Exercise 2

A process is described by two variables $x_1; x_2$

The last two samples of size n=25 yielded the following mean values.

$$\bar{x}_1 = 58; \bar{x}_2 = 32$$
$$\bar{x}_1 = 60; \bar{x}_2 = 33$$

Previous observations allowed us to estimate the following quantities:

$$\underline{\bar{\bar{\mathbf{x}}}} = (55 \quad 30)^T$$
$$\underline{\mathbf{S}} = \begin{pmatrix} 200 & 130 \\ 130 & 120 \end{pmatrix}$$

Determine if the last two samples of the process are IC by using the Hotelling's control chart with $\alpha = 0.05$

SOLUTION

In [ ]:
```
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy import stats

# Define the data
n = 25          # number of replicates (sample size)
p = 2           # number of random variables
alpha = 0.05    # significance level

data = pd.DataFrame({'xbar1': [58, 60], 'xbar2': [32, 33]}, index=['sample1', 'sam
data.head()
```

Out[ ]:

	xbar1	xbar2
sample1	58	32
sample2	60	33

In [ ]:
```
# define the sample mean vector as a pandas series
Xbarbar = pd.Series({'xbar1': 55, 'xbar2': 30})
# define the covariance matrix as a pandas dataframe
S = pd.DataFrame([[200, 130],
                  [130, 120]],
                  columns=['xbar1', 'xbar2'],
                  index=['xbar1', 'xbar2'])
```

> Now let's calculate the Hotelling T^2 statistics for the 1st and 2nd sample

In [ ]:
```
S_inv = np.linalg.inv(S) # inverse of the covariance matrix
```

```
# Calculate the Hotelling's T2 statistic of the sample 1
T2_1 = n * (data.loc['sample1'] - Xbarbar).transpose().dot(S_inv).dot(data.loc['sa

# Calculate the Hotelling's T2 statistic of the sample 2
T2_2 = n * (data.loc['sample2'] - Xbarbar).transpose().dot(S_inv).dot(data.loc['sa

print("T2_1 = %.3f" % T2_1)
print("T2_2 = %.3f" % T2_2)
```

```
T2_1 = 1.127
T2_2 = 3.169
```

If we assume that the number of historical samples (m) is large, we can compute the control limit as follows:

$$UCL = \chi^2_\alpha(p) = \chi^2_{0.05}(2) = 5.9915$$

In [ ]:

```
# Calculate the UCL of the Hotelling's T2 statistic
UCL = stats.chi2.ppf(1-alpha, p)

print("UCL = %.4f" % UCL)
```

```
UCL = 5.9915
```

> Therefore both the samples are IC

> Assuming now that we know the number of samples ($m = 20$), we could design the T^2 control chart by using the F distribution (control limits for future observations). In this case UCL will be:
>
> $$UCL = c_2(m, n, p)F_\alpha(p, m(n-1) - (p-1))$$
>
> where c_2 is a constant that depends on m, n and p:
>
> $$c_2(m, n, p) = \frac{p(n-1)(m+1)}{m(n-1) - (p-1)}$$

In [ ]:

```
m = 20          # number of samples

c2 = (p*(n-1)*(m+1))/(m*(n-1)-(p-1))

UCL_new = c2*stats.f.ppf(1-alpha, p, (m*(n-1)-(p-1)))

print("New UCL = %.4f" % UCL_new)
```

```
New UCL = 6.3438
```

> Therefore both the samples are IC also in this case