

算法设计与应用基础：作业 1

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提交说明

- 请将作业以 **PDF** 附件形式发送到邮箱: algo2020@163.com
- 邮件主题及作业文件统一命名: 第几次作业 _ 学号 _ 姓名, 如, 1_18XXXXXX_ 张
- 编程题一般是 OJ 平台 **LeetCode** 上的题目, 点击题名即可跳转到题目对应的页面。对于编程题, 要求在作业中写出四项内容: 算法思路, 复杂度分析, 代码和 Accepted 截图。

作业

1. Show that

$$\log(n!) = \Theta(n \log n)$$

(Hint: To show an upper bound, compare $n!$ with n^n . To show a lower bound, compare it with $(n/2)^{n/2}$.)

2. Compute $\gcd(210, 588)$ two different ways: by finding the factorization of each number, and by using Euclid's algorithm.
3. In the RSA cryptosystem, Alice's public key (N, e) is available to everyone. Suppose that her private key d is compromised and becomes known to Eve. Show that if $e = 3$ (a common choice) then Eve can efficiently factor N .
4. **Length of Longest Fibonacci Subsequence**

A sequence X_1, X_2, \dots, X_n is fibonacci-like if:

- $n \geq 3$
- $X_i + X_{i+1} = X_{i+2}$, for all $i + 2 \leq n$

Given a **strictly increasing** array A of positive integers forming a sequence, find the **length** of the longest fibonacci-like subsequence of A . If one does not exist, return 0.

(Recall that a subsequence is derived from another sequence A by deleting any number of elements (including none) from A , without changing the order of the remaining elements. For example, $[3, 5, 8]$ is a subsequence of $[3, 4, 5, 6, 7, 8]$.)

Example:

Input : [1, 2, 3, 4, 5, 6, 7, 8]

Output : 5

Explanation: The longest subsequence that is fibonacci-like: [1, 2, 3, 5, 8].

5. Insertion Sort List

Sort a linked list using insertion sort.

Algorithm of Insertion Sort:

- (a) Insertion sort iterates, consuming one input element each repetition, and growing a sorted output list.
- (b) At each iteration, insertion sort removes one element from the input data, finds the location it belongs within the sorted list, and inserts it there.
- (c) It repeats until no input elements remain.

Example:

Input: 4- > 2- > 1- > 3

Output: 1- > 2- > 3- > 4

6. Merge k Sorted Lists

Merge k sorted linked lists and return it as one sorted list. Analyze and describe its complexity.

Example:

Input:

[
1- > 4- > 5,
1- > 3- > 4,
2- > 6
]

Output: 1- > 1- > 2- > 3- > 4- > 4- > 5- > 6