Week 13 Manipulator Kinematics

① Link-connection Description:

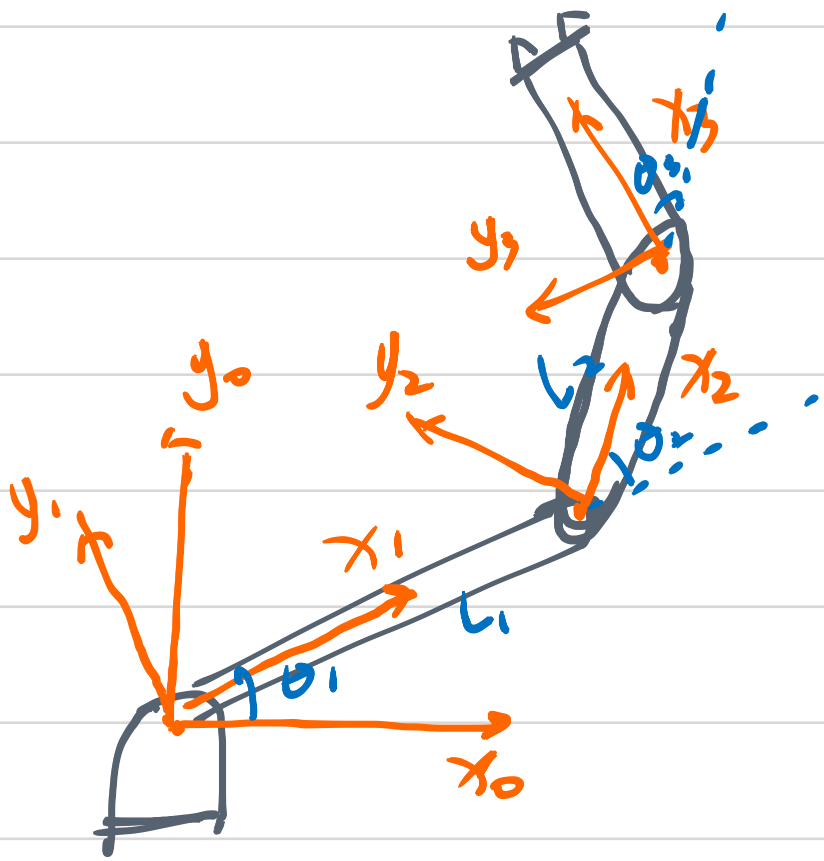
a_{i-1} α_{i-1} d_i θ_i

a_i, α_i 看 a_i 与 a_{i+1}

d_i, θ_i 看 i 与 $i-1$

θ, α. 后向前转

Example:



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

对. z: 转轴方向

x_i: i 与 $i+1$ 之轴共平面公垂线

y_i: 右手系

② 变化矩阵:

$$
{}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i \cdot c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} \cdot d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

12/02/07 Week 14 Study Note.

Inverse Manipulator Kinematics:

☆ desired position. orientation

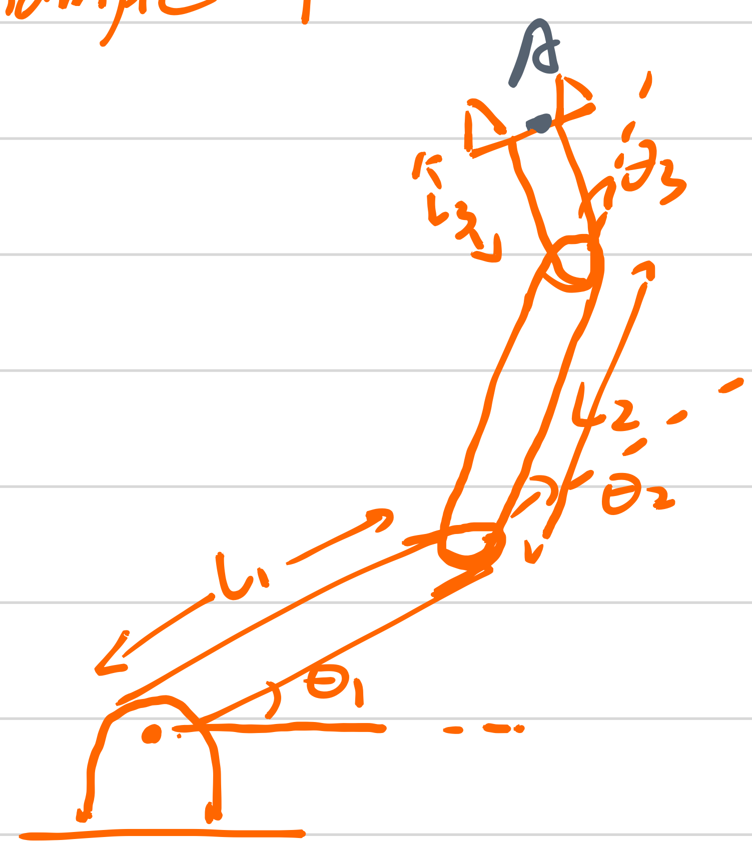
$\xrightarrow{?}$ set of joint angles

There are two way to obtain set of joint angles.

1. Algebraic Solution 2. Geometric Solution

Method 1.

• Example 1



for A.

because we know the $\overset{x.\,y.}{\overline{\text{position}}}$ and $\overset{\Phi}{\overline{\text{orientation}}}$ we can get the matrix.

$$ {}^{B}_{w}T = \begin{bmatrix} c\Phi & -s\Phi & 0 & x \\ s\Phi & c\Phi & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1) $$

because we know the parameter of each part, we can get the marix.

$$ {}^{3}_{0}T = \begin{bmatrix} c(\theta_1+\theta_2+\theta_3) & -s(\ +\ +\ ) & 0 & L_1c\theta_1+L_2c(\theta_1+\theta_2) \\ s(\theta_1+\theta_2+\theta_3) & c(\ +\ +\ ) & 0 & L_1s\theta_1+L_2s(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2) $$

由 (1) . (2) we can know:

$$\begin{cases} \cos\phi = \cos(\theta_1+\theta_2+\theta_3) & (3) \\ \sin\phi = \sin(\theta_1+\theta_2+\theta_3) & (4) \\ x = l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) & (5) \\ y = l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) & (6) \end{cases}$$

for θ_2

又由 (5) (6) we can know:

$$x^2+y^2 = l_1^2+l_2^2+2l_1l_2\cos\theta_2$$

$$\therefore \cos\theta_2 = \frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2} \Rightarrow \underline{\theta_2} = \operatorname{arccos}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$

for θ_1

由 (5) . (6)

$$\begin{cases} x = l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) \\ y = l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) \end{cases}$$

in order to 化简，令 $\Longrightarrow \begin{cases} k_1 = l_1 + l_2\cos\theta_2 \\ k_2 = l_2\sin\theta_2 \end{cases}$

原式： $\begin{cases} x = k_1\cos\theta_1 - k_2\sin\theta_1 \\ y = k_1\sin\theta_1 + k_2\cos\theta_1 \end{cases}$

$\Downarrow$ (没看懂推导)

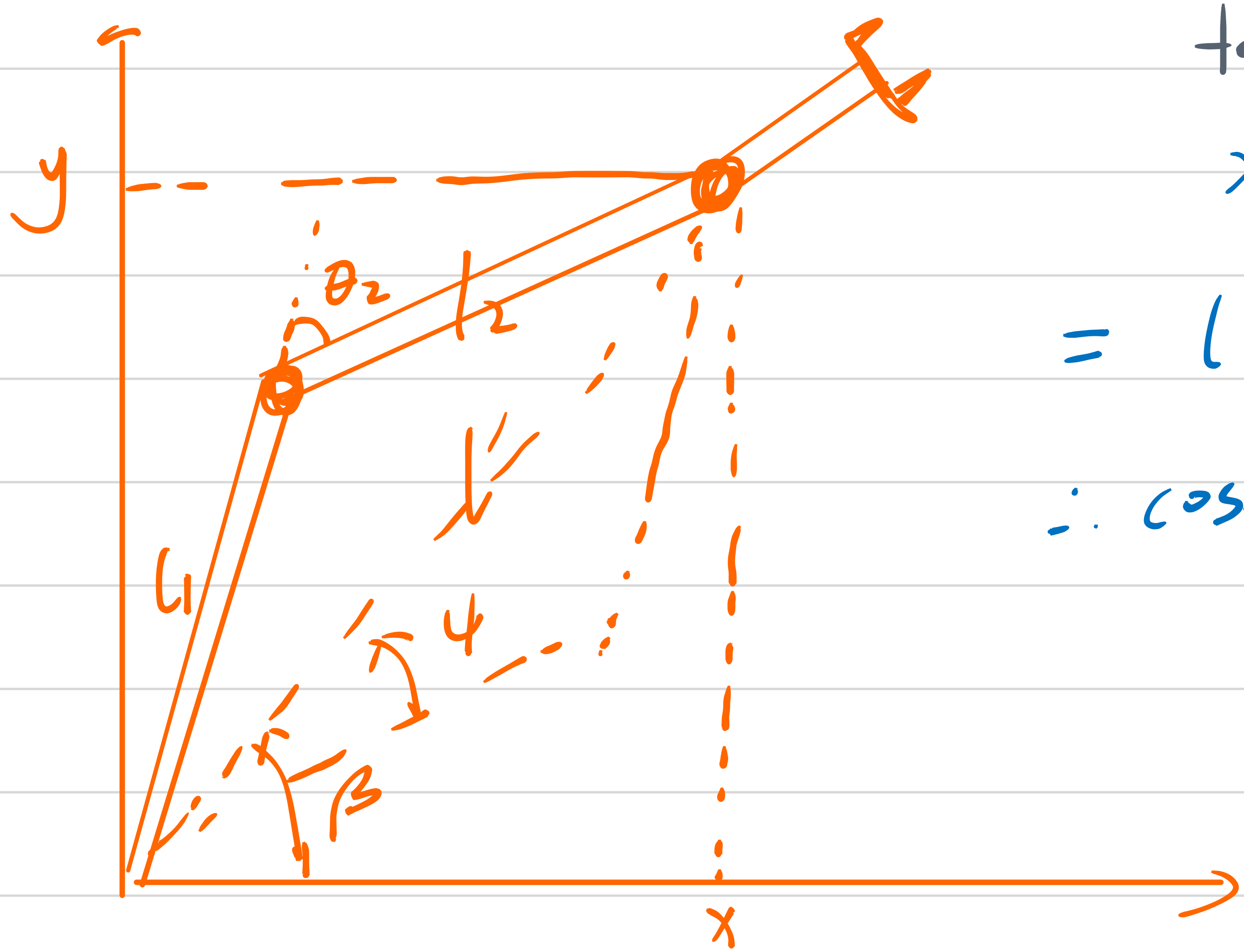
$$\theta_1 = \arctan\left(\frac{y}{x}\right) - \arctan\left(\frac{k_2}{k_1}\right)$$

for θ_3 :

$$\theta_1 + \theta_2 + \theta_3 = \arccos(\phi)$$

$$\therefore \theta_3 = \arccos(\phi) - \theta_1 - \theta_2$$

Method 2:



for θ_2.

$$x^2+y^2=(l')^2$$

$$= l_1^2+l_2^2-2l_1l_2\cos(180^\circ-\theta_2)$$

$$\therefore \cos\theta_2 = \frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}$$

for θ_1

$$\theta_1 = \beta \pm \psi$$

$$\beta = \arctan\left(\frac{y}{x}\right)$$

$$\cos\psi = \frac{l'^2+l_1^2-l_2^2}{2l_1\cdot l'} \quad \therefore l' = \sqrt{\cancel{\theta_1^2}l_1^2+l_2^2}$$

Planar Kinematics and Kinetics of Rigid Body

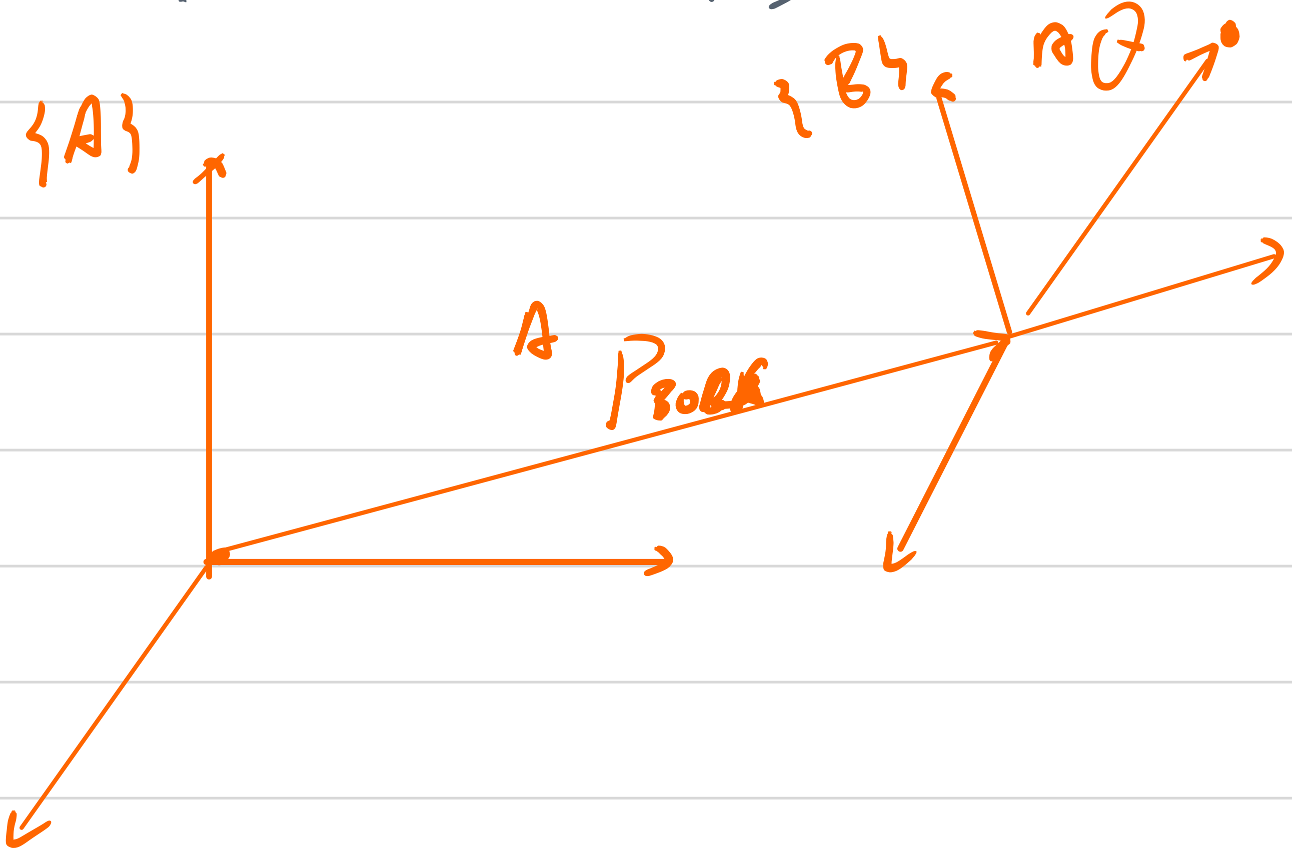
1. 表示方法：

$$\begin{cases} {}^{B}({}^{B}v_Q) = {}^{B}v_Q \\ {}^{A}({}^{B}v_Q) = {}^{A}_{B}R\,{}^{B}v_Q \end{cases}$$

${}^{B}({}^{B}v_Q) = {}^{B}v_Q \Rightarrow$ Q物体在B坐标系下表示

${}^{A}({}^{B}v_Q) = {}^{A}_{B}R\,{}^{B}v_Q \Rightarrow$ Q物体在B坐标系中相对A坐标系的速度

2：物体 + 坐标系平移（平移速度）



$${}^{A}V_{Q} = \underbrace{{}^{A}V_{BORG}}_{\text{坐标轴}} + \underbrace{{}^{A}_{B}R\,{}^{B}V_{Q}}_{\text{物体}}$$

3：物体 + 坐标系转动（转动）

$${}^{A}V_{Q} = \underbrace{{}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q}_{\text{坐标轴}} + \underbrace{{}^{A}_{B}R\,{}^{B}V_{Q}}_{\text{物体}}$$