

Robot Mapping

A Short Introduction to the Bayes Filter and Related Models

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AIS Autonomous Intelligent Systems

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State Estimation

- Estimate the state x of a system given observations z and controls u
- Goal:**

$$p(\underline{x} \mid \underline{z}, u)$$

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Recursive Bayes Filter 1

$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$

Definition of the belief

x_t z_t, u_t

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Recursive Bayes Filter 2

$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t}) \\ = \eta \underbrace{p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) p(x_t \mid z_{1:t-1}, u_{1:t})}$$

Bayes' rule

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Recursive Bayes Filter 3

$$\begin{aligned}
 \text{bel}(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta \underline{p(z_t \mid x_t)} p(x_t \mid z_{1:t-1}, u_{1:t})
 \end{aligned}$$

Markov assumption

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Recursive Bayes Filter 4

$$\begin{aligned}
 \text{bel}(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} \underline{p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t})} \\
 &\quad \underline{p(x_{t-1} \mid z_{1:t-1}, u_{1:t})} dx_{t-1}
 \end{aligned}$$

Law of total probability

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Recursive Bayes Filter 5

$$\begin{aligned}
 \text{bel}(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \\
 &\quad p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) dx_{t-1} \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} \underline{p(x_t \mid x_{t-1}, u_t)} p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) dx_{t-1}
 \end{aligned}$$

Markov assumption

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Recursive Bayes Filter 6

$$\begin{aligned}
 \text{bel}(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \\
 &\quad p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) dx_{t-1} \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) dx_{t-1} \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) p(x_{t-1} \mid z_{1:t-1}, \underline{u_{1:t-1}}) dx_{t-1}
 \end{aligned}$$

Markov assumption

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Recursive Bayes Filter 7

$$\begin{aligned}
 bel(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t) p(x_t \mid z_{1:t-1}, u_{1:t}) \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \\
 &\quad p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) dx_{t-1} \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) dx_{t-1} \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) p(x_{t-1} \mid z_{1:t-1}, u_{1:t-1}) dx_{t-1} \\
 &= \eta p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \underline{bel(x_{t-1})} dx_{t-1}
 \end{aligned}$$

Recursive term

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Prediction and Correction Step

- Bayes filter can be written as a two step process

- Prediction step**

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

- Correction step**

$$bel(x_t) = \eta p(z_t \mid x_t) \overline{bel}(x_t)$$

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Motion and Observation Model

- Prediction step

$$\overline{bel}(x_t) = \int \underline{p(x_t \mid u_t, x_{t-1})} bel(x_{t-1}) dx_{t-1}$$

motion model

- Correction step

$$bel(x_t) = \eta \underline{p(z_t \mid x_t)} \overline{bel}(x_t)$$

sensor or observation model

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Different Realizations

- The Bayes filter is a **framework** for recursive state estimation
- There are **different realizations**
- Different properties**
 - Linear vs. non-linear models for motion and observation models
 - Gaussian distributions only?
 - Parametric vs. non-parametric filters
 - ...

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In this Course

- **Kalman filter & friends**

- Gaussians
- Linear or linearized models

- **Particle filter**

- Non-parametric
- Arbitrary models (sampling required)

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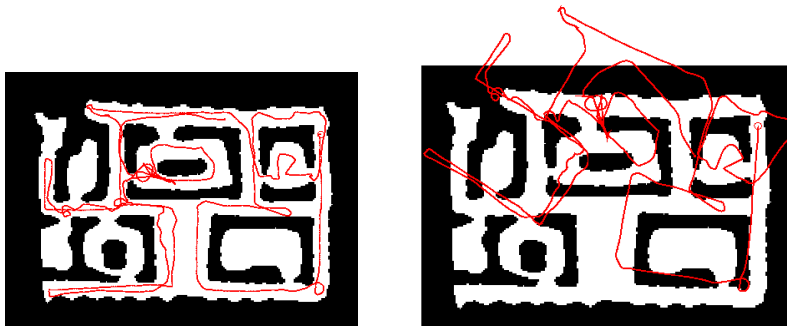
Motion Model

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

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Robot Motion Models

- Robot motion is inherently uncertain
- How can we model this uncertainty?



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Probabilistic Motion Models

- Specifies a posterior probability that action u carries the robot from x to x' .

$$p(x_t \mid u_t, x_{t-1})$$

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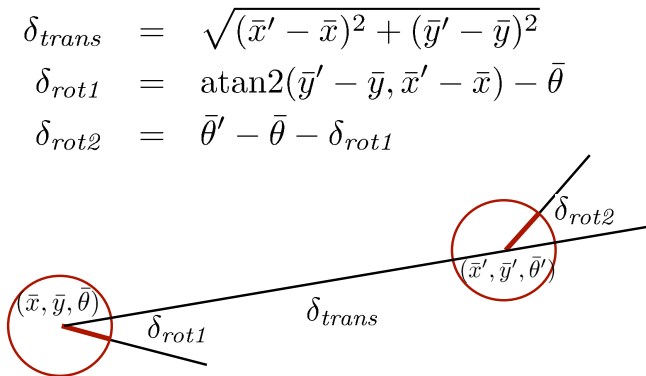
Typical Motion Models

- In practice, one often finds two types of motion models:
 - **Odometry-based**
 - **Velocity-based**
- Odometry-based models for systems that are equipped with wheel encoders
- Velocity-based when no wheel encoders are available

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Odometry Model

- Robot moves from $(\bar{x}, \bar{y}, \bar{\theta})$ to $(\bar{x}', \bar{y}', \bar{\theta}')$
- Odometry information $u = (\delta_{rot1}, \delta_{trans}, \delta_{rot2})$

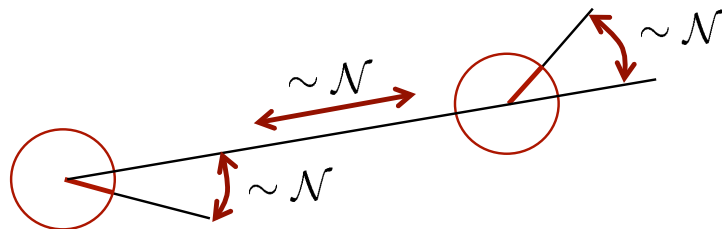


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Probability Distribution

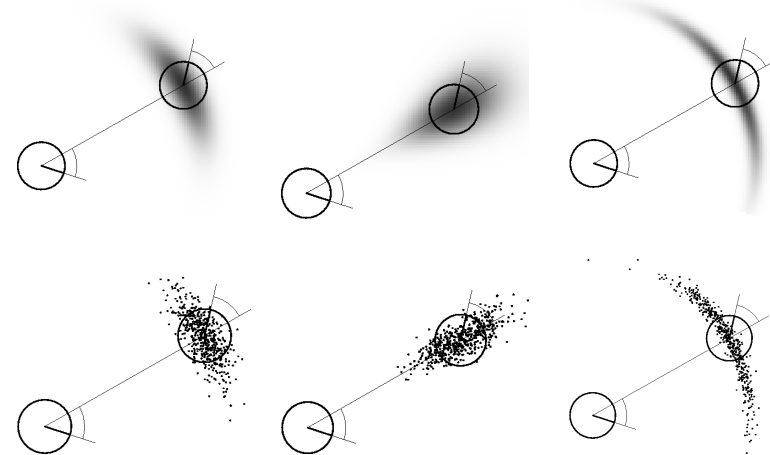
- Noise in odometry $u = (\delta_{rot1}, \delta_{trans}, \delta_{rot2})$
- Example: Gaussian noise

$$u \sim \mathcal{N}(0, \Sigma)$$



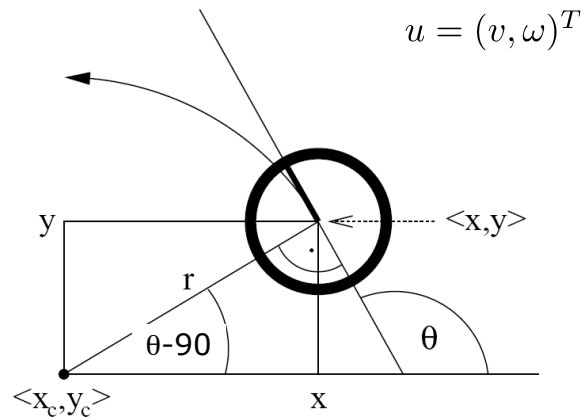
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Examples (Odometry-Based)



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Velocity-Based Model



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Motion Equation

- Robot moves from (x, y, θ) to (x', y', θ')
- Velocity information $u = (v, \omega)$

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v}{\omega} \sin \theta + \frac{v}{\omega} \sin(\theta + \omega \Delta t) \\ \frac{v}{\omega} \cos \theta - \frac{v}{\omega} \cos(\theta + \omega \Delta t) \\ \omega \Delta t \end{pmatrix}$$

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Problem of the Velocity-Based Model

- Robot moves on a circle
- The circle constrains the final orientation
- Fix:** introduce an additional noise term on the final orientation

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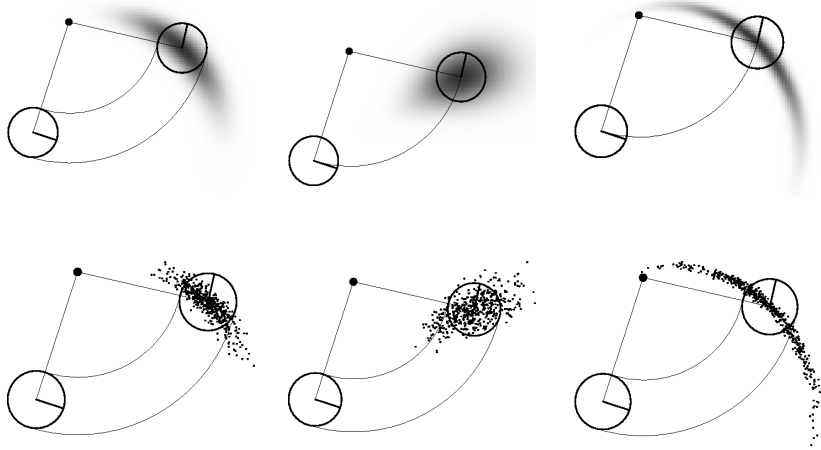
Motion Including 3rd Parameter

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v}{\omega} \sin \theta + \frac{v}{\omega} \sin(\theta + \omega \Delta t) \\ \frac{v}{\omega} \cos \theta - \frac{v}{\omega} \cos(\theta + \omega \Delta t) \\ \omega \Delta t + \gamma \Delta t \end{pmatrix}$$

Term to account for the final rotation

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Examples (Velocity-Based)



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Sensor Model

$$bel(x_t) = \eta \boxed{p(z_t \mid x_t)} \overline{bel}(x_{t-1})$$

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Model for Laser Scanners

- Scan z consists of K measurements.

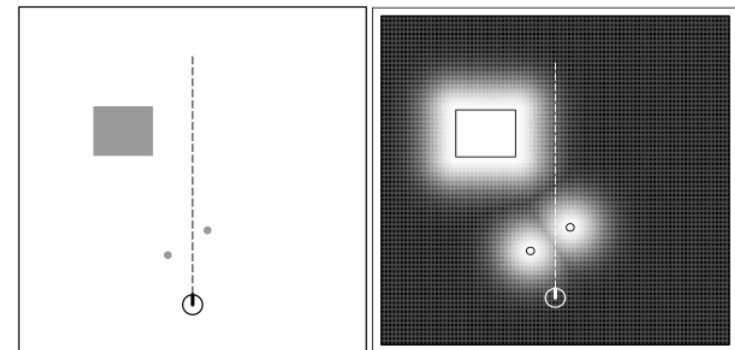
$$z_t = \{z_t^1, \dots, z_t^k\}$$

- Individual measurements are independent given the robot position

$$p(z_t \mid x_t, m) = \prod_{i=1}^k p(z_t^i \mid x_t, m)$$

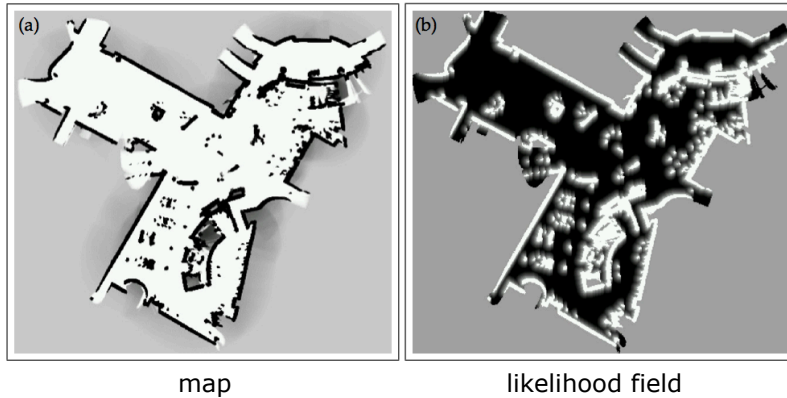
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Beam-Endpoint Model



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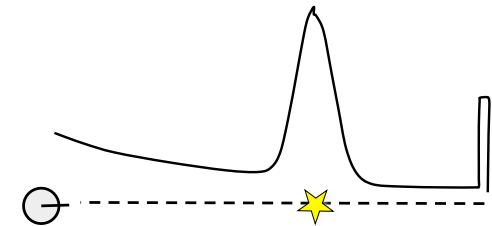
Beam-Endpoint Model



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Ray-cast Model

- Ray-cast model considers the first obstacle long the line of sight
- Mixture of four models



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Model for Perceiving Landmarks with Range-Bearing Sensors

- Range-bearing $z_t^i = (r_t^i, \phi_t^i)^T$
- Robot's pose $(x, y, \theta)^T$
- Observation of feature j at location $(m_{j,x}, m_{j,y})^T$

$$\begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \end{pmatrix} + Q_t$$

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Summary

- Bayes filter is a framework for state estimation
- Motion and sensor model are the central models in the Bayes filter
- Standard models for robot motion and laser-based range sensing

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Literature

On the Bayes filter

- Thrun et al. "Probabilistic Robotics", Chapter 2
- Course: Introduction to Mobile Robotics, Chapter 5

On motion and observation models

- Thrun et al. "Probabilistic Robotics", Chapters 5 & 6
- Course: Introduction to Mobile Robotics, Chapters 6 & 7