Halo Rarity-Tapster

Kieran Thomas

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In this short write up I consider how a prototypical Bell state behaves in our Rarity-Tapster set up. We consider two input modes which exactly match the Bragg condition (i.e. are separated by exactly 2 lattice wave vectors), in this case either a_+ and b_+ or the minus versions in Fig. 1. We can write the coupling Hamiltonian (in basis $\{a_{+/-}, b_{+/-}\}$) as

$$\hat{H} = \frac{\hbar\Omega}{2}\begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix}, \tag{1}$$

where $\Omega/2\pi$ is the two-photon Rabi frequency and ϕ is the phase difference between the two lasers forming the Bragg lattice. The evolution operator then takes the form

$$\hat{U}(t,\phi) = e^{-i\hat{H}t/\hbar} = \begin{pmatrix} \cos(\Omega t/2) & -ie^{-i\phi}\sin(\Omega t/2) \\ -ie^{i\phi}\sin(\Omega t/2) & \cos(\Omega t/2). \end{pmatrix} \tag{2}$$

The dynamics of our the Rarity-Tapster setup, see Fig. 1, can then be modeled as the application of a π-pulse and $\pi/2$-pulse, i.e. $\hat{U}(\pi/2\Omega,\phi)\hat{U}(\pi/\Omega,\phi_D)$, which can be written as

$$\hat{U}(\pi/2\Omega,\phi)\hat{U}(\pi/\Omega,\phi_D) = \hat{A} = \begin{pmatrix} e^{-i(\phi-\phi_D)} & ie^{-i\phi_D} \\ ie^{i\phi_D} & e^{i(\phi-\phi_D)} \end{pmatrix}. \tag{3}$$

Where ϕ is the phase of the beam splitter and ϕ_D is the phase of the mirror. In order to obtain the input modes as functions of the output modes we invert this matrix,

$$\hat{A}^{-1} = \frac{1}{2}\begin{pmatrix} e^{i(\phi-\phi_D)} & -ie^{-i\phi_D} \\ -ie^{i\phi_D} & e^{-i(\phi-\phi_D)} \end{pmatrix} \tag{4}$$

As the phase of the mirror does not affect the dynamics we can chose it for convenience to be $\phi_D = \pi/2$

$$\hat{A}^{-1} = \frac{1}{2}\begin{pmatrix} -ie^{i\phi} & -i\times -i \\ -i\times i & ie^{-i\phi} \end{pmatrix} \tag{5}$$

$$= \frac{1}{2}\begin{pmatrix} -ie^{i\phi} & -1 \\ 1 & ie^{-i\phi} \end{pmatrix} \tag{6}$$

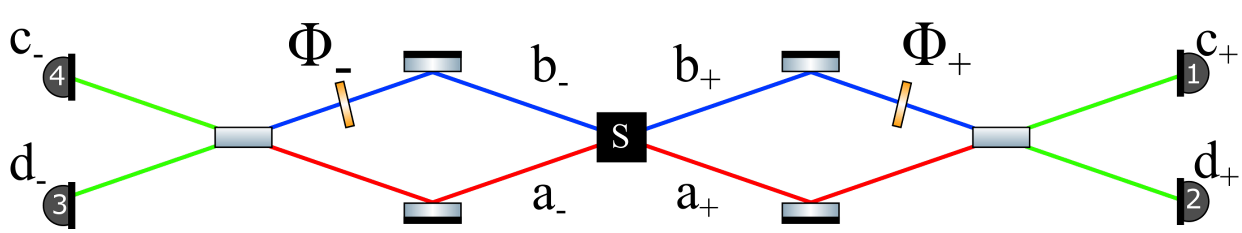

Figure 1: A diagrammatic representation of our Rarity-Tapster set up. The b-modes can be considered to be from one halo while the a-modes are from the other.

Now initial state $|\psi\rangle$ is the classic bell state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00,11\rangle + |11,00\rangle\right) \tag{7}$$

$$= \frac{1}{\sqrt{2}}\left(a_-^\dagger a_+^\dagger\, |0,0,0,0\rangle_{a-,a+,b-,b+} + \hat{b}_-^\dagger \hat{b}_+^\dagger\, |0,0,0,0\rangle_{a-,a+,b-,b+}\right) \tag{8}$$

and we can write our input modes in terms of our output modes as follows,

$$\begin{pmatrix} a_+ \\ b_+ \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -ie^{i\phi_+} & -1 \\ 1 & ie^{-i\phi_+} \end{pmatrix}\begin{pmatrix} c_+ \\ d_+ \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} a_- \\ b_- \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -ie^{i\phi_-} & -1 \\ 1 & ie^{-i\phi_-} \end{pmatrix}\begin{pmatrix} c_- \\ d_- \end{pmatrix} \tag{10}$$

We can hence write the output state as

$$|\psi'\rangle = \frac{1}{2\sqrt{2}}\left((-ie^{i\phi_-}c_- - d_-)(-ie^{i\phi_+}c_+ - d_+)\,|0\rangle + (c_- + ie^{-i\phi_-}d_-)(c_+ + ie^{-i\phi_+}d_+)\,|0\rangle\right) \tag{11}$$

$$= \frac{1}{2\sqrt{2}}((-e^{i(\phi_-+\phi_+)}c_-c_+ + ie^{i\phi_-}c_-d_+ + ie^{i\phi_+}c_+d_- + d_-d_+)\,|0\rangle + ... \tag{12}$$

$$(c_-c_+ + ie^{-i\phi_-}d_-c_+ + ie^{-i\phi_+}d_+c_- - e^{-i(\phi_-+\phi_+)}d_-d_+)\,|0\rangle) \tag{13}$$

$$= \frac{1}{2\sqrt{2}}\Big[(-e^{i(\phi_-+\phi_+)} + 1)c_-c_+ + \tag{14}$$

$$(ie^{i\phi_-} + ie^{-i\phi_+})d_-c_+ + \tag{15}$$

$$(ie^{i\phi_+} + ie^{-i\phi_-})d_+c_- + \tag{16}$$

$$(1 - e^{-i(\phi_-+\phi_+)})d_-d_+\Big]\,|0\rangle \tag{17}$$

Thus for the four possible output states we get the following probabilities

$$P(c_-c_+) = \frac{1}{2}\sin\left(\frac{\phi_- + \phi_+}{2}\right)^2 \tag{18}$$

$$P(d_-c_+) = \frac{1}{2}\cos\left(\frac{\phi_- + \phi_+}{2}\right)^2 \tag{19}$$

$$P(d_+c_-) = \frac{1}{2}\cos\left(\frac{\phi_- + \phi_+}{2}\right)^2 \tag{20}$$

$$P(d_-d_+) = \frac{1}{2}\sin\left(\frac{\phi_- + \phi_+}{2}\right)^2. \tag{21}$$

So all our states only depend on the sum $\phi_- + \phi_+$ and thus a global phase shift should cause a measurable effect.

To see the effect of mode occupancy we consider the full state:

$$|\Psi\rangle_{a,b} = (1-\lambda^2)\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\lambda^{(m+k)}\,|k,k,m,m\rangle_{a-,a+,b-,b+}. \tag{22}$$

Applying techniques as described above we find that the back to back g^2 functions of the respective halos (correlations between modes c_+ and c_- or d_+ and d_- in Fig. 1) should vary as

$$g^2_{BB} = \left(\frac{1}{\lambda^2}\right)\cos\left(\frac{\phi_- + \phi_+}{2}\right)^2 + 1. \tag{23}$$

Hence mode occupancy will decrease the signal, but not at a rate greater than it reduces the normal back-to-back correlation function.