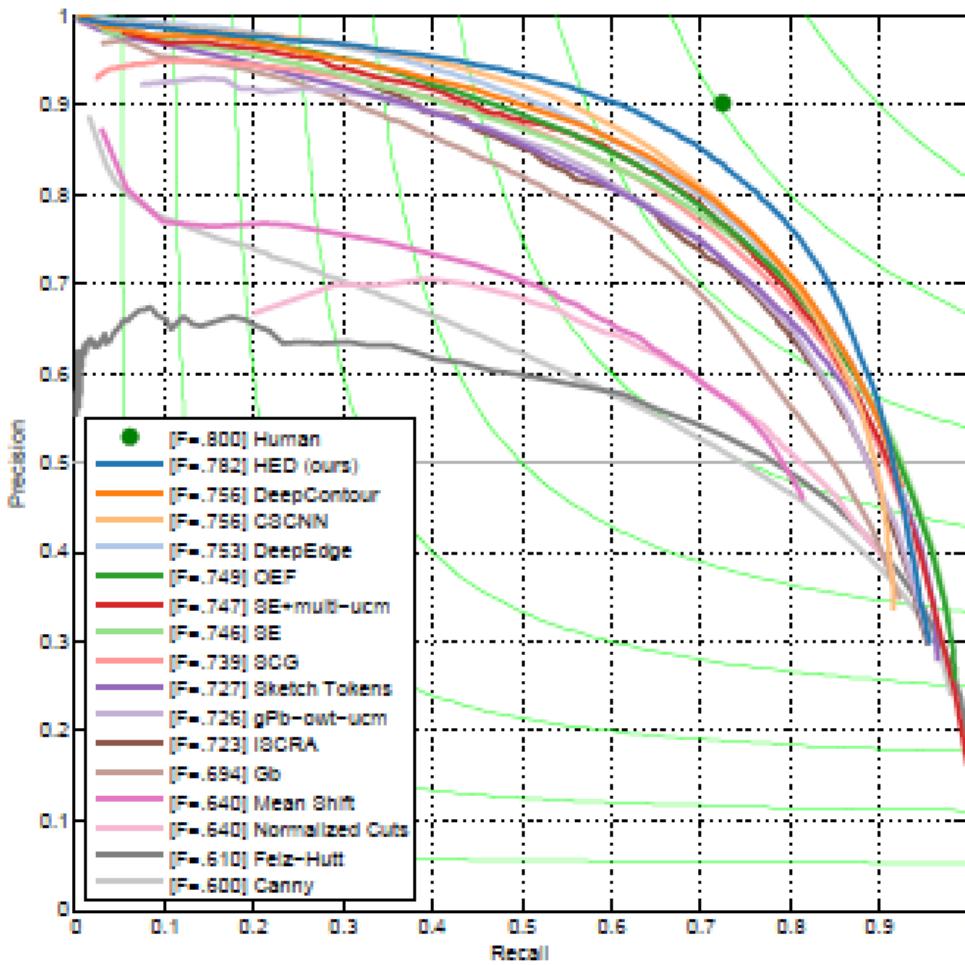
A PARTIAL EXPLANATION?



First and second layer features of a Convolutional Neural Net:

- They can be understood as performing multiscale filtering.
- The weights and thresholds are chosen by the optimization procedure.

50 YEARS OF EDGE DETECTION



- Convolution operators respond to steep smooth shading.
- Parametric matchers tend to reject non ideal edges.
- Arbitrary thresholds and scale sizes are required.
- Learning-based methods need exhaustive databases.

→ **Progress but edge detection remains an open problem.**