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Northwestern University

MSDS 460-DL : Decision Analytics

ASSIGNMENT 1: Linear Programming Example

The Diet Problem Revisited (Spring 2024)

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Project Link: https://github.com/Hconsolo/MSDS460-Assignment_01_

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Introduction

In this assignment, we will be discussing the use of linear programming to solve the Stigler diet problem, with the help of Python (Dantzig 1990). The problem aims to find a diet that meets specific nutritional requirements while minimizing the cost. We can achieve this by optimizing a linear objective function with linear constraints, which will give us the optimal combination of food items that meet our requirements at the lowest possible cost.

We have used food prices for six meals and considered eight nutritional requirements to solve this problem. Our primary focus was to find the minimum-cost diet that would fulfill the necessary nutrients. In our research, we paid special attention to four specific nutrients: Vitamin D, Calcium, Iron, and Potassium. We did not include Sodium, Protein, and Energy in our study because these nutrients are commonly found in packaged foods or meat, poultry, and fish, which are good sources of protein. By excluding them, we were able to narrow down our research and make more informed decisions about our diet.

Table 01 lists the constraints for each nutrient. We have also included a list of items that are good sources of essential nutrients according to the Dietary Guidelines for Americans (2024). The goal is that if any constraint cannot be quickly met with the proposed meals, we can add another item to provide the missing nutrient.

Table 01: Daily nutrient constraints for the diet problem together with important food items to consider.

Component	Max/Min	Daily Amount and measure	Important Food items
Sodium	Maximum	5,000 milligrams (mg)	Not applicable
Energy	Minimum	2,000 Calories (kilocalories, kcal)	Not applicable
Protein	Minimum	50 grams (g)	Not applicable
Vitamin D	Minimum	20 micrograms (mcg)	Salmon, Milk, Soy Beverage
Calcium	Minimum	1,300 milligrams (mg)	Yogurt, Milk, Tofu
Iron	Minimum	18 milligrams (mg)	Spinach, Ready-to-eat cereal, Oyster
Potassium	Minimum	4,700 milligrams (mg)	Yogurt, Milk, Fish

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Method

For this task, we employed Python and the following third part libraries to help with basic data handling tasks and to solve the the LP problem.

- Pandas - Data Wrangling and Processing
- Matplotlib & Seaborn - Data Visualization
- PuIP - Provides a modeling framework and LP optimization tool

We have collected information on the nutritional value of five meals that are easily available online. These meals are Kraft Mac & Cheese, Amy's Thai Pad Thai, Fresh Express Chopped Caesar Salad Kit, Spinach Scramble, Salmon, Rice, and Broccoli. To ensure that all the necessary nutrients are covered, we have also included milk as a sixth meal option to be consumed during the day, since it is cheap and provides calcium, vitamin D, and potassium (Dietary Guidelines for Americans 2020). We chose pre-packaged and easy-to-prepare meals that require no more than five ingredients or have ingredients readily available in the package. We obtained the prices and nutritional information from Mariano's website (2024), and Table 02 summarizes this information along with the pre-packaged meal prices or the pro-rated cost per meal. The prices do not include any costs associated with food preparation, such as utility costs or labor. For the recipes that required preparation, we calculate the prorated costs and nutritional content based on the key ingredients based on the recipes from online websites. On the repository of this assignment one can find the nutritional values on a table, files containing the nutrition information and prices from Mariano’s website.

Table 02: Nutrition facts and prices for the meal options per serving.

Food Item (meal)	Price ($)	Energy (calories)	Sodium (mg)	Protein (g)	Vitamin D (mcg)	Calcium (mg)	Iron (mg)	Potassium (mg)
Kraft Mac & Cheese	0.46	250	560	9	0	110	2.5	330

Food Item (meal)	Price ($)	Energy (calories)	Sodium (mg)	Protein (g)	Vitamin D (mcg)	Calcium (mg)	Iron (mg)	Potassium (mg)
Amy's Thai Pad Thai	6.79	410	760	12	0	90	3.9	360
Fresh Express Chopped Caesar Salad Kit	1.8	160	310	3	0	90	0.9	190
Spinach Scramble	1.16	161.67	458.17	13.08	2	86.68	2.65	351.42
Salmon, Rice, and Broccolis	3.83	322.5	549.25	27.5	12.3	40.75	1.15	618.5
Fat Free Skim Milk	0.46	80	120	8	100	300	0	390

We optimized the problem using three different sets of constraints. These are the models we used:

1. Model 01: Only the constraints listed in Table 01 were used.
2. Model 02: We used the constraints in Table 01 and added a requirement for at least one serving per week (applicable to all meals).
3. Model 03: We used the constraints in Table 01, which required at least one serving of meal per week and set a maximum limit of 28 servings per week (to promote a diverse diet).

To convert this problem to the standard form, we used simple mathematical transformations to convert the minimization problem into a maximization problem as per Camarena (2024). Below is the standard formulation of the problem.

```
\* Diet_Problem_Standard *\
Maximize
OBJ: - 6.79 Amy's_Thai_Pad_Thai - 0.463333333333 Fat_Free_Skim_Milk
 - 1.796 Fresh_Express_Chopped_Caesar_Salad_Kit
 - 0.463333333333 Kraft_Mac_&_Cheese
 - 3.8313953666 Salmon,_Rice,_and_Broccolis - 1.16026357773 Spinach_Scramble
Subject To
Calcium_(mg): - 90 Amy's_Thai_Pad_Thai - 300 Fat_Free_Skim_Milk
 - 90 Fresh_Express_Chopped_Caesar_Salad_Kit - 110 Kraft_Mac_&_Cheese
 - 40.75 Salmon,_Rice,_and_Broccolis - 86.6752916667 Spinach_Scramble <= 9100
Energy_(kcal): - 410 Amy's_Thai_Pad_Thai - 80 Fat_Free_Skim_Milk
 - 160 Fresh_Express_Chopped_Caesar_Salad_Kit - 250 Kraft_Mac_&_Cheese
 - 322.5 Salmon,_Rice,_and_Broccolis - 161.666666667 Spinach_Scramble <= 14000
Iron_(mg): - 3.9 Amy's_Thai_Pad_Thai
 - 0.9 Fresh_Express_Chopped_Caesar_Salad_Kit - 2.5 Kraft_Mac_&_Cheese
 - 1.15 Salmon,_Rice,_and_Broccolis - 2.64541666667 Spinach_Scramble <= 126
Potassium_(mg): - 360 Amy's_Thai_Pad_Thai - 390 Fat_Free_Skim_Milk
 - 190 Fresh_Express_Chopped_Caesar_Salad_Kit - 330 Kraft_Mac_&_Cheese
 - 618.5 Salmon,_Rice,_and_Broccolis - 351.416666667 Spinach_Scramble <= 32900
```

```
Protein_(g): - 12 Amy's_Thai_Pad_Thai - 8 Fat_Free_Skim_Milk
 - 3 Fresh_Express_Chopped_Caesar_Salad_Kit - 9 Kraft_Mac_&_Cheese
 - 27.5 Salmon,_Rice,_and_Broccolis - 13.0791666667 Spinach_Scramble <= 350
Sodium_(mg): - 760 Amy's_Thai_Pad_Thai - 120 Fat_Free_Skim_Milk
 - 310 Fresh_Express_Chopped_Caesar_Salad_Kit - 560 Kraft_Mac_&_Cheese
 - 549.25 Salmon,_Rice,_and_Broccolis - 458.166666667 Spinach_Scramble
 <= 35000
Vitamin_D_(mcg): - 100 Fat_Free_Skim_Milk - 12.3 Salmon,_Rice,_and_Broccolis
 - 2 Spinach_Scramble <= 140
_C1: - Amy's_Thai_Pad_Thai <= -1
_C10: Kraft_Mac_&_Cheese <= 28
_C11: Salmon,_Rice,_and_Broccolis <= 28
_C12: Spinach_Scramble <= 28
_C2: - Fat_Free_Skim_Milk <= -1
_C3: - Fresh_Express_Chopped_Caesar_Salad_Kit <= -1
_C4: - Kraft_Mac_&_Cheese <= -1
_C5: - Salmon,_Rice,_and_Broccolis <= -1
_C6: - Spinach_Scramble <= -1
_C7: Amy's_Thai_Pad_Thai <= 28
_C8: Fat_Free_Skim_Milk <= 28
_C9: Fresh_Express_Chopped_Caesar_Salad_Kit <= 28
End
```

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Results

We employed the optimization simplex algorithm with PulP to devise a dietary model, the solutions of which are illustrated in Figures 01 and 02. In Model 01, we found that the solution consisted of only two meals: milk and Kraft Mac and Cheese. It's crucial to underline that the solution fails to converge without milk, highlighting the pivotal role of this food item in maintaining a balanced diet.

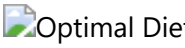

Figure 01: Optimal weekly portion for each food item (meal).

Moreover, when we stipulated the condition of having at least one meal per food item, the cost rose by almost 12 dollars. Once again, the simplex solution selected milk and Kraft Mac and Cheese, with one serving for the other meals. Shifting to Model 03, we observed that the weekly cost nearly doubled when we aimed to achieve a more diverse diet while maintaining the same nutrient targets. This indicates that a diversified diet can have a significant impact on one's budget. Interestingly, the solution found Salmon with rice and broccoli more attractive than Amy's Thai Pad Thai and Fresh Express Chopped Caesar Salad Kit. This suggests that preparing our meals is better than buying ready-to-heat frozen food or ready-to-eat meal kits.

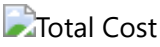

Figure 02: Optimal weekly cost for all consumed food item (meal).

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Conclusion

In this paper, we employed linear programming techniques to analyze a simplified Diet Problem, which is a well-known LP task. To ensure quick convergence of the solution, we took valuable insights from the Dietary Guidelines for Americans (2020). During the exercise, we gathered recipes and nutritional information from the Marianos website to create six meal options. We also included milk in the list of meals due to its high nutrient content, especially for calcium.

Based on Model 01, the optimal solution suggests that a diet with less variety, consisting of milk and mac and cheese, is the best. As we imposed more restrictions on the problem, the total cost of the solution worsened, which was expected. This indicates that a diversified diet can have a significant impact on one's budget. Our study suggests that preparing our meals is better than buying ready-to-heat frozen food or ready-to-eat meal kits.

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Reference

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