Analytic expressions for ANU BEC collision experiment

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These notes correct typographical errors in Appendix B of Phys. Rev. A 91, 052114 (2015).

I. SETUP

Let's start from Eq. (B4) of Ref. [1],

$$\frac{G^{(2)}(\mathbf{k},\mathbf{k}')}{\bar{n}^2} = 1+\frac{h}{2}\left[1 \pm \cos(\Phi + \varphi(\mathbf{k},\mathbf{k}'))\right]\prod_d e^{-(\mathbf{k}+\mathbf{k}')^2/2\sigma_d^2}, \quad (1)$$

which describes the two-point correlations out the output of the Rarity-Tapster scheme. Here, σ_d is the back-to-back (BB) correlation length and h the correlation strength. The correlation is normalized by the nominally constant halo density $\bar{n}$. The phase Φ encodes the chosen phase-shifts of the Rarity-Tapster scheme (e.g., $\Phi = \phi_L \pm \phi_R$ depending on how we encode them) and $\varphi(\mathbf{k},\mathbf{k}')$ encodes some dependence on the phase relation between the two scattered pairs (see Eqs. (B2) and (B3) of Ref. [1]).

We can compute the experimentally observed coincidence counts, C_{ij}, by integrating,

$$C_{ij} = \int_V (\mathbf{k}_i) d^3\mathbf{k} \int_V (\mathbf{k}_j) d^3\mathbf{k}' \frac{G^{(2)}(\mathbf{k},\mathbf{k}')}{\cdot} \quad (2)$$

and then typically are interested in the correlation coefficient,

$$E(\Phi) = \frac{C_{14}+C_{23}-C_{12}-C_{34}}{C_{14}+C_{23}+C_{12}+C_{34}}|_\Phi. \quad (3)$$

II. NO DEPHASING

If we first ignore the dephasing contribution, $\varphi(\mathbf{k},\mathbf{k}') = 0$, then we have

$$C_{ij} = \bar{n}^2 V + \frac{\bar{n}^2 h}{2}\left[1 \pm \cos(\Phi)\right]\prod_d \left(2\sigma_d^2\alpha_d\right), \quad (4)$$

where $V = \prod_d \Delta k_d$ for the detection bins of width Δk_d and $d = x, y, z$, and

$$\alpha_d = e^{-2\lambda_d^2} - 1 + \sqrt{2\pi}\lambda_d \mathrm{erf}(\sqrt{2}\lambda_d), \quad (5)$$

for $\lambda_d = \Delta k_d/(2\sigma_d)$. Note the additional factor $2\sigma_d^2$ in Eq. (4) for C_{ij}, which corrects Eq. (B8) of Ref. [1].

Next, we substitute Eq. (4) into Eq. (3) to obtain,

$$E(\Phi) = \frac{h\prod_d(2\sigma_d^2\alpha_d)}{2V^2 + h\prod_d(2\sigma_d^2\alpha_d)}\cos(\Phi), \quad (6)$$

or in terms of the dimensionless ratio λ_d,

$$E(\Phi) = \frac{h\prod_d \alpha_d}{16\prod_d \lambda_d^2 + h\prod_d \alpha_d}\cos(\Phi). \quad (7)$$

This corrects Eq. (B10) from Ref. [1], and is equivalent to Kieran's derivation with the alternative definition of α_d.

I have independently used the numbers provided in Kieran's note, and I agree that in the weak and tight trap cases one obtains $E(0) \simeq 0.81$ and $E(0) \simeq 0.45$, respectively.

III. DEPHASING

Instead, we can also keep the dephasing contribution with

$$\varphi(\mathbf{k},\mathbf{k}') = A\left[8|\mathbf{k}_L|^2 - 4\mathbf{k}_L\cdot(\mathbf{k}-\mathbf{k}')\right], \quad (8)$$
$$A = \frac{\hbar}{2m}\left(\Delta t_{\mathrm{free}} - t_2\right) + \frac{\sigma_{g,z}}{2|\mathbf{k}_0|\sqrt{\pi}}.$$

Here, $\sigma_{g,z}$ is related to the characteristic width of the initial BEC along the axis it is split ($\hat{z}$).

The evaluation of Eq. (2) is now more involved, but after some manipulation one obtains,

$$C_{ij} = \bar{n}^2 V + \frac{\bar{n}^2 h}{2}\left[1 \pm \cos(\Phi)\right]\alpha_x\alpha_y\beta_z\prod_d\left(2\sigma_d^2\right), \quad (9)$$

This form assumes the Bragg pulses act along $\hat{z}$ and

$$\begin{aligned}\beta_d = i\sqrt{\frac{\pi}{2}}&\frac{e^{-8A^2|\mathbf{k}_L|^2\sigma_d^2}}{8\sigma_d A|\mathbf{k}_L|} \\ &\times\left[e^{-4iA|\mathbf{k}_L|\Delta k}\mathrm{erf}\left(\frac{\Delta k - 4iA|\mathbf{k}_L|\sigma_d^2}{\sqrt{2}\sigma_d}\right)\right. \\ &\quad - e^{4iA|\mathbf{k}_L|\Delta k}\mathrm{erf}\left(\frac{\Delta k + 4iA|\mathbf{k}_L|\sigma_d^2}{\sqrt{2}\sigma_d}\right) \\ &\quad\left. + 2\cos(4A|\mathbf{k}_L|\Delta k)\mathrm{erf}(i2\sqrt{2}A|\mathbf{k}_L|\sigma_d)\right]. \quad (10)\end{aligned}$$

The expression for β_d deviates by a small sign error in the Erf functions and a prefactor (due to the $2\sigma_d^2$ bug previously mentioned for α_d) from Eq. (B9) of Ref. ([1]. One can check that in the limit $A \to 0$ [equivalent to

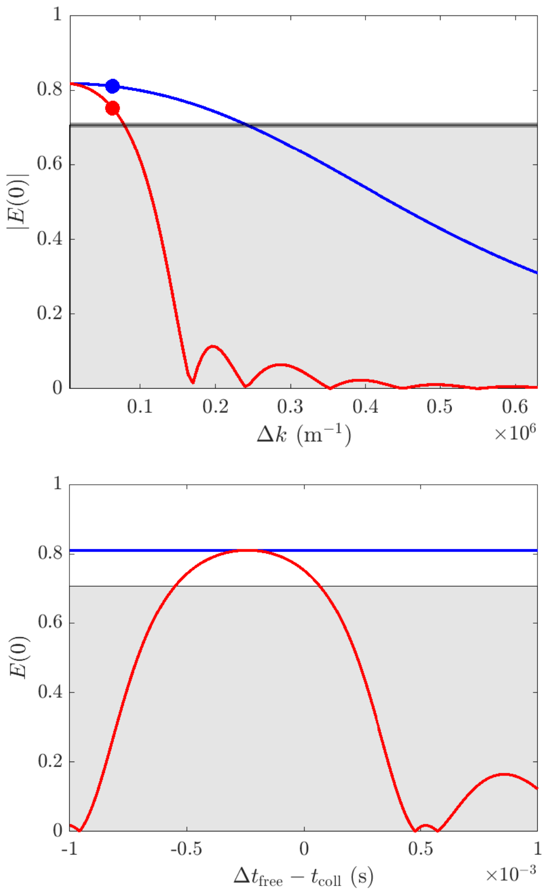

FIG. 1. Weak trap calculations. (Top) Amplitude of correlation coefficient, $|E(0)|$, as a function of detection bin size. Both the ideal result Eq. (7) (blue line) and the more realistic Eq. (11) are compared. (Bottom) Influence of free propagation time and thus dephasing (β_d correction) on $|E(0)|$. Same colors as previous.

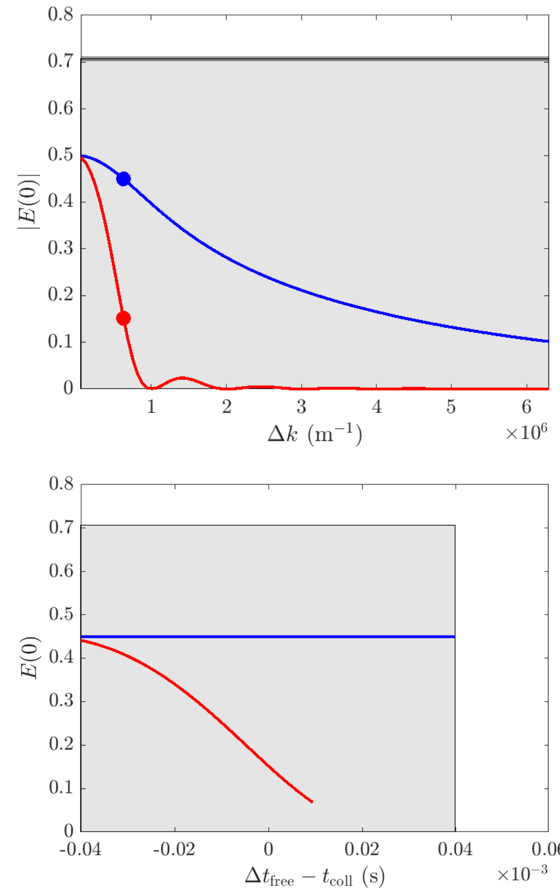

FIG. 2. Tight trap calculations. (Top) Amplitude of correlation coefficient, $|E(0)|$, as a function of detection bin size. Both the ideal result Eq. (7) (blue line) and the more realistic Eq. (11) are compared. (Bottom) Influence of free propagation time and thus dephasing (β_d correction) on $|E(0)|$. Same colors as previous.

removing the $\varphi(\mathbf{k}, \mathbf{k}')$ contribution] we correctly obtain $\beta_d \to \alpha_d$.

Again, we use Eq. (9) to construct the correlation coefficient:

$$E(\Phi) = \frac{h\alpha_x \alpha_y \beta_z}{16 \prod_d \lambda_d^2 + h\alpha_x \alpha_y \beta_z} \cos(\Phi). \tag{11}$$

IV. APPROXIMATE CALCULATION

In Figs. 1 and 2 we present approximate results for the weak and tight trap scenarios considered in Kieran's notes. In both cases parameters are estimated based on previous emails and notes:

- $N = 2 \times 10^5$
- $|\mathbf{k}_0| = |\mathbf{k}_L| = 4.1 \times 10^6$ m^{-1}
- Collsion duration: $t_{\rm coll} = 1$ ms (weak trap) and $t_{\rm coll} = 40$ μs (tight trap)
- $\sigma_{g,z} \approx 0.6 R_z$ where R_z is the Thomas-Fermi radius of the initial BEC along the splitting direction.

The numbers for N and particularly $\sigma_{g,z}$ are crude guesses, and factors of two in these impact the quantitative results.

In the top panels of Figs. 1 and 2 we plot the dependence of the amplitude of the correlation coefficient, $|E(0)|$ on the detection bin volume and assume a free propagation time matched to the collision duration, $\Delta t_{\rm free} = t_{\rm coll}$. The blue solid lines indicate the idealized calculation with no dephasing contribution, Eq. (7), while the red solid lines include dephasing, Eq. (11). In the weak trap scenario dephasing is predicted to introduce only a minor correction, whereas in the tight trap the amplitude $|E(0)|$ is suppressed much more strongly.

The lower panels of Figs. 1 and 2 shows the expected dependence of $|E(0)|$ on the mismatch between the free propagation and collision durations. For the weak trap the optimal free propagation time Δt_{free} is slightly less than the collision duration, but for the tight trap the effect of dephasing during the collision process is such that $\Delta t_{\text{free}} \to 0$ is optimal.

[1] R. J. Lewis-Swan and K. V. Kheruntsyan, Phys. Rev. A **91**, 052114 (2015).