Exercise 5

A pencil producer performs a process control by using 3 quality characteristics:

1. Pencil diameter
2. Ultimate tensile strength
3. The ease of sliding on paper (glide)

Data referring to 20 samples of size $n = 3$ are reported in the file `ESE8_ex5.csv`.

Design an Hotelling T^2 control chart.

> ## Solution

In [ ]:
```python
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy import stats
import pandas as pd

# Import the data
data = pd.read_csv('ESE8_ex5.csv')

# Inspect the data
data.head()
```

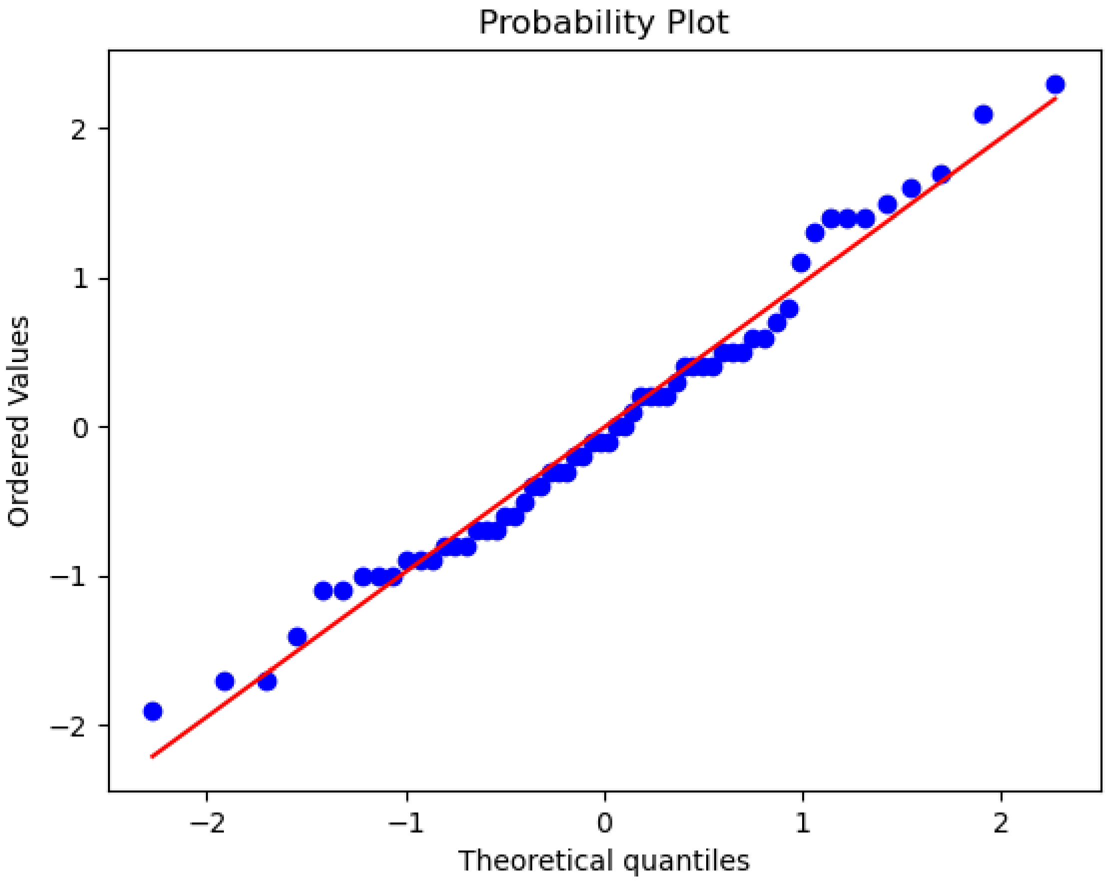
Out[ ]:

	Sample	Diameter 1	Diameter 2	Diameter 3	Tensile strength 1	Tensile strength 2	Tensile strength 3	Glide 1	Glide 2	Glide 3
0	1	2.3	-0.1	-1.4	3.48	2.72	2.28	5.85	5.53	4.46
1	2	-1.7	0.0	0.1	2.19	2.90	2.74	4.25	5.51	4.24
2	3	0.4	-0.1	0.4	2.89	2.81	3.24	6.83	5.01	4.65
3	4	-0.6	0.5	-0.7	2.66	2.98	2.51	6.00	6.40	4.47
4	5	0.2	0.4	-0.9	2.76	2.52	2.69	4.60	4.70	5.55

> First of all we check the Marginal Normality

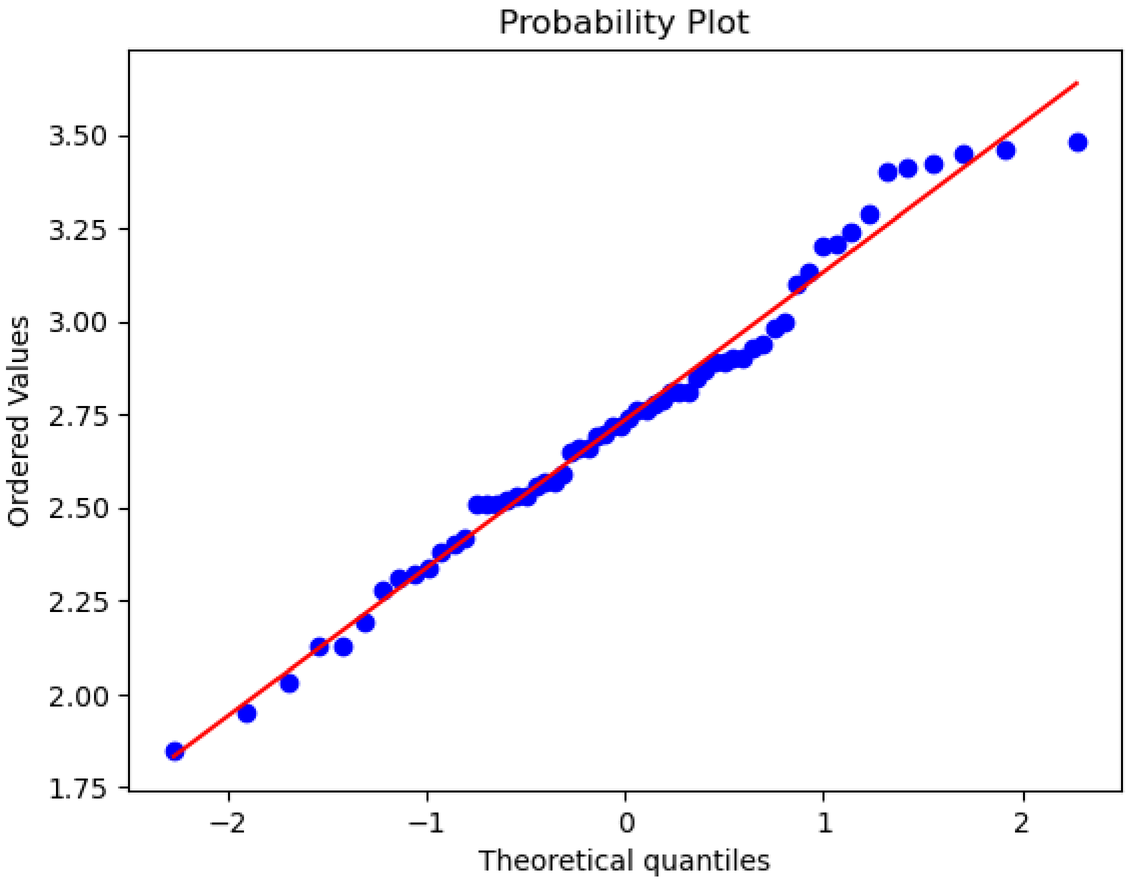
In [ ]:
```python
diameter = data[['Diameter 1', 'Diameter 2', 'Diameter 3']]

# Perform the Shapiro-Wilk test on the Diameter
_, pval1_SW = stats.shapiro(diameter)
print('Shapiro-Wilk test on Diameter p-value = %.3f' % pval1_SW)

# Plot the qqplot
stats.probplot(diameter.stack(), dist="norm", plot=plt)
plt.show()
```

```
Shapiro-Wilk test on Diameter p-value = 0.348
```



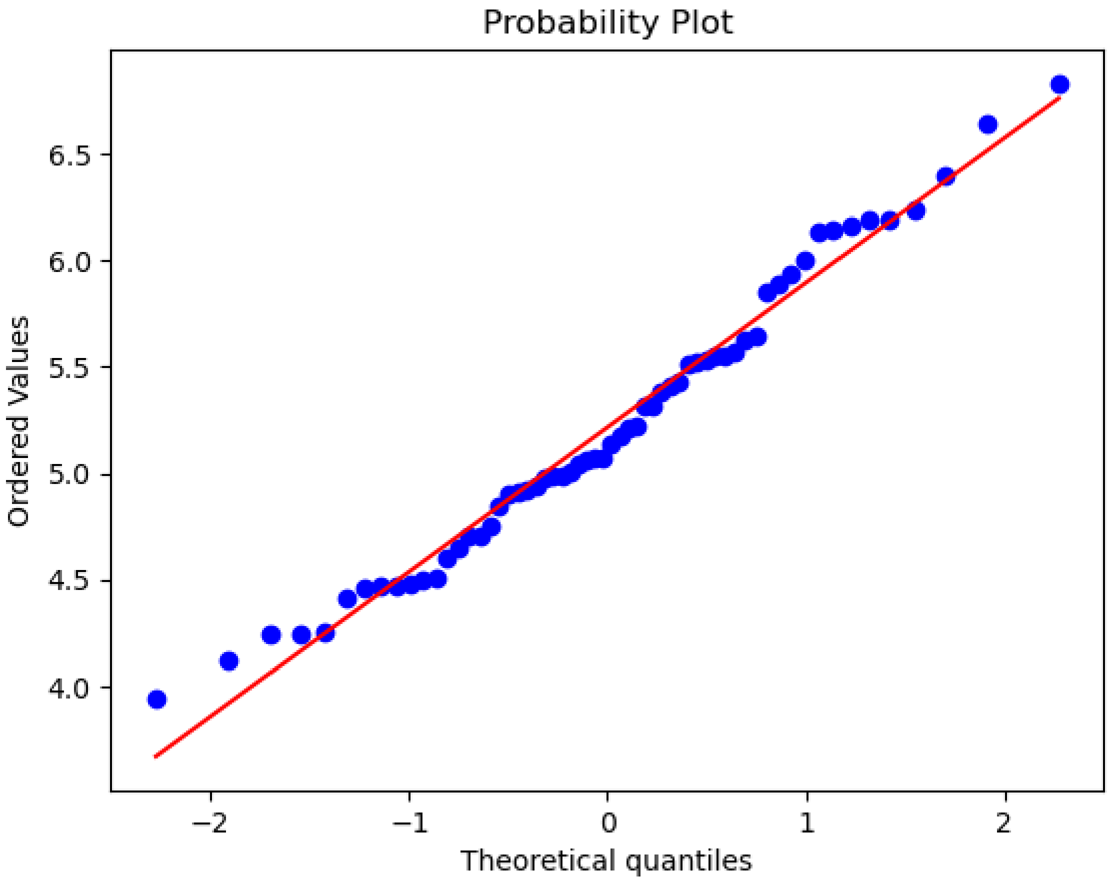
In [ ]:
```
uts = data[['Tensile strength 1', 'Tensile strength 2', 'Tensile strength 3']]

# Perform the Shapiro-Wilk test on the Tensile strength
_, pval2_SW = stats.shapiro(uts)
print('Shapiro-Wilk test on Tensile strenght p-value = %.3f' % pval2_SW)

# Plot the qqplot
stats.probplot(uts.stack(), dist="norm", plot=plt)
plt.show()
```

```
Shapiro-Wilk test on Tensile strenght p-value = 0.433
```



In [ ]:
```
glide = data[['Glide 1', 'Glide 2', 'Glide 3']]

# Perform the Shapiro-Wilk test on the Glide
_, pval3_SW = stats.shapiro(glide)
print('Shapiro-Wilk test on Glide p-value = %.3f' % pval3_SW)

# Plot the qqplot
stats.probplot(glide.stack(), dist="norm", plot=plt)
plt.show()
```

```
Shapiro-Wilk test on Glide p-value = 0.352
```



In [ ]:
```python
# Create a new dataframe to store the sample mean
sample_mean = pd.DataFrame()
sample_mean['diameter'] = diameter.mean(axis=1)
sample_mean['uts'] = uts.mean(axis=1)
sample_mean['glide'] = glide.mean(axis=1)

# Calculate the grand mean
Xbarbar = sample_mean.mean()
print(Xbarbar)
```

```
diameter   -0.006667
uts         2.735500
glide       5.215333
dtype: float64
```

In [ ]:
```python
# Create a new dataframe to store the stacked data
data_stack = pd.DataFrame()
data_stack[['sample', 'diameter']] = diameter.transpose().melt()
data_stack['uts'] = uts.transpose().melt()['value']
data_stack['glide'] = glide.transpose().melt()['value']

data_stack.head(9)
```

Out[ ]:

	sample	diameter	uts	glide
0	0	2.3	3.48	5.85
1	0	-0.1	2.72	5.53
2	0	-1.4	2.28	4.46
3	1	-1.7	2.19	4.25
4	1	0.0	2.90	5.51
5	1	0.1	2.74	4.24
6	2	0.4	2.89	6.83
7	2	-0.1	2.81	5.01
8	2	0.4	3.24	4.65

In [ ]:

```
# Compute the variance and covariance matrix of each group (sample)
cov_matrix = data_stack.groupby('sample').cov()

cov_matrix.head(9)
```

Out[ ]:

sample		diameter	uts	glide
0	**diameter**	3.523333	1.139333	1.217000
	uts	1.139333	0.368533	0.397000
	glide	1.217000	0.397000	0.529900
1	**diameter**	1.023333	0.363500	0.332833
	uts	0.363500	0.138700	0.182050
	glide	0.332833	0.182050	0.533433
2	**diameter**	0.083333	0.042500	0.121667
	uts	0.042500	0.052300	-0.128700
	glide	0.121667	-0.128700	1.365733

In [ ]:

```
# Compute the mean covariance matrix
S = cov_matrix.groupby(level=1).mean()

print(S)
```

```
          diameter       uts     glide
diameter  0.910167  0.229467  0.271600
glide     0.271600  0.138368  0.473593
uts       0.229467  0.134328  0.138368
```

> Attention! The indeces are now in alphabetic order. We need to reorder them in the order of the variables to get the correct variance/covariance matrix.

In [ ]:

```
# Reorder the indeces of S to match the order of the columns
# get the names of the columns
cols = S.columns.tolist()

S = S.reindex(columns=cols, index=cols)
```

```
print(S)
```

```
          diameter       uts     glide
diameter  0.910167  0.229467  0.271600
uts       0.229467  0.134328  0.138368
glide     0.271600  0.138368  0.473593
```

> Now we can compute the T^2 Control Chart

In [ ]:

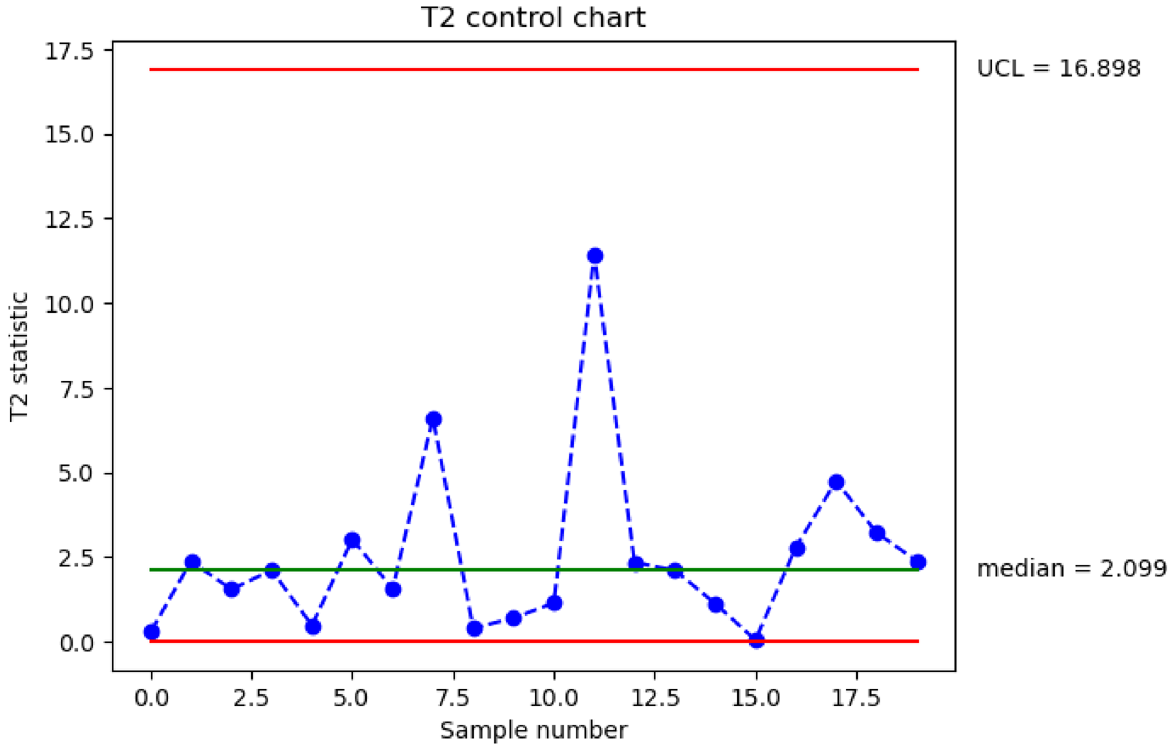
```
p = 3            # number of random variables
m = len(data)    # number of samples
n = 3            # number of replicates (sample size)
alpha = 0.0027   # significance level

S_inv = np.linalg.inv(S)

# Create a new dataframe to store the T2 statistics
data_CC = sample_mean.copy()
data_CC['T2'] = np.nan

for i in range(m):
    data_CC['T2'].iloc[i] = n * (sample_mean.iloc[i] - Xbarbar).transpose().dot(S_i

# Now we can add the UCL, CL and LCL to the dataframe
data_CC['T2_UCL'] = (p*(m-1)*(n-1))/(m*(n-1)-(p-1))*stats.f.ppf(1-alpha, 3, m*n-m+
data_CC['T2_CL'] = data_CC['T2'].median()
data_CC['T2_LCL'] = 0

# Add one column to test if the sample is out of control
data_CC['T2_TEST'] = np.where((data_CC['T2'] > data_CC['T2_UCL']), data_CC['T2'], 

# Inspect the dataset
data_CC.head()
```

Out[ ]:

	diameter	uts	glide	T2	T2_UCL	T2_CL	T2_LCL	T2_TEST
0	0.266667	2.826667	5.280000	0.278341	16.898109	2.09881	0	NaN
1	-0.533333	2.610000	4.666667	2.363377	16.898109	2.09881	0	NaN
2	0.233333	2.980000	5.496667	1.533608	16.898109	2.09881	0	NaN
3	-0.266667	2.716667	5.623333	2.098853	16.898109	2.09881	0	NaN
4	-0.100000	2.656667	4.950000	0.467668	16.898109	2.09881	0	NaN

In [ ]:

```
# Plot the T2 control chart
plt.title('T2 control chart')
plt.plot(data_CC['T2'], color='b', linestyle='--', marker='o')
plt.plot(data_CC['T2_UCL'], color='r')
plt.plot(data_CC['T2_CL'], color='g')
plt.plot(data_CC['T2_LCL'], color='r')
plt.ylabel('T2 statistic')
plt.xlabel('Sample number')
# add the values of the control limits on the right side of the plot
plt.text(len(data_CC)+.5, data_CC['T2_UCL'].iloc[0], 'UCL = {:.3f}'.format(data_CC
plt.text(len(data_CC)+.5, data_CC['T2_CL'].iloc[0], 'median = {:.3f}'.format(data_C
# highlight the points that violate the alarm rules
plt.plot(data_CC['T2_TEST'], linestyle='none', marker='s', color='r', markersize=10
plt.show()
```


T2 control chart
UCL = 16.898
median = 2.099
T2 statistic
Sample number
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5