

Case study: airline revenue management

15.093: Optimization

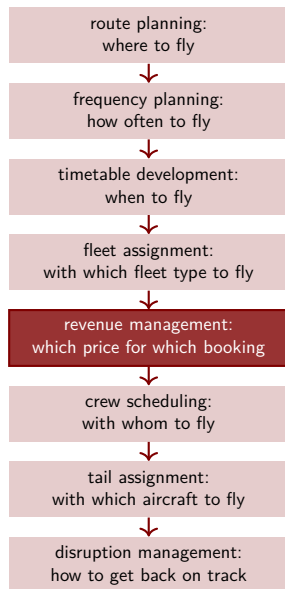
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Alexandre Jacquillat

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Massachusetts Institute of Technology



Introduction and background

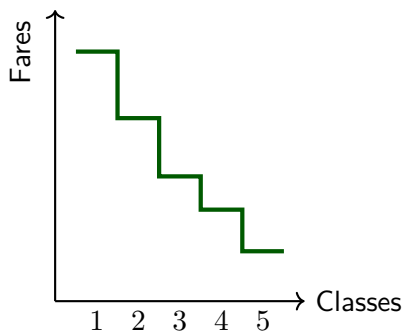
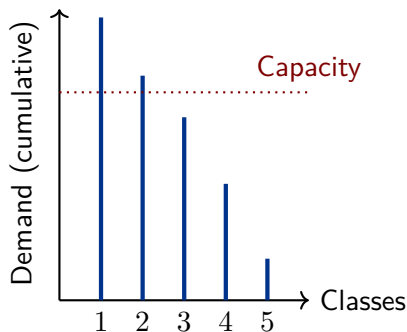
Optimization in the airline industry



DL5681 ¹ 6:35am → 8:02am 1h 27m BOS Nonstop EWR <small>¹ DL 5681 is operated by Republic Airways DBA Delta Connection</small> Details Seats	Sold Out	Main (K) From \$328 Round Trip 1 left at this price	Comfort+® (W) From \$378 Round Trip 1 left at this price	First (Z) From \$458 Round Trip 1 left at this price
DL5612 ¹ 7:00am → 8:31am 1h 31m BOS Nonstop LGA <small>¹ DL 5612 is operated by Republic Airways DBA Delta Connection</small> Details Seats	Sold Out	Main (L) From \$388 Round Trip 3 left at this price	Comfort+® (W) From \$428 Round Trip 3 left at this price	First (I) From \$508 Round Trip 3 left at this price
DL5719 ¹ 8:00am → 9:28am 1h 28m BOS Nonstop LGA <small>¹ DL 5719 is operated by Republic Airways DBA Delta Connection</small> Details Seats	Sold Out	Main (U) From \$358 Round Trip	Comfort+® (W) From \$398 Round Trip	First (Z) From \$478 Round Trip 3 left at this price
DL5702 ¹ 8:55am → 10:20am 1h 25m BOS Nonstop JFK <small>¹ DL 5702 is operated by Republic Airways DBA Delta Connection</small> Details Seats	Sold Out	Main (L) From \$428 Round Trip	Comfort+® (W) From \$478 Round Trip	First (I) From \$558 Round Trip 3 left at this price
DL5661 ¹ 9:00am → 10:26am 1h 26m BOS Nonstop LGA <small>¹ DL 5661 is operated by Republic Airways DBA Delta Connection</small> Details Seats	Sold Out	Main (T) From \$328 Round Trip 1 left at this price	Comfort+® (W) From \$398 Round Trip 1 left at this price	First (Z) From \$478 Round Trip 1 left at this price

Airline pricing and revenue management

- Two broad sets of algorithms and software solutions:
 1. Pricing: creation of “fare classes”, each with a fare level and restrictions
 2. Revenue management: closing and opening of fare classes
- Higher revenue for airlines with little impact on operating costs



Principles of airline revenue management

Early bookings

- Generally: leisure travelers with lower willingness to pay
- Low fares to spur demand
- Limit early sales to protect capacity for later sales



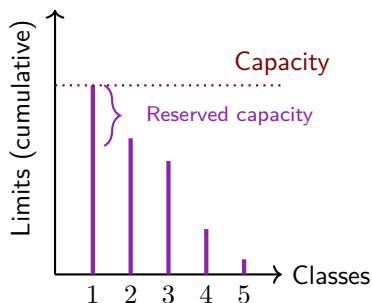
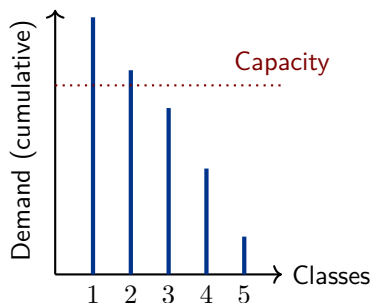
Late bookings

- Generally: business travelers with higher willingness to pay
- High fares to increase revenue
- Reserve sufficient capacity to avoid sellouts



Analytics of airline revenue management

- Market segmentation: passengers with different willingness to pay (e.g., leisure vs. business travelers)
 - Pricing and revenue management toward price discrimination
- Revenue management as quantity control at the core of pricing software in the airline industry (and way beyond)



Leg-based revenue management

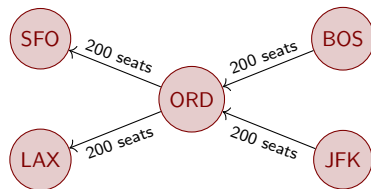
- Principle: sell low-fare tickets as long as price exceeds the expected revenue loss from rejecting higher-fare requests due to full flights
- Implementation via booking limits on lower classes

Class	Advance?	Min. stay	Change fee?	Fare	Demand		Open?
					Mean	SD	
Y	—	—	—	\$1,150	19	8	YES
B	3 days	—	—	\$950	12	6	YES
M	7 days	Sat.	—	\$800	24	9	YES
Q	14 days	Sat.	Yes	\$700	29	10	NO
V	21 days	Weekend	Yes	\$550	49	14	NO

- Extensions to embed more advanced capabilities
 - Overbooking: more bookings than capacity due to “no-shows”
 - Passenger choice: incentivize sell-ups and disincentivize buy-downs
 - Ancillaries: embed revenue from baggage fees, on-board purchases, etc.
 - Personalized pricing: adjust offerings based on passenger-level data
 - **Network revenue management: manage multiple flights simultaneously**

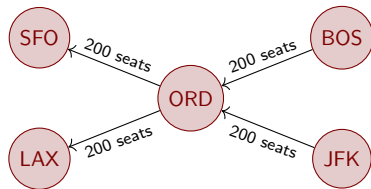
Network RM: small-scale problem

Data and decisions



From...	To...	Demand		Fare	
		Q	Y	Q	Y
BOS	ORD	25	20	\$200	\$230
BOS	SFO	55	40	\$320	\$420
BOS	LAX	65	25	\$400	\$490
JFK	ORD	24	16	\$250	\$290
JFK	SFO	65	50	\$410	\$550
JFK	LAX	40	35	\$450	\$550
ORD	SFO	21	20	\$200	\$230
ORD	LAX	25	14	\$250	\$300

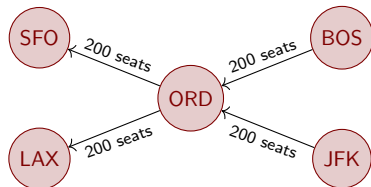
Data and decisions



From...	To...	Demand		Fare	
		Q	Y	Q	Y
BOS	ORD	25	20	\$200	\$230
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JFK	SFO	65	50	\$410	\$550
JFK	LAX	40	35	\$450	\$550
ORD	SFO	21	20	\$200	\$230
ORD	LAX	25	14	\$250	\$300

- Simplifying assumption: deterministic demand on each market
- Key complexity from the dichotomy of origin-destination demand (e.g., BOS-ORD-SFO) vs. flight-based supply (e.g., BOS-ORD; ORD-SFO)

Data and decisions



From...	To...	Demand		Fare	
		Q	Y	Q	Y
BOS	ORD	25	20	\$200	\$230
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JFK	LAX	40	35	\$450	\$550
ORD	SFO	21	20	\$200	\$230
ORD	LAX	25	14	\$250	\$300

- Simplifying assumption: deterministic demand on each market
- Key complexity from the dichotomy of origin-destination demand (e.g., BOS-ORD-SFO) vs. flight-based supply (e.g., BOS-ORD; ORD-SFO)
- Decision variables:

x_{BO}^Y, x_{BO}^Q : number of seats for BOS-ORD in class Y,Q

x_{BS}^Y, x_{BS}^Q : number of seats for BOS-ORD-SFO in class Y,Q

x_{BL}^Y, x_{BL}^Q : number of seats for BOS-ORD-LAX in class Y,Q

etc.

Optimization formulation

- Maximize total revenue across the network
- Comply with aircraft capacity on each flight
- Comply with passenger demand in each origin-destination market and in each fare class

Formulation

$$\begin{aligned} \max \quad & 200x_{BO}^Q + 230x_{BO}^Y + \cdots + 250x_{OL}^Q + 300x_{OL}^Y \\ \text{s.t.} \quad & x_{BO}^Q + x_{BO}^Y + x_{BS}^Q + x_{BS}^Y + x_{BL}^Q + x_{BL}^Y \leq 200 \\ & x_{JO}^Q + x_{JO}^Y + x_{JS}^Q + x_{JS}^Y + x_{JL}^Q + x_{JL}^Y \leq 200 \\ & x_{BS}^Q + x_{BS}^Y + x_{JS}^Q + x_{JS}^Y + x_{OS}^Q + x_{OS}^Y \leq 200 \\ & x_{BL}^Q + x_{BL}^Y + x_{JL}^Q + x_{JL}^Y + x_{OL}^Q + x_{OL}^Y \leq 200 \\ & x_{BO}^Q \leq 25, \quad x_{BO}^Y \leq 20, \quad \cdots \quad x_{OL}^Q \leq 25, \quad x_{OL}^Y \leq 14 \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

Optimal solution

Market-based solution at the origin-destination level

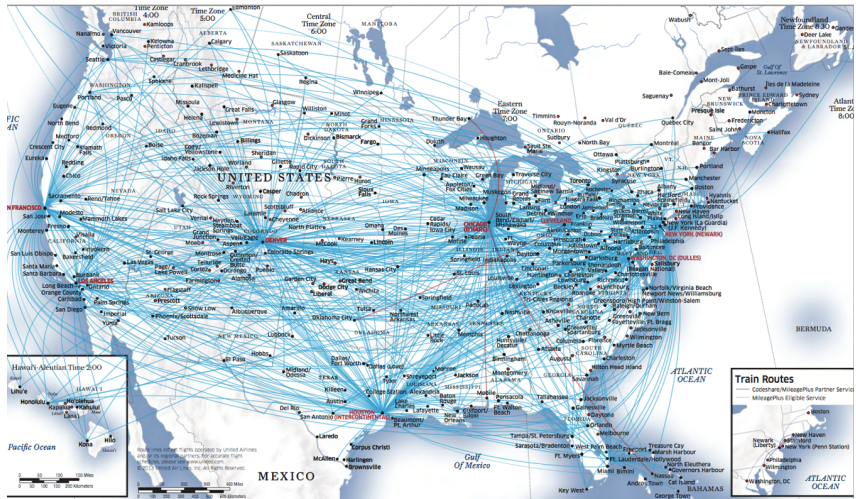
From...	To...	Demand		Fare		Sales	
		Q	Y	Q	Y	Q	Y
BOS	ORD	25	20	\$200	\$230	25	20
BOS	SFO	55	40	\$320	\$420	25	40
BOS	LAX	65	25	\$400	\$490	65	25
JFK	ORD	24	16	\$250	\$290	19	16
JFK	SFO	65	50	\$410	\$550	44	50
JFK	LAX	40	35	\$450	\$550	36	35
ORD	SFO	21	20	\$200	\$230	21	20
ORD	LAX	25	14	\$250	\$300	25	14

Flight-based solution at the segment level

From...	To...	Capacity	Demand			Sales		
			Q	Y	Total	Q	Y	Total
BOS	ORD	200	145	85	230	115	85	200
JFK	ORD	200	129	101	230	99	101	200
ORD	SFO	200	141	110	251	90	110	200
ORD	LAX	200	130	74	204	126	74	200

Network RM: large-scale problem

Real-world airline networks



Large-scale optimization formulation

- Network structure:
 - Set of flights $i = 1, \dots, m$
 - Set of itineraries $j = 1, \dots, n$, with \mathcal{J}_i of itineraries using flight i
 - Set of fare classes $k = 1, \dots, K$
- Data: passenger demand D_j^k , price p_j^k , aircraft capacity C_i
- Decision variables:

x_j^k : number of seats on itinerary j in class k

Formulation (Network revenue management)

$$\begin{aligned} \max \quad & \sum_{j=1}^n \sum_{k=1}^K p_j^k x_j^k \\ \text{s.t.} \quad & \sum_{j \in \mathcal{J}_i} \sum_{k=1}^K x_j^k \leq C_i, \quad \forall i = 1, \dots, m \\ & x_j^k \leq D_j^k, \quad \forall j = 1, \dots, n, \quad \forall k = 1, \dots, K \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

Edge of optimization

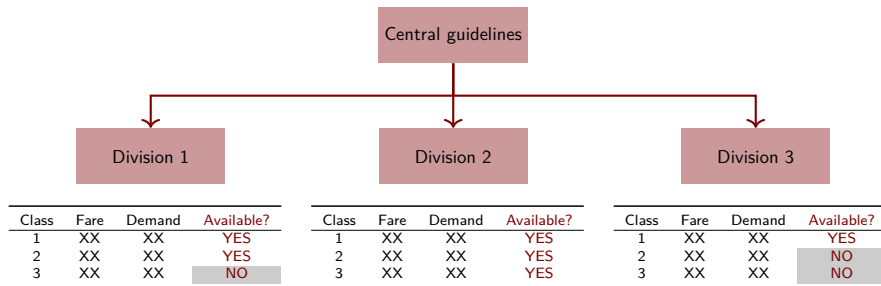
- Implementation with 2007 network from Delta Airlines: thousands of flights, tens of thousands of itineraries, dozens of fare classes
 - Large-scale optimization: 200,000 variables and 200,000 constraints
 - Revenue management balances non-stop and connecting demand
 - Estimated daily revenue in the optimal solution: \$65.37 million
 - Sequential benchmark: priority to non-connecting passengers
 1. Non-stop revenue management
 2. One-stop revenue management with remaining capacity on every flight
- Edge of optimization: \$1.04 million extra daily revenue (+1.6%)

Metric	Optimized	Benchmark	Change
Non-stop revenue	\$41.35M	\$42.25M	-2.13%
One-stop revenue	\$24.02M	\$22.08M	+8.80%
Total revenue	\$65.37M	\$64.33M	+1.62%

Bid price implementation

Network revenue management in practice

- Network revenue management optimization can provide significant benefits by managing demand and capacity across the network
 - Yet, network revenue management models suffer from core limitations
 - Reliance on deterministic demand forecasts, with estimation errors
 - Many markets with tiny forecasts, offering little guidance in practice
 - No flexibility for revenue managers to open and close fare classes
- Can we design decentralized rules to guide revenue management?



Intuition from small-scale example

- Each ticket sold brings revenue via the airfare but each ticket creates an “opportunity cost” for the airline by utilizing aircraft capacity
- Example on our small-scale instance
 - BOS-ORD utilizes capacity on a single flight: cost of \$160
 - BOS-LAX utilizes capacity on two flights: cost of $\$160 + \$200 = \$360$
 - BOS-LAX tickets are more “costly” to the airline than BOS-ORD ones
 - The BOS-LAX ticket needs to be more expensive than the BOS-ORD ticket to be “accepted” by the revenue management system

From...	To...	Demand			Seats			Dual
		Q	Y	Total	Q	Y	Total	
BOS	ORD	145	85	230	115	85	200	\$160
JFK	ORD	129	101	230	99	101	200	\$250
ORD	SFO	141	110	251	90	110	200	\$160
ORD	LAX	130	74	204	126	74	200	\$200

Bid price control: an itinerary view

Definition (bid price of itinerary)

Bid price of itinerary = Dual variable of capacity constraints of all flights

- Bid price: Cost to the airline of the “last” seat sold on the itinerary
- Accept bookings as long as the fare is higher than the bid price

From...	To...	Demand		Fare		Bid price	Sales	
		Q	Y	Q	Y		Q	Y
BOS	ORD	25	20	\$200	\$230	\$160	25	20
BOS	SFO	55	40	\$320	\$420	\$320	25	40
BOS	LAX	65	25	\$400	\$490	\$360	65	25
JFK	ORD	24	16	\$250	\$290	\$250	19	16
JFK	SFO	65	50	\$410	\$550	\$410	44	50
JFK	LAX	40	35	\$450	\$550	\$450	36	35
ORD	SFO	21	20	\$200	\$230	\$160	21	20
ORD	LAX	25	14	\$250	\$300	\$200	25	14

Bid price control: a flight view

Definition (bid price of itinerary)

Bid price of flight = Dual variable of capacity constraint of the flight

- Bid price: cost to the airline of the “last” seat sold on the flight
 - The value of a booking is equal to the fare minus the “cost” (i.e., dual variable of capacity constraint on “other” flights)
- Accept bookings as long as value is higher than its bid price of flight

Example on the BOS-ORD flight (bid price: \$160)

From...	To...	Demand		Fare		Cost	Value		Sales	
		Q	Y	Q	Y		Q	Y	Q	Y
BOS	ORD	25	20	\$200	\$230	\$0	\$200	\$230	25	20
BOS	SFO	55	40	\$320	\$420	\$160	\$160	\$260	25	40
BOS	LAX	65	25	\$400	\$490	\$200	\$200	\$290	65	25

Dual formulation

- Separation of direct itineraries \mathcal{J}^D and connecting itineraries \mathcal{J}^C
 - Each itinerary $j \in \mathcal{J}^D$ is associated with a single flight $f(j)$
 - Each itinerary $j \in \mathcal{J}^C$ is associated with two flights $g(j)$ and $h(j)$
- Dual variables:
 - λ_i : dual variable of capacity constraint on flight $i = 1, \dots, m$
 - μ_j^k : dual variable of demand constraint on itinerary $j = 1, \dots, n$ for fare class $k = 1, \dots, K$

Formulation (Network revenue management: dual problem)

$$\begin{aligned} \min \quad & \sum_{i=1}^m C_i \lambda_i + \sum_{j=1}^n \sum_{k=1}^K D_j^k \mu_j^k \\ \text{s.t.} \quad & \lambda_{f(j)} + \mu_j^k \geq p_j^k, \quad \forall j \in \mathcal{J}^D, \quad \forall k = 1, \dots, K \\ & \lambda_{g(j)} + \lambda_{h(j)} + \mu_j^k \geq p_j^k, \quad \forall j \in \mathcal{J}^C, \quad \forall k = 1, \dots, K \\ & \lambda, \mu \geq 0 \end{aligned}$$

Bid price formulas

- Bid price of flight i :

$$BP_i^F = \lambda_i$$

- Bid price of itinerary j :

$$BP_j^I = \begin{cases} \lambda_{f(j)} & \text{if } j \in \mathcal{J}^D \\ \lambda_{g(j)} + \lambda_{h(j)} & \text{if } j \in \mathcal{J}^C \end{cases}$$

- Example: bid price of direct BOS-LGA itinerary is the dual variable of the capacity constraint on the BOS-LGA flight
- Example: bid price of connecting BOS-ATL-HOU itinerary is the sum of the dual variables of the capacity constraints BOS-ATL and ATL-HOU

Bid price formulas

- Bid price of flight i :

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- Example: bid price of direct BOS-LGA itinerary is the dual variable of the capacity constraint on the BOS-LGA flight
- Example: bid price of connecting BOS-ATL-HOU itinerary is the sum of the dual variables of the capacity constraints BOS-ATL and ATL-HOU

Formulation (dual formulation using itinerary bid prices)

$$\begin{aligned} \min \quad & \sum_{i=1}^m C_i \lambda_i + \sum_{j=1}^n \sum_{k=1}^K D_j^k \mu_j^k \\ \text{s.t.} \quad & BP_j^I + \mu_j^k \geq p_j^k, \quad \forall j \in \mathcal{J}, \quad k = 1, \dots, K \\ & \lambda, \mu \geq 0 \end{aligned}$$

Large-scale example: dual prices of capacity constraints



Complementary slackness in action

Dual constraints: $BP_j^I + \mu_j^k \geq p_j^k, \forall j \in \mathcal{J}, k = 1, \dots, K$

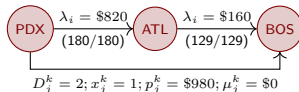
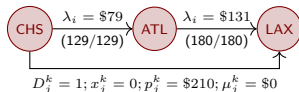
Duals

$x_j^k = 0$

$x_j^k > 0$

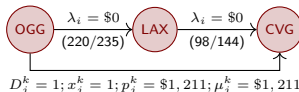
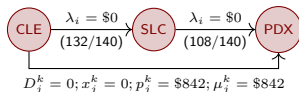
$$BP_j^I = p_j^k$$

$$\mu_j^k = 0$$



$$\mu_j^k = p_j^k$$

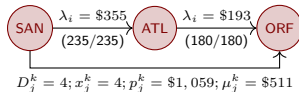
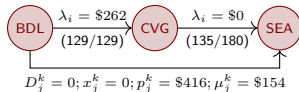
$$BP_j^I = 0$$



$$BP_j^I + \mu_j^k = p_j^k$$

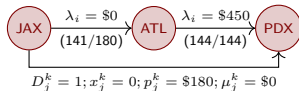
$$BP_j^I > 0$$

$$\mu_j^k > 0$$



$$BP_j^I + \mu_j^k > p_j^k$$

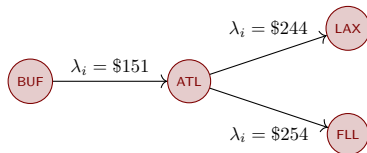
$$\mu_j^k = 0$$



Bid price control: an itinerary view

Dual constraints: $\mu_j^k \geq p_j^k - BP_j^I, \forall j \in \mathcal{J}, k = 1, \dots, K$

→ Accept bookings as long as fare is higher than bid price of itinerary



Cl.	BUF-ATL			ATL-LAX			ATL-FLL			BUF-LAX			BUF-FLL		
	p	D	x	p	D	x	p	D	x	p	D	x	p	D	x
1	\$489	2	2	\$1,498	6	6	\$539	1	1	\$1,117	2	2	\$808	0	0
2	\$435	2	2	\$983	10	10	\$426	1	1	\$1,078	0	0	\$714	0	0
3	\$371	3	3	\$858	7	7	\$358	2	2	\$847	2	2	\$704	0	0
4	\$355	3	3	\$854	9	9	\$329	2	2	\$578	2	2	\$673	1	1
5	\$305	4	4	\$642	13	13	\$258	2	2	\$497	1	1	\$494	2	2
6	\$256	4	4	\$621	18	18	\$236	2	0	\$493	2	2	\$425	0	0
7	\$242	7	7	\$559	15	15	\$207	3	0	\$424	1	1	\$358	2	0
8	\$169	5	5	\$464	20	20	\$180	3	0	\$386	4	0	\$265	1	0
9	\$151	9	2	\$359	26	26	\$146	5	0	\$333	3	0	\$248	1	0
10	\$146	8	0	\$357	26	26	\$138	4	0	\$282	3	0	\$240	1	0
11	\$127	6	0	\$305	17	17	\$99	3	0	\$186	2	0	\$226	2	0
12	\$90	3	0	\$171	12	0	\$83	1	0	\$178	4	0	\$199	0	0

Bid price control: a (wrong) flight view

- Wrong implementation: sort all passengers using each flight by fare
 - e.g., on BUF-ATL flight: BUF-ATL, BUF-LAX, BUF-FLL passengers
- No effective flight-level quantity control via nested fare classes

Example: BUF-ATL (part 1)

Class	Market	p	D	x
1	BUF-LAX	\$1,117	2	2
2	BUF-LAX	\$1,078	0	0
3	BUF-LAX	\$847	2	2
1	BUF-FLL	\$808	0	0
2	BUF-FLL	\$714	0	0
3	BUF-FLL	\$704	0	0
4	BUF-FLL	\$673	1	1
4	BUF-LAX	\$578	2	2
5	BUF-LAX	\$497	1	1
5	BUF-FLL	\$494	2	2
6	BUF-LAX	\$493	2	2
1	BUF-ATL	\$489	2	2
2	BUF-ATL	\$435	2	2
6	BUF-FLL	\$425	0	0
7	BUF-LAX	\$424	1	1
8	BUF-LAX	\$386	4	0
3	BUF-ATL	\$371	3	3
7	BUF-FLL	\$358	2	0

Example: BUF-ATL (part 2)

Class	Market	p	D	x
4	BUF-ATL	\$355	3	3
9	BUF-LAX	\$333	3	0
5	BUF-ATL	\$305	4	4
10	BUF-LAX	\$282	3	0
8	BUF-FLL	\$265	1	0
6	BUF-ATL	\$256	4	4
9	BUF-FLL	\$248	1	0
7	BUF-ATL	\$242	7	7
10	BUF-FLL	\$240	1	0
11	BUF-FLL	\$226	2	0
12	BUF-FLL	\$199	0	0
11	BUF-LAX	\$186	2	0
12	BUF-LAX	\$178	4	0
8	BUF-ATL	\$169	5	5
9	BUF-ATL	\$151	9	2
10	BUF-ATL	\$146	8	0
11	BUF-ATL	\$127	6	0
12	BUF-ATL	\$90	3	0

Bid price control: a (correct) flight view

- Adjust fares by subtracting the cost due to network effects
- Accept bookings as long as value is higher than the bid price of flight
- Effective flight-level quantity control via nested fare classes

Example: BUF-ATL (part 1)

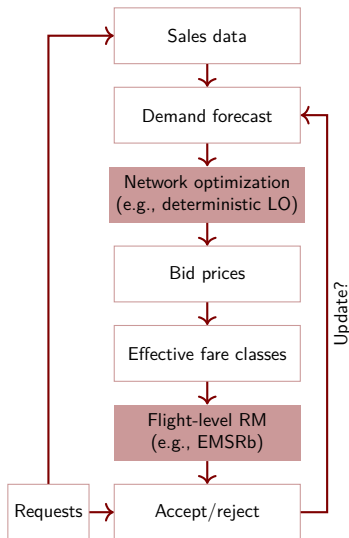
Cl.	Market	p	Cost	Value	D	x
1	BUF-LAX	\$1,117	\$244	\$873	2	2
2	BUF-LAX	\$1,078	\$244	\$834	0	0
3	BUF-LAX	\$847	\$244	\$603	2	2
1	BUF-FLL	\$808	\$254	\$554	0	0
1	BUF-ATL	\$489	\$0	\$489	2	2
2	BUF-FLL	\$714	\$254	\$460	0	0
3	BUF-FLL	\$704	\$254	\$450	0	0
2	BUF-ATL	\$435	\$0	\$435	2	2
4	BUF-FLL	\$673	\$254	\$419	1	1
3	BUF-ATL	\$371	\$0	\$371	3	3
4	BUF-ATL	\$355	\$0	\$355	3	3
4	BUF-LAX	\$578	\$244	\$334	2	2
5	BUF-ATL	\$305	\$0	\$305	4	4
6	BUF-ATL	\$256	\$0	\$256	4	4
5	BUF-LAX	\$497	\$244	\$253	1	1
6	BUF-LAX	\$493	\$244	\$249	2	2
7	BUF-ATL	\$242	\$0	\$242	7	7
5	BUF-FLL	\$494	\$254	\$240	2	2

Example: BUF-ATL (part 2)

Cl.	Market	p	Cost	Value	D	x
7	BUF-LAX	\$424	\$244	\$180	1	1
6	BUF-FLL	\$425	\$244	\$171	0	0
8	BUF-ATL	\$169	\$0	\$169	5	5
9	BUF-ATL	\$151	\$0	\$151	9	2
10	BUF-ATL	\$146	\$0	\$146	8	0
8	BUF-LAX	\$386	\$244	\$142	4	0
11	BUF-ATL	\$127	\$0	\$127	6	0
7	BUF-FLL	\$358	\$254	\$104	2	0
12	BUF-ATL	\$90	\$0	\$90	3	0
9	BUF-LAX	\$333	\$244	\$89	3	0
10	BUF-LAX	\$282	\$244	\$38	3	0
8	BUF-FLL	\$265	\$254	\$11	1	0
9	BUF-FLL	\$248	\$254	-\$6	1	0
10	BUF-FLL	\$240	\$254	-\$14	1	0
11	BUF-FLL	\$226	\$254	-\$28	2	0
12	BUF-FLL	\$199	\$254	-\$55	0	0
11	BUF-LAX	\$186	\$244	-\$58	2	0
12	BUF-LAX	\$178	\$244	-\$66	4	0

Conclusion

Network revenue management



- Goals of network revenue management
 1. Protect demand from high-fare classes
 2. Enable sales to high-fare connecting passengers, by avoiding sellouts
 3. Prioritize high-yield local passengers over low-yield connecting passengers
- Bid prices: opportunity cost of extra sale
 - Dual prices of relevant capacity constraints
- Flight-level revenue management systems, while capturing network-wide effects
- Revenue gains of network RM around 1-2% over flight-based RM
- Many extensions on network optimization models and flight-level RM systems

Network revenue management: impact

"I believe that yield management is the single most important technical development in transportation management since we entered the era of airline deregulation in 1979. Without yield management we were often faced with two unsatisfactory responses in a price competitive marketplace. We could match deeply discounted fares and risk diluting our entire inventory, or we could not match and certainly lose marketshare. Yield management gave us a third alternative—match deeply discounted fares on a portion of our inventory and close deeply discounted inventory when it is profitable to save space for later booking higher value customers.

The development of the American Airline's yield-management system has been long and sometimes difficult, but this investment has paid off. We estimate that yield management has generated \$1.4 billion in incremental revenue in the last three years alone. This is not a one-time benefit. We expect yield management to generate at least \$500 million annually for the foreseeable future."

—R. L. Crandall, CEO of American Airlines, 1992.