The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Science A Physics

Lectures 1-3:

Answers to Additional Problems: Describing Motion, Kinematics and Causing Motion

General Problem Solving Skills

Problem Solving

MODEL Make simplifying assumptions.

VISUALIZE Use:

- **Pictorial representation**
- **Graphical representation**

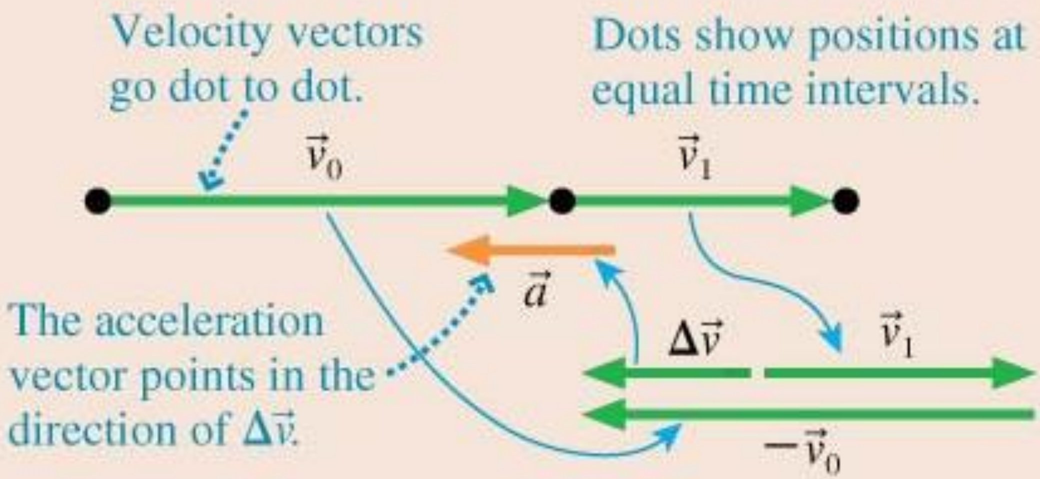
SOLVE Use a **mathematical representation** to find numerical answers.

ASSESS Does the answer have the proper units? Does it make sense?

General Problem Solving Skills

Motion Diagrams

- Help visualize motion.
- Provide a tool for finding acceleration vectors.



▶ These are the average velocity and the average acceleration vectors.

General Problem Solving Skills

The **particle model** represents a moving object as if all its mass were concentrated at a single point.

Position locates an object with respect to a chosen coordinate system. Change in position is called displacement.

Velocity is the rate of change of the position vector $\vec{r}$.

Acceleration is the rate of change of the velocity vector $\vec{v}$.

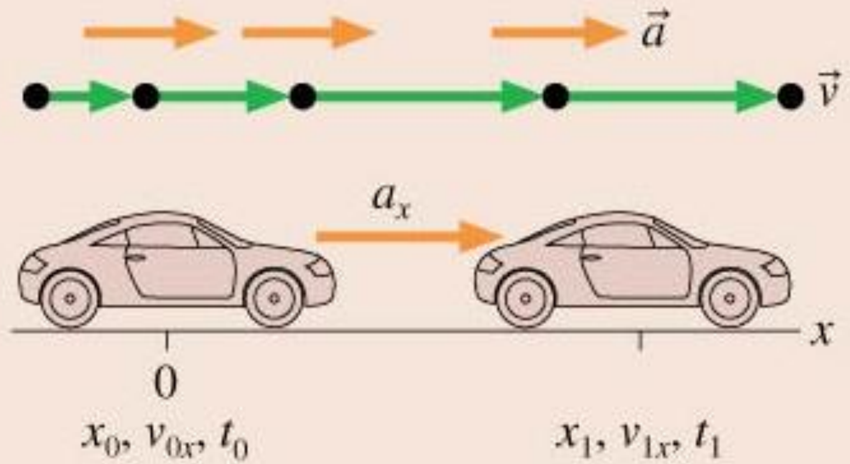
An object has an acceleration if it

- Changes speed and/or
- Changes direction.

General Problem Solving Skills

Pictorial Representation

1. Draw a motion diagram.
2. Establish coordinates.
3. Sketch the situation.
4. Define symbols.
5. List knowns.
6. Identify desired unknown.



Known

$x_0 = v_{0x} = t_0 = 0$

$a_x = 2.0\ \text{m/s}^2 \quad t_1 = 2.0\ \text{s}$

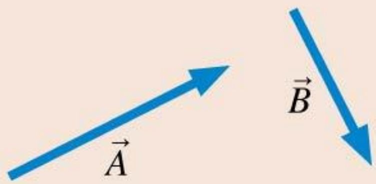
Find

x_1

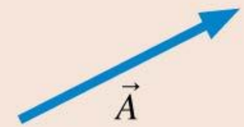
Vector Addition

TACTICS BOX 1.1 **Vector addition**

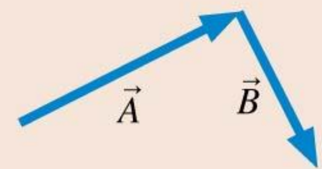
To add $\vec{B}$ to $\vec{A}$:



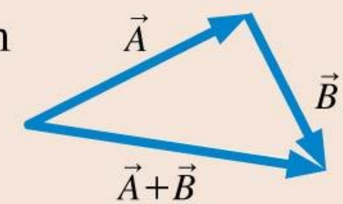
1. Draw $\vec{A}$.



2. Place the tail of $\vec{B}$ at the tip of $\vec{A}$.



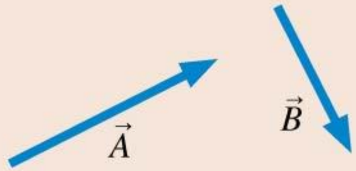
3. Draw an arrow from the tail of $\vec{A}$ to the tip of $\vec{B}$. This is vector $\vec{A} + \vec{B}$.



Vector Subtraction

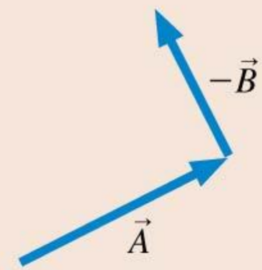
TACTICS BOX 1.2 **Vector subtraction**

To subtract $\vec{B}$ from $\vec{A}$:

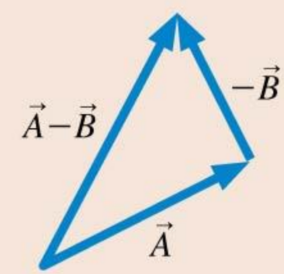


1. Draw $\vec{A}$.

2. Place the tail of $-\vec{B}$ at the tip of $\vec{A}$.



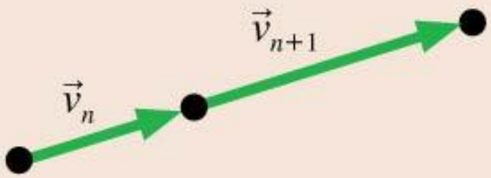
3. Draw an arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. This is vector $\vec{A} - \vec{B}$.



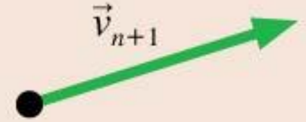
The Acceleration Vector

TACTICS BOX 1.3 **Finding the acceleration vector**

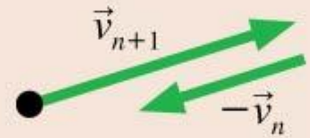
To find the acceleration as the velocity changes from $\vec{v}_n$ to $\vec{v}_{n+1}$, we must determine the *change* of velocity $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$.



1. Draw the velocity vector $\vec{v}_{n+1}$.

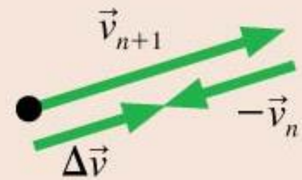


2. Draw $-\vec{v}_n$ at the tip of $\vec{v}_{n+1}$.

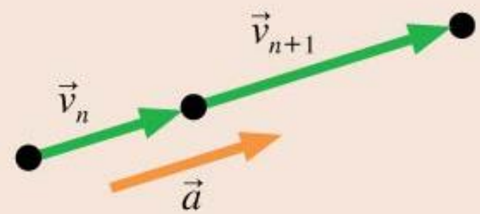


The Acceleration Vector

❸ Draw $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$
$= \vec{v}_{n+1} + (-\vec{v}_n)$
This is the direction of $\vec{a}$.



❹ Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_n$ and $\vec{v}_{n+1}$.



- Notice that the acceleration vectors goes beside the dots, not beside the velocity vectors.
- That is because each acceleration vector is the difference between two velocity vectors on either side of a dot.

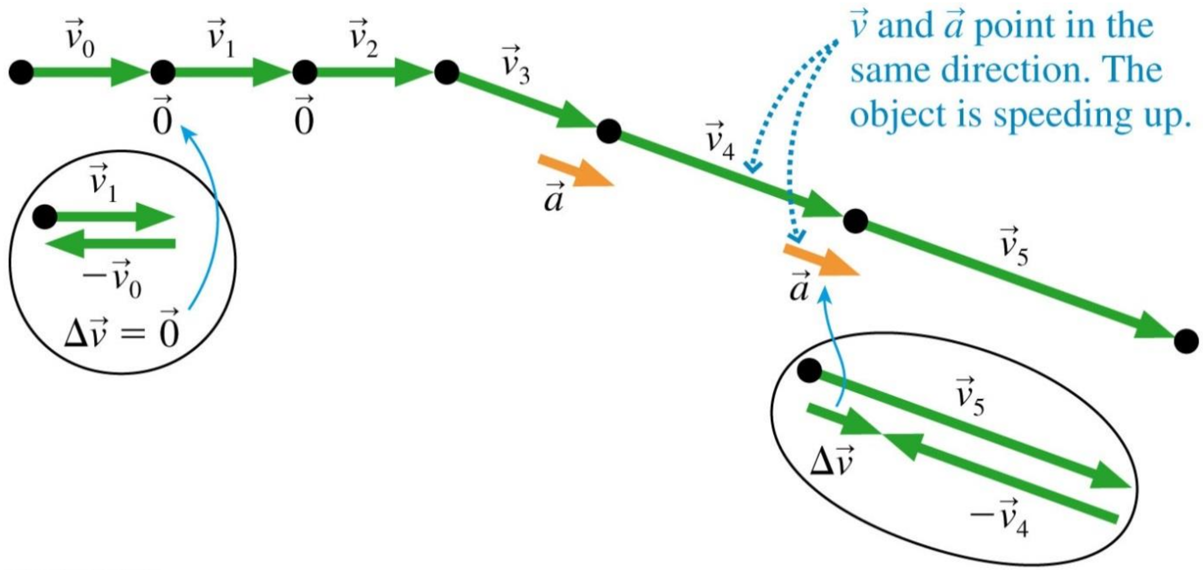
Skiing through the Woods

Q.1 A skier glides along smooth, horizontal snow at constant speed, then speeds up going down a hill. Draw the skier's motion diagram.

Model:

Represent the skier as a particle. It's reasonable to assume that the downhill slope is a straight line. Although the motion as a whole is not linear, we can treat the skier's motion as two separate linear motions.

Skiing through the Woods



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Model:

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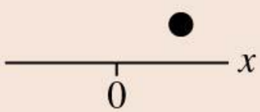
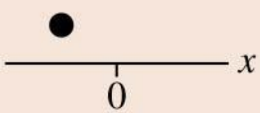
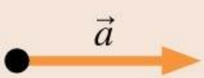
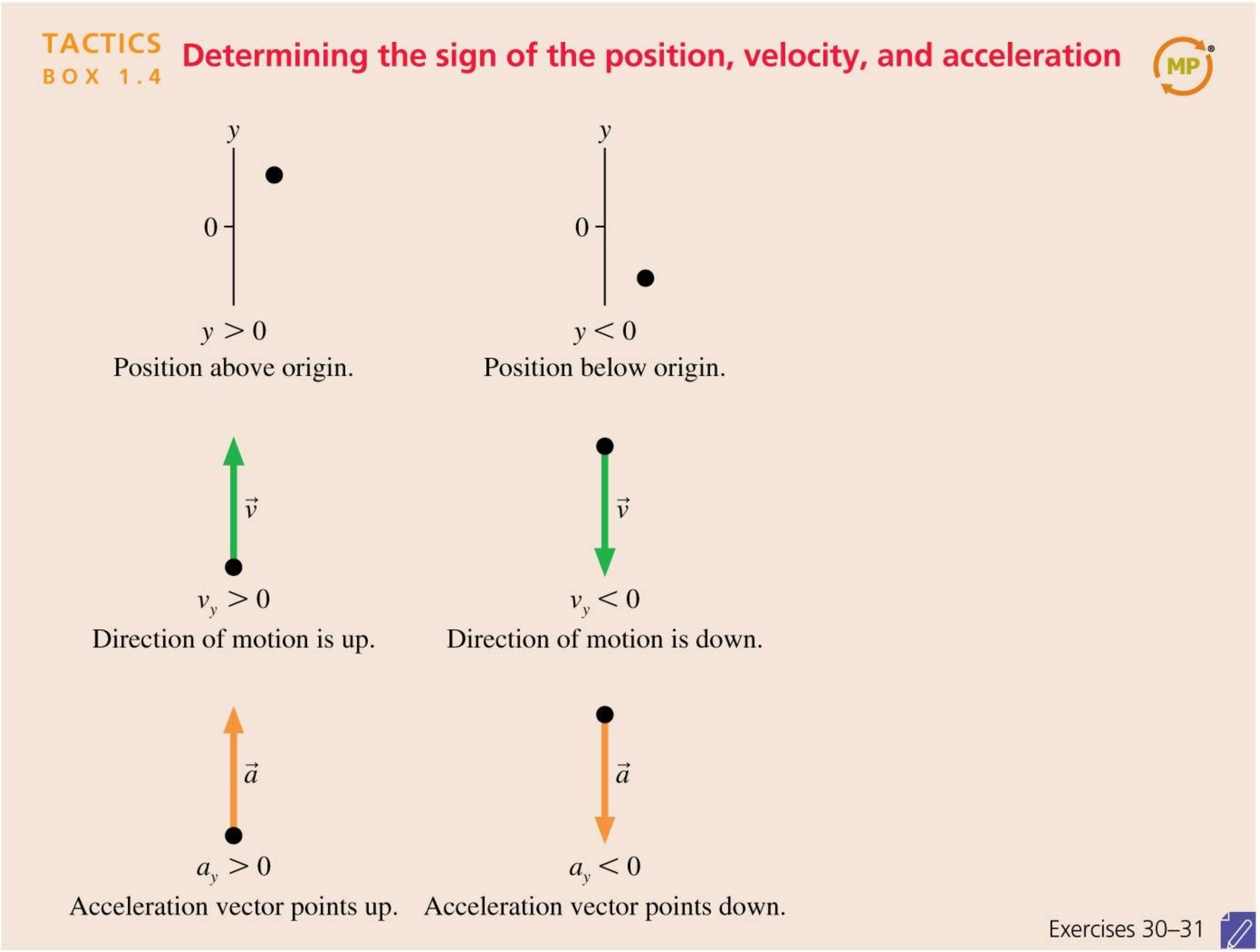
Determining the Sign of the Position, Velocity and Acceleration

TACTICS BOX 1.4 **Determining the sign of the position, velocity, and acceleration**

	$x > 0$	Position to right of origin.
	$x < 0$	Position to left of origin.
	$v_x > 0$	Direction of motion is to the right.
	$v_x < 0$	Direction of motion is to the left.
	$a_x > 0$	Acceleration vector points to the right.
	$a_x < 0$	Acceleration vector points to the left.

Exercises 30–31

Determining the Sign of the Position, Velocity and Acceleration



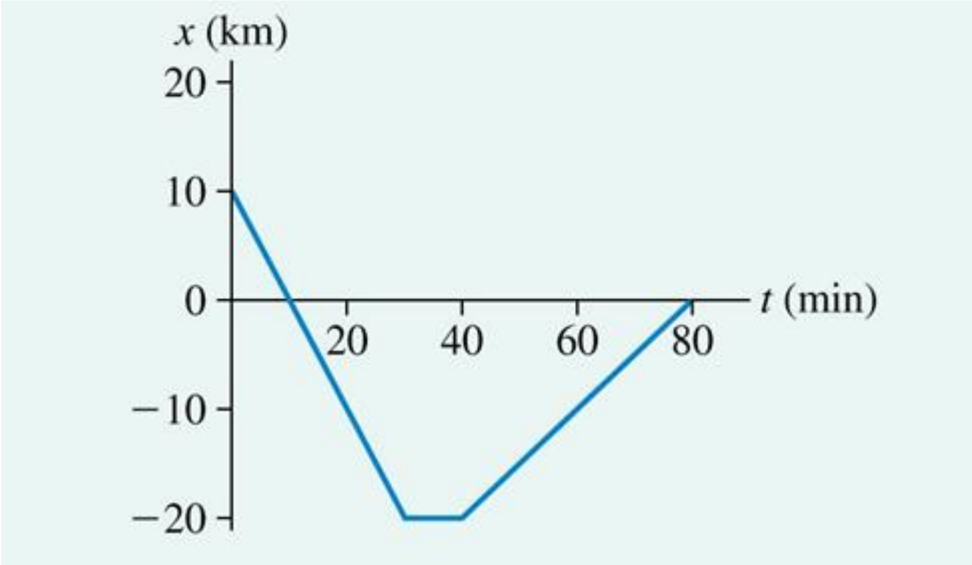
Determining the Sign of the Position, Velocity and Acceleration

TACTICS BOX 1.4 **Determining the sign of the position, velocity, and acceleration**

- The sign of position (x or y) tells us *where* an object is.
- The sign of velocity (v_x or v_y) tells us *which direction* the object is moving.
- The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, *not* whether the object is speeding up or slowing down.

Exercises 30–31

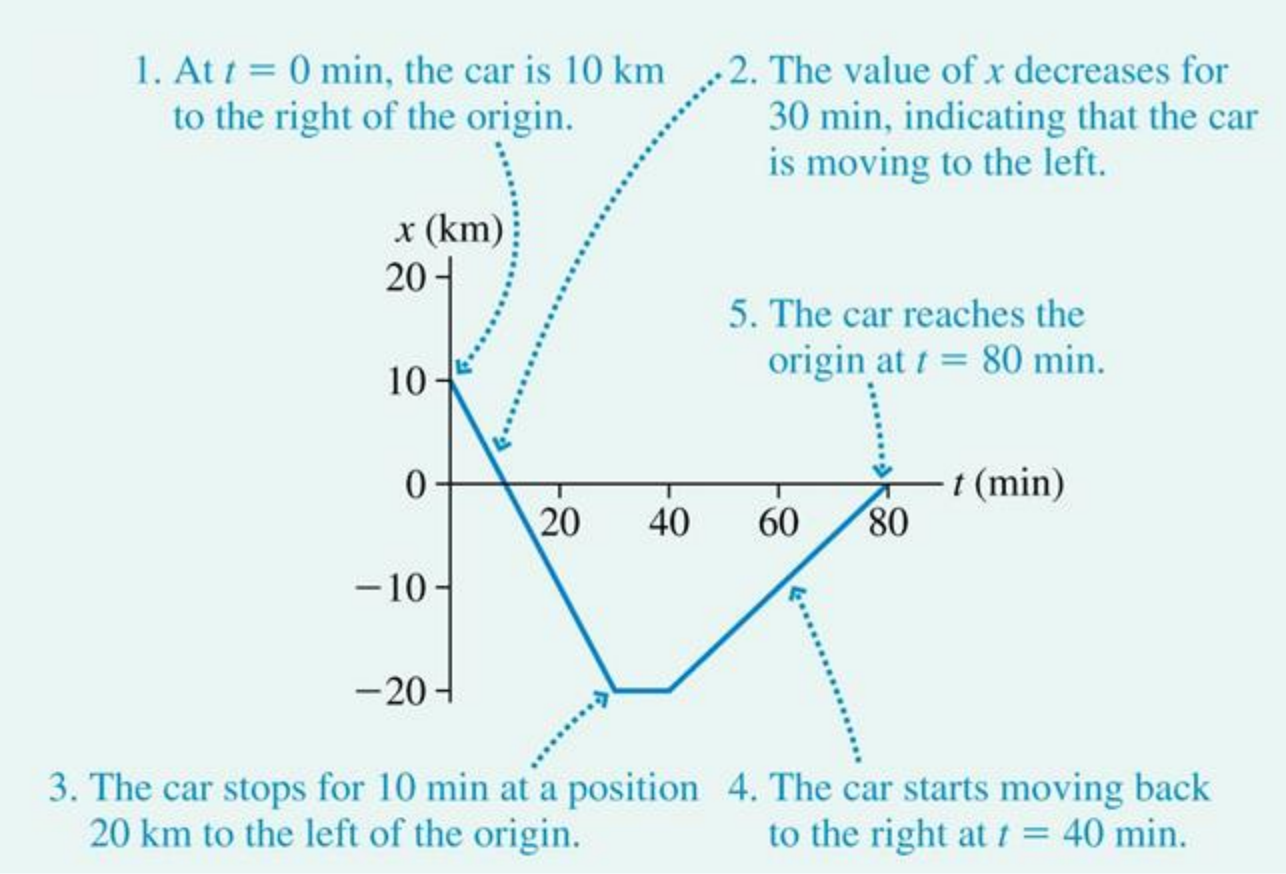
Interpreting a Position Graph



Q.2 The graph in the figure represents the motion of a car along a straight road. Describe the motion of the car.

MODEL: Represent the car as a particle.

Interpreting a Position Graph



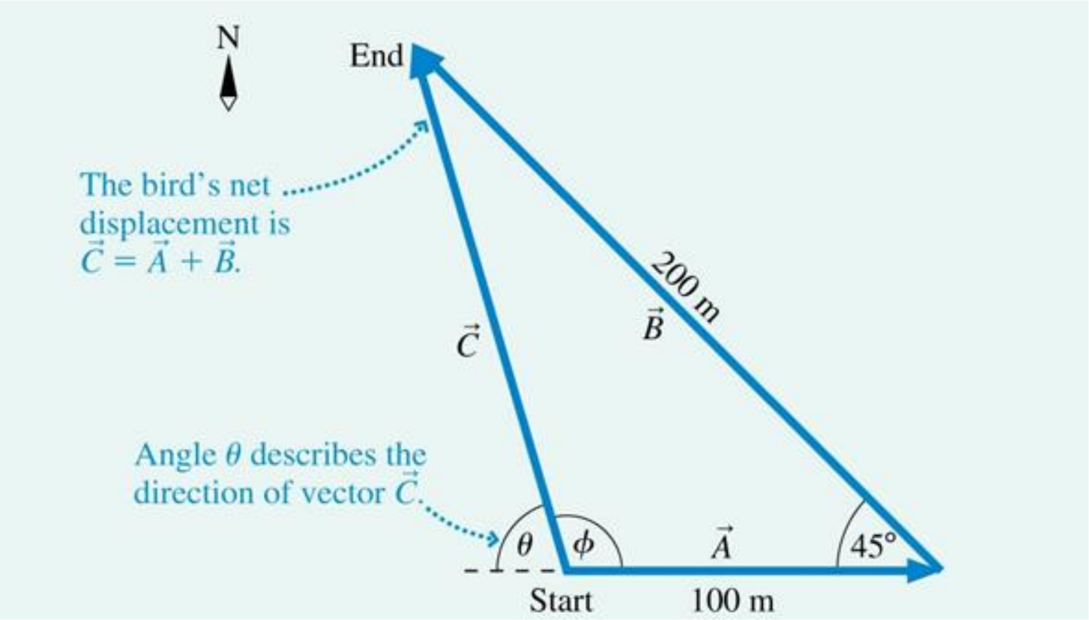
VISUALISE:

As the figure shows, the graph represents a car that travels to the left for 30 minutes, stops for 10 minutes, then travels back to the right for 40 minutes.

Using Graphical Addition to Find a Displacement

Q.3 A bird flies 100 m due east from a tree, then 200 m northwest (that is, $45°$ north of west).
What is the bird's net displacement?

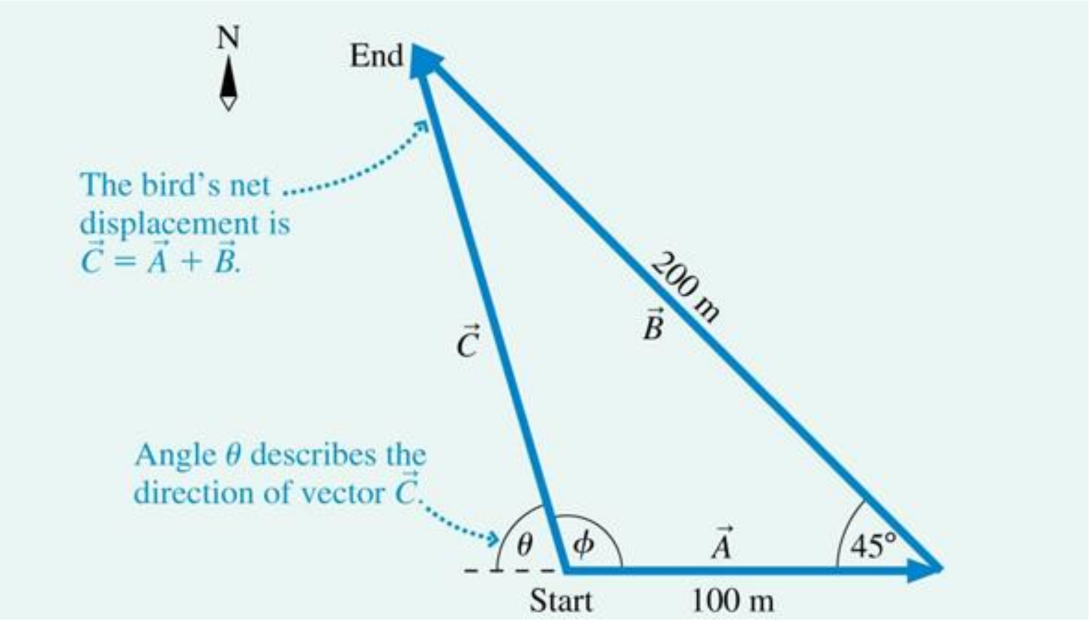
Using Graphical Addition to Find a Displacement



VISUALISE:

The figure above shows the two individual displacements, which we've called $\vec{A}$ and $\vec{B}$. The net displacement is the vector sum $\vec{C} = \vec{A} + \vec{B}$, which is found graphically.

Using Graphical Addition to Find a Displacement



VISUALISE:

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Using Graphical Addition to Find a Displacement

SOLVE :

The two displacements are $\vec{A}$ = (100 m, east) and $\vec{B}$ = (200 m, northwest). The net displacement $\vec{C} = \vec{A} + \vec{B}$ is found by drawing a vector from the initial to the final position.

We can find the magnitude of $\vec{C}$ by using the law of cosines from trigonometry:

$$C^2 = A^2 + B^2 - 2AB\cos 45°$$
$$= (100\text{ m})^2 + (200\text{ m})^2 - 2(100\text{ m})(200\text{ m})\cos 45°$$
$$= 21{,}720\text{ m}^2$$

Thus, $C = \sqrt{21{,}720\text{ m}^2} = 147$ m. Then a second use of the law of cosines can determine angle φ (the Greek letter phi):

$$B^2 = A^2 + C^2 - 2AC\cos\varphi$$
$$\varphi = \cos^{-1}[(A^2 + C^2 - B^2)/(2AC)] = 106°$$

It is easier to describe $\vec{C}$ with the angle Θ = 180° – φ = 74°.
The bird's net displacement is

$$\vec{C} = (147\text{ m}, 74° \text{ north of west})$$

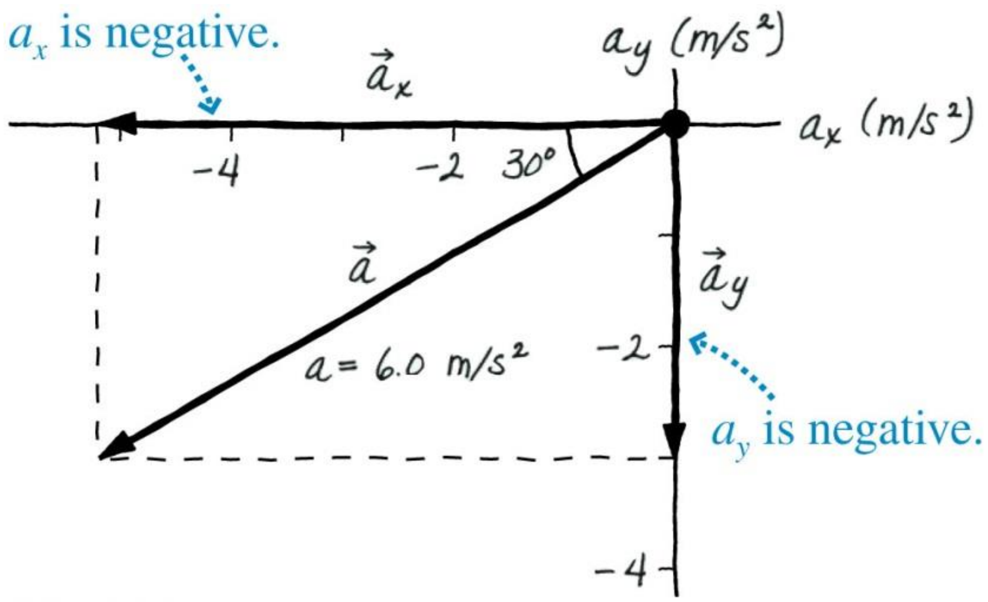
Vector Components

TACTICS BOX 3.1 **Determining the components of a vector**

1. The absolute value $|A_x|$ of the x-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The *sign* of A_x is positive if $\vec{A}_x$ points in the positive x-direction, negative if $\vec{A}_x$ points in the negative x-direction.
3. The y-component A_y is determined similarly.

Exercises 10–18

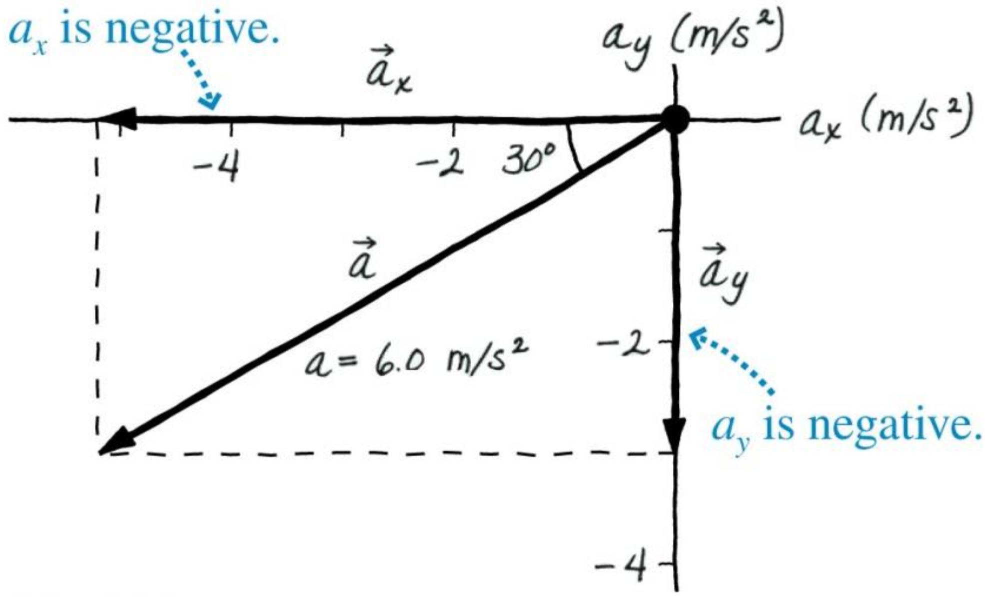
Finding the Components of an Acceleration Vector



VISUALISE:

It's important to be able to decompose vectors. The figure above shows the original vector $\vec{a}$ decomposed into components parallel to the axes. Notice that the axes are 'acceleration axes', not xy-axes, because we're measuring an acceleration vector.

Finding the Components of an Acceleration Vector



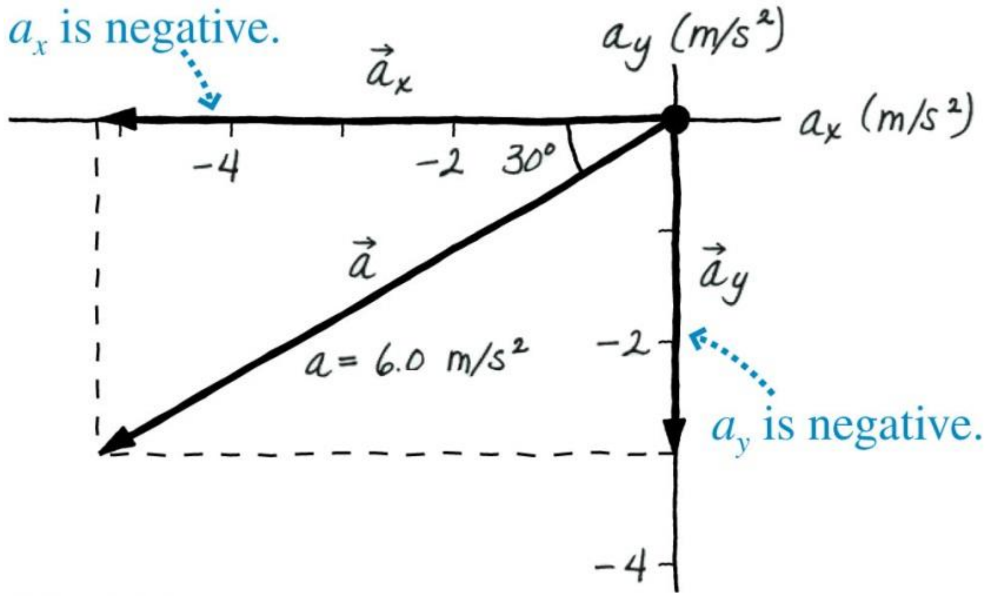
SOLVE:

The acceleration vector $\vec{a}$ = (6.0 m/s^2, 30° below the negative x-axis) points to the left (negative x-direction) and down (negative y-direction), so the components a_x and a_y are both negative:

$$a_x = -a\cos 30° = -(6.0 \text{ m/s}^2)\cos 30° = -5.2 \text{ m/s}^2$$

$$a_y = -a\sin 30° = -(6.0 \text{ m/s}^2)\sin 30° = -3.0 \text{ m/s}^2$$

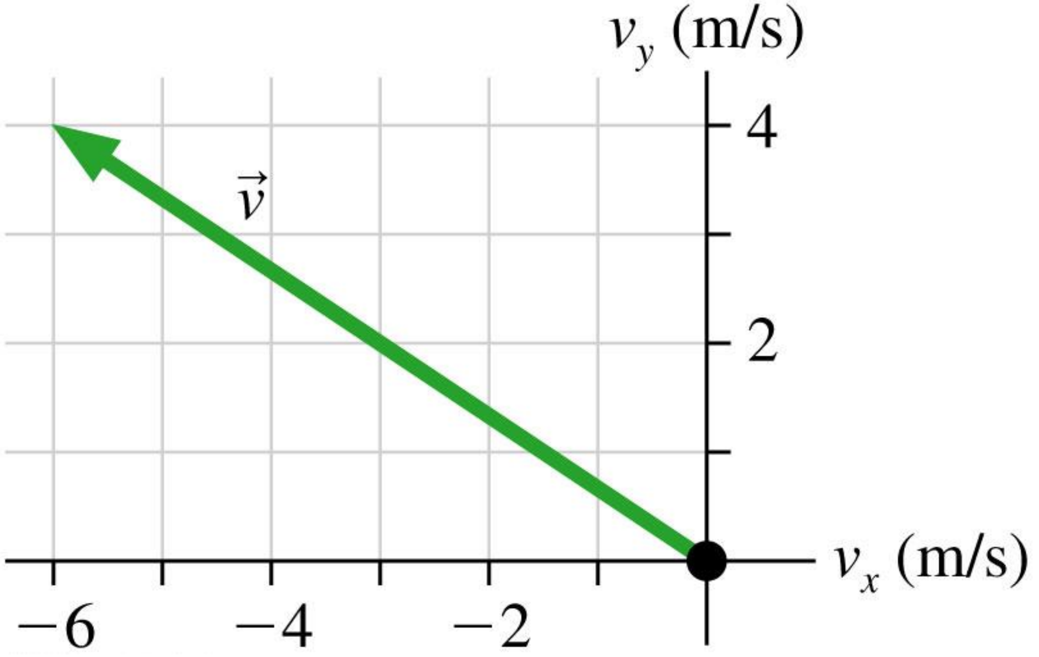
Finding the Components of an Acceleration Vector



SOLVE:

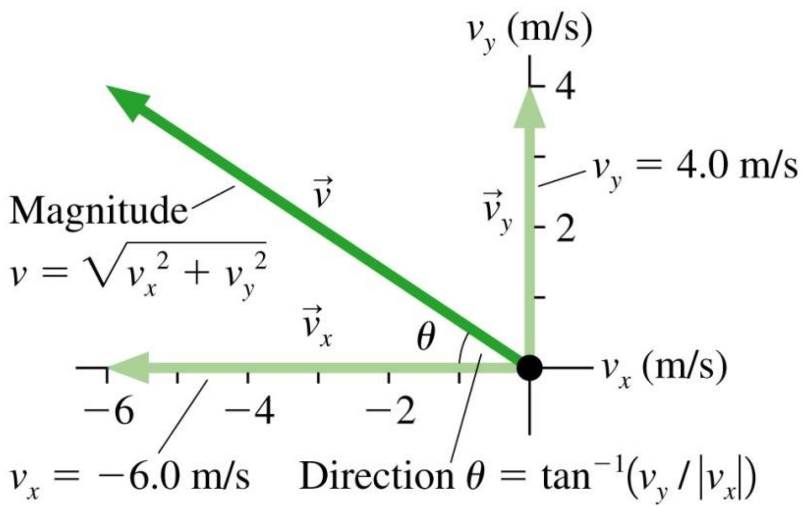
The units of a_x and a_y are the same as the units of vector $\vec{a}$. Notice that we had to insert the minus signs manually by observing that the vector points left and down.

Finding the Direction of Motion



Q.4 The figure above shows a car's velocity vector $\vec{v}$. Determine the car's speed and direction of motion.

Finding the Direction of Motion



VISUALISE:

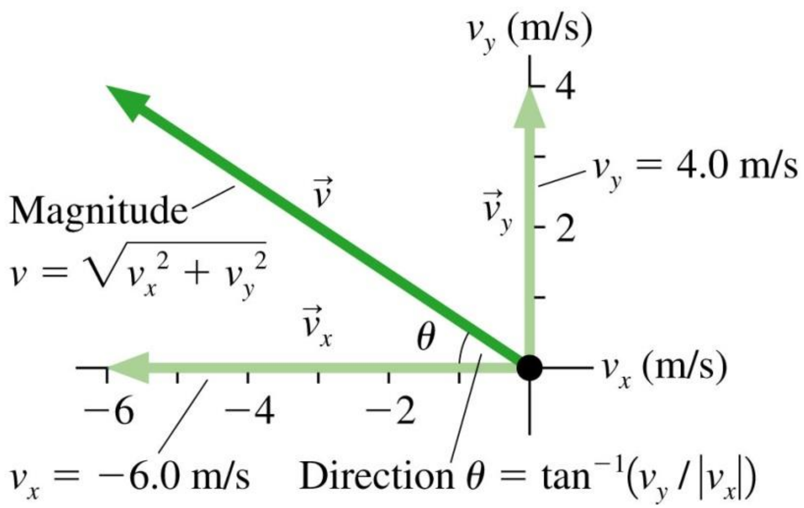
The figure above shows the components v_x and v_y and defines an angle Θ with which we can use to specify the direction of motion.

SOLVE:

We can read the components of $\vec{v}$ directly from the axes: v_x = -6.0 m/s and v_y = 4.0 m/s. Notice that v_x is negative. This is enough information to find the car's speed v, which is the magnitude of $\vec{v}$:

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(-6.0 \text{ m/s})^2 + (4.0 \text{ m/s})^2} = 7.2 \text{ m/s}$$

Finding the Direction of Motion



From trigonometry, angle Θ is

$$\theta = \tan^{-1}\left(\frac{v_y}{|v_x|}\right) = \tan^{-1}\left(\frac{4.0\ \mathrm{m/s}}{6.0\ \mathrm{m/s}}\right) = 34°$$

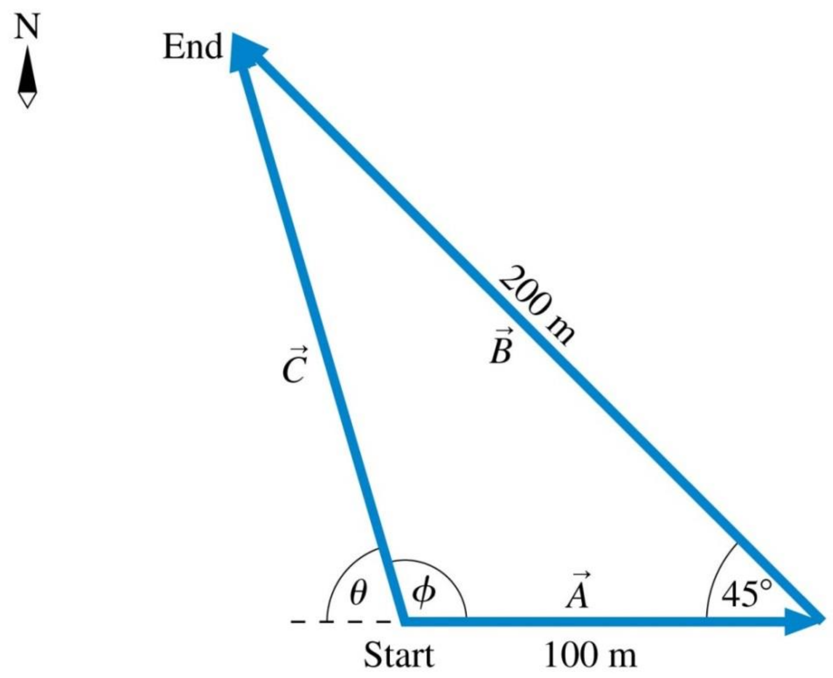
The absolute value signs are necessary because v_x is a negative number.
The velocity vector $\vec{v}$ can be written in terms of the speed and the direction of motion as

$\vec{v}$ = (7.2 m/s, 34° above the negative x-axis)

Or, if the axes are aligned to north,

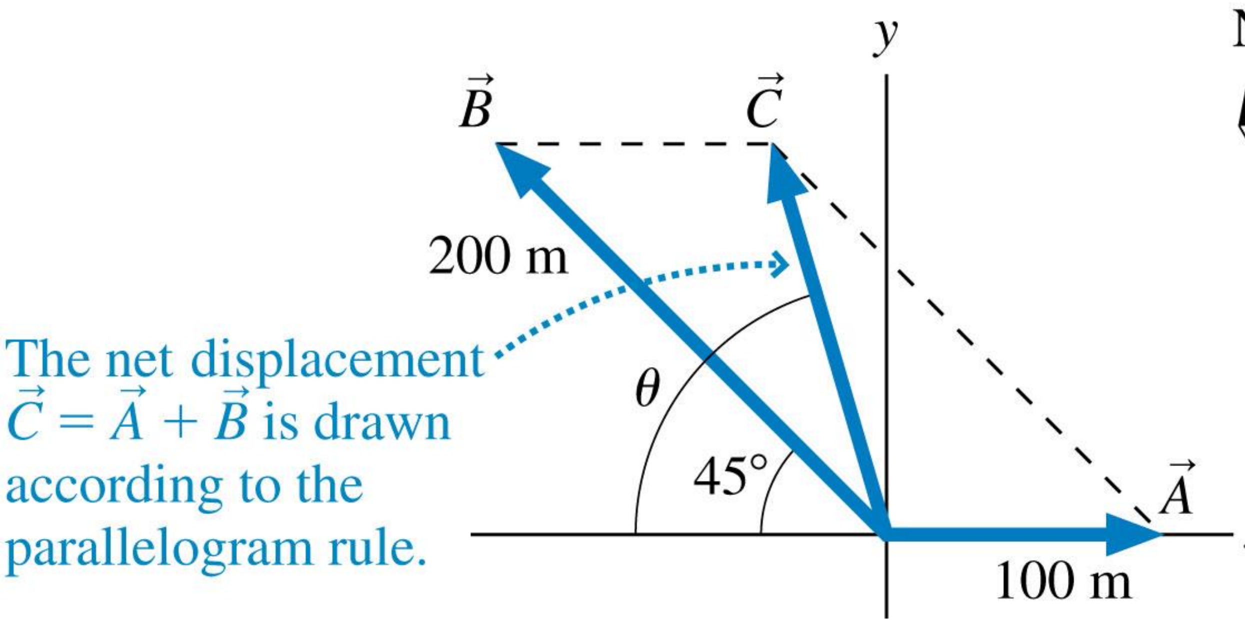
$\vec{v}$ = (7.2 m/s, 34° north of west)

Using Algebraic Addition to Find a Displacement



Q.5 In question 3, we considered a bird that flew 100 m to the east, and then 200 m to the northwest. In this question, we use the algebraic addition of vectors to find the bird's net displacement.

Using Algebraic Addition to Find a Displacement

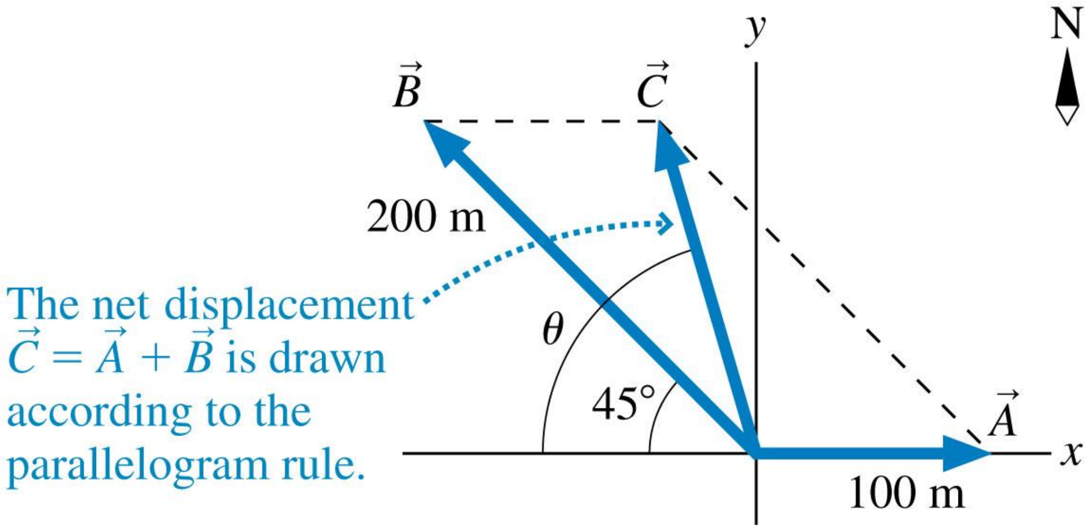


VISUALISE:

The figure above shows displacement vectors $\vec{A}$ = (100 m, east) and $\vec{B}$ = (200 m, northwest).

We draw vectors tip-to-tail to add them graphically, but it's usually easier to draw them all from the origin if we are going to use algebraic addition.

Using Algebraic Addition to Find a Displacement



SOLVE:

To add the vectors algebraically, we must know their components. From the figure these are seen to be

$$\vec{A} = 100\hat{\imath}\text{ m}$$

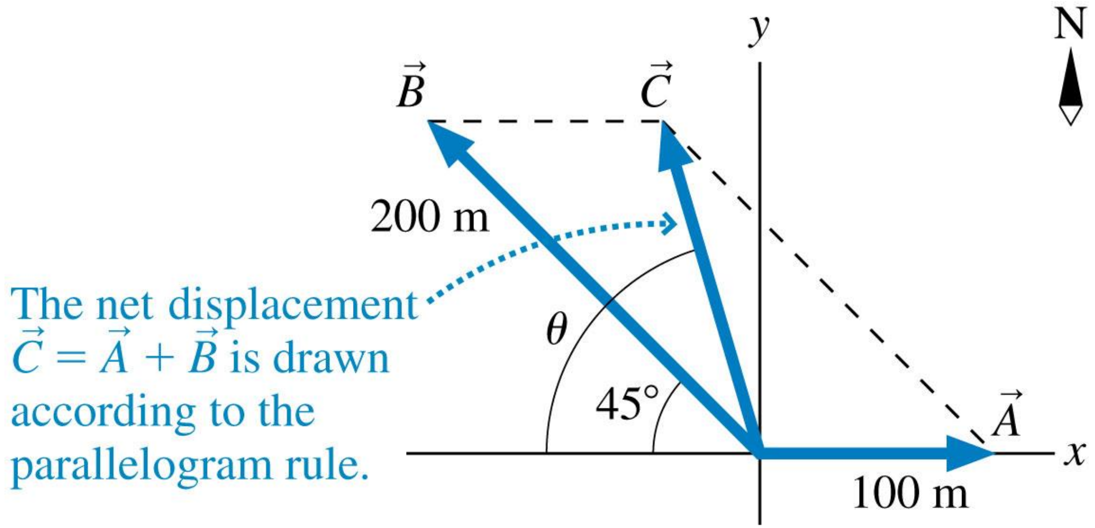
$$\vec{B} = (-200\cos 45°\hat{\imath} + 200\sin 45°\hat{\jmath})\text{m} = (-141\hat{\imath} + 141\hat{\jmath})\text{m}$$

Notice that vector quantities must include units. Adding $\vec{A}$ and $\vec{B}$ by components gives

$$\vec{C} = \vec{A} + \vec{B} = 100\hat{\imath}\text{ m} + (-141\hat{\imath} + 141\hat{\jmath})\text{m}$$

$$= (100\text{ m} - 141\text{m})\hat{\imath} + (141\text{m})\hat{\jmath} = (-41\hat{\imath} + 141\hat{\jmath})\text{m}$$

Using Algebraic Addition to Find a Displacement



SOLVE:

However, we need to calculate the magnitude and direction of $\vec{C}$ if we want to compare this result to our earlier answer. The magnitude of $\vec{C}$ is

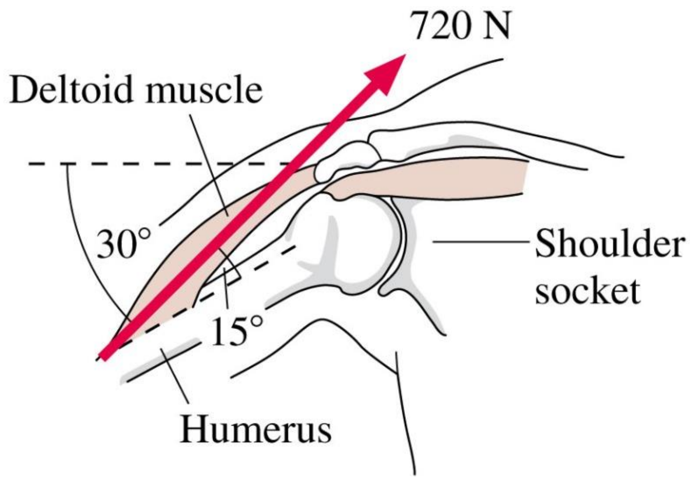
$$C = \sqrt{{C_x}^2 + {C_y}^2} = \sqrt{(-41.0\text{ m/s})^2 + (141.0\text{ m/s})^2} = 147\text{ m/s}$$

The angle Θ is

$$\theta = \tan^{-1}\left(\frac{C_y}{|C_x|}\right) = \tan^{-1}\left(\frac{141.0\text{ m/s}}{41.0\text{ m/s}}\right) = 74°$$

Thus $\vec{C}$= (147 m, 74° north of west).

Muscle and Bone



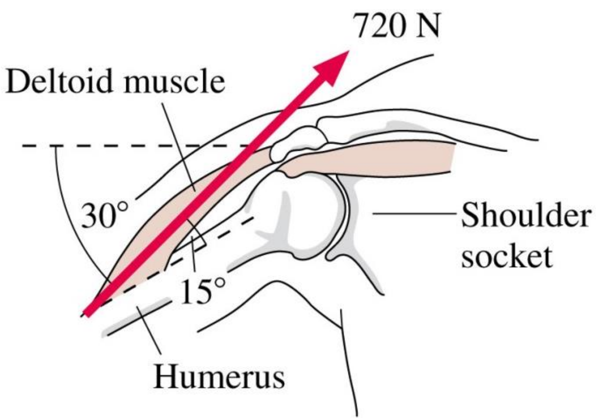
Q.6

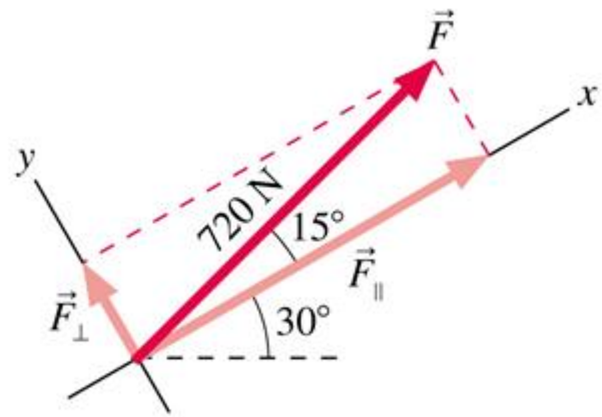
The deltoid – the round muscle across the top of your upper arm – allows you to lift your arm away from your side. It does so by pulling on an attachment point on the humerus, the upper arm bone, at an angle of 15° with respect to the humerus.

If you hold your arm at an angle of 30° below the horizontal, the deltoid must pull with a force of 720 N to support the weight of your arm, as shown in the figure.

What are the components of the muscle force parallel to and perpendicular to the bone?

Muscle and Bone



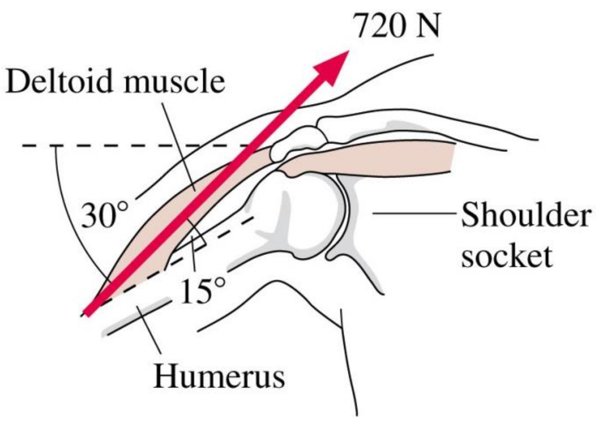


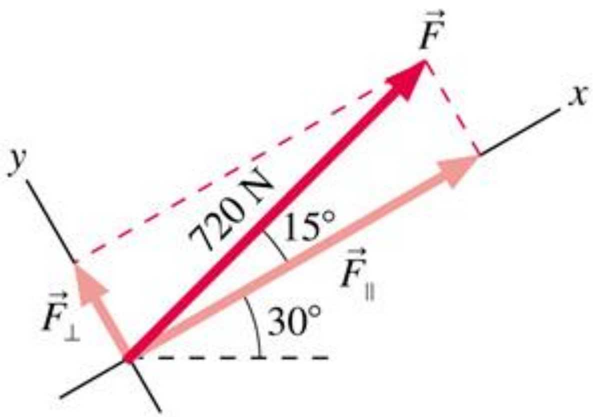
Q.6

Figure (b) shows a tilted coordinate system with the *x*-axis parallel to the humerus. The force $\vec{F}$ is shown from the *x*-axis.

The component of force parallel to the bone, which we can denote $F_{\parallel}$, is equivalent to the *x*-component: $F_{\parallel} = F_x$. Similarly, the component of force perpendicular to the bone is $F_{\perp} = F_y$.

Muscle and Bone





Q.6

SOLVE:

From the geometry of Figure b, we can see that

$F_{\parallel} = F\cos 15^\circ = (720\text{ N})\cos 15^\circ = 695\text{ N}$

$F_{\perp} = F\sin 15^\circ = (720\text{ N})\sin 15^\circ = 186\text{ N}$

ASSESS:

The muscle pulls nearly parallel to the bone, so we expected $F_{\parallel} \approx$ 720 N and $F_{\perp} << F_{\parallel}$. Thus our results seem reasonable.

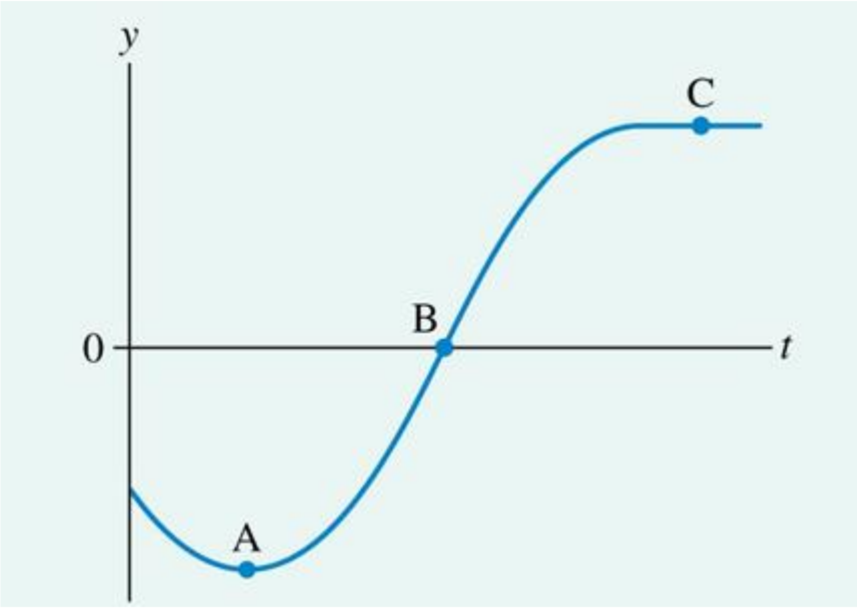
TACTICS BOX 2.1

Interpreting position-versus-time graphs

1. Steeper slopes correspond to faster speeds.
2. Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).
3. The slope is a ratio of intervals, $\Delta x/\Delta t$, not a ratio of coordinates. That is, the slope is *not* simply x/t.
4. We are distinguishing between the *actual* slope and the *physically meaningful* slope. If you were to use a ruler to measure the rise and the run of the graph, you could compute the actual slope of the line as drawn on the page. That is not the slope to which we are referring when we equate the velocity with the slope of the line. Instead, we find the *physically meaningful* slope by measuring the rise and run using the scales along the axes. The "rise" Δx is some number of meters; the "run" Δt is some number of seconds. The physically meaningful rise and run include units, and the ratio of these units gives the units of the slope.

Exercises 1–3

Finding Velocity from Position Graphically



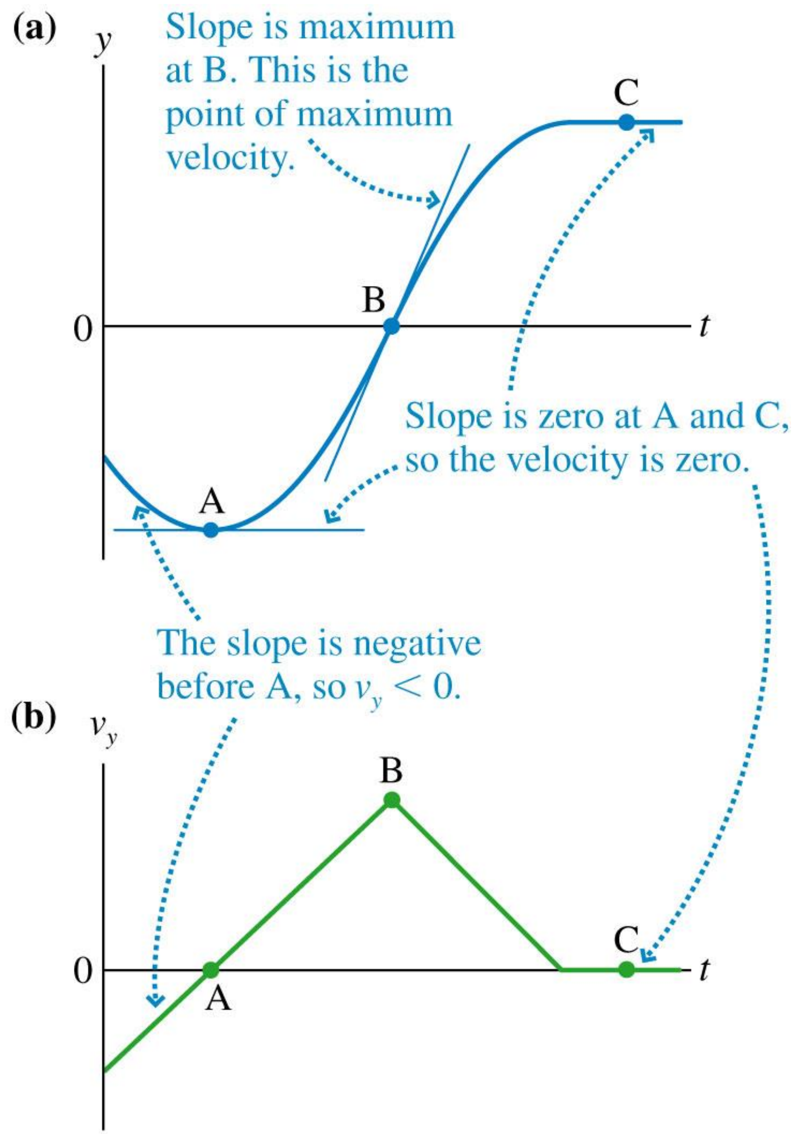
Q.7 The figure shows the position-versus-time graph of an elevator.

a. At which labeled point or points does the elevator have the least speed?

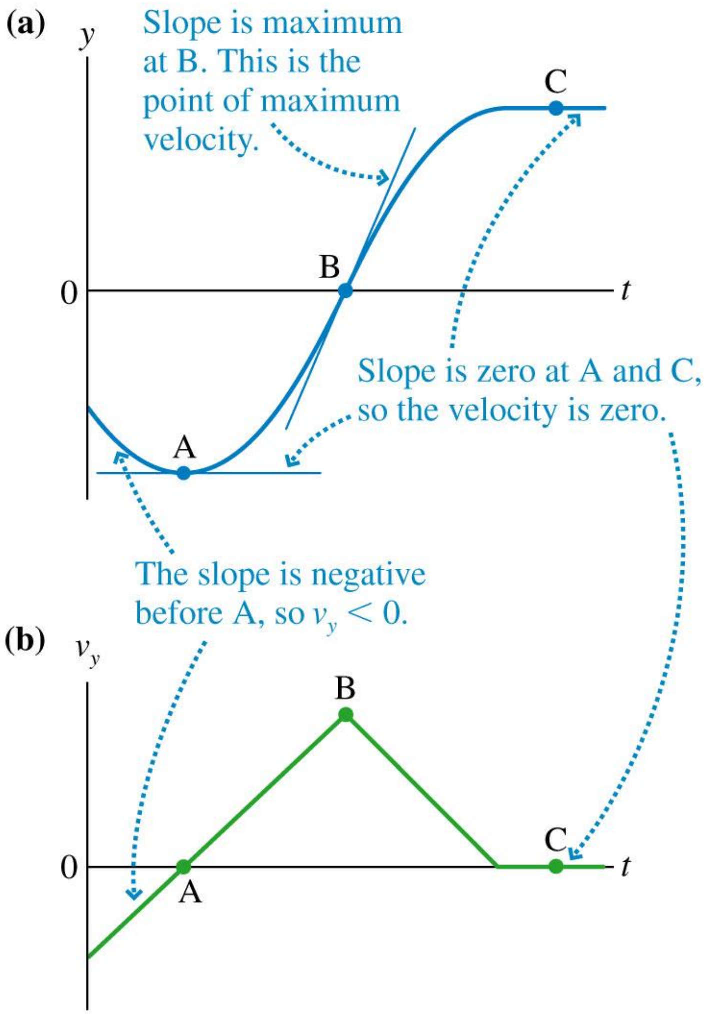
b. At which point or points does the elevator have the maximum velocity?

c. Sketch an approximate position-versus-time graph for the elevator.

Finding Velocity from Position Graphically



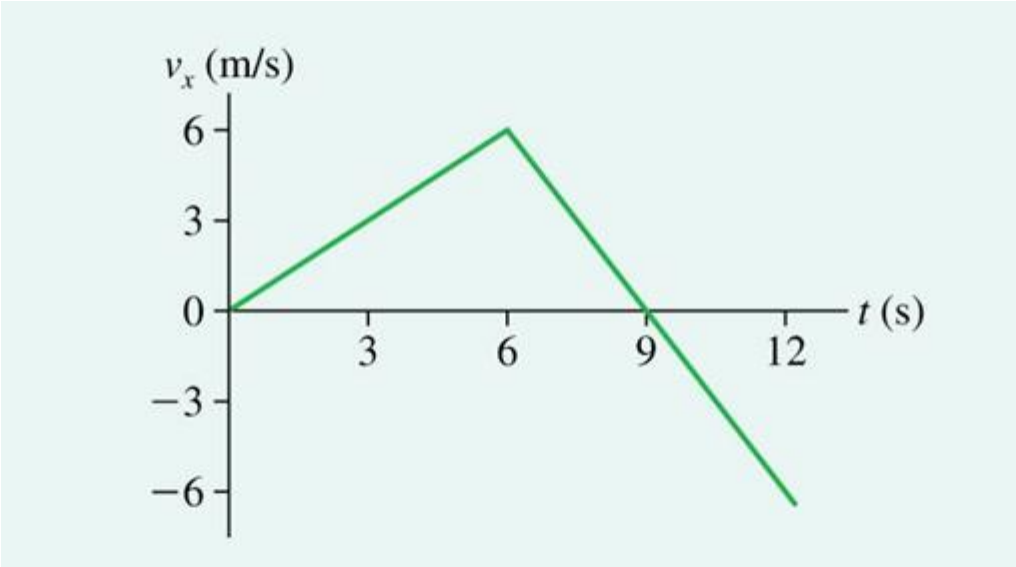
Finding Velocity from Position Graphically



b. The elevator has the maximum velocity at point B.

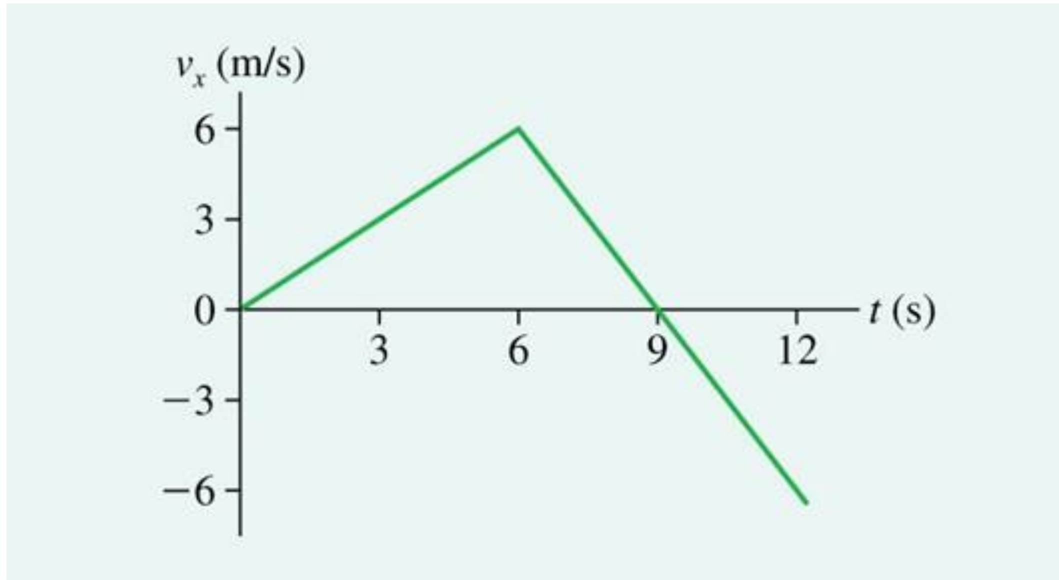
c. Although we cannot find an exact velocity-versus-time graph, we can see that the slope, and hence v_y, is initially negative, becomes zero at point A, rises to a maximum value at point B, decreases back to zero a little before point C, then remains at zero thereafter. Thus, part b of the figure shows, at least approximately, the elevator's velocity-versus-time graph.

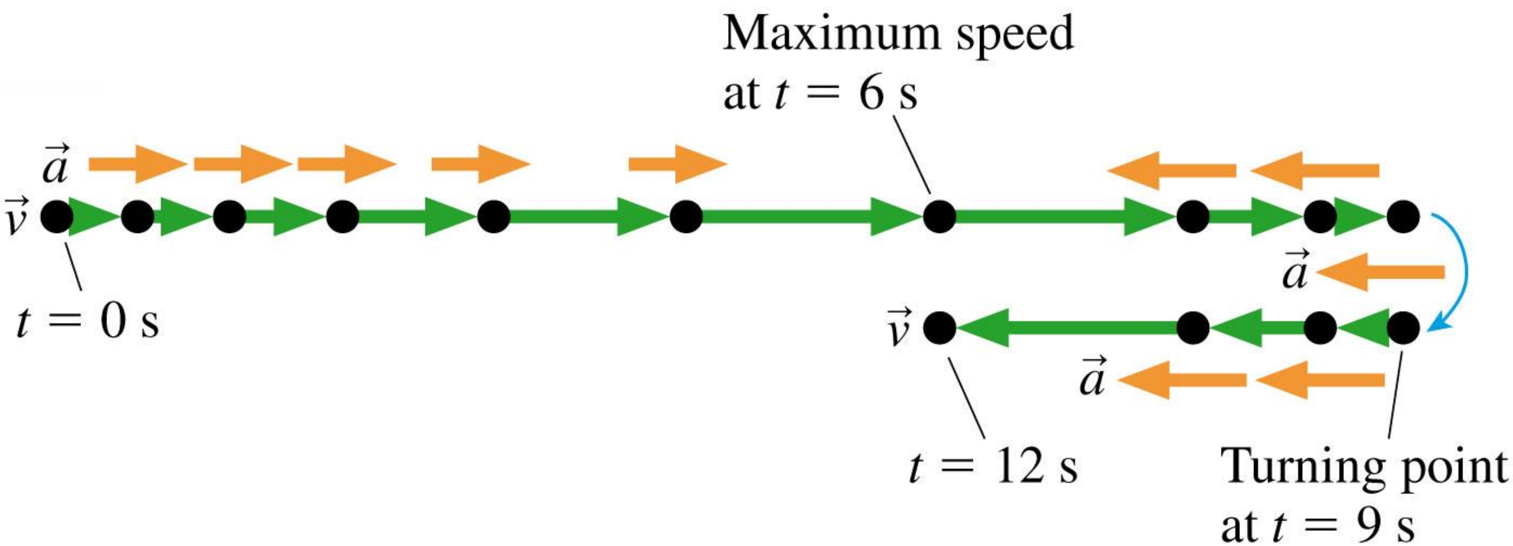
Running the Court



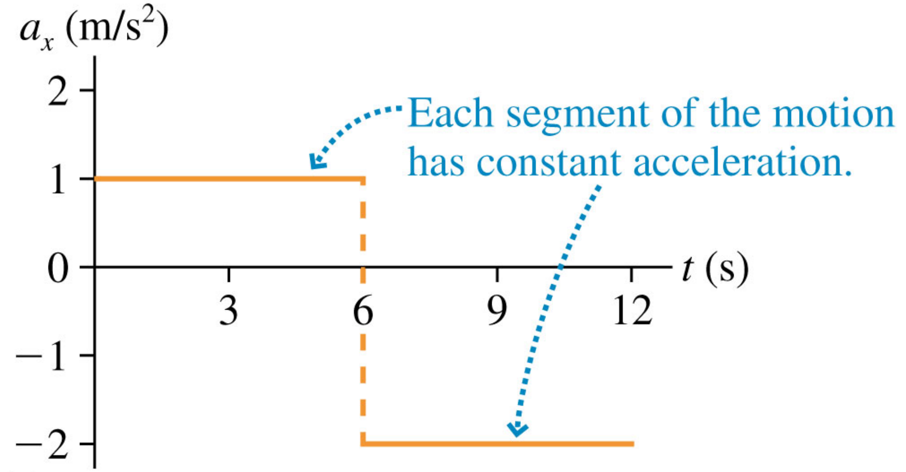
Q.8 A basket player starts at the left end of the court and moves with the velocity shown in the figure below. Draw a motion diagram and an acceleration-versus-time graph for the basket player.

Running the Court





Running the Court



Acceleration is the slope of the velocity graph. For the first 6 s, the slope has a constant value

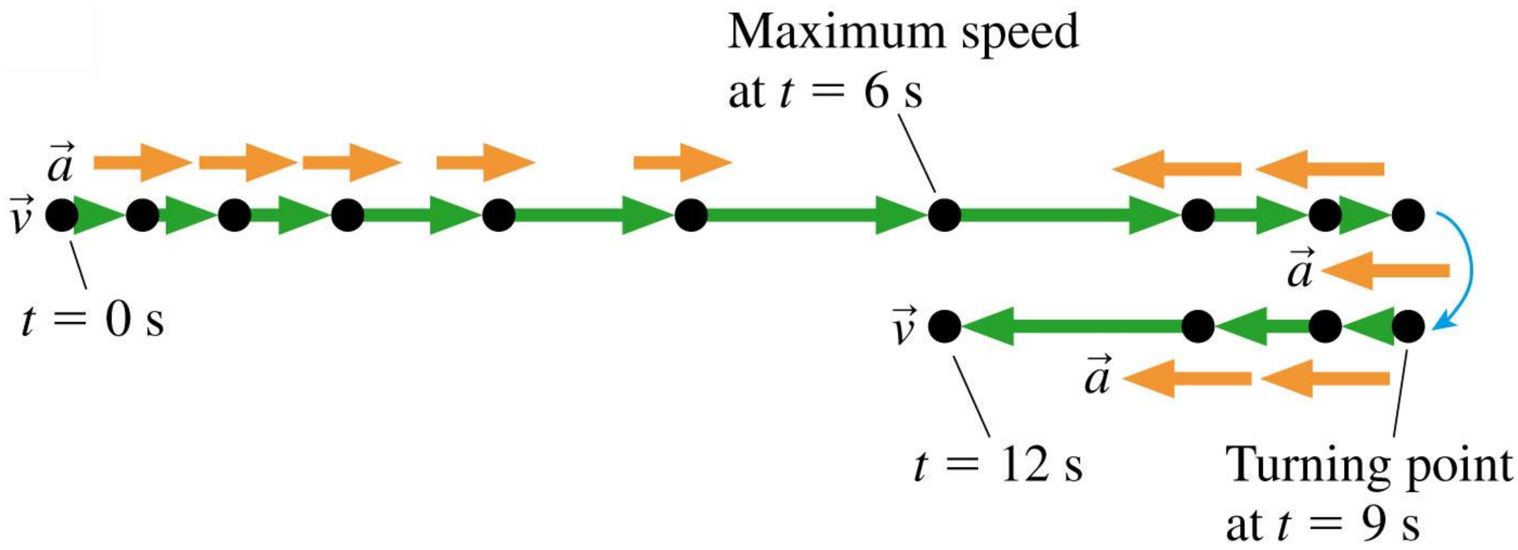
$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{6.0 \text{ m/s}}{6.0 \text{ s}} = 1.0 \text{ m/s}^2$$

The velocity then decreases by 12 m/s during the 6 s interval from t = 6 s to t = 12 s, so

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{-12.0 \text{ m/s}}{6.0 \text{ s}} = -2.0 \text{ m/s}^2$$

The acceleration graph for these 12 s is shown in the figure above. Notice that there is no change in the acceleration at t = 9 s, the turn point.

Running the Court



The **sign of a_x** does **not tell** us whether the object is speeding up or slowing down. The basketball player is slowing down from *t* = 6 s to *t* = 9 s, then speeding up from *t* = 6 s to *t* = 12 s. Nonetheless, his acceleration is negative during this entire interval because his acceleration vector, as seen in the motion diagram, always point to the left.

PROBLEM-SOLVING STRATEGY 2.1

Kinematics with constant acceleration

MODEL Use the particle model. Make simplifying assumptions.

VISUALIZE Use different representations of the information in the problem.

- Draw a *pictorial representation.* This helps you assess the information you are given and starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these two representations as needed.

SOLVE The mathematical representation is based on the three kinematic equations

$$v_{\mathrm{f}s} = v_{\mathrm{i}s} + a_s\,\Delta t$$

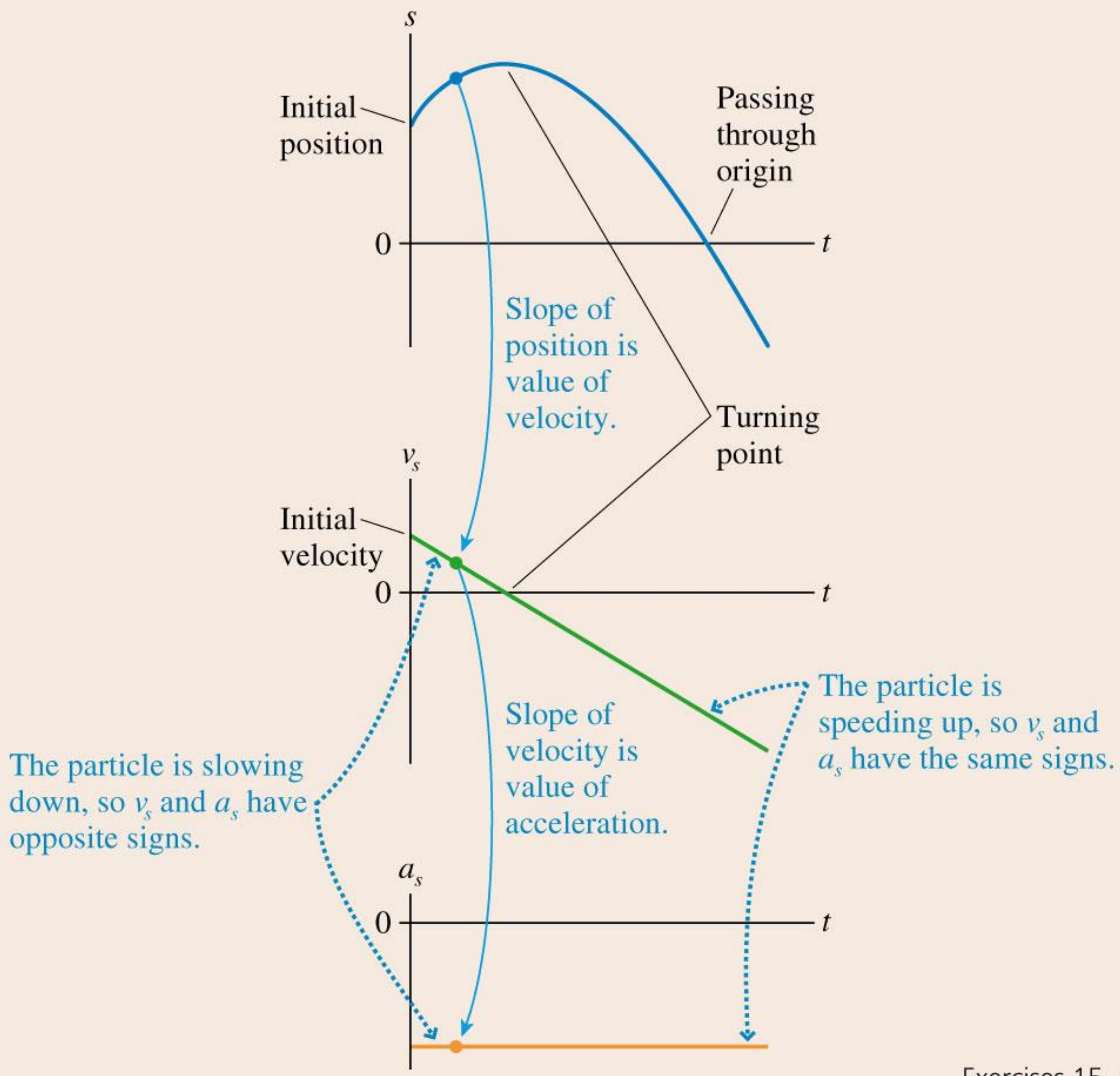
$$s_\mathrm{f} = s_\mathrm{i} + v_{\mathrm{i}s}\,\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$

$$v_{\mathrm{f}s}{}^2 = v_{\mathrm{i}s}{}^2 + 2a_s\,\Delta s$$

- Use x or y, as appropriate to the problem, rather than the generic s.
- Replace i and f with numerical subscripts defined in the pictorial representation.
- Uniform motion with constant velocity has $a_s = 0$.

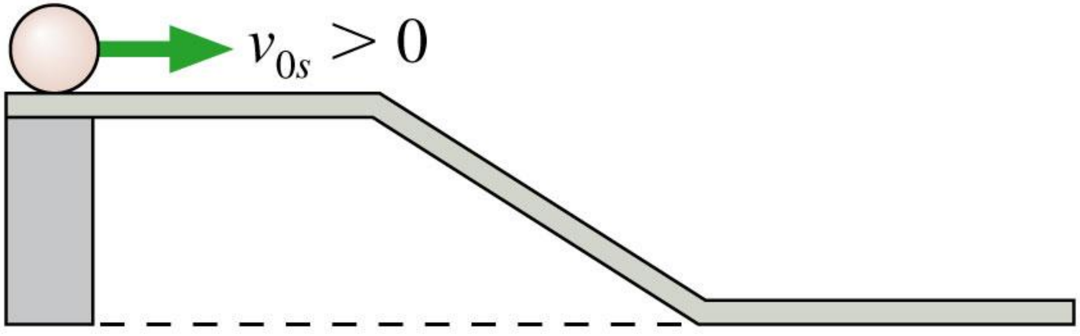
ASSESS Is your result believable? Does it have proper units? Does it make sense?

TACTICS BOX 2.2 **Interpreting graphical representations of motion**



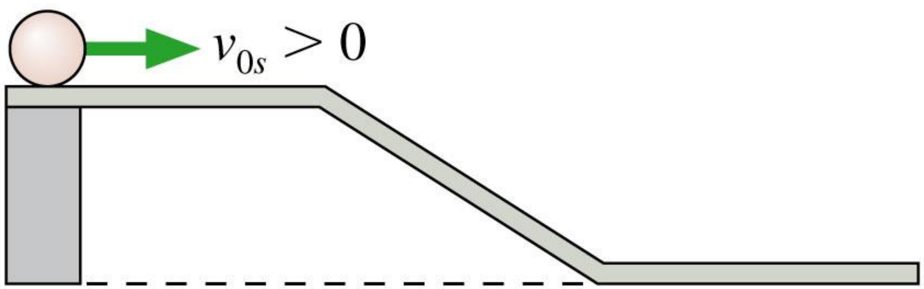
Exercises 15, 16, 22

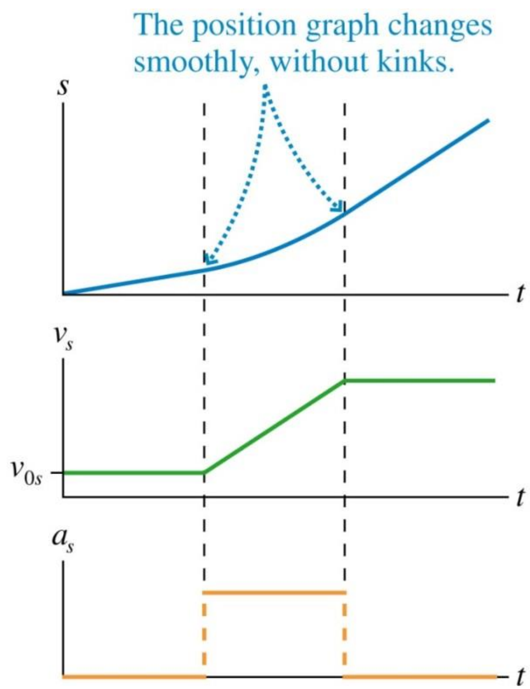
From Track to Graphs



Q.9 Draw the position, velocity, and acceleration graphs for the ball on the frictionless track in the above figure.

From Track to Graphs

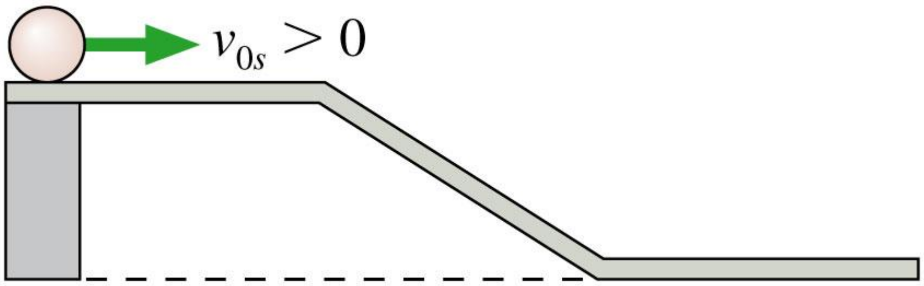


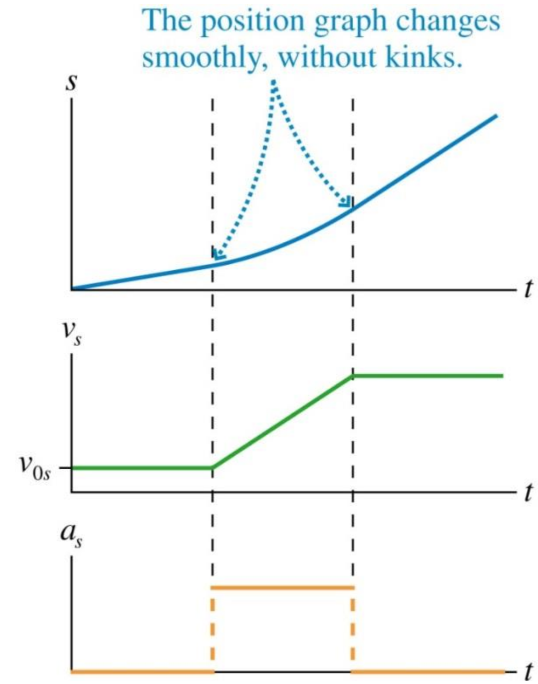


VISUALSE:

It is often easiest to begin with the velocity. Here the ball starts with an initial velocity v_{0s}. There is no acceleration on the horizontal surface ($a_s = 0$ if $\Theta = 0°$), so the velocity remains constant until the ball reaches the slope. The slope is an inclined plane that, as we have learned, has constant acceleration. The velocity increases linearly with time during constant-acceleration motion. The ball returns to constant-velocity motion after reaching the bottom horizontal segment. The middle graph of the above figure shows the velocity.

From Track to Graphs

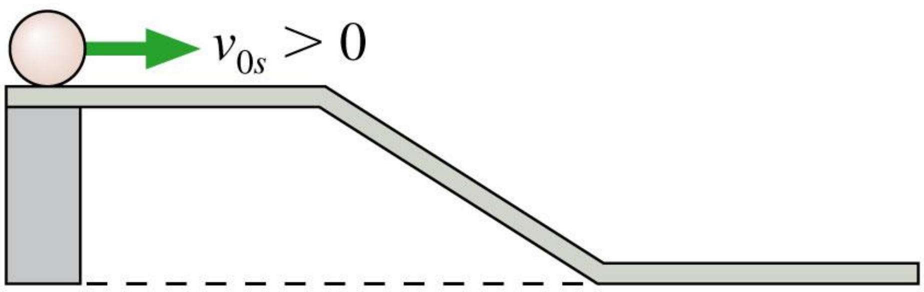


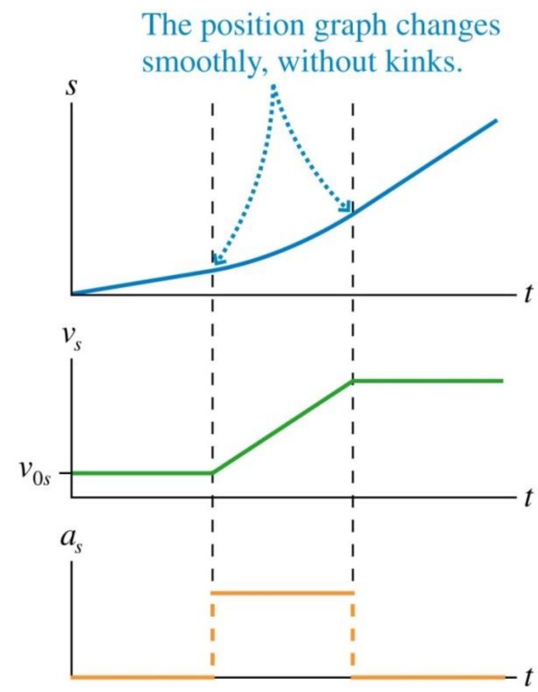


VISUALSE:

We have enough information to draw the acceleration graph. We noted that the acceleration is zero while the ball is on the horizontal segment, and a_s has a constant positive value on the slope. These accelerations are consistent with the slope of the velocity graph: zero slope, then positive slope, then a return to zero slope. The acceleration cannot *really* change instantly from zero to a nonzero value, but the change can be so quick that we do not see it on the time scale of the graph. That is what the vertical dotted lines imply.

From Track to Graphs

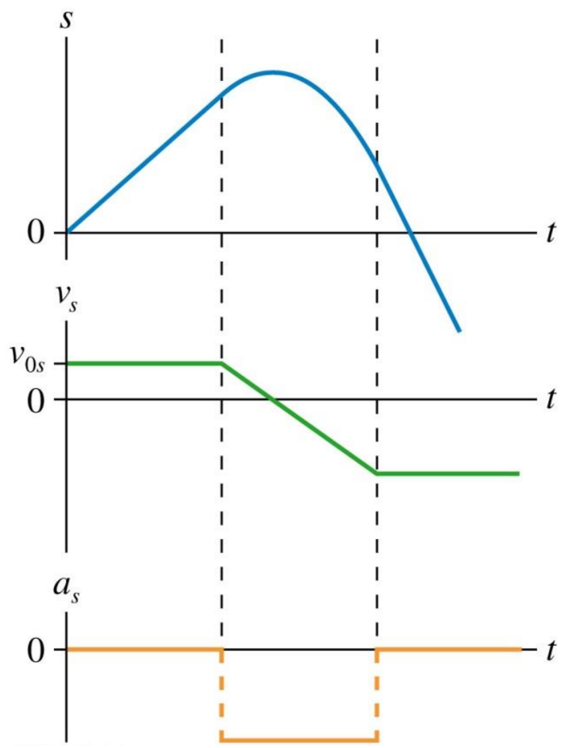




VISUALSE:

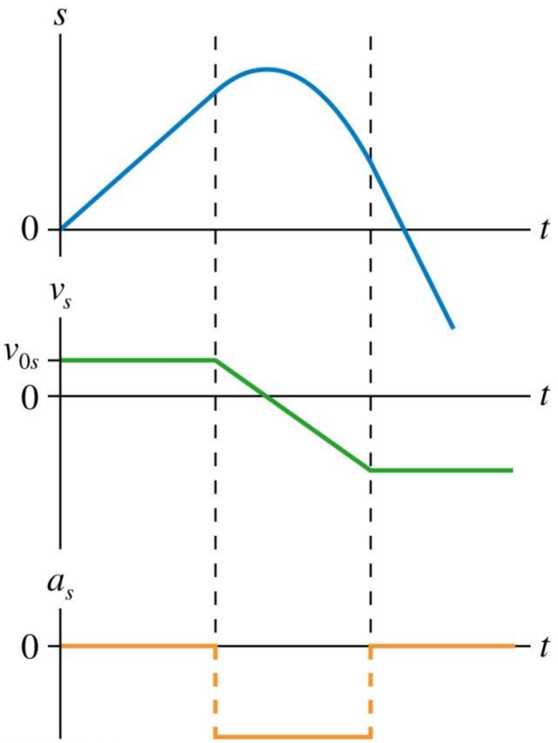
Finally, we need to find the position-versus-time graph. The position increases linearly with time during the first segment at constant velocity. It also does so during the third segment of motion, but with a steeper slope to indicate a faster velocity. In between, while the acceleration is nonzero but constant, the position graph has a *parabolic* shape.

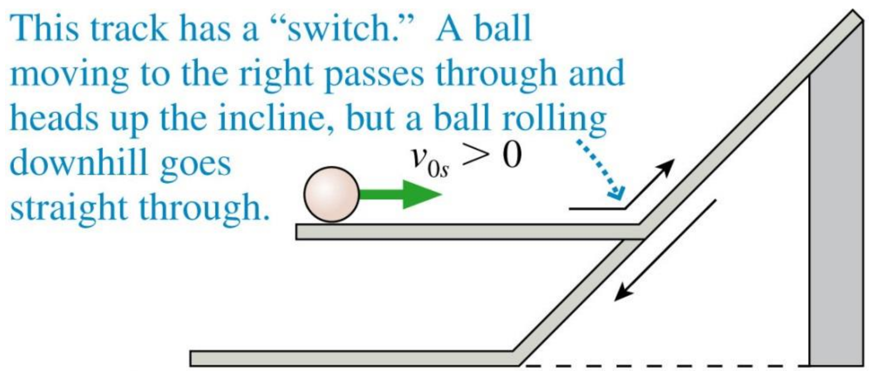
From Graphs to Track



Q.10 The above figure shows a set of motion graphs for a ball moving on a track. Draw a picture of the track and describe the ball's initial condition. Each segment of the track is straight, but the segments may be tilted.

From Graphs to Track

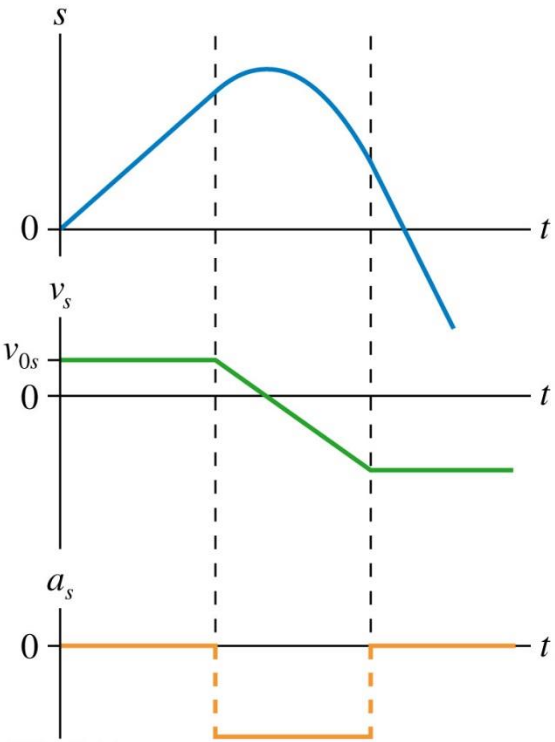


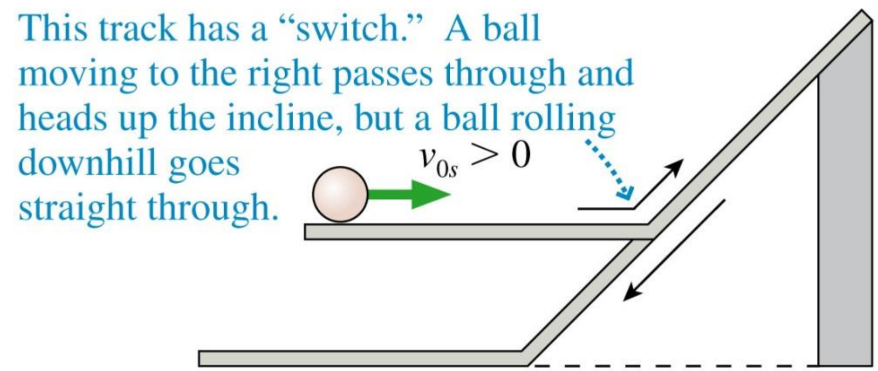


VISUALISE:

Let's begin by examining the velocity graph. The ball starts with initial velocity $v_{0s} > 0$ and maintains this velocity for awhile; there's no acceleration. Thus the ball must start out rolling to the right on a horizontal track. At the end of the motion, the ball is again rolling on a horizontal track (no acceleration, constant velocity), but it's rolling to the *left* because v_s is negative.

From Graphs to Track





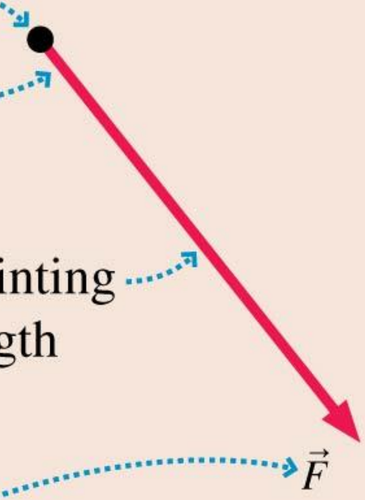
VISUALISE:

Further, the final speed ($|v_s|$) is greater that the initial speed. The middle section of the graph shows us what happens. The ball starts slowing with constant acceleration (rolling uphill), reaches a turning point (s is maximum, $v_s = 0$), then speeds up in the opposite direction (rolling downhill). This is still a negative acceleration because the ball is speeding up in the negative s-direction. It must roll farther downhill than it had rolled uphill before reaching a horizontal section of track.

Drawing Force Vectors

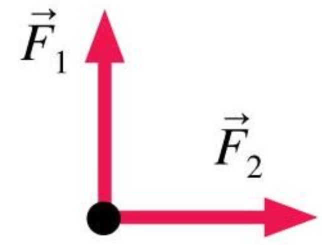
TACTICS BOX 5.1 **Drawing force vectors**

1. Represent the object as a particle.
2. Place the *tail* of the force vector on the particle.
3. Draw the force vector as an arrow pointing in the proper direction and with a length proportional to the size of the force.
4. Give the vector an appropriate label.

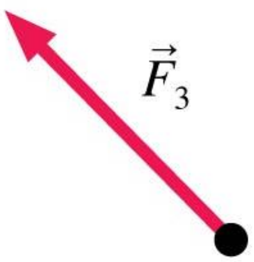


Combining Forces

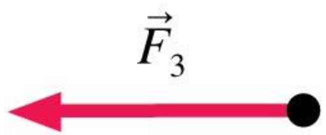
Q.11 The net force on an object points to the left. Two of three forces are shown. Which is the missing third force?



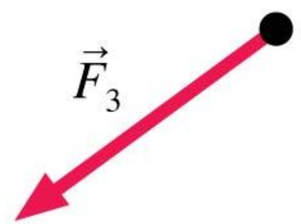
Two of the three forces exerted on an object



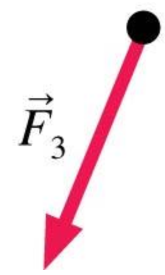
A.



B.



C.



D.

Combining Forces

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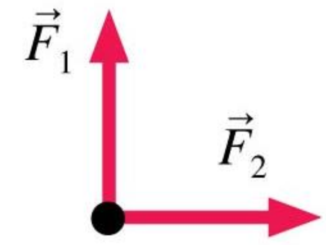
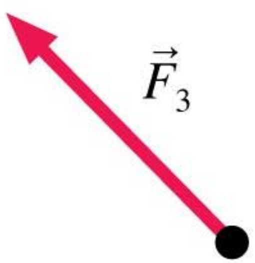
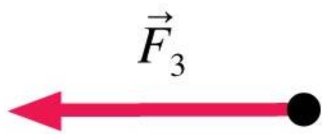


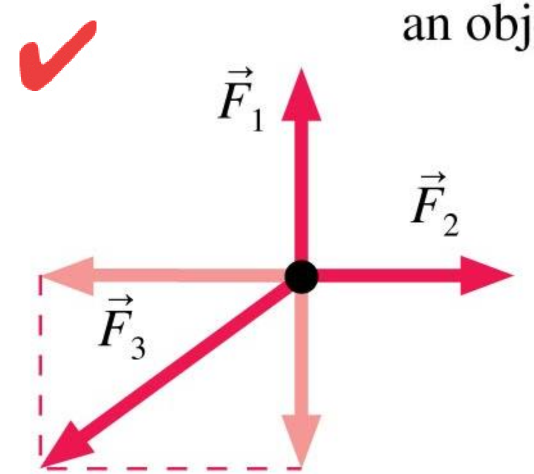
Two of the three forces exerted on an object



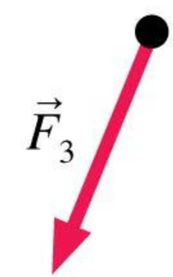
A.



B.



C.



D.

Vertical components cancel

Identifying Forces

TACTICS BOX 5.2 Identifying forces

1. **Identify the object of interest.** This is the object whose motion you wish to study.
2. **Draw a picture of the situation.** Show the object of interest and all other objects—such as ropes, springs, or surfaces—that touch it.
3. **Draw a closed curve around the object.** Only the object of interest is inside the curve; everything else is outside.

Exercises 3–8

Identifying Forces

TACTICS BOX 5.2 Identifying forces

❹ **Locate every point on the boundary of this curve where other objects touch the object of interest.** These are the points where *contact forces* are exerted on the object.

❺ **Name and label each contact force acting on the object.** There is at least one force at each point of contact; there may be more than one. When necessary, use subscripts to distinguish forces of the same type.

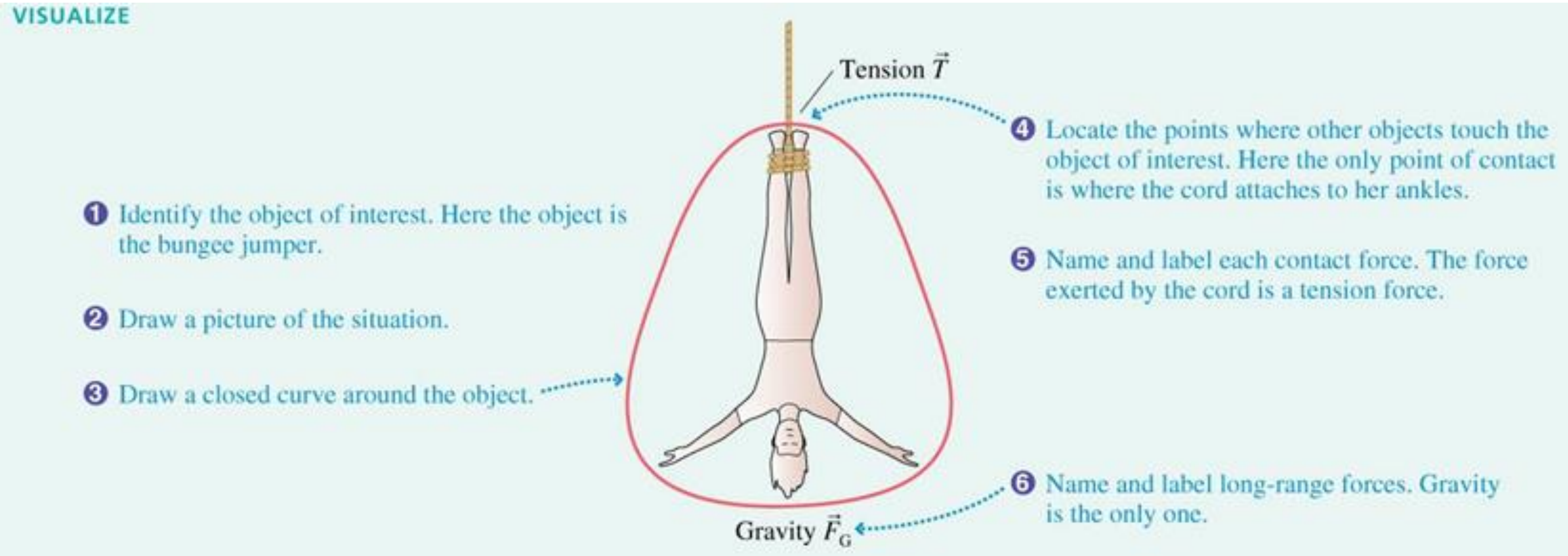
❻ **Name and label each long-range force acting on the object.** For now, the only long-range force is the gravitational force.

Exercises 3–8

Forces on a Bungee Jumper

Q.12 A bungee jumper has leapt off a bridge, and is nearing the bottom of her fall. What forces are being exerted on the jumper?

Forces on a Bungee Jumper

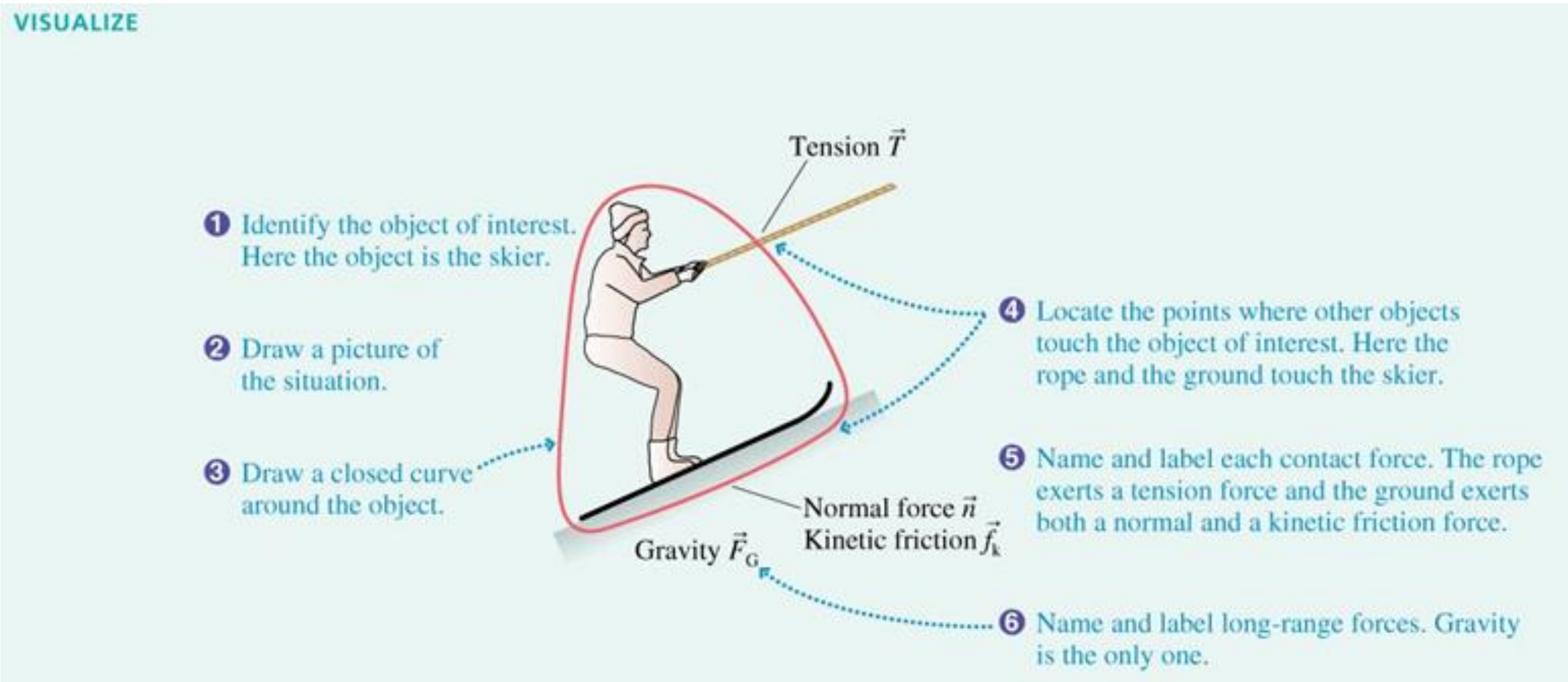


Q.12 A bungee jumper has leapt off a bridge, and is nearing the bottom of her fall. What forces are being exerted on the jumper?

Forces on a Skier

Q.13 A skier is being towed up a snow-covered hill by a tow rope. What forces are being exerted on the skier?

Forces on a Skier

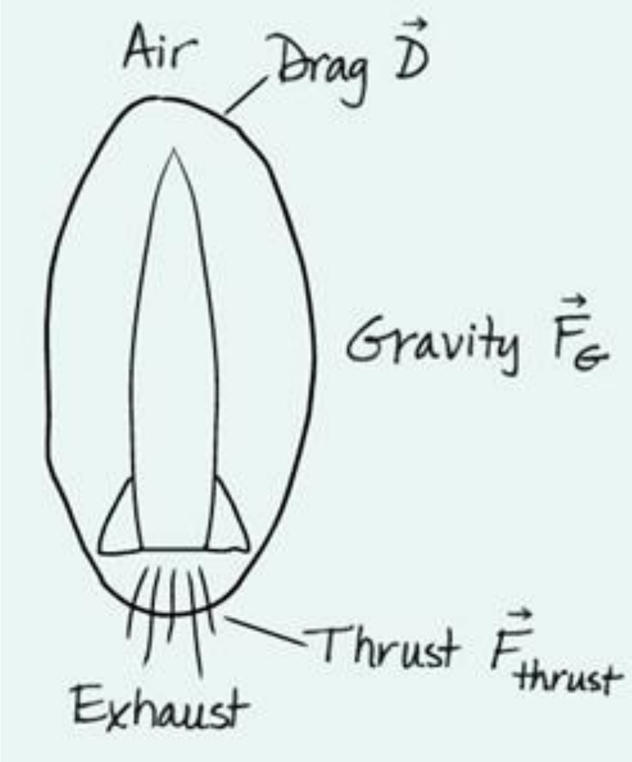


Q.13 A skier is being towed up a snow-covered hill by a tow rope. What forces are being exerted on the skier?

Forces on a Rocket

Q.14 A rocket is being launched to place a new satellite in orbit. Air resistance is not negligible. What forces are being exerted on the rocket?

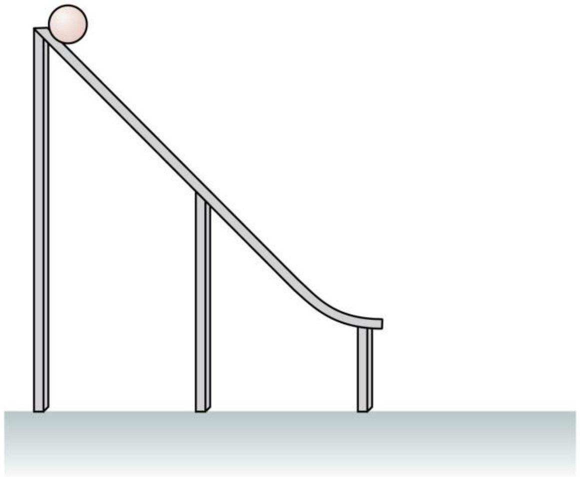
Forces on a Rocket



Q.14 A rocket is being launched to place a new satellite in orbit. Air resistance is not negligible. What forces are being exerted on the rocket?

Forces

Q.15 A ball rolls down an incline and off a horizontal ramp. Ignoring air resistance, what force or forces act on the ball as it moves through the air just after leaving the horizontal ramp?



a) The weight of the ball acting vertically down.

b) A horizontal force that maintains the motion.

c) A force whose direction changes as the direction of motion changes.

d) The weight of the ball and a horizontal force.

e) The weight of the ball and a force in the direction of motion.

Forces

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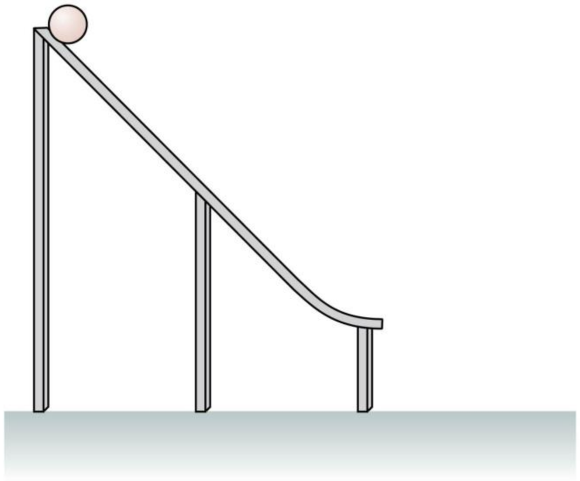


a) **The weight of the ball acting vertically down.**

b) A horizontal force that maintains the motion.

c) A force whose direction changes as the direction of motion changes.

d) The weight of the ball and a horizontal force.

e) The weight of the ball and a force in the direction of motion.

Drawing a Free-body Diagram

TACTICS BOX 5.3 Drawing a free-body diagram

1. **Identify all forces acting on the object.** This step was described in Tactics Box 5.2.
2. **Draw a coordinate system.** Use the axes defined in your pictorial representation.
3. **Represent the object as a dot at the origin of the coordinate axes.** This is the particle model.

Exercises 24–29

Drawing a Free-body Diagram

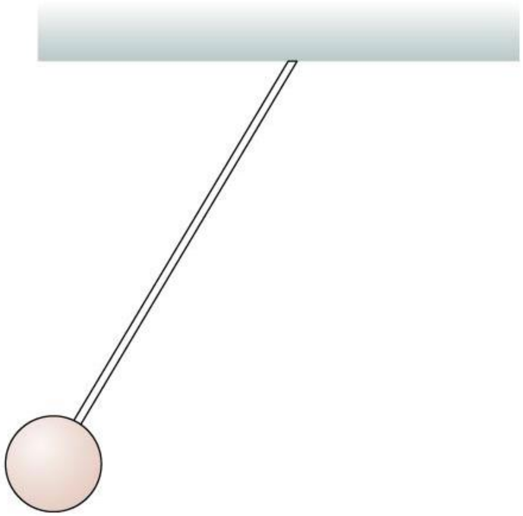
TACTICS BOX 5.3 **Drawing a free-body diagram**

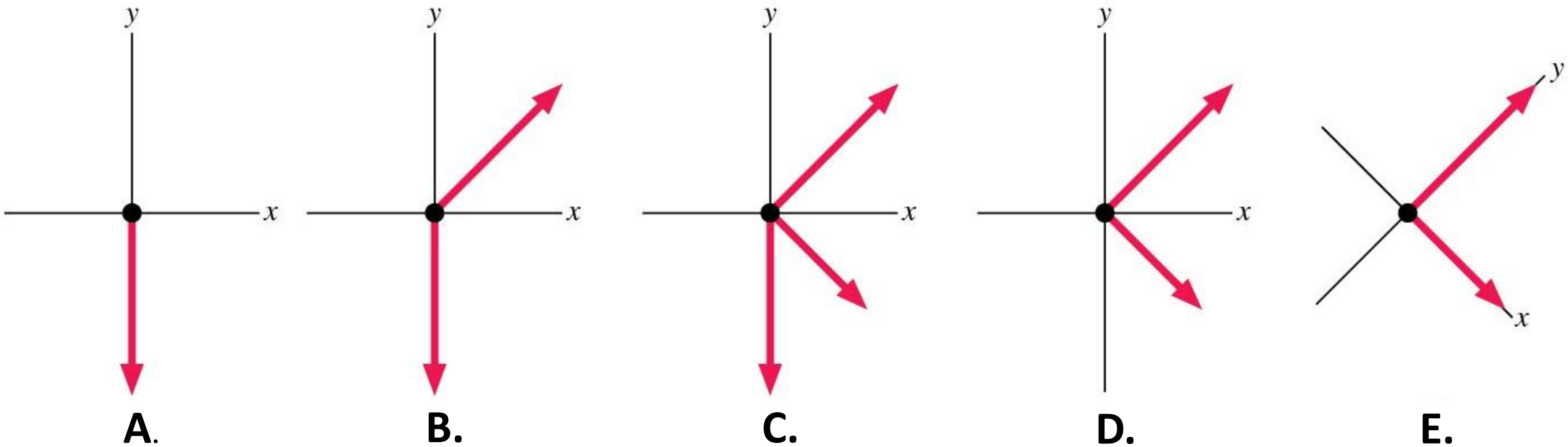
4. **Draw vectors representing each of the identified forces.** This was described in Tactics Box 5.1. Be sure to label each force vector.
5. **Draw and label the *net force* vector $\vec{F}_{net}$.** Draw this vector beside the diagram, not on the particle. Or, if appropriate, write $\vec{F}_{net} = \vec{0}$. Then check that $\vec{F}_{net}$ points in the same direction as the acceleration vector $\vec{a}$ on your motion diagram.

Exercises 24–29

Free-body Diagrams

Q.16 A ball, hanging from the ceiling by a string, is pulled back and released. Which is the correct free-body diagram just after its release?





Free-body Diagrams

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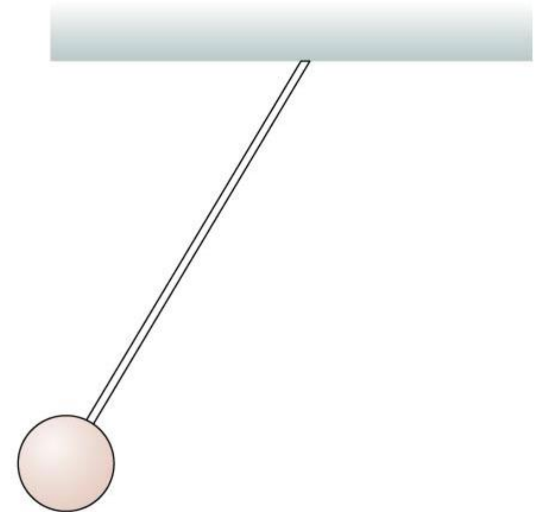
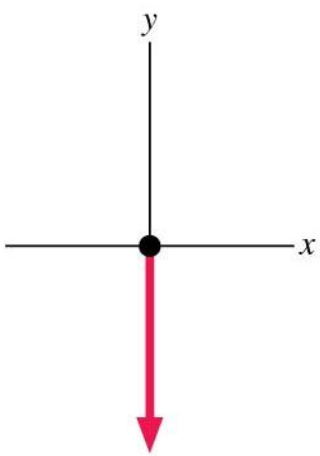
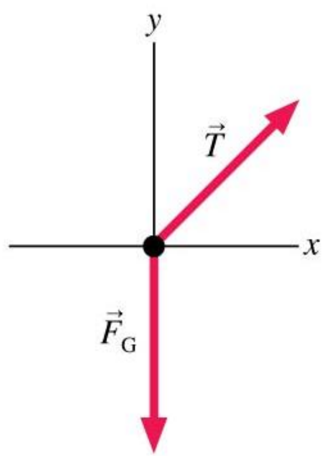
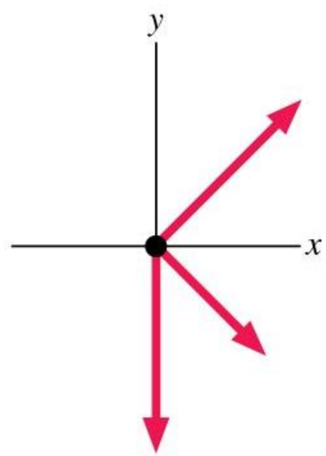




A.



B.



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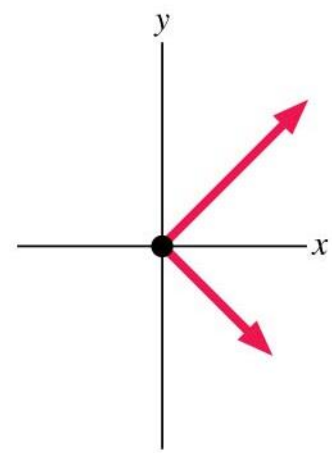
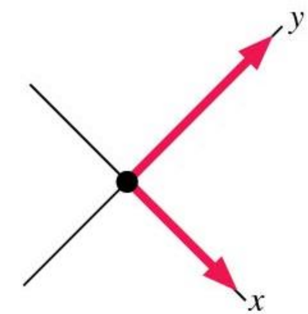


D.

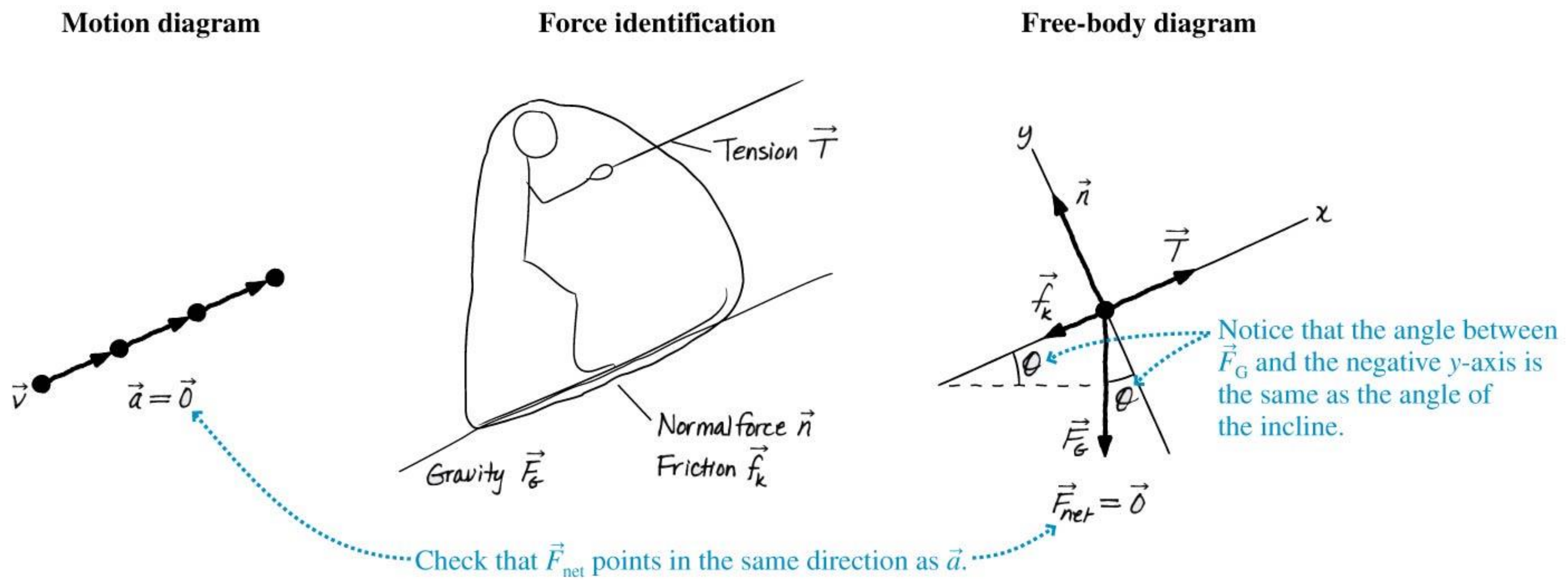


E.

Free-body Diagrams

Q.17 A tow rope pulls a skier up a snow-covered hill at a constant speed. Draw a pictorial representation of the skier.

Free-body Diagrams

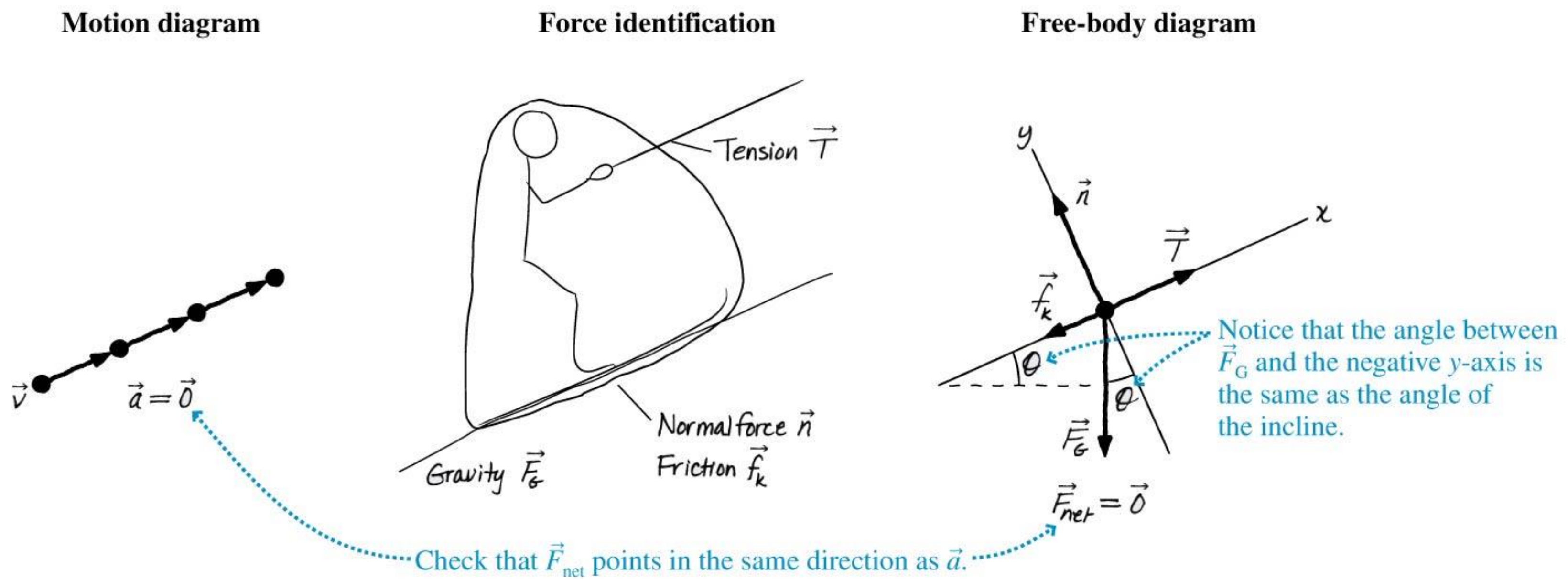


MODEL:

This is question 3 again, with the additional information that the skier is moving at constant speed. The skier is treated as a particle in dynamic equilibrium.

If we were doing a kinematics problem, the pictorial representation would use a tilted coordinate system with the x-axis parallel to the slope, so we use these same tilted coordinate axes for the free-body diagram.

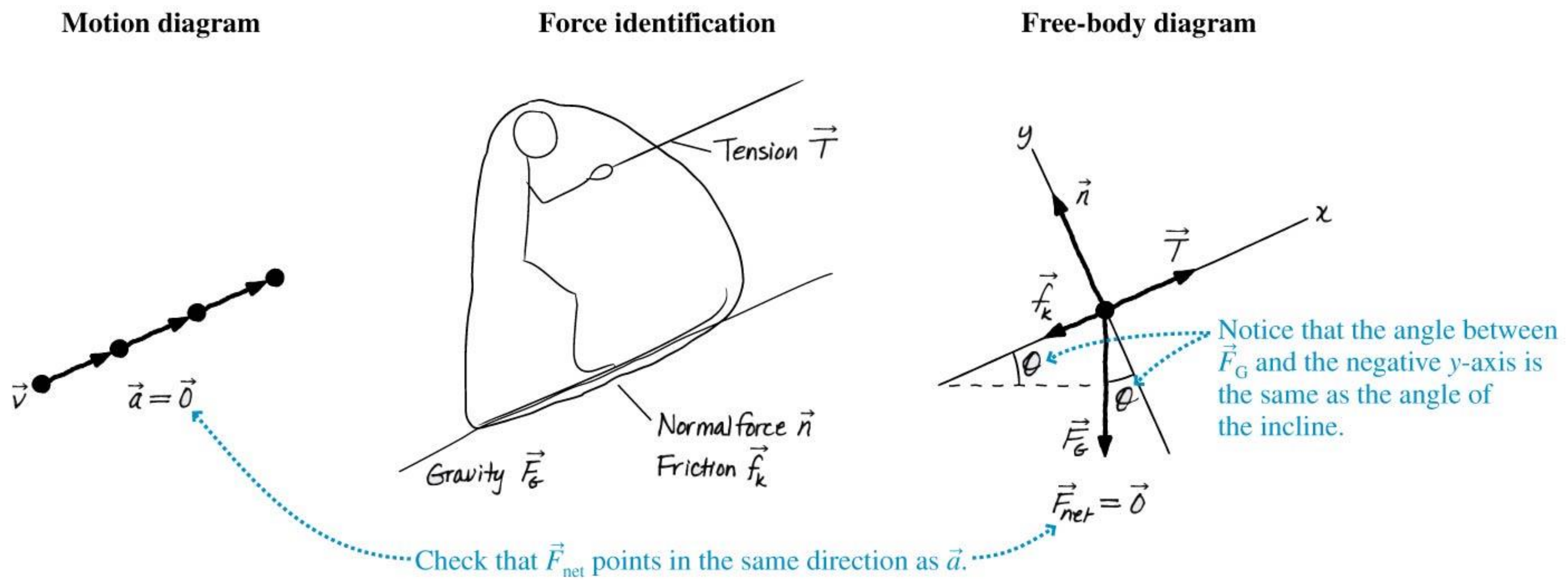
Free-body Diagrams



ASSESS:

We have shown $\vec{T}$ pulling parallel to the slope and $\vec{f}_k$, which opposes the direction of motion, pointing down the slope. $\vec{n}$ is perpendicular to the surface and thus along the *y*-axis.

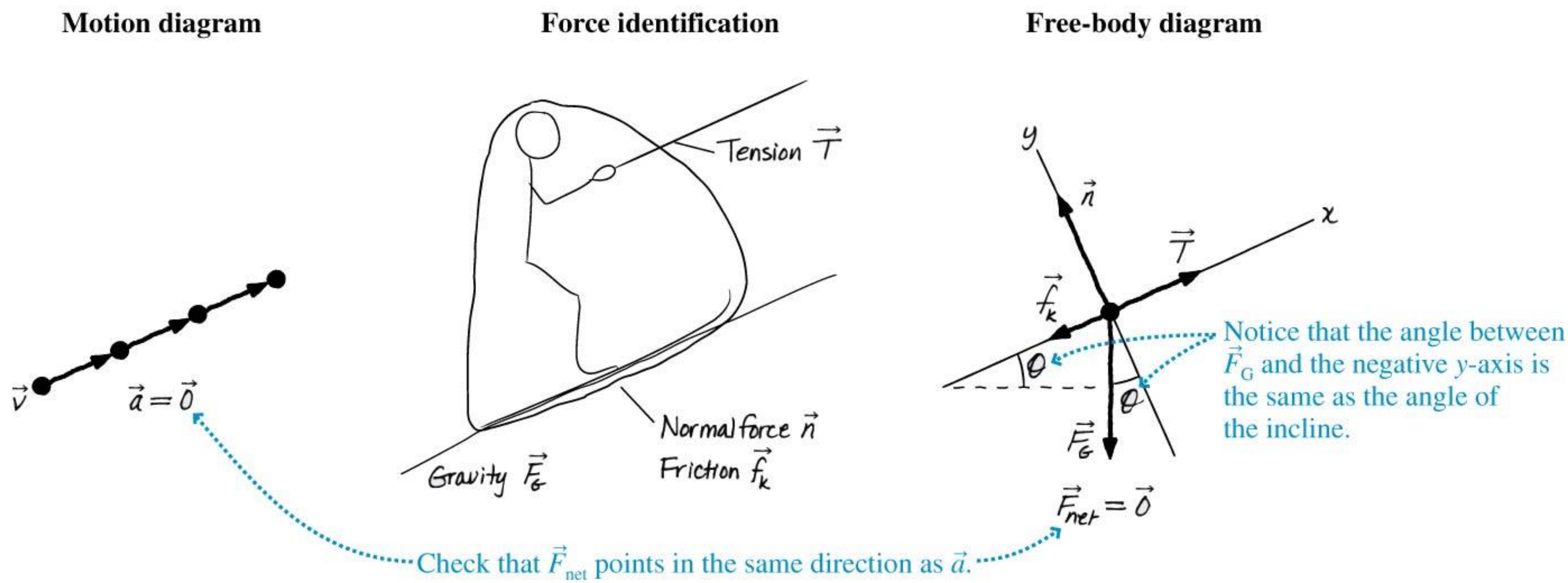
Free-body Diagrams



ASSESS:

Finally, and this is important, the gravitational force $\vec{F}_G$ is *vertically* downward, not along the negative *y*-axis. In fact, you should convince yourself from the geometry that the angle θ between the $\vec{F}_G$ vector and the negative *y*-axis is the same as the angle θ of the incline above the horizontal.

Free-body Diagrams



ASSESS:

The skier moves in a straight line with constant speed, so $\vec{a}$ = $\vec{0}$ and, from Newton's first law, $\vec{F}_{\text{net}}$ = $\vec{0}$. Thus we have drawn the vectors such that the *y*-component of $\vec{F}_{\text{G}}$ is equal in magnitude to $\vec{n}$. Similarly, $\vec{T}$ must be large enough to match the negative x-component of both $\vec{f}_{\text{k}}$ and $\vec{F}_{\text{G}}$.

PROBLEM-SOLVING STRATEGY 6.1

Equilibrium problems

MODEL Make simplifying assumptions. When appropriate, represent the object as a particle.

VISUALIZE

- Establish a coordinate system, define symbols, and identify what the problem is asking you to find. This is the process of translating words into symbols.
- Identify *all* forces acting on the object and show them on a free-body diagram.
- These elements form the **pictorial representation** of the problem.

PROBLEM-SOLVING STRATEGY 6.1 **Equilibrium problems**

SOLVE The mathematical representation is based on Newton's first law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = \vec{0}$$

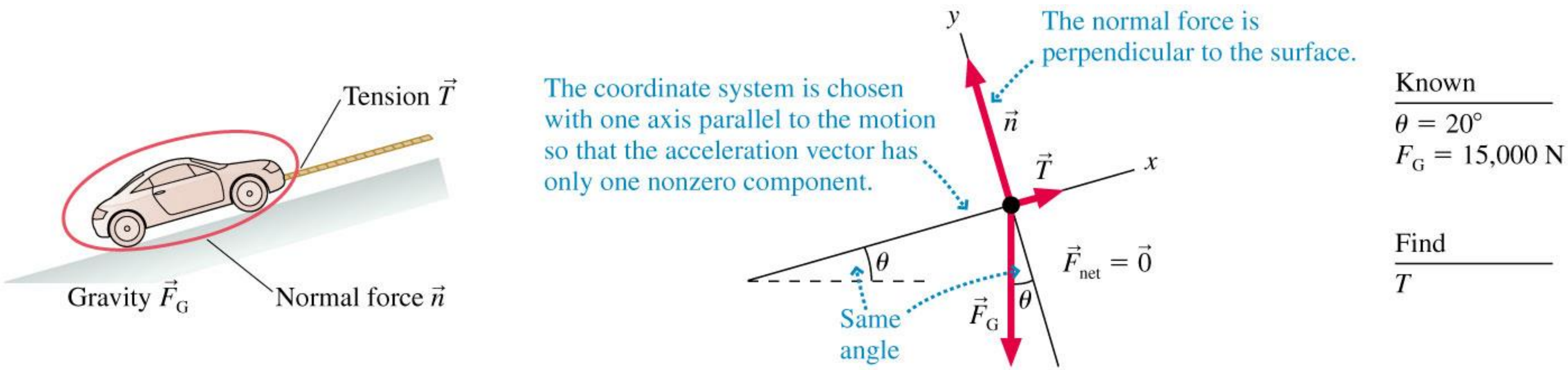
The vector sum of the forces is found directly from the free-body diagram.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Towing a Car up a Hill

Q.18 A car with a weight of 15,000 N is being towed up a 20° slope at constant velocity. Friction is negligible. The tow rope is rated at 6000 N maximum tension. Will it break?

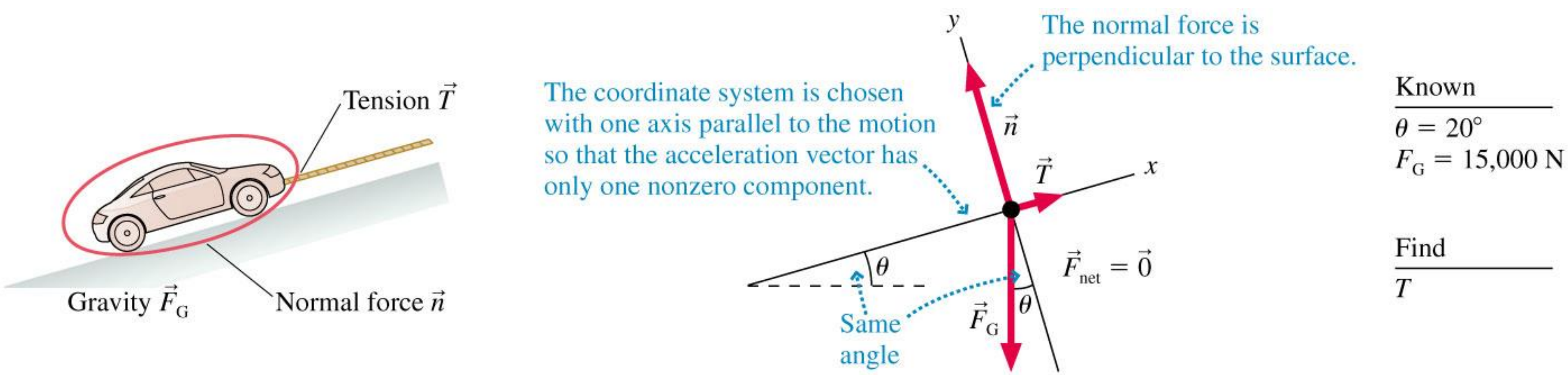
A Dynamic Equilibrium Problem



MODEL:

We'll treat the car as a particle in dynamic equilibrium.

A Dynamic Equilibrium Problem

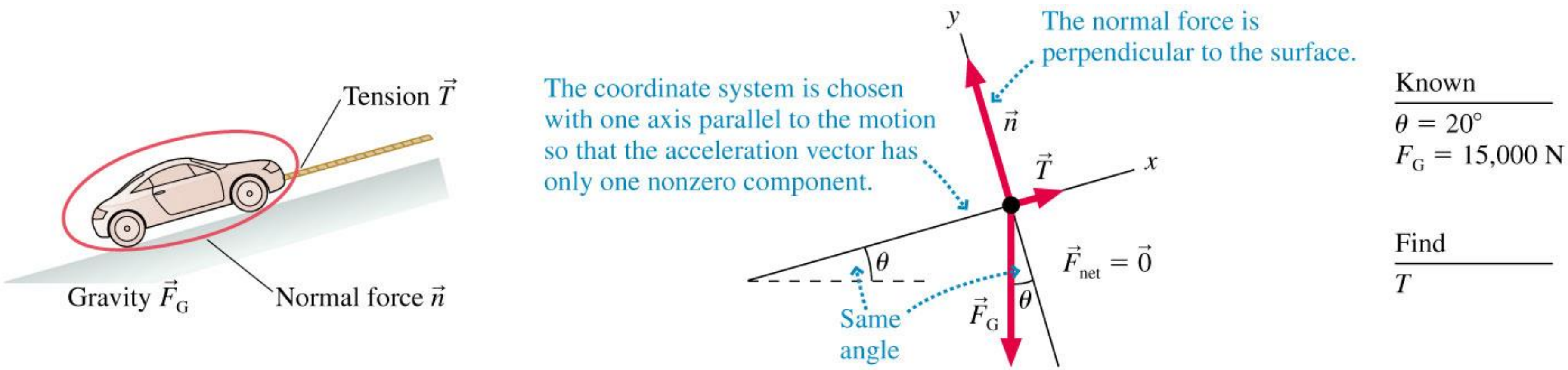


SOLVE:

The free-body diagram shows forces $\vec{T}$, $\vec{n}$ and $\vec{F}_G$ acting on the car. Newton's first law is

$$(F_{net})_x = \sum F_x = T_x + n_x + (F_G)_x = 0$$
$$(F_{net})_y = \sum F_y = T_y + n_y + (F_G)_y = 0$$

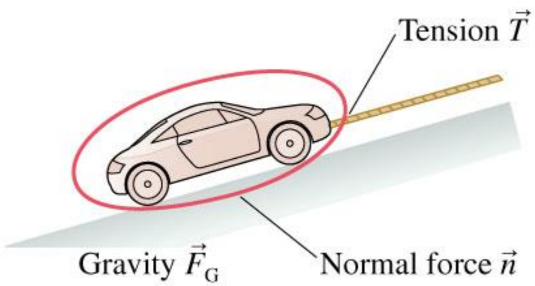
A Dynamic Equilibrium Problem

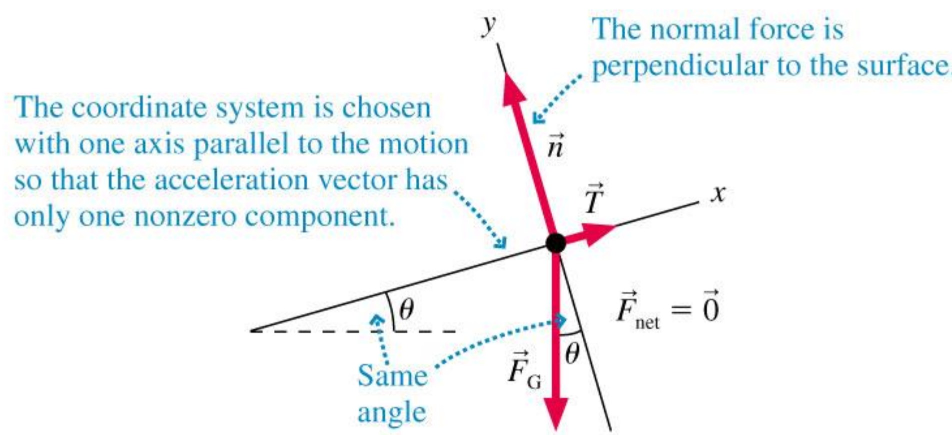


SOLVE:

From here on, we'll use $\sum F_x$ and $\sum F_y$ without the label i, as a simple shorthand notation to indicate that we're adding all the x-components and all the y-components of the forces.

A Dynamic Equilibrium Problem





Known
$\theta = 20°$
$F_G = 15{,}000$ N

Find
T

SOLVE:

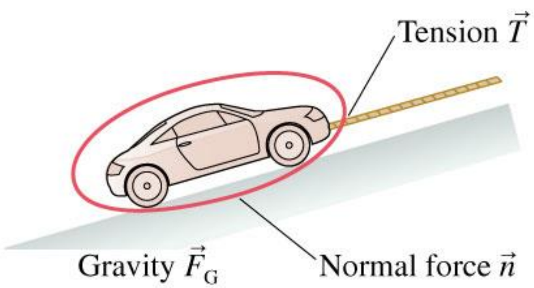
We can deduce the components directly from the free-body diagram:

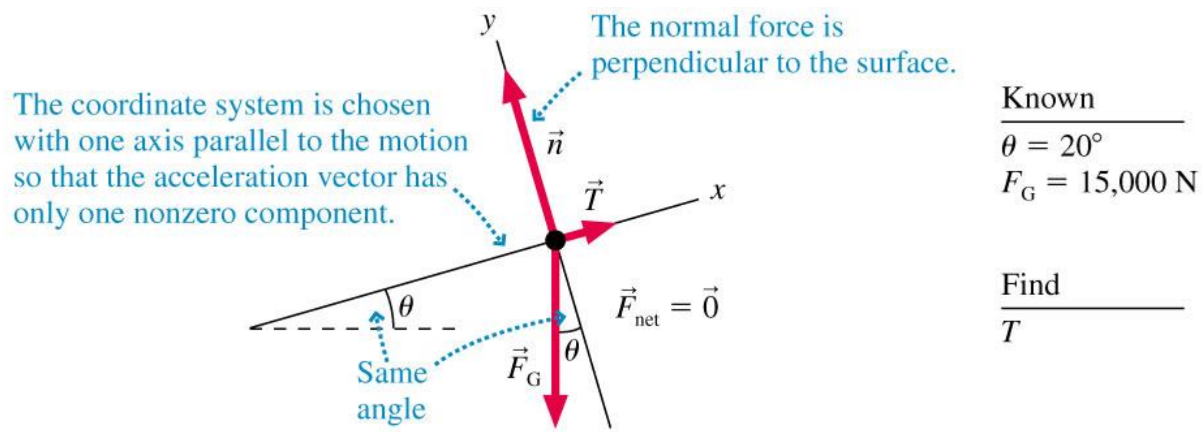
$T_x = T$ $\qquad$ $T_y = 0$

$n_x = 0$ $\qquad$ $n_y = n$

$(F_G)_x = -F_G \sin\theta$ $\qquad$ $(F_G)_y = -F_G \cos\theta$

A Dynamic Equilibrium Problem





SOLVE:

With these components, the first law becomes

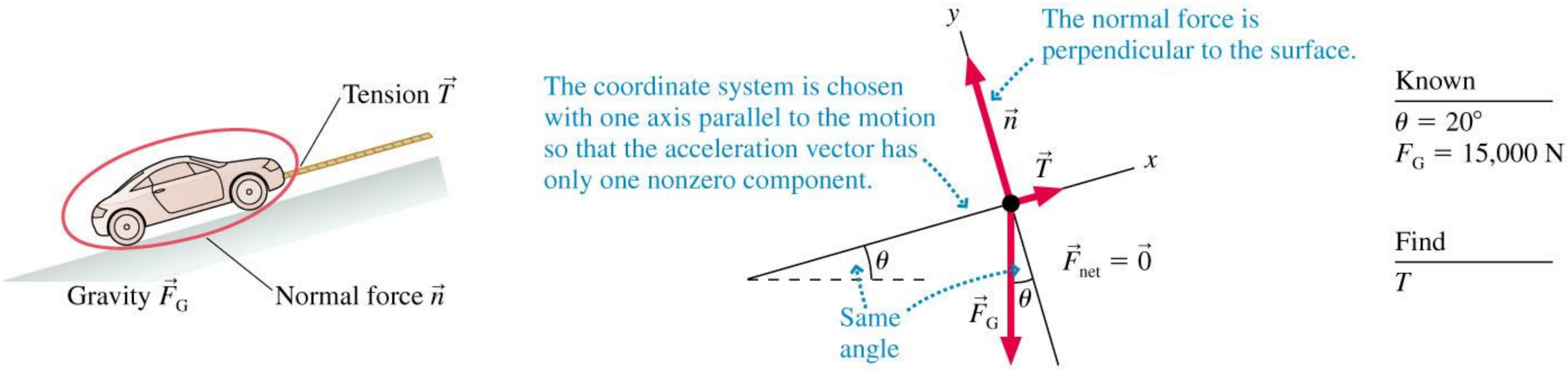
$$T - F_G \sin\theta = 0$$
$$n - F_G \cos\theta = 0$$

The first of these can be rewritten as

$$T = F_G \sin\theta = (15{,}000N) \sin 20° = 5{,}100N$$

Because $T < 6000N$, we conclude that the rope will **not** break. It turned out that we did not need the y-component equation in this problem.

A Dynamic Equilibrium Problem



ASSESS:

Because there's no friction, it would not take any tension force to keep the car rolling along a horizontal surface ($\theta = 0°$). At the other extreme, $\theta = 90°$, the tension force would need to equal the car's weight ($T = 15{,}000\ N$) to lift the car straight up at constant velocity.

The tension force for a 20° slope should be somewhere in between, and 5100 N is a little less than half the weight of the car. That our result is reasonable doesn't prove it's right, but we have at least ruled out careless errors that give unreasonable results.

PROBLEM-SOLVING STRATEGY 6.2

Dynamics problems

MODEL Make simplifying assumptions.

VISUALIZE Draw a **pictorial representation.**

- Show important points in the motion with a sketch, establish a coordinate system, define symbols, and identify what the problem is trying to find.
- Use a motion diagram to determine the object's acceleration vector $\vec{a}$.
- Identify all forces acting on the object *at this instant* and show them on a free-body diagram.
- It's OK to go back and forth between these steps as you visualize the situation.

PROBLEM-SOLVING STRATEGY 6.2

Dynamics problems

SOLVE The mathematical representation is based on Newton's second law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

The vector sum of the forces is found directly from the free-body diagram. Depending on the problem, either

- Solve for the acceleration, then use kinematics to find velocities and positions; or
- Use kinematics to determine the acceleration, then solve for unknown forces.

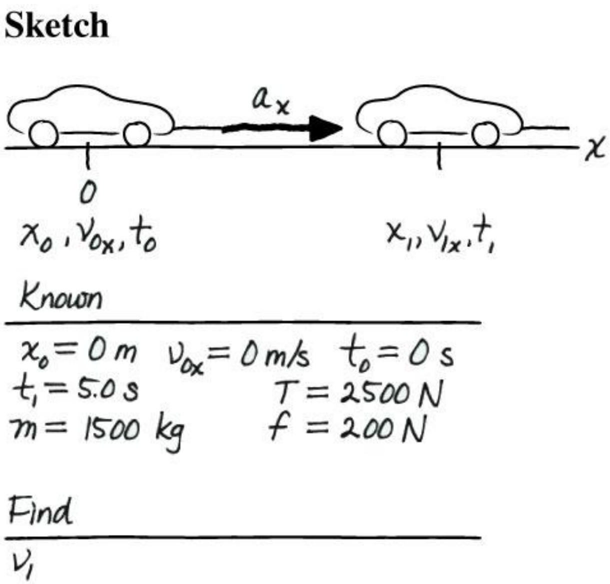
ASSESS Check that your result has the correct units, is reasonable, and answers the question.

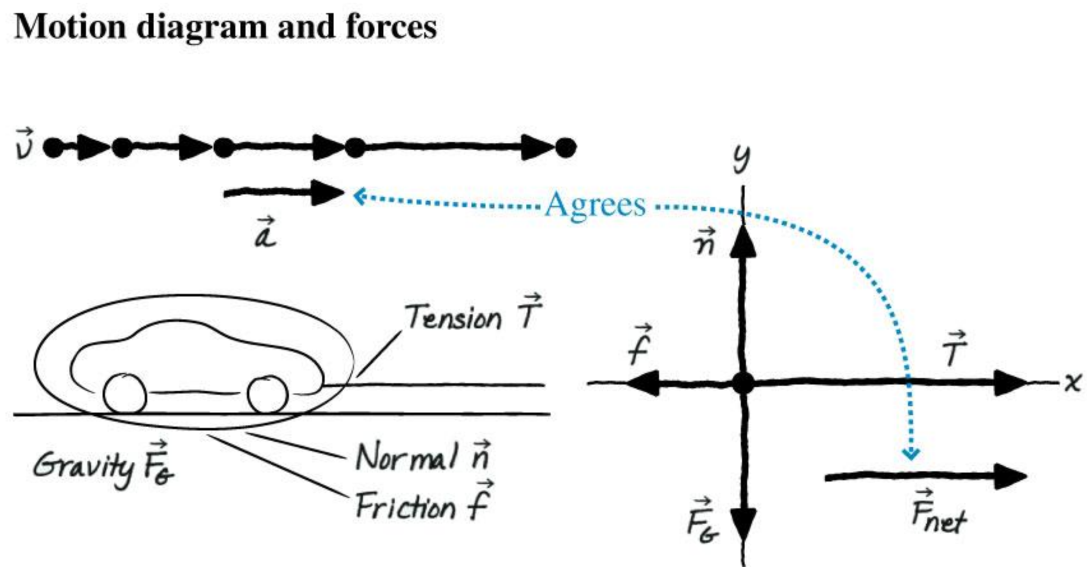
Exercise 22

Speed of a Towed Car

Q.19 A 1500 kg car is pulled by a tow truck. The tension in the tow rope is 2500 N, and a 200 N friction force opposes the motion. If the car starts from rest, what is its speed after 5.0 seconds?

Speed of a Towed Car

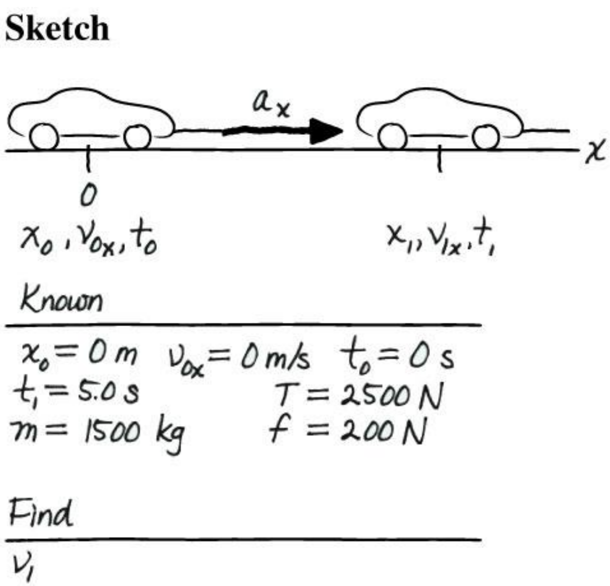


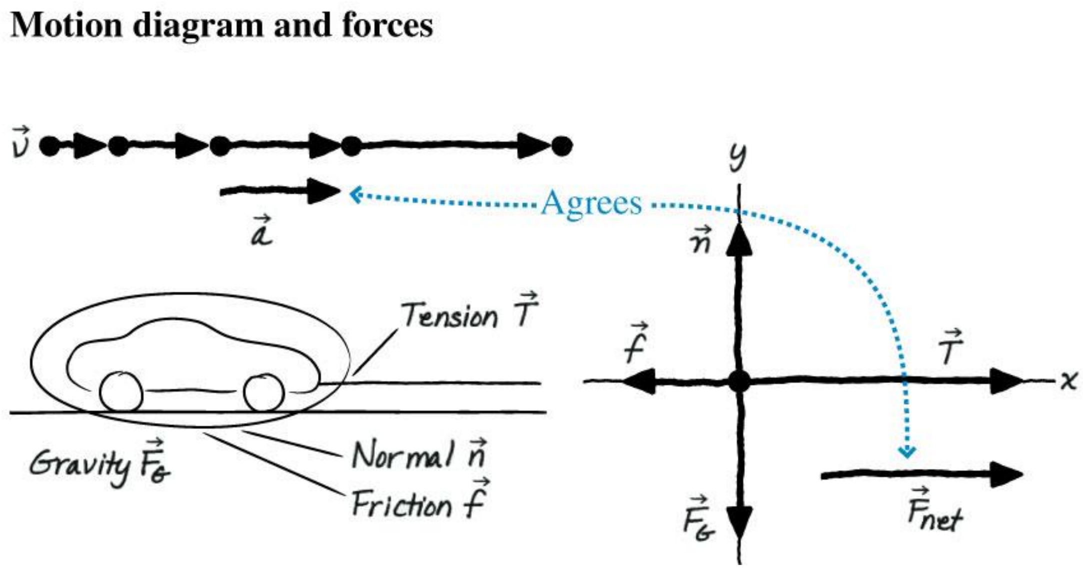


Q.19 A 1500 kg car is pulled by a tow truck. The tension in the tow rope is 2500 N, and a 200 N friction force opposes the motion. If the car starts from rest, what is its speed after 5.0 seconds?

MODEL: We'll treat the car as an accelerating particle. We'll assume, as part of our interpretation of the problem, that the road is horizontal, and that the direction of the motion is to the right.

Dynamics Problems





SOLVE:

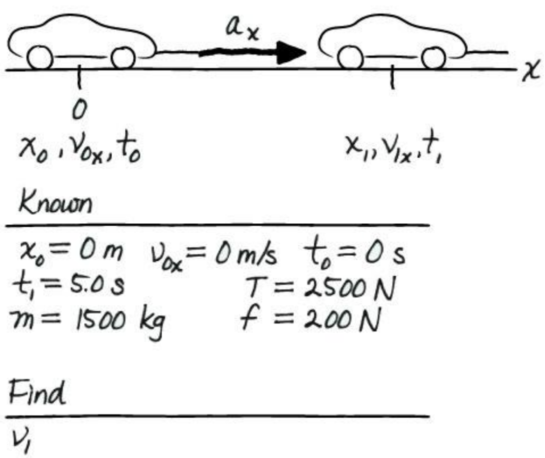
We begin with Newton's second law:

$$(F_{net})_x = \sum F_x = T_x + f_x + n_x + (F_G)_x = ma_x$$

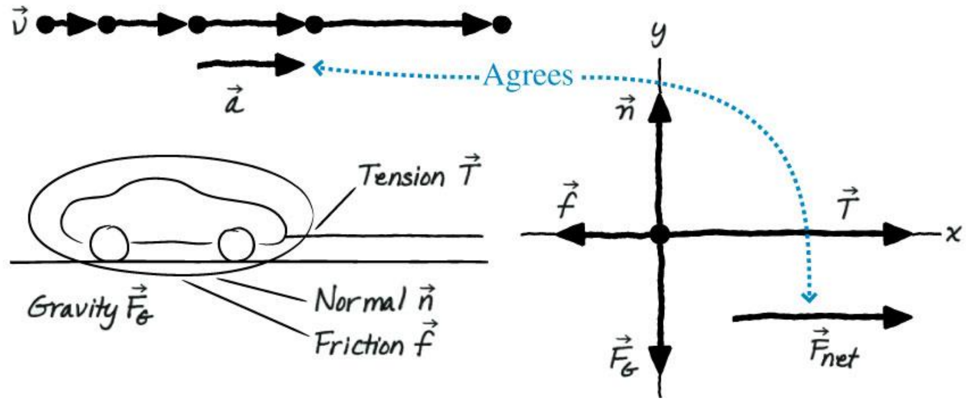
$$(F_{net})_y = \sum F_y = T_y + f_y + n_y + (F_G)_y = ma_y$$

Dynamics Problems

Sketch



Motion diagram and forces



SOLVE:

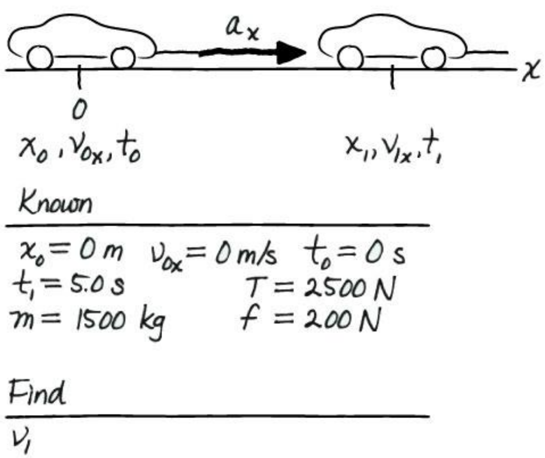
We begin with Newton's second law:

$$(F_{net})_x = \sum F_x = T_x + f_x + n_x + (F_G)_x = ma_x$$
$$(F_{net})_y = \sum F_y = T_y + f_y + n_y + (F_G)_y = ma_y$$

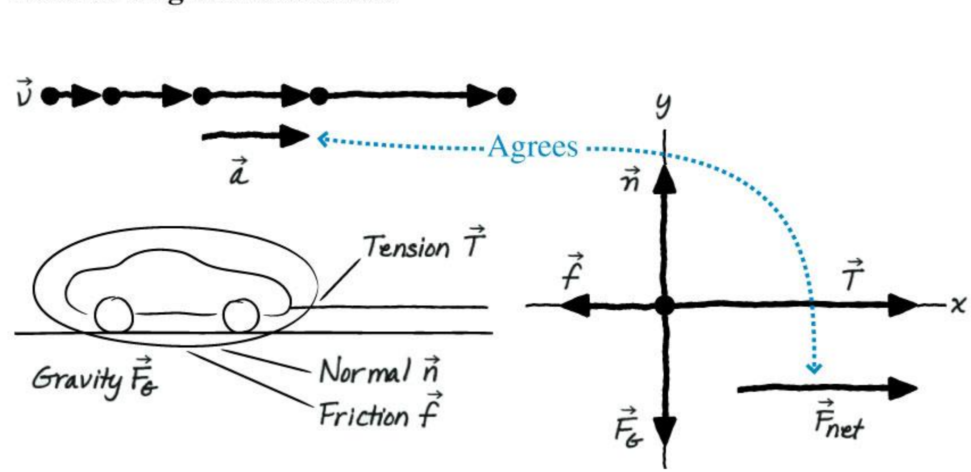
All four forces acting on the car have been included in the vector sum. The equations are perfectly general, with + signs everywhere, because the four vectors are added to give and $\vec{F}_{net}$.

Dynamics Problems

Sketch



Motion diagram and forces



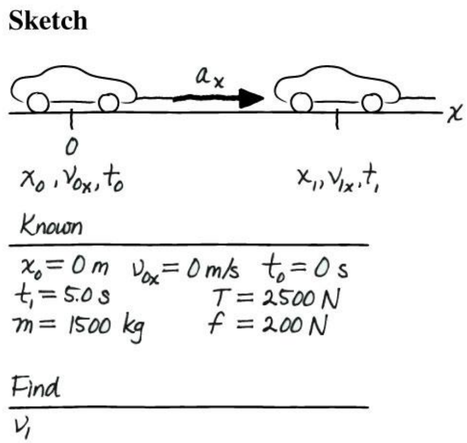
SOLVE:

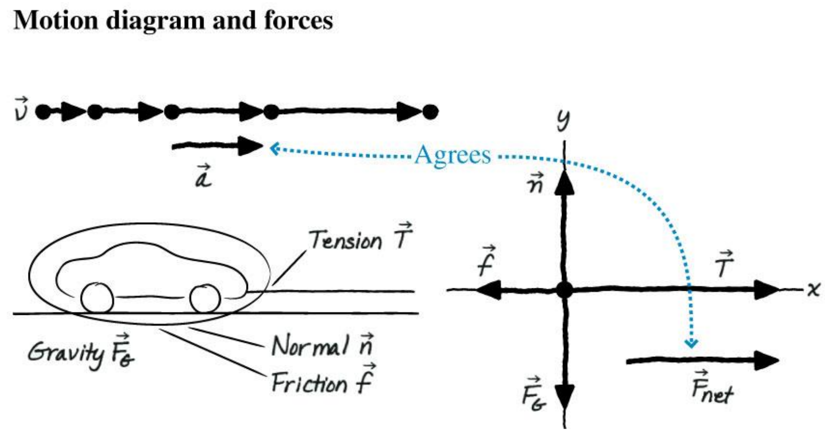
We begin with Newton's second law:

$$(F_{net})_x = \sum F_x = T_x + f_x + n_x + (F_G)_x = ma_x$$
$$(F_{net})_y = \sum F_y = T_y + f_y + n_y + (F_G)_y = ma_y$$

We can now 'read' the vector components from the free-body diagram:

$$T_x = +T \qquad T_y = 0 \qquad n_x = 0 \qquad n_y = +n$$
$$f_x = -f \qquad f_y = 0 \qquad (F_G)_x = 0 \qquad (F_G)_y = -F_G$$





The signs depend on which way the vectors point. Substituting these into the second-law equations and dividing by m gives

$$a_x = \frac{1}{m}(T - f)$$

$$= \frac{1}{1500\,kg}(2500\,N\ - 200\,N) = 1.53 m/s^2$$

$$a_y = \frac{1}{m}(n - F_G)$$

Because a_x is a constant $1.53 m/s^2$, we can finish by using constant-acceleration kinematics to find the velocity:

$$v_{1x} = v_{0x} + a_x \Delta t$$

$$= 0 + (1.53 m/s^2)(5.0 s) = 7.7 m/s$$

The problem asked for the speed after 5.0s, which is $v_{1x} = 7.7 m/s$.

Make Sure the Cargo Doesn't Slide

Q.20 A 100 kg box of dimensions 50 cm x 50 cm x 50cm is in the back of a flatbed truck. The coefficients of friction between the box and the bed of the truck are $\mu_s = 0.40$ and $\mu_k = 0.20$. What is the maximum acceleration the truck can have without the box slipping?

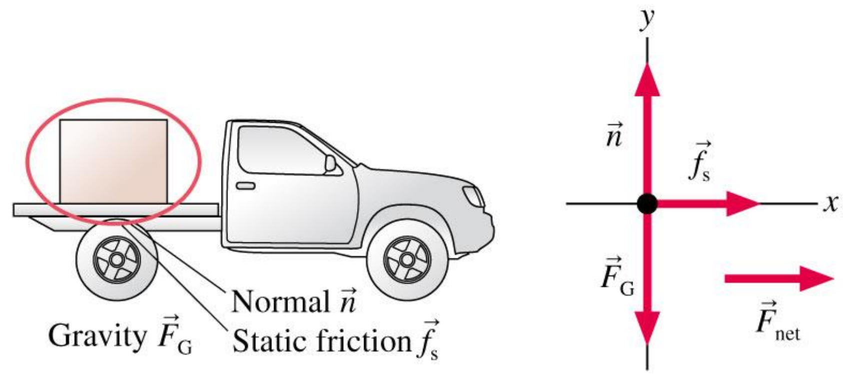
Make Sure the Cargo Doesn't Slide

Known

$m = 100$ kg
Box dimensions 50 cm × 50 cm × 50 cm
$\mu_s = 0.40$ $\mu_k = 0.20$

Find

Acceleration at which box slips



MODEL:

- Let the box, which we'll model as a particle, be the object of interest.
- Only the truck exerts contact forces on the box.
- The box does not slip relative to the truck.

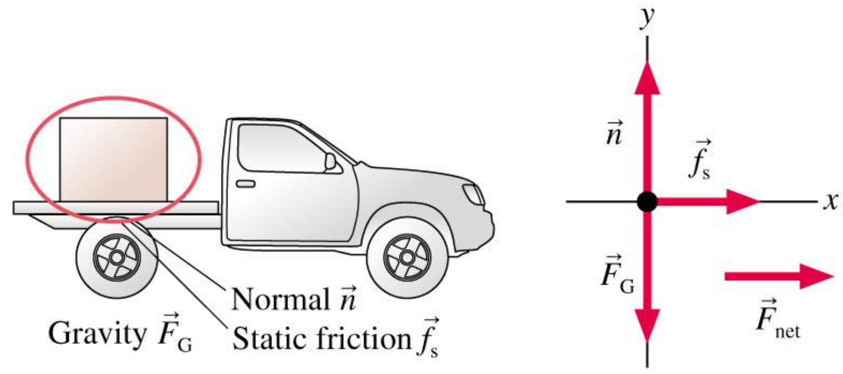
Make Sure the Cargo Doesn't Slide

Known

$m = 100$ kg
Box dimensions 50 cm × 50 cm × 50 cm
$\mu_s = 0.40 \quad \mu_k = 0.20$

Find

Acceleration at which box slips



MODEL

- If the truck bed were frictionless, the box would slide backward as seen in the truck's reference frame as the truck accelerates.
- The force that prevents sliding is **static friction**.
- The box must accelerate forward with the truck: $a_{box} = a_{truck}$.

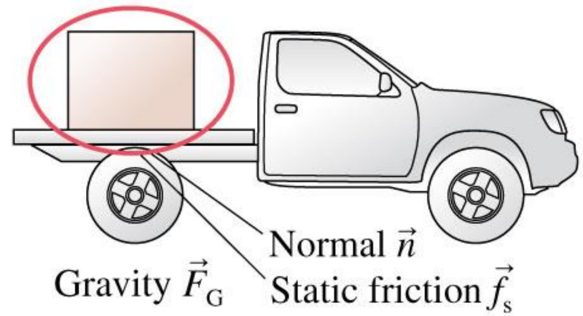
Make Sure the Cargo Doesn't Slide

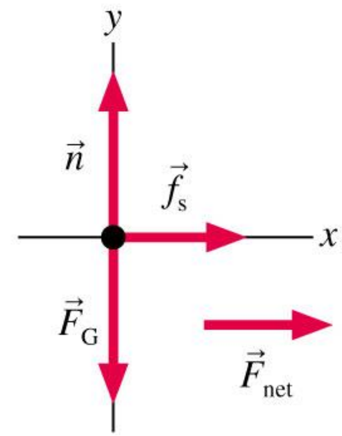
Known

$m = 100$ kg
Box dimensions 50 cm × 50 cm × 50 cm
$\mu_s = 0.40 \quad \mu_k = 0.20$

Find

Acceleration at which box slips





VISUALISE:

There is only one horizontal force on the box, $\overrightarrow{f_s}$, and it points in the forward direction to accelerate the box.
Notice that we're solving the problem with the ground as our reference frame. Newton's laws are not valid in the accelerating truck because it is not an inertial reference frame.

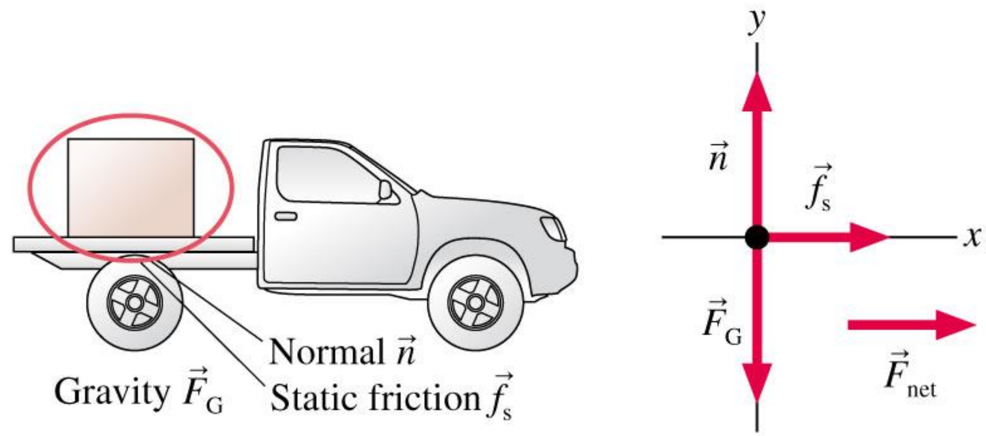
Make Sure the Cargo Doesn't Slide

Known

$m = 100$ kg
Box dimensions 50 cm × 50 cm × 50 cm
$\mu_s = 0.40 \quad \mu_k = 0.20$

Find

Acceleration at which box slips



SOLVE :

Newton's second law, which we can 'read' from the free-body diagram, is

$$\sum F_x = f_s = ma_x$$
$$\sum F_y = n - F_G = n - mg = ma_y = 0$$

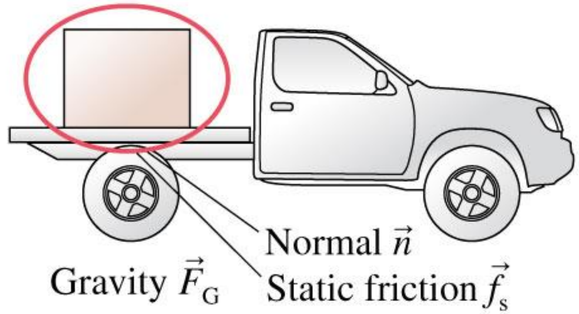
Make Sure the Cargo Doesn't Slide

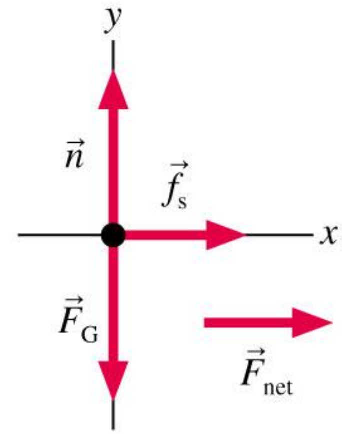
Known

$m = 100$ kg
Box dimensions 50 cm $\times$ 50 cm $\times$ 50 cm
$\mu_s = 0.40 \quad \mu_k = 0.20$

Find

Acceleration at which box slips





SOLVE :

$$\sum F_x = f_s = ma_x$$
$$\sum F_y = n - F_G = n - mg = ma_y = 0$$

Now, static friction, you will recall, can be any value between 0 and $f_{s\,max}$. If the truck accelerates slowly, so that the box doesn't slip, then $f_s < f_{s\,max}$. However, we're interested in the acceleration a_{max} at which the box begins to slip. This is the acceleration at which f_s reaches its maximum possible value

$$f_s = f_{s\,max} = \mu_s n$$

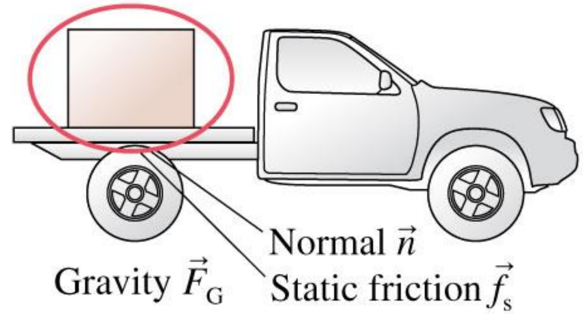
Make Sure the Cargo Doesn't Slide

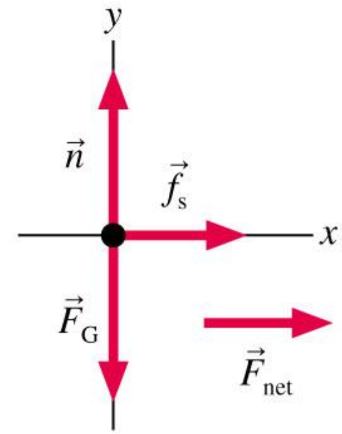
Known

$m = 100$ kg
Box dimensions 50 cm × 50 cm × 50 cm
$\mu_s = 0.40 \quad \mu_k = 0.20$

Find

Acceleration at which box slips





SOLVE :

$$f_s = f_{s\,max} = \mu_s n$$

The y-equation of the second law and the friction model combine to give $f_{s\,max} = \mu_s mg$. Substituting this into the x-equation, and noting that a_x is now a_{max}, we find

$$a_{max} = \frac{f_{s\,max}}{m} = \mu_s g = 3.9 m/s^2$$

The truck must keep its acceleration less than $3.9 m/s^2$ if slipping is to be avoided.

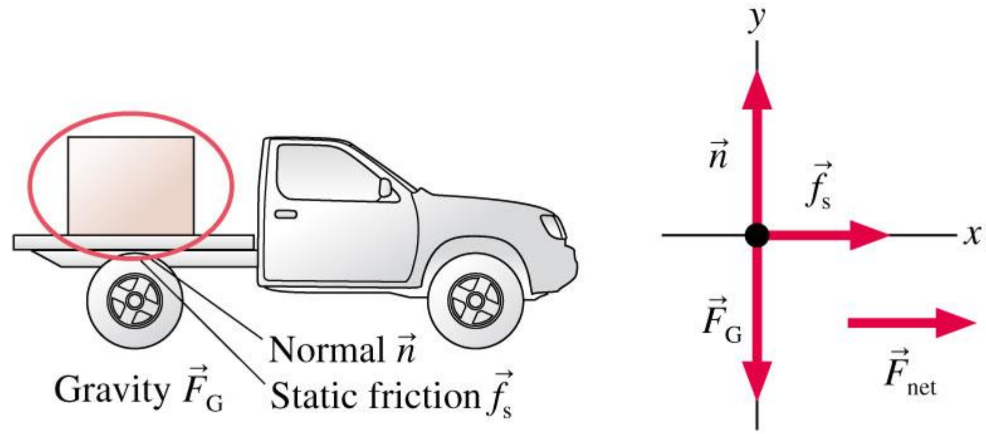
Make Sure the Cargo Doesn't Slide

Known

$m = 100$ kg
Box dimensions 50 cm $\times$ 50 cm $\times$ 50 cm
$\mu_s = 0.40$ $\mu_k = 0.20$

Find

Acceleration at which box slips



ASSESS:

$3.9 m/s^2$ is about one-third of g. You may have noticed that items in a car or truck are likely to tip over when you start or stop, but they slide only if you really 'floor it' and accelerate very quickly. So this answer seems reasonable. Notice that the dimensions of the crate were not needed. Real-word situations rarely have exactly the information you need, no more and no less.

TACTICS BOX 7.1 Analyzing interacting objects

❶ **Represent each object as a circle.** Place each in the correct position relative to other objects.

- Give each a name and a label.
- The surface of the earth (contact forces) and the entire earth (long-range forces) should be considered separate objects. Label the **e**ntire **e**arth EE.
- Ropes and pulleys often need to be considered objects.

TACTICS BOX 7.1

Analyzing interacting objects

❷ **Identify interactions.** Draw connecting lines between the circles to represent interactions.

- Draw *one* line for each interaction. Label it with the type of force.
- Every interaction line connects two and only two objects.
- There can be at most two interactions at a surface: a force parallel to the surface (e.g., friction) and a force perpendicular to the surface (e.g., a normal force).
- The entire earth interacts only by the long-range gravitational force.

TACTICS BOX 7.1 **Analyzing interacting objects**

3. **Identify the system.** Identify the objects of interest; draw and label a box enclosing them. This completes the interaction diagram.
4. **Draw a free-body diagram for each object in the system.** Include only the forces acting *on* each object, not forces exerted by the object.
   - Every interaction line crossing the system boundary is one external force acting on the object. The usual force symbols, such as $\vec{n}$ and $\vec{T}$, can be used.
   - Every interaction line within the system represents an action/reaction pair of forces. There is one force vector on *each* of the objects, and these forces always point in opposite directions. Use labels like $\vec{F}_{\text{A on B}}$ and $\vec{F}_{\text{B on A}}$.
   - Connect the two action/reaction forces—which must be on *different* free-body diagrams—with a dashed line.

Exercises 1–7

Newton's 3rd Law

Q.21 A mosquito runs head-on into a truck. Splat! Which is true during the collision?

a) The mosquito exerts more force on the truck than the truck exerts on the mosquito.
b) The truck exerts more force on the mosquito than the mosquito exerts on the truck.
c) The mosquito exerts the same force on the truck as the truck exerts on the mosquito.
d) The truck exerts a force on the mosquito but the mosquito does not exert a force on the truck.
e) The mosquito exerts a force on the truck but the truck does not exert a force on the mosquito.

Newton's 3rd Law

Q.21 A mosquito runs head-on into a truck. Splat! Which is true during the collision?

a) The mosquito exerts more force on the truck than the truck exerts on the mosquito.

b) The truck exerts more force on the mosquito than the mosquito exerts on the truck.

c) The mosquito exerts the same force on the truck as the truck exerts on the mosquito.

d) The truck exerts a force on the mosquito but the mosquito does not exert a force on the truck.

e) The mosquito exerts a force on the truck but the truck does not exert a force on the mosquito.

Newton's 3rd Law

Q.22 The same mosquito runs head-on into a truck. Which is also true during the collision?

a) The magnitude of the mosquito's acceleration is larger than that of the truck.

b) The magnitude of the truck's acceleration is larger than that of the mosquito.

c) The magnitude of the mosquito's acceleration is the same as that of the truck.

d) The truck accelerates but the mosquito does not.

e) The mosquito accelerates but the truck does not.

Q.22 The same mosquito runs head-on into a truck. Which is also true during the collision?

a) **The magnitude of the mosquito's acceleration is larger than that of the truck.**

b) The magnitude of the truck's acceleration is larger than that of the mosquito.

c) The magnitude of the mosquito's acceleration is the same as that of the truck.

d) The truck accelerates but the mosquito does not.

e) The mosquito accelerates but the truck does not.

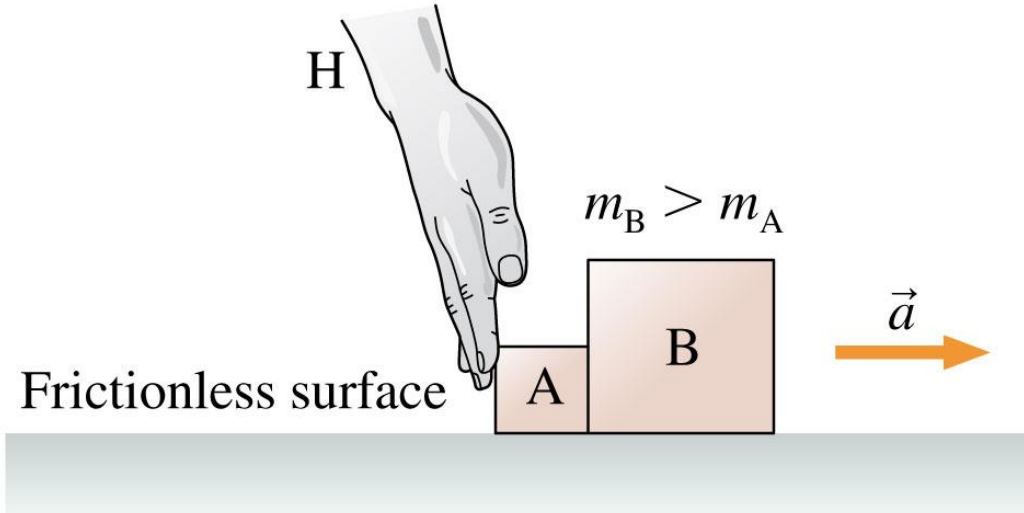
Same for both

Newton's second law: $$a = \frac{F}{m}$$

Huge difference

Don't confuse cause and effect! The same force can have very different effects.

The Force on Accelerating Boxes

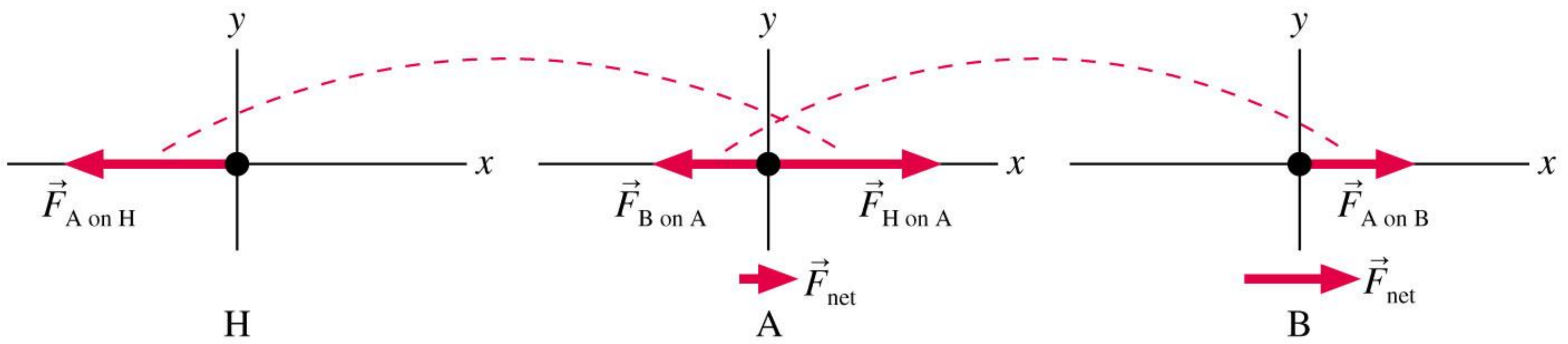


Q.23 The hand shown in the figure above pushes boxes A and B to the right across a frictionless table. The mass of B is larger than the mass of A.

a. Draw free-body diagrams of A, B, and the hand H, showing only the horizontal forces. Connect action/reaction pairs with dashed lines.

b. Rank in order, from largest to smallest, the horizontal forces shown on your free-body diagrams.

The Force on Accelerating Boxes



VISUALISE:

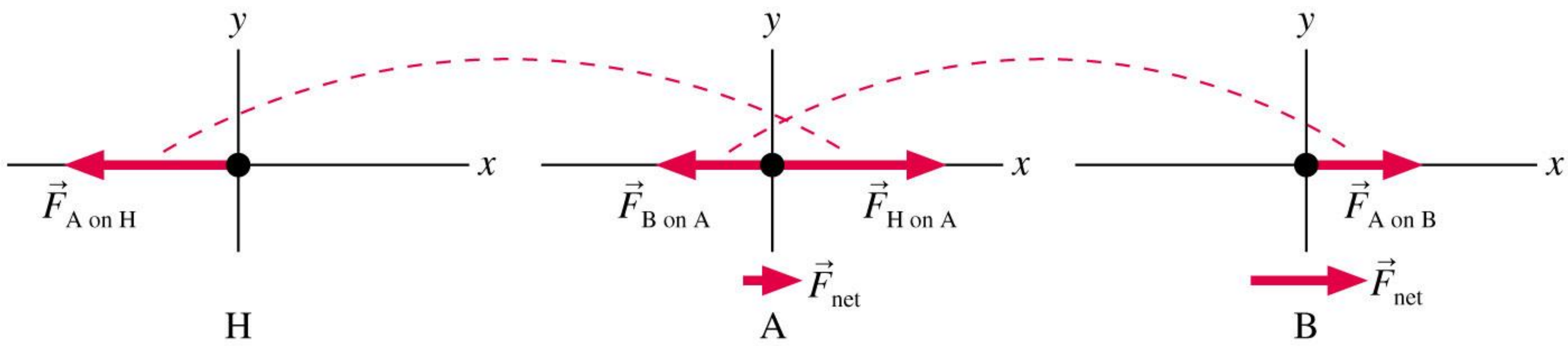
a. The hand H pushes on box A, and A pushes back on H. Thus, $\vec{F}_{B\ on\ A}$ and $\vec{F}_{A\ on\ B}$ are an action/reaction pair.

Similarly, A pushes on B and B pushes back on A.

The hand H does not touch box B, so there is no interaction between them. There is no friction.

The figure above shows the four horizontal forces and identifies two action/reaction pairs. Notice that each force is shown on the free-body diagram of the object that it acts on.

The Force on Accelerating Boxes



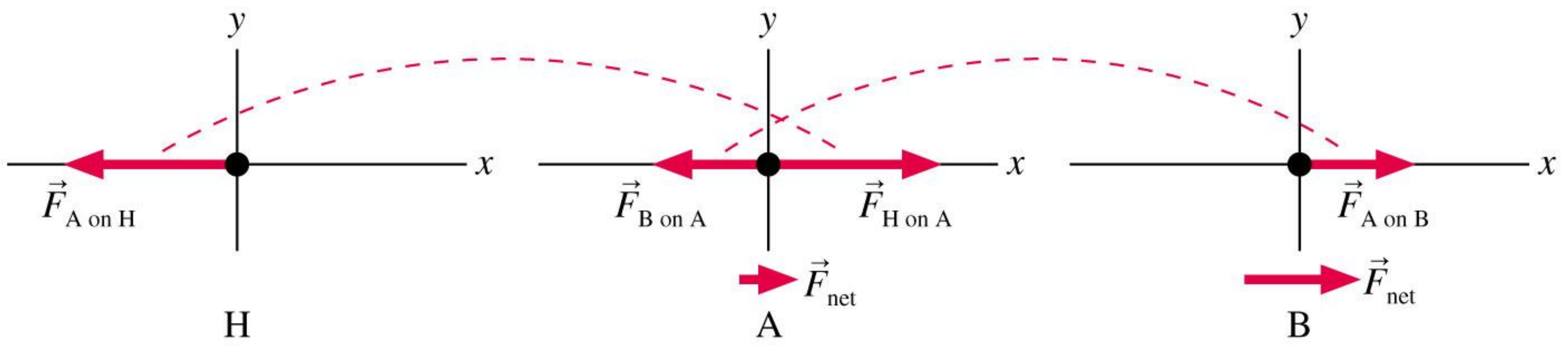
VISUALISE:

b. According to Newton's third law, $F_{A\ on\ H} = F_{H\ on\ A}$ and $F_{A\ on\ B} = F_{B\ on\ A}$.

But the third law is not our only tool. The boxes are accelerating to the right, because there's no friction, so Newton's second law tells us that box A must have a net force to the right. Consequently, $F_{H\ on\ A} > F_{B\ on\ A}$. Thus

$$F_{A\ on\ H} = F_{H\ on\ A} > F_{B\ on\ A} = F_{A\ on\ B}$$

The Force on Accelerating Boxes



VISUALISE:

$F_{A\ on\ H} = F_{H\ on\ A} > F_{B\ on\ A} = F_{A\ on\ B}$

ASSESS:

You might have expected $F_{A\ on\ B}$ to be larger than $F_{H\ on\ A}$ because $m_B > m_A$. It's true that the net force on B is larger than the net force on A, but we have to reason more closely to judge the individual forces. Notice how we used both the second and the third laws to answer this question.

PROBLEM-SOLVING STRATEGY 7.1

Interacting-objects problems

MODEL Identify which objects are part of the system and which are part of the environment. Make simplifying assumptions.

VISUALIZE Draw a pictorial representation.

- Show important points in the motion with a sketch. You may want to give each object a separate coordinate system. Define symbols and identify what the problem is trying to find.
- Identify acceleration constraints.
- Draw an interaction diagram to identify the forces on each object and all action/reaction pairs.
- Draw a *separate* free-body diagram for each object. Each shows only the forces acting *on* that object, not forces exerted by the object.
- Connect the force vectors of action/reaction pairs with dashed lines. Use subscript labels to distinguish forces that act independently on more than one object.

PROBLEM-SOLVING STRATEGY 7.1

Interacting-objects problems

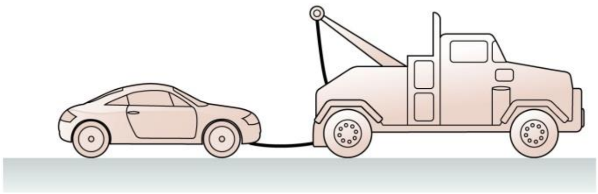
SOLVE Use Newton's second and third laws.

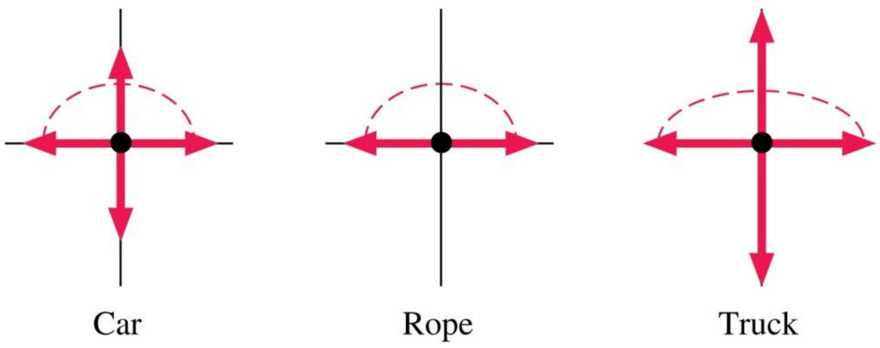
- Write the equations of Newton's second law for *each* object, using the force information from the free-body diagrams.
- Equate the magnitudes of action/reaction pairs.
- Include the acceleration constraints, the friction model, and other quantitative information relevant to the problem.
- Solve for the acceleration, then use kinematics to find velocities and positions.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Newton's 3rd Law

Q.24 What, if anything, is wrong with these free-body diagrams for a truck towing a car at steady speed? The truck is heavier than the car and the rope is massless.





a) Nothing is wrong.
b) One or more forces have the wrong length.
c) One of more forces have the wrong direction.
d) One or more action/reaction pairs are wrong.
e) Both B and D.

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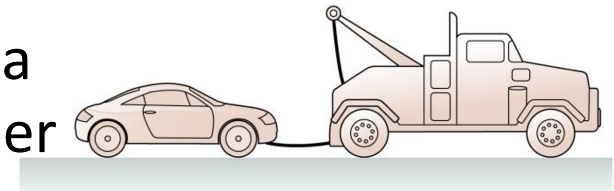

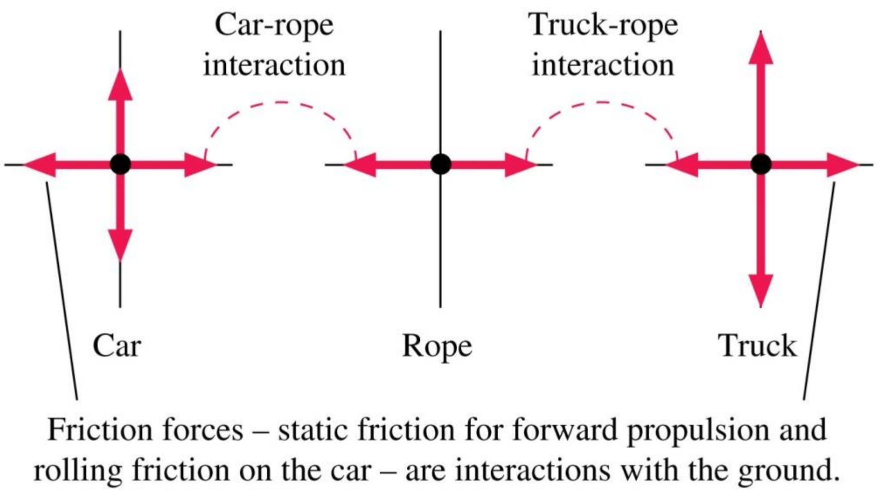

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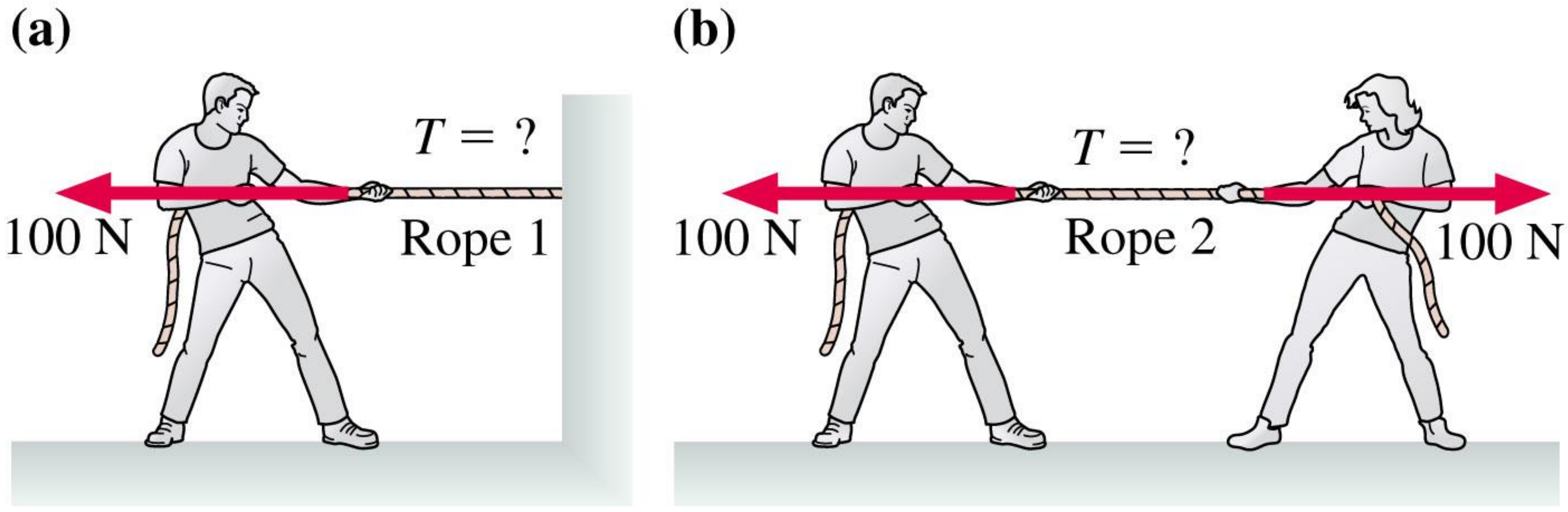
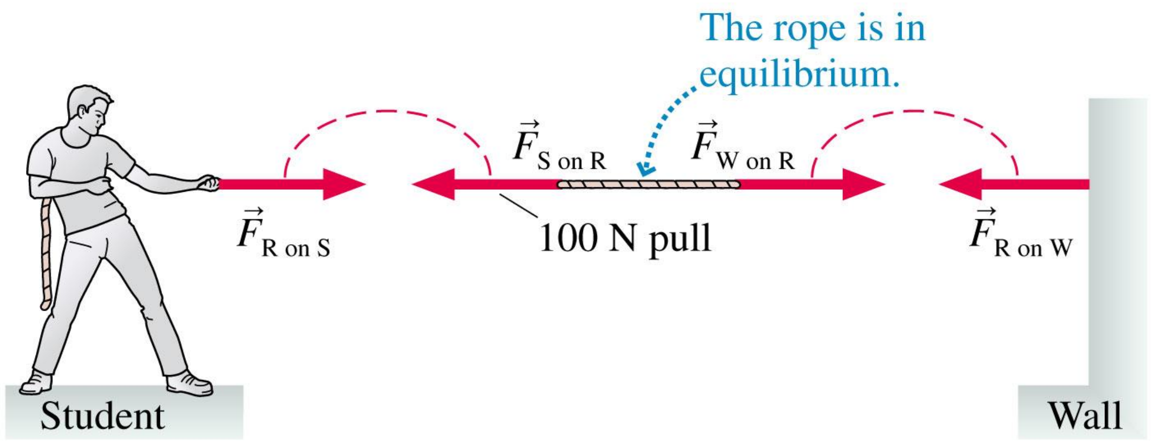
Pulling a Rope



Q.25 Figure (a) shows a student pulling horizontally with a 100 N force on a rope that is attached to a wall. In figure (b) two students in a tug-of-war pull on opposite ends of a rope with 100 N each. Is the tension in the second rope larger than, smaller than, or the same as that in the first rope?

Pulling a Rope

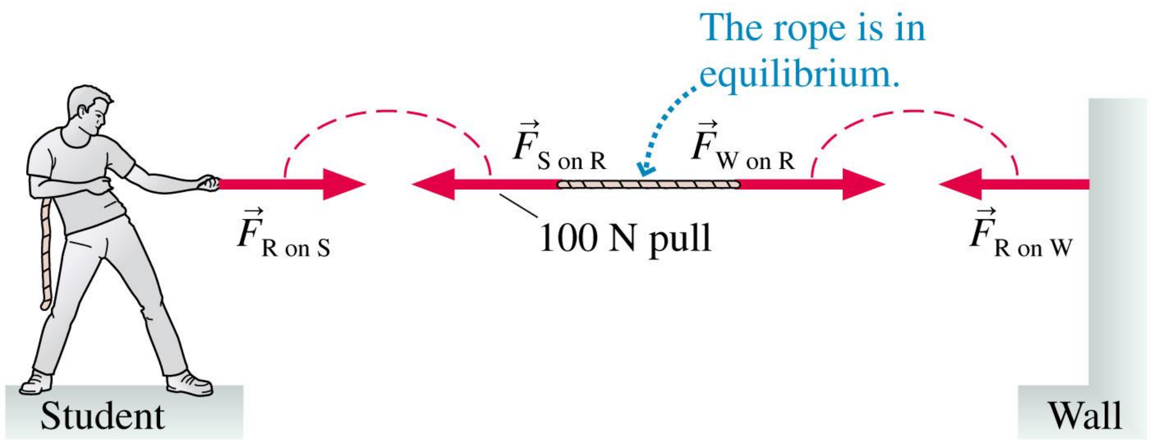


SOLVING A

Surely pulling on a rope from both ends causes more tension than pulling on one end. Right? Before jumping to conclusion, let's analyse the situation carefully.

Figure (a) shows the first student, the rope, and the wall as separate, interacting objects. Force $\vec{F}_{S\ on\ R}$ is the student pulling on the rope, so it has magnitude 100 N.

Pulling a Rope

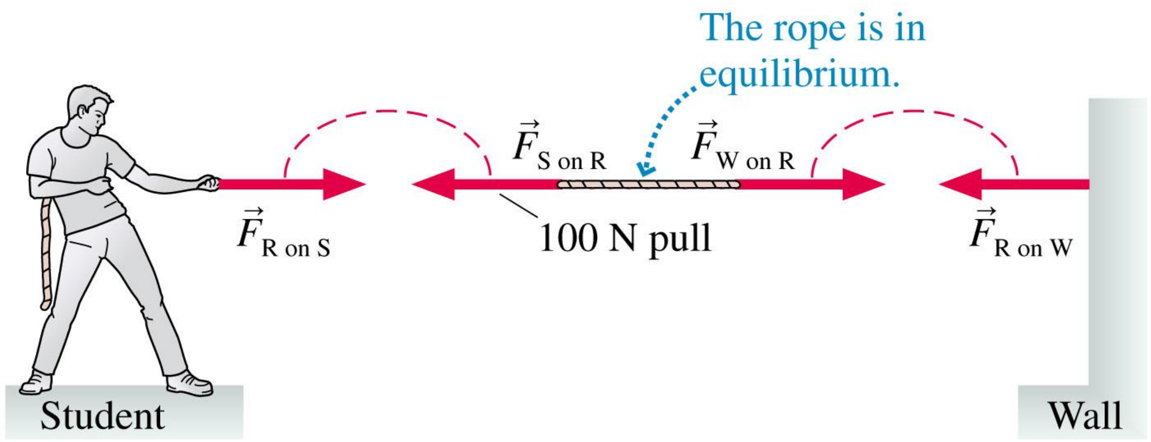


SOLVING A

Forces $\vec{F}_{S\ on\ R}$ and $\vec{F}_{R\ on\ S}$ are an action/reaction pair and must have equal magnitudes. Similarly for forces $\vec{F}_{W\ on\ R}$ and $\vec{F}_{R\ on\ W}$. Finally, because the rope is in static equilibrium, force $\vec{F}_{W\ on\ R}$ has to balance force $\vec{F}_{S\ on\ R}$. Thus

$F_{R\ on\ W}$ = $F_{W\ on\ R}$ = $F_{S\ on\ R}$ = $F_{R\ on\ S}$ = 100 N

Pulling a Rope

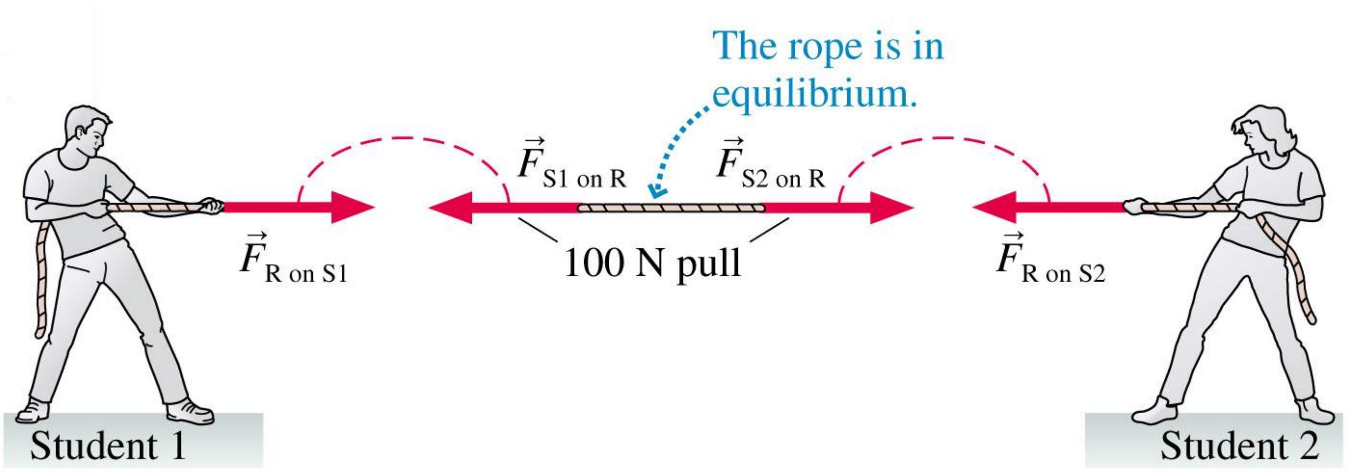


SOLVING A

The first and third equalities are Newton's third law; the second equality is Newton's first law for the rope.

Forces $\vec{F}_{R\ on\ S}$ and $\vec{F}_{R\ on\ W}$ are the pulling forces exerted by the rope and are what we *mean* by 'the tension in the rope'. Thus the tension in the first rope is 100 N.

Pulling a Rope



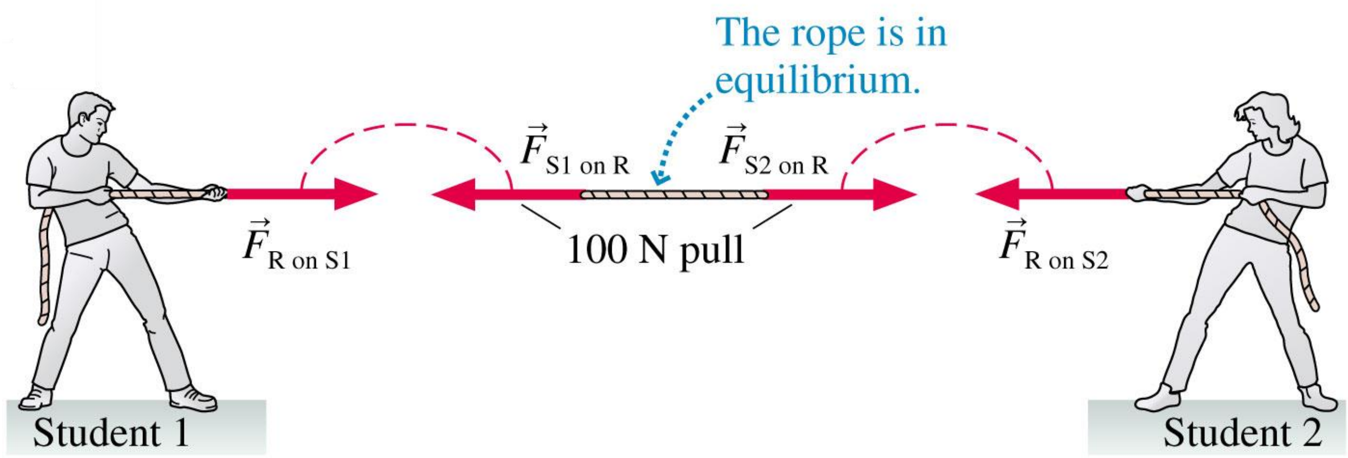
SOLVING B

In figure b, we repeat the analysis for the rope pulled by the two students. Each student pulls with 100 N, so $F_{S1\ on\ R}$ = 100 N and $F_{S2\ on\ R}$ = 100 N.

Just as before, there are two action/reaction pairs, and the rope is in static equilibrium. Thus

$F_{R\ on\ S2}$ = $F_{S2\ on\ R}$ = $F_{S1\ on\ R}$ = $F_{R\ on\ S1}$ = 100 N

Pulling a Rope

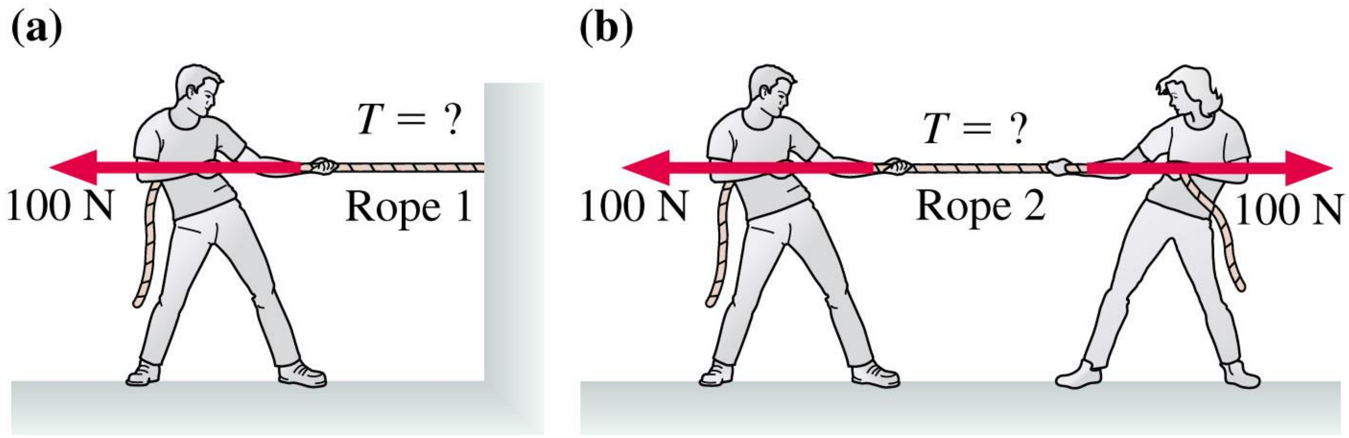


SOLVING B

The tension in the rope (the pulling forces $\vec{F}_{R\ on\ S1}$ and $\vec{F}_{R\ on\ S2}$) is still 100 N!

You may have assumed that the student on the right in Figure b is doing something to the rope that the wall in Figure a does not do. But our analysis finds that the wall, just like the student, pulls to the right with 100 N. The rope doesn't care whether it's pulled by a wall or a hand. It experiences the same forces in both cases, so the rope's tension is the same in both.

Pulling a Rope



ASSESS:

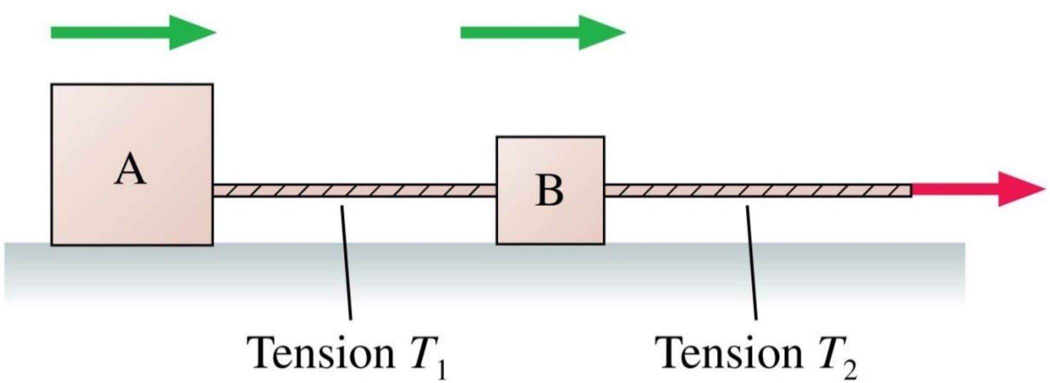
Ropes and strings exert forces at **both** ends. The force with which they pull (and thus the force pulling on them at each end) is the tension in the rope. Tension is not the sum of the pulling forces.

The rope's tension is the same in both situations.

Tension Forces

Q.26 Boxes A and B are being pulled to the right on a frictionless surface. Box A has a larger mass than B. How do the two tension forces compare?

a) $T_1 > T_2$
b) $T_1 = T_2$
c) $T_1 < T_2$
d) Not enough information to tell.



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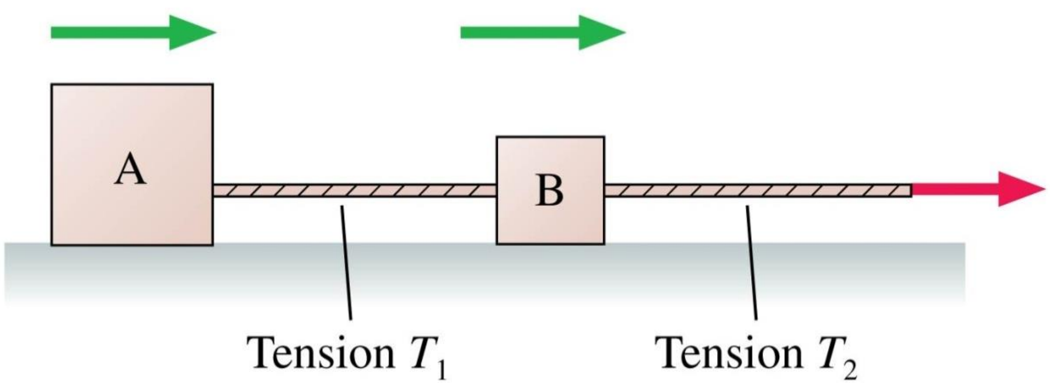
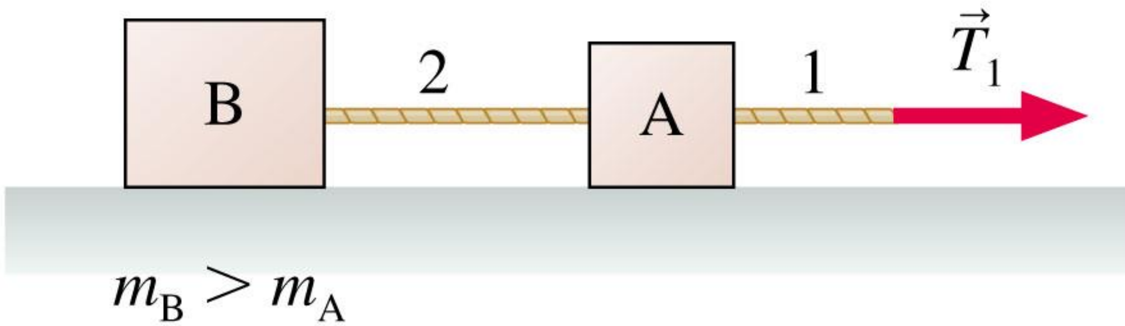
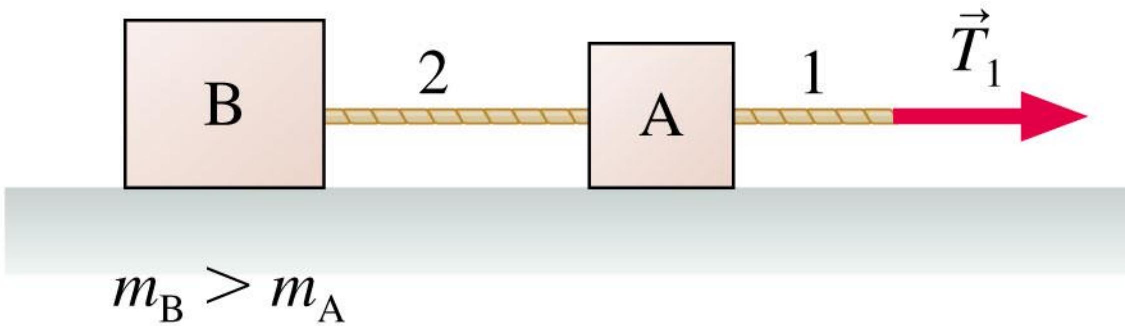


Comparing Two Tensions



Q.27 Blocks A and B in the figure below are connected by massless string 2 and pulled across a frictionless table by massless string 1. B has a larger mass than A. Is the tension in string 2 larger than, smaller than, or equal to the tension in string 1?

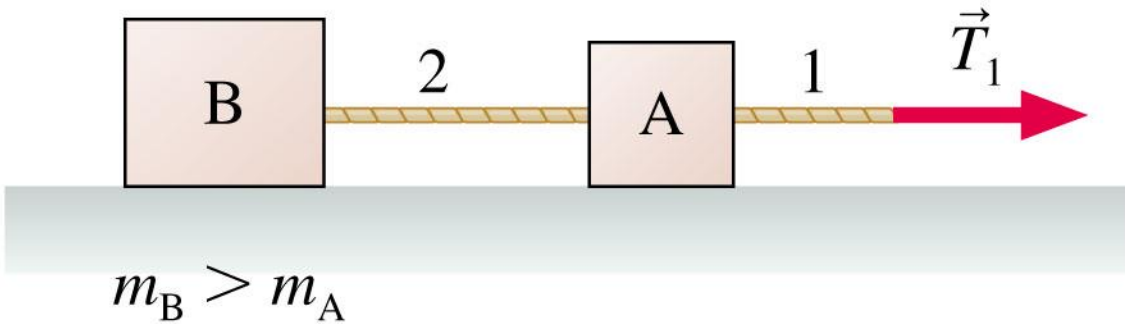
Comparing Two Tensions



MODEL:

The massless string approximation allows us to treat A and B as if they interact directly with each other. The blocks are accelerating because there's a force to the right and no friction.

Comparing Two Tensions



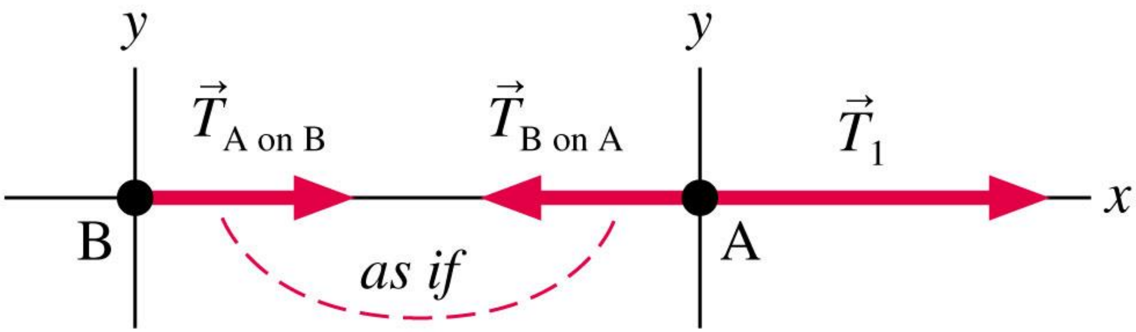
SOLVE:

B has larger mass, so it may be tempting to conclude that string 2, which pulls B, has a greater tension than string 1, which pulls A.

The flaw in this reasoning is that Newton’s second law tells us only about the net force. The net force on B is larger than the net force on A, but the net force on A **is not** just the tension $\vec{T}_1$ in the forward direction.

The tension in string 2 also pulls backward on A!

Comparing Two Tensions



SOLVE:

The figure above shows the horizontal forces in this frictionless situation. Forces $\vec{T}_{A\ on\ B}$ and $\vec{T}_{B\ on\ A}$ act **as if** they are an action/reaction pair.

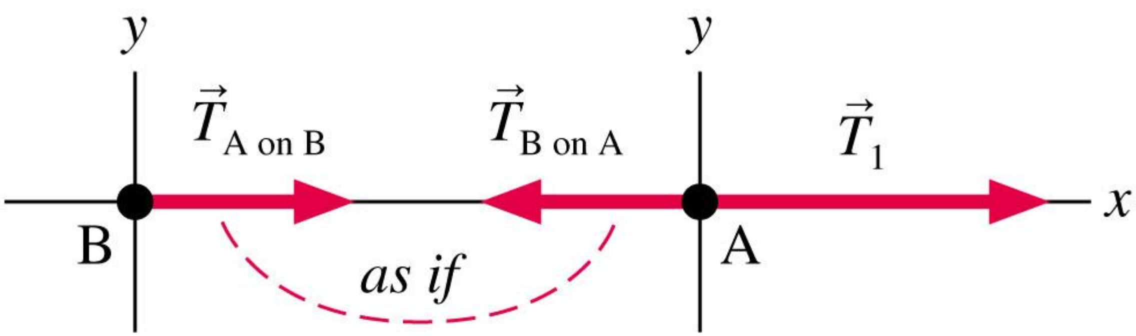
For Newton's third law,

$T_{A\ on\ B} = T_{B\ on\ A} = T_2$

Where T_2 is the tension in string 2. From Newton's second law, the net force on A is

$1(T_{\text{A net}})_1 = T_1 - T_{B\ on\ A} = T_1 - T_2 = m_A a_{A1}$

Comparing Two Tensions



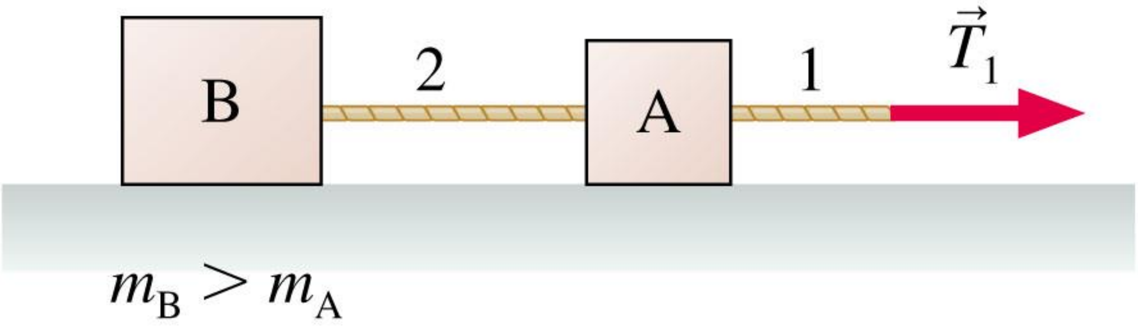
SOLVE:

The net force on A is the difference in tensions. The blocks are accelerating to the right, making $a_{A1} > 0$, so

$$T_1 > T_2$$

The tension in string 2 is smaller than the tension in string 1.

Comparing Two Tensions



$$T_1 > T_2$$

ASSESS:

This is not an intuitively obvious result. A careful study of the reasoning in this example is worthwhile. An alternative analysis would note that $\vec{T}_1$ accelerates **both** blocks, of combined mass (m_A+m_B), whereas $\vec{T}_2$ accelerates only block B. Thus string 1 must have the larger tension.