CS435 Assignment 3 - Retargeting

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1 Theory Questions

1. Where $T_L = 2$ and $T_H = 4$:

   (a) 4-way connectivity:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

   (b) 8-way connectivity:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

2. For a circle centered at (a,b) with radius r, below is the piece-wise such that given a pixel location (x,y), f(x,y) will give the blending factor:

$$f(x,y) = \begin{cases} 1 & (x-a)^2 + (y-b)^2 \leq r^2 \\ 1 - \frac{1}{2\pi\sigma^2} e^{-(\frac{\sqrt{(x-r)^2+(y-r)^2}}{2\sigma^2})} & otherwise \end{cases}$$

   If the pixel is on or within the circle, this means the pixel value is only taken from the first image, I_1. As a result, the first piece-wise function will be used, where the blending factor will be equal to 1. Otherwise, if the pixel is outside the circle, the blending factor will be equal to the result from the second piece-wise.

3. Given the grayscale image, I, what is the value of a pixel at the location x = 2.7 and y = 3.1?

$$I = \begin{bmatrix} 2 & 3 & 4 & 5 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 3 & 5 & 1 & 2 \\ 4 & 4 & 4 & 4 & 6 \\ 4 & 5 & 2 & 0 & 2 \\ 2 & 3 & 3 & 0 & 3 \end{bmatrix}$$

(a) By using Nearest Neighbor, the closest coordinate to (2.7, 3.1) is (3.0, 3.0). As a result, the value of a pixel at (2.7, 3.1) would be $\boxed{5}$.

(b) By using bilinear interpolation, we know that the coordinate p = (2.7, 3.1) is surrounded by the following 4 coordinates: A = (2, 3) = 3, B = (3, 3) = 5, C = (2, 4) = 4, and D = (3, 4) = 4.

We will also need to use the Manhattan Distance Formula:

$$d(a, b) = |b_0 - a_0| + |b_1 - a_1|$$

$$f(p|A) = f(A)\left(\frac{\frac{1}{d(A,p)}}{\frac{1}{d(A,p)} + \frac{1}{d(B,p)} + \frac{1}{d(C,p)} + \frac{1}{d(D,p)}}\right)$$

$$f(p|A) = 3\left(\frac{\frac{1}{|2.7-2|+|3.1-3|}}{\frac{1}{|2.7-2|+|3.1-3|} + \frac{1}{|2.7-3|+|3.1-3|} + \frac{1}{|2.7-2|+|3.1-4|} + \frac{1}{|2.7-3|+|3.1-4|}}\right)$$

$$f(p|A) = 3\left(\frac{\frac{1}{0.8}}{\frac{1}{0.8} + \frac{1}{0.4} + \frac{1}{1.6} + \frac{1}{1.2}}\right)$$

$$\boxed{f(p|A) = 0.72}$$

$$f(p|B) = f(B)\left(\frac{\frac{1}{d(B,p)}}{\frac{1}{d(A,p)} + \frac{1}{d(B,p)} + \frac{1}{d(C,p)} + \frac{1}{d(D,p)}}\right)$$

$$f(p|B) = 5\left(\frac{\frac{1}{|2.7-3|+|3.1-3|}}{\frac{1}{|2.7-2|+|3.1-3|} + \frac{1}{|2.7-3|+|3.1-3|} + \frac{1}{|2.7-2|+|3.1-4|} + \frac{1}{|2.7-3|+|3.1-4|}}\right)$$

$$f(p|B) = 5\left(\frac{\frac{1}{0.4}}{\frac{1}{0.8} + \frac{1}{0.4} + \frac{1}{1.6} + \frac{1}{1.2}}\right)$$

$$\boxed{f(p|B) = 2.40}$$

$$f(p|C) = f(C)\left(\frac{\frac{1}{d(C,p)}}{\frac{1}{d(A,p)}+\frac{1}{d(B,p)}+\frac{1}{d(C,p)}+\frac{1}{d(D,p)}}\right)$$

$$f(p|C) = 4\left(\frac{\frac{1}{|2.7-2|+|3.1-4|}}{\frac{1}{|2.7-2|+|3.1-3|}+\frac{1}{|2.7-3|+|3.1-3|}+\frac{1}{|2.7-2|+|3.1-4|}+\frac{1}{|2.7-3|+|3.1-4|}}\right)$$

$$f(p|C) = 4\left(\frac{\frac{1}{1.6}}{\frac{1}{0.8}+\frac{1}{0.4}+\frac{1}{1.6}+\frac{1}{1.2}}\right)$$

$$\boxed{f(p|C) = 0.48}$$

$$f(p|D) = f(D)\left(\frac{\frac{1}{d(D,p)}}{\frac{1}{d(A,p)}+\frac{1}{d(B,p)}+\frac{1}{d(C,p)}+\frac{1}{d(D,p)}}\right)$$

$$f(p|D) = 4\left(\frac{\frac{1}{|2.7-2|+|3.1-4|}}{\frac{1}{|2.7-2|+|3.1-3|}+\frac{1}{|2.7-3|+|3.1-3|}+\frac{1}{|2.7-2|+|3.1-4|}+\frac{1}{|2.7-3|+|3.1-4|}}\right)$$

$$f(p|D) = 4\left(\frac{\frac{1}{1.2}}{\frac{1}{0.8}+\frac{1}{0.4}+\frac{1}{1.6}+\frac{1}{1.2}}\right)$$

$$\boxed{f(p|D) = 0.64}$$

$$f(p) = f(p|A) + f(p|B) + f(p|C) + f(p|D)$$

$$f(p) = 0.72 + 2.40 + 0.48 + 0.64$$

$$\boxed{f(p) = 4.24}$$

4. Original Energy of Image Matrix:

$$\begin{bmatrix} 2 & 3 & 4 & 5 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 3 & 5 & 1 & 2 \\ 4 & 4 & 4 & 4 & 6 \\ 4 & 5 & 2 & 0 & 2 \\ 2 & 3 & 3 & 0 & 3 \end{bmatrix}$$

(a) Optimal Seam Matrix:

$$\begin{bmatrix} 2 & 3 & 4 & 5 & 1 \\ 3 & 2 & 5 & 3 & 2 \\ 6 & 5 & 7 & 3 & 4 \\ 9 & 9 & 7 & 7 & 9 \\ 13 & 12 & 9 & 7 & 9 \\ 14 & 12 & 10 & 7 & 10 \end{bmatrix}$$

(b) Optimal Seams

$$\begin{bmatrix} 2 & 3 & 4 & 5 & \underline{\mathbf{1}} \\ 3 & 2 & 5 & 3 & \underline{\mathbf{2}} \\ 6 & 5 & 7 & \underline{\mathbf{3}} & 4 \\ 9 & 9 & 7 & \underline{\mathbf{7}} & 9 \\ 13 & 12 & 9 & \underline{\mathbf{7}} & 9 \\ 14 & 12 & 10 & \underline{\mathbf{7}} & 10 \end{bmatrix} \textit{ or } \begin{bmatrix} 2 & 3 & 4 & 5 & \underline{\mathbf{1}} \\ 3 & 2 & 5 & 3 & \underline{\mathbf{2}} \\ 6 & 5 & 7 & \underline{\mathbf{3}} & 4 \\ 9 & 9 & \underline{\mathbf{7}} & 7 & 9 \\ 13 & 12 & 9 & \underline{\mathbf{7}} & 9 \\ 14 & 12 & 10 & \underline{\mathbf{7}} & 10 \end{bmatrix}$$

2 Image Resizing

Figure 1: Background Image 1 - Original (960 x 540)

Figure 2: Background Image 1 - Nearest Neighbor Sampling (480 x 270)

Figure 3: Background Image 1 - Nearest Neighbor Sampling (240 x 135)

Figure 4: Background Image 1 - Linear Interpolation (480 x 270)

Figure 5: Background Image 1 - Linear Interpolation (240 x 135)

Figure 6: Background Image 2 - Original (1024 x 640)

Figure 7: Background Image 2 - Nearest Neighbor Sampling (512 x 320)

Figure 8: Background Image 2 - Nearest Neighbor Sampling (256 x 160)

Figure 9: Background Image 2 - Linear Interpolation (512 x 320)

Figure 10: Background Image 2 - Linear Interpolation (256 x 160)

3 Energy Function

By using the following Gaussian Filter Function:

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

And the following derivative kernels:

$$d_x = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 1/2 & 0 & -1/2 \\ 1/2 & 0 & -1/2 \end{bmatrix}$$

$$d_y = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 0 & 0 & 0 \\ -1/2 & -1/2 & -1/2 \end{bmatrix}$$

for a $3x3$ kernel where $\sigma = 1$, we get the following Gaussian Matrix:

$$\begin{bmatrix} 0.07511 & 0.12380 & 0.07511 \\ 0.12380 & 0.20410 & 0.12380 \\ 0.07511 & 0.12380 & 0.07511 \end{bmatrix}$$

By using the above Gaussian Matrix, we get the following energy images:

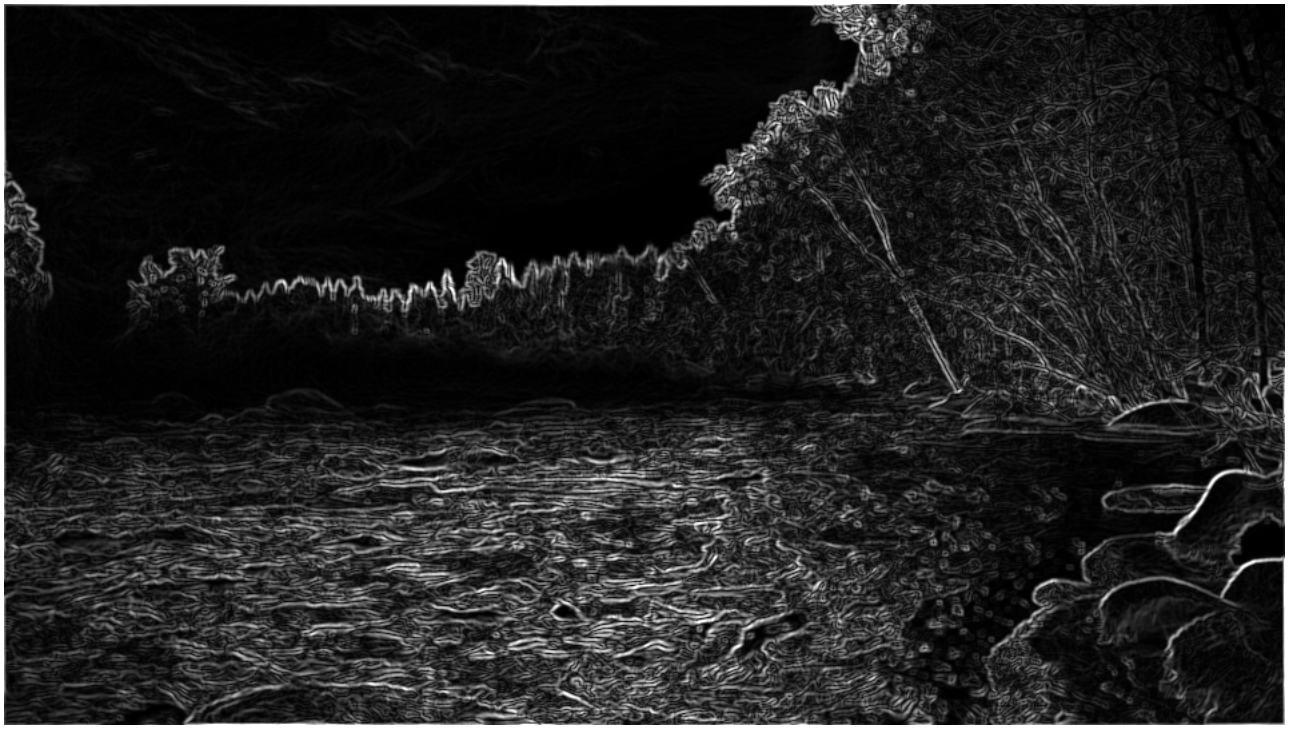

Figure 11: Background Image 1 - Image Energy

Figure 12: Background Image 2 - Image Energy

4 Optimal Seam

Figure 13: Background Image 1 - Optimal Seam Overlay

Figure 14: Background Image 2 - Optimal Seam Overlay

5 Seam Carving

Please find the seam carving videos located in the 'results' folder under the respective sub folder named after each image.