

# Object-oriented scientific programming with C++

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Lecture 3

## Overview

During last several lectures, we learned polymorphism, that is, inheritance of one class from another class

- Implementation of common functionality in **base class** (possibly realised as **abstract class** that cannot even be instantiated)
- Derivation of specialised class(es) from the base class that
  - implement the missing functionality (**pure virtual functions**)
  - override generic functionality by specialised variants (**virtual functions**)
  - reuse all other functionality from the base class

## Overview

Today, a more careful view on polymorphism

- **Static polymorphism**: static binding/method overloading
- **Dynamic polymorphism**: dynamic binding/method overriding Auto functionality of C++11/14/17.

## Template meta programming

- A powerful variant of static polymorphism

## Quiz: Guess what happens

In [2]:

```
struct Base {  
    virtual void hello() const = 0;  
    virtual ~Base() {} // virtual destructor for proper cleanup  
};  
  
struct Derived : public Base {  
    void hello() const override // Correct function name to override the base class method  
    {  
        std::cout << "Hallo\n";  
    }  
};
```

Quiz: Guess what happens

Run the main function to verify your guess

In [3]:

```
Derived d; // Create an instance of Derived  
Base* ptr = &d; // Pointer to Base type, pointing to Derived instance  
ptr->hello(); // Virtual dispatch ensures Derived::hello() is called
```

Hallo

## Discussion

Is there another way to call the function? Think about what we have learned in the past two weeks.

**HINT:** the above example called the function by using a pointer, what we have learned at the same time with pointer?

## Discussion

Is there another way to call the function? Think about what we have learned in the past two weeks.

In [4]:

```
Derived d; // Create an instance of Derived
Base& ref = d; // Reference to Base type, referring to Derived instance

ref.hello();
```

Hallo

## Quiz: Guess what happens

```
struct Base {  
    virtual void hello() const = 0;  
};  
struct Derived : public Base {  
    void hello() const  
    {  
        std::cout << "Hallo\n";  
    }  
};
```



## Quiz: Guess what happens

```
struct Base {  
    virtual void hello() const = 0;  
};  
struct Derived : public Base {  
    void hello() const  
    {  
        std::cout << "Hallo\n";  
    }  
};
```

- Structurally correct in terms of C++ syntax.
- Derived properly overrides the pure virtual function `hello()` from `Base`, allowing instances of `Derived` to be created and used polymorphically through pointers or references to `Base`.

## Quiz: Guess what happens

```
struct Base {  
    virtual void hello() const {  
        std::cout << "Hello\n";  
    }  
};  
struct Derived : public Base {  
    void hallo() const //Good luck with debugging  
    {  
        std::cout << "Hallo\n";  
    }  
};
```

## Quiz: Guess what happens

```
struct Base {  
    virtual void hello() const {  
        std::cout << "Hello\n";  
    }  
};  
struct Derived : public Base {  
    void hallo() const //Good luck with debugging  
    {  
        std::cout << "Hallo\n";  
    }  
};
```

- The Derived struct inherits from Base and introduces a new function hallo() that prints "Hallo\n" to the standard output.
- However, Derived does not override the hello() function from Base; it simply provides an additional function.

The override keyword

C++11 introduces the override keyword to explicitly state that a function from the base class shall be overridden

*Modify the program below yourself to see the different with Quiz var2*

In [5]:

```
struct Base {  
    virtual void hello() const = 0;  
    virtual ~Base() {} // virtual destructor for proper cleanup  
};  
struct Derived : public Base {  
    void hello() const override {std::cout << "Hallo\n";}   
};  
  
Derived d; // Create an instance of Derived  
d.hello(); // Calls the overridden hello() method in Derived  
Base* b = &d; // Since hello() is a virtual function, you can also call it through a Base pointer  
b->hello(); // This will also call Derived::hello()  
            //NOTE: If the function to be overridden does not exist in the base class an error is thrown by  
            the compiler
```

Hallo  
Hallo

The override keyword

The overriding function must exactly match the signature of the function from the base class

```
struct Base { virtual void hello() const { std::cout &&& "Hello\n"; } };  
struct Derived : public Base { void hello() const override { std::cout &&& "Hallo\n"; }  
};  
struct Derived2 : public Base { void hello() override { std::cout &&& "Hallo\n"; } };
```

The final keyword

C++11 introduces the final keyword to explicitly state that a function from the base class must not be overridden

```
struct Base { virtual void hello() const final { std::cout &&& "Hello\n"; } };
```

If a derived class tries to override the function hello an error is thrown by the compiler

The override and final keywords

The override and final keywords are often used together

```
struct Base { // Pure virtual function must be overridden in a derived class virtual void  
hello() const = 0; }; struct Derived : public Base { // Override pure virtual function and  
make it final void hello() const override final { std::cout &&& "Hello\n"; } }; struct  
Derived2 : public Derived { // Must not try to override function hello };
```

## Overloading vs. overriding

Implementation of method or operator with **identical** name but a **different interface**, e.g.

```
double operator*(const  
Vector & other) const {...}
```

overloads

```
double operator*(double  
other) const {...}
```

Implementation of method or operator with **identical** name and the **same interface** in a derived class or structure, e.g.

```
struct Derived : public Base {  
    void hello(...) const override  
    {...}  
};
```

overrides

```
struct Base {  
    virtual void hello(...) const  
    {...};  
};
```



## Overloading vs. overriding

- **Static polymorphism is**
  - *more efficient* at run time but
  - *less flexible* because all decisions must be made at compile time

- **Dynamic polymorphism is**
  - *less efficient* at run time but
  - *more flexible* because decisions are made at run time

```
Derived d; d.welcome();
```

```
struct Base {  
    virtual void hello() const = 0;  
    virtual void welcome() const {  
        cout << "Say\n";  
        hello();  
    }  
};  
  
struct Derived : public Base {  
    void hello() const override {  
        cout << "Hello\n";  
    }  
};
```

Implementation time! Task: Calculator

Write a class (or for demonstrating purposes a hierarchy of classes) that provide(s) a member function to calculate the **sum of two and three integer values**, respectively

- Use *static* polymorphism: method overloading
- Use *dynamic* polymorphism: method overriding

## Static polymorphism

Method overloading (at compile time), the StaticCalc has two member functions with identical names but different interface; compiler can decide at compile time which of the two functions should be used

In [6]:

```
struct StaticCalc {  
    int sum(int a, int b) const {  
        return a + b;  
    }  
    // Overloaded function to sum three integers  
    int sum(int a, int b, int c) const { return sum(sum(a,b),c);}  
};  
  
StaticCalc statC;  
std::cout << "Static sum of two integers: " << statC.sum(1, 2) << std::endl;  
std::cout << "Static sum of three integers: " << statC.sum(1, 2, 3) << std::endl;
```

```
Static sum of two integers: 3  
Static sum of three integers: 6
```

## Static polymorphism

### Method overloading (not working!)

```
struct StaticCalcBug { int sum(int a, int b) { return a+b; } void sum(int a, int b) {  
std::cout<<< a+b <<<std::endl;} };
```

- Difference must be in the interface of the arguments passed. Same name with different parameter lists is allowed (either the number of parameters or their types must differ).
- You cannot have two functions that only differ by their return type. Because the return type could be cast into another type

Static polymorphism

Method overloading (also not working!)

```
struct StaticCalcBug2 { int sum(int a, int b) { return a+b; } int sum(int c, int d) { return c+d; } };
```

- The interface only refers to the types and not to the names

Static polymorphism - Laaaaaast! counter example

Method overloading (still not working!)

```
struct StaticCalcBugLast { int sum(int a, int b) { return a+b; } int sum(int a, int b, int c=1) { return a+b+c; } };
```

- Be careful with default values. Here, the compiler cannot decide unambiguously which of the two functions should be used in the case `sum(arg0, arg1)`

Rule of thumb:

If it is not crystal clear to you which function should be used then the compiler will fail, too.



## Static polymorphism (master version1)

Method overloading: decision about which method to call is made at compile time; hence the compiler can decide to inline code to improve performance (no overhead due to function calls and copying of data to and from the stack!)

```
std::cout << statC.sum(1,2) << std::endl;  
std::cout << statC.sum(1,2,3) << std::endl;
```

becomes

```
std::cout << (1+2) << std::endl;  
std::cout << ((1+2)+3) << std::endl;
```



## Static polymorphism (master version2)

Method overloading: since static polymorphism takes place at compile time, the inline specifier can be used to explicitly 'suggest' to the compiler to inline the function

```
struct StaticCalcMaster { inline int sum(int a, int b) { return a+b; } inline int sum(int a, int b, int c) { return sum(sum(a,b),c); } };
```

## Dynamic polymorphism

Method overloading: reimplementation of a function inherited from a base class with new function body and same interface!

In [7]:

```
struct BaseCalc {  
    virtual int sum2(int a, int b) { return a+b; }  
    int sum3(int a, int b, int c)  
        { return sum2(sum2(a,b),c); }  
};  
struct DerivedCalc : public BaseCalc {  
    int sum2(int a, int b) override final { return b+a; }  
};
```

In [8]:

```
BaseCalc baseCalc;
DerivedCalc derivedCalc;
// Using the base class to sum two numbers
std::cout << "BaseCalc sum2: " << baseCalc.sum2(1, 2) << std::endl;
// Using the base class to sum three numbers
std::cout << "BaseCalc sum3: " << baseCalc.sum3(1, 2, 3) << std::endl;
// Using the derived class to sum two numbers, this will use the overridden method
std::cout << "DerivedCalc sum2: " << derivedCalc.sum2(1, 2) << std::endl;
// Using the derived class to sum three numbers, this will use the base
// class method sum3, which in turn will use the overridden sum2 method
// from DerivedCalc
std::cout << "DerivedCalc sum3: " << derivedCalc.sum3(1, 2, 3) << std::endl;
```

```
BaseCalc sum2: 3
BaseCalc sum3: 6
DerivedCalc sum2: 3
DerivedCalc sum3: 6
```

Dynamic polymorphism(where things can go wrong)

Method overloading: a common pitfall is to forget the virtual specifier in the base class to indicate that the `sum2` function from the base class can be overridden in a derived class

```
struct BaseCalc { int sum2(int a, int b) { return a+b; } int sum3(int a, int b, int c){  
return sum2(sum2(a,b),c); } }; struct DerivedCalc : public BaseCalc { int sum2(int a, int  
b) { return b+a; } };
```

Dynamic polymorphism(where things can go wrong)

Take home notes, read after the lecture. Here's what happens if you forget `virtual` specifier

- Method Hiding: In `DerivedCalc`, the `sum2` method hides the `sum2` method from `BaseCalc`. This means that if you have an object of `DerivedCalc` and call `sum2`, it will use `DerivedCalc`'s `sum2`. However, if you have a pointer or reference to `BaseCalc` that actually points to a `DerivedCalc` object and call `sum2`, it will still use `BaseCalc`'s `sum2`. This is because the method is not virtual and hence does not support polymorphic behavior.

- No Polymorphism: Without the virtual keyword in BaseCalc, sum2 in DerivedCalc doesn't exhibit polymorphic behavior, crucial for method overriding in C++.

Rule of thumb: These bugs are hard to find (they often remain unrecognised) and can be prevented by using the override keyword in C++11. It explicitly indicates that a function is intended to override a virtual function in a base class.

## Dynamic polymorphism - general suggestions

Method overriding: decision about which virtual method to call is made at run time; hence inlining is not possible

Common design pattern

- Specify expected minimal functionality of a group of classes in abstract base class via pure virtual member functions
- Implement generic common functionality of a group of classes in abstract base class via virtual member functions
- Implement specific functionality of a particular derived class by overriding the pure virtual member function

Example: inner product space

In linear algebra, an inner product space is a vector space  $V$  that is equipped with a special mapping (inner product)

$$\langle \cdot, \cdot \rangle: V \times V \rightarrow \mathbb{R} \text{ or } \mathbb{C}$$

Inner product spaces have a naturally induced norm

$$\|x\| = \sqrt{\langle x, x \rangle}$$



Example: inner product space

Class `InnerProductSpaceBase` declares inner product as pure virtual and implements the naturally induced norm

```
struct InnerProductBase { virtual double inner_product(... x,... y) = 0; double norm(... x) { return inner_product(x,x); } };
```

Derived `InnerProductSpace` class implements inner product

```
struct InnerProductSpace : public InnerProductSpaceBase { double inner_product(... x, ... y) = { return x*y; } };
```

Implementation time! Task: Calculator2

Extend the calculator class so that it can handle numbers of integer, float and double type at the same time

- Prevent manual code duplication
- Prevent explicit type casting
- Make use of auto-functionality (C++11/14/17)
- Make use of template meta programming

## Vanilla implementation in C++

In [9]:

```
#include <iostream>
struct Calc2 {
    int sum(int a, int b) { return a + b; }
    int sum(int a, int b, int c) { return sum(sum(a, b), c); }
};
Calc2 calculator;
// Using the sum method with two arguments
std::cout << "Sum of 1 and 2: " << calculator.sum(1, 2) << std::endl;
// Using the sum method with three arguments
std::cout << "Sum of 1, 2, and 3: " << calculator.sum(1, 2, 3) << std::endl;
```

Sum of 1 and 2: 3  
Sum of 1, 2, and 3: 6

## Automatic return type deduction (C++11)

- Explicit definition of the function return type

```
int sum(int a, int b){ return a+b; }
```

- Automatic function return type (since C++11)

```
auto sum(int a, int b) -> decltype(a+b) { return a+b; }
```

- By using decltype, the return type of the sum function is determined automatically as the type of operator+(a,b)

## Automatic return type deduction

decltype specifier (C++11) queries the type of an expression

In [10]:

```
#include <iostream>
struct Calc2 {
    auto sum(int a, int b) -> decltype(a + b) {return a + b;}
    auto sum(int a, int b, int c) -> decltype(sum(sum(a, b), c)) {
        return sum(sum(a, b), c);
    }
};

Calc2 C;
std::cout << C.sum(1, 2) << std::endl;    // Outputs: 3
std::cout << C.sum(1, 2, 3) << std::endl; // Outputs: 6
```

3  
6

Out[10]:

@0x7ffb425f6de0

## Automatic type deduction (C++14)

C++14 deduces the type of parameters automatically

```
auto sum(int a, int b) // no -> decltype(...) needed { return a+b; }  
auto sum(int a, int b, int c) // no -> decltype(...) needed { return sum(sum(a,b),c); }
```

Remark: This C++14 feature helps to improve readability of the code and prevents deduction errors (due to forgotten/ inconsistent deduction rule by the programmer) but it does not solve the problem of being able to pass arguments of different types to the same function

## Function templates

Template meta programming is the standard technique to deal with arbitrary (= generic) function parameters

Function templates: allow you to implement so-called parameterized functions for generic parameter types

```
template<typename R, typename A, typename B> R sum(A a, B b) {return a+b; }
```

## Function templates

Types must be specified explicitly when the function is called

```
int s1 = sum<int, int, int>(1, 2); double s2 = sum<double, double, int>(1.2, 2);  
double s3 = sum<double, float, double>(1.4, 2.2);
```

This can be slightly simplified by using the auto specifier

```
auto s1 = sum<int, int, int>(1, 2); auto s2 = sum<double, double, int>(1.2, 2); auto  
s3 = sum<double, float, double>(1.4, 2.2);
```



## Function templates

C++11: automatic return type deduction

```
template<typename A, typename B> auto sum(A a, B b) -> decltype(a+b){ return a+b; }
```

C++14: automatic type deduction

```
template<typename A, typename B> auto sum(A a, B b){ return a+b; }
```

Usage

```
auto s1 = sum<int, int>(1, 2);
```

## Function templates

How to convert this function into a templated function

```
int sum(int a, int b, int c)
{
    return sum(sum(a,b), c);
}
```

## Function templates

Use explicit return type parameter (ugly!) -> NO `template<typename R, typename A, typename B, typename C> auto sum(A a, B b, C c) { return sum<R,C>(sum<A,B>(a,b), c); }` Guess what this function call will return

`auto s1 = sum<int, double,double,double>(1.1,2.2,3.3)`

## Function templates

Use a smart combination of templates and auto specifier

```
template<typename A, typename B> auto sum(A a, B b) -> decltype(a+b) // omit in C++14 {return a+b; }
```

```
template<typename A, typename B, typename C> auto sum(A a, B b, C c) { return sum<decltype(sum<A,B>(a,b)),C>(sum<A,B>(a,b), c); }
```

## Function templates

But wait, C++ can deduce the type of the template argument from the given argument automatically

```
template<typename A, typename B> auto sum(A a, B b) -> decltype(a+b) // omit in  
C++14 {return a+b; } template<typename A, typename B, typename C> auto sum(A  
a, B b, C c) { return sum(sum(a,b), c); }
```

## Function templates

We can call the generic sum functions as follows:

```
auto s1 = sum<int, int> (1, 2); auto s2 = sum<double, int> (1.2, 2); auto s3 =  
sum<float, double>(1.4f, 2.2);
```

Or, even simpler, as follows:

```
auto s1 = sum (1, 2); auto s2 = sum (1.2, 2); auto s3 = sum (1.4f, 2.2)
```

## Function templates

- Since the compiler needs to duplicate code and substitute A,B,C for each combination of templated types both the compile time and the size of the executable will increase
- Template meta programming is simplest if the code resides in header files only; later we will see how to use template meta-programming together with pre-compiled libraries

Implementation time! Task: generic Vector class

Write a Vector class that can store real values and complex values and supports the following operations:

- Addition of two vectors of the same length
- Multiplication of a vector with a scalar
- Dot product of a vector with another one In all operations, the involved objects (vectors/scalar) can be of different types, e.g., double/float



## Vector class prototype

## Implementation of Vector-of-double class

In [71]:

```
#include <iostream>
class Vector{
private:
    double* data;
    int n;
public:
    //----- Constructors below -----
    //
    Vector() : n(0), data(nullptr) {} // Default constructor

    Vector(int n) : n(n), data(new double[n]) {} // Constructor with size

    ~Vector(){ n=0; delete[] data; } // Destructor
    //----- method and operators -----
    --//

    // Addition (+=) operator
    Vector& operator+=(const Vector& other) {
        if (n != other.n) {
            throw std::invalid_argument("Vectors must be of the same size");
        }
        for (auto i = 0; i < n; i++) {
            data[i] += other.data[i];
        }
        return *this;
    }

    // Scalar multiplication (*=) operator
    Vector& operator*=(double scalar) {
        for (int i = 0; i < n; i++) {
            data[i] *= scalar;
        }
    }
}
```

```

        return *this;
    }

    // Dot product
    double dot(const Vector& other) const {
        if (n != other.n) {
            throw std::invalid_argument("Vectors must be of the same size");
        }
        double sum = 0;
        for (auto i = 0; i < n; i++) {
            sum += data[i] * other.data[i];
        }
        return sum;
    }

    // Subscript operator
    double& operator[](int index) {
        if (index >= n || index < 0) {
            throw std::out_of_range("Index out of range");
        }
        return data[index];
    }
};

```

## Brainstorming

Function templates alone will not help since the type of a class attribute needs to be templated -> class templates

Some member functions can be implemented generically, e.g., addition of two vectors and multiplication of a vector with a scalar value since they are the same for all types

Some member functions must be implemented in different manners for real and complex values -> specialisation

$$x \cdot y = \sum_{i=1}^n x_i y_i, \quad x, y \in \mathbb{R}, \quad x \cdot y = \sum_{i=1}^n x_i \bar{y}_i \quad x, y \in \mathbb{C}$$

Class template: anything vector class

Implementation of Vector-of-anything class -- creating a generic vector that can store elements of any type.

CAUTION: We are going to leave the presentation mode to have an overview of the entire program (without the main function).

In [72]:

```
#include <iostream>
#include <stdexcept>

template<typename T>
class Vector {
private:
    T* data;
    int n;

public:
    //-----Constructors below-----//
    // Default constructor
    Vector() : data(nullptr), n(0) {}

    // Constructor with size
    Vector(int n) : n(n), data(new T[n]()) {}

    // Destructor
    ~Vector() {
        delete[] data;
    }

    //-----End of constructors, go to next page for explanation-----//

    //-----Methods and operators below-----//
    /
    // Addition (+=) operator
```

```

Vector<T>& operator+=(const Vector<T>& other) {
    if (n != other.n) {
        throw std::invalid_argument("Vectors must be of the same size");
    }
    for (int i = 0; i < n; i++) {
        data[i] += other.data[i];
    }
    return *this;
}

// Scalar multiplication (*=) operator
Vector<T>& operator*=(T scalar) {
    for (int i = 0; i < n; i++) {
        data[i] *= scalar;
    }
    return *this;
}

// Dot product
T dot(const Vector<T>& other) const {
    if (n != other.n) {
        throw std::invalid_argument("Vectors must be of the same size");
    }
    T sum = 0;
    for (int i = 0; i < n; i++) {
        sum += data[i] * other.data[i];
    }
    return sum;
}

//-----Combined class template and function template below-----//
template<typename S>
Vector<T>& operator*=(S scalar){
    for (auto i=0; i<n; i++)
        data[i] *= scalar;
    return *this;
}

// Non-const subscript operator
T& operator[](int index) {
    if (index < 0 || index >= n) {
        throw std::out_of_range("Index out of range");
    }
    return data[index];
}

```

```
// Const version of subscript operator
const T& operator[](int index) const {
    if (index < 0 || index >= n) {
        throw std::out_of_range("Index out of range");
    }
    return data[index];
}
};
```

## Class template

Template parameter must be explicitly specified

```
Vector<int> x(10); // Vector-of-int with length 10  
Vector<double> y; // Empty Vector-of-double  
Vector<float> z(5); // Vector-of-float with length 5
```

Remark: if you want to pass a Vector-of-anything to a function in the templated class Vector you have to write

```
Vector<T> v  
Vector<T>& v
```

instead of

```
Vector v  
Vector& v
```

## Class template

With the **class template parameter T** we can do

In [70]:

```
Vector<int> x1(5);  
Vector<int> x2(5);  
  
// Initialize x1 and x2 with some values  
for (int i = 0; i < 5; ++i) {  
    x1[i] = i + 1; // x1 = [1, 2, 3, 4, 5]  
    x2[i] = 2 * (i + 1); // x2 = [2, 4, 6, 8, 10]  
}  
  
x1 += x2; // x1 = [3, 6, 9, 12, 15]  
x1 *= 2; // x1 = [6, 12, 18, 24, 30]  
int dot_product = x1.dot(x2); // Dot product of x1 and x2  
  
std::cout << "Dot product:" << dot_product;
```

Dot product:660



How about?

```
Vector<int> x2(5);  
x2 *= (double)1.2;
```

Intermezzo

```
template<typename T> class Vector { ... template<typename S> Vector<T>&  
operator*=(S scalar) { for (auto i=0; i<n; i++) data[i] *= scalar; return *this; } };
```

## Intermezzo

At first glance, this seems to be more flexible

```
Vector<double> x1(5); x1 = x1*(int)(5);
```

But be really careful since strange things can happen

```
Vector<int> x1(5); x1 = x1*(double)(5);
```

Rule of thumb: before using extensive template meta programming like this think about all(!) implications

## Specialization

The dot product needs special treatment since

```
T dot(const Vector<T>& other) const { T d=0; for (auto i=0; i<n; i++) d += data[i]*other.data[i]; return d; }
```

- Lacks the complex conjugate of other and yields the wrong return type in case of complex-valued vectors

Remedy: implement a specialised variant for this case

## Specialization

Generic dot product implemented in Vector class

```
#include <complex> template<typename T> class Vector { ... T dot(const  
Vector<T>& other) const {...} };
```

This function is used whenever no specialized implementation for a concrete type is available

## Specialization

Specialized dot product for Vectors-of-complex-complex

```
template<float> Vector<std::complex<float>> >::dot(const  
Vector<std::complex<float>>& other) const { std::complex<float> d=0; //  
special treatment of dot product for (auto i=0; i<n; i++) d +=  
data[i]*std::conj(other.data[i]); return d; }
```

## Specialization

Current implementation yields

```
Vector<float> x1(5), x2(5); auto x1.dot(x2); // calls generic implementation
```

```
Vector<std::complex<float>> x1(5), x2(5); auto x1.dot(x2); // calls generic  
implementation
```

```
Vector<std::complex<double>> > z1(5), z2(5); auto z1.dot(z2); // calls generic  
implementation
```

```
auto x1.dot(y1); // does not even compile
```

Outlook on next session

C++ allows you to **partially specialize** class templates

```
template<typename S> std::complex<S> Vector<std::complex<S>> >:: dot(const  
Vector<std::complex<S>> > other) const { std::complex<S> d=0; for (auto i=0; i<n;  
i++) d += data[i]*std::conj(other.data[i]); return d; }
```

Note that this code will not compile. We will see why and learn remedies. Welcome to where template magic begins!

In [ ]:

Processing math: 100%