# Machine learning: Part 1

- Supervised learning
- Decision tree learning
- Statistical learning
- Learning from complete Data

*Slides based on those of Pascal Poupart

# What is Machine Learning?

A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E. [Mitchell, 1997]

# Common learning tasks

- Supervised learning: Given some example input – output pairs, learn a function that maps from input to output.
- Unsupervised learning: Find natural classes for examples
- Reinforcement learning: determine what to do based on a series of rewards or punishments

# Examples

- Checker (reinforcement learning):
  - T: playing checker
  - P: percent of games won against an opponent
  - E: playing practice games against itself
- Handwriting recognition (supervised learning):
  - T: recognize handwritten words within images
  - P: percent of words correctly recognized
  - E: database of handwritten words with given classifications
- Customer profiling (分析) (unsupervised learning):
  - T: cluster customers based on transaction patterns
  - P: homogeneity (同种性) of clusters
  - E: database of customer transactions

# Representation

- Representation of the learned information is important
  - Determines how the learning algorithm will work
- Common representations:
  - Linear weighted polynomials
  - Propositional logic
  - First order logic
  - Bayes nets
  - ...

# Supervised learning

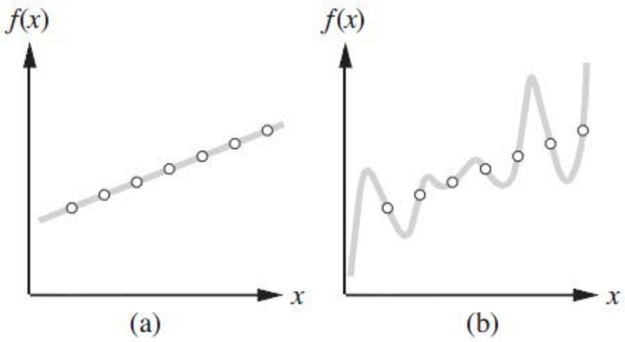
- Definition: Given a training set of $N$ example input – output pairs $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$, where each $y_j$ was generated by an unknown function $y = f(x)$, discover a function $h$ that approximates the true function $f$.
- The function $h$ is a hypothesis.
- Learning is a search through the space of possible hypotheses for one that will perform well, even on new examples beyond the training set.

# Classification and regression

- When the output $y$ is one of a finite set of values, the learning problem is called classification (分类).
- Called Boolean or binary classification, if there are only two values.
- When $y$ is a number, the learning problem is called regression (回归).

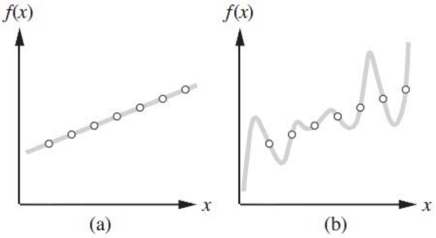
# A regression example

- Fitting a function of a single variable to some data points
- A hypothesis is consistent if it agrees with all the data
- A linear hypothesis and a degree 7 polynomial hypothesis



# Hypothesis space

- Hypothesis space: set of all hypotheses $h$ under consideration
- *e.g.*, set of polynomials
- How to choose from among multiple consistent hypotheses?
- Prefer the simplest hypothesis consistent with the data.
- This principle is called Ockham's razor (奥坎姆剃刀), which is against all sorts of complications.
- *e.g.*, (a) should be preferred to (b).

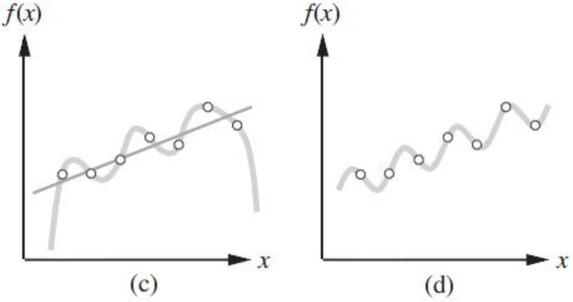


# Generalization

- A good hypothesis will generalize (泛化) well, *i.e.*, predict unseen examples well
- In general, there is a tradeoff between complex hypotheses that fit the training data well and simpler hypotheses that may generalize better

# An example

- No consistent straight line for this data set
- Require a degree-6 polynomial for an exact fit
- Can be fitted exactly by a simple function of the form $ax + b + c\sin(x)$



# Realizability

- Finding a consistent hypothesis depends on the hypothesis space
- We say that a learning problem is realizable (可实现的) if the hypothesis space contains the true function.
- Unfortunately, we cannot always tell whether a given learning problem is realizable, because the true function is not known.

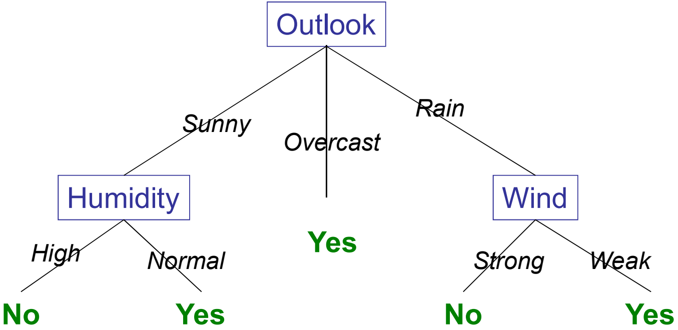
# Realizability

- Why not let $H$ be the class of all Java programs, or Turing machines, since every computable function can be represented by some Turing machine?
- There is a tradeoff between the expressiveness of a hypothesis space and the complexity of finding a good hypothesis within that space.
- *e.g.*, fitting a straight line to data is easy; fitting high-degree polynomials is harder; and fitting Turing machines is in general undecidable.

# Decision trees

- Represent a function that takes as input a vector of attribute values and returns a “decision” —a single output value.
- Reach the decision by performing a sequence of tests.
- Nodes: labeled with attributes
- Edges: labeled with attribute values
- Leaves: labeled with output values

# An example (playing tennis)



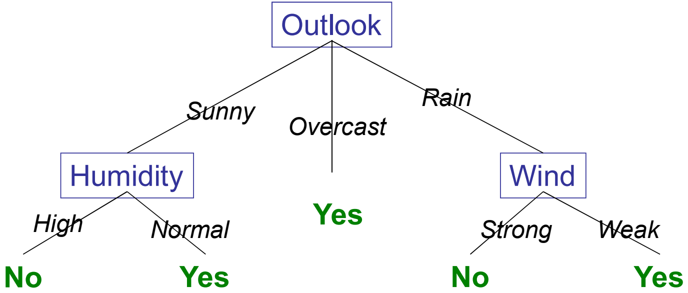

An instance

<Outlook=Sunny, Temp=Hot, Humidity=High, Wind=Strong>

Classification: No

# Decision tree representation

Decision trees can represent disjunctions of conjunctions of constraints on attribute values




(Outlook=Sunny $\wedge$ Humidity=Normal)
$\vee$ (Outlook=Overcast)
$\vee$ (Outlook=Rain $\wedge$ Wind=Weak)

# Decision tree representation

- Any Boolean function can be written as a decision tree
- By allowing each row in the truth table correspond to a path in the tree
- Can often use small trees
- Some functions require exponentially large trees
- *e.g.*, the majority function, which returns true iff more than half of the inputs are true,
- No representation efficient for all functions
- With $n$ Boolean attributes, there are $2^{2^n}$ Boolean functions

# Decision tree learning

- Aim: find a small tree consistent with the training examples
- Idea: choose "most significant" attribute as root of (sub)tree

**function** DECISION-TREE-LEARNING(*examples*, *attributes*, *parent_examples*) **returns** a tree

  **if** *examples* is empty **then return** PLURALITY-VALUE(*parent_examples*)
  **else if** all *examples* have the same classification **then return** the classification
  **else if** *attributes* is empty **then return** PLURALITY-VALUE(*examples*)
  **else**
    $A \leftarrow \mathrm{argmax}_{a \in attributes}$ IMPORTANCE($a$, *examples*)
    *tree* $\leftarrow$ a new decision tree with root test $A$
    **for each** value $v_k$ of $A$ **do**
      $exs \leftarrow \{e : e \in examples$ **and** $e.A = v_k\}$
      *subtree* $\leftarrow$ DECISION-TREE-LEARNING(*exs*, *attributes* $- A$, *examples*)
      add a branch to *tree* with label $(A = v_k)$ and subtree *subtree*
    **return** *tree*

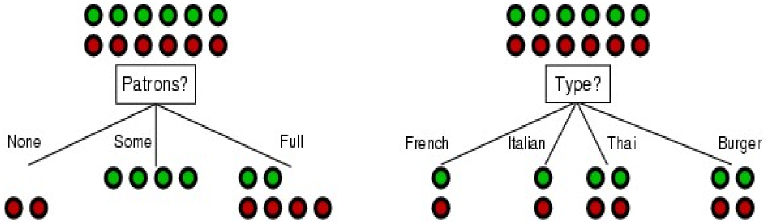
Plurality-value(examples) returns the majority classification of the examples

# An example: restaurant

| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *Wait* |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

# Choosing an attribute

- Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"




- *Patrons?* is a better choice

# Using information theory

- We will use the notion of information gain (信息增益), which is defined in terms of entropy (熵), the fundamental quantity in information theory.
- Entropy is a measure of the uncertainty of a random variable; acquisition of information corresponds to a reduction in entropy.
- A random variable with only one value has no uncertainty and thus its entropy is defined as zero.
- A flip of a fair coin has "1 bit" of entropy.
- The roll of a fair four-sided die has 2 bits of entropy, because it takes 2 bits to describe one of 4 equally probable choices.

# Entropy

- The entropy of a random variable $V$ with values $v_k$, each with probability $P(v_k)$:

$$H(V) = -\sum_k P(v_k) \log_2 P(V_k)$$

- The entropy of a Boolean random variable that is true with probability $q$:

$$B(q) = -(q \log_2 q + (1-q) \log_2 (1-q))$$

- If a training set contains p positive examples and n negative examples, then the entropy of the goal attribute on the whole set is

$$H(Goal) = B(\frac{p}{p+n})$$

# Information gain

- An attribute $A$ with $d$ distinct values divides the training set $E$ into subsets $E_1, \ldots, E_d$.
- Each subset $E_k$ has $p_k$ positive examples and $n_k$ negative examples,
- So the expected entropy remaining after testing attribute $A$ is

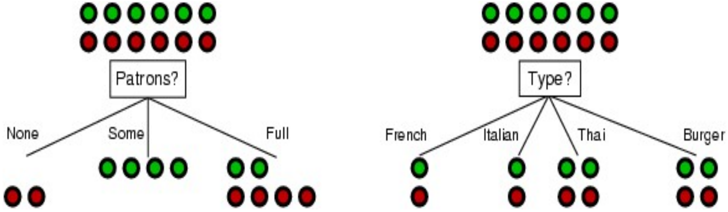
$$Remainder(A) = \sum_{k=1}^{d} \frac{p_k + n_k}{p + n} B(\frac{p_k}{p_k + n_k}).$$

- The information gain (IG) from the attribute test on A is the expected reduction in entropy:

$$Gain(A) = B(\frac{p}{p + n}) - Remainder(A)$$

- Choose the attribute with the largest IG

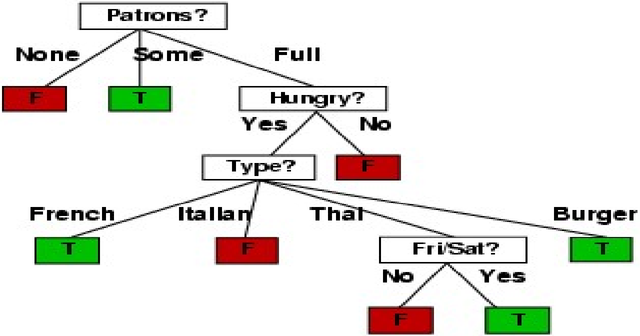
# An example




- For the training set, $p = n = 6$, $B(6/12) = 1$
- $Gain(Pat) = 1 - [\frac{2}{12}B(\frac{0}{2}) + \frac{4}{12}B(\frac{4}{4}) + \frac{6}{12}B(\frac{2}{6})] \approx 0.541$
- $Gain(Type) = 1 - [\frac{2}{12}B(\frac{1}{2}) + \frac{2}{12}B(\frac{1}{2}) + \frac{4}{12}B(\frac{2}{4}) + \frac{4}{12}B(\frac{2}{4})] = 0$
- So Patrons is a better attribute to split on.
- In fact, Patrons has the maximum gain of any of the attributes and would be chosen by the DTL algorithm as the root.

# An example

Decision tree learned from the 12 examples:



# Performance of a learning algorithm

- A learning algorithm is good if it produces a hypothesis that does a good job of predicting classifications of unseen examples
- Verify performance with a test set
    - Collect a large set of examples
    - Divide into 2 disjoint sets: training set and test set
    - Learn hypothesis $h$ with training set
    - Measure percentage of correctly classified examples by $h$ in the test set
    - Repeat 2-4 for different randomly selected training sets of varying sizes

# Learning curves

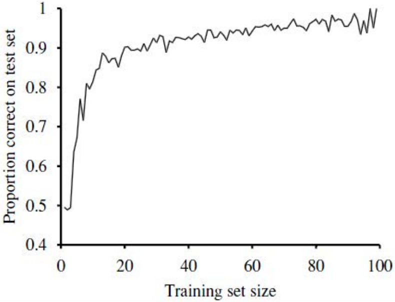



**Figure 18.7** A learning curve for the decision tree learning algorithm on 100 randomly generated examples in the restaurant domain. Each data point is the average of 20 trials.

# Overfitting

- Decision-tree grows until all training examples are perfectly classified
- But what if
  - Data is noisy
  - Training set is too small to give a representative sample of the target function
- May lead to Overfitting!
  - Common problem with most learning algorithms

# Overfitting (过度拟合)

- Definition: Given a hypothesis space $H$, a hypothesis $h \in H$ is said to overfit the training data if there exists some alternative hypothesis $h' \in H$ such that $h$ has smaller error than $h'$ over the training examples but $h'$ has smaller error than $h$ over the entire distribution of instances
- Avoiding overfitting for DTL: Decision tree pruning: Eliminating nodes that are not clearly relevant.

# K-fold Cross-validation (交叉验证)

- Split data in two parts, one for training, one for testing the accuracy of a hypothesis
- Run $k$ experiments, each time putting aside $1/k$ of the data to test on
- Take the average test set score of the $k$ rounds
- Popular values for $k$ are 5 and 10

# Exercise

Perform DTL on the following dataset, where $D$ is the output

| A | B | C | D |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 |

# Candy example

- Favorite candy sold in two flavors: Cherry (yum), Lime (ugh)
- Same wrapper for both flavors
- Sold in bags with different ratios:
  - 100% cherry
  - 75% cherry + 25% lime
  - 50% cherry + 50% lime
  - 25% cherry + 75% lime
  - 100% lime
- You bought a bag of candy but don't know its flavor ratio
- After eating $k$ candies:
  - What's the flavor ratio of the bag?
  - What will be the flavor of the next candy?

# Candy example

- Hypothesis H: probabilistic theory of the world
  - $h_1$: 100% cherry
  - $h_2$: 75% cherry + 25% lime
  - $h_3$: 50% cherry + 50% lime
  - $h_4$: 25% cherry + 75% lime
  - $h_5$: 100% lime
- Data D: evidence about the world
  - $d_1$: 1st candy is cherry
  - $d_2$: 2nd candy is lime
  - $d_3$: 3rd candy is lime
  - ...

# Bayesian Learning

- Prior: $Pr(H)$
- Likelihood: $Pr(d|H)$
- Evidence: $d = \langle d_1, d_2, \ldots, d_n \rangle$
- Computing the posterior using Bayes'Theorem:

$$Pr(H|d) = \alpha Pr(d|H) Pr(H)$$

# Bayesian Prediction

- Suppose we want to make a prediction about an unknown quantity $X$ (*i.e.*, the flavor of the next candy)

$$P(X|d) = \sum_i P(X|d, h_i)P(h_i|d) = \sum_i P(X|h_i)P(h_i|d)$$

- Predictions are weighted averages of the predictions of the individual hypotheses
- Hypotheses serve as "intermediaries" between raw data and prediction

# Candy Example

- Hypothesis H:
  - $h_1$: 100% cherry
  - $h_2$: 75% cherry + 25% lime
  - $h_3$: 50% cherry + 50% lime
  - $h_4$: 25% cherry + 75% lime
  - $h_5$: 100% lime
- Assume prior $P(H) = \langle 0.1, 0.2, 0.4, 0.2, 0.1 \rangle$
- Assume candies are i.i.d. (identically and independently distributed), *i.e.*, $P(d|h) = \Pi_j P(d_j|h)$
- Suppose first 10 candies all taste lime:
  - $P(d|h_5) = 1^{10} = 1$,
  - $P(d|h_3) = 0.5^{10} = 0.00097$
  - $P(d|h_1) = 0^{10} = 0$

# Bayesian prediction examples

Let $d_1 = \langle lime \rangle$, and $d_2 = \langle lime, lime \rangle$

| hypothesis | $h_1$ | $h_2$ | $h_3$ | $h_4$ | $h_5$ |
|---|---|---|---|---|---|
| $P(lime\|h_i)$ | 0 | 0.25 | 0.5 | 0.75 | 1 |
| $P(h_i)$ | 0.1 | 0.2 | 0.4 | 0.2 | 0.1 |
| $P(d_1\|h_i)$ | 0 | 0.25 | 0.5 | 0.75 | 1 |
| $P(d_2\|h_i)$ | 0 | $0.25^2$ | $0.5^2$ | $0.75^2$ | 1 |

- $P(d_1) = \Sigma_i P(d_1|h_i)P(h_i) = 0.5$
- $P(lime|d_1) = \frac{1}{P(d_1)} \Sigma_i P(lime|h_i)P(d_1|h_i)P(h_i) = 0.65$
- $P(d_2) = \Sigma_i P(d_2|h_i)P(h_i) = 0.325$
- $P(lime|d_2) = \frac{1}{P(d_2)} \Sigma_i P(lime|h_i)P(d_2|h_i)P(h_i) = 0.654$

# Bayesian predictions with data generated from $h_5$

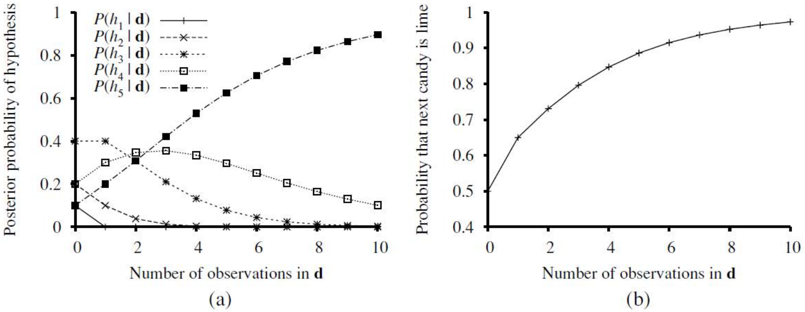



**Figure 20.1** (a) Posterior probabilities $P(h_i \mid d_1, \ldots, d_N)$ from Equation (20.1). The number of observations $N$ ranges from 1 to 10, and each observation is of a lime candy. (b) Bayesian prediction $P(d_{N+1} = lime \mid d_1, \ldots, d_N)$ from Equation (20.2).

# Bayesian learning properties

- Optimal (*i.e.*, given prior, no other prediction is correct more often than the Bayesian one)
- No overfitting (all hypotheses weighted and considered)
- There is a price to pay:
  - When hypothesis space is large, Bayesian learning may be intractable
  - *i.e.*, sum (or integral) over hypothesis often intractable
- Solution: approximate Bayesian learning

# Maximum a posteriori (极大后验,MAP)

- Idea: make prediction based on most probable hypothesis
  - $h_{\text{MAP}} = \text{argmax}_{h_i} P(h_i|d)$
  - $P(X|d) \approx P(X|h_{\text{MAP}})$

- In contrast, Bayesian learning makes prediction based on all hypotheses weighted by their probability

# Candy Example (MAP)

- Prediction after
  - 1 lime: $h_{\text{MAP}} = h_3$, $Pr(lime|h_{\text{MAP}}) = 0.5$
  - 2 limes: $h_{\text{MAP}} = h_4$, $Pr(lime|h_{\text{MAP}}) = 0.75$
  - 3 limes: $h_{\text{MAP}} = h_5$, $Pr(lime|h_{\text{MAP}}) = 1$
  - 4 limes: $h_{\text{MAP}} = h_5$, $Pr(lime|h_{\text{MAP}}) = 1$
  - ...
- After only 3 limes, it correctly selects $h_5$
- But what if correct hypothesis is $h_4$?
- After 3 limes, MAP incorrectly predicts $h_5$
  - MAP yields $P(lime|h_{MAP}) = 1$
  - Bayesian learning yields $P(lime|d) = 0.8$

# MAP properties

- MAP prediction less accurate than Bayesian prediction since it relies only on one hypothesis $h_{\text{MAP}}$
- But MAP and Bayesian predictions converge as data increases
- Controlled overfitting (prior can be used to penalize complex hypotheses)
- Finding $h_{MAP}$ may be intractable:
  - $h_{MAP} = \text{argmax}_h P(h|d)$
  - Optimization may be difficult

# MAP computation

- Optimization:
  - $h_{\text{MAP}} = \text{argmax}_h P(h|d) = \text{argmax}_h P(h)P(d|h) = \text{argmax}_h P(h)\Pi_i P(d_i|h)$
- Product induces non-linear optimization
- Take the log to linearize optimization
  - $h_{\text{MAP}} = \text{argmax}_h \log P(h) + \sum_i \log P(d_i|h)$

# Maximum Likelihood (极大似然,ML)

- Idea: simplify MAP by assuming uniform prior (*i.e.*, $P(h_i) = P(h_j)$ for all $i, j$)
  - $h_{\text{MAP}} = \text{argmax}_h P(h)P(d|h)$
  - $h_{\text{ML}} = \text{argmax}_h P(d|h)$
- Make prediction based on $h_{\text{ML}}$ only:
  - $P(X|d) \approx P(X|h_{\text{ML}})$

# ML properties

- ML prediction less accurate than Bayesian and MAP predictions since it ignores prior info and relies only on one hypothesis $h_{\text{ML}}$
- But ML, MAP and Bayesian predictions converge as data increases
- Subject to overfitting (no prior to penalize complex hypothesis that could exploit statistically insignificant data patterns)
- Finding $h_{\text{ML}}$ is often easier than $h_{\text{MAP}}$
  - $h_{\text{ML}} = \text{argmax}_h \sum_i \log P(d_i|h)$

# Statistical Learning: summary

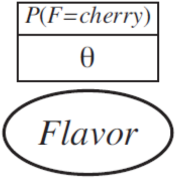
- Given: Prior $Pr(H)$, evidence $d = \langle d_1, d_2, \ldots, d_n \rangle$, and likelihood $Pr(d|H)$, make prediction on $P(X|d)$
- i.i.d. asuumption: $P(d|h) = \Pi_j P(d_j|h)$
- Bayesian learning: $P(X|d) = \sum_i P(X|h_i)P(h_i|d)$
- MAP learning: $P(X|d) \approx P(X|h_{\text{MAP}})$, where $h_{\text{MAP}} = \text{argmax}_{h_i} P(h_i|d) = \text{argmax}_{h_i} P(h_i)P(d|h_i)$
- ML learning: $P(X|d) \approx P(X|h_{\text{ML}})$, where $h_{\text{ML}} = \text{argmax}_{h_i} P(d|h_i)$
- ML, MAP and Bayesian predictions converge as data increases

# Statistical Learning

- Complete data:
  - When data has multiple attributes, all attributes are known
  - Easy
- Incomplete data:
  - When data has multiple attributes, some attributes are unknown
  - Harder

# Simple ML example

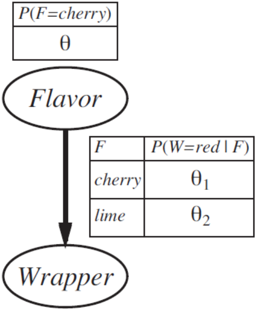
- Hypothesis $h_\theta$
  - $P(cherry) = \theta$ and $P(lime) = 1 - \theta$
- Data $d$:
  - $c$ cherries and $l$ limes




- $P(d|h_\theta) = \theta^c(1-\theta)^l$
- $\log P(d|h_\theta) = c\log\theta + l\log(1-\theta)$
- $d(logP(d|h_\theta))/d\theta = c/\theta - l/(1-\theta)$
- $c/\theta - l/(1-\theta) = 0 \Rightarrow \theta = c/(c+l)$

# More complicated ML example

- Hypothesis $h_{\theta,\theta_1,\theta_2}$
- Data $d$:
  - $c$ cherries: $g_c$ green and $r_c$ red
  - $l$ limes: $g_l$ green and $r_l$ red



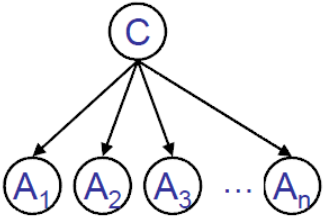

- $P(d|h_{\theta,\theta_1,\theta_2}) = \theta^c(1-\theta)^l\theta_1^{r_c}(1-\theta_1)^{g_c}\theta_2^{r_l}(1-\theta_2)^{g_l}$
- $c/\theta - l/(1-\theta) = 0 \Rightarrow \theta = c/(c+l)$
- $r_c/\theta_1 - g_c/(1-\theta_1) = 0 \Rightarrow \theta_1 = r_c/(r_c+g_c)$
- $r_l/\theta_2 - g_l/(1-\theta_2) = 0 \Rightarrow \theta_2 = r_l/(r_l+g_l)$

# Laplace Smoothing

- An important case of overfitting happens when there is no sample for a certain outcome
  - *e.g.*, no cherries eaten so far
  - $P(cherry) = \theta = c/(c+l) = 0$
  - Zero prob. are dangerous: they rule out outcomes
- Solution: Laplace (add-one) smoothing
  - Add 1 to all counts
  - $P(cherry) = \theta = (c+1)/(c+l+2) > 0$
  - Much better results in practice

# Naive Bayes models

- Want to predict a class $C$ based on attributes $A_1, \ldots, A_n$
- Parameters:
  - $\theta = P(C = true)$
  - $\theta_{i1} = P(A_i = true | C = true)$
  - $\theta_{i2} = P(A_i = true | C = false)$
- Assumption: $A_i$'s are independent given C



# An example: restaurant

| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $Alt$ | $Bar$ | $Fri$ | $Hun$ | $Pat$ | $Price$ | $Rain$ | $Res$ | $Type$ | $Est$ | $Wait$ |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

# Naive Bayes learning

- Notation: $p = \#(c), n = \#(-c), p_i^+ = \#(c, a_i)$, $n_i^+ = \#(c, -a_i), p_i^- = \#(-c, a_i), n_i^- = \#(-c, -a_i)$
- $P(d|h) = \theta^p (1-\theta)^n \Pi_i \theta_{i1}^{p_i^+} \theta_{i2}^{p_i^-} (1-\theta_{i1})^{n_i^+} (1-\theta_{i2})^{n_i^-}$
- $\theta = p/(p+n)$, $\theta_{i1} = p_i^+/(p_i^+ + n_i^+)$, $\theta_{i2} = p_i^-/(p_i^- + n_i^-)$,
- $P(C|a_1, \ldots, a_n) = \alpha P(C) \Pi_i P(a_i|C)$
- Choose the most likely class

# Naive Bayes vs decision trees

Less accurate since the true hypothesis, which is a decision tree, is not representable exactly using a naive Bayes model.

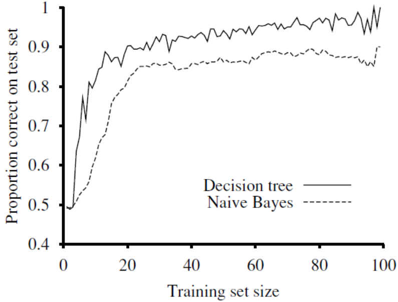



**Figure 20.3** The learning curve for naive Bayes learning applied to the restaurant problem from Chapter 18; the learning curve for decision-tree learning is shown for comparison.

# Bayesian network parameter learning (ML)

- Parameters $\theta_{V,pa(V)=\mathbf{v}}$:
  - CPTs: $\theta_{V,pa(V)=\mathbf{v}} = P(V|pa(V)=\mathbf{v})$
- Data **d**:
  - $d_1$ : $\langle V_1=v_{1,1}, V_2=v_{2,1}, \ldots, V_n = v_{n,1}\rangle$
  - $d_2$ : $\langle V_1=v_{1,2}, V_2=v_{2,2}, \ldots, V_n = v_{n,2}\rangle$
  - ...
- Maximum likelihood:
  - Set $\theta_{V,pa(V)=\mathbf{v}}$ to the relative frequencies of the values of V given the values **v** of the parents of V

$$\theta_{V,pa(V)=\mathbf{v}} = \#(V,pa(V)=\mathbf{v}) \,/\, \#(pa(V)=\mathbf{v})$$

# Exercise

对一个新的输入$A = 0, B = 0, C = 1$，朴素贝叶斯分类器将会怎样预测$D$?

| A | B | C | D |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 |

# Exercise: Candy example

- Prior $P(H) = \langle 0.1, 0.2, 0.4, 0.2, 0.1 \rangle$
- Evidence $d = \langle lime, cherry, lime \rangle$
- Make predictions using Bayesian, MAP and ML learning