Instructions: Complete this problem with your team. The problem needs to be solved using a *numerical method* that was presented in class. Built-in functions may only supplement the numerical method of choice, not replace it. Your Python code needs to be well-commented. The results need to be clearly presented and include relevant labels. You will be submitting the ipynb file, html file, report, and field manual. AI and internet resources are NOT allowed to be used to solve this problem.

Problem Statement:

Background

The goal of artillery is to fire a shell so that it hits a specific target. If we ignore the effects of air resistance the differential equations describing the acceleration of the shell are very simple:

$$\frac{dv_x}{dt} = 0 \quad and \quad \frac{dv_z}{dt} = -g$$

where v_x and v_z are the velocities in the x and z directions respectively and g is the acceleration due to gravity. We can use these equations to show that the resulting trajectory is parabolic. How do we do this? *I encourage you to answer this question before moving forward*. Once we know this, we can calculate the initial speed v_o and angle θ_o above the horizontal necessary for the shell to reach the target. We will find that the maximum range will always result from an angle of $\theta_o = 45°$.

The effects of air resistance are significant when the shell must travel a large distance or when the speed is large. If we modify the equations to include a simple model of air resistance the governing equations become

$$\frac{dv_x}{dt} = cv_x\sqrt{v_x^2 + v_z^2} \quad and \quad \frac{dv_z}{dt} = -g - cv_z\sqrt{v_x^2 + v_z^2}$$

where the constant c depends on the shape and density of the shell and the density of air. For this project assume that $c = 10^{-3}\ m^{-1}$. To calculate the components of the position vector, recall that since the derivative of position, s(t), is velocity we have

$$s_x(t) = \int_0^t v_x(t)dt \quad and \quad s_z(t) = \int_0^t v_z(t)dt$$

Problem

The year is approximately 1805 and Napoleon Bonaparte hired you as an artillery officer to assist him in the Battle of Ulm. Napoleon, known for his mathematical prowess, has recognized you as a mathematical savant and tasked you with the responsibility to project trajectories using numerical methods. For this problem, cannon balls will be firing at a target that is a distance Δx away. As the artillery officer choose the distance (e.g., 0.5 miles).

a. Develop a method for estimating v_0 and θ_0 with reasonable accuracy given the exact range to the target, Δx. Your method needs to be simple enough to use in real time on the battle field without the aid of a computer. (Be sure you can persuade Napoleon that your numerical solution is sufficiently accurate. *He is tough to persuade*.)
b. Discuss the sensitivity in your solutions to variations in the constant c.
c. Extend this problem to make it more realistic. A few possible extensions are listed below but please do not restrict yourselves to this list. You must make at least one modification to the problem to make it more realistic.
    - Consider the effects of targets at different altitudes Δz.
    - Consider moving targets.
    - Consider headwinds and/or tailwinds.
    - Consider winds coming from an angle outside the xz-plane.
    - Consider shooting the cannon from a boat with the target on shore (consider the waves).

The final product of this project will be:

- a **technical report** describing your method to a mathematically sophisticated audience
- a **field manual** instructing the artillery officer how to use your method
- **html** and **ipynb** files which show the solutions to parts a) – c)

Bon Courage!