Exercise 3

We need to measure three dimensions x,y and z on a mechanical part, which are individually distributed as a normal with the following parameters:

$\mu_x = 10; \sigma_x^2 = 1$ $\mu_y = 12; \sigma_y^2 = 1.5$ $\mu_z = 13; \sigma_z^2 = 2$

We don't have any information about their correlation.

The sample size is $n=25$.

Determine the control region that guarantees a joint (family) ARL(0) larger or equal to 500.

SOLUTION

$$ARL(0) = \frac{1}{\alpha_{FAM}} \geq 500 \Rightarrow \alpha_{FAM} \leq \frac{1}{500}$$

From the Bonferroni's inequality we know that:

$$\alpha_{FAM} \leq p \cdot \alpha$$

$$p \cdot \alpha = \frac{1}{500} \Rightarrow \alpha = \frac{1}{p \cdot 500}$$

$$p' = 3 \Rightarrow \alpha = \frac{1}{1500} = 6.67 \cdot 10^{-4}$$

> From this we can design three univariate CC as:

$$\begin{cases} UCL_i = \mu_i + Z_{\frac{\alpha}{2}} \dfrac{\sigma_i}{\sqrt{n}} \\ CL_i = \mu_i \\ LCL_i = \mu_i - Z_{\frac{\alpha}{2}} \dfrac{\sigma_i}{\sqrt{n}} \end{cases}$$

In [ ]:
```python
# Import the libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
import pandas as pd

# Define the parameters
ARL = 500
p = 3           # number of random variables
n = 25          # number of replicates (sample size)

mu_x = 10
var_x = 1

mu_y = 12
var_y = 1.5

mu_z = 13
var_z = 2

# Calculate the Z
alpha_FAM = 1 / ARL
alpha = alpha_FAM / p
Zalpha = norm.ppf(1-alpha/2)

print("alpha_FAM = %f " % alpha_FAM)
print("alpha = %f " % alpha)
print("Z alpha/2 = %.3f " % Zalpha)
```

```
alpha_FAM = 0.002000
alpha = 0.000667
Z alpha/2 = 3.403
```

> We can now compute the limits of the three CC

In [ ]:
```python
# Compute the control limits of the variable x CC
UCL_x = mu_x + Zalpha*np.sqrt(var_x)/np.sqrt(n)
CL_x = mu_x
LCL_x = mu_x - Zalpha*np.sqrt(var_x)/np.sqrt(n)

# Compute the control limits of the variable y CC
UCL_y = mu_y + Zalpha*np.sqrt(var_y)/np.sqrt(n)
CL_y = mu_y
LCL_y = mu_y - Zalpha*np.sqrt(var_y)/np.sqrt(n)

# Compute the control limits of the variable z CC
UCL_z = mu_z + Zalpha*np.sqrt(var_z)/np.sqrt(n)
CL_z = mu_z
LCL_z = mu_z - Zalpha*np.sqrt(var_z)/np.sqrt(n)

# Print a table with the control limits
data = pd.DataFrame({'UCL': [UCL_x, UCL_y, UCL_z],
                        'CL': [CL_x, CL_y, CL_z],
                        'LCL': [LCL_x, LCL_y, LCL_z]}, index = ['x', 'y', 'z'])

print(data)
```

```
         UCL  CL        LCL
x  10.680587  10   9.319413
y  12.833545  12  11.166455
z  13.962495  13  12.037505
```