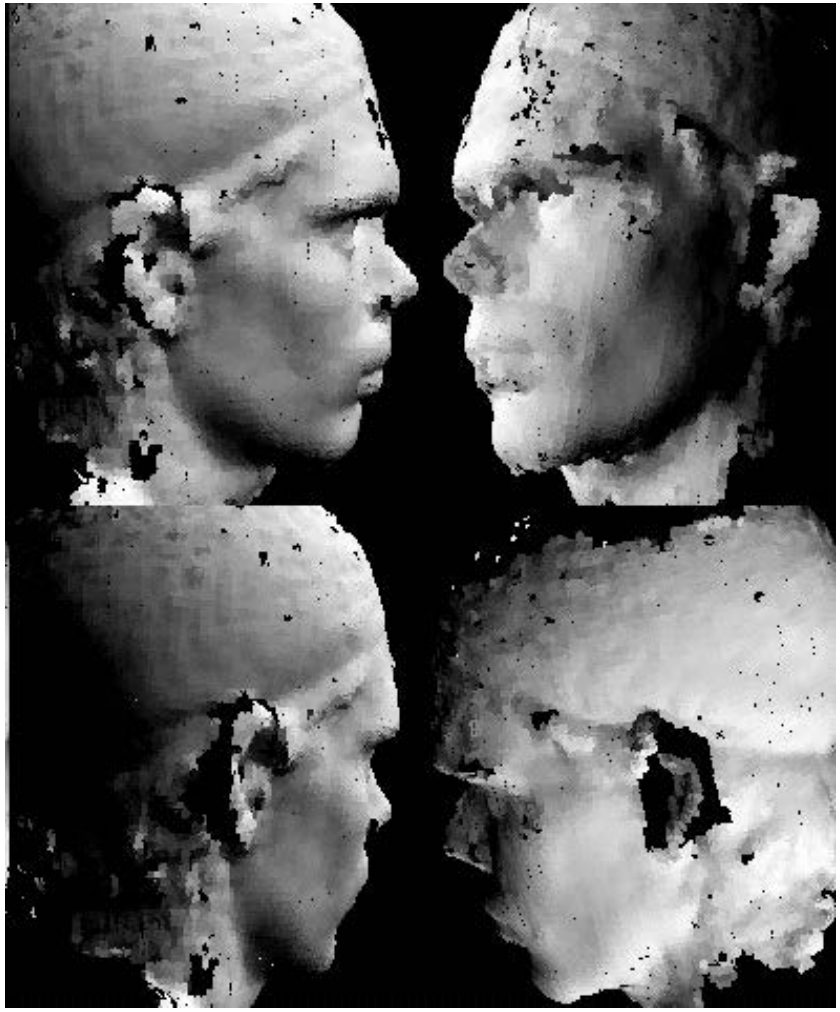
SHAPE FROM X

One image:

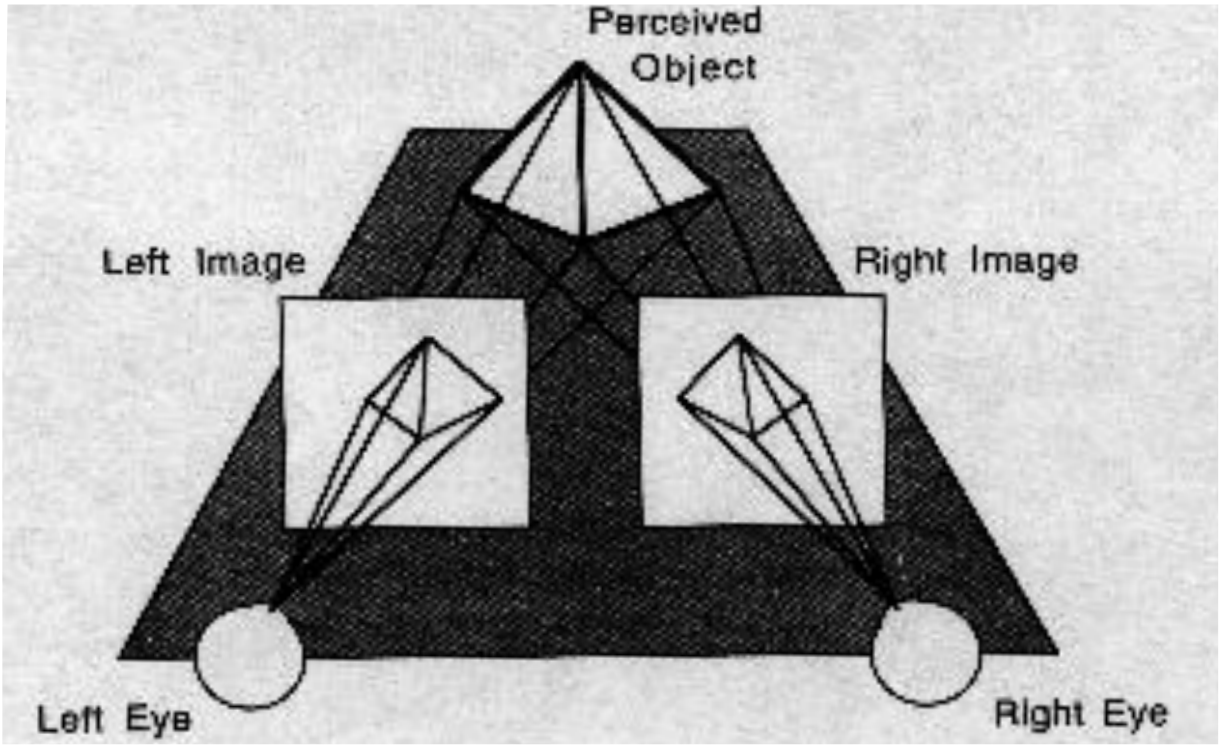
- Texture
- Shading

Two images or more:

- **Stereo**
- Contours
- Motion



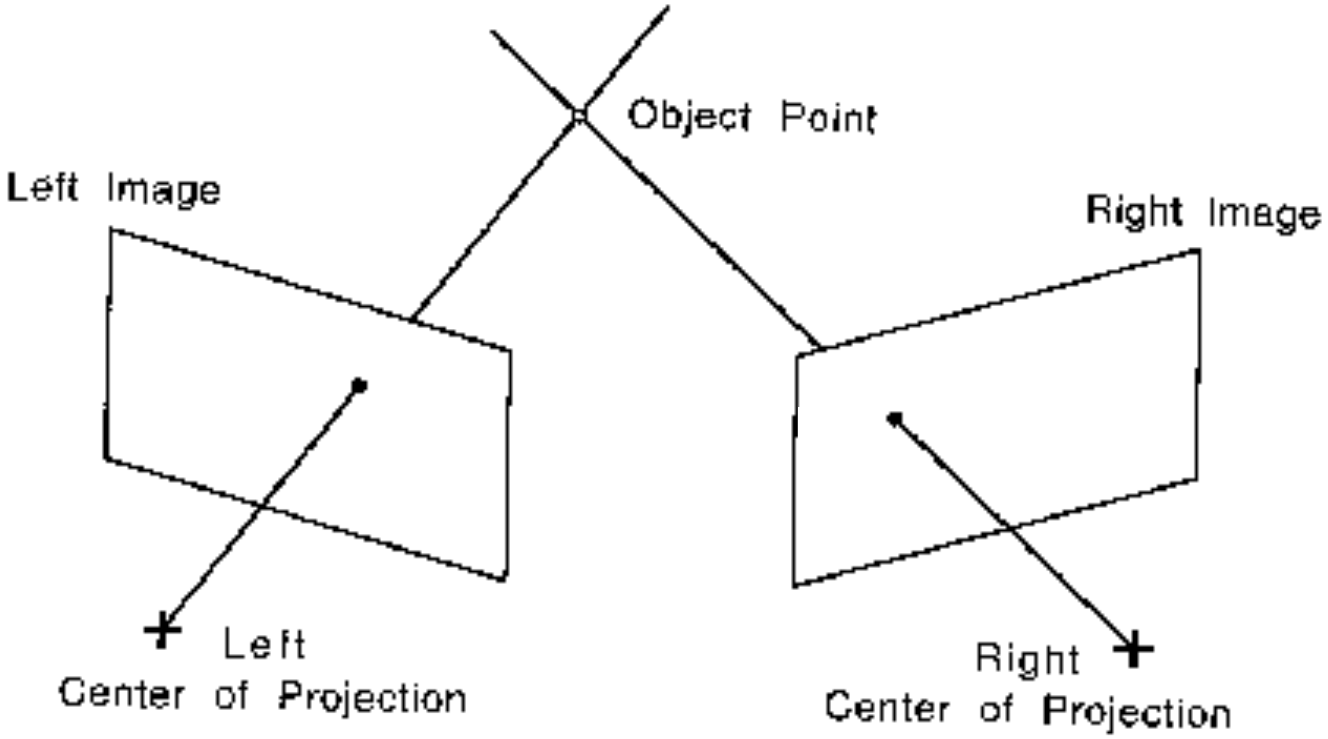
GEOMETRIC STEREO



Depth from two or more images:

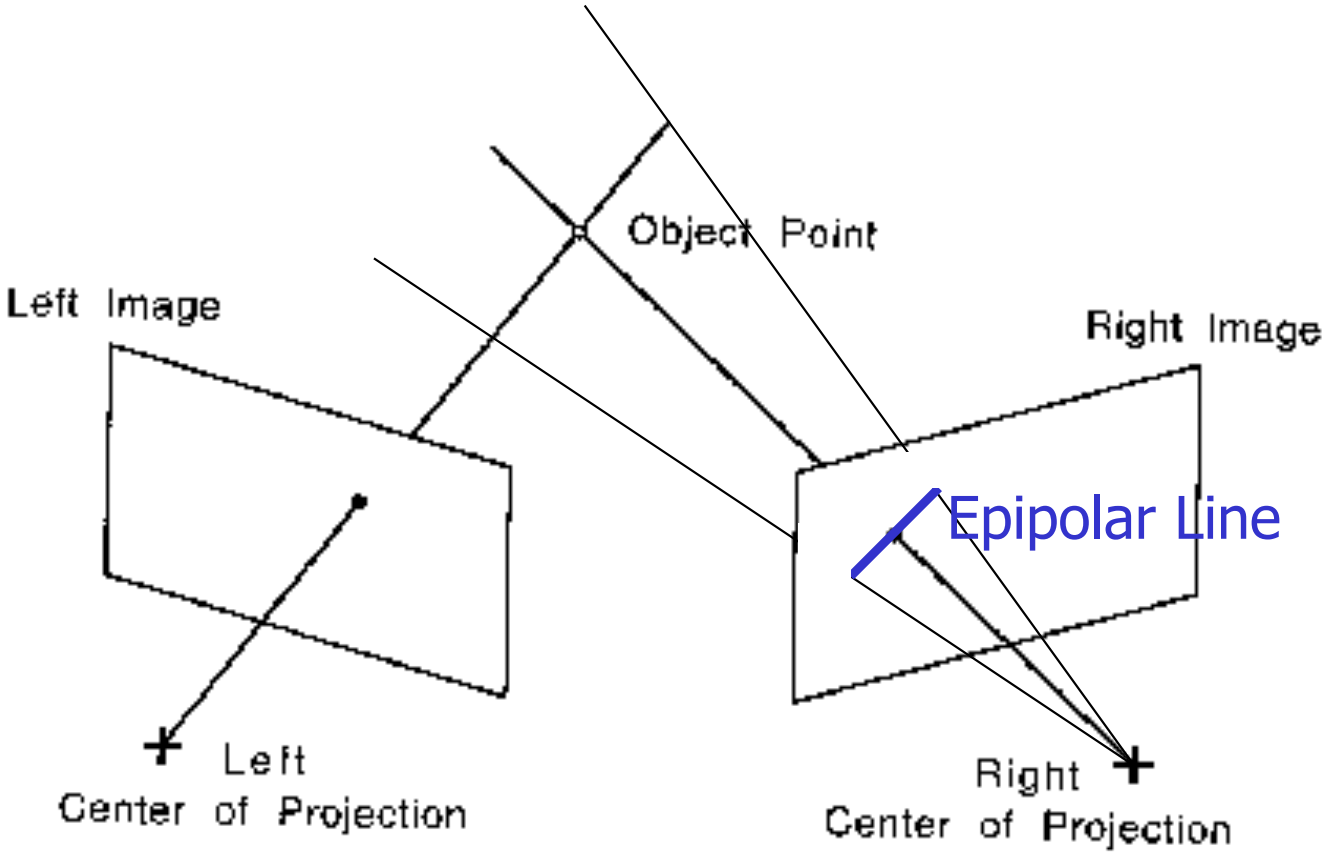
- Geometry of image pairs
- Establishing correspondences

TRIANGULATION



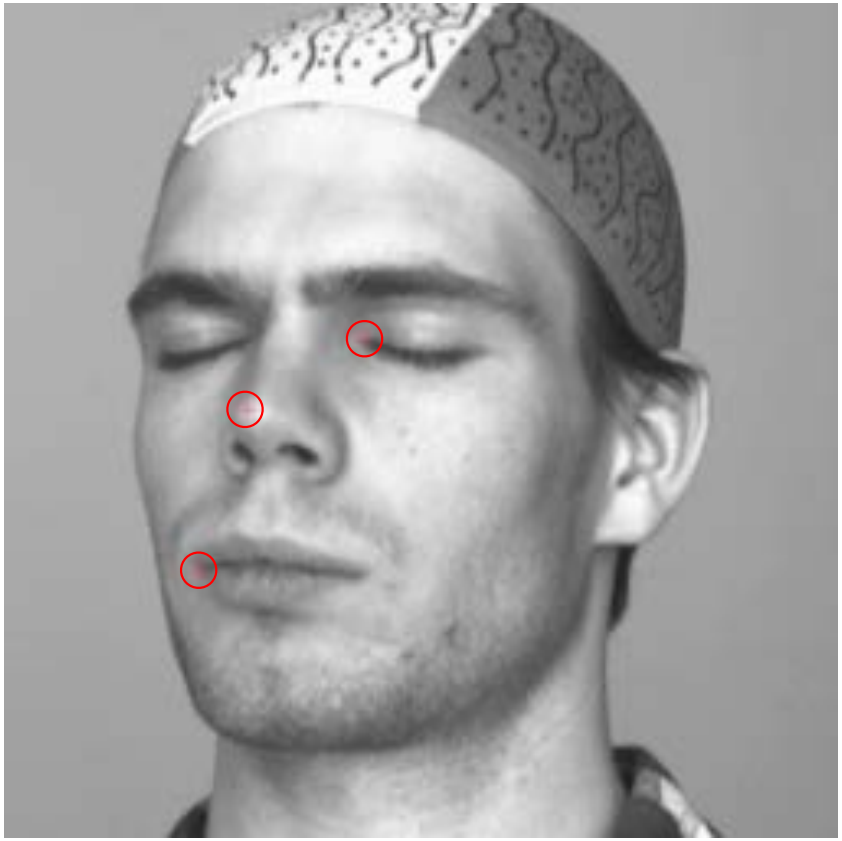
Geometric Stereo: Depth from two images

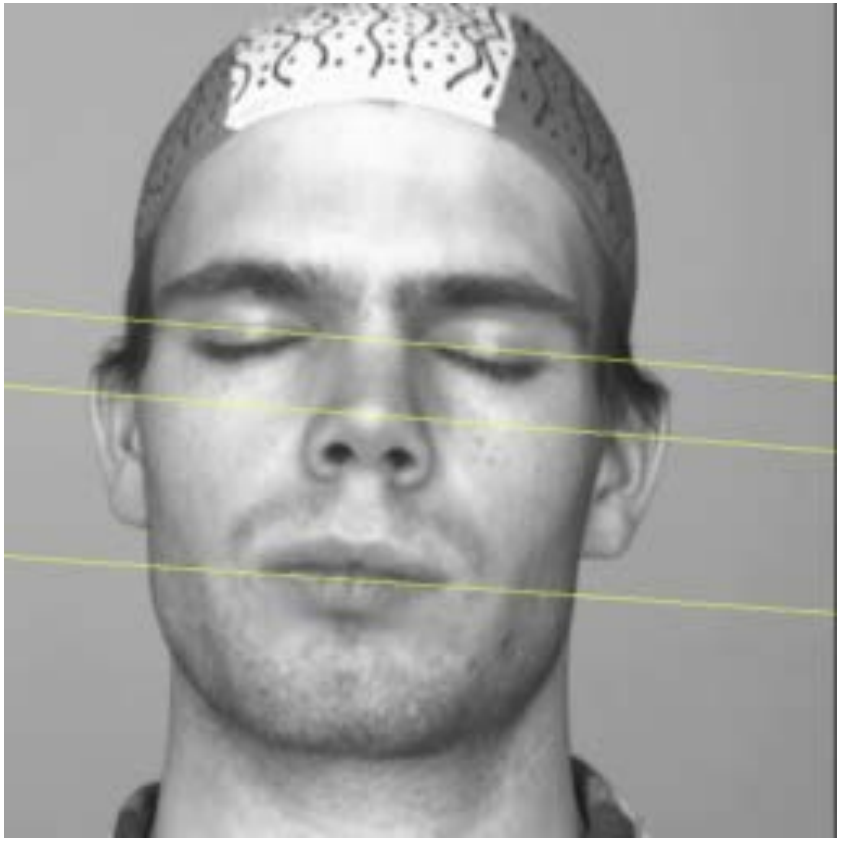
EPIPOLAR LINE



Line on which the corresponding point must lie.

EPIPOLAR LINES

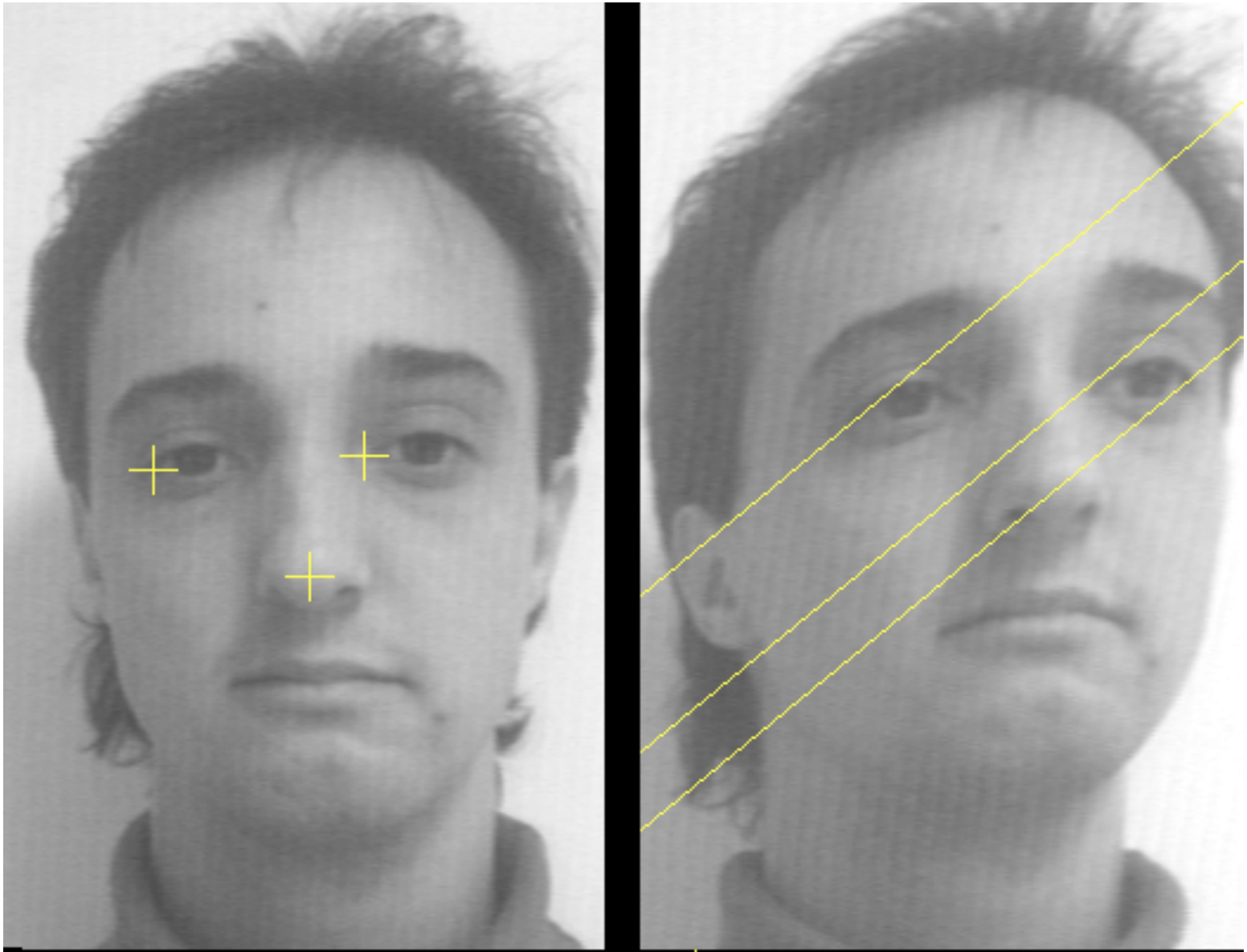




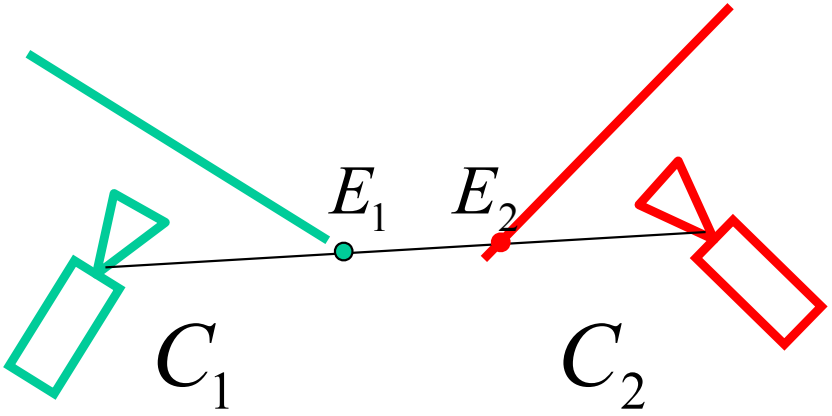
Three points shown as red crosses.

Corresponding epipolar lines.

EPIPOLAR LINES



EPIPOLE



Point at which **all** epipolar lines intersect:

- Located at the intersection of line joining optical centers and image plane.

FUNDAMENTAL MATRIX

There is 3×3 matrix F such that for all corresponding points $\mathbf{x} \leftrightarrow \mathbf{x}'$

$$\mathbf{x}'^{T}\mathbf{F}\mathbf{x} = 0.$$

Therefore, the epipolar line corresponding to $\mathbf{x}$ is $\mathbf{l} = \mathbf{F}\mathbf{x}$.

Given a set of n point matches,we write

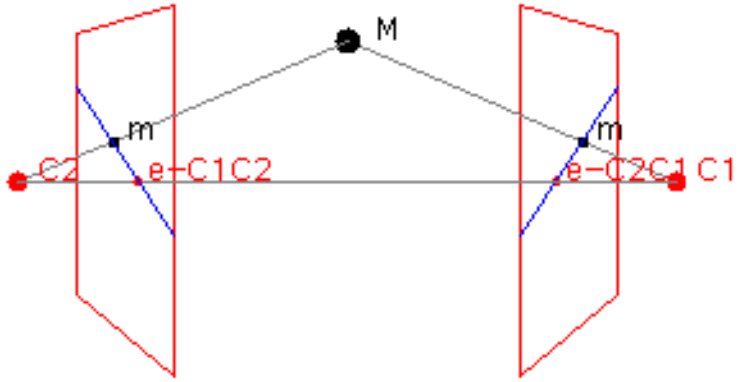
$$\begin{bmatrix} u_1'u_1 & u_1'v_1 & u_1' & v_1'u_1 & v_1'v_1 & v_1' & u_1 & v_1 & 1 \\ : & : & : & : & : & : & : & : & : \\ u_n'u_n & u_n'v_n & u_n' & v_n'u_n & v_n'v_n & v_n' & u_n & v_n & 1 \end{bmatrix} \mathbf{f} = 0.$$

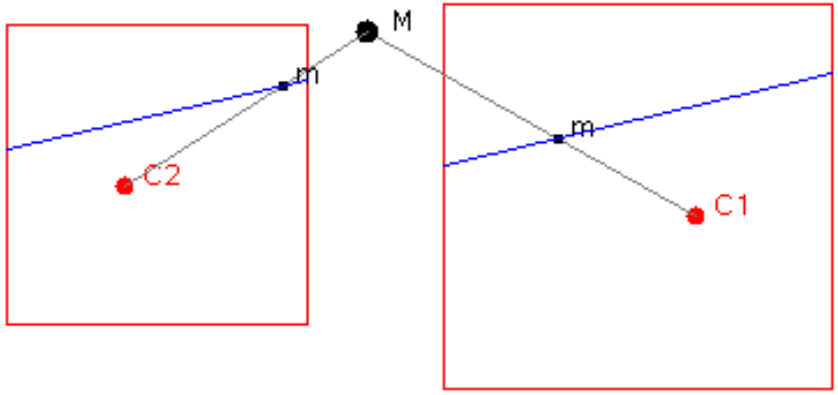
$\rightarrow$DLT or non – linear minimization.

Hartley, Chap 9.

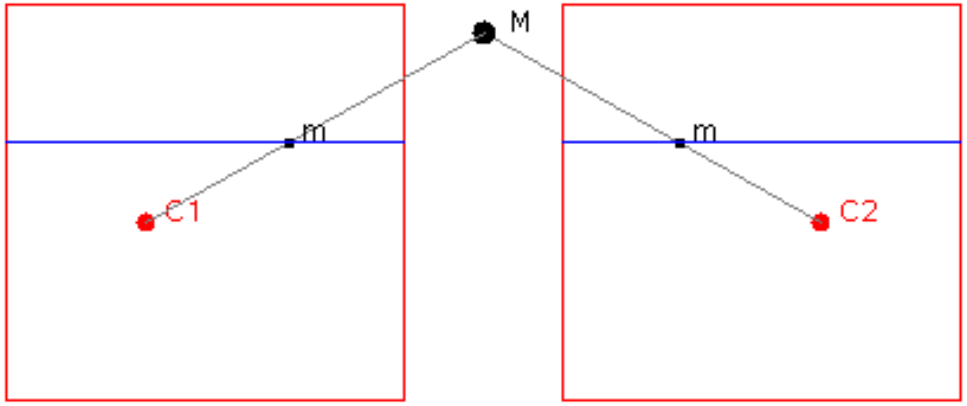
EPIPOLAR GEOMETRY

In general:



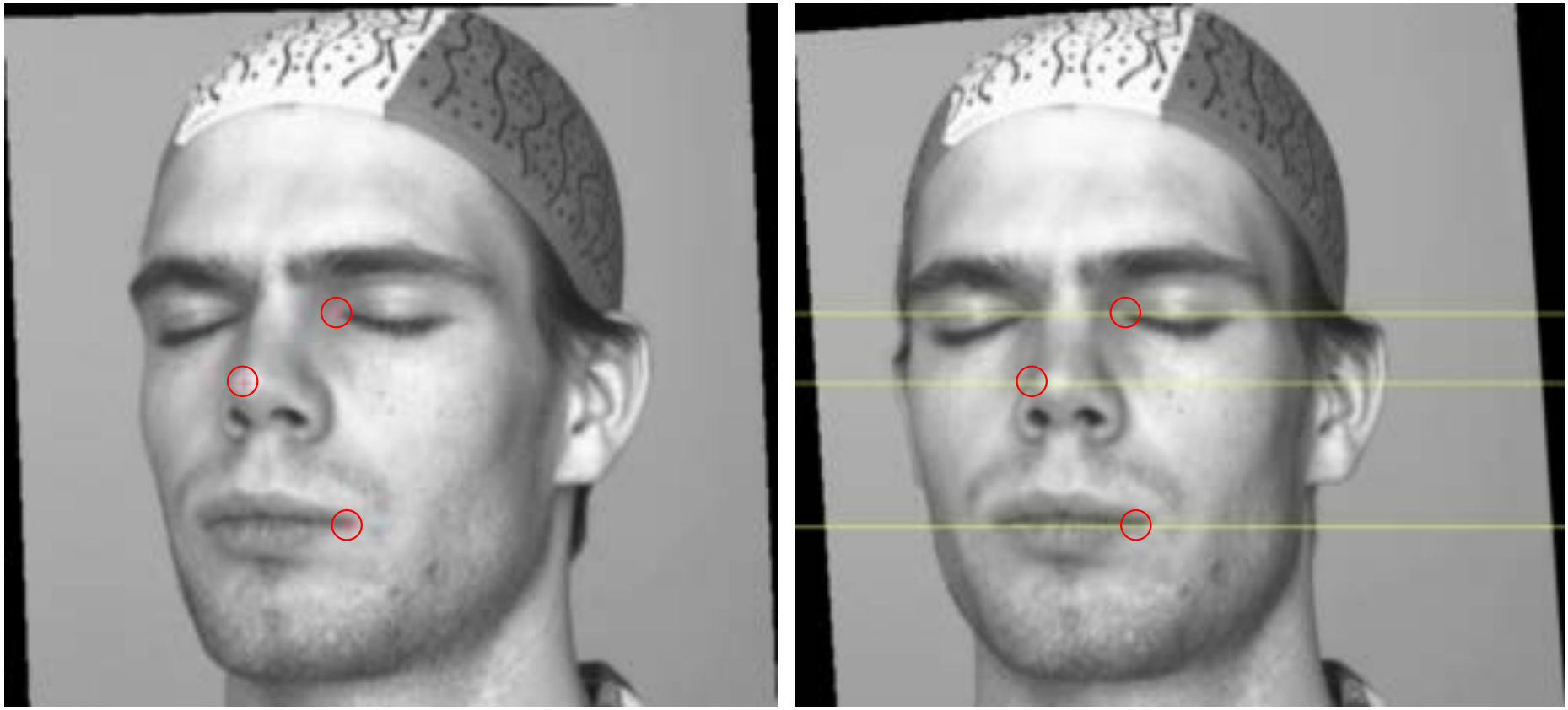


Parallel image planes



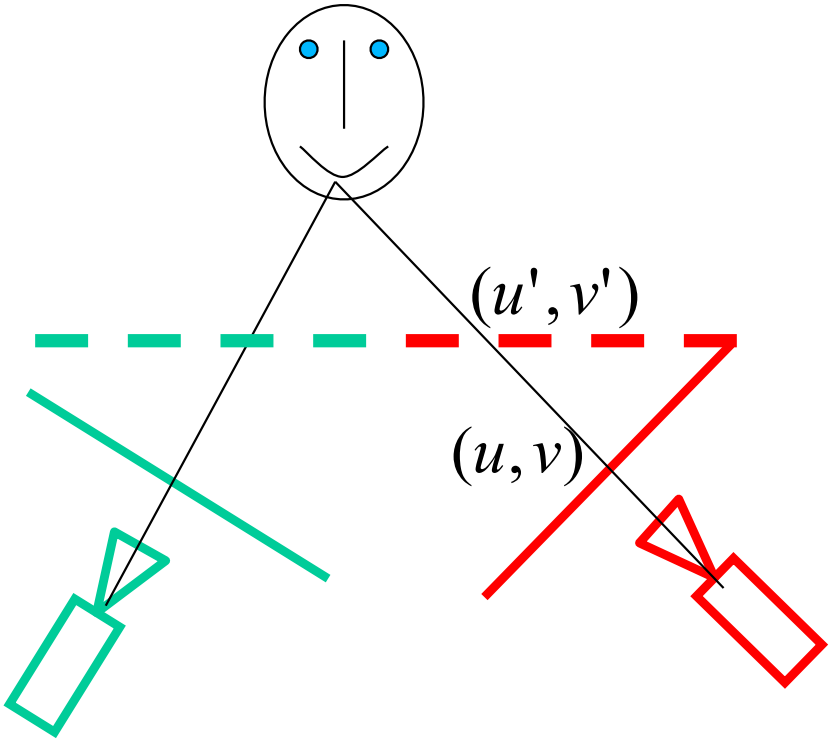
Horizontal baseline

RECTIFICATION



Parallel epipolar lines

RECTIFICATION



$$\begin{bmatrix} U' \\ V' \\ W' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

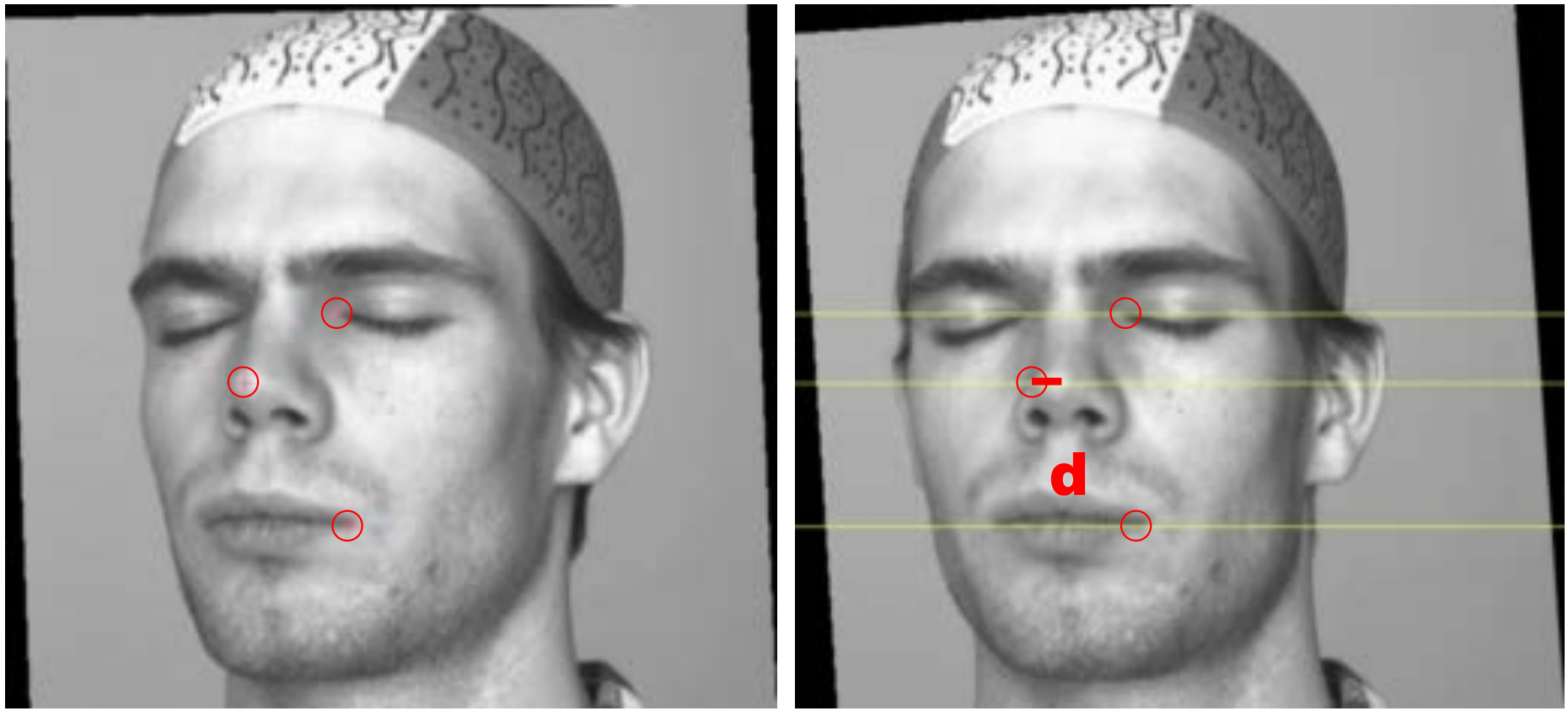
$$u' = U'/W'$$

$$v' = V'/W'$$

Reprojection into parallel virtual image planes:

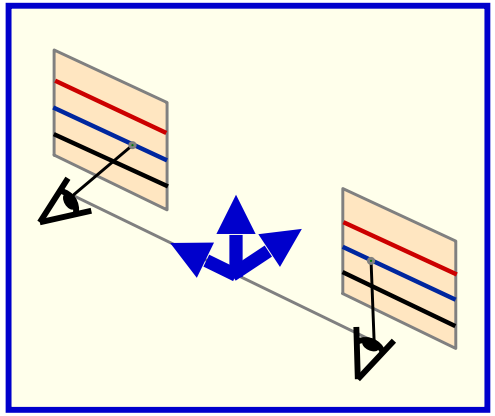
- Linear operation in projective coordinates
- Real-time implementation possible

DISPARITY



Horizontal shift along epipolar line, inversely proportional to distance.

DISPARITY VS DEPTH



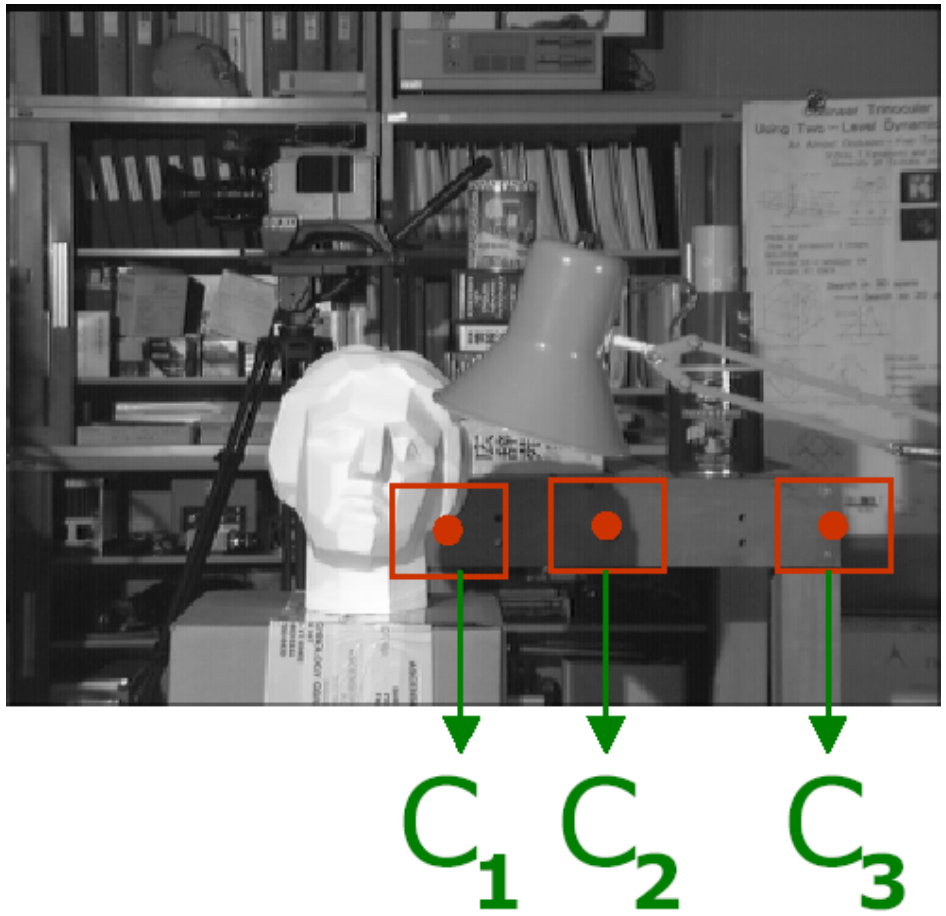
$$u_l = \frac{f(X - b/2)}{Z}, v_l = \frac{fY}{Z}$$

$$u_r = \frac{f(X + b/2)}{Z}, v_l = \frac{fY}{Z}$$

$$d = f\frac{b}{Z}$$

→ Disparity is inversely proportional to depth.

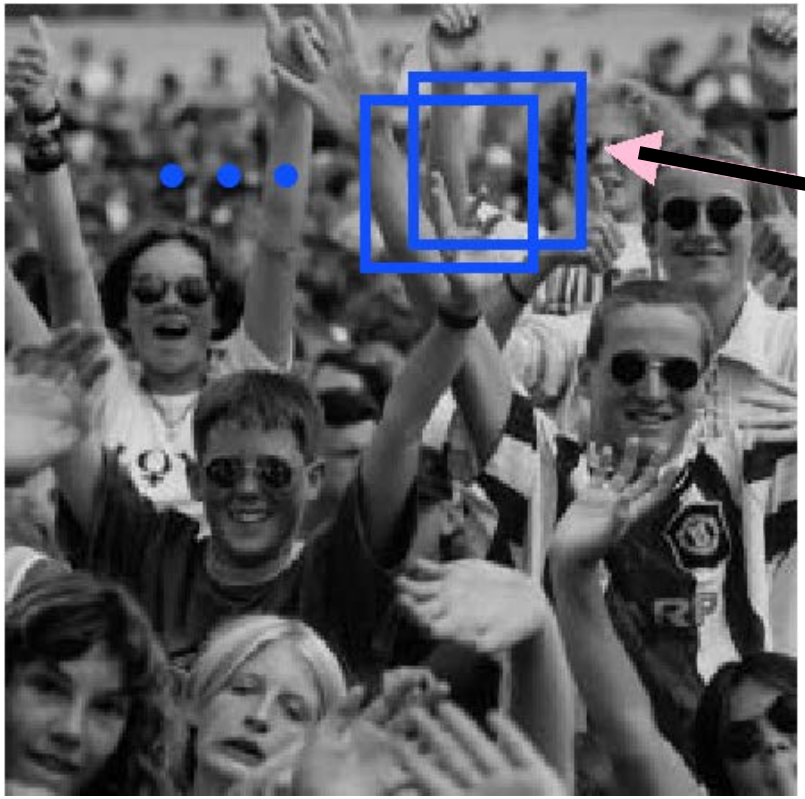
WINDOW BASED APPROACH



- Compute a cost for each C_n location.
- Pick the lowest cost one.

FINDING A PATTERN IN AN IMAGE

Straightforward Approach



Pattern

Move pattern everywhere and compare with image.

But how?

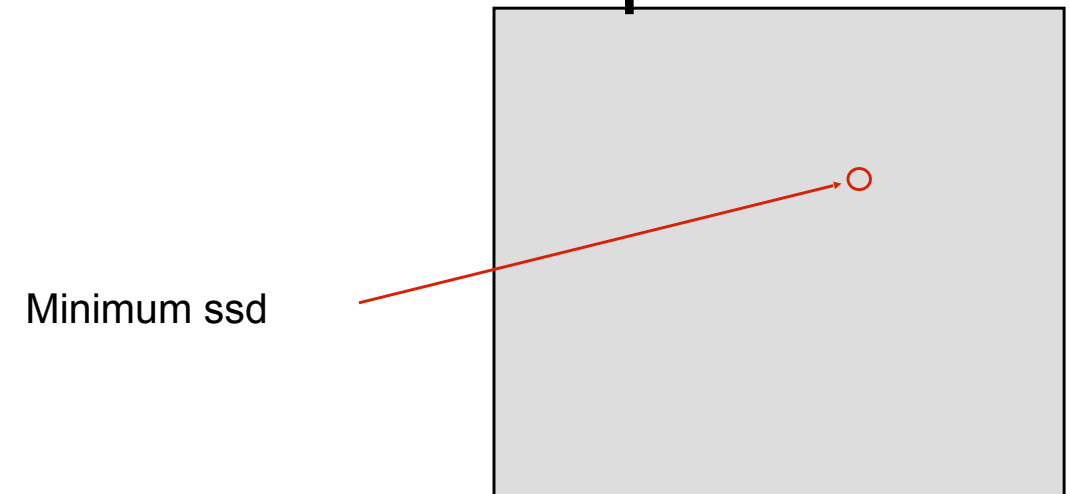
SUM OF SQUARE DIFFERENCES

Subtract pattern and image pixel by pixel and add squares:

$$ssd(u,v) = \sum_{(x,y)\in N}\left[I(u+x,v+y) - P(x,y)\right]^2$$

If identical ssd=0, otherwise ssd >0

→Look for minimum of ssd with respect to u and v.



CORRELATION

$$ssd(u,v) = \sum_{(x,y)\in N}\left[I(u+x,v+y) - P(x,y)\right]^2$$

$$= \sum_{(x,y)\in N} I(u+x,v+y)^2 + \sum_{(x,y)\in N} P(x,y)^2 - 2\sum_{(x,y)\in N} I(u+x,v+y)P(x,y)$$

Sum of squares of the window (positive term)

Sum of squares of the pattern (CONSTANT term)

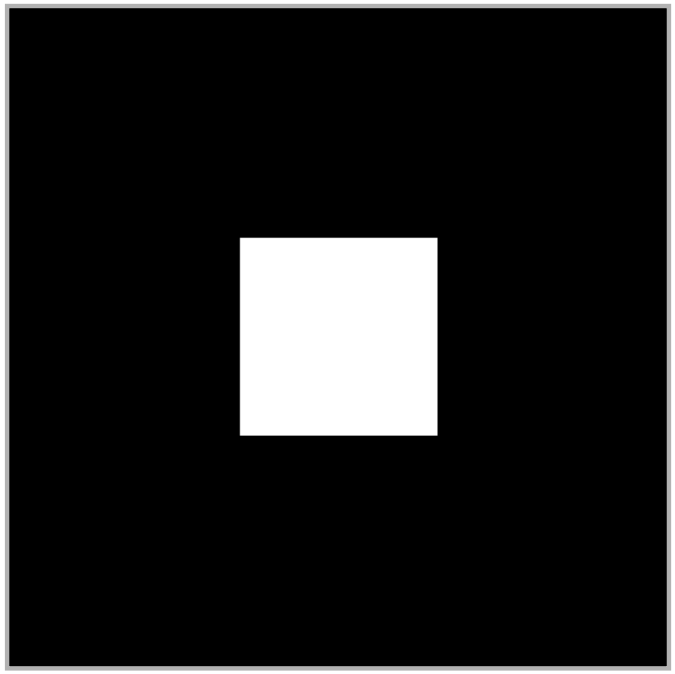
Correlation

ssd(u,v) is minimized when correlation is largest

→ Correlation measures similarity

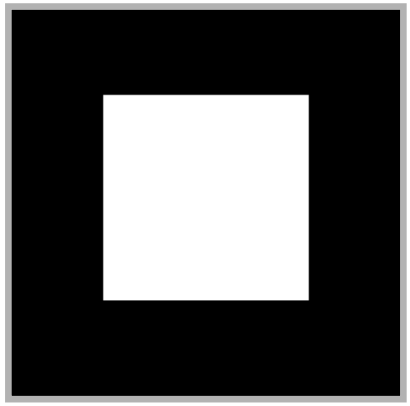
SIMPLE EXAMPLE

I



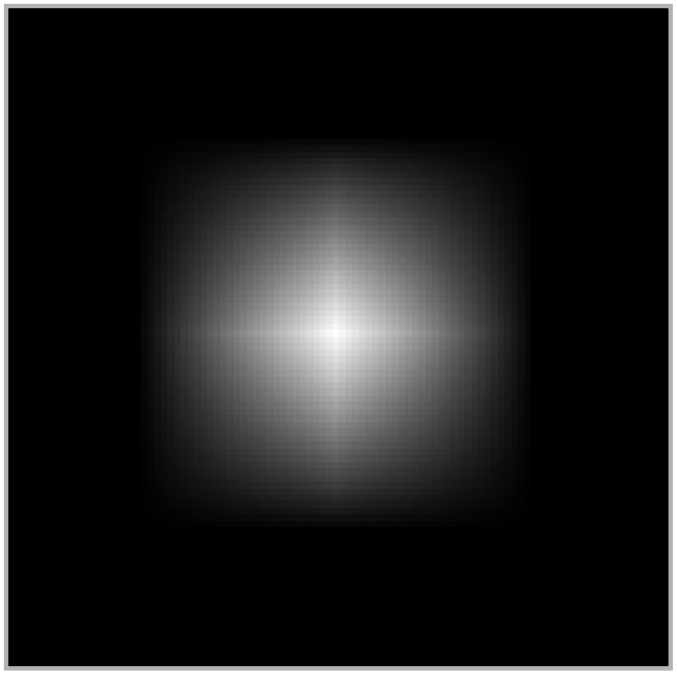
*

P



=

I correlated with P

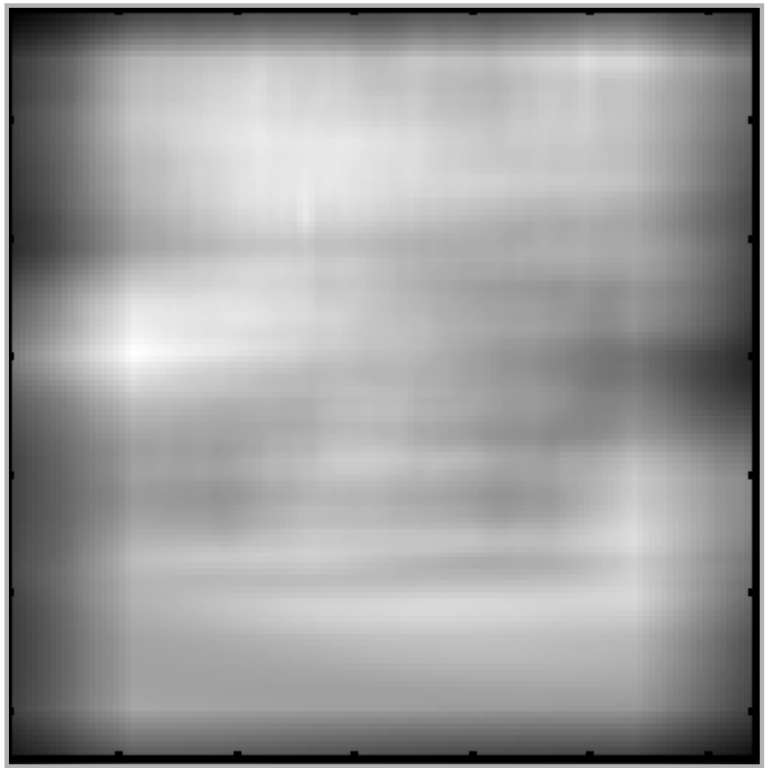


NOT SO SIMPLE EXAMPLE

Image

Pattern

Correlation



- Correlation value depends on the local gray levels of the pattern and image window.
- Need to normalize.

NORMALIZED CROSS CORRELATION

$$ncc(u,v) = \frac{\sum_{(x,y)\in N}\left[I(u+x,v+y)-\bar{I}\right]\left[P(x,y)-\bar{P}\right]}{\sqrt{\sum_{(x,y)\in N}\left[I(u+x,v+y)-\bar{I}\right]^2 \sum_{(x,y)\in N}\left[P(x,y)-\bar{P}\right]^2}}$$

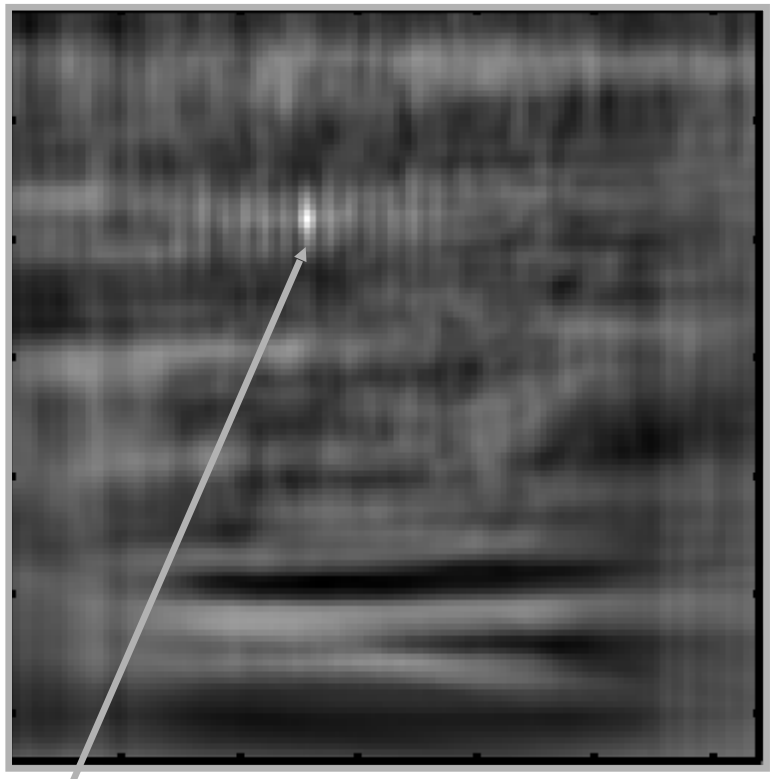
- Between -1 and 1
- Invariant to linear transforms
- Independent of the average gray levels of the pattern and the image window

NORMALIZED EXAMPLE

Image

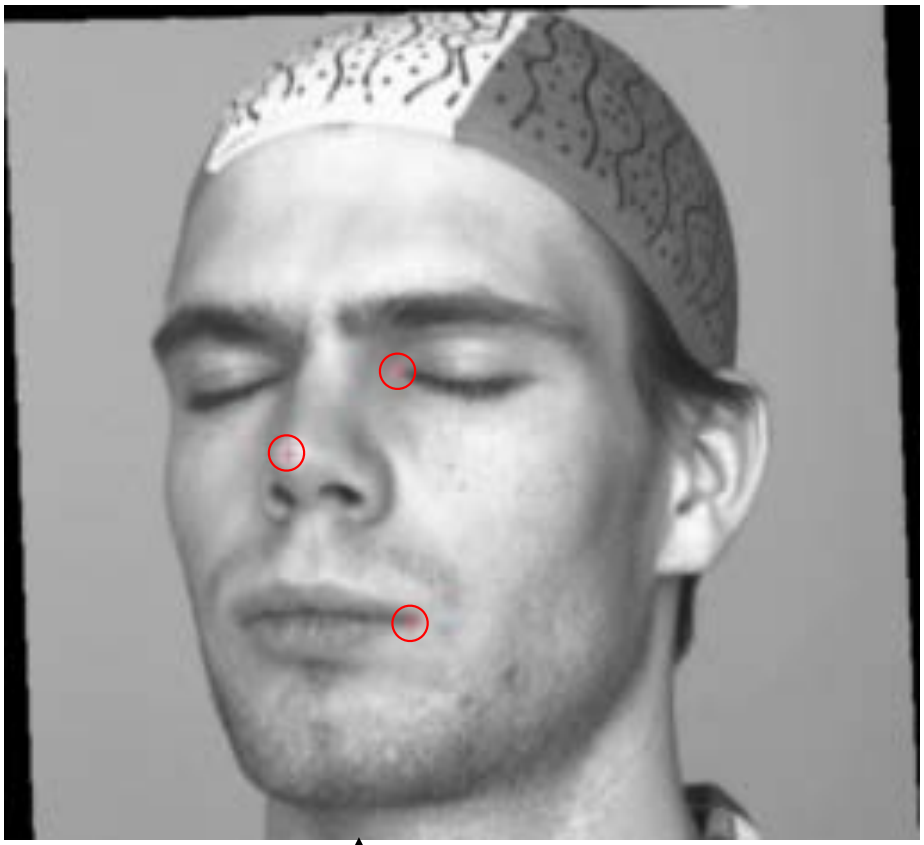
Pattern

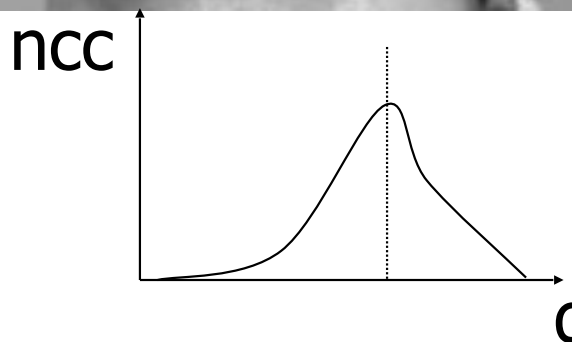
Normalized Correlation



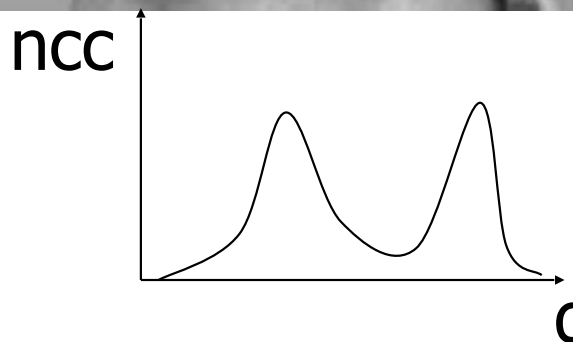
Point of maximum correlation

SEARCHING ALONG EPIPOLAR LINES





or

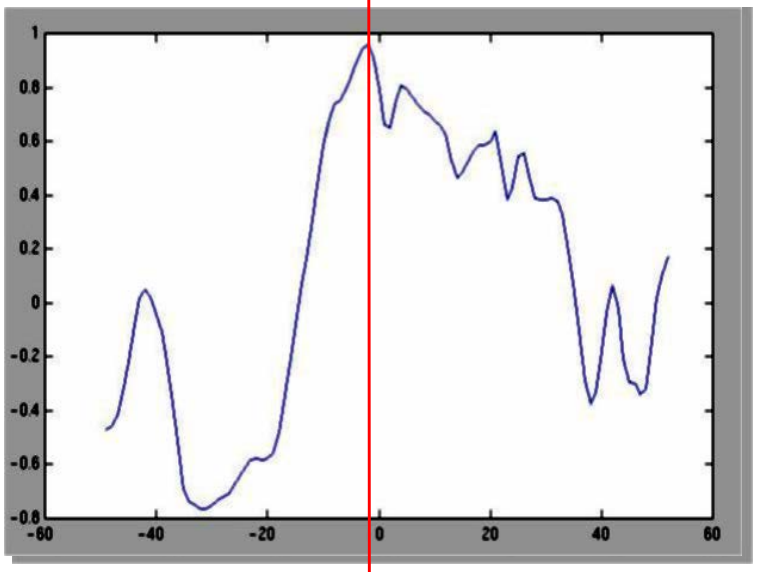


OUTDOOR SCENE

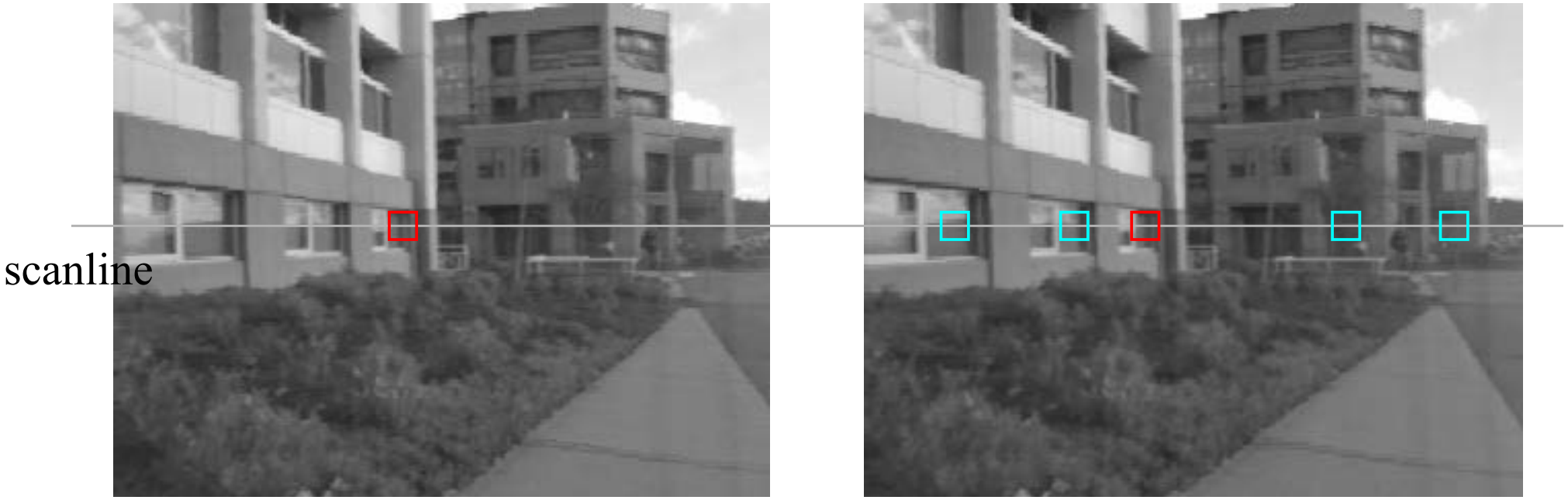
scanline

SSD

NCCR

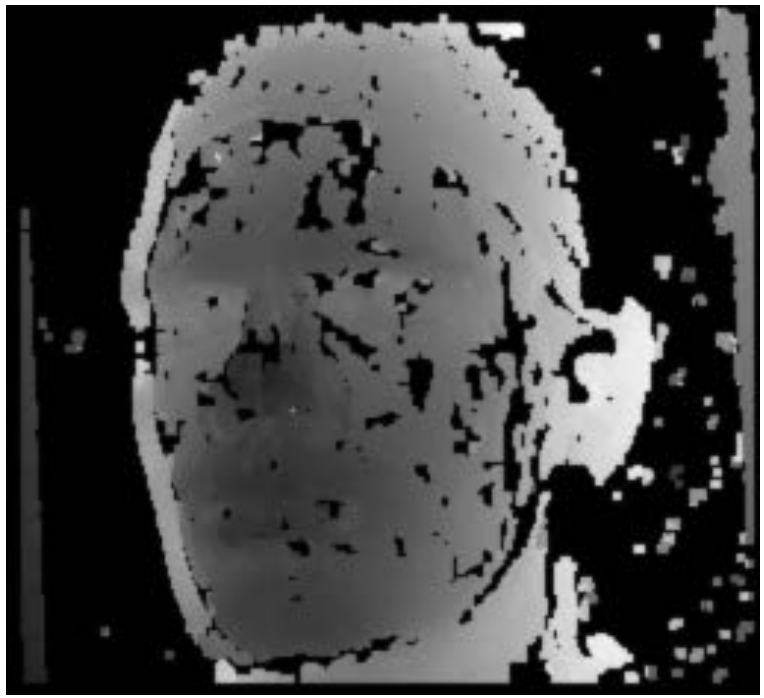


AMBIGUITIES



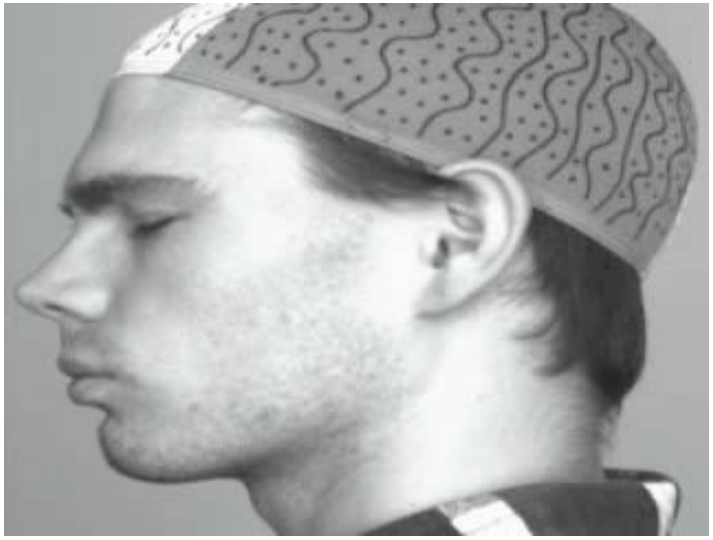
Repetitive patterns, textureless areas, and occlusions cause problems.

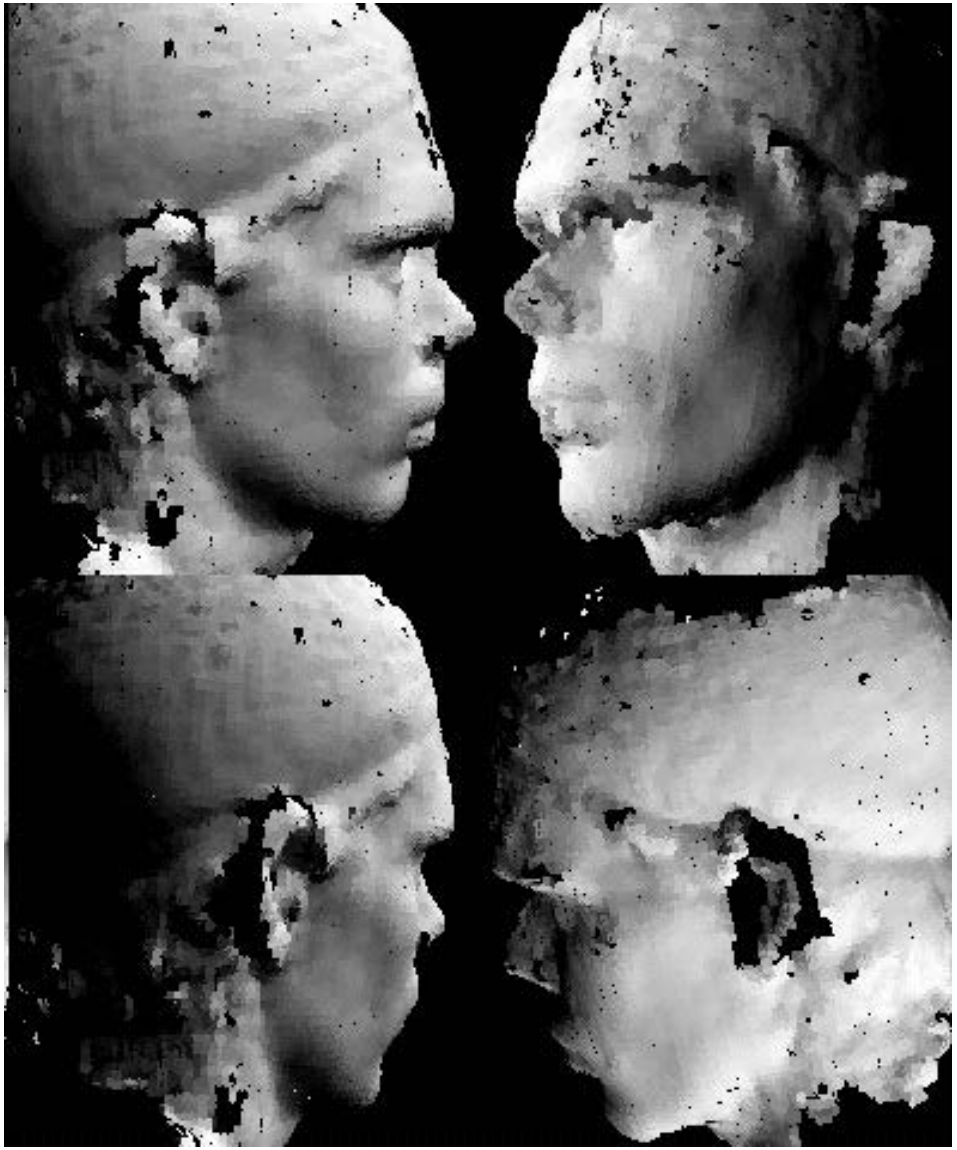
DISPARITY MAP



Black pixels: No disparity.

SHAPE FROM VIDEO



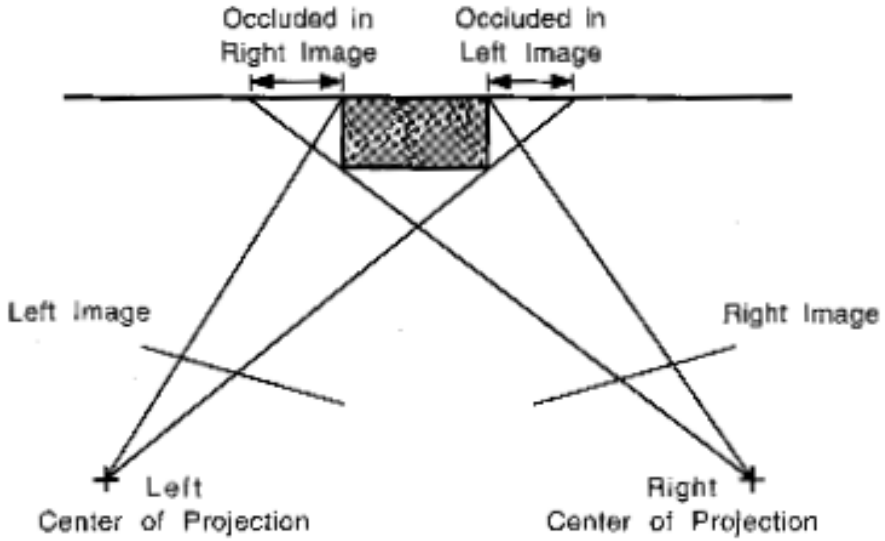


Treat consecutive images as stereo pairs.

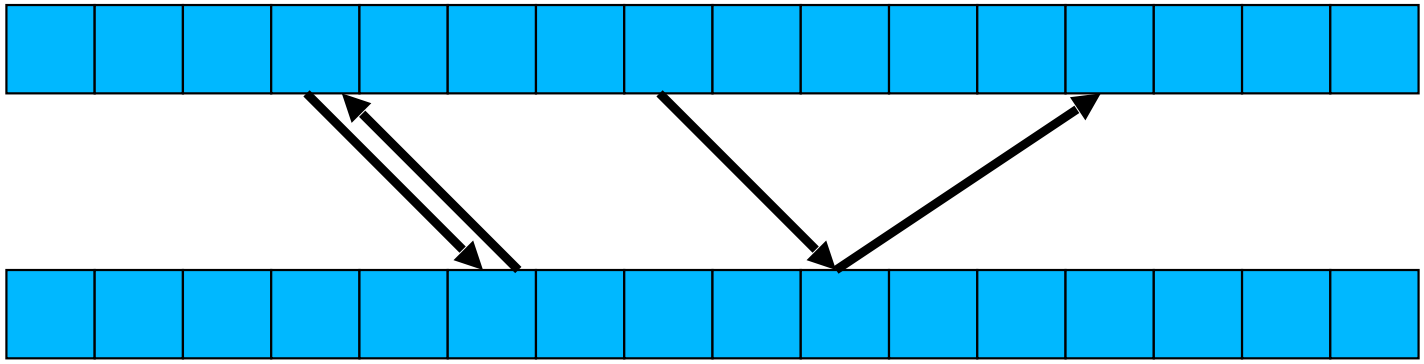
1. Compute disparity maps.
2. Merge 3-D point clouds.
3. Represent as particles.

Fua. IJCV'97

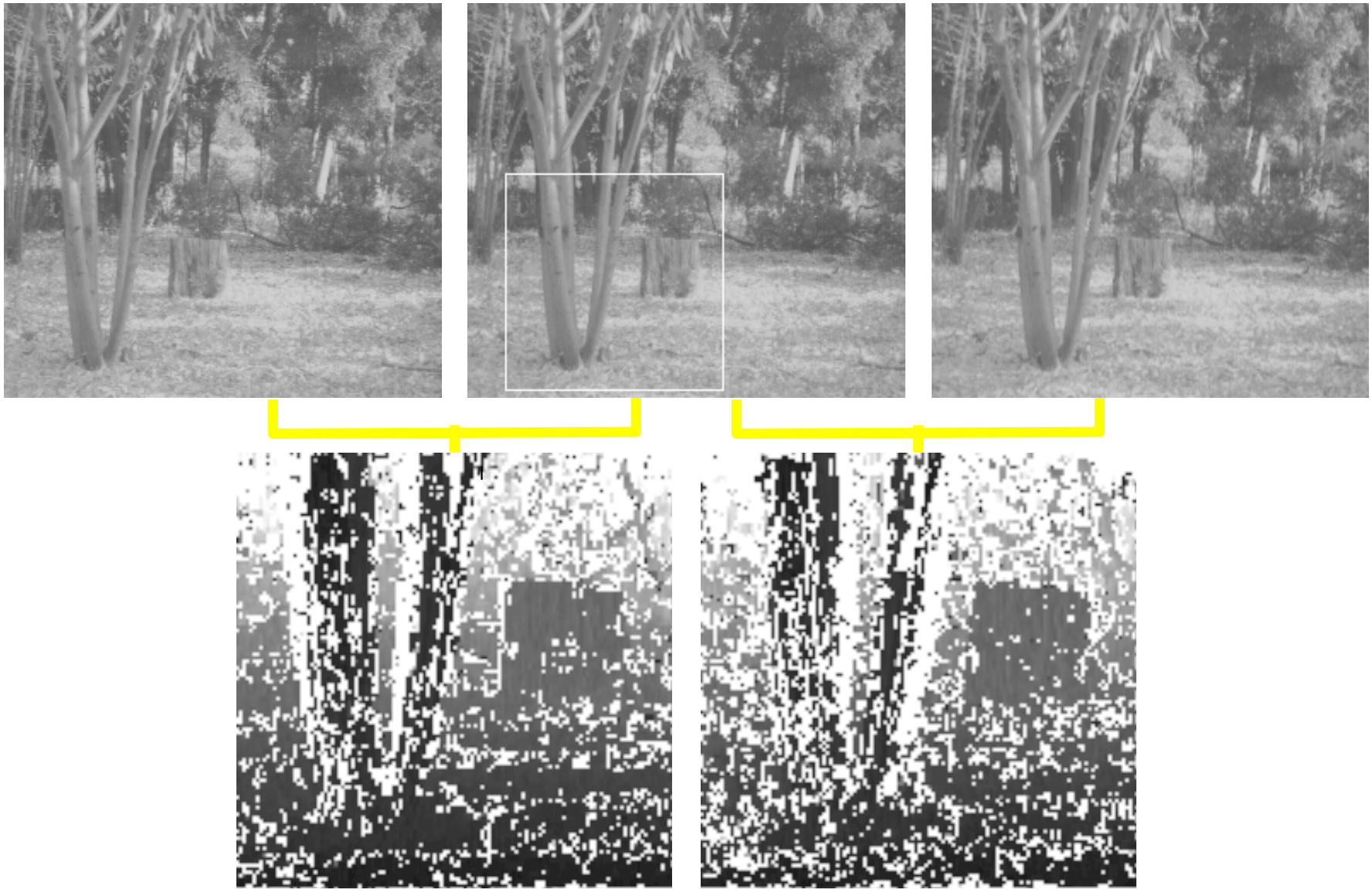
OCCLUSIONS



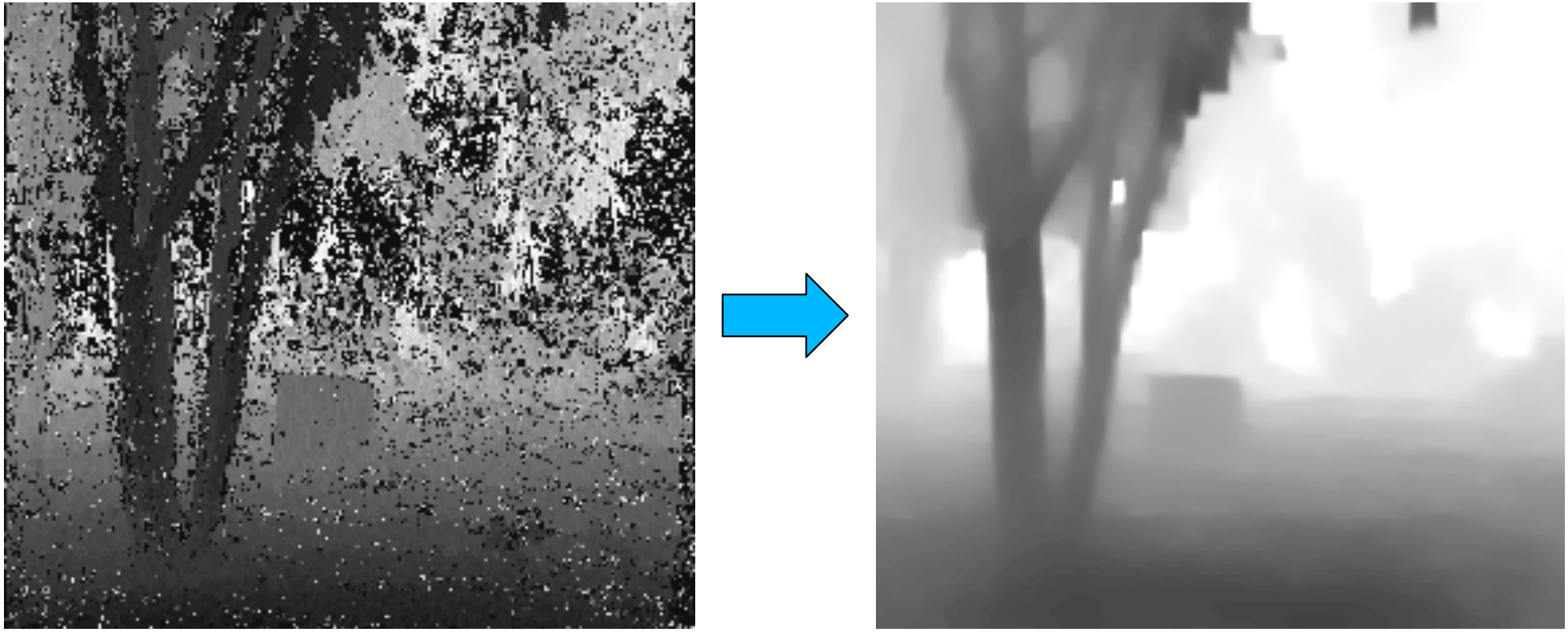
→ Consistency test



GROUND LEVEL STEREO



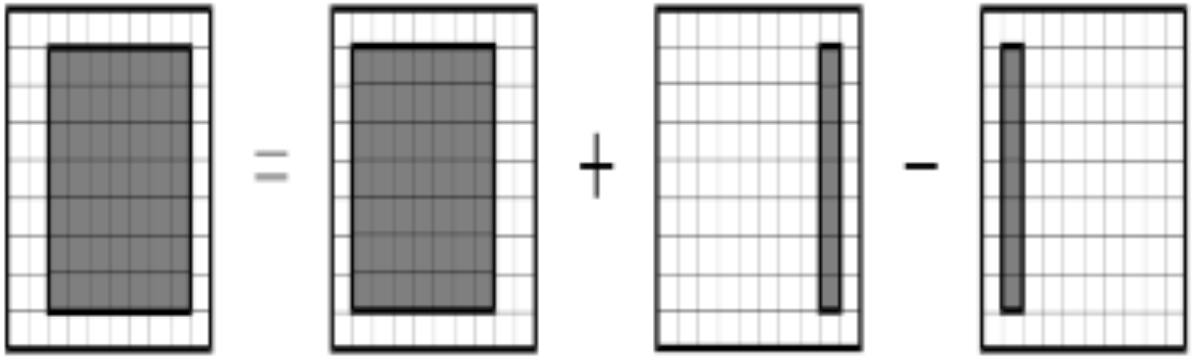
COMBINING DISPARITY MAPS



- Merging several disparity maps.
- Smoothing the resulting map.

Fua, MVA'91

REAL-TIME IMPLEMENTATION



$$C(x,y,d) \propto \frac{\sum_{i,j} I_1(x+i,y+j) \times I_2(x+d+i,y+j)}{\sqrt{\sum_{i,j} I_2(x+d+i,y+j)^2}}$$

$$C(x+1,y,d) \propto \frac{\sum_{i,j} I_1(x+1+i,y+j) \times I_2(x+1+d+i,y+j)}{\sqrt{\sum_{i,j} I_2(x+1+d+i,y+j)^2}}$$

$$\propto \frac{\sum_{i',j} I_1(x+i',y+j) \times I_2(x+d+i',y+j)}{\sqrt{\sum_{i,j} I_2(x+d+i',y+j)^2}}$$

VARIATIONAL APPROACH

$$\mathcal{C} = \int s(w - w_0)^2 + \lambda_x (\frac{\partial w}{\partial x})^2 + \lambda_y (\frac{\partial w}{\partial y})^2$$

$$s = \text{Correlation score if } w_0 \text{ has been measured, 0 otherwise.}$$

$$\lambda_x = c_x f(\frac{\partial I}{\partial x})$$

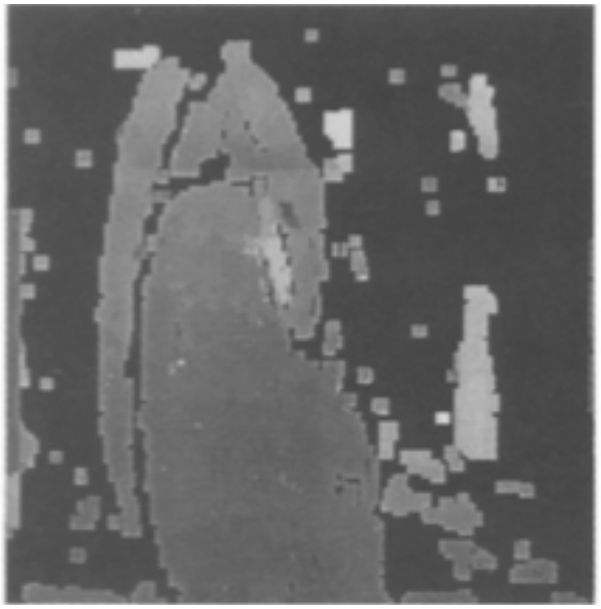
$$\lambda_y = c_y f(\frac{\partial I}{\partial y})$$

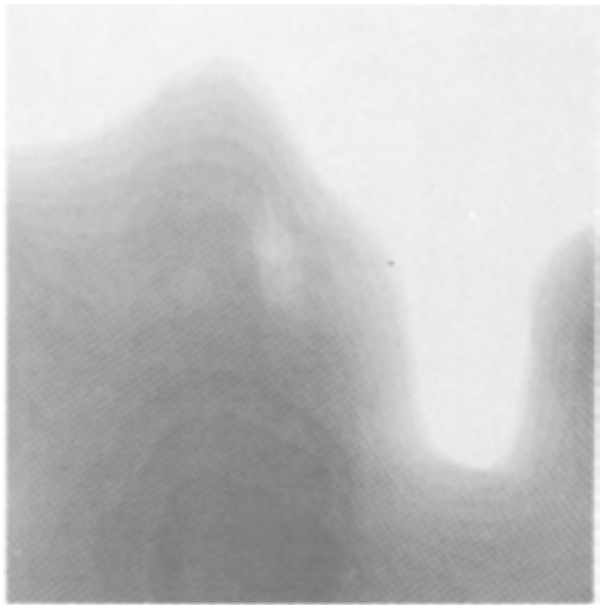
$$f(x) = \begin{cases} 1 & \text{if} \quad x < x_0 \\ \frac{x_1 - x}{x_1 - x_0} & \text{if} \quad x_0 < x < x_1 \\ 0 & \text{if} \quad x_1 < x \end{cases}$$

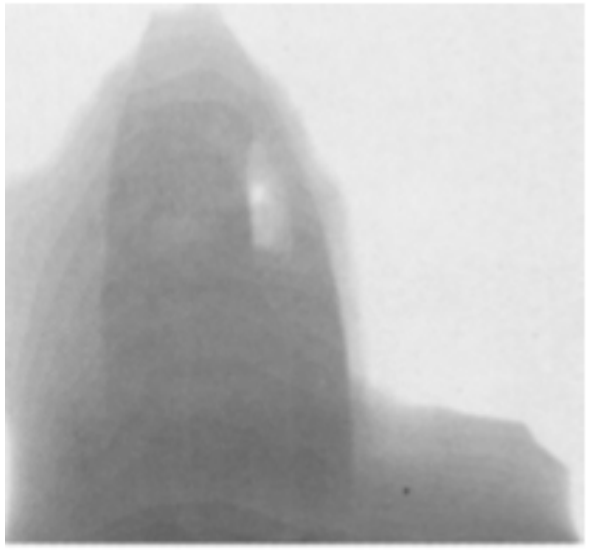
DISCRETIZATION

$$
\begin{aligned}
\mathcal{C} &= \sum_{ij} s_{ij}(w_{ij} - w_{0ij})^2 + \lambda_x \sum_{ij}(w_{i+1,j} - w_{i,j})^2 + \lambda_y \sum_{ij}(w_{i,j+1} - w_{i,j})^2 \\
&= (W - W_0)^t S(W - W_0) + W^t K W \\
\Rightarrow \quad \frac{\partial \mathcal{C}}{\partial W} &= 0 \\
\Rightarrow \quad (K + S)W &= SW_0
\end{aligned}
$$

PRESERVING DISCONTINUITIES



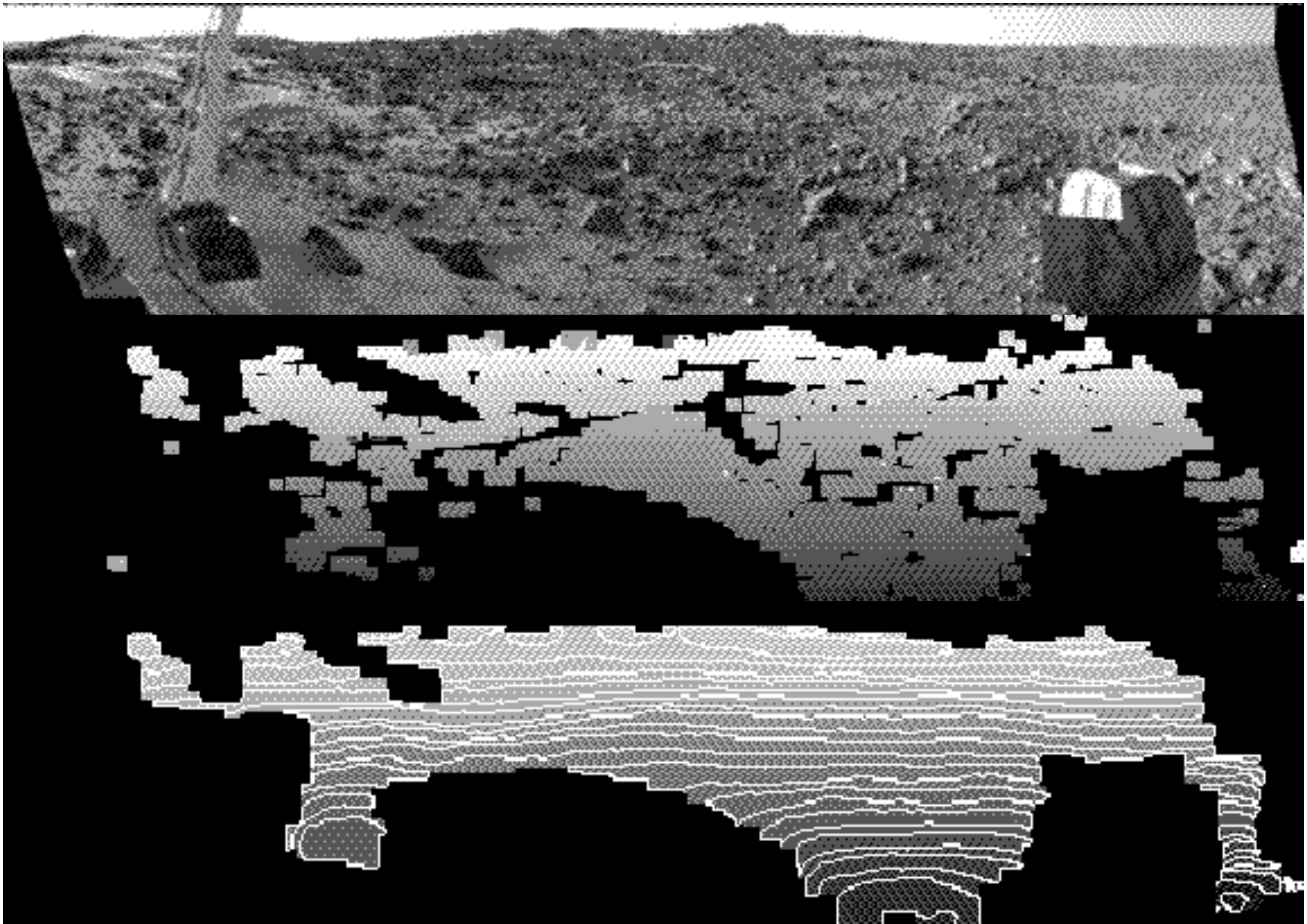




$$\lambda_x = f(\frac{\partial I}{\partial x}) f(\frac{\partial w}{\partial x})^2$$

$$\lambda_y = f(\frac{\partial I}{\partial y}) f(\frac{\partial w}{\partial y})^2$$

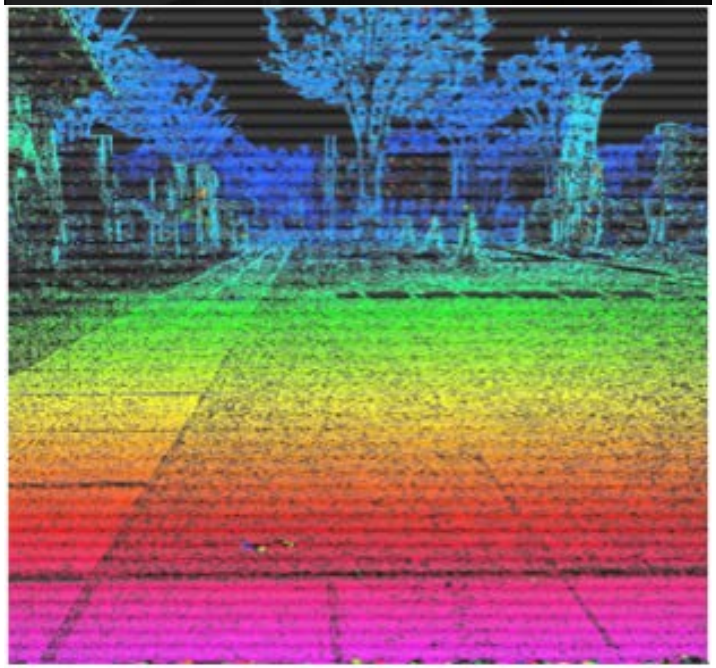
THEN ....



1993:
256x256,
60 disps,
7 fps.

Faugeras et al.,
INRIA'93

… AND MORE RECENTLY



Subaru's EyeSight System

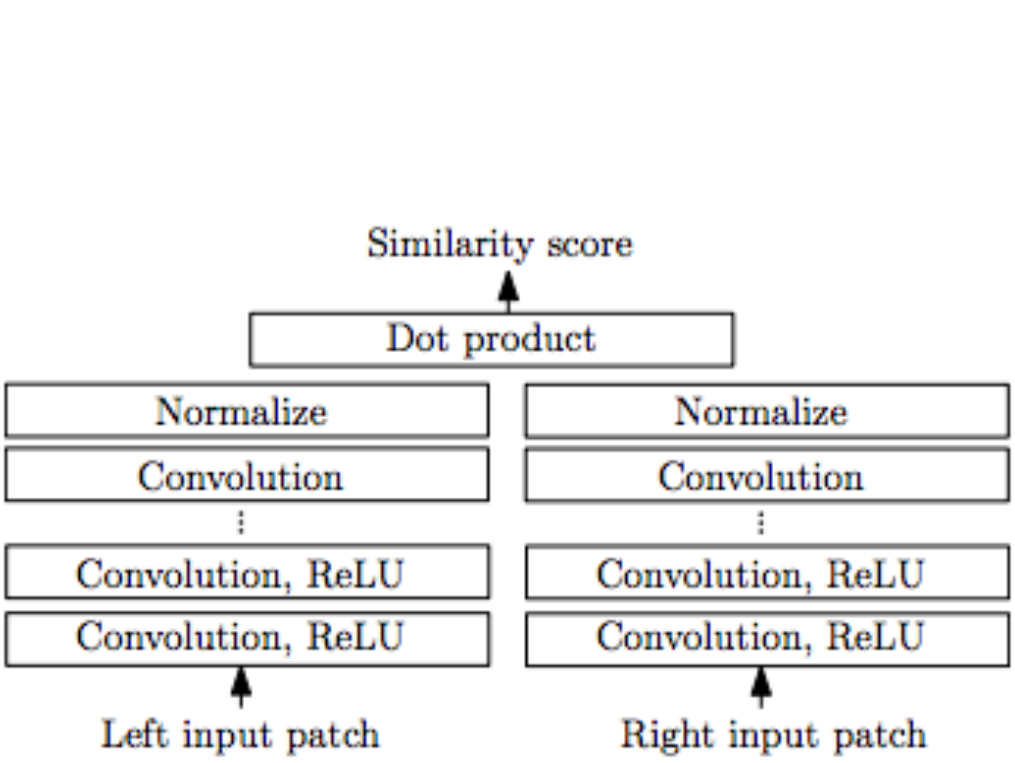
http://www.gizmag.com/subaru-new-eyesight-stereoscopic-vision-system/14879/

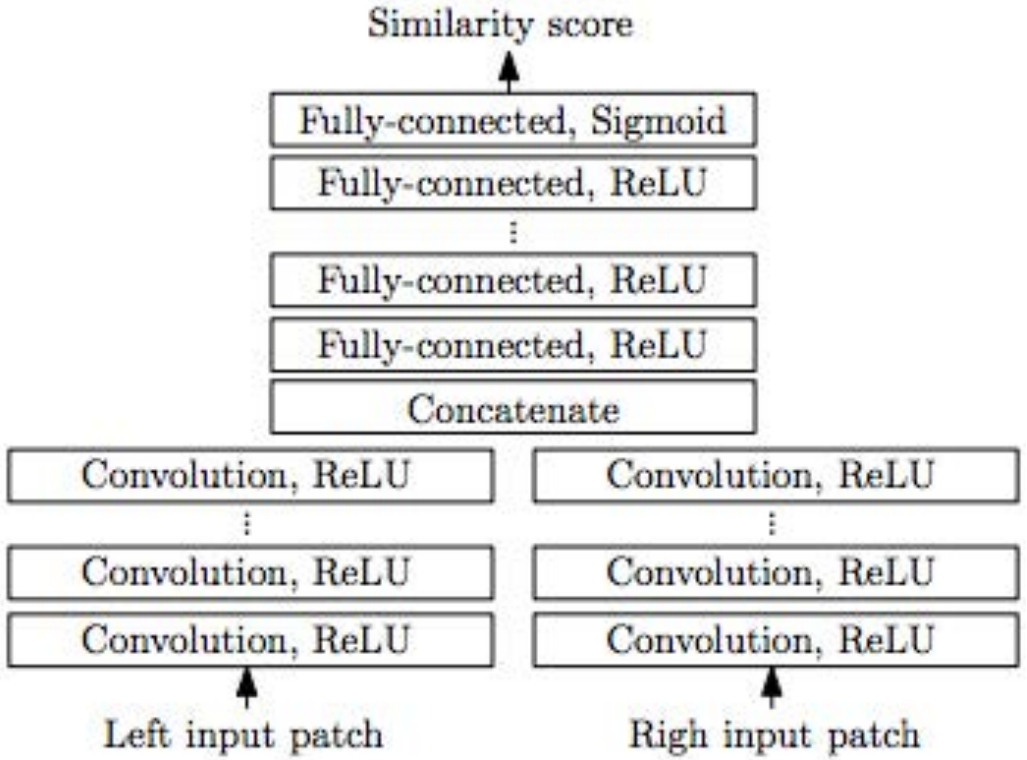
2011:
1312x688,
176 disps,
160 fps.

Saneyoshi, CMVA'11

... AND EVEN MORE RECENTLY

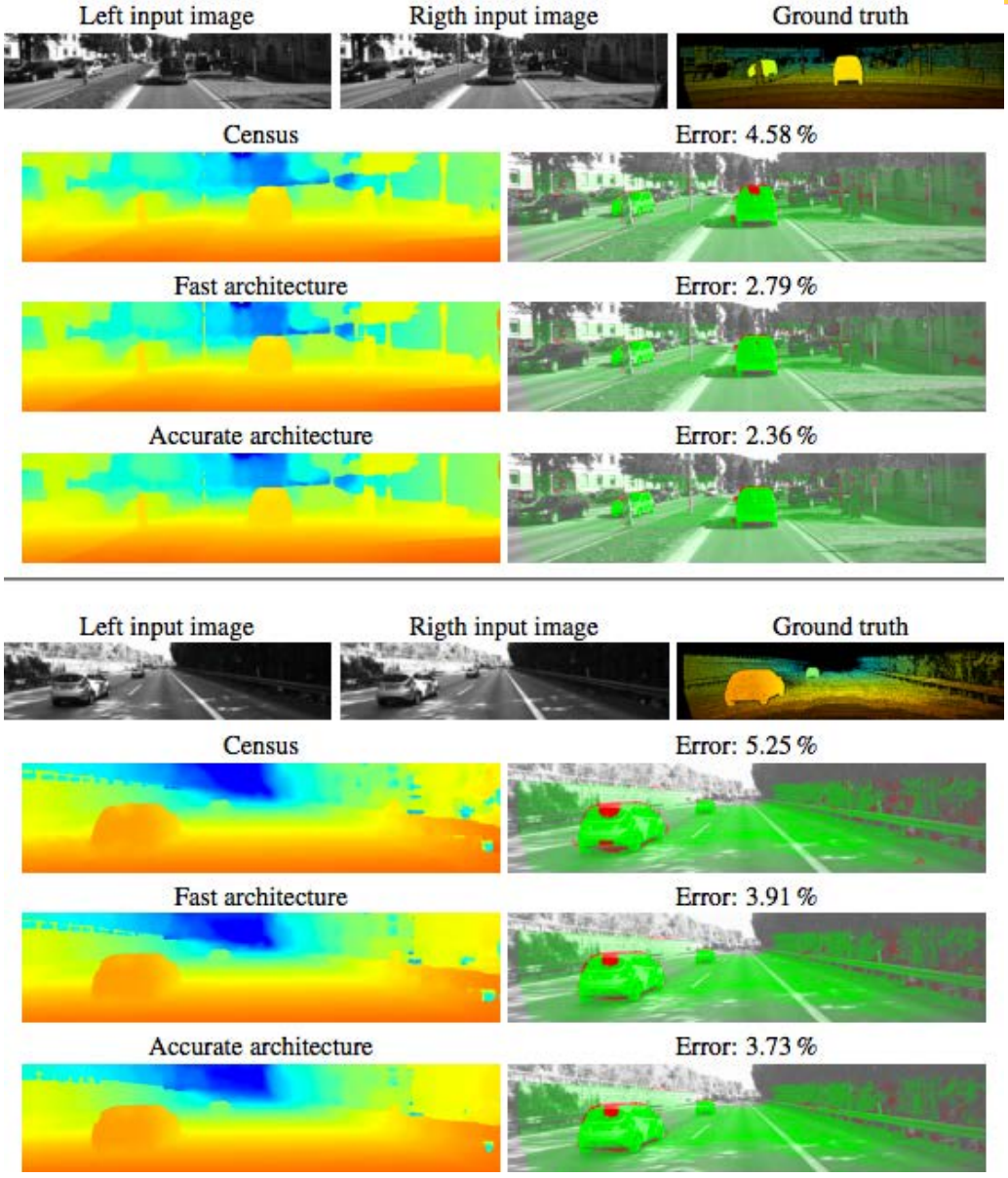
Train Siamese nets to return a similarity score.





Zbontar and Lecun, 2015

COMPARATIVE RESULTS



Improved performance on test data but

- How well will it generalize to unseen images?
- Is it worth the much heavier computational load?

Time will tell.

WINDOW SIZE

Small windows:

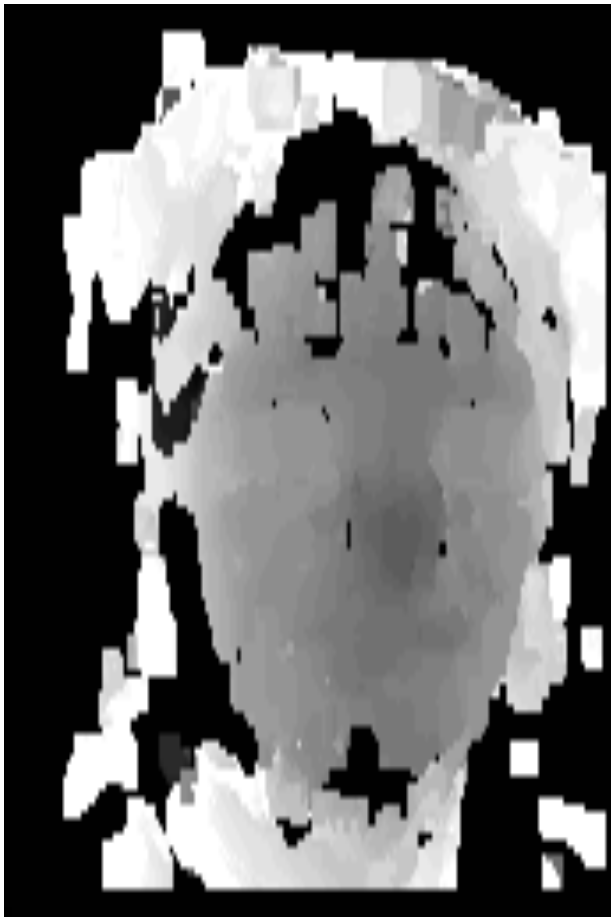
- Good precision
- Sensitive to noise

Large windows:

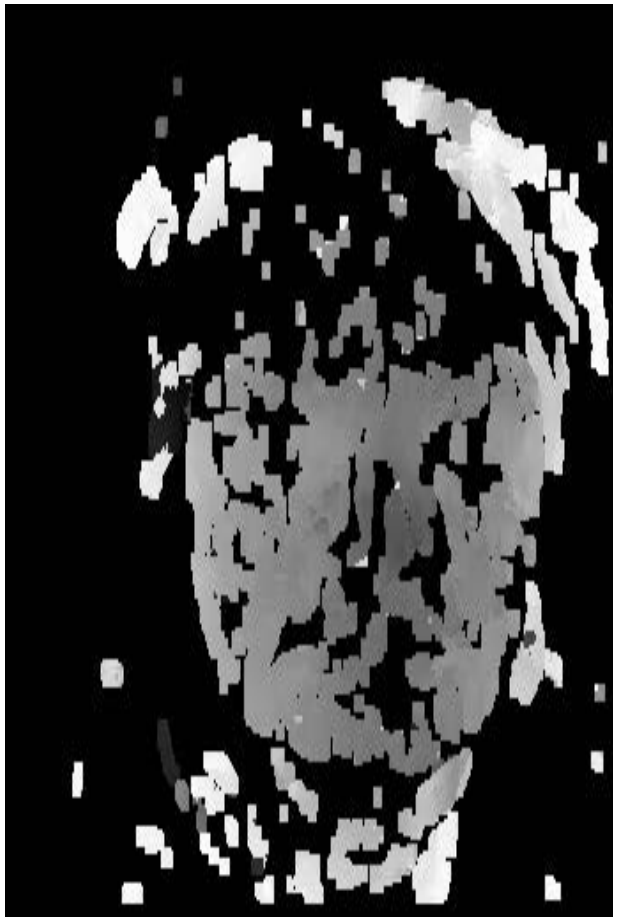
- Diminished precision
- Increased robustness to noise

→ Same kind of trade-off as for edge-detection.

WINDOW SIZE



15x15



7x7

SCALE-SPACE REVISITED

Gaussian pyramid

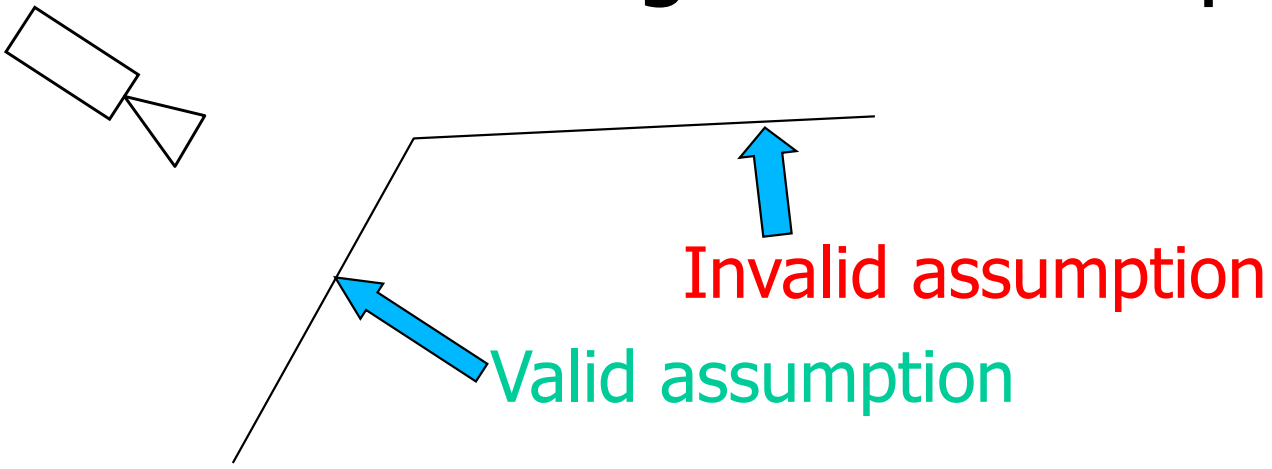
Difference of Gaussians

- Using a small window on a reduced image is equivalent to using a large one on the original image.
- Using difference of Gaussian images is an effective way of achieving normalization.

→It becomes natural to use results obtained using low resolution images to guide the search at higher resolution.

FRONTO-PARALLEL ASSUMPTION

The disparity is assumed to be the same in the whole correlation window, which is equivalent to assuming constant depth.



→ Ok when the surface faces the camera but breaks down otherwise.

MULTI-VIEW STEREO

Multi-view reconstruction setup

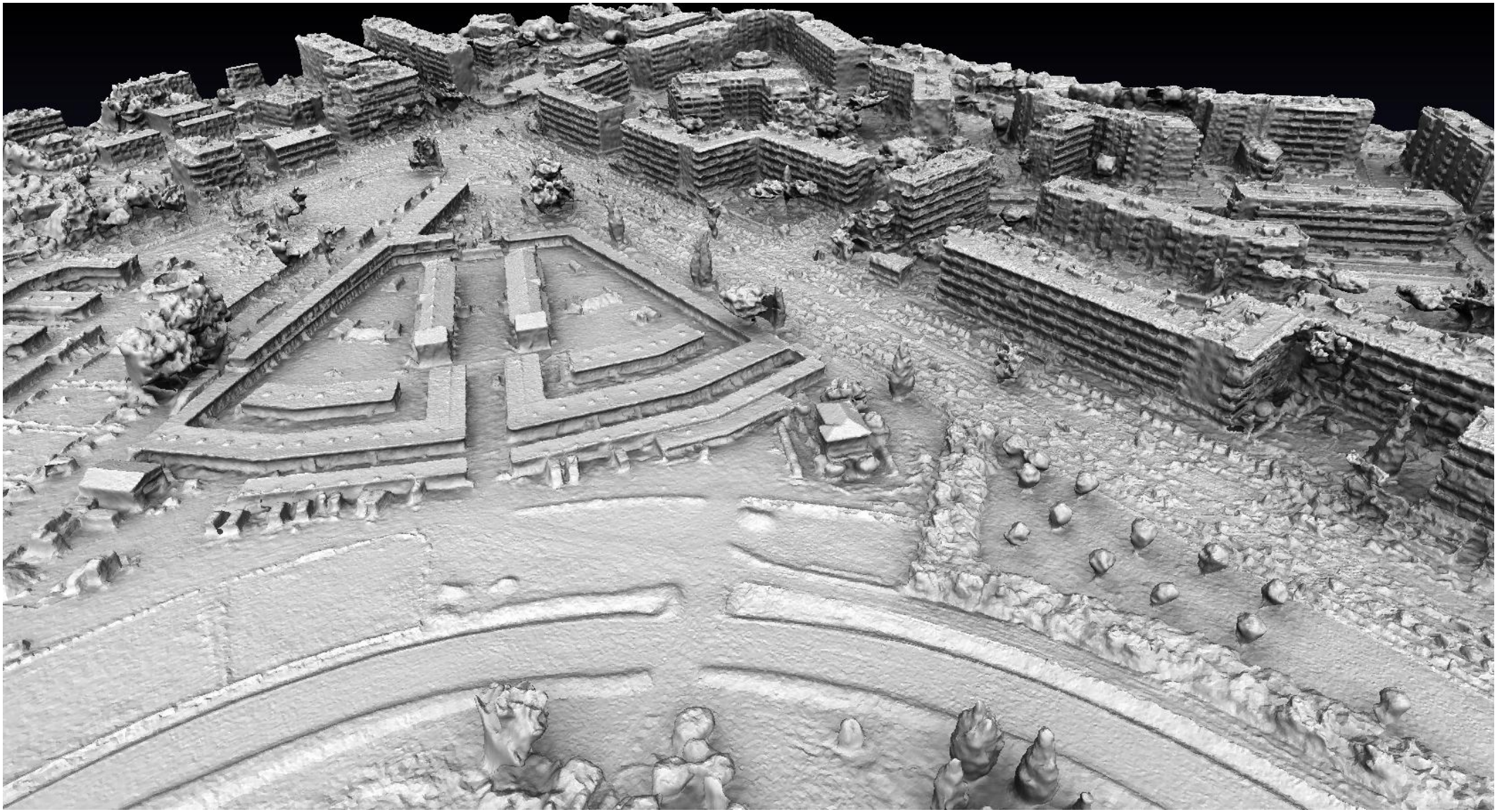
Textured 3D Model

Shaded 3D Model

➢ Adjust correlation window shapes to handle orientation.

Furukawa&Ponce ECCV'06

MULTI-VIEW STEREO



Aganj et al. ICCV'09

SMALL DRONES

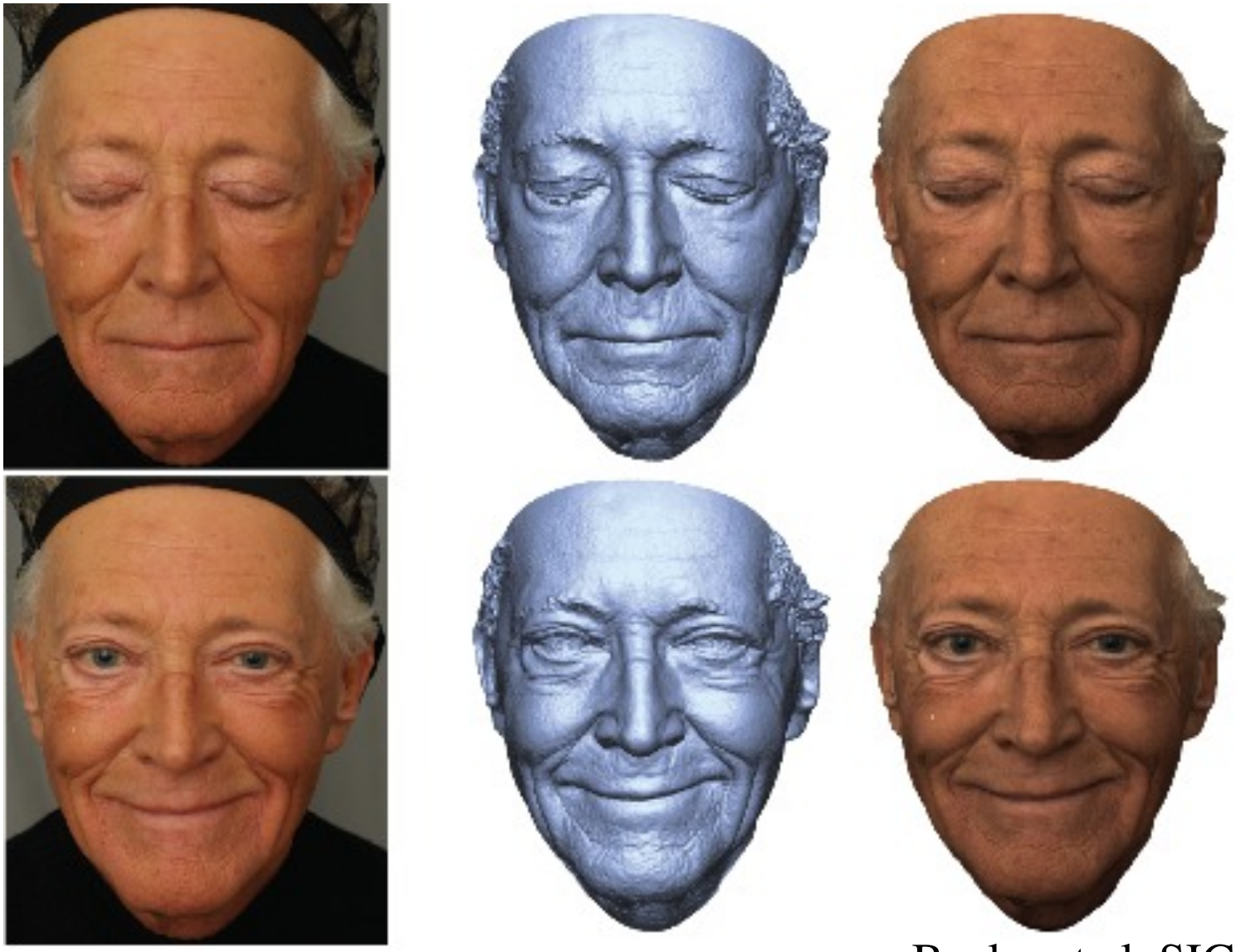
SenseFly:
www.sensefly.com

Gatewing:
www.gatewing.com

MATTERHORN

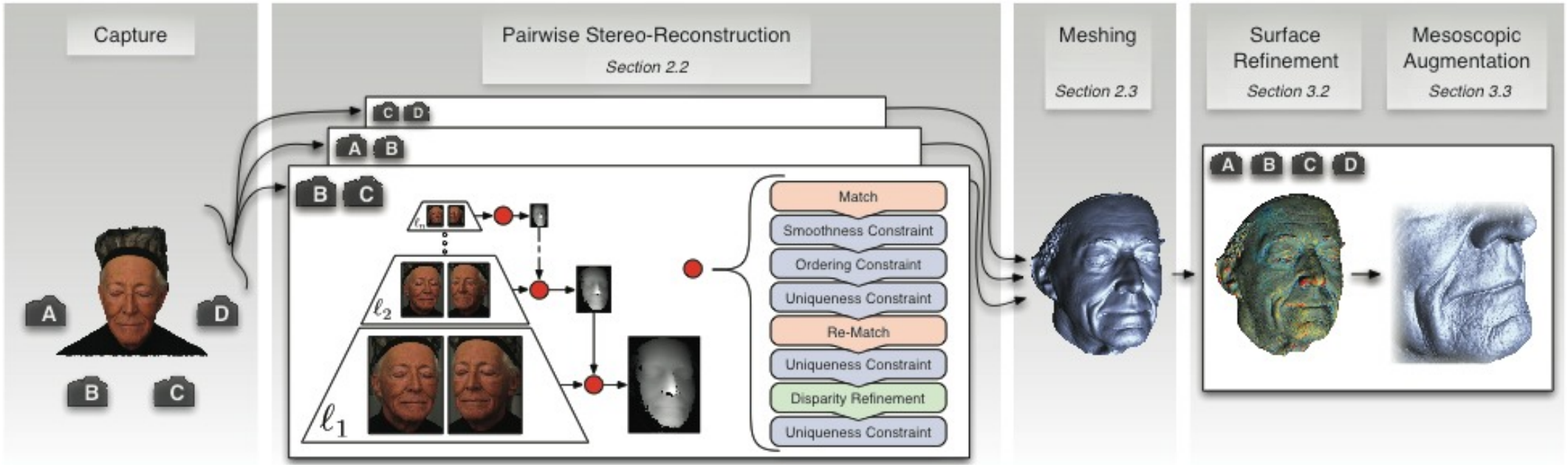
Drone: www.sensefly.com Mapping: www.pix4d.com

FACE RECONSTRUCTION



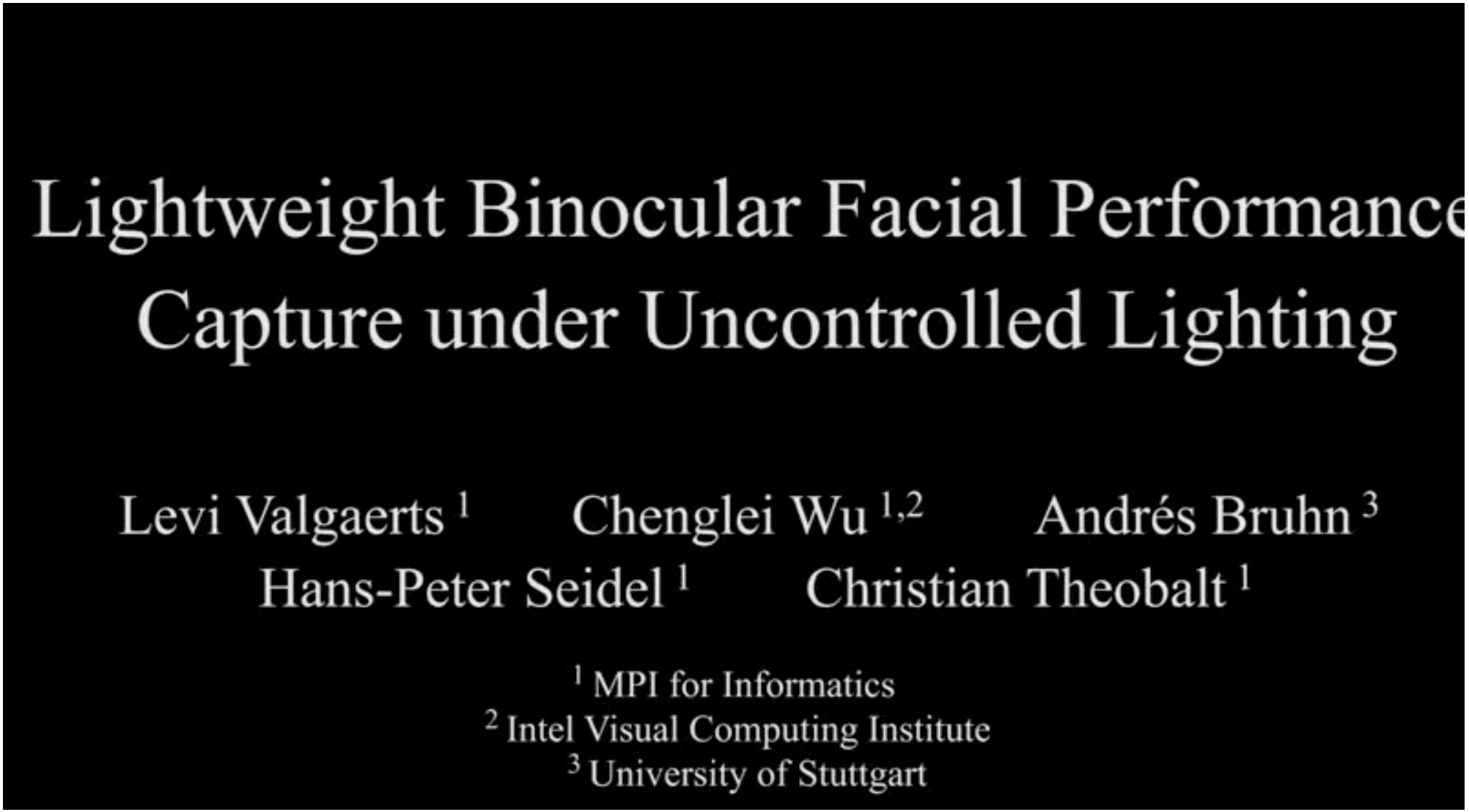
Beeler et al. SIGGRAPH'10

FACE RECONSTRUCTION



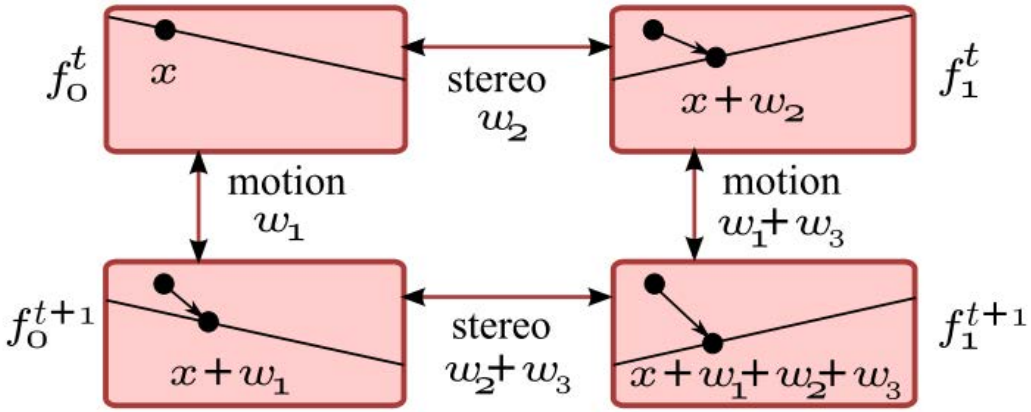
Beeler et al. SIGGRAPH'10

DYNAMIC SHAPE

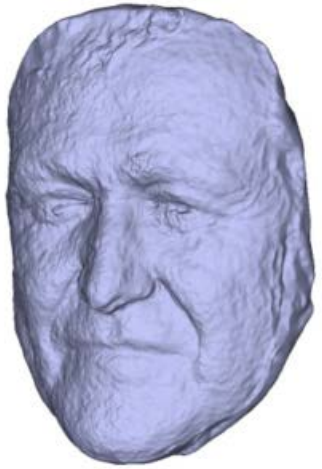


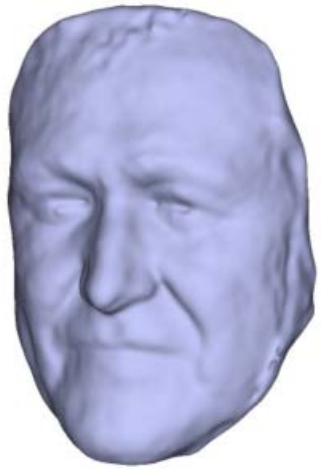
Valgaerts et al. SIGGRAPH Asia'12

SCENE FLOW



Correspondences across cameras and across time



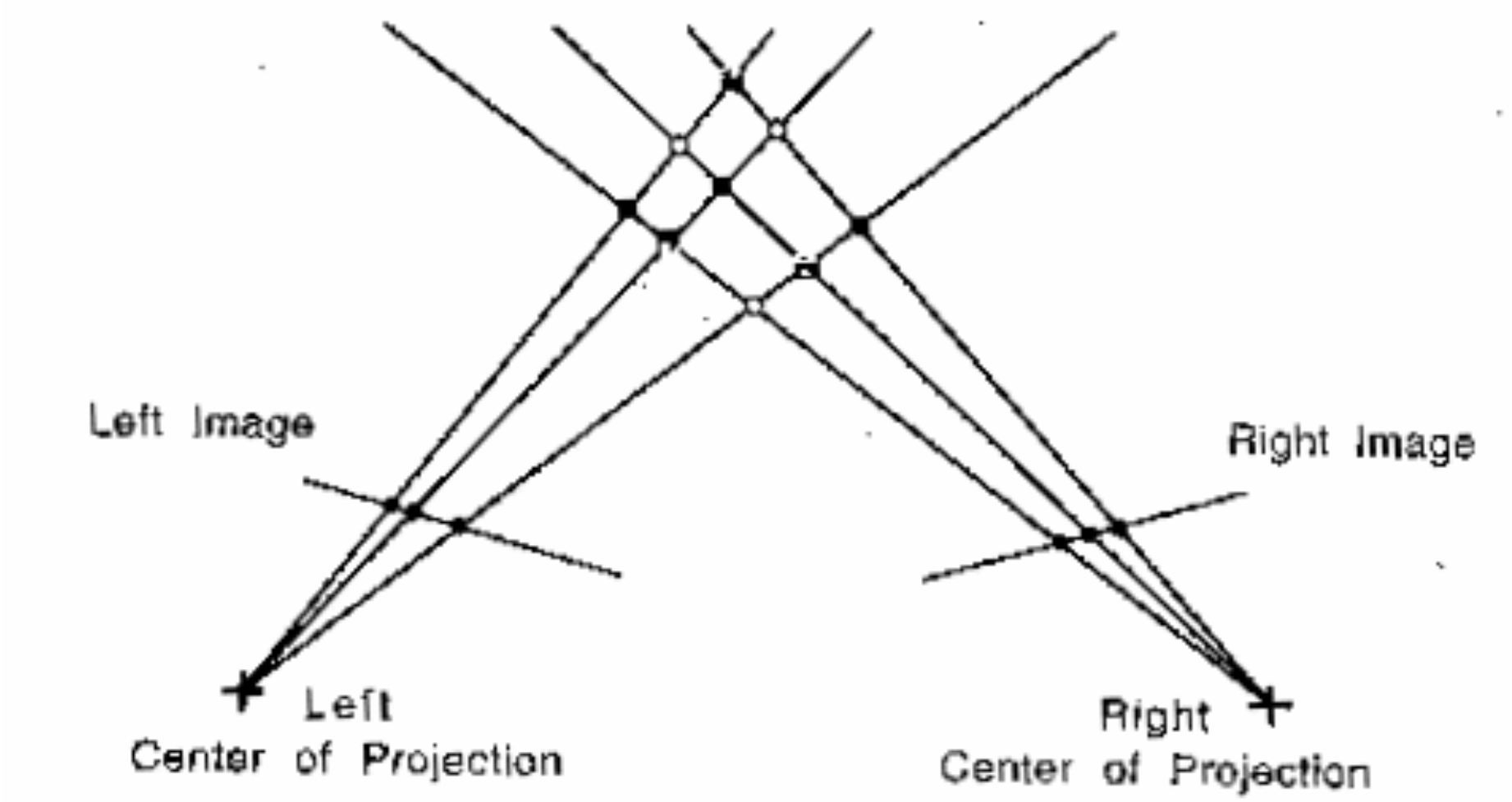


Stereo Only Stereo + Flow

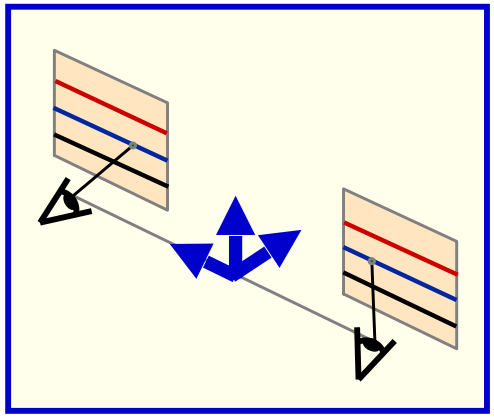
SHAPE FROM SHADING

Shape-from-shading is used to refine the shape and provide high-frequency details.

UNCERTAINTY



PRECISION vs BASELINE



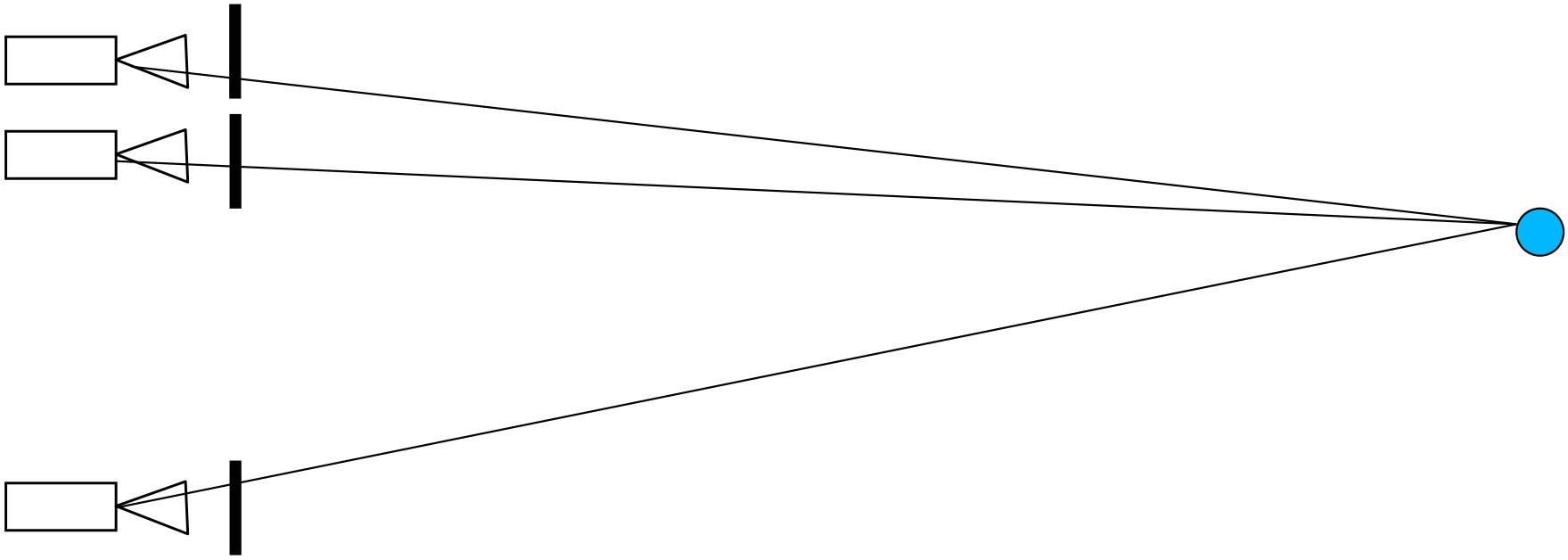
$$d = f\frac{b}{Z}$$

$$\Rightarrow Z = f\frac{b}{d}$$

$$\Rightarrow \frac{\partial Z}{\partial d} = -f\frac{b}{d^2} = -\frac{Z^2}{fb}$$

- Beyond a certain depth stereo stops being useful.
- Precision is inversely proportional to baseline length.

SHORT vs LONG BASELINE



Long baseline:

- Harder to match
- More occlusions
- Better precision

Short baseline:

- Good matches
- Few occlusions
- Poor precision

MARS ROVER

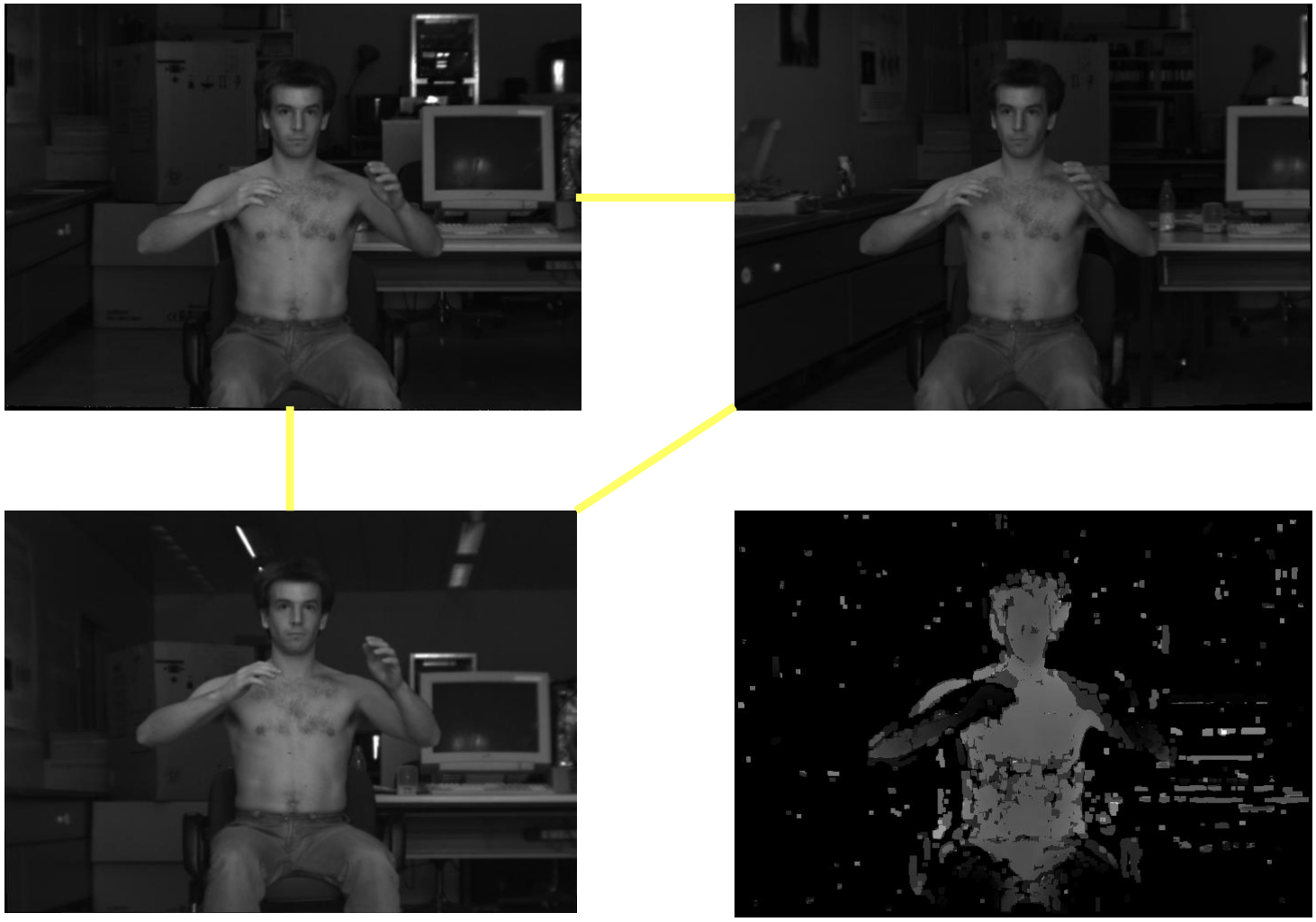
There are four cameras!

VIDEO-BASED MOTION CAPTURE

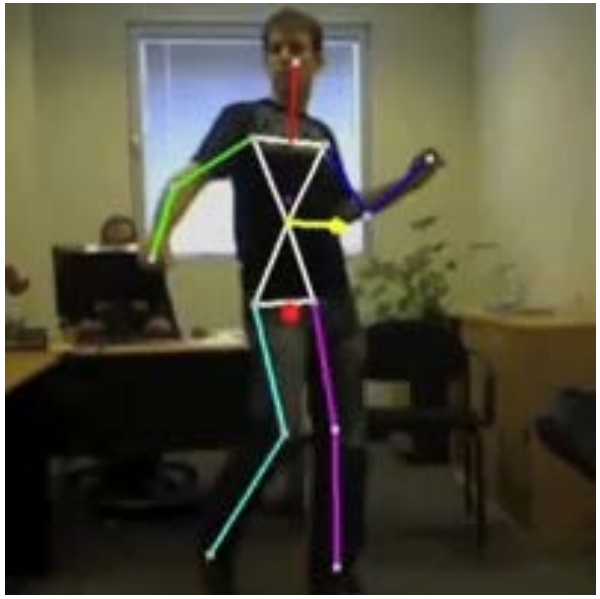
Fitting an articulated body model to stereo data.

Plankers & Fua, PAMI'03

TRINOCULAR STEREO



KINECT: STRUCTURED LIGHT

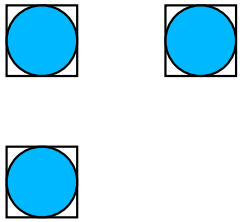


- The Kinect camera projects a IR pattern and measures depth from its distortion.
- Same principle but the second camera is replaced by the projector.

MULTI-CAMERA CONFIGURATIONS

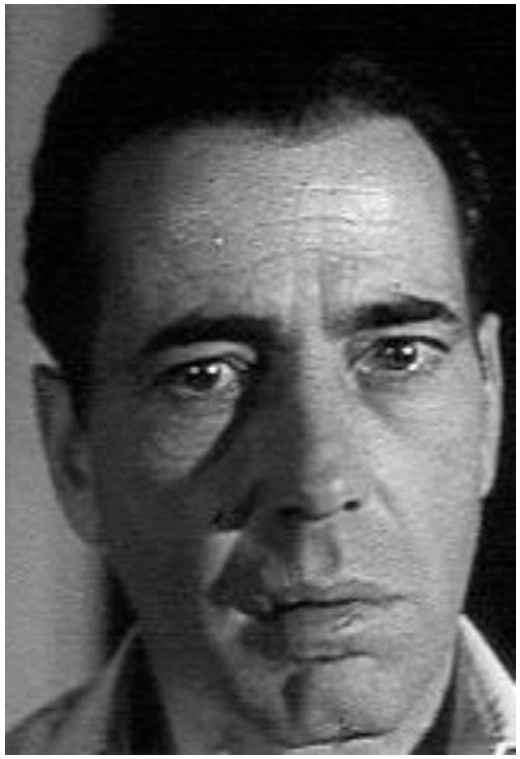
3 cameras give both robustness and precision

4 cameras give additional redundancy



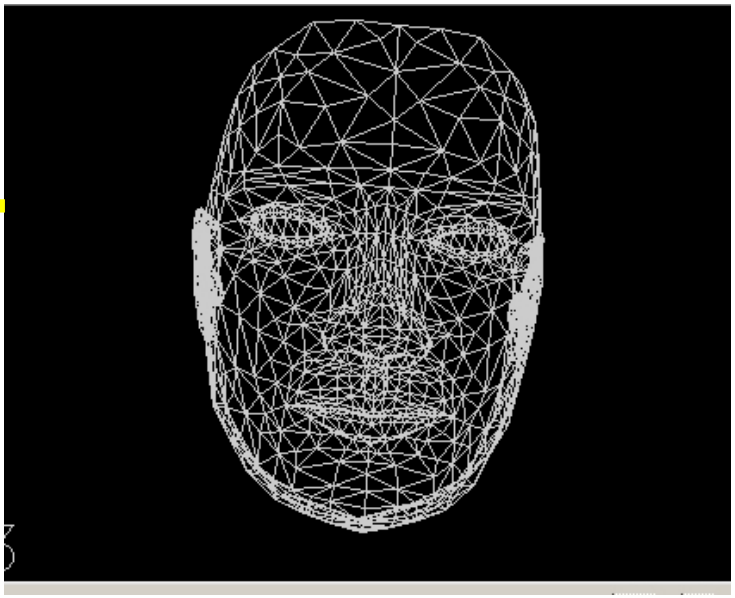
3 cameras in a T arrangement allow the system to see vertical lines.

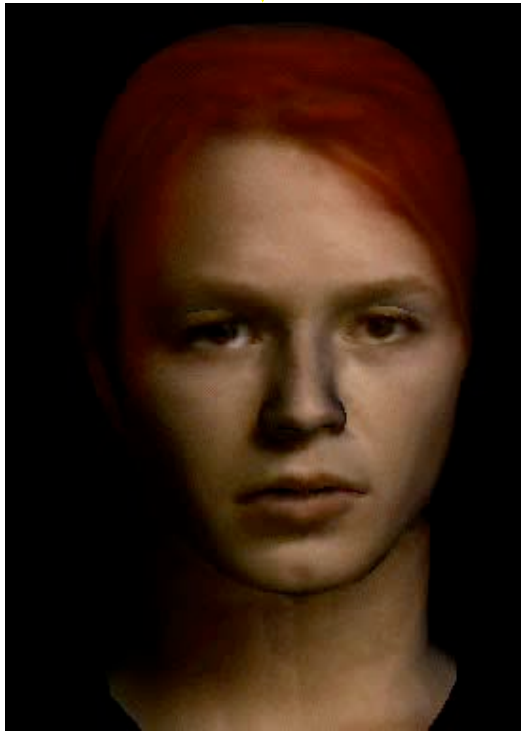
FACES FROM LOW-RESOLUTION VIDEOS



- No calibration data
- Relatively little texture
- Difficult lighting

SIMPLE FACE MODEL





PCA FACE MODEL

$$S = \overline{S} + \sum_{i=1}^{99} a_i S_i$$

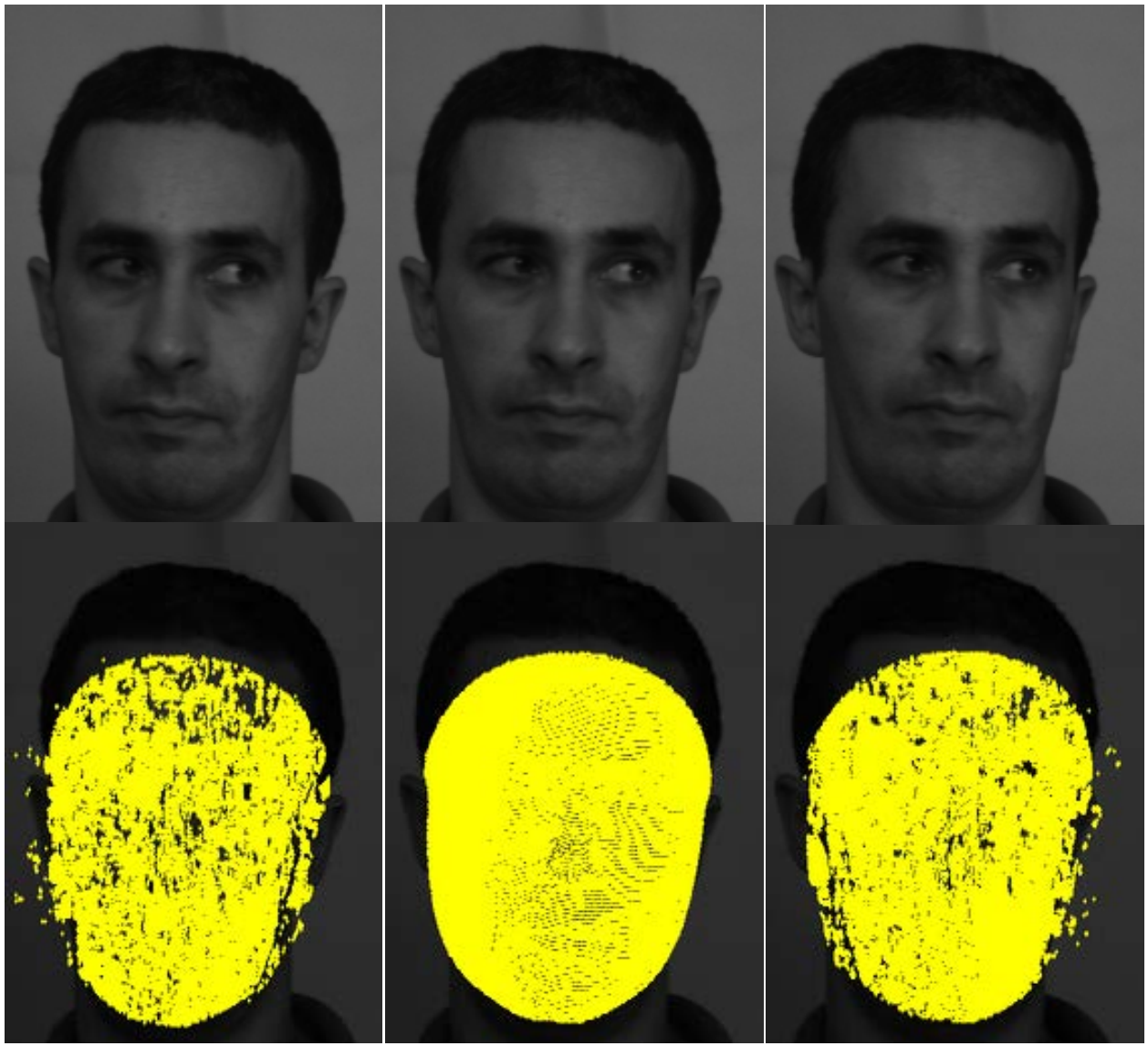
$\overline{S}$: Average shape

S_i: Shape vector

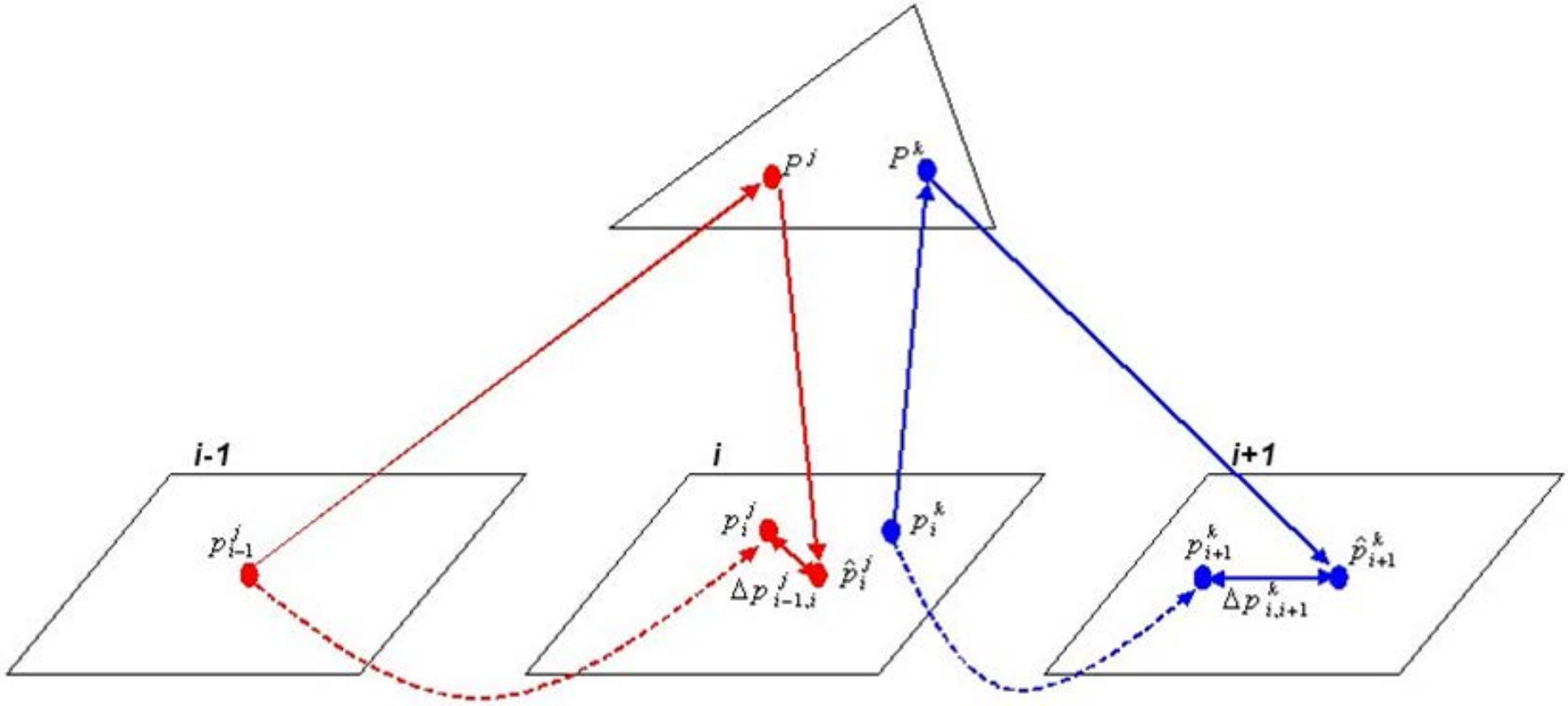
a_i: Shape coefficients

V. Blanz and T. Vetter, “A Morphable Model for the Synthesis of 3-D Faces” in Computer Graphics, SIGGRAPH Proceedings, Los Angeles, CA, August 1999.

CORRESPONDENCES

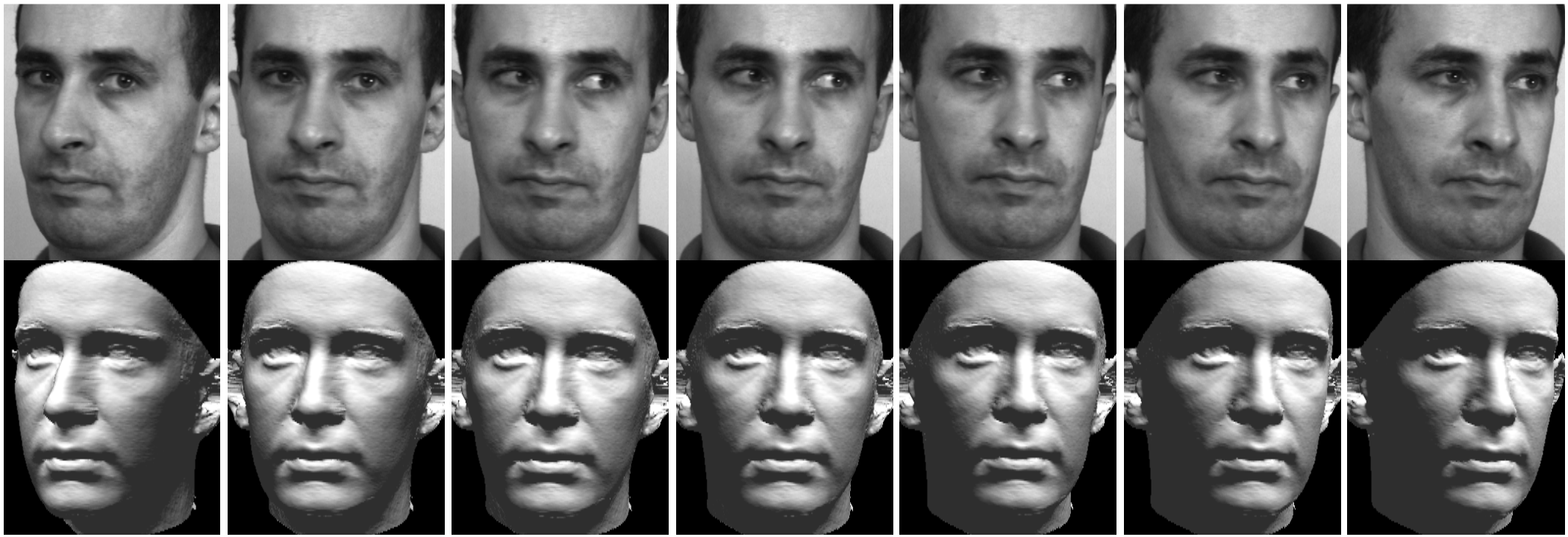


TRANSFER FUNCTION



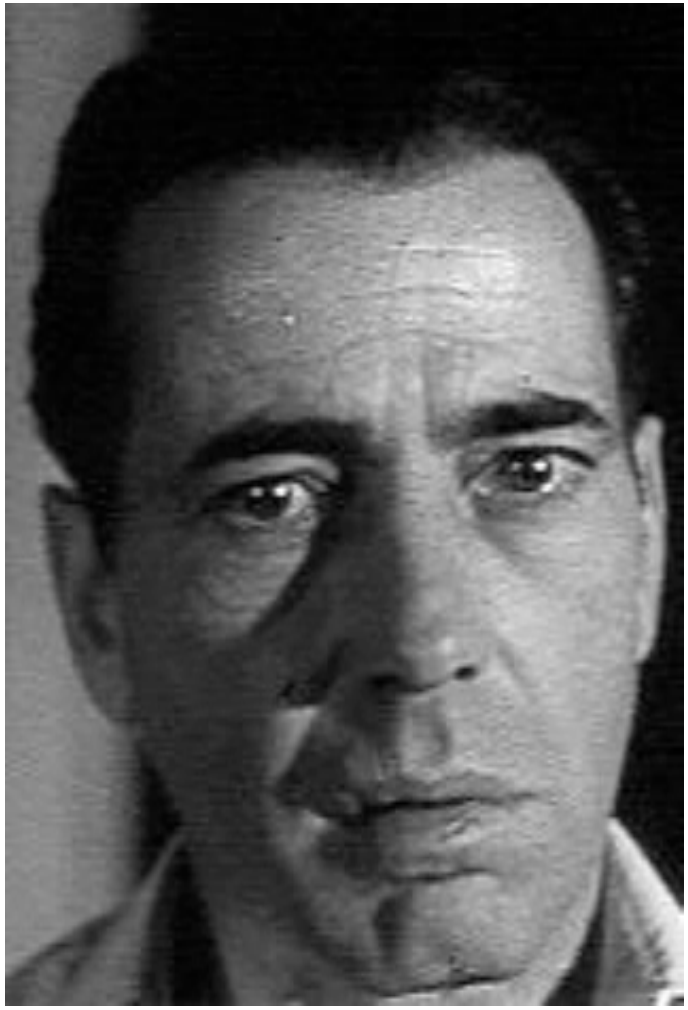
$$F_3(A, C_{i-1}, C_i, C_{i+1}) = \sum_{j \in Q_{i-1}} \left\| \Delta p_{i-1,i}^j \right\|^2 + \sum_{k \in Q_i} \left\| \Delta p_{i,i+1}^k \right\|^2$$

MODEL BASED BUNDLE ADJUSTMENT

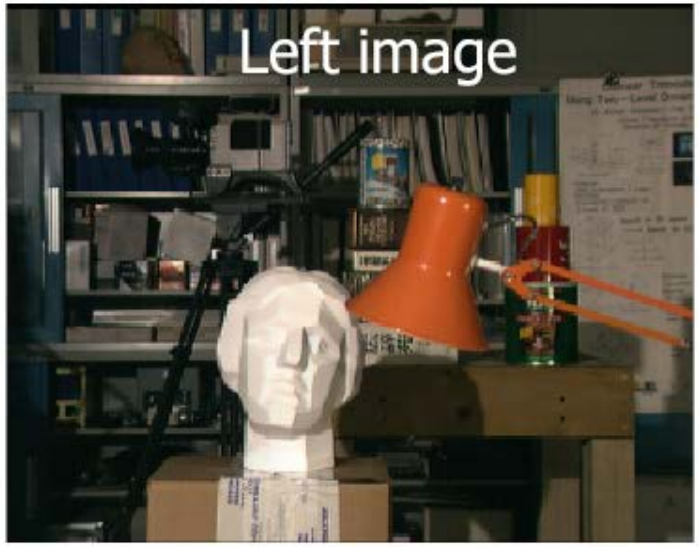


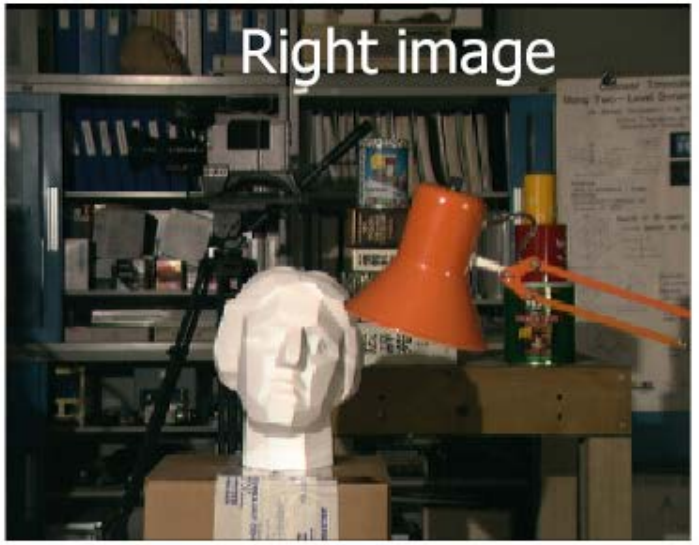
→ Median accuracy greater than 0.5mm

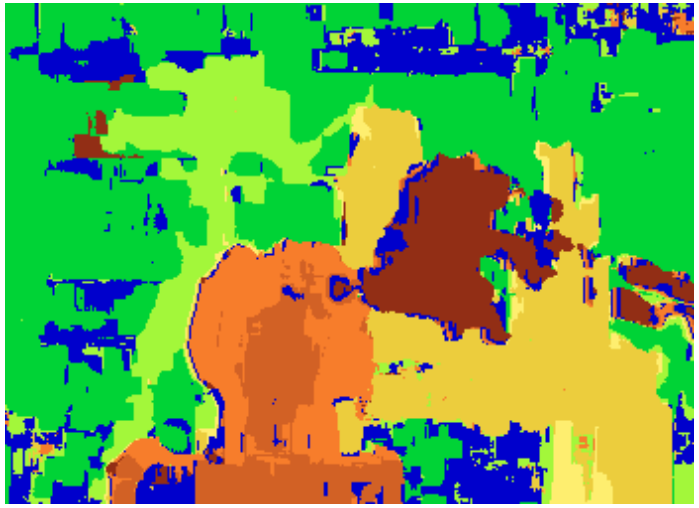
MODEL FROM OLD MOVIE



LIMITATIONS OF WINDOW BASED METHODS





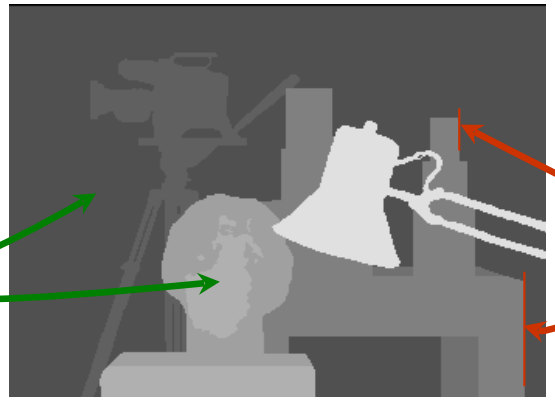


Ground truth

Correlation result

ENERGY MINIMIZATION

Disparity continuous in most places,



except at depth discontinuities

1. Matching pixels should have similar intensities.
2. Most nearby pixels should have similar disparities

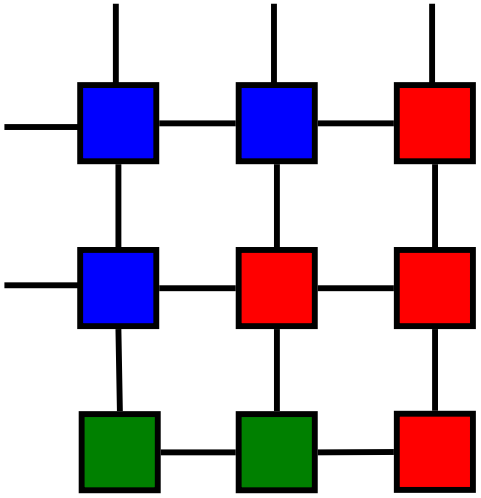
→ Minimize

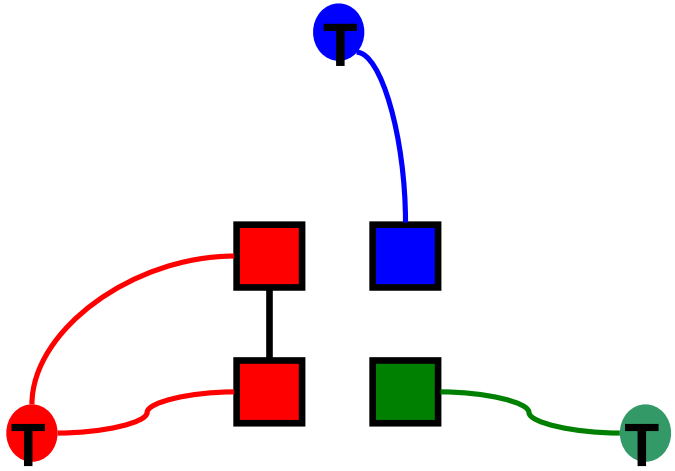
$$\sum \left[I_2(x + D(x,y), y) - I_1(x,y) \right]^2$$

$$+ l \sum \left[D(x+1,y) - D(x,y) \right]^2$$

$$+ m \sum \left[D(x,y+1) - D(x,y) \right]^2$$

GRAPH CUTS





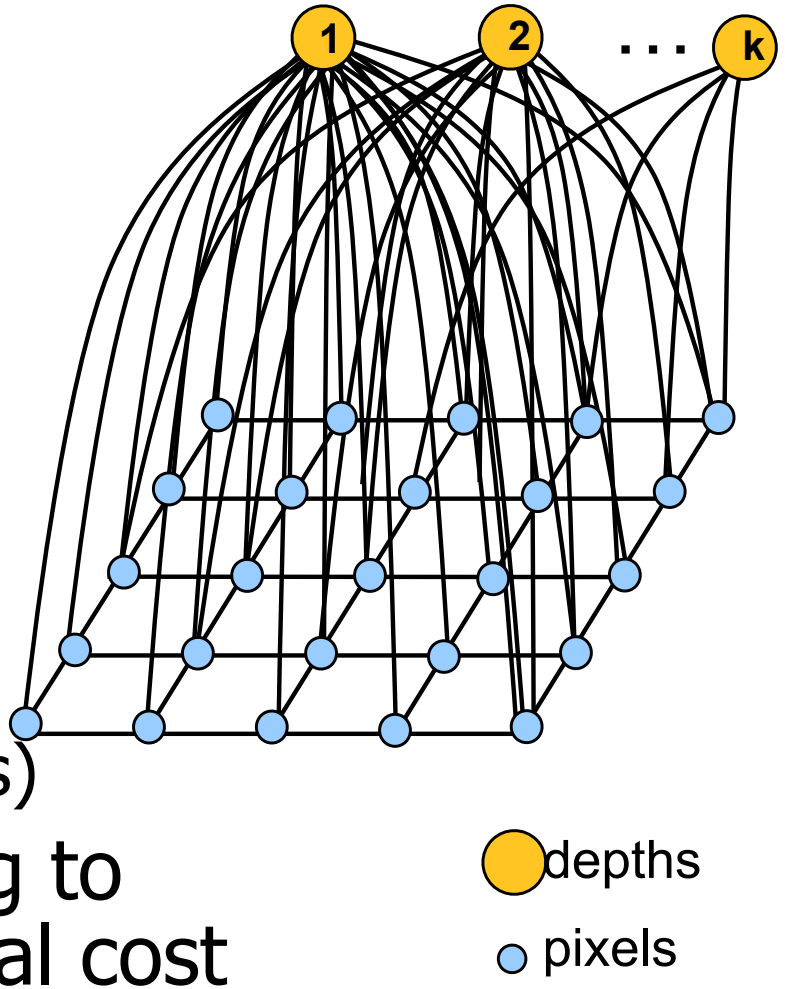
1. Stereo is a labeling problem
2. Graph cut corresponds to a labeling.

→ **Assign edge weights cleverly so that the min-weight cut gives the minimum energy!**

GRAPH CUT OPTIMIZATION

Construct a graph including

- Nodes:
  - Pixels (in first image)
  - k discrete disparity values
- Edges:
  - From every pixel node to a depth node (data term)
  - Neighboring nodes (smoothness)
- Assign weights corresponding to pixel intensities to get a global cost function



MULTI WAY CUT

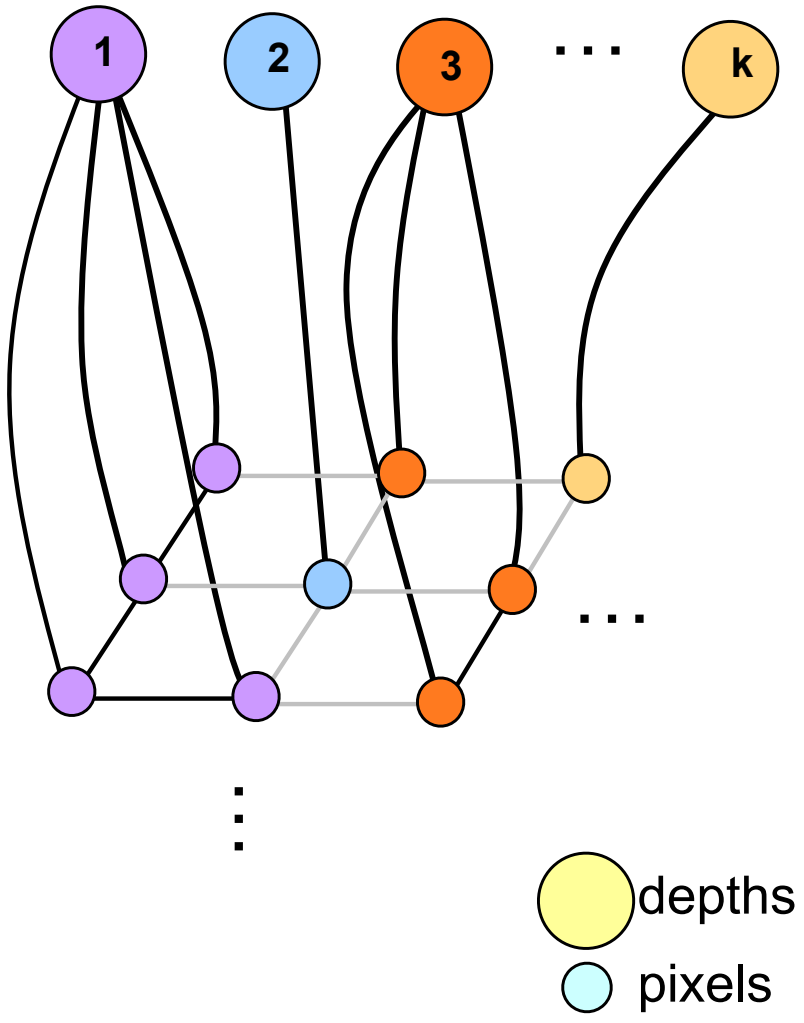
Goal:

Every pixel remains connected to one depth node.

Edges between neighboring nodes only if they are connected to same depth node.

- Nodes are assigned the depth that they are connected to.

- Multiway cut is NP-complete, solve iteratively.

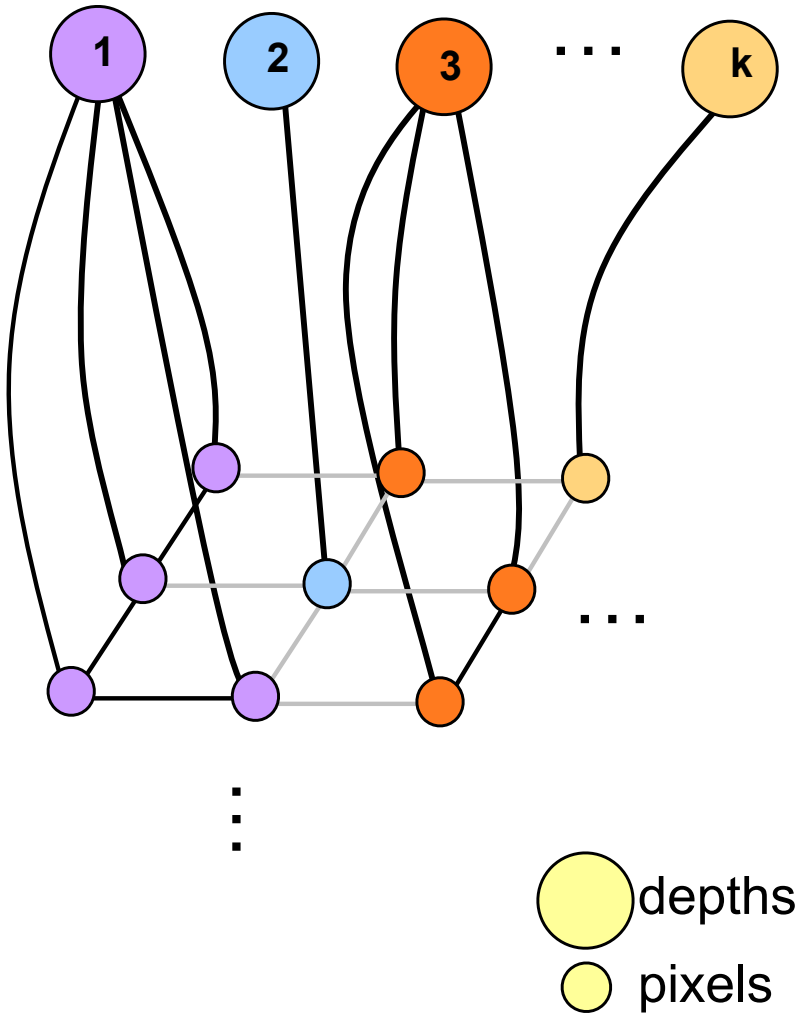


α-β **SWAP**

- Nodes labeled α or β can switch their labels.

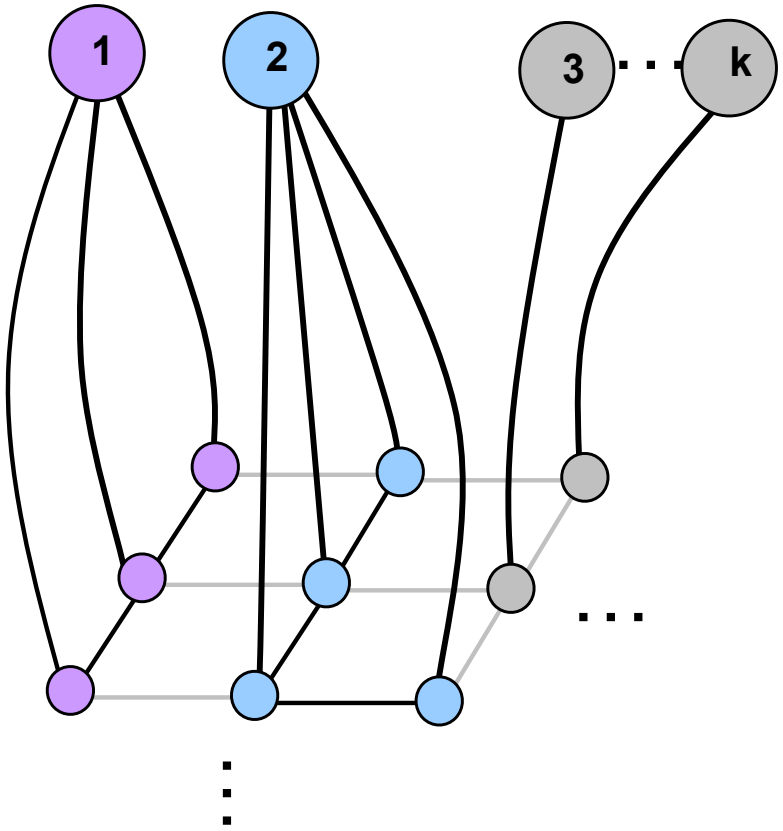
- Edges between neighbors are updated according to the new labeling.

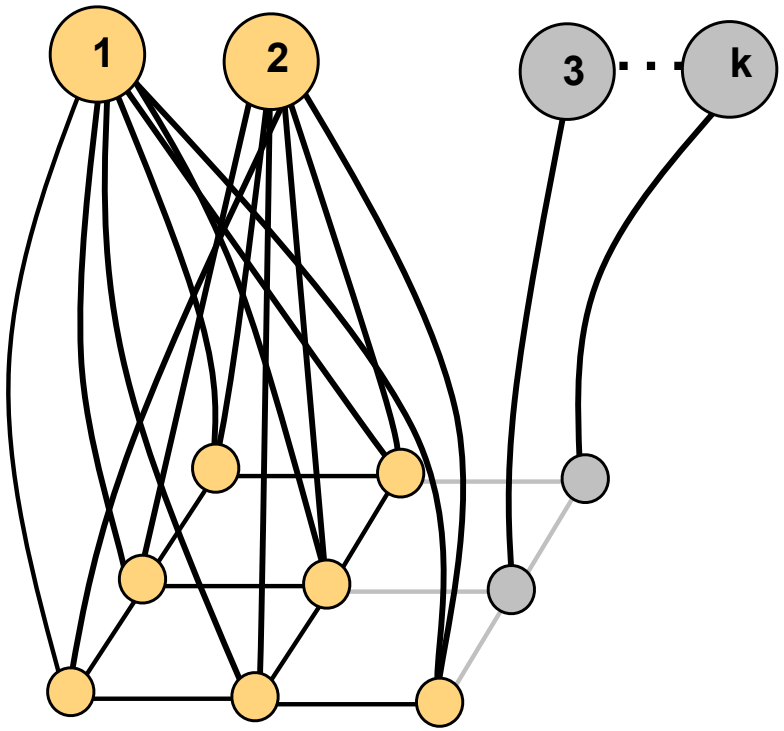
- Other edges remain unchanged.



Boykov et al, ICCV '99

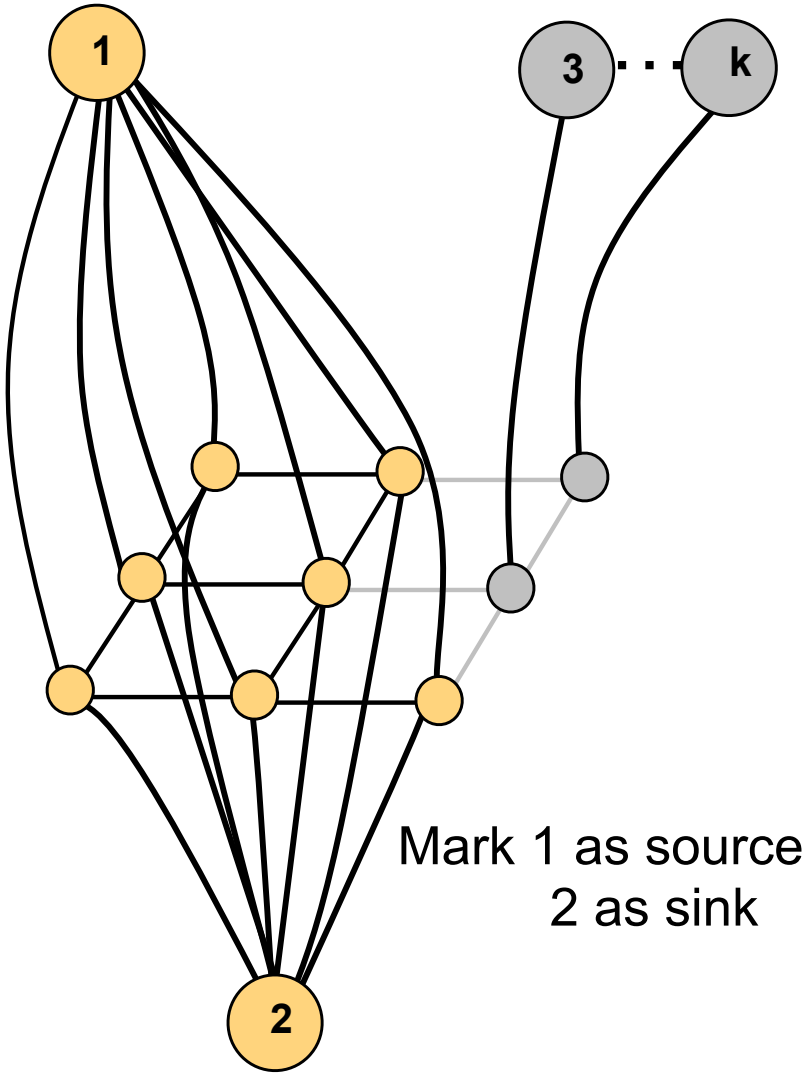
EXAMPLE: 1-2 SWAP



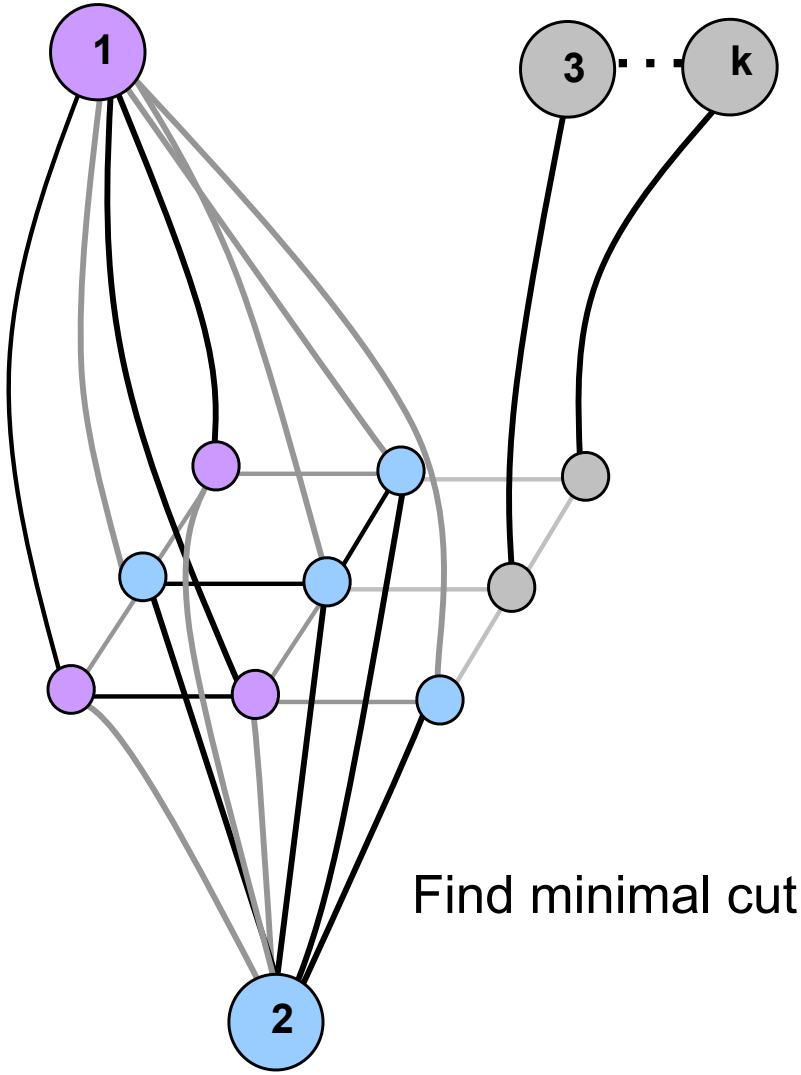


Connect the nodes labeled 1 or 2 to both labels

EXAMPLE: 1-2 SWAP

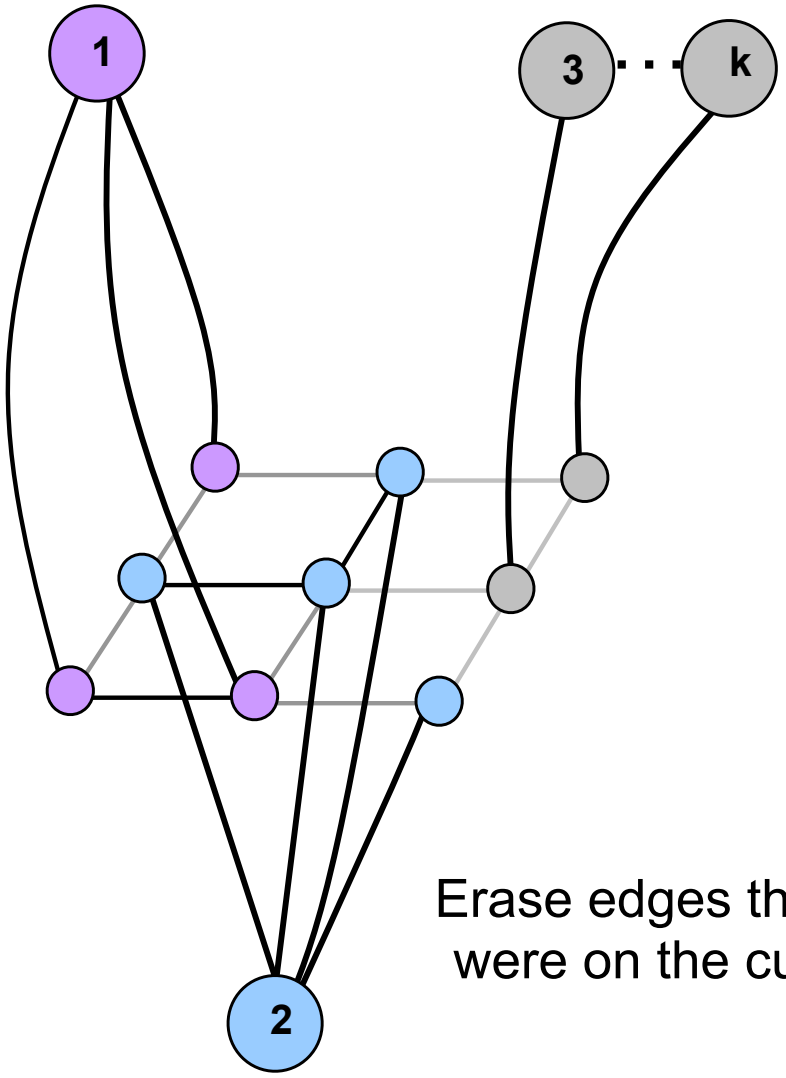


Mark 1 as source and
2 as sink

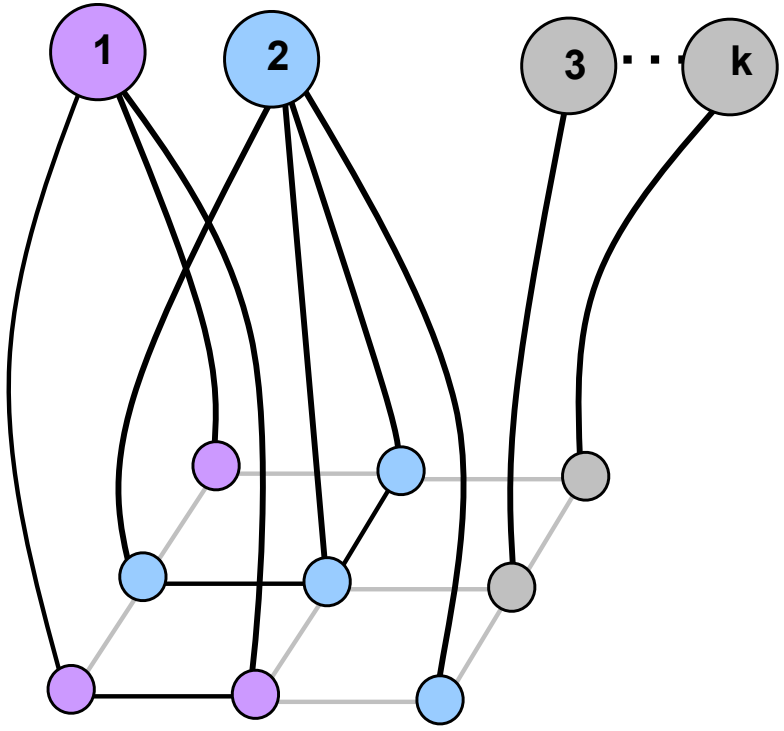


Find minimal cut

EXAMPLE: 1-2 SWAP

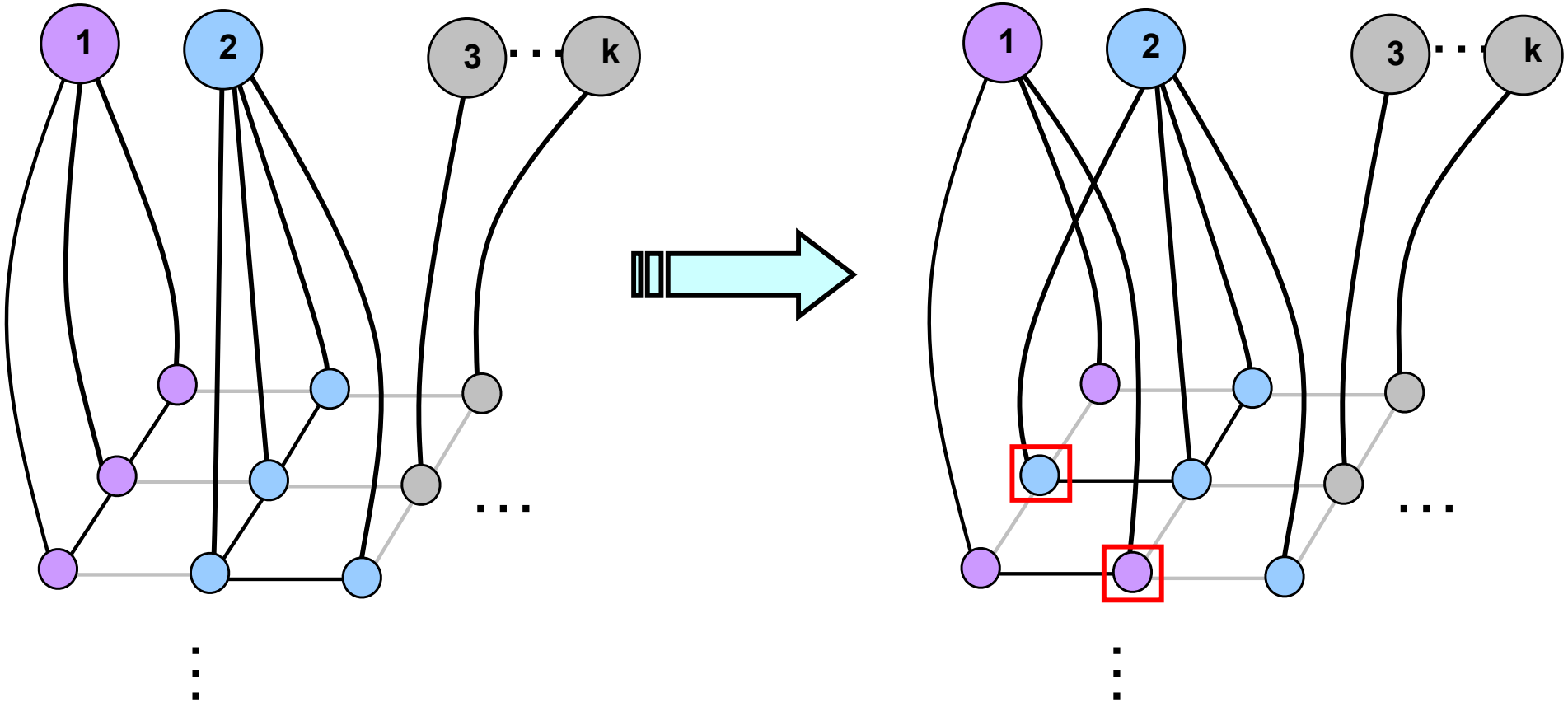


Erase edges that were on the cut



Result: a new labeling of the 1,2 nodes

EXAMPLE: 1-2 SWAP



GRAPH CUT ALGORITHM

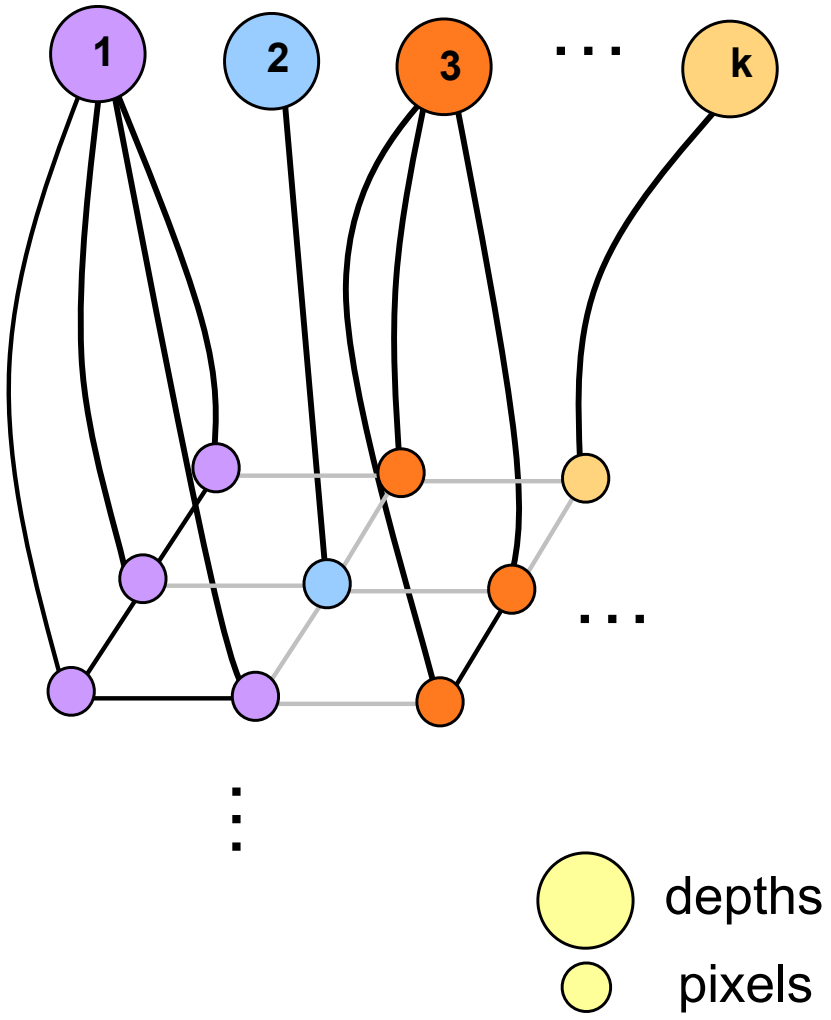
1. Start with an arbitrary labeling
2. For every pair $\{\alpha, \beta\}$ in $\{1,\ldots,k\}$
   - Find the α-β swap that minimizes the function
   - Update the graph by adding and erasing edges
3. Quit when no pair improves the cost function
4. Induce pixel labels

α-Expansion

- Nodes having a label different than α can either keep it or switch to α.

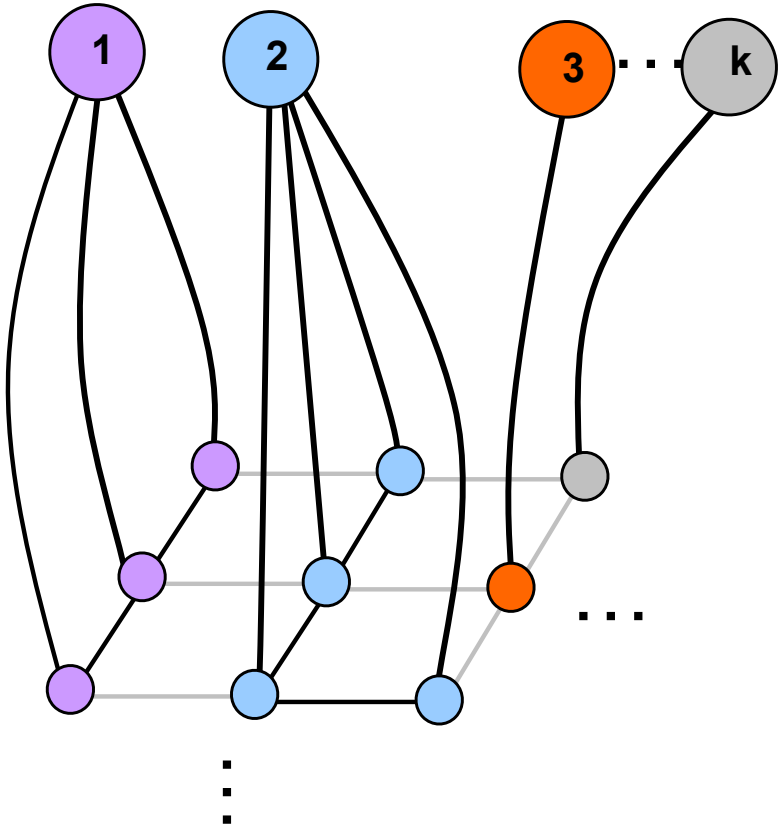
- Edges between neighbors are updated according to the new labeling.

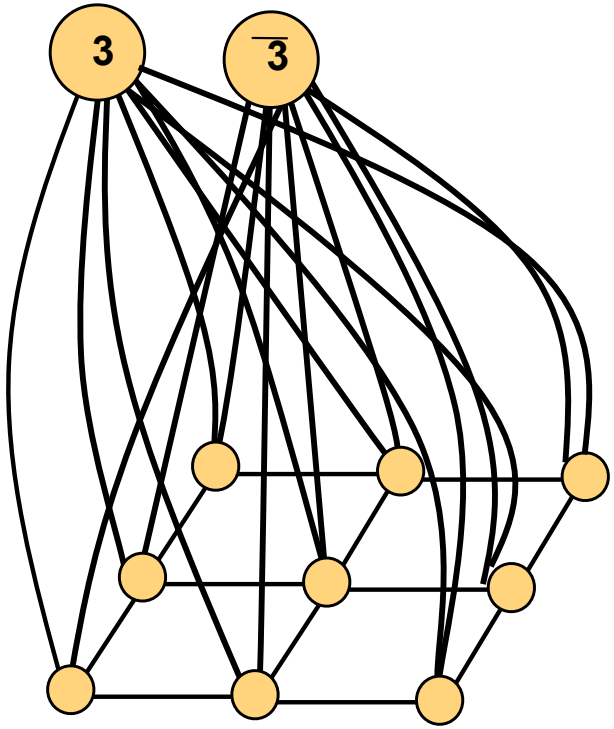
- Other edges remain unchanged.



Boykov et al, ICCV '99

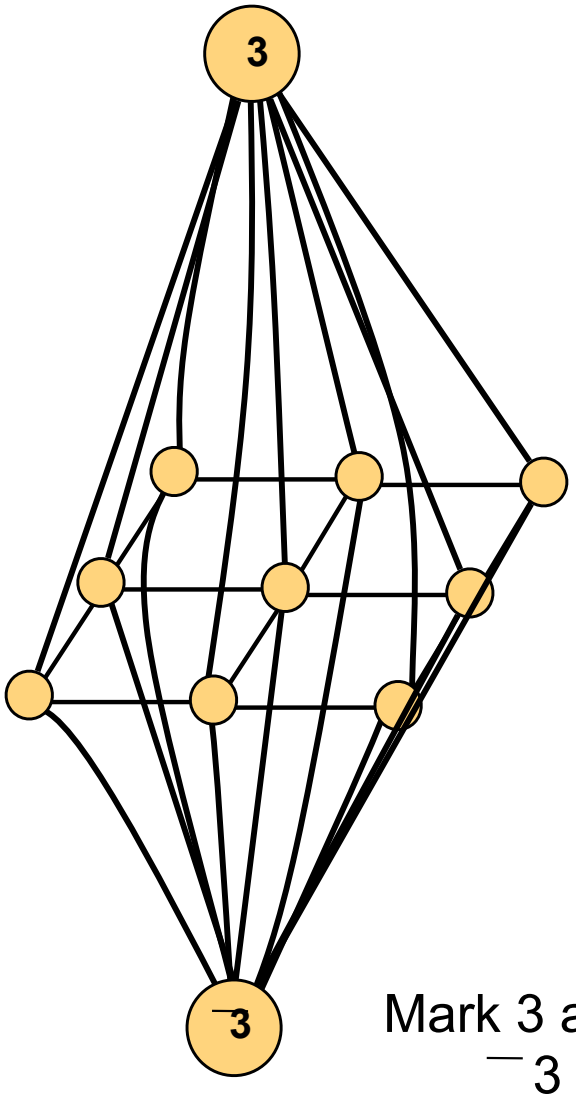
EXAMPLE: 3-EXPANSION





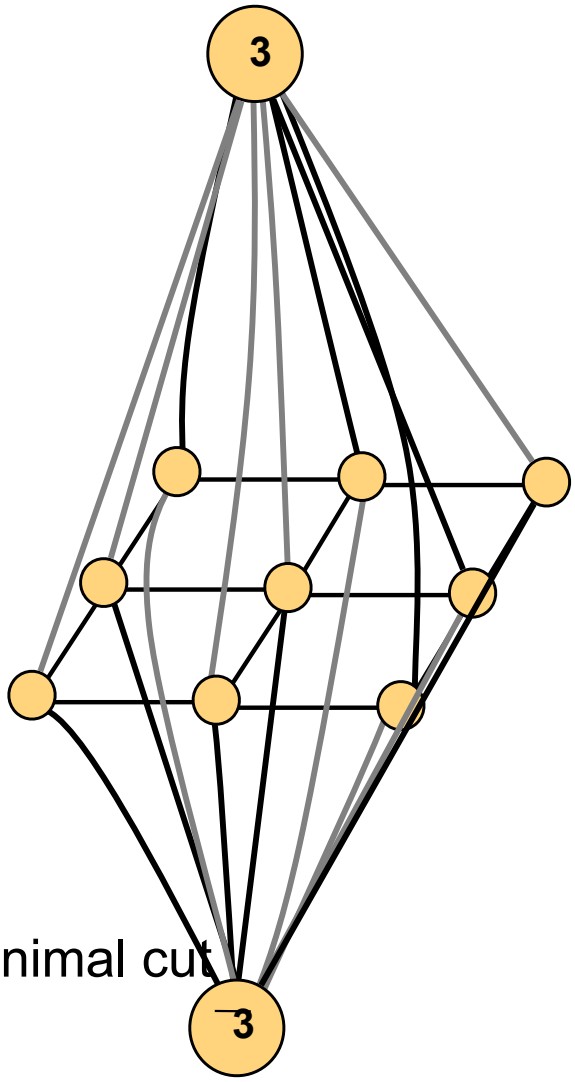
Connect all nodes to
both 3 $\overline{a}$nd 3

EXAMPLE: 3-EXPANSION

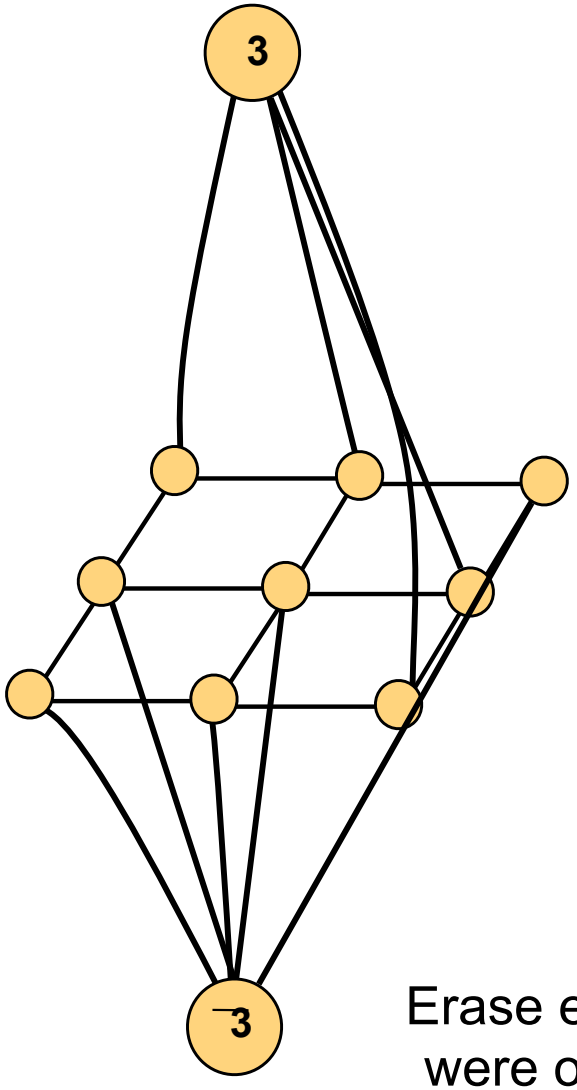


Mark 3 as source and
$\overline{3}$ as sink

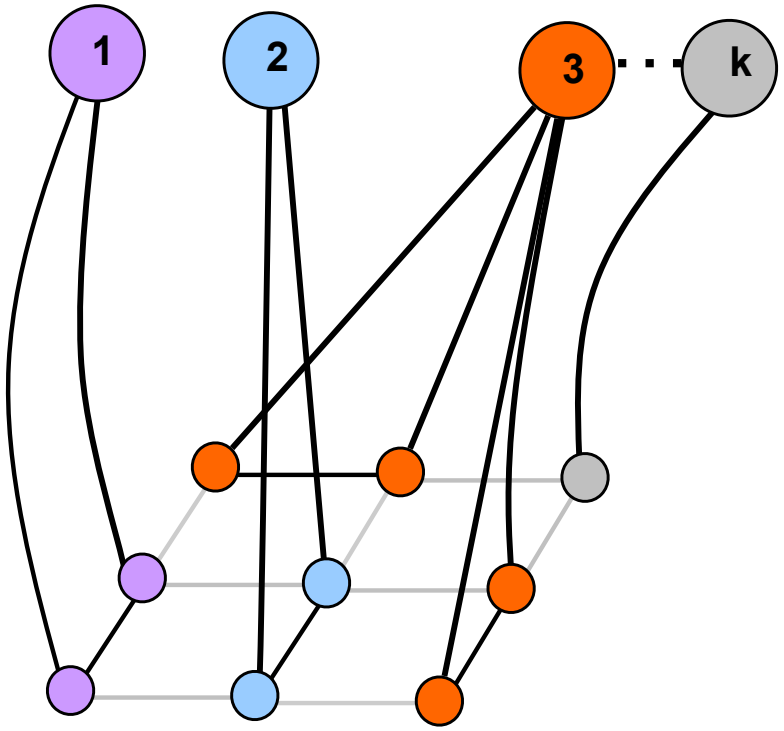
Find minimal cut



EXAMPLE: 3-EXPANSION

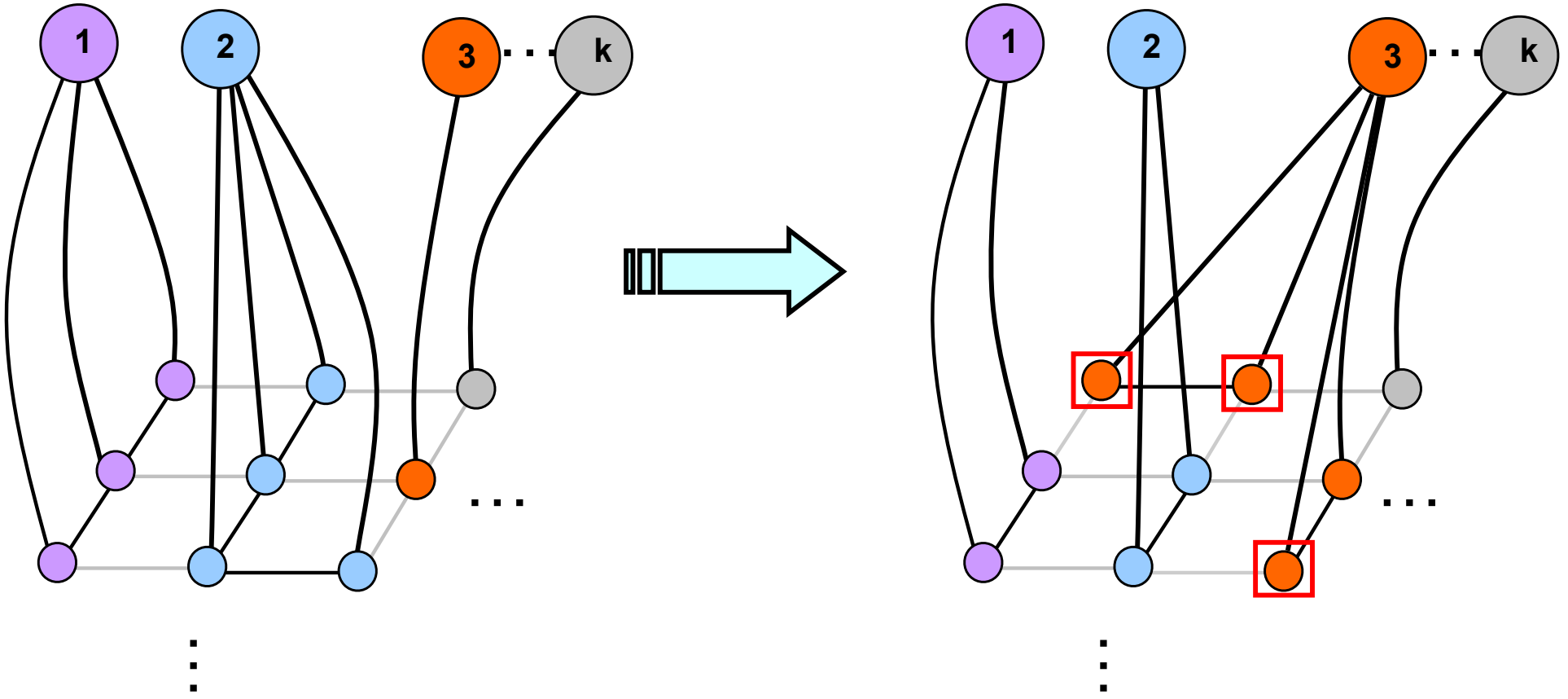


Erase edges that were on the cut



Result: 3-expansion

EXAMPLE: 3-EXPANSION



GRAPH CUT ALGORITHM

1. Start with an arbitrary labeling
2. For every label α in {1,…,L}
   - Find the α-Expansion that minimizes the function
   - Update the graph by adding and erasing edges
3. Quit when no expansion improves the cost
4. Induce pixel labels

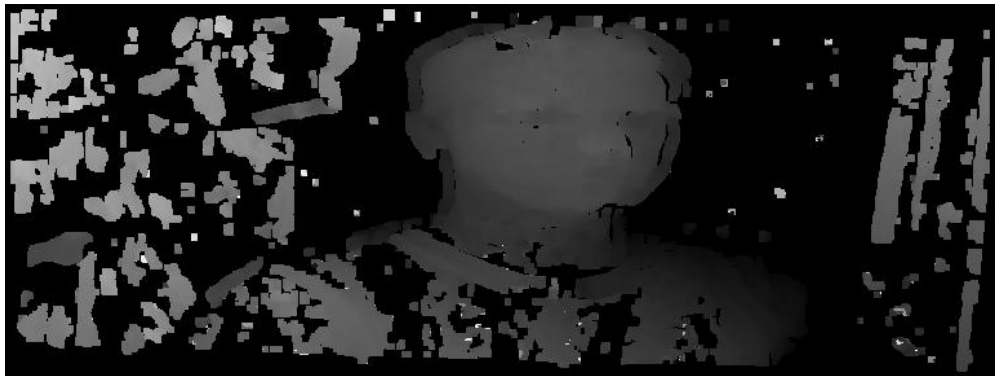
α–β SWAP vs α–EXPANSION

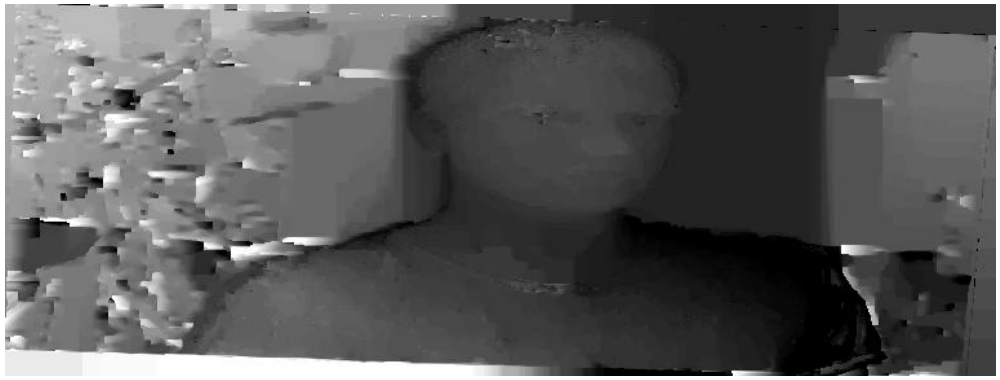
	Pair-Wise Penalty	Optimality
α–β Swap	Semi-metric	No guarantee
α–Expansion	Metric	Twice global optimum

- α–Expansion guarantees a solution whose energy is at most twice the global optimum but requires the pairwise term to satisfy the triangular inequality.

- α–β Swap offers no such guarantee but can deal with more generic pairwise terms.

NCC vs GRAPH CUTS



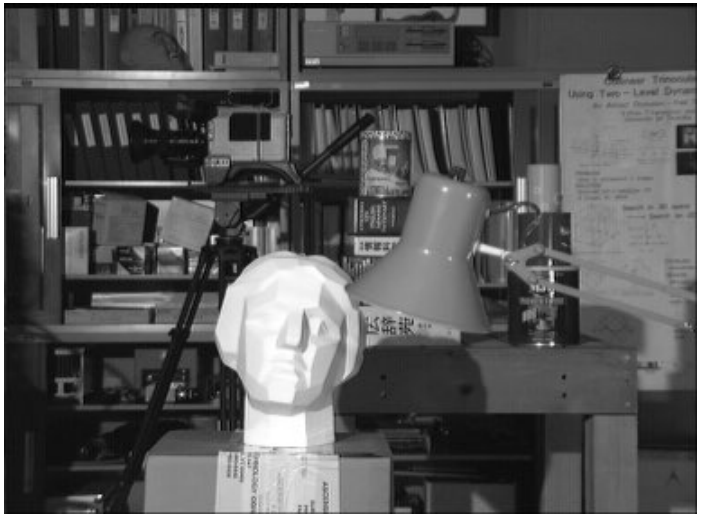


Normalized correlation

Graph Cuts

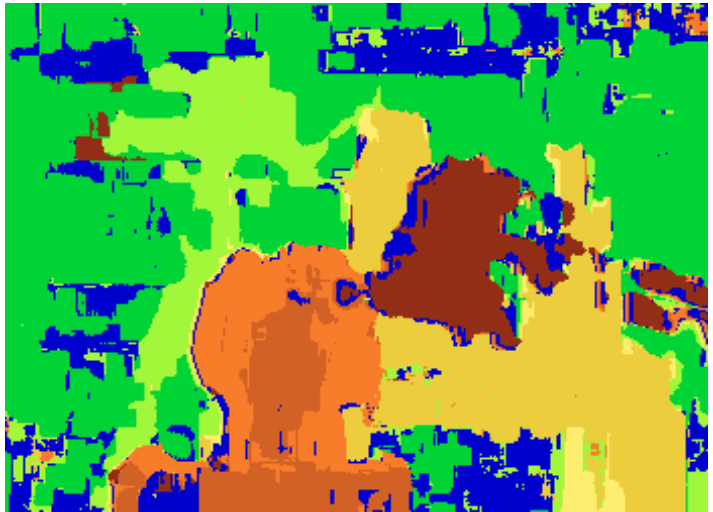
NCC vs GRAPH CUTS

left image

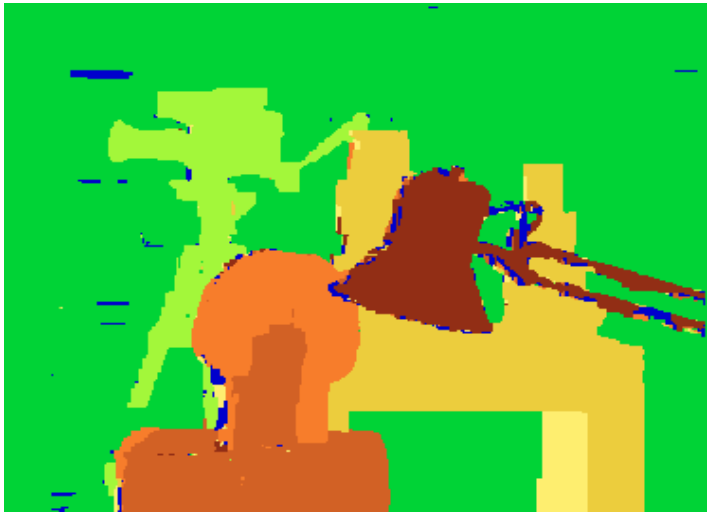


true disparities

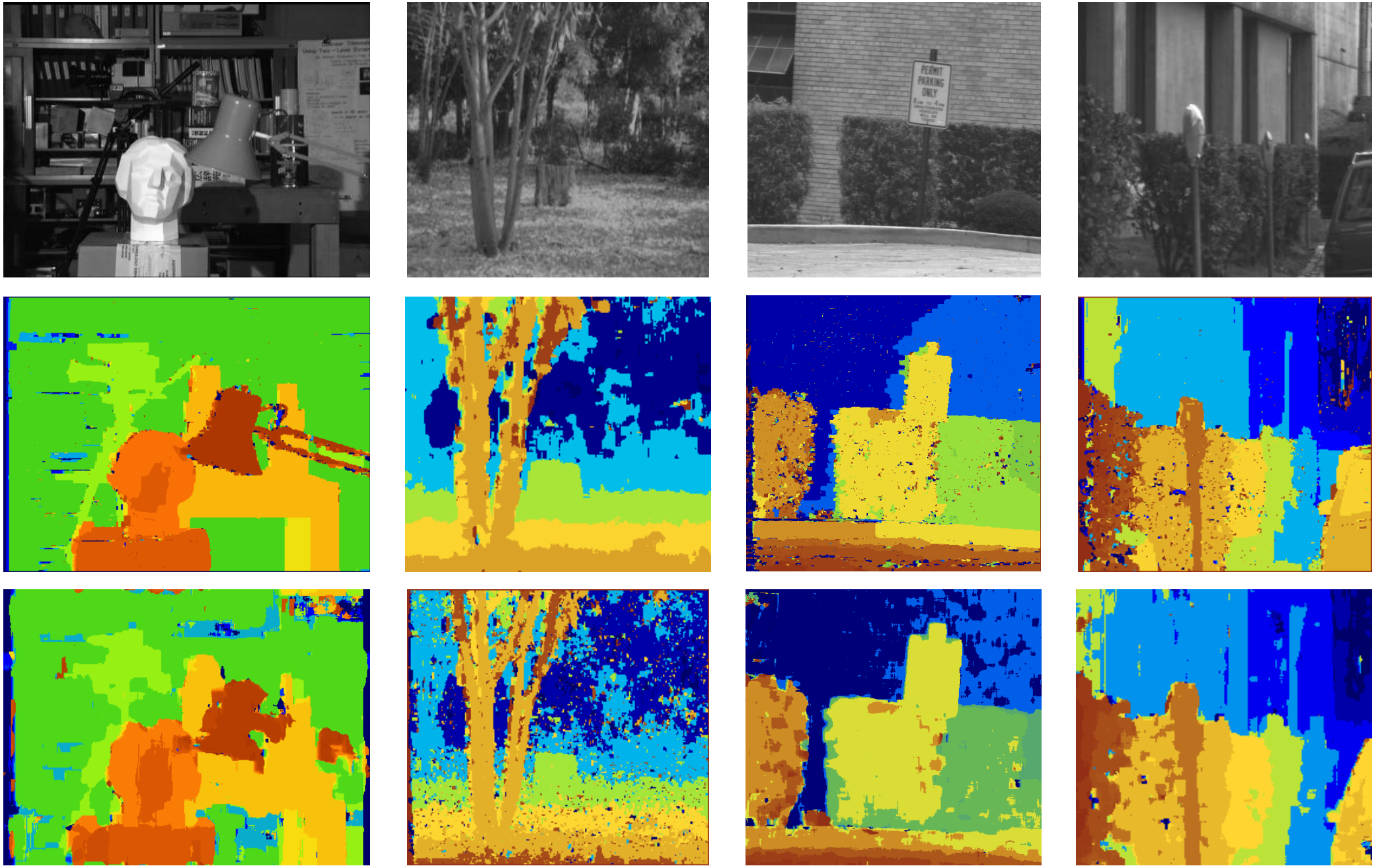
Normalized correlation



Graph Cuts



GRAPH CUT RESULTS



STRENGTHS AND LIMITATIONS

Strengths:

- Practical method for recovering depth.
- Runs in real-time on ordinary hardware.

Limitations:

- Requires multiple views.
- Only applicable to reasonably textured objects.