15.093: Optimization **Fall 2023**

Assignment 3: Duality in linear optimization

Assigned: October 03; Due: October 11.

Exercise 4.5 from *Introduction to Linear Optimization*, Bertsimas & Tsisiklis [30 pts]

Exercise 4.9 from *Introduction to Linear Optimization*, Bertsimas & Tsisiklis [30 pts]

Problem: simple rules for order fulfillment [40 pts]

Consider the transportation problem. Let $i = 1, \cdots, m$ index facilities and $j = 1, \cdots, n$ index customers. We denote by s_i the supply at facility i; by d_j the demand from customer j; and by c_{ij} the cost of shipping a unit from facility i to customer j. We define the following decision variable:

$$x_{ij} = \text{ quantity shipped from facility } i =, 1 \cdots, m \text{ to customer } j = 1, \cdots, n$$

The transportation problem minimizes the total shipment costs, subject to supply and demand constraints. It is formulated mathematically as follows:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} & \\
\text{s.t.} \quad & \sum_{i=1}^{m} x_{ij} \geq d_j, & \forall j = 1, \cdots, n \\
& \sum_{j=1}^{n} x_{ij} \leq s_i, & \forall i = 1, \cdots, m \\
& x_{ij} \geq 0, & \forall i, j
\end{aligned}$$

We consider a geographical area that can be modeled as a 10×10 miles rectangle, with 100 facilities and 500 customers. We assume that the unit shipment cost c_{ij} is proportional to the Euclidean distance between facility i and customer j. If facility i is located in the point of coordinates (X_i, Y_i) and if customer j is located in the point of coordinates (X_j, Y_j), then the distance from i to j is given by $\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$.

You have access to the following data files:

- facilities.csv: A matrix of size 100×3 that indicates, for each of the 100 facilities, (i) its x-coordinate, in miles (0 to 10), (ii) its y-coordinate, in miles (0 to 10) and (iii) its supply.
- customers.csv: A matrix of size 500×3 that indicates, for each of the 500 customers, (i) its x-coordinate, in miles (0 to 10), (ii) its y-coordinate, in miles (0 to 10) and (iii) its demand.

a. Implement the problem computationally and report the optimal cost. Out of the 500 customers, how many are fully served from the closest facility? Comment briefly.

b. Using duality concepts, propose an index β_{ij} that can be amenable to threshold-based implementation. In other words, propose an index β_{ij} such that each customer $j = 1, \cdots, n$ is served from the facility, or facilities, that minimize β_{ij}.

c. We now assume that products are shipped from facilities through warehouses eventually to customers. We index the warehouses by $k = 1, \cdots, K$ and denote by C_k the capacity of warehouse k (i.e., the maximum quantity it can handle). We denote by c^1_{ik} the unit shipment cost from facility i to warehouse k, and by c^2_{kj} the unit shipment cost from warehouse k to customer j. Modify the problem's formulation to minimize the shipment costs, while adhering to warehouse capacities.

d. Using duality concepts, propose an index γ_{ijk} that can be amenable to threshold-based implementation. In other words, propose an index γ_{ijk} such that each customer $j = 1, \cdots, n$ is served from the facility, or facilities, and the warehouse, or warehouses, that minimize γ_{ijk}.