Problem 1

i) general solution is $u = f(x-2t)$

we can guess that $u = e^{-(x-2t)} H(x-2t) + \frac{1}{1+(x-2t)^2} H(-(x-2t))$

where $H(y) = \begin{cases} 1 & (y \geq 0) \\ 0 & (y < 0) \end{cases}$

we can check that equations and boundary conditions are satisfied

because solution is unique, then it is the solution

ii) $u_t + t u_x = 0 \iff u_{\frac{1}{2}t^2} + u_x = 0$

general solution is $u = f(x - \frac{1}{2}t^2)$

$= u(x - \frac{1}{2}t^2, 0)$

$= e^{x - \frac{1}{2}t^2}$

iii) $2^2 - 4 \times 1 \times 1 = 0 \Rightarrow$ parabolic equation

suppose that $x = p+q \quad y = p-q$

then $\partial_p = \partial_x + \partial_y$

$\partial_q = \partial_x - \partial_y$

$\Rightarrow \quad k \partial_p^2 \phi = \partial_q \phi$

$\Rightarrow \quad \partial_p^2 \phi = \frac{1}{k} \partial_q \phi$

Problem 2.

suppose that $u = \sum_{l=-\infty}^{\infty} e^{ilx} f(t)$

$\Rightarrow f'(t) + \alpha \cdot i l f = \beta(-l^2) f + \gamma(-il^3) f - \delta l^4 f$

$\Rightarrow f'(t) + (i\alpha l + \beta l^2 + i\gamma l^3 + \delta l^4) f(t) = 0$

$\Rightarrow f(t) = e^{-(i\alpha l + \beta l^2 + i\gamma l^3 + \delta l^4)t}$

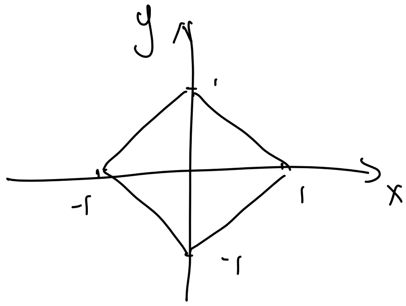
$\Rightarrow u = \sum_{l=-\infty}^{\infty} e^{ilx} e^{-(i\alpha l + \beta l^2 + i\gamma l^3 + \delta l^4)t}$

because $u(x,0) = \frac{1}{4}(-e^{3x} - e^{-3x} + e^{x} + e^{-x})$

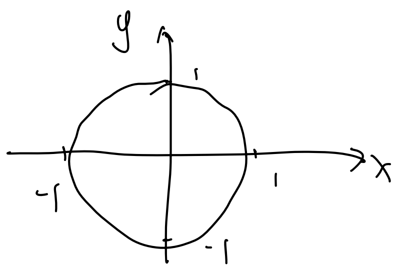
$\Rightarrow u(x,t) = \frac{1}{4} e^{x} e^{-(i\alpha + \beta + i\gamma + \delta)t} + \frac{1}{4} e^{-x} e^{-(-i\alpha + \beta - i\gamma + \delta)t}$
$- \frac{1}{4} e^{3x} e^{-(3i\alpha + 9\beta + 27i\gamma + 81\delta)t} - \frac{1}{4} e^{-3x} e^{-(-3i\alpha + 9\beta - 27i\gamma + 81\delta)t}$

Problem 3.

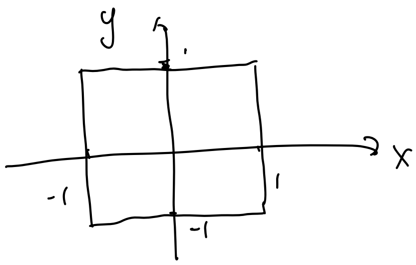
i) $p=1$



$p=2$



$p=\infty$



ii) for $\max_{\underline{w}} \{ \underline{w}^T \underline{v} : \|\underline{w}\|_2 = 1 \}$

according to Cauchy-inequality, $w^T v \le \|w\|_2 \|v\|_2$

iff $w = \alpha v$, $\le$ becomes $=$,

So we have $\max_{\underline{w}} \{ \underline{w}^T \underline{v} : \|\underline{w}\|_2 = 1 \} = \|\underline{v}\|_2$

for $\max_{\underline{w}} \{ \underline{w}^T \underline{v} : \|\underline{w}\|_\infty = 1 \}$

$w^T \underline{v} \le \sum_i |w_i||v_i| \le \sum_i \|\underline{w}\|_\infty |v_i| = \|w\|_\infty \|v\|_1$

iff $w_i = 1$ for all i, $\le$ becomes $=$

So we have $\max_{\underline{w}} \{ \underline{w}^T \underline{v} : \|\underline{w}\|_\infty = 1 \} = \|\underline{v}\|_1$

for $\max_{\underline{w}} \{ \underline{w}^T \underline{v} : \|\underline{w}\|_1 = 1 \}$

$w^T \underline{v} \le \sum_i |w_i||v_i| \le \left(\sum_i |w_i|\right) \max_i |v_i| = \|w\|_1 \|v\|_\infty$

iff $w_i = \begin{cases} 1 & i = \arg\max_i |v_i| \\ 0 & \text{other} \end{cases}$, $\le$ becomes $=$

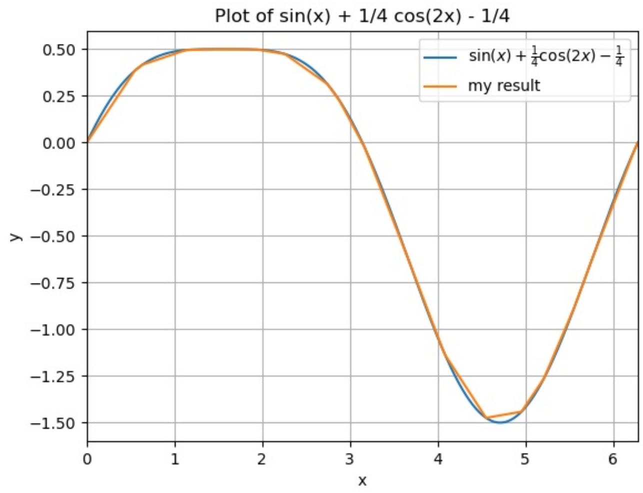
So we have $\max_{\underline{w}} \{ \underline{w}^T \underline{v} : \|\underline{w}\|_1 = 1 \} = \|\underline{v}\|_\infty$

Problem 3.

i)

```
1th order, forward
[-2.08333333  4.         -3.          1.33333333 -0.25      ]
2th order, forward
[ 2.91666667 -8.66666667  9.5        -4.66666667  0.91666667]
3th order, forward
[ -2.5   9.  -12.    7.   -1.5]
1th order, backward
[ 0.25       -1.33333333  3.         -4.          2.08333333]
2th order, backward
[ 0.91666667 -4.66666667  9.5        -8.66666667  2.91666667]
3th order, backward
[ 1.5 -7.  12.  -9.   2.5]
1th order, center
[ 0.08333333 -0.66666667  0.          0.66666667 -0.08333333]
1th order, center
[ 0.08333333 -0.66666667  0.          0.66666667 -0.08333333]
2th order, center
[-0.08333333  1.33333333 -2.5         1.33333333 -0.08333333]
3th order, center
[-0.5  1.   0.  -1.   0.5]
```

ii)



Plot of sin(x) + 1/4 cos(2x) - 1/4
$\sin(x) + \frac{1}{4}\cos(2x) - \frac{1}{4}$
my result
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
y
0
1
2
3
4
5
6
x