

代码库

上海交通大学

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目录

1	数论	2
1.1	快速求逆元	2
1.2	扩展欧几里德算法	2
1.3	中国剩余定理	2
1.4	Miller Rabin 素数测试	3
1.5	Pollard Rho 大数分解	4
1.6	快速数论变换	5
1.7	直线下整点个数	6
2	数值	6
2.1	快速傅立叶变换	6
2.2	单纯形法求解线性规划	7
2.3	自适应辛普森	9
3	数据结构	10
3.1	平衡的二叉查找树	10
3.1.1	Treap	10
3.1.2	Splay	11
3.2	坚固的数据结构	14
3.2.1	坚固的线段树	14
3.2.2	坚固的平衡树	15
3.2.3	坚固的字符串	17
3.2.4	坚固的左偏树	21
3.3	树上的魔术师	22
3.3.1	轻重树链剖分	22
3.3.2	Link Cut Tree	23
3.4	k-d 树	28
4	图论	32
4.1	强连通分量	32
4.2	2-SAT 问题	33
4.3	二分图最大匹配	34

4.3.1	Hungary 算法	34
4.3.2	Hopcroft Karp 算法	35
4.4	二分图最大权匹配	36
4.5	最大流	38
4.6	上下界网络流	39
4.6.1	无源汇的上下界可行流	40
4.6.2	有源汇的上下界可行流	40
4.6.3	有源汇的上下界最大流	40
4.6.4	有源汇的上下界最小流	40
4.7	最小费用最大流	40
4.7.1	稀疏图	40
4.7.2	稠密图	42
4.8	一般图最大匹配	44
4.9	无向图全局最小割	47
4.10	有根树的同构	48
4.11	哈密尔顿回路 (ORE 性质的图)	49
5	字符串	52
5.1	模式匹配	52
5.1.1	KMP 算法	52
5.1.2	扩展 KMP 算法	52
5.1.3	AC 自动机	53
5.2	后缀三姐妹	54
5.2.1	后缀数组	54
5.2.2	后缀自动机	56
5.3	回文三兄弟	56
5.3.1	Manacher 算法	56
5.3.2	回文树	57
5.4	循环串最小表示	58
6	计算几何	59
6.1	二维基础	59
6.1.1	点类	59
6.1.2	凸包	60
6.1.3	最近点对	61
6.2	多边形	62
6.2.1	判断点在多边形内部	62
7	其他	63
7.1	某年某月某日是星期几	63
7.2	枚举大小为 k 的子集	63
7.3	环状最长公共子串	63

8	Java	65
8.1	基础模板	65
9	数学	66
9.1	常用数学公式	66
9.1.1	求和公式	66
9.1.2	斐波那契数列	67
9.1.3	错排公式	67
9.1.4	莫比乌斯函数	67
9.1.5	伯恩赛德引理	67
9.1.6	五边形数定理	68
9.1.7	树的计数	68
9.1.8	欧拉公式	68
9.1.9	皮克定理	69
9.1.10	牛顿恒等式	69
9.2	平面几何公式	69
9.2.1	三角形	69
9.2.2	四边形	70
9.2.3	正 n 边形	70
9.2.4	圆	70
9.2.5	棱柱	71
9.2.6	棱锥	71
9.2.7	棱台	71
9.2.8	圆柱	72
9.2.9	圆锥	72
9.2.10	圆台	72
9.2.11	球	72
9.2.12	球台	73
9.2.13	球扇形	73
9.3	立体几何公式	73
9.3.1	球面三角公式	73
9.3.2	四面体体积公式	73

1 数论

1.1 快速求逆元

返回结果:

$$x^{-1}(\text{mod})$$

使用条件: $x \in [0, \text{mod})$ 并且 x 与 mod 互质

```
long long inverse(const long long &x, const long long &mod) {  
    if (x == 1) {  
        return 1;  
    } else {  
        return (mod - mod / x) * inverse(mod % x, mod) % mod;  
    }  
}
```

1.2 扩展欧几里德算法

返回结果:

$$ax + by = \gcd(a, b)$$

时间复杂度: $\mathcal{O}(n \log n)$

```
void solve(const long long &a, const long long &b, long long &x, long long &y) {  
    if (b == 0) {  
        x = 1;  
        y = 0;  
    } else {  
        solve(b, a % b, x, y);  
        x -= a / b * y;  
        std::swap(x, y);  
    }  
}
```

1.3 中国剩余定理

返回结果:

$$x \equiv r_i(\text{mod } p_i) \quad (0 \leq i < n)$$

使用条件: p_i 无需两两互质

时间复杂度: $\mathcal{O}(n \log n)$

```
bool solve(int n, std::pair<long long, long long> input[],  
           std::pair<long long, long long> &output) {  
    output = std::make_pair(1, 1);
```

```

for (int i = 0; i < n; ++i) {
    long long number, useless;
    euclid(output.second, input[i].second, number, useless);
    long long divisor = std::__gcd(output.second, input[i].second);
    if ((input[i].first - output.first) % divisor) {
        return false;
    }
    number *= (input[i].first - output.first) / divisor;
    fix(number, input[i].second);
    output.first += output.second * number;
    output.second *= input[i].second / divisor;
    fix(output.first, output.second);
}
return true;
}

```

1.4 Miller Rabin 素数测试

```

const int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};

bool check(const long long &prime, const long long &base) {
    long long number = prime - 1;
    for (; ~number & 1; number >>= 1);
    long long result = power_mod(base, number, prime);
    for (; number != prime - 1 && result != 1 && result != prime - 1; number <<= 1) {
        result = multiply_mod(result, result, prime);
    }
    return result == prime - 1 || (number & 1) == 1;
}

bool miller_rabin(const long long &number) {
    if (number < 2) {
        return false;
    }
    if (number < 4) {
        return true;
    }
    if (~number & 1) {
        return false;
    }
    for (int i = 0; i < 12 && BASE[i] < number; ++i) {
        if (!check(number, BASE[i])) {
            return false;
        }
    }
}

```

```

    }
}
return true;
}

```

1.5 Pollard Rho 大数分解

时间复杂度: $\mathcal{O}(n^{1/4})$

```

long long pollard_rho(const long long &number, const long long &seed) {
    long long x = rand() % (number - 1) + 1, y = x;
    for (int head = 1, tail = 2; ; ) {
        x = multiply_mod(x, x, number);
        x = add_mod(x, seed, number);
        if (x == y) {
            return number;
        }
        long long answer = std::__gcd(abs(x - y), number);
        if (answer > 1 && answer < number) {
            return answer;
        }
        if (++head == tail) {
            y = x;
            tail <= 1;
        }
    }
}

void factorize(const long long &number, std::vector<long long> &divisor) {
    if (number > 1) {
        if (miller_rabin(number)) {
            divisor.push_back(number);
        } else {
            long long factor = number;
            for (; factor >= number;
                factor = pollard_rho(number, rand() % (number - 1) + 1));
            factorize(number / factor, divisor);
            factorize(factor, divisor);
        }
    }
}

```

1.6 快速数论变换

返回结果:

$$c_i = \sum_{0 \leq j \leq i} a_j \cdot b_{i-j} \pmod{mod} \quad (0 \leq i < n)$$

使用说明: *magic* 是 *mod* 的原根

时间复杂度: $\mathcal{O}(n \log n)$

```
void solve(long long number[], int length, int type) {
    for (int i = 1, j = 0; i < length - 1; ++i) {
        for (int k = length; j ^= k >>= 1, ~j & k; );
        if (i < j) {
            std::swap(number[i], number[j]);
        }
    }
    long long unit_p0;
    for (int turn = 0; (1 << turn) < length; ++turn) {
        int step = 1 << turn, step2 = step << 1;
        if (type == 1) {
            unit_p0 = power_mod(MAGIC, (MOD - 1) / step2, MOD);
        } else {
            unit_p0 = power_mod(MAGIC, MOD - 1 - (MOD - 1) / step2, MOD);
        }
        for (int i = 0; i < length; i += step2) {
            long long unit = 1;
            for (int j = 0; j < step; ++j) {
                long long &number1 = number[i + j + step];
                long long &number2 = number[i + j];
                long long delta = unit * number1 % MOD;
                number1 = (number2 - delta + MOD) % MOD;
                number2 = (number2 + delta) % MOD;
                unit = unit * unit_p0 % MOD;
            }
        }
    }
}

void multiply() {
    for (; lowbit(length) != length; ++length);
    solve(number1, length, 1);
    solve(number2, length, 1);
    for (int i = 0; i < length; ++i) {
        number[i] = number1[i] * number2[i] % MOD;
    }
}
```

```

solve(number, length, -1);
for (int i = 0; i < length; ++i) {
    answer[i] = number[i] * power_mod(length, MOD - 2, MOD) % MOD;
}
}

```

1.7 直线下整点个数

返回结果：

$$\sum_{0 \leq i < n} \lfloor \frac{a + b \cdot i}{m} \rfloor$$

使用条件： $n, m > 0, a, b \geq 0$

时间复杂度： $\mathcal{O}(n \log n)$

```

long long solve(const long long &n, const long long &a,
               const long long &b, const long long &m) {
    if (b == 0) {
        return n * (a / m);
    }
    if (a >= m) {
        return n * (a / m) + solve(n, a % m, b, m);
    }
    if (b >= m) {
        return (n - 1) * n / 2 * (b / m) + solve(n, a, b % m, m);
    }
    return solve((a + b * n) / m, (a + b * n) % m, m, b);
}

```

2 数值

2.1 快速傅立叶变换

返回结果：

$$c_i = \sum_{0 \leq j \leq i} a_j \cdot b_{i-j} \quad (0 \leq i < n)$$

时间复杂度： $\mathcal{O}(n \log n)$

```

void solve(Complex number[], int length, int type) {
    for (int i = 1, j = 0; i < length - 1; ++i) {
        for (int k = length; j ^= k >= 1, ~j & k; );
        if (i < j) {
            std::swap(number[i], number[j]);
        }
    }
}

```



```

}
Complex unit_p0;
for (int turn = 0; (1 << turn) < length; ++turn) {
    int step = 1 << turn, step2 = step << 1;
    double p0 = PI / step * type;
    sincos(p0, &unit_p0.imag(), &unit_p0.real());
    for (int i = 0; i < length; i += step2) {
        Complex unit = 1;
        for (int j = 0; j < step; ++j) {
            Complex &number1 = number[i + j + step];
            Complex &number2 = number[i + j];
            Complex delta = unit * number1;
            number1 = number2 - delta;
            number2 = number2 + delta;
            unit = unit * unit_p0;
        }
    }
}

void multiply() {
    for (; lowbit(length) != length; ++length);
    solve(number1, length, 1);
    solve(number2, length, 1);
    for (int i = 0; i < length; ++i) {
        number[i] = number1[i] * number2[i];
    }
    solve(number, length, -1);
    for (int i = 0; i < length; ++i) {
        answer[i] = (int)(number[i].real() / length + 0.5);
    }
}

```

2.2 单纯形法求解线性规划

返回结果：

$$\max\{c_{1 \times m} \cdot x_{m \times 1} \mid x_{m \times 1} \geq 0_{m \times 1}, a_{n \times m} \cdot x_{m \times 1} \leq b_{n \times 1}\}$$

```

std::vector<double> solve(const std::vector<std::vector<double> > &a,
                        const std::vector<double> &b, const std::vector<double> &c) {
    int n = (int)a.size(), m = (int)a[0].size() + 1;
    std::vector<std::vector<double> > value(n + 2, std::vector<double>(m + 1));
    std::vector<int> index(n + m);
}

```

```

int r = n, s = m - 1;
for (int i = 0; i < n + m; ++i) {
    index[i] = i;
}
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m - 1; ++j) {
        value[i][j] = -a[i][j];
    }
    value[i][m - 1] = 1;
    value[i][m] = b[i];
    if (value[r][m] > value[i][m]) {
        r = i;
    }
}
for (int j = 0; j < m - 1; ++j) {
    value[n][j] = c[j];
}
value[n + 1][m - 1] = -1;
for (double number; ; ) {
    if (r < n) {
        std::swap(index[s], index[r + m]);
        value[r][s] = 1 / value[r][s];
        for (int j = 0; j <= m; ++j) {
            if (j != s) {
                value[r][j] *= -value[r][s];
            }
        }
        for (int i = 0; i <= n + 1; ++i) {
            if (i != r) {
                for (int j = 0; j <= m; ++j) {
                    if (j != s) {
                        value[i][j] += value[r][j] * value[i][s];
                    }
                }
                value[i][s] *= value[r][s];
            }
        }
    }
    r = s = -1;
    for (int j = 0; j < m; ++j) {
        if (s < 0 || index[s] > index[j]) {
            if (value[n + 1][j] > eps || value[n + 1][j] > -eps && value[n][j] > eps) {

```

```

        s = j;
    }
}
if (s < 0) {
    break;
}
for (int i = 0; i < n; ++i) {
    if (value[i][s] < -eps) {
        if (r < 0
            || (number = value[r][m] / value[r][s] - value[i][m] / value[i][s]) < -eps
            || number < eps && index[r + m] > index[i + m]) {
            r = i;
        }
    }
}
if (r < 0) {
    // Solution is unbounded.
    return std::vector<double>();
}
}
if (value[n + 1][m] < -eps) {
    // No solution.
    return std::vector<double>();
}
std::vector<double> answer(m - 1);
for (int i = m; i < n + m; ++i) {
    if (index[i] < m - 1) {
        answer[index[i]] = value[i - m][m];
    }
}
return answer;
}

```

2.3 自适应辛普森

```

double area(const double &left, const double &right) {
    double mid = (left + right) / 2;
    return (right - left) * (calc(left) + 4 * calc(mid) + calc(right)) / 6;
}

double simpson(const double &left, const double &right,
               const double &eps, const double &area_sum) {

```

```

    double mid = (left + right) / 2;
    double area_left = area(left, mid);
    double area_right = area(mid, right);
    double area_total = area_left + area_right;
    if (std::abs(area_total - area_sum) < 15 * eps) {
        return area_total + (area_total - area_sum) / 15;
    }
    return simpson(left, mid, eps / 2, area_left)
        + simpson(mid, right, eps / 2, area_right);
}

double simpson(const double &left, const double &right, const double &eps) {
    return simpson(left, right, eps, area(left, right));
}

```

3 数据结构

3.1 平衡的二叉查找树

3.1.1 Treap

```

class Node {
public:
    Node *child[2];
    int key;
    int size, priority;

    Node(Node *left, Node *right, int key) : key(key), size(1), priority(rand()) {
        child[0] = left;
        child[1] = right;
    }

    void update() {
        size = child[0]->size + 1 + child[1]->size;
    }
};

Node *null;

void rotate(Node *&x, int dir) {
    Node *y = x->child[dir];
    x->child[dir] = y->child[dir ^ 1];
    y->child[dir ^ 1] = x;
}

```

```

    x->update();
    y->update();
    x = y;
}

void insert(Node *&x, int key) {
    if (x == null) {
        x = new Node(null, null, key);
    } else {
        insert(x->child[key > x->key], key);
        if (x->child[key > x->key]->priority < x->priority) {
            rotate(x, key > x->key);
        }
        x->update();
    }
}

void remove(Node *&x, int key) {
    if (x->key != key) {
        remove(x->child[key > x->key], key);
    } else if (x->child[0] == null && x->child[1] == null) {
        x = null;
        return;
    } else {
        int dir = x->child[0]->priority > x->child[1]->priority;
        rotate(x, dir);
        remove(x->child[dir ^ 1], key);
    }
    x->update();
}

void build() {
    null = new Node(NULL, NULL, 0);
    null->child[0] = null->child[1] = null;
    null->size = 0;
    null->priority = RAND_MAX;
}

```

3.1.2 Splay

```

class Node {
public:
    Node *child[2], *father;

```

```

    int size;
    int key;

    Node(int key = 0);

    int side() {
        return father->child[1] == this;
    }

    void set(Node *son, int dir) {
        child[dir] = son;
        son->father = this;
    }

    void modify();

    void update() {
        size = child[0]->size + 1 + child[1]->size;
    }

    void release();
};

Node *null, *root;

Node::Node(int key) : size(1), key(key) {
    child[0] = child[1] = father = null;
}

void Node::modify() {
    if (this == null) {
        return;
    }
}

void rotate(Node *x) {
    int dir = x->side();
    Node *p = x->father;
    p->release();
    x->release();
    p->set(x->child[dir ^ 1], dir);
    p->father->set(x, p->side());
}

```

```

    x->set(p, dir ^ 1);
    if (p == root) {
        root = x;
    }
    p->update();
    x->update();
}

void splay(Node *x, Node *target = null) {
    for (x->release(); x->father != target; ) {
        if (x->father->father == target) {
            rotate(x);
        } else {
            x->side() == x->father->side()
                ? (rotate(x->father), rotate(x))
                : (rotate(x), rotate(x));
        }
    }
    x->update();
}

Node* kth(int size) {
    Node *x = root;
    for (; x->child[0]->size + 1 != size; ) {
        x->release();
        if (x->child[0]->size + 1 > size) {
            x = x->child[0];
        } else {
            size -= x->child[0]->size + 1;
            x = x->child[1];
        }
    }
    return x;
}

void select(int left, int right) {
    splay(kth(right + 2));
    splay(kth(left), root);
}

void insert(int pos, int n, int key[]) {
    select(pos, pos - 1);

```

```

Node *x = root->child[0];
for (int i = 0; i < n; ++i) {
    Node *now = new Node(key[i]);
    x->set(now, 1);
    x = now;
}
splay(x);
}

```

```

void solve(int left, int right) {
    select(left, right);
    root->child[0]->child[1]->solve();
    root->child[0]->update();
    root->update();
}

```

```

void build() {
    null = new Node();
    null->size = 0;
    root = new Node();
    Node *blank = new Node();
    root->set(blank, 1);
    splay(blank);
}

```

3.2 坚固的数据结构

3.2.1 坚固的线段树

```

class Node {
public:
    Node *left, *right;
    int value;

    Node(Node *left, Node *right, int value) : left(left), right(right), value(value) {}

    Node* modify(int l, int r, int ql, int qr, int delta);
    int query(int l, int r, int qx);
};

Node* null;

Node* Node::modify(int l, int r, int ql, int qr, int delta) {

```



```

    if (qr < l || r < ql) {
        return this;
    }
    if (ql <= l && r <= qr) {
        return new Node(this->left, this->right, this->value + delta);
    }
    int mid = l + r >> 1;
    return new Node(this->left->modify(l, mid, ql, qr, delta),
                    this->right->modify(mid + 1, r, ql, qr, delta),
                    this->value);
}

int Node::query(int l, int r, int qx) {
    if (qx < l || r < qx) {
        return 0;
    }
    if (qx <= l && r <= qx) {
        return this->value;
    }
    int mid = l + r >> 1;
    return this->left->query(l, mid, qx)
        + this->right->query(mid + 1, r, qx)
        + this->value;
}

void build() {
    null = new Node(NULL, NULL, 0);
    null->left = null->right = null;
}

```

3.2.2 坚固的平衡树

```

class Node {
public:
    Node *left, *right;
    int size;

    Node();
    std::pair<Node*, Node*> split(int size);

    Node* update() {
        size = left->size + 1 + right->size;
        return this;
    }
}

```

```

    }
};

bool random(int a, int b) {
    return rand() % (a + b) < a;
}

Node *null;

Node::Node() : left(null), right(null), size(1) {}

Node* merge(Node *x, Node *y) {
    if (x == null) {
        return y;
    }
    if (y == null) {
        return x;
    }
    if (random(x->size, y->size)) {
        x->right = merge(x->right, y);
        return x->update();
    } else {
        y->left = merge(x, y->left);
        return y->update();
    }
}

std::pair<Node*, Node*> Node::split(int size) {
    if (this == null) {
        return std::make_pair(null, null);
    }
    if (size <= left->size) {
        std::pair<Node*, Node*> result = left->split(size);
        left = null;
        return std::make_pair(result.first, merge(result.second, this->update()));
    } else {
        std::pair<Node*, Node*> result = right->split(size - left->size - 1);
        right = null;
        return std::make_pair(merge(this->update(), result.first), result.second);
    }
}

```

```

void build() {
    null = new Node();
    null->size = 0;
}

```

3.2.3 坚固的字符串

1. ext 库中的 rope

```

#include <ext/rope>

using __gnu_cxx::crope;
using __gnu_cxx::rope;

crope a, b;

int main(void) {
    a = b.substr(pos, len);    // [pos, pos + len)
    a = b.substr(pos);        // [pos, pos]
    b.c_str();                // might lead to memory leaks
    b.delete_c_str();         // delete the c_str that created before
    a.insert(pos, text);      // insert text before position pos
    a.erase(pos, len);        // erase [pos, pos + len)
}

```

2. 可持久化平衡树实现的 rope

```

class Rope {
private:
    class Node {
    public:
        Node *left, *right;
        int size;
        char key;

        Node(char key = 0, Node *left = NULL, Node *right = NULL)
            : key(key), left(left), right(right) {
            update();
        }

        void update() {
            size = (left ? left->size : 0) + 1 + (right ? right->size : 0);
        }
    }
}

```

```

std::string to_string() {
    return (left ? left->to_string() : "") + key
        + (right ? right->to_string() : "");
}
};

bool random(int a, int b) {
    return rand() % (a + b) < a;
}

Node* merge(Node *x, Node *y) {
    if (!x) {
        return y;
    }
    if (!y) {
        return x;
    }
    if (random(x->size, y->size)) {
        return new Node(x->key, x->left, merge(x->right, y));
    } else {
        return new Node(y->key, merge(x, y->left), y->right);
    }
}

std::pair<Node*, Node*> split(Node *x, int size) {
    if (!x) {
        return std::make_pair<Node*, Node*>(NULL, NULL);
    }
    if (size == 0) {
        return std::make_pair<Node*, Node*>(NULL, x);
    }
    if (size > x->size) {
        return std::make_pair<Node*, Node*>(x, NULL);
    }
    if (x->left && size <= x->left->size) {
        std::pair<Node*, Node*> part =
            split(x->left, size);
        return std::make_pair(part.first, new Node(x->key, part.second, x->right));
    } else {
        std::pair<Node*, Node*> part =
            split(x->right, size - (x->left ? x->left->size : 0) - 1);
        return std::make_pair(new Node(x->key, x->left, part.first), part.second);
    }
}

```

```

    }
}

Node* build(const std::string &text, int left, int right) {
    if (left > right) {
        return NULL;
    }
    int mid = left + right >> 1;
    return new Node(text[mid],
                    build(text, left, mid - 1),
                    build(text, mid + 1, right));
}

public:
    Node *root;

    Rope() {
        root = NULL;
    }

    Rope(const std::string &text) {
        root = build(text, 0, (int)text.length() - 1);
    }

    Rope(const Rope &other) {
        root = other.root;
    }

    Rope& operator = (const Rope &other) {
        if (this == &other) {
            return *this;
        }
        root = other.root;
        return *this;
    }

    int size() {
        return root ? root->size : 0;
    }

    void insert(int pos, const std::string &text) {
        if (pos < 0 || pos > size()) {

```

```

        throw "Out of range";
    }
    std::pair<Node*, Node*> part = split(root, pos);
    root = merge(merge(part.first, build(text, 0, (int)text.length() - 1)),
        part.second);
}

void erase(int left, int right) {
    if (left < 0 || left >= size() ||
        right < 1 || right > size()) {
        throw "Out of range";
    }
    if (left >= right) {
        return;
    }
    std::pair<Node*, Node*> part = split(root, left);
    root = merge(part.first, split(part.second, right - left).second);
}

std::string substr(int left, int right) {
    if (left < 0 || left >= size() ||
        right < 1 || right > size()) {
        throw "Out of range";
    }
    if (left >= right) {
        return "";
    }
    return split(split(root, left).second, right - left).first->to_string();
}

void copy(int left, int right, int pos) {
    if (left < 0 || left >= size() ||
        right < 1 || right > size() ||
        pos < 0 || pos > size()) {
        throw "Out of range";
    }
    if (left >= right) {
        return;
    }
    std::pair<Node*, Node*> part = split(root, pos);
    root = merge(merge(part.first,
        split(split(root, left).second, right - left).first),

```

```

        part.second);
    }
};

```

3.2.4 坚固的左偏树

```

class Node {
public:
    Node *left, *right;
    int key, dist;

    Node(int key) : left(NULL), right(NULL), key(key), dist(0) {}

    Node* update() {
        if (!left || (right && left->dist < right->dist)) {
            std::swap(left, right);
        }
        dist = right ? right->dist + 1 : 0;
        return this;
    }
};

Node* merge(Node *x, Node *y) {
    if (!x) {
        return y;
    }
    if (!y) {
        return x;
    }
    if (x->key < y->key) {
        x = new Node(*x);
        x->right = merge(x->right, y);
        return x->update();
    } else {
        y = new Node(*y);
        y->right = merge(x, y->right);
        return y->update();
    }
}

```

3.3 树上的魔术师

3.3.1 轻重树链剖分

```
int father[N], height[N], size[N], son[N], top[N], pos[N], data[N];

void build(int root) {
    std::vector<int> queue;
    father[root] = -1;
    height[root] = 0;
    queue.push_back(root);
    for (int head = 0; head < (int)queue.size(); ++head) {
        int x = queue[head];
        for (int i = 0; i < (int)edge[x].size(); ++i) {
            int y = edge[x][i];
            if (y != father[x]) {
                father[y] = x;
                height[y] = height[x] + 1;
                queue.push_back(y);
            }
        }
    }
}

for (int index = n - 1; index >= 0; --index) {
    int x = queue[index];
    size[x] = 1;
    son[x] = -1;
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (y != father[x]) {
            size[x] += size[y];
            if (son[x] == -1 || size[son[x]] < size[y]) {
                son[x] = y;
            }
        }
    }
}

std::fill(top, top + n, 0);
int counter = 0;
for (int index = 0; index < n; ++index) {
    int x = queue[index];
    if (top[x] == 0) {
        for (int y = x; y != -1; y = son[y]) {
            top[y] = x;
        }
    }
}
```



```

        pos[y] = ++counter;
        data[counter] = value[y];
    }
}
}
build(1, 1, n);
}

void solve(int x, int y) {
    while (true) {
        if (top[x] == top[y]) {
            if (x == y) {
                solve(1, 1, n, pos[x], pos[x]);
            } else {
                if (height[x] < height[y]) {
                    solve(1, 1, n, pos[x], pos[y]);
                } else {
                    solve(1, 1, n, pos[y], pos[x]);
                }
            }
            break;
        }
        if (height[top[x]] > height[top[y]]) {
            solve(1, 1, n, pos[top[x]], pos[x]);
            x = father[top[x]];
        } else {
            solve(1, 1, n, pos[top[y]], pos[y]);
            y = father[top[y]];
        }
    }
}
}

```

3.3.2 Link Cut Tree

```

class Node {
public:
    T data, sum;
    Node *father, *vfather;
    Node *child[2];
    int index;
    bool reverse;

    Node() {}
}

```

```

Node(int index, const T &data) : index(index), data(data) {
    sum = data;
    father = NULL;
    vfather = NULL;
    child[0] = NULL;
    child[1] = NULL;
}

void update() {
    if (child[0]) {
        child[0]->release();
    }
    if (child[1]) {
        child[1]->release();
    }
    sum = (child[0] ? child[0]->sum : T()) + data + (child[1] ? child[1]->sum : T());
}

void release() {
    if (reverse) {
        if (child[0]) {
            child[0]->reverse ^= 1;
        }
        if (child[1]) {
            child[1]->reverse ^= 1;
        }
        std::swap(child[0], child[1]);
        reverse = false;
    }
}

};

void rotate(Node *x, bool dir){
    Node *y = x->father;
    if (y->father) {
        y->father->child[y->father->child[1] == y] = x;
    }
    x->father = y->father;
    x->vfather = y->vfather;
    y->vfather = NULL;
    if (x->child[dir ^ 1]) {

```

```

        x->child[dir ^ 1]->father = y;
    }
    y->child[dir] = x->child[dir ^ 1];
    x->child[dir ^ 1] = y;
    y->father = x;
    y->update();
}

void splay(Node *x, Node *target = NULL){
    for (x->release(); x->father != target; ) {
        Node *y = x->father;
        if (y->father == target){
            y->release();
            x->release();
            bool dir = (y->child[1] == x);
            rotate(x, dir);
        } else {
            y->father->release();
            y->release();
            x->release();
            bool dir = (y->child[1] == x);
            if ((y->father->child[1] == y) == dir) {
                rotate(y, dir);
                rotate(x, dir);
            } else {
                rotate(x, dir);
                rotate(x, dir ^ 1);
            }
        }
    }
    x->update();
}

Node* access(Node *x){
    splay(x);
    if (x->child[0]) {
        x->release();
        x->child[0]->vfather = x;
        x->child[0]->father = NULL;
        x->child[0] = NULL;
        x->update();
    }
}

```

```

Node *y = x;
if (x->vfather) {
    y = access(x->vfather);
    x->vfather->child[0] = x;
    x->father = x->vfather;
    x->vfather->update();
}
splay(x);
return y;
}

void addEdge(Node *x, Node *y) {
    access(x);
    Node *w = access(y);
    splay(x);
    if (x->vfather == w || x == w) {
        throw "Circle exists";
    }
    x->reverse ^= 1;
    access(y);
    y->child[0] = x;
    x->father = y;
    y->update();
}

void eraseEdge(Node *x, Node *y) {
    if (x == y) {
        throw "Not connected";
    }
    access(x);
    if (access(y) == x){
        splay(x, y);
        if (x->child[0]) {
            throw "Not connected";
        }
        y->release();
        y->child[1]->father = NULL;
        y->child[1]->vfather = NULL;
        y->child[1] = NULL;
        y->update();
    } else {
        splay(x);
    }
}

```

```

        if (x->vfather != y || x->child[1]) {
            throw "Not connected";
        }
        access(x);
        x->release();
        x->child[1]->father = NULL;
        x->child[1]->vfather = NULL;
        x->child[1] = NULL;
        x->update();
    }
}

```

```

void modify(Node *x, const T &v) {
    access(x);
    x->data = v;
    x->update();
}

```

```

T query(Node *x, Node *y) {
    access(x);
    Node *u = access(y);
    T sum = u->data;
    splay(u);
    if (u->child[0]) {
        sum = sum + u->child[0]->sum;
    }
    access(x);
    splay(u);
    if (u->child[0]) {
        sum = sum + u->child[0]->sum;
    }
    return sum;
}

```

```

Node* lca(Node *x, Node *y) {
    access(x);
    return access(y);
}

```

```

Node* root(Node *x) {
    access(x);
    for (; ; x = x->child[1]) {

```

```

        x->release();
        if (!x->child[1]) {
            break;
        }
    }
}
return x;
}

void evert(Node *x) {
    access(x);
    x->reverse ^= 1;
}

int n;
std::vector<Node> node;

void build(const std::vector<std::pair<int, int> > &edge, const std::vector<T> &weight) {
    node.clear();
    for (int i = 0; i < n; ++i) {
        node.push_back(Node(i, weight[i]));
    }
    for (int i = 0; i < (int)edge.size(); ++i) {
        int x = edge[i].first, y = edge[i].second;
        if (root(x) == root(y)) {
            throw "Circle exists";
        }
        addEdge(x, y);
    }
}

```

3.4 k-d 树

```

long long norm(const long long &x) {
    // For manhattan distance
    return std::abs(x);
    // For euclid distance
    return x * x;
}

struct Point {
    int x, y, id;

    const int& operator [] (int index) const {

```

```

        if (index == 0) {
            return x;
        } else {
            return y;
        }
    }

friend long long dist(const Point &a, const Point &b) {
    long long result = 0;
    for (int i = 0; i < 2; ++i) {
        result += norm(a[i] - b[i]);
    }
    return result;
}

} point[N];

struct Rectangle {
    int min[2], max[2];

    Rectangle() {
        min[0] = min[1] = INT_MAX;
        max[0] = max[1] = INT_MIN;
    }

    void add(const Point &p) {
        for (int i = 0; i < 2; ++i) {
            min[i] = std::min(min[i], p[i]);
            max[i] = std::max(max[i], p[i]);
        }
    }

    long long dist(const Point &p) {
        long long result = 0;
        for (int i = 0; i < 2; ++i) {
            // For minimum distance
            result += norm(std::min(std::max(p[i], min[i]), max[i]) - p[i]);
            // For maximum distance
            result += std::max(norm(max[i] - p[i]), norm(min[i] - p[i]));
        }
        return result;
    }
};

```

```

struct Node {
    Point seperator;
    Rectangle rectangle;
    int child[2];

    void reset(const Point &p) {
        seperator = p;
        rectangle = Rectangle();
        rectangle.add(p);
        child[0] = child[1] = 0;
    }
} tree[N << 1];

int size, pivot;

bool compare(const Point &a, const Point &b) {
    if (a[pivot] != b[pivot]) {
        return a[pivot] < b[pivot];
    }
    return a.id < b.id;
}

int build(int l, int r, int type = 1) {
    pivot = type;
    if (l >= r) {
        return 0;
    }
    int x = ++size;
    int mid = l + r >> 1;
    std::nth_element(point + l, point + mid, point + r, compare);
    tree[x].reset(point[mid]);
    for (int i = l; i < r; ++i) {
        tree[x].rectangle.add(point[i]);
    }
    tree[x].child[0] = build(l, mid, type ^ 1);
    tree[x].child[1] = build(mid + 1, r, type ^ 1);
    return x;
}

int insert(int x, const Point &p, int type = 1) {
    pivot = type;

```



```

    if (x == 0) {
        tree[++size].reset(p);
        return size;
    }
    tree[x].rectangle.add(p);
    if (compare(p, tree[x].seperator)) {
        tree[x].child[0] = insert(tree[x].child[0], p, type ^ 1);
    } else {
        tree[x].child[1] = insert(tree[x].child[1], p, type ^ 1);
    }
    return x;
}

// For minimum distance
void query(int x, const Point &p, std::pair<long long, int> &answer, int type = 1) {
    pivot = type;
    if (x == 0 || tree[x].rectangle.dist(p) > answer.first) {
        return;
    }
    answer = std::min(answer,
        std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
    if (compare(p, tree[x].seperator)) {
        query(tree[x].child[0], p, answer, type ^ 1);
        query(tree[x].child[1], p, answer, type ^ 1);
    } else {
        query(tree[x].child[1], p, answer, type ^ 1);
        query(tree[x].child[0], p, answer, type ^ 1);
    }
}

std::priority_queue<std::pair<long long, int> > answer;

void query(int x, const Point &p, int k, int type = 1) {
    pivot = type;
    if (x == 0 ||
        ((int)answer.size() == k && tree[x].rectangle.dist(p) > answer.top().first) {
        return;
    }
    answer.push(std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
    if ((int)answer.size() > k) {
        answer.pop();
    }
}

```

```

    if (compare(p, tree[x].seperator)) {
        query(tree[x].child[0], p, k, type ^ 1);
        query(tree[x].child[1], p, k, type ^ 1);
    } else {
        query(tree[x].child[1], p, k, type ^ 1);
        query(tree[x].child[0], p, k, type ^ 1);
    }
}
}

```

4 图论

4.1 强连通分量

```

int stamp, comps, top;
int dfn[N], low[N], comp[N], stack[N];

void tarjan(int x) {
    dfn[x] = low[x] = ++stamp;
    stack[top++] = x;
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (!dfn[y]) {
            tarjan(y);
            low[x] = std::min(low[x], low[y]);
        } else if (!comp[y]) {
            low[x] = std::min(low[x], dfn[y]);
        }
    }
    if (low[x] == dfn[x]) {
        comps++;
        do {
            int y = stack[--top];
            comp[y] = comps;
        } while (stack[top] != x);
    }
}

void solve() {
    stamp = comps = top = 0;
    std::fill(dfn, dfn + n, 0);
    std::fill(comp, comp + n, 0);
    for (int i = 0; i < n; ++i) {

```

```

        if (!dfn[i]) {
            tarjan(i);
        }
    }
}

```

4.2 2-SAT 问题

```

int stamp, comps, top;
int dfn[N], low[N], comp[N], stack[N];

void add(int x, int a, int y, int b) {
    edge[x << 1 | a].push_back(y << 1 | b);
}

void tarjan(int x) {
    dfn[x] = low[x] = ++stamp;
    stack[top++] = x;
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (!dfn[y]) {
            tarjan(y);
            low[x] = std::min(low[x], low[y]);
        } else if (!comp[y]) {
            low[x] = std::min(low[x], dfn[y]);
        }
    }
    if (low[x] == dfn[x]) {
        comps++;
        do {
            int y = stack[--top];
            comp[y] = comps;
        } while (stack[top] != x);
    }
}

bool solve() {
    int counter = n + n + 1;
    stamp = top = comps = 0;
    std::fill(dfn, dfn + counter, 0);
    std::fill(comp, comp + counter, 0);
    for (int i = 0; i < counter; ++i) {
        if (!dfn[i]) {

```

```

        tarjan(i);
    }
}
for (int i = 0; i < n; ++i) {
    if (comp[i << 1] == comp[i << 1 | 1]) {
        return false;
    }
    answer[i] = (comp[i << 1 | 1] < comp[i << 1]);
}
return true;
}

```

4.3 二分图最大匹配

4.3.1 Hungary 算法

时间复杂度: $\mathcal{O}(V \cdot E)$

```

int n, m, stamp;
int match[N], visit[N];

bool dfs(int x) {
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (visit[y] != stamp) {
            visit[y] = stamp;
            if (match[y] == -1 || dfs(match[y])) {
                match[y] = x;
                return true;
            }
        }
    }
    return false;
}

int solve() {
    std::fill(match, match + m, -1);
    int answer = 0;
    for (int i = 0; i < n; ++i) {
        stamp++;
        answer += dfs(i);
    }
    return answer;
}

```

4.3.2 Hopcroft Karp 算法

时间复杂度: $\mathcal{O}(\sqrt{V} \cdot E)$

```
int matchx[N], matchy[N], level[N];

bool dfs(int x) {
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        int w = matchy[y];
        if (w == -1 || level[x] + 1 == level[w] && dfs(w)) {
            matchx[x] = y;
            matchy[y] = x;
            return true;
        }
    }
    level[x] = -1;
    return false;
}

int solve() {
    std::fill(matchx, matchx + n, -1);
    std::fill(matchy, matchy + m, -1);
    for (int answer = 0; ; ) {
        std::vector<int> queue;
        for (int i = 0; i < n; ++i) {
            if (matchx[i] == -1) {
                level[i] = 0;
                queue.push_back(i);
            } else {
                level[i] = -1;
            }
        }
        for (int head = 0; head < (int)queue.size(); ++head) {
            int x = queue[head];
            for (int i = 0; i < (int)edge[x].size(); ++i) {
                int y = edge[x][i];
                int w = matchy[y];
                if (w != -1 && level[w] < 0) {
                    level[w] = level[x] + 1;
                    queue.push_back(w);
                }
            }
        }
    }
}
```

```

    int delta = 0;
    for (int i = 0; i < n; ++i) {
        if (matchx[i] == -1 && dfs(i)) {
            delta++;
        }
    }
    if (delta == 0) {
        return answer;
    } else {
        answer += delta;
    }
}
}

```

4.4 二分图最大权匹配

时间复杂度: $\mathcal{O}(V^4)$

```

int labelx[N], labely[N], match[N], slack[N];
bool visitx[N], visity[N];

bool dfs(int x) {
    visitx[x] = true;
    for (int y = 0; y < n; ++y) {
        if (visity[y]) {
            continue;
        }
        int delta = labelx[x] + labely[y] - graph[x][y];
        if (delta == 0) {
            visity[y] = true;
            if (match[y] == -1 || dfs(match[y])) {
                match[y] = x;
                return true;
            }
        } else {
            slack[y] = std::min(slack[y], delta);
        }
    }
    return false;
}

int solve() {
    for (int i = 0; i < n; ++i) {

```

```

    match[i] = -1;
    labelx[i] = INT_MIN;
    labely[i] = 0;
    for (int j = 0; j < n; ++j) {
        labelx[i] = std::max(labelx[i], graph[i][j]);
    }
}

for (int i = 0; i < n; ++i) {
    while (true) {
        std::fill(visitx, visitx + n, 0);
        std::fill(visity, visity + n, 0);
        for (int j = 0; j < n; ++j) {
            slack[j] = INT_MAX;
        }
        if (dfs(i)) {
            break;
        }
        int delta = INT_MAX;
        for (int j = 0; j < n; ++j) {
            if (!visity[j]) {
                delta = std::min(delta, slack[j]);
            }
        }
        for (int j = 0; j < n; ++j) {
            if (visitx[j]) {
                labelx[j] -= delta;
            }
            if (visity[j]) {
                labely[j] += delta;
            } else {
                slack[j] -= delta;
            }
        }
    }
}

int answer = 0;
for (int i = 0; i < n; ++i) {
    answer += graph[match[i]][i];
}

return answer;
}

```

4.5 最大流

时间复杂度: $\mathcal{O}(V^2 \cdot E)$

```
struct EdgeList {
    int size;
    int last[N];
    int succ[M], other[M], flow[M];
    void clear(int n) {
        size = 0;
        fill(last, last + n, -1);
    }
    void add(int x, int y, int c) {
        succ[size] = last[x];
        last[x] = size;
        other[size] = y;
        flow[size++] = c;
    }
} e;

int n, source, target;
int dist[N], curr[N];

void add(int x, int y, int c) {
    e.add(x, y, c);
    e.add(y, x, 0);
}

bool relabel() {
    std::vector<int> queue;
    for (int i = 0; i < n; ++i) {
        curr[i] = e.last[i];
        dist[i] = -1;
    }
    queue.push_back(target);
    dist[target] = 0;
    for (int head = 0; head < (int)queue.size(); ++head) {
        int x = queue[head];
        for (int i = e.last[x]; ~i; i = e.succ[i]) {
            int y = e.other[i];
            if (e.flow[i ^ 1] && dist[y] == -1) {
                dist[y] = dist[x] + 1;
                queue.push_back(y);
            }
        }
    }
}
```



```

    }
}
return ~dist[source];
}

int dfs(int x, int answer) {
    if (x == target) {
        return answer;
    }
    int delta = answer;
    for (int &i = curr[x]; ~i; i = e.succ[i]) {
        int y = e.other[i];
        if (e.flow[i] && dist[x] == dist[y] + 1) {
            int number = dfs(y, std::min(e.flow[i], delta));
            e.flow[i] -= number;
            e.flow[i ^ 1] += number;
            delta -= number;
        }
        if (delta == 0) {
            break;
        }
    }
    return answer - delta;
}

int solve() {
    int answer = 0;
    while (relabel()) {
        answer += dfs(source, INT_MAX);
    }
    return answer;
}

```

4.6 上下界网络流

$B(u, v)$ 表示边 (u, v) 流量的下界, $C(u, v)$ 表示边 (u, v) 流量的上界, $F(u, v)$ 表示边 (u, v) 的流量。设 $G(u, v) = F(u, v) - B(u, v)$, 显然有

$$0 \leq G(u, v) \leq C(u, v) - B(u, v)$$

4.6.1 无源汇的上下界可行流

建立超级源点 S^* 和超级汇点 T^* ，对于原图每条边 (u, v) 在新网络中连如下三条边： $S^* \rightarrow v$ ，容量为 $B(u, v)$ ； $u \rightarrow T^*$ ，容量为 $B(u, v)$ ； $u \rightarrow v$ ，容量为 $C(u, v) - B(u, v)$ 。最后求新网络的最大流，判断从超级源点 S^* 出发的边是否都满流即可，边 (u, v) 的最终解中的实际流量为 $G(u, v) + B(u, v)$ 。

4.6.2 有源汇的上下界可行流

从汇点 T 到源点 S 连一条上界为 ∞ ，下界为 0 的边。按照无源汇的上下界可行流一样做即可，流量即为 $T \rightarrow S$ 边上的流量。

4.6.3 有源汇的上下界最大流

1. 在有源汇的上下界可行流中，从汇点 T 到源点 S 的边改为连一条上界为 ∞ ，下界为 x 的边。 x 满足二分性质，找到最大的 x 使得新网络存在无源汇的上下界可行流即为原图的最大流。
2. 从汇点 T 到源点 S 连一条上界为 ∞ ，下界为 0 的边，变成无源汇的网络。按照无源汇的上下界可行流的方法，建立超级源点 S^* 和超级汇点 T^* ，求一遍 $S^* \rightarrow T^*$ 的最大流，再将从汇点 T 到源点 S 的这条边拆掉，求一次 $S \rightarrow T$ 的最大流即可。

4.6.4 有源汇的上下界最小流

1. 在有源汇的上下界可行流中，从汇点 T 到源点 S 的边改为连一条上界为 x ，下界为 0 的边。 x 满足二分性质，找到最小的 x 使得新网络存在无源汇的上下界可行流即为原图的最小流。
2. 按照无源汇的上下界可行流的方法，建立超级源点 S^* 与超级汇点 T^* ，求一遍 $S^* \rightarrow T^*$ 的最大流，但是注意这一次不加上汇点 T 到源点 S 的这条边，即不使之改为无源汇的网络去求解。求完后，再加上那条汇点 T 到源点 S 上界 ∞ 的边。因为这条边下界为 0，所以 S^* ， T^* 无影响，再直接求一次 $S^* \rightarrow T^*$ 的最大流。若超级源点 S^* 出发的边全部满流，则 $T \rightarrow S$ 边上的流量即为原图的最小流，否则无解。

4.7 最小费用最大流

4.7.1 稀疏图

时间复杂度： $\mathcal{O}(V \cdot E^2)$

```
struct EdgeList {
    int size;
    int last[N];
    int succ[M], other[M], flow[M], cost[M];
    void clear(int n) {
        size = 0;
        std::fill(last, last + n, -1);
    }
    void add(int x, int y, int c, int w) {
```

```

        succ[size] = last[x];
        last[x] = size;
        other[size] = y;
        flow[size] = c;
        cost[size++] = w;
    }
} e;

int n, source, target;
int prev[N];

void add(int x, int y, int c, int w) {
    e.add(x, y, c, w);
    e.add(y, x, 0, -w);
}

bool augment() {
    static int dist[N], occur[N];
    std::vector<int> queue;
    std::fill(dist, dist + n, INT_MAX);
    std::fill(occur, occur + n, 0);
    dist[source] = 0;
    occur[source] = true;
    queue.push_back(source);
    for (int head = 0; head < (int)queue.size(); ++head) {
        int x = queue[head];
        for (int i = e.last[x]; ~i; i = e.succ[i]) {
            int y = e.other[i];
            if (e.flow[i] && dist[y] > dist[x] + e.cost[i]) {
                dist[y] = dist[x] + e.cost[i];
                prev[y] = i;
                if (!occur[y]) {
                    occur[y] = true;
                    queue.push_back(y);
                }
            }
        }
        occur[x] = false;
    }
    return dist[target] < INT_MAX;
}

```

```

std::pair<int, int> solve() {
    std::pair<int, int> answer = std::make_pair(0, 0);
    while (augment()) {
        int number = INT_MAX;
        for (int i = target; i != source; i = e.other[prev[i] ^ 1]) {
            number = std::min(number, e.flow[prev[i]]);
        }
        answer.first += number;
        for (int i = target; i != source; i = e.other[prev[i] ^ 1]) {
            e.flow[prev[i]] -= number;
            e.flow[prev[i] ^ 1] += number;
            answer.second += number * e.cost[prev[i]];
        }
    }
    return answer;
}

```

4.7.2 稠密图

使用条件：费用非负

时间复杂度： $\mathcal{O}(V \cdot E^2)$

```

struct EdgeList {
    int size;
    int last[N];
    int succ[M], other[M], flow[M], cost[M];
    void clear(int n) {
        size = 0;
        std::fill(last, last + n, -1);
    }
    void add(int x, int y, int c, int w) {
        succ[size] = last[x];
        last[x] = size;
        other[size] = y;
        flow[size] = c;
        cost[size++] = w;
    }
} e;

int n, source, target, flow, cost;
int slack[N], dist[N];
bool visit[N];

```

```

void add(int x, int y, int c, int w) {
    e.add(x, y, c, w);
    e.add(y, x, 0, -w);
}

bool relabel() {
    int delta = INT_MAX;
    for (int i = 0; i < n; ++i) {
        if (!visit[i]) {
            delta = std::min(delta, slack[i]);
        }
        slack[i] = INT_MAX;
    }
    if (delta == INT_MAX) {
        return true;
    }
    for (int i = 0; i < n; ++i) {
        if (visit[i]) {
            dist[i] += delta;
        }
    }
    return false;
}

int dfs(int x, int answer) {
    if (x == target) {
        flow += answer;
        cost += answer * (dist[source] - dist[target]);
        return answer;
    }
    visit[x] = true;
    int delta = answer;
    for (int i = e.last[x]; ~i; i = e.succ[i]) {
        int y = e.other[i];
        if (e.flow[i] > 0 && !visit[y]) {
            if (dist[y] + e.cost[i] == dist[x]) {
                int number = dfs(y, std::min(e.flow[i], delta));
                e.flow[i] -= number;
                e.flow[i ^ 1] += number;
                delta -= number;
            }
            if (delta == 0) {
                dist[x] = INT_MIN;
            }
        }
    }
}

```

```

        return answer;
    }
    } else {
        slack[y] = std::min(slack[y], dist[y] + e.cost[i] - dist[x]);
    }
}
}
return answer - delta;
}

```

```

std::pair<int, int> solve() {
    flow = cost = 0;
    std::fill(dist, dist + n, 0);
    do {
        do {
            fill(visit, visit + n, 0);
        } while (dfs(source, INT_MAX));
    } while (!relabel());
    return std::make_pair(flow, cost);
}

```

4.8 一般图最大匹配

时间复杂度: $\mathcal{O}(V^3)$

```

int match[N], belong[N], next[N], mark[N], visit[N];
std::vector<int> queue;

int find(int x) {
    if (belong[x] != x) {
        belong[x] = find(belong[x]);
    }
    return belong[x];
}

void merge(int x, int y) {
    x = find(x);
    y = find(y);
    if (x != y) {
        belong[x] = y;
    }
}

```

```

int lca(int x, int y) {
    static int stamp = 0;
    stamp++;
    while (true) {
        if (x != -1) {
            x = find(x);
            if (visit[x] == stamp) {
                return x;
            }
            visit[x] = stamp;
            if (match[x] != -1) {
                x = next[match[x]];
            } else {
                x = -1;
            }
        }
        std::swap(x, y);
    }
}

void group(int a, int p) {
    while (a != p) {
        int b = match[a], c = next[b];
        if (find(c) != p) {
            next[c] = b;
        }
        if (mark[b] == 2) {
            mark[b] = 1;
            queue.push_back(b);
        }
        if (mark[c] == 2) {
            mark[c] = 1;
            queue.push_back(c);
        }
        merge(a, b);
        merge(b, c);
        a = c;
    }
}

void augment(int source) {
    queue.clear();

```

```

for (int i = 0; i < n; ++i) {
    next[i] = visit[i] = -1;
    belong[i] = i;
    mark[i] = 0;
}
mark[source] = 1;
queue.push_back(source);
for (int head = 0; head < (int)queue.size() && match[source] == -1; ++head) {
    int x = queue[head];
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (match[x] == y || find(x) == find(y) || mark[y] == 2) {
            continue;
        }
        if (mark[y] == 1) {
            int r = lca(x, y);
            if (find(x) != r) {
                next[x] = y;
            }
            if (find(y) != r) {
                next[y] = x;
            }
            group(x, r);
            group(y, r);
        } else if (match[y] == -1) {
            next[y] = x;
            for (int u = y; u != -1; ) {
                int v = next[u];
                int mv = match[v];
                match[v] = u;
                match[u] = v;
                u = mv;
            }
            break;
        } else {
            next[y] = x;
            mark[y] = 2;
            mark[match[y]] = 1;
            queue.push_back(match[y]);
        }
    }
}
}

```



```

}

int solve() {
    std::fill(match, match + n, -1);
    for (int i = 0; i < n; ++i) {
        if (match[i] == -1) {
            augment(i);
        }
    }
    int answer = 0;
    for (int i = 0; i < n; ++i) {
        answer += (match[i] != -1);
    }
    return answer;
}

```

4.9 无向图全局最小割

时间复杂度: $\mathcal{O}(V^3)$

注意事项: 处理重边时, 应该对边权累加

```

int node[N], dist[N];
bool visit[N];

int solve(int n) {
    int answer = INT_MAX;
    for (int i = 0; i < n; ++i) {
        node[i] = i;
    }
    while (n > 1) {
        int max = 1;
        for (int i = 0; i < n; ++i) {
            dist[node[i]] = graph[node[0]][node[i]];
            if (dist[node[i]] > dist[node[max]]) {
                max = i;
            }
        }
        int prev = 0;
        memset(visit, 0, sizeof(visit));
        visit[node[0]] = true;
        for (int i = 1; i < n; ++i) {
            if (i == n - 1) {
                answer = std::min(answer, dist[node[max]]);
            }
        }
    }
}

```

```

        for (int k = 0; k < n; ++k) {
            graph[node[k]][node[prev]] =
                (graph[node[prev]][node[k]] += graph[node[k]][node[max]]);
        }
        node[max] = node[--n];
    }
    visit[node[max]] = true;
    prev = max;
    max = -1;
    for (int j = 1; j < n; ++j) {
        if (!visit[node[j]]) {
            dist[node[j]] += graph[node[prev]][node[j]];
            if (max == -1 || dist[node[max]] < dist[node[j]]) {
                max = j;
            }
        }
    }
}
}
return answer;
}

```

4.10 有根树的同构

时间复杂度: $\mathcal{O}(V \log V)$

```

const unsigned long long MAGIC = 4423;

unsigned long long magic[N];
std::pair<unsigned long long, int> hash[N];

void solve(int root) {
    magic[0] = 1;
    for (int i = 1; i <= n; ++i) {
        magic[i] = magic[i - 1] * MAGIC;
    }
    std::vector<int> queue;
    queue.push_back(root);
    for (int head = 0; head < (int)queue.size(); ++head) {
        int x = queue[head];
        for (int i = 0; i < (int)son[x].size(); ++i) {
            int y = son[x][i];
            queue.push_back(y);
        }
    }
}

```

```

    }
}
for (int index = n - 1; index >= 0; --index) {
    int x = queue[index];
    hash[x] = std::make_pair(0, 0);

    std::vector<std::pair<unsigned long long, int> > value;
    for (int i = 0; i < (int)son[x].size(); ++i) {
        int y = son[x][i];
        value.push_back(hash[y]);
    }
    std::sort(value.begin(), value.end());

    hash[x].first = hash[x].first * magic[1] + 37;
    hash[x].second++;
    for (int i = 0; i < (int)value.size(); ++i) {
        hash[x].first = hash[x].first * magic[value[i].second] + value[i].first;
        hash[x].second += value[i].second;
    }
    hash[x].first = hash[x].first * magic[1] + 41;
    hash[x].second++;
}
}

```

4.11 哈密尔顿回路 (ORE 性质的图)

ORE 性质:

$$\forall x, y \in V \wedge (x, y) \notin E \quad s.t. \quad deg_x + deg_y \geq n$$

返回结果: 从顶点 1 出发的一个哈密尔顿回路

使用条件: $n \geq 3$

```

int left[N], right[N], next[N], last[N];

void cover(int x) {
    left[right[x]] = left[x];
    right[left[x]] = right[x];
}

int adjacent(int x) {
    for (int i = right[0]; i <= n; i = right[i]) {
        if (graph[x][i]) {
            return i;
        }
    }
}

```

```

    }
    return 0;
}

std::vector<int> solve() {
    for (int i = 1; i <= n; ++i) {
        left[i] = i - 1;
        right[i] = i + 1;
    }
    int head, tail;
    for (int i = 2; i <= n; ++i) {
        if (graph[1][i]) {
            head = 1;
            tail = i;
            cover(head);
            cover(tail);
            next[head] = tail;
            break;
        }
    }
    while (true) {
        int x;
        while (x = adjacent(head)) {
            next[x] = head;
            head = x;
            cover(head);
        }
        while (x = adjacent(tail)) {
            next[tail] = x;
            tail = x;
            cover(tail);
        }
        if (!graph[head][tail]) {
            for (int i = head, j; i != tail; i = next[i]) {
                if (graph[head][next[i]] && graph[tail][i]) {
                    for (j = head; j != i; j = next[j]) {
                        last[next[j]] = j;
                    }
                    j = next[head];
                    next[head] = next[i];
                    next[tail] = i;
                    tail = j;
                }
            }
        }
    }
}

```

```

        for (j = i; j != head; j = last[j]) {
            next[j] = last[j];
        }
        break;
    }
}

next[tail] = head;
if (right[0] > n) {
    break;
}

for (int i = head; i != tail; i = next[i]) {
    if (adjacent(i)) {
        head = next[i];
        tail = i;
        next[tail] = 0;
        break;
    }
}

std::vector<int> answer;
for (int i = head; ; i = next[i]) {
    if (i == 1) {
        answer.push_back(i);
        for (int j = next[i]; j != i; j = next[j]) {
            answer.push_back(j);
        }
        answer.push_back(i);
        break;
    }
    if (i == tail) {
        break;
    }
}

return answer;
}

```

5 字符串

5.1 模式匹配

5.1.1 KMP 算法

```
void build(char *pattern) {
    int length = (int)strlen(pattern + 1);
    fail[0] = -1;
    for (int i = 1, j; i <= length; ++i) {
        for (j = fail[i - 1]; j != -1 && pattern[i] != pattern[j + 1]; j = fail[j]);
        fail[i] = j + 1;
    }
}

void solve(char *text, char *pattern) {
    int length = (int)strlen(text + 1);
    for (int i = 1, j; i <= length; ++i) {
        for (j = match[i - 1]; j != -1 && text[i] != pattern[j + 1]; j = fail[j]);
        match[i] = j + 1;
    }
}
```

5.1.2 扩展 KMP 算法

返回结果：

$$next_i = lcp(text, text_{i...n-1})$$

```
void solve(char *text, int length, int *next) {
    int j = 0, k = 1;
    for (; j + 1 < length && text[j] == text[j + 1]; j++);
    next[0] = length - 1;
    next[1] = j;
    for (int i = 2; i < length; ++i) {
        int far = k + next[k] - 1;
        if (next[i - k] < far - i + 1) {
            next[i] = next[i - k];
        } else {
            j = std::max(far - i + 1, 0);
            for (; i + j < length && text[j] == text[i + j]; j++);
            next[i] = j;
            k = i;
        }
    }
}
```

```

    }
}

```

5.1.3 AC 自动机

```

class Node {
public:
    Node *child[256], *fail;
    int counter;

    Node() : fail(NULL), counter(0) {
        memset(child, NULL, sizeof(child));
    }
};

void insert(Node *x, char *text) {
    int length = (int)strlen(text);
    for (int i = 0; i < length; ++i) {
        int token = (int)text[i];
        if (!x->child[token]) {
            x->child[token] = new Node();
        }
        x = x->child[token];
    }
    x->counter++;
}

void build() {
    std::vector<Node*> queue;
    queue.push_back(root->fail = root);
    for (int head = 0; head < (int)queue.size(); ++head) {
        Node *x = queue[head];
        for (int token = 0; token < 256; ++token) {
            if (x->child[token]) {
                x->child[token]->fail = (x == root) ? root : x->fail->child[token];
                x->child[token]->counter += x->child[token]->fail->counter;
                queue.push_back(x->child[token]);
            } else {
                x->child[token] = (x == root) ? root : x->fail->child[token];
            }
        }
    }
}

```

5.2 后缀三姐妹

5.2.1 后缀数组

```
int array[N], rank[N], height[N];
int counter[N], new_array[N], new_rank[N][2];
int log2[N], value[N][20];

void build(char *text, int n) {
    memset(counter, 0, sizeof(counter));
    for (int i = 0; i < n; ++i) {
        counter[(int)text[i]]++;
    }
    for (int i = 0; i < 256; ++i) {
        counter[i + 1] += counter[i];
    }
    for (int i = 0; i < n; ++i) {
        rank[i] = counter[(int)text[i]] - 1;
    }
    for (int length = 1; length < n; length <= 1) {
        for (int i = 0; i < n; ++i) {
            new_rank[i][0] = rank[i];
            new_rank[i][1] = i + length < n ? rank[i + length] + 1 : 0;
        }
        memset(counter, 0, sizeof(counter));
        for (int i = 0; i < n; ++i) {
            counter[new_rank[i][1]]++;
        }
        for (int i = 0; i < n; ++i) {
            counter[i + 1] += counter[i];
        }
        for (int i = n - 1; i >= 0; --i) {
            new_array[--counter[new_rank[i][1]]] = i;
        }
        memset(counter, 0, sizeof(counter));
        for (int i = 0; i < n; ++i) {
            counter[new_rank[i][0]]++;
        }
        for (int i = 0; i < n; ++i) {
            counter[i + 1] += counter[i];
        }
        for (int i = n - 1; i >= 0; --i) {
            array[--counter[new_rank[new_array[i]][0]]] = new_array[i];
        }
    }
}
```



```

    }
    rank[array[0]] = 0;
    for (int i = 0; i + 1 < n; ++i) {
        rank[array[i + 1]] = rank[array[i]] +
            (new_rank[array[i]][0] != new_rank[array[i + 1]][0]
             || new_rank[array[i]][1] != new_rank[array[i + 1]][1]);
    }
}

for (int i = 0, length = 0; i < n; ++i) {
    if (rank[i]) {
        int j = array[rank[i] - 1];
        while (i + length < n && j + length < n
                && text[i + length] == text[j + length]) {
            length++;
        }
        height[rank[i]] = length;
        if (length) {
            length--;
        }
    }
}

for (int i = 2; i <= n; ++i) {
    log2[i] = log2[i >> 1] + 1;
}

for (int i = 1; i < n; ++i) {
    value[i][0] = height[i];
}

for (int step = 1; (1 << step) <= n; ++step) {
    for (int i = 1; i + (1 << step) <= n; ++i) {
        value[i][step] = std::min(value[i][step - 1],
                                    value[i + (1 << step - 1)][step - 1]);
    }
}

}

int lcp(int left, int right) {
    if (left > right) {
        std::swap(left, right);
    }
    int step = log2[right - left];
    return std::min(value[left + 1][step], value[right - (1 << step) + 1][step]);
}

```

5.2.2 后缀自动机

```
class Node {
public:
    Node *child[256], *parent;
    int length;

    Node(int length = 0) : parent(NULL), length(length) {
        memset(child, NULL, sizeof(child));
    }

    Node* extend(Node *start, int token) {
        Node *p = this;
        Node *np = new Node(length + 1);
        for (; p && !p->child[token]; p = p->parent) {
            p->child[token] = np;
        }
        if (!p) {
            np->parent = start;
        } else {
            Node *q = p->child[token];
            if (p->length + 1 == q->length) {
                np->parent = q;
            } else {
                Node *nq = new Node(p->length + 1);
                memcpy(nq->child, q->child, sizeof(q->child));
                nq->parent = q->parent;
                np->parent = q->parent = nq;
                for (; p && p->child[token] == q; p = p->parent) {
                    p->child[token] = nq;
                }
            }
        }
        return np;
    }
};
```

5.3 回文三兄弟

5.3.1 Manacher 算法

```
void manacher(char *text, int length) {
    palindrome[0] = 1;
```

```

for (int i = 1, j = 0; i < length; ++i) {
    if (j + palindrome[j] <= i) {
        palindrome[i] = 0;
    } else {
        palindrome[i] = std::min(palindrome[(j << 1) - i], j + palindrome[j] - i);
    }
    while (i - palindrome[i] >= 0 && i + palindrome[i] < length
        && text[i - palindrome[i]] == text[i + palindrome[i]]) {
        palindrome[i]++;
    }
    if (i + palindrome[i] > j + palindrome[j]) {
        j = i;
    }
}
}

```

5.3.2 回文树

```

class Node {
public:
    Node *child[256], *fail;
    int length;

    Node(int length) : fail(NULL), length(length) {
        memset(child, NULL, sizeof(child));
    }
};

int size;
int text[N];
Node *odd, *even;

Node* match(Node *now) {
    for (; text[size - now->length - 1] != text[size]; now = now->fail);
    return now;
}

bool extend(Node *&last, int token) {
    text[++size] = token;
    Node *now = last;
    now = match(now);
    if (now->child[token]) {
        last = now->child[token];
    }
}

```

```

        return false;
    }
    last = now->child[token] = new Node(now->length + 2);
    if (now == odd) {
        last->fail = even;
    } else {
        now = match(now->fail);
        last->fail = now->child[token];
    }
    return true;
}

void build() {
    text[size = 0] = -1;
    even = new Node(0), odd = new Node(-1);
    even->fail = odd;
}

```

5.4 循环串最小表示

```

int solve(char *text, int length) {
    int i = 0, j = 1, delta = 0;
    while (i < length && j < length && delta < length) {
        char tokeni = text[(i + delta) % length];
        char tokenj = text[(j + delta) % length];
        if (tokeni == tokenj) {
            delta++;
        } else {
            if (tokeni > tokenj) {
                i += delta + 1;
            } else {
                j += delta + 1;
            }
            if (i == j) {
                j++;
            }
            delta = 0;
        }
    }
    return std::min(i, j);
}

```

6 计算几何

6.1 二维基础

6.1.1 点类

```
int sgn(const double &x, const double &eps = 1e-8) {
    return (x < -eps) ? -1 : (x > eps);
}

class Point {
public:
    double x, y;

    Point() {}

    Point(const double &x, const double &y) : x(x), y(y) {}

    double len() const {
        return hypot(x, y);
    }

    friend Point operator + (const Point &a, const Point &b) {
        return Point(a.x + b.x, a.y + b.y);
    }

    friend Point operator - (const Point &a, const Point &b) {
        return Point(a.x - b.x, a.y - b.y);
    }

    friend Point operator * (const Point &a, const double &k) {
        return Point(a.x * k, a.y * k);
    }

    friend Point operator / (const Point &a, const double &k) {
        return Point(a.x / k, a.y / k);
    }

    friend bool operator == (const Point &a, const Point &b) {
        return sgn(a.x - b.x) == 0 && sgn(a.y - b.y) == 0;
    }

    friend bool operator < (const Point &a, const Point &b) {
```

```

        if (sgn(a.x - b.x) == 0) {
            return sgn(a.y - b.y) < 0;
        } else {
            return sgn(a.x - b.x) < 0;
        }
    }

    friend double det(const Point &a, const Point &b) {
        return a.x * b.y - a.y * b.x;
    }

    friend double det(const Point &a, const Point &b, const Point &c) {
        return det(b - a, c - a);
    }

    friend double dot(const Point &a, const Point &b) {
        return a.x * b.x + a.y * b.y;
    }

    friend double dot(const Point &a, const Point &b, const Point &c) {
        return dot(b - a, c - a);
    }

    friend Point intersect(const Point &a, const Point &b, const Point &c, const Point &d) {
        double s1 = det(a, b, c);
        double s2 = det(a, b, d);
        return (s1 * d - s2 * c) / (s1 - s2);
    }
};

```

6.1.2 凸包

```

std::vector<Point> convex_hull(std::vector<Point> point) {
    std::sort(point.begin(), point.end());
    std::vector<Point> convex;
    {
        std::vector<Point> stack;
        for (int i = 0; i < (int)point.size(); ++i) {
            while ((int)stack.size() >= 2 &&
                sgn(det(stack[(int)stack.size() - 2], stack.back(), point[i])) <= 0) {
                stack.pop_back();
            }
            stack.push_back(point[i]);
        }
    }
    return convex;
}

```

```

    }
    for (int i = 0; i < (int)stack.size(); ++i) {
        convex.push_back(stack[i]);
    }
}
{
    std::vector<Point> stack;
    for (int i = (int)point.size() - 1; i >= 0; --i) {
        while ((int)stack.size() >= 2 &&
            sgn(det(stack[(int)stack.size() - 2], stack.back(), point[i])) <= 0) {
            stack.pop_back();
        }
        stack.push_back(point[i]);
    }
    for (int i = 1; i < (int)stack.size() - 1; ++i) {
        convex.push_back(stack[i]);
    }
}
return convex;
}

```

6.1.3 最近点对

```

bool comparex(const Point &a, const Point &b) {
    return sgn(a.x - b.x) < 0;
}

bool comparey(const Point &a, const Point &b) {
    return sgn(a.y - b.y) < 0;
}

double solve(const std::vector<Point> &point, int left, int right) {
    if (left == right) {
        return INF;
    }
    if (left + 1 == right) {
        return dist(point[left], point[right]);
    }
    int mid = left + right >> 1;
    double result = std::min(solve(left, mid), solve(mid + 1, right));
    std::vector<Point> candidate;
    for (int i = left; i <= right; ++i) {
        if (std::abs(point[i].x - point[mid].x) <= result) {

```

```

        candidate.push_back(point[i]);
    }
}

std::sort(candidate.begin(), candidate.end(), comparey);
for (int i = 0; i < (int)candidate.size(); ++i) {
    for (int j = i + 1; j < (int)candidate.size(); ++j) {
        if (std::abs(candidate[i].y - candidate[j].y) >= result) {
            break;
        }
        result = std::min(result, dist(candidate[i], candidate[j]));
    }
}

return result;
}

double solve(std::vector<Point> point) {
    std::sort(point.begin(), point.end(), comparex);
    return solve(point, 0, (int)point.size() - 1);
}

```

6.2 多边形

6.2.1 判断点在多边形内部

```

bool point_on_line(const Point &p, const Point &a, const Point &b) {
    return sgn(det(p, a, b)) == 0 && sgn(dot(p, a, b)) <= 0;
}

bool point_in_polygon(const Point &p, const std::vector<Point> &polygon) {
    int counter = 0;
    for (int i = 0; i < (int)polygon.size(); ++i) {
        Point a = polygon[i], b = polygon[(i + 1) % (int)polygon.size()];
        if (point_on_line(p, a, b)) {
            // Point on the boundary are excluded.
            return false;
        }
        int x = sgn(det(a, p, b));
        int y = sgn(a.y - p.y);
        int z = sgn(b.y - p.y);
        counter += (x > 0 && y <= 0 && z > 0);
        counter -= (x < 0 && z <= 0 && y > 0);
    }
    return counter;
}

```



```
}
```

7 其他

7.1 某年某月某日是星期几

```
int solve(int year, int month, int day) {
    int answer;
    if (month == 1 || month == 2) {
        month += 12;
        year--;
    }
    if ((year < 1752) || (year == 1752 && month < 9) ||
        (year == 1752 && month == 9 && day < 3)) {
        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4 + 5) % 7;
    } else {
        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4
                  - year / 100 + year / 400) % 7;
    }
    return answer;
}
```

7.2 枚举大小为 k 的子集

使用条件: $k > 0$

```
void solve(int n, int k) {
    for (int comb = (1 << k) - 1; comb < (1 << n); ) {
        // ...
        int x = comb & -comb, y = comb + x;
        comb = (((comb & ~y) / x) >> 1) | y;
    }
}
```

7.3 环状最长公共子串

```
int n, a[N << 1], b[N << 1];

bool has(int i, int j) {
    return a[(i - 1) % n] == b[(j - 1) % n];
}

const int DELTA[3][2] = {{0, -1}, {-1, -1}, {-1, 0}};
```

```

int from[N][N];

int solve() {
    memset(from, 0, sizeof(from));
    int ret = 0;
    for (int i = 1; i <= 2 * n; ++i) {
        from[i][0] = 2;
        int left = 0, up = 0;
        for (int j = 1; j <= n; ++j) {
            int upleft = up + 1 + !!from[i - 1][j];
            if (!has(i, j)) {
                upleft = INT_MIN;
            }
            int max = std::max(left, std::max(upleft, up));
            if (left == max) {
                from[i][j] = 0;
            } else if (upleft == max) {
                from[i][j] = 1;
            } else {
                from[i][j] = 2;
            }
            left = max;
        }
        if (i >= n) {
            int count = 0;
            for (int x = i, y = n; y; ) {
                int t = from[x][y];
                count += t == 1;
                x += DELTA[t][0];
                y += DELTA[t][1];
            }
            ret = std::max(ret, count);
            int x = i - n + 1;
            from[x][0] = 0;
            int y = 0;
            while (y <= n && from[x][y] == 0) {
                y++;
            }
            for (; x <= i; ++x) {
                from[x][y] = 0;
                if (x == i) {

```

```

        break;
    }
    for (; y <= n; ++y) {
        if (from[x + 1][y] == 2) {
            break;
        }
        if (y + 1 <= n && from[x + 1][y + 1] == 1) {
            y++;
            break;
        }
    }
}
}
return ret;
}

```

8 Java

8.1 基础模板

```

import java.io.*;
import java.util.*;
import java.math.*;

public class Main {
    public static void main(String[] args) {
        InputStream inputStream = System.in;
        OutputStream outputStream = System.out;
        InputReader in = new InputReader(inputStream);
        PrintWriter out = new PrintWriter(outputStream);
        Task solver = new Task();
        solver.solve(0, in, out);
        out.close();
    }
}

class Task {
    public void solve(int testNumber, InputReader in, PrintWriter out) {

    }
}

```

```

class InputReader {
    public BufferedReader reader;
    public StringTokenizer tokenizer;

    public InputReader(InputStream stream) {
        reader = new BufferedReader(new InputStreamReader(stream), 32768);
        tokenizer = null;
    }

    public String next() {
        while (tokenizer == null || !tokenizer.hasMoreTokens()) {
            try {
                tokenizer = new StringTokenizer(reader.readLine());
            } catch (IOException e) {
                throw new RuntimeException(e);
            }
        }
        return tokenizer.nextToken();
    }

    public int nextInt() {
        return Integer.parseInt(next());
    }

    public long nextLong() {
        return Long.parseLong(next());
    }
}

```

9 数学

9.1 常用数学公式

9.1.1 求和公式

1. $\sum_{k=1}^n (2k-1)^2 = \frac{n(4n^2-1)}{3}$
2. $\sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2}\right]^2$
3. $\sum_{k=1}^n (2k-1)^3 = n^2(2n^2-1)$
4. $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
5. $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$

6. $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$
7. $\sum_{k=1}^n k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
8. $\sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$

9.1.2 斐波那契数列

1. $fib_0 = 0, fib_1 = 1, fib_n = fib_{n-1} + fib_{n-2}$
2. $fib_{n+2} \cdot fib_n - fib_{n+1}^2 = (-1)^{n+1}$
3. $fib_{-n} = (-1)^{n-1} fib_n$
4. $fib_{n+k} = fib_k \cdot fib_{n+1} + fib_{k-1} \cdot fib_n$
5. $gcd(fib_m, fib_n) = fib_{gcd(m,n)}$
6. $fib_m | fib_n^2 \Leftrightarrow n fib_n | m$

9.1.3 错排公式

1. $D_n = (n-1)(D_{n-2} + D_{n-1})$
2. $D_n = n! \cdot (1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!})$

9.1.4 莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & \text{若 } n = 1 \\ (-1)^k & \text{若 } n \text{ 无平方数因子, 且 } n = p_1 p_2 \dots p_k \\ 0 & \text{若 } n \text{ 有大于1的平方数因数} \end{cases}$$

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{若 } n = 1 \\ 0 & \text{其他情况} \end{cases}$$

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g\left(\frac{n}{d}\right)$$

$$g(x) = \sum_{n=1}^{[x]} f\left(\frac{x}{n}\right) \Leftrightarrow f(x) = \sum_{n=1}^{[x]} \mu(n) g\left(\frac{x}{n}\right)$$

9.1.5 伯恩赛德引理

设 G 是一个有限群, 作用在集合 X 上. 对每个 g 属于 G , 令 X^g 表示 X 中在 g 作用下的不动元素, 轨道数 (记作 $|X/G|$) 由如下公式给出:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

9.1.6 五边形数定理

设 $p(n)$ 是 n 的拆分数, 有

$$p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k-1} p\left(n - \frac{k(3k-1)}{2}\right)$$

9.1.7 树的计数

1. 有根树计数: $n+1$ 个结点的有根树的个数为

$$a_{n+1} = \frac{\sum_{j=1}^n j \cdot a_j \cdot S_{n,j}}{n}$$

其中,

$$S_{n,j} = \sum_{i=1}^{n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$$

2. 无根树计数: 当 n 为奇数时, n 个结点的无根树的个数为

$$a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$$

当 n 为偶数时, n 个结点的无根树的个数为

$$a_n - \sum_{i=1}^{n/2} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$$

3. n 个结点的完全图的生成树个数为

$$n^{n-2}$$

4. 矩阵-树定理: 图 G 由 n 个结点构成, 设 $A[G]$ 为图 G 的邻接矩阵、 $D[G]$ 为图 G 的度数矩阵, 则图 G 的不同生成树的个数为 $C[G] = D[G] - A[G]$ 的任意一个 $n-1$ 阶主子式的行列式值。

9.1.8 欧拉公式

平面图形的顶点个数、边数和面的个数有如下关系:

$$V - E + F = C + 1$$

其中, V 是顶点的数目, E 是边的数目, F 是面的数目, C 是组成图形的连通部分的数目。当图是单连通图的时候, 公式简化为:

$$V - E + F = 2$$

9.1.9 皮克定理

给定顶点坐标均是整点（或正方形格点）的简单多边形，其面积 A 和内部格点数目 i 、边上格点数目 b 的关系：

$$A = i + \frac{b}{2} - 1$$

9.1.10 牛顿恒等式

设

$$\prod_{i=1}^n (x - x_i) = a_n + a_{n-1}x + \cdots + a_1x^{n-1} + a_0x^n$$

$$p_k = \sum_{i=1}^n x_i^k$$

则

$$a_0 p_k + a_1 p_{k-1} + \cdots + a_{k-1} p_1 + k a_k = 0$$

特别地，对于

$$|\mathbf{A} - \lambda \mathbf{E}| = (-1)^n (a_n + a_{n-1}\lambda + \cdots + a_1\lambda^{n-1} + a_0\lambda^n)$$

有

$$p_k = \text{Tr}(\mathbf{A}^k)$$

9.2 平面几何公式

9.2.1 三角形

1. 半周长

$$p = \frac{a + b + c}{2}$$

2. 面积

$$S = \frac{a \cdot H_a}{2} = \frac{ab \cdot \sin C}{2} = \sqrt{p(p-a)(p-b)(p-c)}$$

3. 中线

$$M_a = \frac{\sqrt{2(b^2 + c^2) - a^2}}{2} = \frac{\sqrt{b^2 + c^2 + 2bc \cdot \cos A}}{2}$$

4. 角平分线

$$T_a = \frac{\sqrt{bc \cdot [(b+c)^2 - a^2]}}{b+c} = \frac{2bc}{b+c} \cos \frac{A}{2}$$

5. 高线

$$H_a = b \sin C = c \sin B = \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2}$$

6. 内切圆半径

$$\begin{aligned} r &= \frac{S}{p} = \frac{\arcsin \frac{B}{2} \cdot \sin \frac{C}{2}}{\sin \frac{B+C}{2}} = 4R \cdot \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \\ &= \sqrt{\frac{(p-a)(p-b)(p-c)}{p}} = p \cdot \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2} \end{aligned}$$

7. 外接圆半径

$$R = \frac{abc}{4S} = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$$

9.2.2 四边形

D_1, D_2 为对角线, M 为对角线中点连线, A 为对角线夹角, p 为半周长

$$1. a^2 + b^2 + c^2 + d^2 = D_1^2 + D_2^2 + 4M^2$$

$$2. S = \frac{1}{2} D_1 D_2 \sin A$$

3. 对于圆内接四边形

$$ac + bd = D_1 D_2$$

4. 对于圆内接四边形

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$$

9.2.3 正 n 边形

R 为外接圆半径, r 为内切圆半径

1. 中心角

$$A = \frac{2\pi}{n}$$

2. 内角

$$C = \frac{n-2}{n} \pi$$

3. 边长

$$a = 2\sqrt{R^2 - r^2} = 2R \cdot \sin \frac{A}{2} = 2r \cdot \tan \frac{A}{2}$$

4. 面积

$$S = \frac{nar}{2} = nr^2 \cdot \tan \frac{A}{2} = \frac{nR^2}{2} \cdot \sin A = \frac{na^2}{4 \cdot \tan \frac{A}{2}}$$

9.2.4 圆

1. 弧长

$$l = rA$$

2. 弦长

$$a = 2\sqrt{2hr - h^2} = 2r \cdot \sin \frac{A}{2}$$

3. 弓形高

$$h = r - \sqrt{r^2 - \frac{a^2}{4}} = r(1 - \cos \frac{A}{2}) = \frac{1}{2} \cdot \arctan \frac{A}{4}$$

4. 扇形面积

$$S_1 = \frac{rl}{2} = \frac{r^2 A}{2}$$

5. 弓形面积

$$S_2 = \frac{rl - a(r - h)}{2} = \frac{r^2}{2}(A - \sin A)$$

9.2.5 棱柱

1. 体积

$$V = Ah$$

A 为底面积, h 为高

2. 侧面积

$$S = lp$$

l 为棱长, p 为直截面周长

3. 全面积

$$T = S + 2A$$

9.2.6 棱锥

1. 体积

$$V = Ah$$

A 为底面积, h 为高

2. 正棱锥侧面积

$$S = lp$$

l 为棱长, p 为直截面周长

3. 正棱锥全面积

$$T = S + 2A$$

9.2.7 棱台

1. 体积

$$V = (A_1 + A_2 + \sqrt{A_1 A_2}) \cdot \frac{h}{3}$$

A_1, A_2 为上下底面积, h 为高

2. 正棱台侧面积

$$S = \frac{p_1 + p_2}{2} l$$

p_1, p_2 为上下底面周长, l 为斜高

3. 正棱台全面积

$$T = S + A_1 + A_2$$

9.2.8 圆柱

1. 侧面积

$$S = 2\pi rh$$

2. 全面积

$$T = 2\pi r(h + r)$$

3. 体积

$$V = \pi r^2 h$$

9.2.9 圆锥

1. 母线

$$l = \sqrt{h^2 + r^2}$$

2. 侧面积

$$S = \pi rl$$

3. 全面积

$$T = \pi r(l + r)$$

4. 体积

$$V = \frac{\pi}{3} r^2 h$$

9.2.10 圆台

1. 母线

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

2. 侧面积

$$S = \pi(r_1 + r_2)l$$

3. 全面积

$$T = \pi r_1(l + r_1) + \pi r_2(l + r_2)$$

4. 体积

$$V = \frac{\pi}{3}(r_1^2 + r_2^2 + r_1 r_2)h$$

9.2.11 球

1. 全面积

$$T = 4\pi r^2$$

2. 体积

$$V = \frac{4}{3}\pi r^3$$

9.2.12 球台

1. 侧面积

$$S = 2\pi r h$$

2. 全面积

$$T = \pi(2rh + r_1^2 + r_2^2)$$

3. 体积

$$V = \frac{\pi h[3(r_1^2 + r_2^2) + h^2]}{6}$$

9.2.13 球扇形

1. 全面积

$$T = \pi r(2h + r_0)$$

h 为球冠高, r_0 为球冠底面半径

2. 体积

$$V = \frac{2}{3}\pi r^2 h$$

9.3 立体几何公式

9.3.1 球面三角公式

设 a, b, c 是边长, A, B, C 是所对的二面角, 有余弦定理

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

正弦定理

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

三角形面积是 $A + B + C - \pi$

9.3.2 四面体体积公式

U, V, W, u, v, w 是四面体的 6 条棱, U, V, W 构成三角形, $(U, u), (V, v), (W, w)$ 互为对棱, 则

$$V = \frac{\sqrt{(s-2a)(s-2b)(s-2c)(s-2d)}}{192uvw}$$

其中

$$\left\{ \begin{array}{lcl} a & = & \sqrt{xYZ}, \\ b & = & \sqrt{yZX}, \\ c & = & \sqrt{zXY}, \\ d & = & \sqrt{xyz}, \\ s & = & a + b + c + d, \\ X & = & (w - U + v)(U + v + w), \\ x & = & (U - v + w)(v - w + U), \\ Y & = & (u - V + w)(V + w + u), \\ y & = & (V - w + u)(w - u + V), \\ Z & = & (v - W + u)(W + u + v), \\ z & = & (W - u + v)(u - v + W) \end{array} \right.$$