CS435 Assignment 2 - High Dynamic Range (HDR) Images

Harry Chong

2 February 2021

1 Theory Questions

1. $3x3$ Median Matrix:

   After taking the median for every $3x3$, we getting the following:

$$\boxed{\begin{bmatrix} 6 & 6 & 4 & 4 & 3 & 2 \\ 3 & 4 & 4 & 4 & 3 & 4 \\ 3 & 4 & 4 & 3 & 2 & 3 \\ 2 & 2 & 3 & 4 & 5 & 5 \\ 2 & 3 & 5 & 6 & 5 & 4 \\ 4 & 4 & 5 & 6 & 4 & 4 \end{bmatrix}}$$

2. Denote the center of a $3x3$ kernel with a coordinate of (0,0):

$$\begin{bmatrix} \text{(-1, -1)} & \text{(0, -1)} & \text{(1, -1)} \\ \text{(-1, 0)} & \text{(0, 0)} & \text{(1, 0)} \\ \text{(-1, 1)} & \text{(0, 1)} & \text{(1, 1)} \end{bmatrix}$$

   And by using the simplified Laplace probability density function where x represents the distance from the center, $\mu = 0$ and $b = 1$:

$$f(x|\mu, b) = \frac{1}{2b}e^{\frac{-|x-\mu|}{b}} = \frac{1}{2}e^{-|x|}$$

   We can get the following matrix:

$$\begin{bmatrix} 0.0676 & 0.1839 & 0.0676 \\ 0.1839 & 0.5000 & 0.1839 \\ 0.0676 & 0.1839 & 0.0676 \end{bmatrix}$$

   However, the sum of all the values in the matrix is 1.5060. To normalize the matrix, divide each element by 1.5060 to get the following answer:

$$\boxed{\begin{bmatrix} 0.0448 & 0.1221 & 0.0448 \\ 0.1221 & 0.3320 & 0.1221 \\ 0.0448 & 0.1221 & 0.0448 \end{bmatrix}}$$

3. Reference equations used to calculate the gradient:

$$G_x = \frac{\partial I}{\partial x}$$
$$G_y = \frac{\partial I}{\partial y}$$
$$|G| = \sqrt{G_x^2 + G_y^2}$$
$$\theta = tan^{-1}(\frac{G_y}{G_x})$$

We first must obtain $\frac{\partial I}{\partial x}$ and $\frac{\partial I}{\partial y}$ by using the following equations:

$$\frac{\partial I}{\partial x} = \frac{\partial}{\partial x} I$$
$$\frac{\partial I}{\partial y} = \frac{\partial}{\partial y} I$$

This results in the following:

$$\frac{\partial I}{\partial x} = \begin{bmatrix} -6.3333 & 1.66667 & 6.3333 \\ -10.6667 & \boxed{-0.6667} & 10.6667 \\ -9.3333 & -1.3333 & 9.3333 \end{bmatrix}$$
$$\frac{\partial I}{\partial y} = \begin{bmatrix} -4.3333 & -8.6667 & -5.3333 \\ 0.0000 & \boxed{-3.3333} & -3.0000 \\ 4.3333 & 8.6667 & 5.3333 \end{bmatrix}$$

With each matrix representing the gradient, we can calculate the magnitude and angle of the gradient at the center element:

$$\boxed{G = \sqrt{G_x^2 + G_y^2} = \sqrt{(-0.6667)^2 + (-3.3333)^2} = 3.3993}$$
$$\boxed{\theta = \tan^{-1}(\frac{-3.3333}{-0.6667}) = 78.69°}$$

2 Plotting pixel value vs log exposure

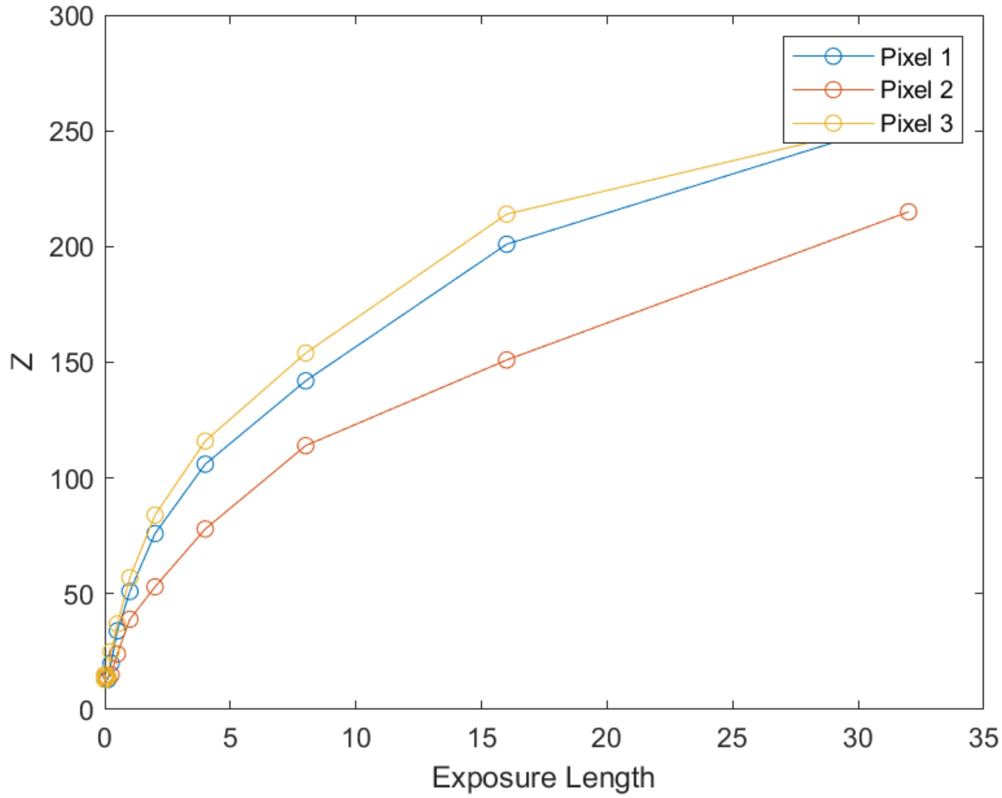

Figure 1: Observed red intensity vs Exposure Length

3 Finding and Plotting a Log Irradiance Function

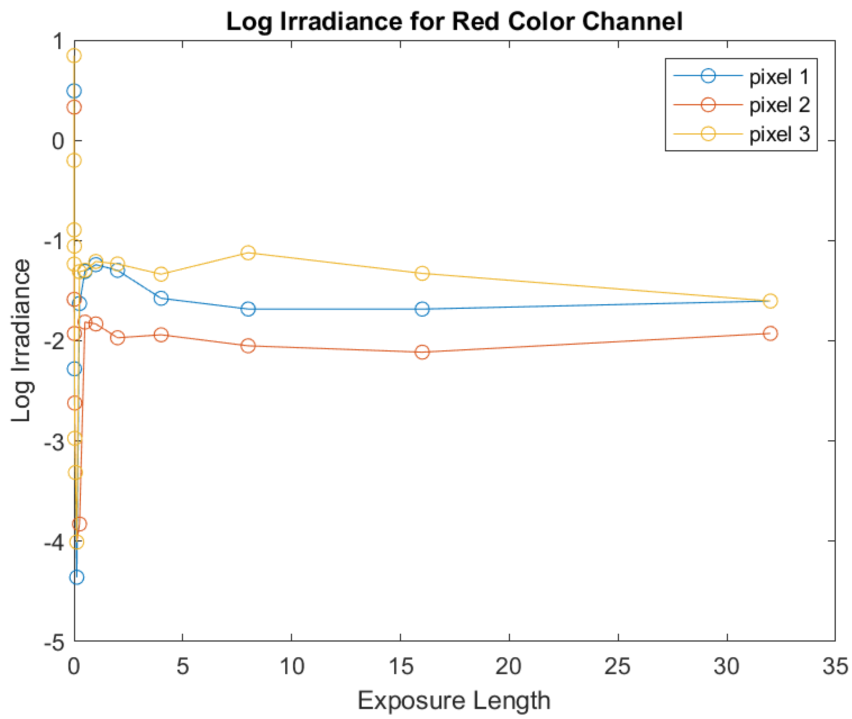

Figure 2: Red Channel Log Irradiance vs Exposure Length

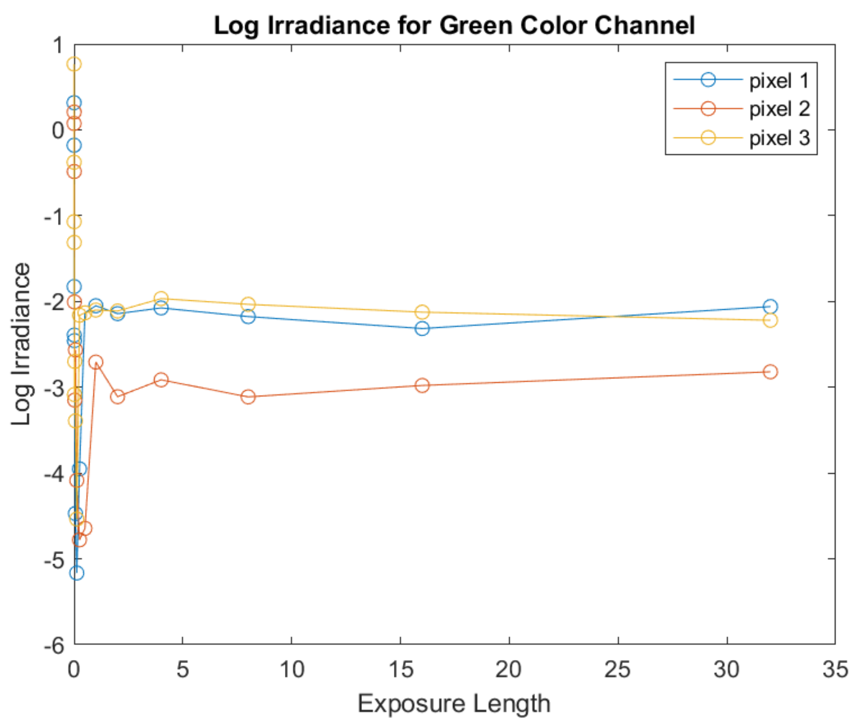

Figure 3: Green Channel Log Irradiance vs Exposure Length

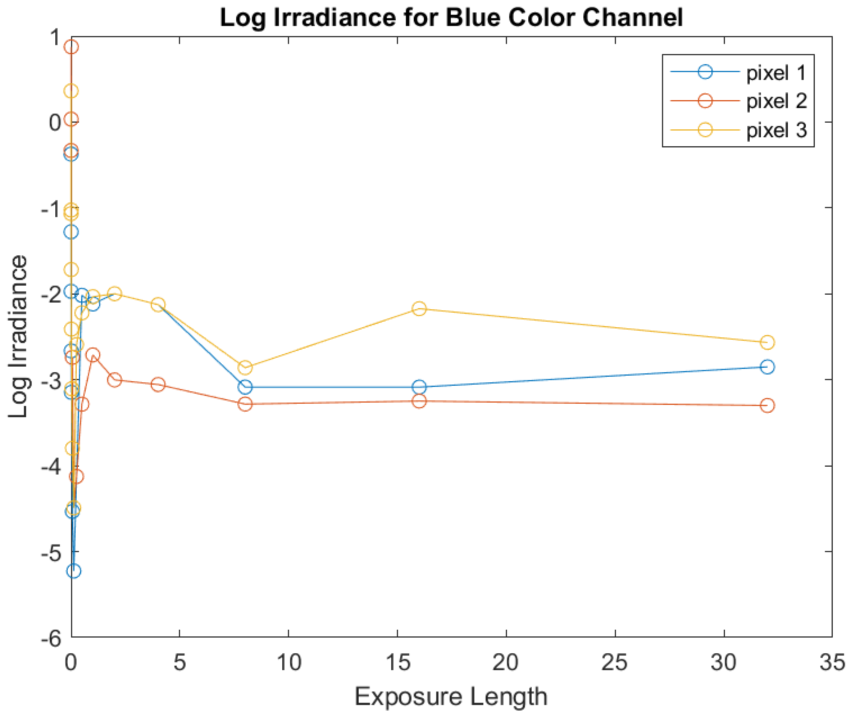

Figure 4: Blue Channel Log Irradiance vs Exposure Length

4 Generate HDR Image and Tonemap

Figure 5: Generated HDR Image

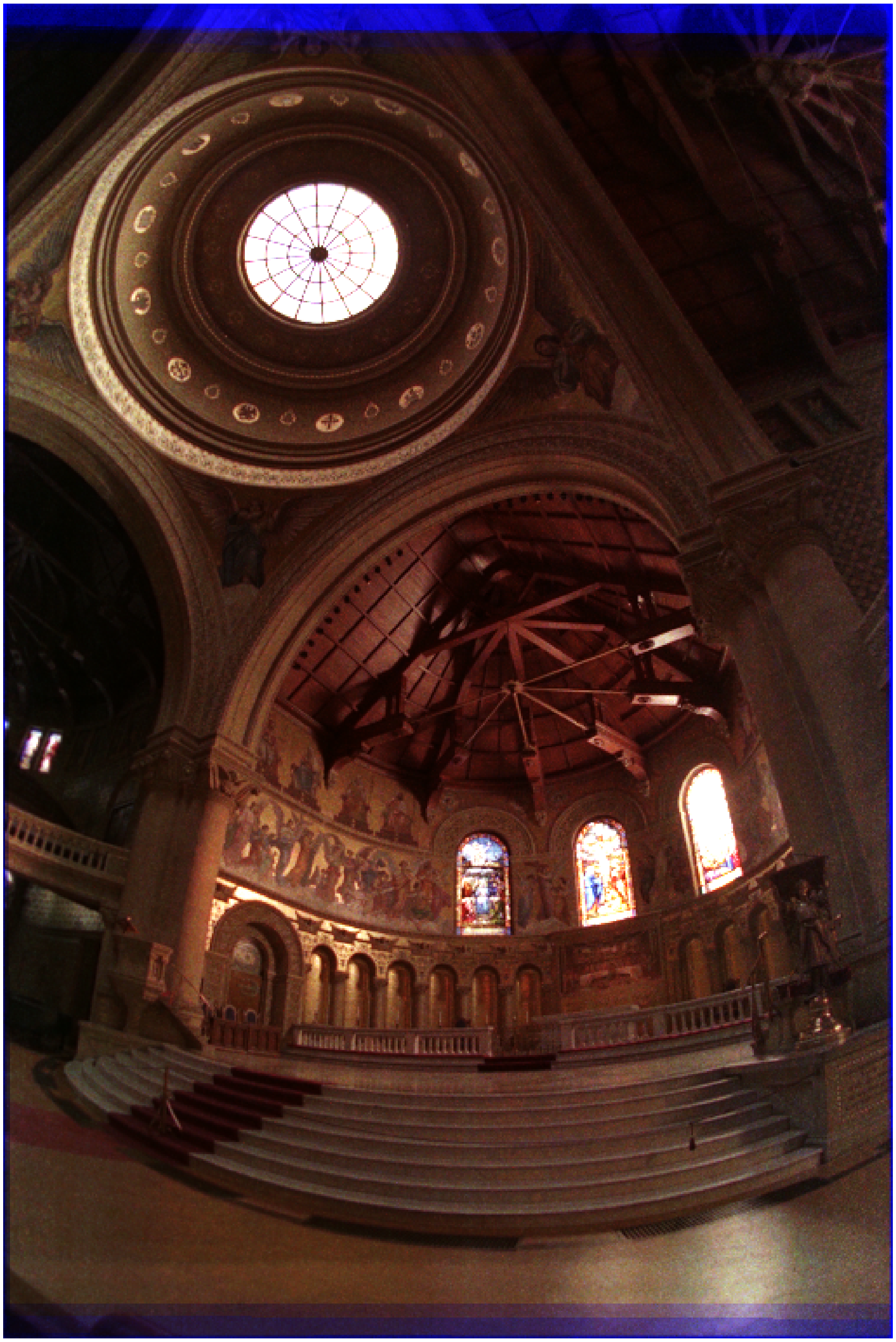

Figure 6: Generated Tonemap (SDR) Image on the HDR Image