Principles of Compiler Construction

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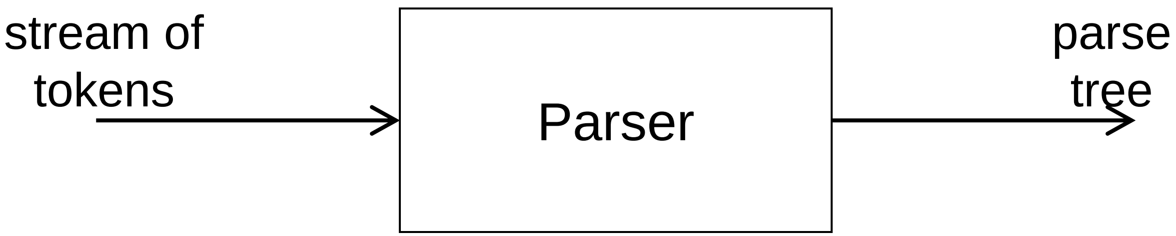
Lecture 4. Top-Down Parsing

1. Introduction to Parsing
2. Top-Down Parsing
3. Rewriting Grammars
4. Top-Down Parser with Backtracking
5. Recursive Descent Predictive Parser

1. Introduction to Parsing

- Parser: input, process, and output (IPO)

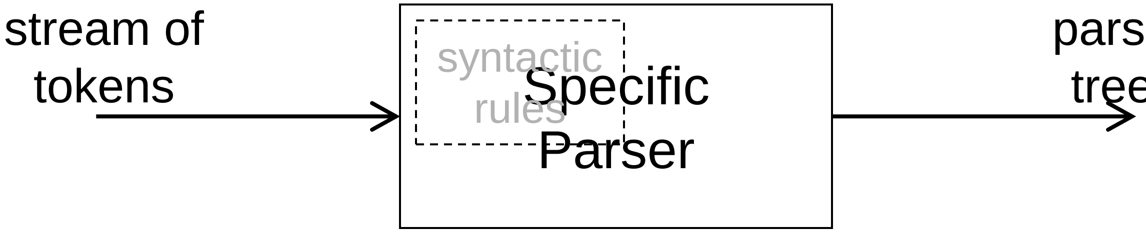
Logical vs. Physical *Logical vs. Physical*



Two questions must be answered.

Structure of a Parser

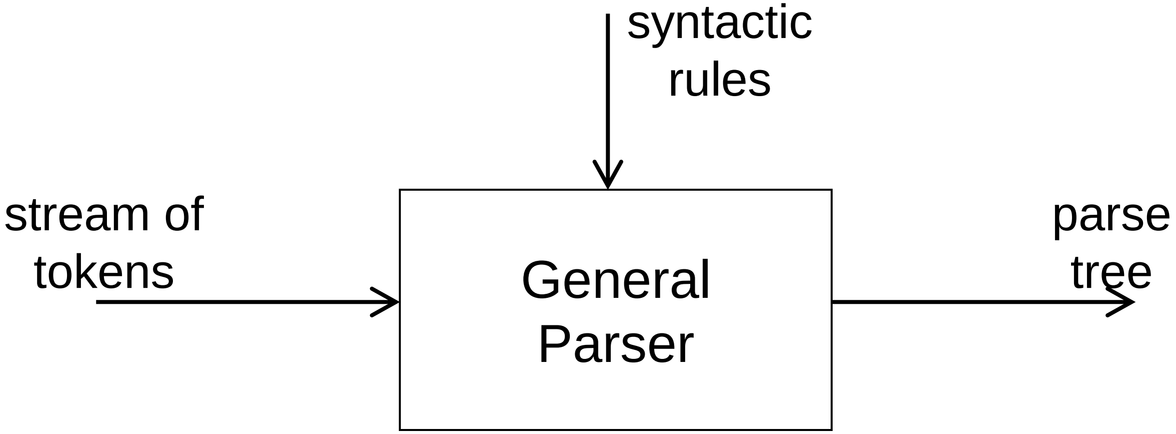
- Implicit syntactic rules



Specific to some predefined language.

Structure of a Parser (cont')

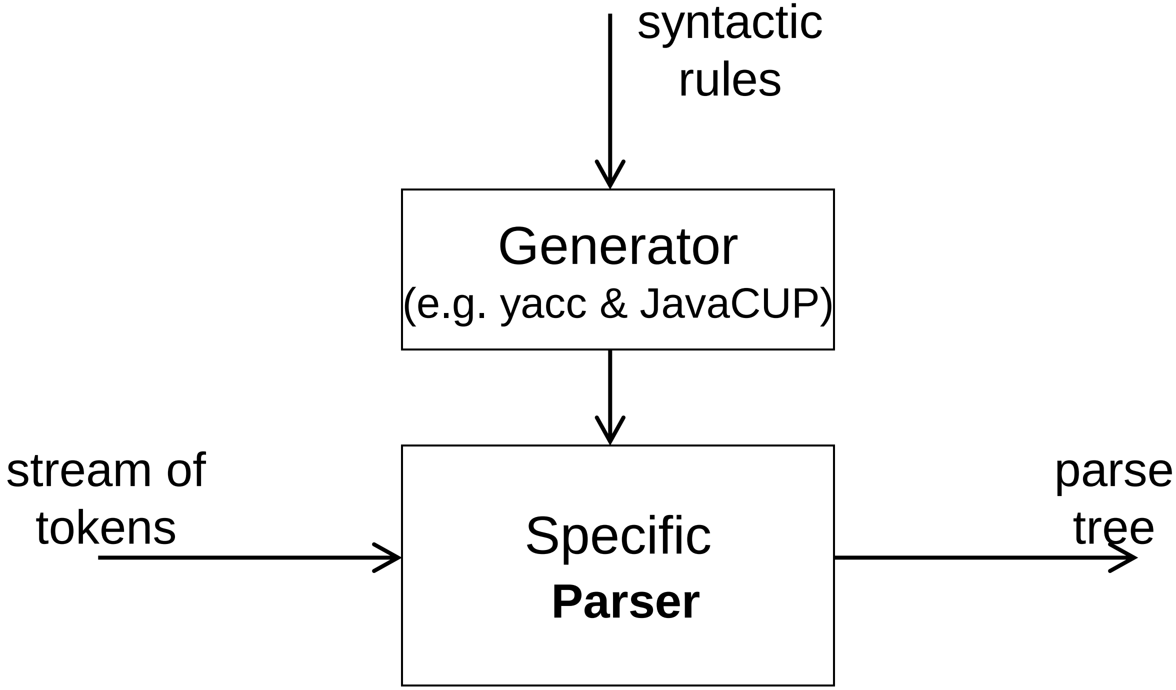
- Explicit syntactic rules (interpretation model)



No hard-coding of language-specific code.

Structure of a Parser (cont')

- Explicit syntactic rules (compilation model)



Review

- The pipe between a parser and a scanner
  - Method invocation (procedure call)
  - Logical vs. physical
- Context-free grammars
  - Parse tree, derivation, and reduction
  - Ambiguity

Capability of Context-Freedom

- What languages it can generate?
  - $L_0 = \{a^n b^n \mid n \geq 1\}$
    - Abstraction of some problem in practice
- What languages it can not generate?
  - $L_1 = \{\omega c \omega \mid \omega \in (a \mid b)^* \wedge a, b, c \in \Sigma\}$
    - Abstraction of some problem in practice
    - How to solve the problem?
  - $L_2 = \{a^n b^m c^n d^m \mid n \geq 1 \wedge m \geq 1\}$
    - Abstraction of some problem in practice
    - How to solve the problem?

2. Top-Down Parsing

- Parsing strategies
  - Top-down parsing
    - How to choose a unique production in multiple candidates?
  - Bottom-up parsing
    - How to find the handle in a sentential form?

Top-Down Parsing: Motivation

- A motivating example

$$
\begin{array}{lcl}
\textit{type} & \rightarrow & \textit{simple} \\
 & | & \wedge\ \textbf{id} \\
 & | & \textbf{array}\ [\ \textit{simple}\ ]\ \textbf{of}\ \textit{type} \\
\textit{simple} & \rightarrow & \textbf{integer} \\
 & | & \textbf{char} \\
 & | & \textbf{num}\ \ \textbf{dotdot}\ \ \textbf{num}
\end{array}
$$

  - `array [1..10] of integer`

Parsing Process (Initial)

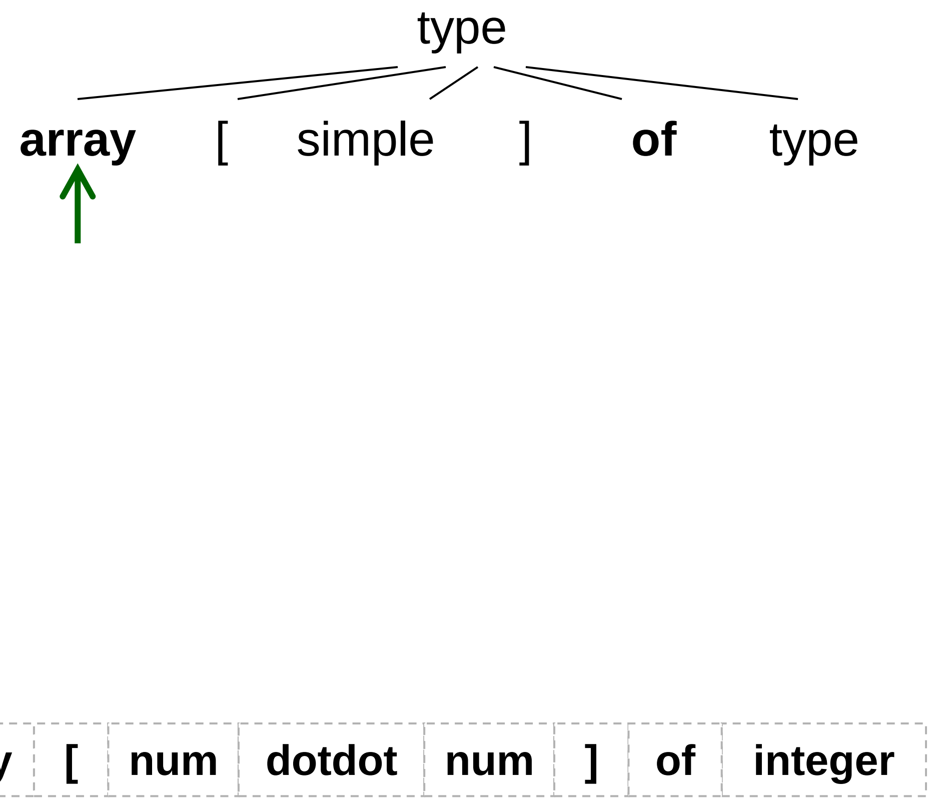
*Derive with "type → **array [** simple **] of** type"*

type

array	**[**	**num**	**dotdot**	**num**	**]**	**of**	**integer**

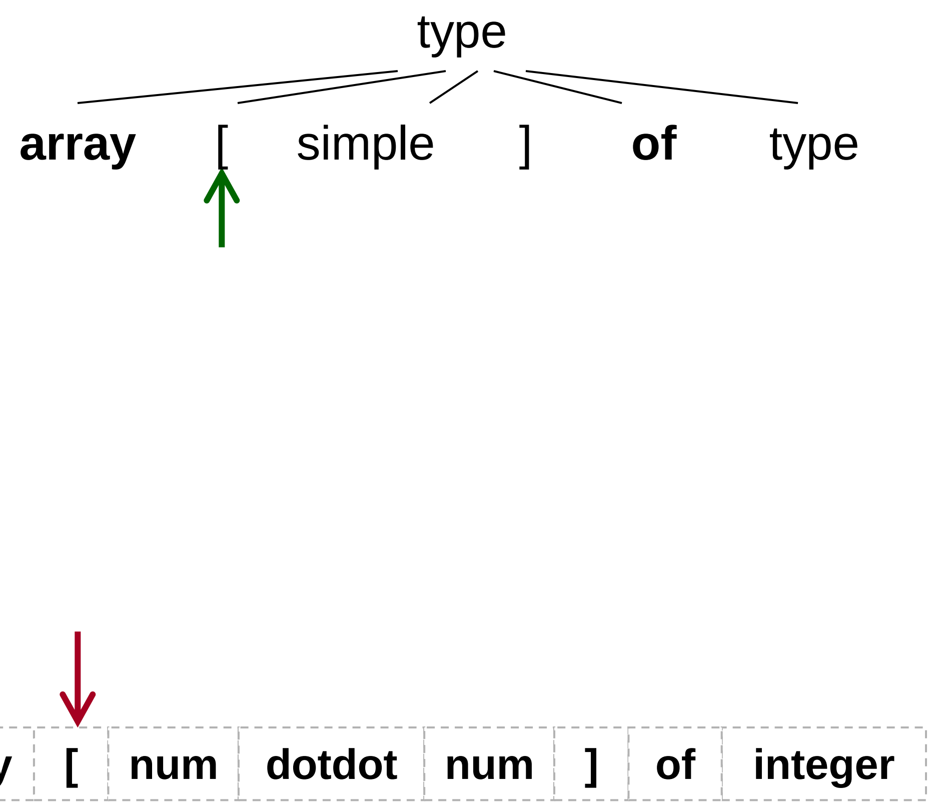
Parsing Process (Action 1)

Match !



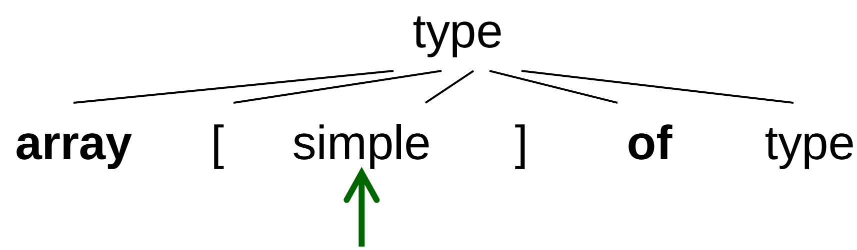
Parsing Process (Action 2)

Match !



Parsing Process (Action 3)

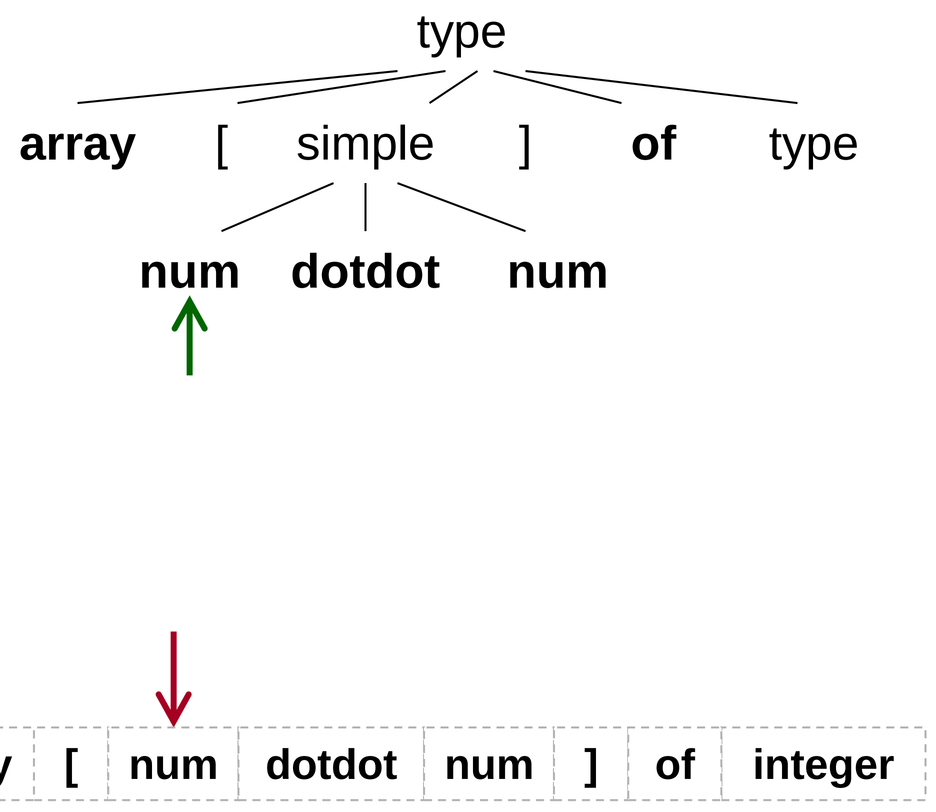
*Derive with "simple → **num dotdot num**"*



array	**[**	**num**	**dotdot**	**num**	**]**	**of**	**integer**

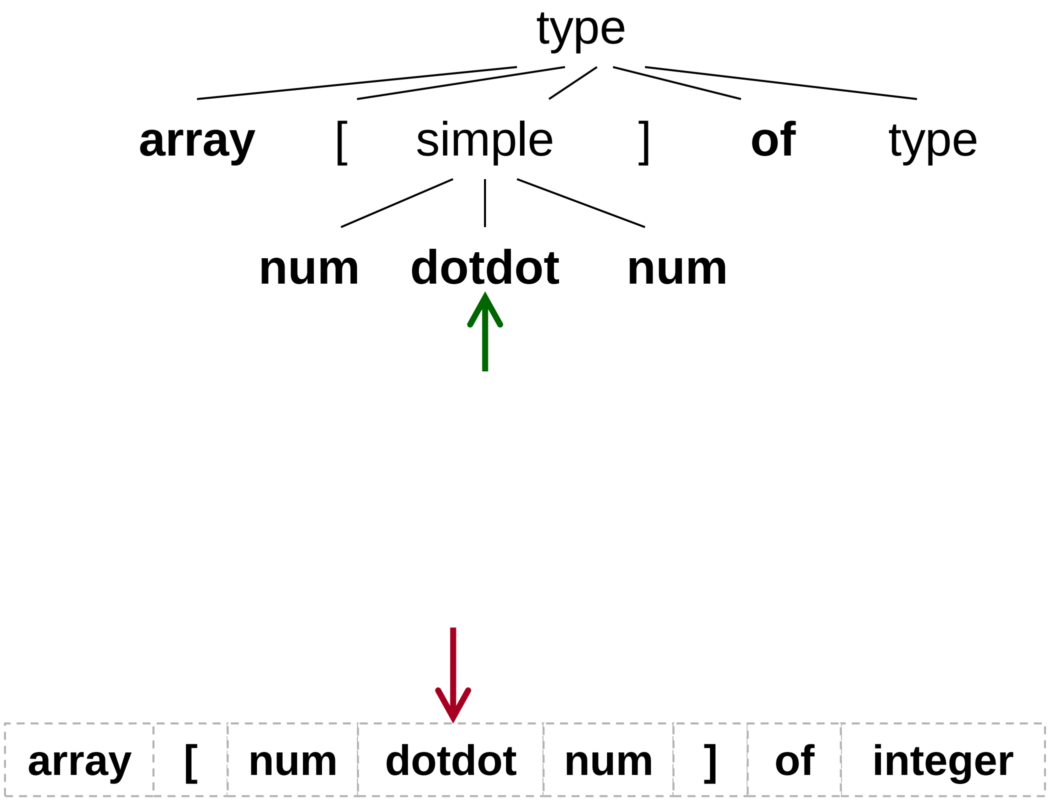
Parsing Process (Action 4)

Match !



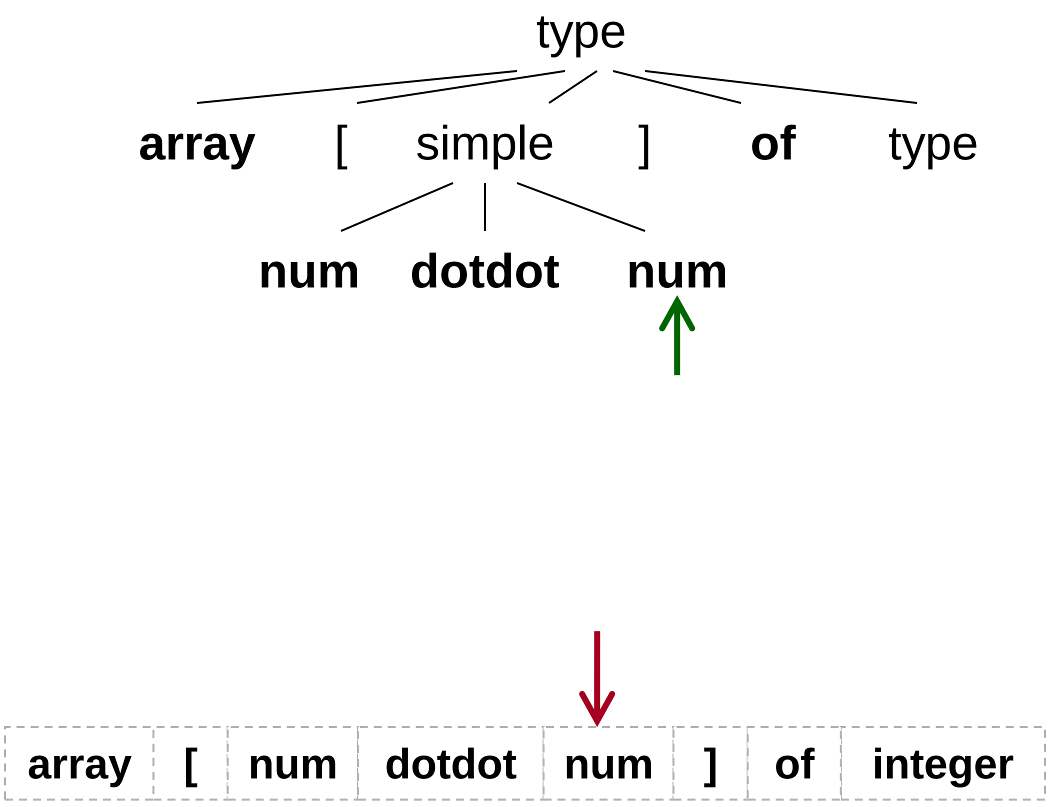
Parsing Process (Action 5)

Match !



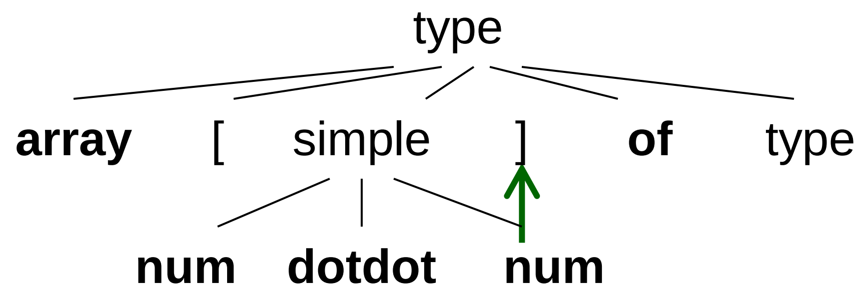
Parsing Process (Action 6)

Match !



Parsing Process (Action 7)

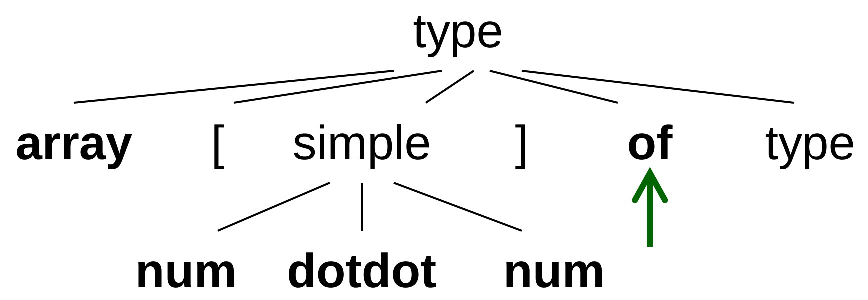
Match !



array	**[**	**num**	**dotdot**	**num**	**]**	**of**	**integer**

Parsing Process (Action 8)

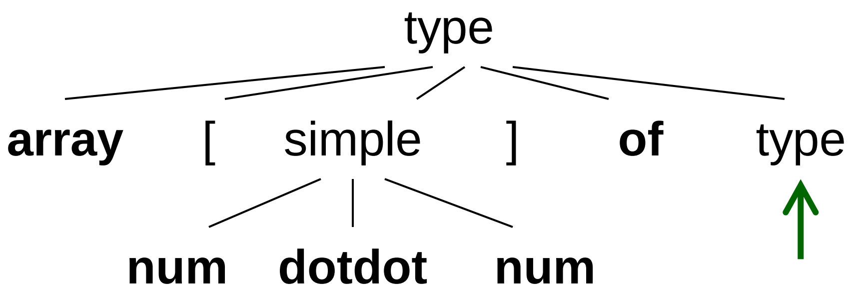
Match !



array	**[**	**num**	**dotdot**	**num**	**]**	**of**	**integer**

Parsing Process (Action 9)

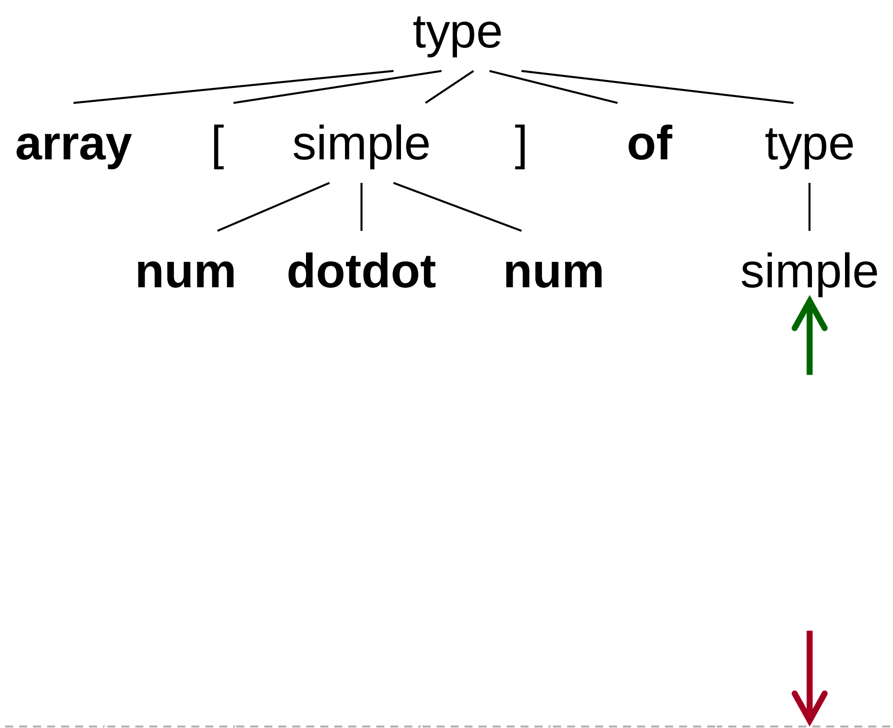
Derive with "type → simple"



array	**[**	**num**	**dotdot**	**num**	**]**	**of**	**integer**

Parsing Process (Action 10)

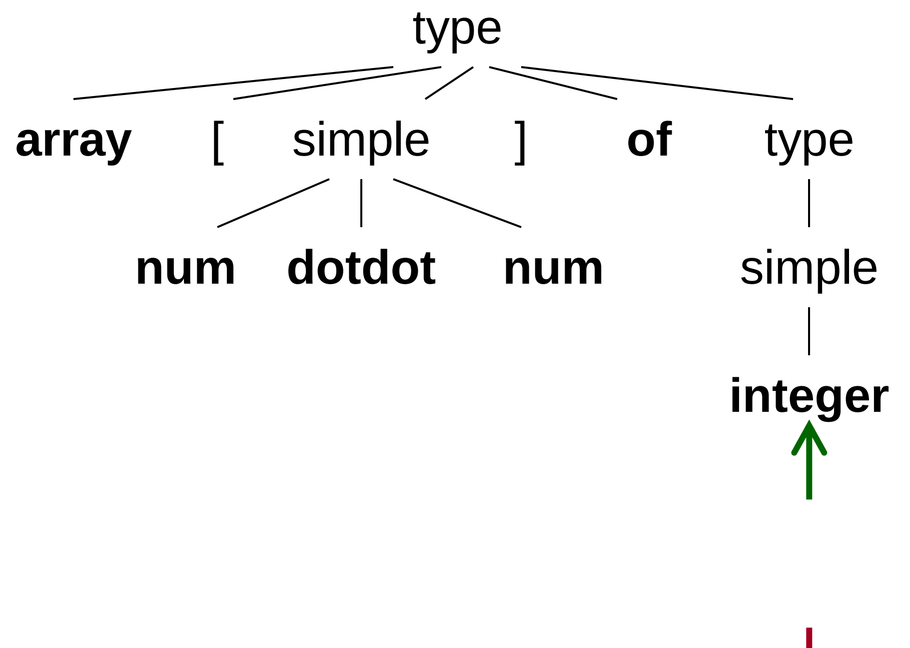
*Derive with "simple → **integer**"*



array	**[**	**num**	**dotdot**	**num**	**]**	**of**	**integer**

Parsing Process (Action 11)

Match and Accept !



array	[	num	dotdot	num	]	of	integer

Top-Down Parsers: Perspectives

- Perspective 1:
  parsing capability vs. efficiency
  - Top-down parser with backtracking
  - Predictive parser

Two **orthogonal** perspectives

Top-Down Parsers: Perspectives (cont')

- Perspective 2:
  parser implementation
  - Recursive descent parser
    - A parser with backtracking
    - A predictive parser
  - Table-driven parser
    - Non-recursive programs with an explicit stack and a parsing table.
    - An approach to automation (usually predictive)

3. Rewriting Grammars

Grammar Transformation

- Resolving ambiguities
  - Trade-off and consequence
- Elimination of ε-productions
  - Systematic elimination of left recursions
- Elimination of left recursions
  - Avoid infinite loop in top-down parsing
- Left-factoring
  - Avoid backtracking in top-down parsing

☑ Resolving Ambiguities

- Review: ambiguities in practice
  - Expression
  - Dangling-else
- Resolving ambiguities
  - Ad hoc constraints
  - Trade-off and consequence

Ambiguous Expressions

- Ambiguous grammar

$$
\begin{array}{lcl}
\mathit{expr} & \to & \mathit{expr} + \mathit{expr} \\
& | & \mathit{expr} \; \mathbf{*} \; \mathit{expr} \\
& | & \mathbf{(} \; \mathit{expr} \; \mathbf{)} \; | \; \mathbf{n}
\end{array}
$$

- Unambiguous grammar

$$
\begin{array}{lcl}
\mathit{expr} & \to & \mathit{expr} + \mathit{term} \; | \; \mathit{term} \\
\mathit{term} & \to & \mathit{term} \; \mathbf{*} \; \mathit{factor} \; | \; \mathit{factor} \\
\mathit{factor} & \to & \mathbf{(} \; \mathit{expr} \; \mathbf{)} \; | \; \mathbf{n}
\end{array}
$$

Rewriting Rules

- Ad hoc but heuristic rewriting rules
  - Rules for precedence
  - Rules for associativity

Dangling-**else** Problem

- Grammar

$$
\begin{aligned}
\textit{stmt} \;\rightarrow\;& \textbf{if}\ \textit{expr}\ \textbf{then}\ \textit{stmt} \\
\mid\;& \textbf{if}\ \textit{expr}\ \textbf{then}\ \textit{stmt}\ \textbf{else}\ \textit{stmt} \\
\mid\;& \textbf{other}
\end{aligned}
$$

- Example

$\texttt{\textbf{if}}\ E_1\ \texttt{\textbf{then}}\ S_1\ \texttt{\textbf{else}}\ \underline{\texttt{\textbf{if}}\ E_2\ \texttt{\textbf{then}}\ S_2\ \texttt{\textbf{else}}\ S_3}$

- Ambiguity

$\texttt{\textbf{if}}\ E_1\ \texttt{\textbf{then}}\ \underline{\texttt{\textbf{if}}\ E_2\ \texttt{\textbf{then}}\ S_1\ \texttt{\textbf{else}}\ S_2}$

$\texttt{\textbf{if}}\ E_1\ \texttt{\textbf{then}}\ \underline{\texttt{\textbf{if}}\ E_2\ \texttt{\textbf{then}}\ S_1}\ \texttt{\textbf{else}}\ S_2$

Unambiguous Grammar

- Additional disambiguation rule
  - *Each* ***else*** *is matched with the closest unmatched* ***then****.*
- Unambiguous grammar

$$
\begin{array}{lll}
\textit{stmt} & \rightarrow & \textit{matched\_stmt} \mid \textit{open\_stmt} \\
\textit{matched\_stmt} & \rightarrow & \textbf{if}\ \textit{expr}\ \textbf{then}\ \textit{matched\_stmt}\ \textbf{else}\ \textit{matched\_stmt} \\
 & \mid & \textbf{other} \\
\textit{open\_stmt} & \rightarrow & \textbf{if}\ \textit{expr}\ \textbf{then}\ \textit{stmt} \\
 & \mid & \textbf{if}\ \textit{expr}\ \textbf{then}\ \textit{matched\_stmt}\ \textbf{else}\ \textit{open\_stmt}
\end{array}
$$

- Example

  if E_1 **then** <u>**if** E_2 **then** S_1 **else** S_2</u>

☑ Eliminating ε-Productions

Some textbook defines ε–free grammar with only the first restriction

- An ε-free grammar
  - No production body is ε (ε-production), or
  - The only ε-production is $S \rightarrow \varepsilon$, and S does not appear in the body of any productions.
- Elimination algorithm
  - For every production $A \rightarrow X_1\ X_2\ \ldots\ X_n$, where $X_i \in \Sigma \cup N$, $1 \leq i \leq n$
  - Add new productions $A \rightarrow a_1\ a_2\ \ldots\ a_n$, where
    - $\neg\ (X_i \Rightarrow^* \varepsilon) \Rightarrow (a_i = X_i)$
    - $(X_i \Rightarrow^* \varepsilon) \Rightarrow (a_i = X_i \vee a_i = \varepsilon)$
    - $\exists 1 \leq i \leq n.\ a_i \neq \varepsilon$

Eliminating ε-Productions: Example 1

- Original grammar

$$S \rightarrow A\,\mathbf{a} \mid \mathbf{b}$$

$$A \rightarrow A\,\mathbf{c} \mid S\,\mathbf{d} \mid \varepsilon$$

- Rewriting grammar

$$S \rightarrow A\,\mathbf{a} \mid \mathbf{a} \mid \mathbf{b}$$

$$A \rightarrow A\,\mathbf{c} \mid \mathbf{c} \mid S\,\mathbf{d}$$

Eliminating ε-Productions: Example 2

- Original grammar

  $S \rightarrow \mathbf{a}\, S\, \mathbf{b}\, S \mid \mathbf{b}\, S\, \mathbf{a}\, S \mid \varepsilon$

- Equivalent ε-free grammar

  $S' \rightarrow S \mid \varepsilon$

  $S \rightarrow \mathbf{a}\, S\, \mathbf{b}\, S \mid \mathbf{a}\, \mathbf{b}\, S \mid \mathbf{a}\, S\, \mathbf{b} \mid \mathbf{a}\, \mathbf{b}$

  $\quad\; \mid \mathbf{b}\, S\, \mathbf{a}\, S \mid \mathbf{b}\, \mathbf{a}\, S \mid \mathbf{b}\, S\, \mathbf{a} \mid \mathbf{b}\, \mathbf{a}$

Augmented grammar

Theorem on ε–Free Grammars

- Given any context-free grammar G, there exists an ε-free grammar G', so that $L(G) - \{\varepsilon\} = L(G')$
  - $\varepsilon \notin L(G) \Rightarrow L(G) = L(G')$
  - The only difference between G and G' is the productions.

☑ Eliminating Left Recursions

- Simple immediate left recursion
  - $A \rightarrow A\alpha \mid \beta$
  - $A \rightarrow \beta A'$
    $A' \rightarrow \alpha A' \mid \varepsilon$

From left recursion to right recursion

- Elimination algorithm
  - $A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \ldots \mid A\alpha_m$
    $\mid \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$
  - $A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \ldots \mid \beta_n A'$
    $A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \ldots \mid \alpha_m A' \mid \varepsilon$

Immediate Left Recursion: Example

- Original grammar

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \mathbf{n}$$

- Equivalent grammar without left recursions

$$E \rightarrow T\,E'$$
$$E' \rightarrow + T\,E' \mid \varepsilon$$
$$T \rightarrow F\,T'$$
$$T' \rightarrow * F\,T' \mid \varepsilon$$
$$F \rightarrow (E) \mid \mathbf{n}$$

Systematic Elimination

- Preconditions
  - No cycles, e.g. $A \Rightarrow^+ A$
  - No ε-productions, e.g. $A \to \varepsilon$

Why?

- Elimination algorithm

Arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.

for $i = 1$ **to** n **do begin**

  for $j = 1$ **to** $i - 1$ **do begin**

    Replace each production of the form $A_i \to A_j\gamma$

    by the production $A_i \to \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$,

    where $A_j \to \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$ *are all current* A_j*-productions*

  end

  Eliminate the immediate left recursion among the A_i*-productions*

end

Systematic Elimination: Example 1

- Original grammar

$$S \rightarrow A\,\mathbf{a} \mid \mathbf{b}$$
$$A \rightarrow A\,\mathbf{c} \mid S\,\mathbf{d} \mid \varepsilon$$

- Rewriting grammar
  - Eliminating ε-productions:

$$S \rightarrow A\,\mathbf{a} \mid \mathbf{a} \mid \mathbf{b}$$
$$A \rightarrow A\,\mathbf{c} \mid \mathbf{c} \mid S\,\mathbf{d}$$

  - Eliminating left recursions, ordered by S, A:

$$S \rightarrow A\,\mathbf{a} \mid \mathbf{a} \mid \mathbf{b}$$
$$\cancel{A \rightarrow A\,\mathbf{c} \mid \mathbf{c} \mid A\,\mathbf{a}\,\mathbf{d} \mid \mathbf{a}\,\mathbf{d} \mid \mathbf{b}\,\mathbf{d}}$$
$$A \rightarrow \mathbf{c}\,A' \mid \mathbf{a}\,\mathbf{d}\,A' \mid \mathbf{b}\,\mathbf{d}\,A'$$
$$A' \rightarrow \mathbf{c}\,A' \mid \mathbf{a}\,\mathbf{d}\,A' \mid \varepsilon$$

Systematic Elimination: Example 2

- Original grammar

$$
\begin{aligned}
S &\rightarrow A\,\mathbf{c} \mid \mathbf{c} \\
A &\rightarrow B\,\mathbf{b} \mid \mathbf{b} \\
B &\rightarrow S\,\mathbf{a} \mid \mathbf{a}
\end{aligned}
$$

- Rewriting with different order

Ordered by **S**, **A**, **B**:

$$
\begin{aligned}
S &\rightarrow A\,\mathbf{c} \mid \mathbf{c} \\
A &\rightarrow B\,\mathbf{b} \mid \mathbf{b} \\
B &\rightarrow \underline{A\,\mathbf{c}}\,\mathbf{a} \mid \underline{\mathbf{c}}\,\mathbf{a} \mid \mathbf{a} \\
B &\rightarrow \underline{B\,\mathbf{b}}\,\mathbf{c}\,\mathbf{a} \mid \underline{\mathbf{b}}\,\mathbf{c}\,\mathbf{a} \mid \mathbf{c}\,\mathbf{a} \mid \mathbf{a} \\
B &\rightarrow \mathbf{b}\,\mathbf{c}\,\mathbf{a}\,B' \mid \mathbf{c}\,\mathbf{a}\,B' \mid \mathbf{a}\,B' \\
B' &\rightarrow \mathbf{b}\,\mathbf{c}\,\mathbf{a}\,B' \mid \varepsilon
\end{aligned}
$$

Ordered by **B**, **A**, **S**:

$$
\begin{aligned}
B &\rightarrow S\,\mathbf{a} \mid \mathbf{a} \\
A &\rightarrow S\,\mathbf{a}\,\mathbf{b} \mid \mathbf{a}\,\mathbf{b} \mid \mathbf{b} \\
S &\rightarrow \underline{S\,\mathbf{a}\,\mathbf{b}}\,\mathbf{c} \mid \underline{\mathbf{a}\,\mathbf{b}}\,\mathbf{c} \mid \underline{\mathbf{b}}\,\mathbf{c} \mid \mathbf{c} \\
S &\rightarrow \mathbf{a}\,\mathbf{b}\,\mathbf{c}\,S' \mid \mathbf{b}\,\mathbf{c}\,S' \mid \mathbf{c}\,S' \\
S' &\rightarrow \mathbf{a}\,\mathbf{b}\,\mathbf{c}\,S' \mid \varepsilon
\end{aligned}
$$

Equivalent

☑ Left Factoring

- Simple left factoring
  - $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$
  - $A \rightarrow \alpha A'$
    $A' \rightarrow \beta_1 \mid \beta_2$
- Left factoring algorithm
  - $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots \mid \alpha\beta_n \mid \gamma$
  - $A \rightarrow \alpha A' \mid \gamma$
    $A' \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$

Left Factoring: Example

- Original grammar

$$S \rightarrow \textbf{if}\ E\ \textbf{then}\ S \mid \textbf{if}\ E\ \textbf{then}\ S\ \textbf{else}\ S \mid \textbf{other}$$

$$E \rightarrow \textbf{bool}$$

- After left factoring

$$S \rightarrow \textbf{if}\ E\ \textbf{then}\ S\ S' \mid \textbf{other}$$

$$S' \rightarrow \textbf{else}\ S \mid \varepsilon$$

$$E \rightarrow \textbf{bool}$$

Conclusions: Why Rewriting?

- For all parsing techniques
  - Resolving ambiguities: why?
- Only for top-down parsing
  - Eliminating ε-productions: why?
  - Eliminating left recursions: why?
  - Left factoring: why?

4. Top-down Parser with Backtracking

- Trade-off and consequence
  - Pros: powerful to handle most CFG.
  - Cons: complex, and low efficiency.
- Only used to demonstrate the idea of top-down parsing
  - Why a left recursion leads to infinite loop?
  - The meaning of the first symbol that a nonterminal can derive.

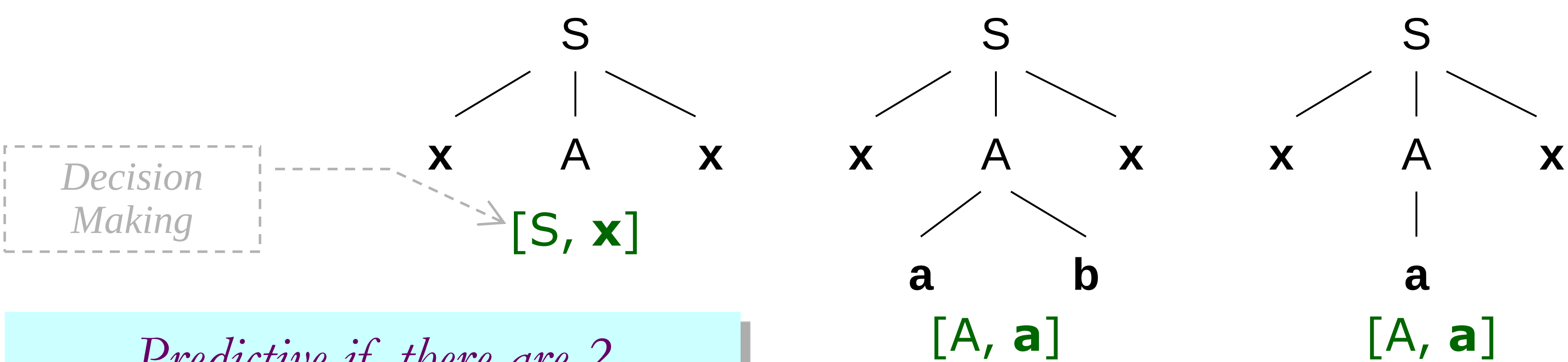
Making Decisions on Actions

- Given the following grammar

  $S \rightarrow \mathbf{x}\ A\ \mathbf{x}$

  $A \rightarrow \mathbf{a}\ \mathbf{b}\ |\ \mathbf{a}\ |\ \mathbf{b}$

- Parsing with only one lookahead
  - Sentence: **`xax`**



Predictive if there are 2 lookaheads

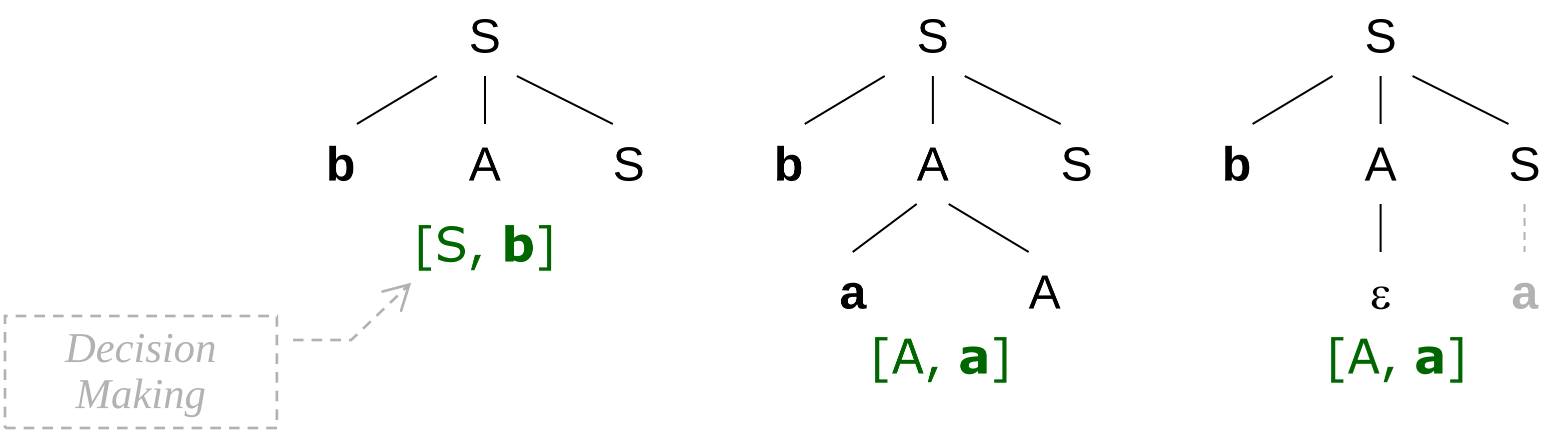
Decision on Actions (cont')

- Given the following grammar

  $S \rightarrow \mathbf{b}\, A\, S \mid \mathbf{a}$

  $A \rightarrow \mathbf{a}\, A \mid \varepsilon$

- Backtracking caused by an ε-production
  - Sentence: **ba**



Four Actions in Top-Down Parsing

- General actions
  - Accept
  - Error
- Actions specific to top-down parsing
  - match
  - derive

5. Recursive Descent Predictive Parser

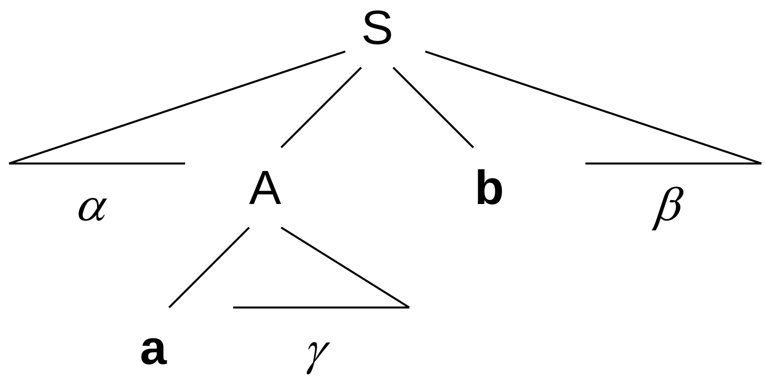
- How can a parser be predictive?
  - Sufficient and necessary conditions for the grammar.
  - How to select a unique production candidate with regard to the lookahead.
- An approach suitable for manual development of a top-down parser.

Development Phrases

- Development of a recursive descent predictive parser
  1. Resolve ambiguities in the grammar.
  2. Eliminate left recursions in the grammar.
  3. Left factor the grammar.
  4. Construct transition diagrams from the grammar.
  5. Reduce the transition diagrams (ad hoc and optional).
  6. Write the parser using the transition diagrams as blue print.

FIRST() and FOLLOW()

- Purposes of these two functions
  - **a** $\in$ FIRST(A) and **b** $\in$ FOLLOW(A)



Function FIRST()

- Intent: while choosing one of $A \rightarrow \alpha \mid \beta$ with regard to the lookahead **x**, we must have $\text{FIRST}(\alpha) \cap \text{FIRST}(\beta) = \varnothing$.
- Syntax (signature)
  - $\text{FIRST}: (\Sigma \cup N)^* \rightarrow 2^{\Sigma \cup \{\varepsilon\}}$
- Semantics
  - $\{\boldsymbol{x} \mid \alpha \Rightarrow^* \boldsymbol{x}\ldots, \boldsymbol{x} \in \Sigma\} \subseteq \text{FIRST}(\alpha)$
  - $\alpha \Rightarrow^* \varepsilon \;\Rightarrow\; \varepsilon \in \text{FIRST}(\alpha)$
    - Why do we permit ε in $\text{FIRST}(\alpha)$?

Function FIRST() (cont')

- Given $X \in \Sigma \cup N$, we have FIRST(X)
  - $X \in \Sigma \Rightarrow \text{FIRST}(X) = \{X\}$
  - $X \in N \wedge X \rightarrow Y_1 Y_2 \ldots Y_n \Rightarrow$
    - $\exists 1 \le i \le n.\ \mathbf{a} \in \text{FIRST}(Y_i)$
      $\wedge\ \forall 1 \le j \le i-1.\ \varepsilon \in \text{FIRST}(Y_j)$
      $\Rightarrow \mathbf{a} \in \text{FIRST}(X)$
    - $\forall 1 \le k \le n.\ \varepsilon \in \text{FIRST}(Y_k)$
      $\Rightarrow \varepsilon \in \text{FIRST}(X)$
      - So we have $X \rightarrow \varepsilon \Rightarrow \varepsilon \in \text{FIRST}(X)$

Function FIRST() (cont')

- Given $\alpha = X_1X_2...X_n \in (\Sigma \cup N)^*$, we have FIRST($\alpha$)
  - $\mathrm{FIRST}(X_1) - \{\varepsilon\} \subseteq \mathrm{FIRST}(\alpha)$
  - $\forall 2 \le i \le n.\ \forall 1 \le j \le i-1.\ \varepsilon \in \mathrm{FIRST}(X_j)$
    $\Rightarrow \mathrm{FIRST}(X_i) - \{\varepsilon\} \subseteq \mathrm{FIRST}(\alpha)$
  - $\forall 1 \le i \le n.\ \varepsilon \in \mathrm{FIRST}(X_i) \Rightarrow \varepsilon \in \mathrm{FIRST}(\alpha)$

Overloading

Function FOLLOW()

- Intent: while choosing one of $A \to \alpha \mid \varepsilon$ with regard to the lookahead **x**, we must have FIRST(α) $\cap$ FOLLOW(A) = $\varnothing$.
- Syntax (signature)
  - FOLLOW: $N \to 2^{\Sigma \cup \{\$\}}$, *where a special symbol* **$** *indicates the end of input token stream.*
- Semantics
  - $\{\boldsymbol{x} \mid S \Rightarrow^{*} \alpha A \boldsymbol{x} \beta \wedge \boldsymbol{x} \in \Sigma\} \subseteq \text{FOLLOW(A)}$
  - $S \Rightarrow^{*} \alpha A \;\Rightarrow\; \boldsymbol{\$} \in \text{FOLLOW(A)}$

Function FOLLOW() (cont')

- Given X $\in$ N, we have FOLLOW(X)
  - $X = S \Rightarrow \mathbf{\$} \in FOLLOW(X)$
  - $A \rightarrow \alpha X\beta \Rightarrow FIRST(\beta) - \{\varepsilon\} \subseteq FOLLOW(X)$
  - $(A \rightarrow \alpha X) \vee (A \rightarrow \alpha X\beta \wedge \varepsilon \in FIRST(\beta))$
    $\Rightarrow FOLLOW(A) \subseteq FOLLOW(X)$

Example 1

- Given the following grammar

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \varepsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \varepsilon \\
F & \rightarrow & \mathbf{(}\ E\ \mathbf{)} \mid \mathbf{n}
\end{array}
$$

- We have
  - FIRST(F) = FIRST(T) = FIRST(E) = {**(**, **n**}
  - FIRST(T') = {*****, ε}
  - FIRST(E') = {**+**, ε}
  - FOLLOW(E) = FOLLOW(E') = {**)**, **$**}
  - FOLLOW(T) = FOLLOW(T') = {**+**, **)**, **$**}
  - FOLLOW(F) = {**+**, *****, **)**, **$**}

Example 2

- Given the following grammar

$$
\begin{aligned}
A &\rightarrow B\ C \\
B &\rightarrow A\ \mathbf{x} \mid \mathbf{x} \mid \varepsilon \\
C &\rightarrow \mathbf{y}\ C \mid \mathbf{y}
\end{aligned}
$$

- We have

Symbol	FIRST	FOLLOW
A	x y	x $
B	x y ε	y
C	y	x $

Heuristics in manual calculation:
Left vs. **right**
Top-down vs. **bottom-up**

Conditions for Predictive Parsing

- Sufficient and necessary conditions
- CFG G is LL(1) iif whenever A $\rightarrow \alpha \mid \beta$ are two distinct productions of G, the following conditions hold
  - $\mathrm{FIRST}(\alpha) \cap \mathrm{FIRST}(\beta) = \varnothing$
  - $\varepsilon \in \mathrm{FIRST}(\beta) \Rightarrow$
    $\mathrm{FIRST}(\alpha) \cap \mathrm{FOLLOW}(A) = \varnothing$

Discussions

- What benefits from the LL(1) conditions?
  - Decision of the derive/error action in the parser with regard to the current nonterminal and a single lookahead.
  - Only 0 or 1 production will be chosen.
- They are not predictive (why?)
  - Ambiguous grammars
  - Grammars with left recursions
  - Grammars with left factors

Transition Diagram

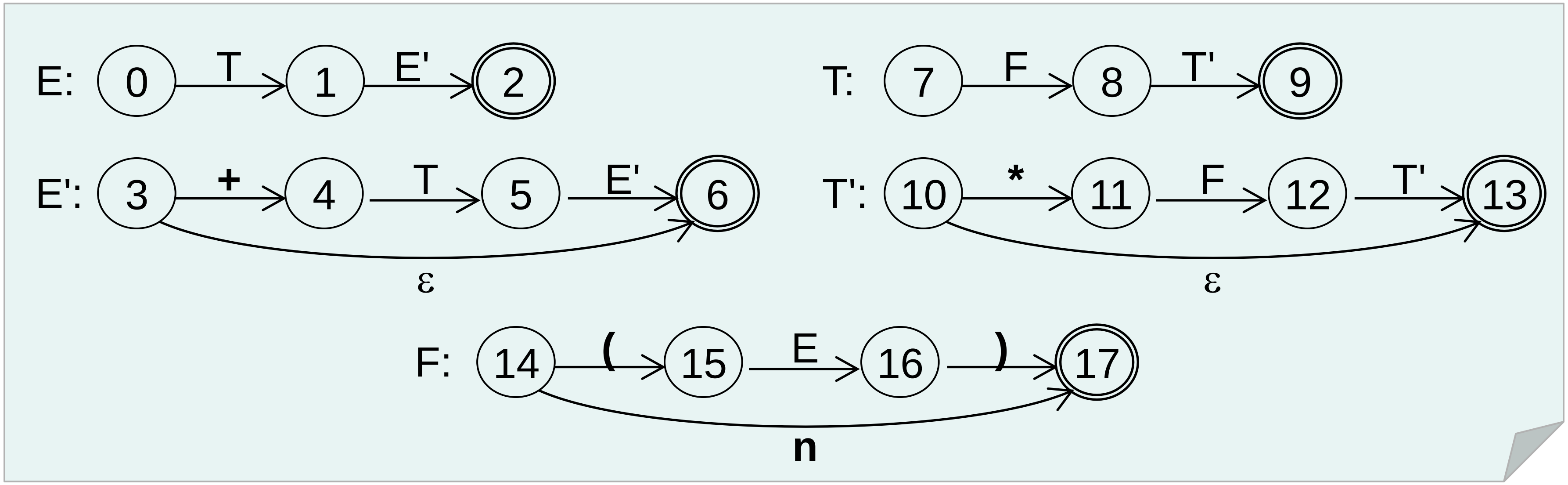
- Visualize the predictive parser
  - For each nonterminal A
    - Create an initial and final state.
    - For each $A \rightarrow X_1X_2...X_n$, create a path from the initial state to the final state, with edges labeled $X_1, X_2, ..., X_n$.
      - If $A \rightarrow \varepsilon$, the path is labeled ε.

Transition Diagram: Example

- Given the following grammar

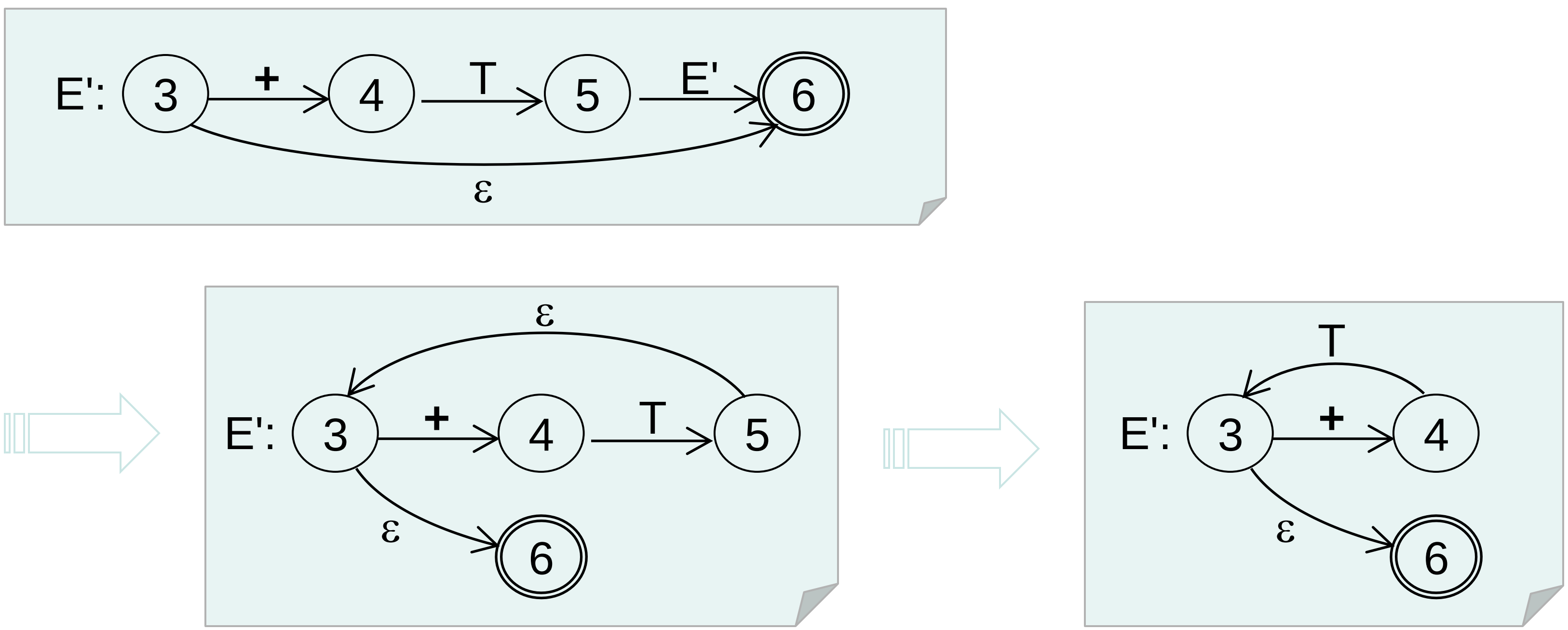
$$
\begin{array}{lllllll}
E & \rightarrow & T\,E' & \quad & E' & \rightarrow & +\,T\,E' \mid \varepsilon \\
T & \rightarrow & F\,T' & \quad & T' & \rightarrow & *\,F\,T' \mid \varepsilon \\
F & \rightarrow & (\,E\,) \mid \mathbf{n} & & & &
\end{array}
$$

- Transform to transition diagrams

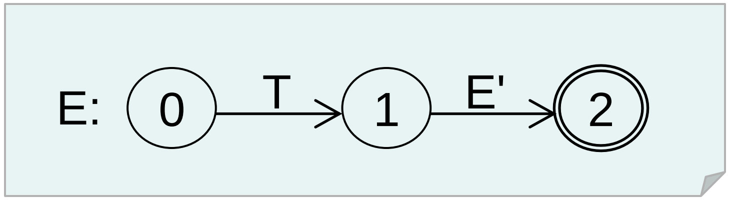


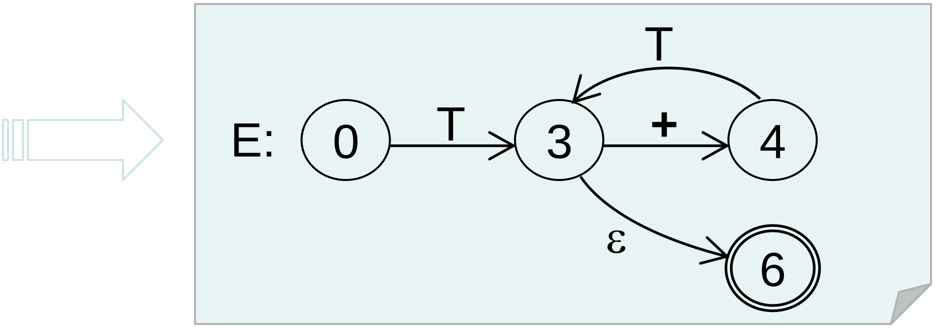
Reduction of Transition Diagrams

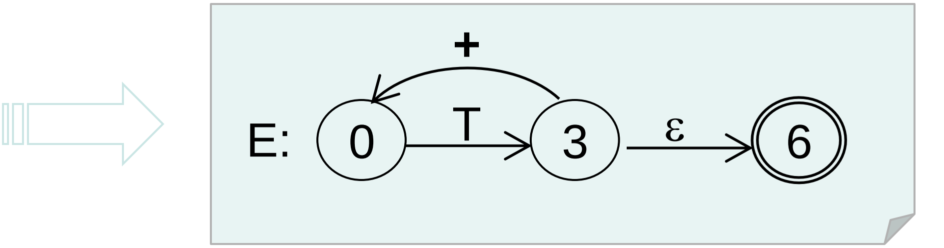
- Ad hoc rules: iterative substitution



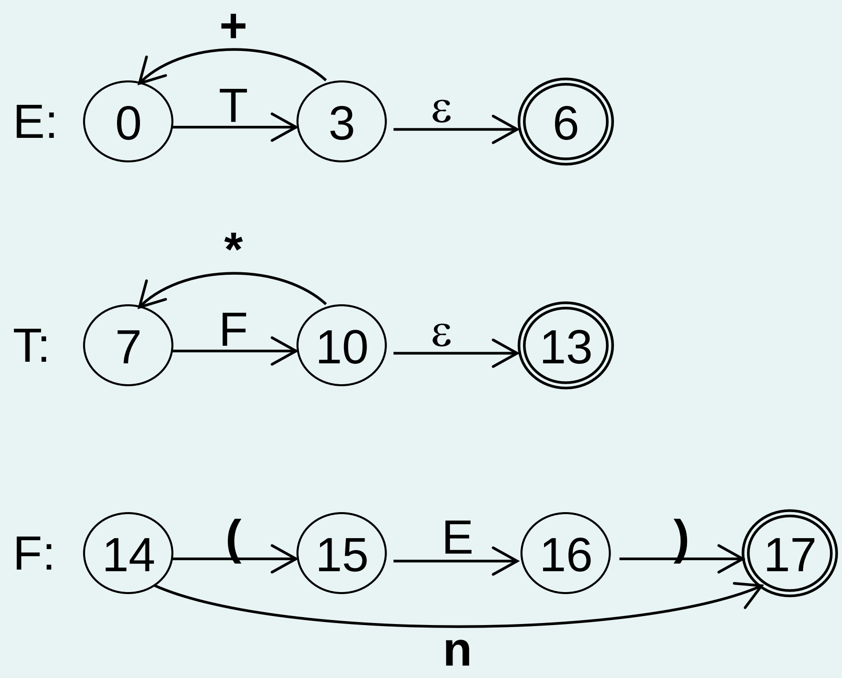
Reduction of Transition Diagrams (cont')







Reduced Transition Diagrams



Discussion

- Transition diagrams: (recursive descent predictive) parsing vs. scanning
  - Point 1
  - Point 2

Write a Recursive Predictive Parser

- Use the transition diagrams as the blue-print of the parser
  - Write a recursive procedure for each diagram.
    - The procedure for the diagram of S is the main entry.
  - Design the control flow of the procedure mimicking the paths in the diagram, with regard to the lookahead.
    - If the label of an edge in the path is a terminal, perform the **match** action.
    - If the label is a nonterminal, perform the **derive** action, that is invoking the procedure of the nonterminal (recursively).

Coding Rules

- $A \rightarrow a\ B\ b$

```
void A() {
        match(a);
        B();
        match(b);
}
```

```
void match(Token tok) {
    if (lookahead == tok) {
        lookahead = scanner.getNextToken();
    } else  error();
}
```

Coding Rules (cont')

- $A \rightarrow a\ B\ b\ |\ b\ A\ B$

```
void A() {
      if (lookahead == a) {
            match(a);  B();  match(b)
      } else if (lookahead == b) {
            match(b);  A();  B()
      } else  error()
}
```

Coding Rules (cont')

- A → a B b | b A B | C

```
void A() {
   if (lookahead == a) {
      match(a);  B();  match(b)
   } else if (lookahead == b) {
      match(b);  A();  B()
   } else  C();
}
```

```
void A() {
   if (lookahead == a) {
      match(a);  B();  match(b)
   } else if (lookahead == b) {
      match(b);  A();  B()
   } else if (lookahead in FIRST(C)) {
      C()
   } else  error();
}
```

Coding Rules (cont')

- $A \rightarrow a\ B\ b \mid b\ A\ B \mid \varepsilon$

```
void A() {
   if (lookahead == a) {
      match(a);  B();  match(b)
   } else if (lookahead == b) {
      match(b);  A();  B()
   } else  ;     // do nothing
}
```

```
void A() {
   if (lookahead == a) {
      match(a);  B();  match(b)
   } else if (lookahead == b) {
      match(b);  A();  B()
   } else if (lookahead in FOLLOW(A)) {
      // do nothing, more accurate!
   } else  error();
}
```

Example 1

```
type   → simple
       | ^ id
       | array [ simple ] of type
simple → integer
       | char
       | num dotdot num
```

```
void type() throws SyntacticException {
   if (lookahead.equals(new Token('^'))) {
      match(new Token('^'));
      match(new Token(Token.ID));
   } else if (lookahead.equals(new Token(Token.ARRAY))) {
      match(new Token(Token.ARRAY));
      match(new Token('['));
      simple();
      match(new Token(']'));
      match(new Token(Token.OF));
      type();
   } else  simple();
}
```

Coding is direct and intuitive while using unreduced transition diagrams as a blueprint.

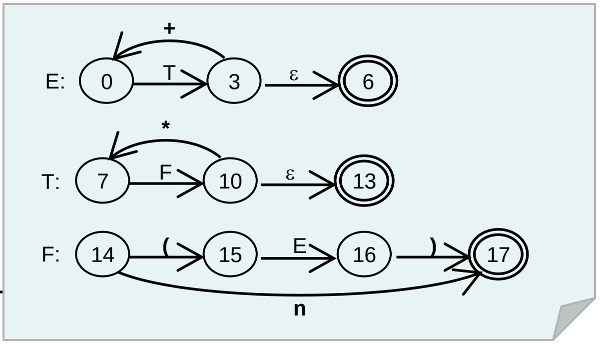
Example 1 (cont')

type → simple
  | ^ **id**
  | **array** [simple] **of** type
simple → **integer**
  | **char**
  | **num dotdot num**

```
void simple() throws SyntacticException {
   if (lookahead.equals(new Token(Token.INTEGER))) {
      match(new Token(Token.INTEGER));
   } else if (lookahead.equals(new Token(Token.CHAR))) {
      match(new Token(Token.CHAR));
   } else if (lookahead.equals(new Token(Token.NUM))) {
      match(new Token(Token.NUM));
      match(new Token(Token.DOTDOT));
      match(new Token(Token.NUM));
   } else  throw new SyntacticException();
}

void match(Token tok) throws SyntacticException {
   if (lookahead.equals(tok))
      lookahead = scanner.getNextToken();
   else  throw new SyntacticException();
}
```

Example 2

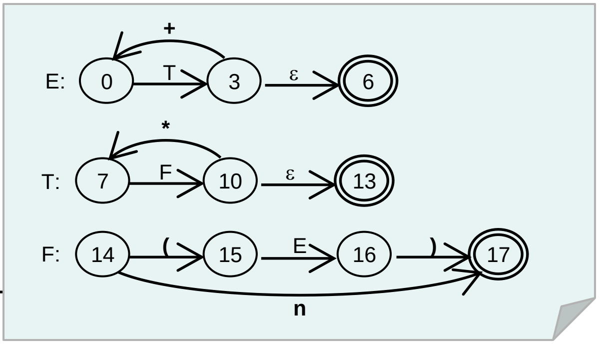



```
void expr() throws SyntacticException {
   term();
   while (lookahead.equals(new Token('+'))) {
      match(new Token('+'));
      term();
   }
}

void term() throws SyntacticException {
   factor();
   while (lookahead.equals(new Token('*'))) {
      match(new Token('*'));
      factor();
   }
}
```

Some tricks are needed while coding with reduced transition diagrams.

Example 2 (cont')




```
void factor() throws SyntacticException {
   if (lookahead.equals(new Token('('))) {
      match(new Token('('));
      expr();
      match(new Token(')'));
   } else if (lookahead.equals(new Token(Token.NUM))) {
      match(new Token(Token.NUM));
   } else {
      throw new SyntacticException();
   }
}
```

Exercise 4.1

- Given the following grammar

$$S \rightarrow (L) \mid \mathbf{a}$$

$$L \rightarrow L, S \mid S$$

  - Eliminate left recursions in the grammar.
  - Draw the transition diagrams for the grammar.
  - Write a recursive descent predictive parser.
  - Indicate the procedure call sequence for an input sentence **`(a, (a, a))`**.

Exercise 4.2

- Consider the context-free grammar

$$S \rightarrow \mathbf{a}\, S\, \mathbf{b}\, S \mid \mathbf{b}\, S\, \mathbf{a}\, S \mid \varepsilon$$

  - Can you construct a predictive parser for the grammar? and why?

Exercise 4.3

- Compute the FIRST and FOLLOW for the start symbol of the following grammar

$$S \rightarrow S\,S + \mid S\,S\,* \mid \mathbf{a}$$

Further Reading

- Dragon Book, 2nd Edition (DBv2)
  - Comprehensive Reading:
    - Section 2.4, 4.1.1–4.1.2 and 4.4.1 for the introduction to top-down parsing.
    - Section 4.2 and 4.3 for context-free grammar and grammar transformations.
    - Section 4.4.2 for function FIRST and FOLLOW.
  - Skip Reading:
    - Section 4.1.3–4.1.4 for error recovery in top-down parsing.

Enjoy the Course!