Role of gravity in an atomic Rarity-Tapster scheme

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We estimate the magnitude of the phase shift due to gravity in an atomic Rarity-Tapster scheme based on four-wave mixing in BEC collisions. Similar to previous notes, we focus on the collision configuration of the current ANU experiment and highlight differences to the previously considered setup of Phys. Rev. A 91, 052114 (2015). We comment on the implications for the experimental observation of a violation of a CHSH-Bell inequality.

Our calculation follows the semi-classical calculation outlined in Kieran's notes, wherein the phase accrued due to gravity is proportional to the action of a particle tracing a classical trajectory through the Rarity-Tapster interferometer. Specifically, we can write

$$\psi = \frac{m}{\hbar} \int_{t_{\rm birth}}^{t_{\rm BS}} \left[\frac{|\mathbf{v}(\tau)|^2}{2} - g z(\tau) \right] d\tau, \tag{1}$$

where $\mathbf{v}$ is the instantaneous velocity of the particle and z the position. In this expression it is assumed gravity is acting along the $-\hat{z}$-axis. The upper limit of the integral is taken to be the time at which the final beam-splitter is applied, $t_{\rm BS}$, while the lower limit corresponds to the time $t_{\rm birth}$ at which the particle was scattered from the condensate.

Before moving into the specific details of the ANU and UQ configurations, it is useful to first outline how this phase due to gravity will arise in the Rarity-Tapster scheme. The key quantity underpinning the CHSH-Bell inequality is the integrated pair-correlation function,

$$C_{ij} = \int_{V(\mathbf{k}_i)} d^3\mathbf{k} \int_{V(\mathbf{k}_j)} d^3\mathbf{k}' \; G^{(2)}(\mathbf{k}, \mathbf{k}'). \tag{2}$$

If we take the case of C_{12} then the relevant density-density correlation in momentum space that is integrated over can be expressed in terms of the initial correlations of the scattered pairs,

$$\begin{aligned} G^{(2)}(\mathbf{k}, \mathbf{k}') = \bar{n}^2 + \frac{1}{4}\Big\{ &|m(\mathbf{k}, \mathbf{k}')|^2 + |m(\mathbf{k}-2\mathbf{k}_L, \mathbf{k}'-2\mathbf{k}_L)|^2 \\ &- m(\mathbf{k}-2\mathbf{k}_L, \mathbf{k}'-2\mathbf{k}_L)^* m(\mathbf{k}, \mathbf{k}') e^{-i\Phi - i\Theta_{\rm free}(\mathbf{k}, \mathbf{k}', \mathbf{k}_L)} \\ &- m(\mathbf{k}, \mathbf{k}')^* m(\mathbf{k}-2\mathbf{k}_L, \mathbf{k}'-2\mathbf{k}_L) e^{i\Phi + i\Theta_{\rm free}(\mathbf{k}, \mathbf{k}', \mathbf{k}_L)} \Big\}, \end{aligned} \tag{3}$$

where $m(\mathbf{k}, \mathbf{k}') = \langle \hat{a}(\mathbf{k}, t_c) \hat{a}(\mathbf{k}', t_c) \rangle$ is the anomalous moment computed at the nominal end of the collision, $t = t_c$. Note that in this form we have implicitly assumed an ANU configuration where $\mathbf{k} \simeq \mathbf{k}_1$, $\mathbf{k}' \simeq \mathbf{k}_2$, $\mathbf{k} - 2\mathbf{k}_L \simeq \mathbf{k}_3$ and $\mathbf{k}' - 2\mathbf{k}_L \approx \mathbf{k}_1$.

The relative phase-shift between the two arms of the interferometer (e.g., the 'left' and 'right' arms) is characterized by Φ, while $\Theta_{\rm free}$ captures any relative phase accrued during free propagation, i.e. $t_c \leq t \leq t_{\rm BS}$ and $\Delta t_{\rm free} = t_{\rm BS} - t_c$. The gravitational phase will appear as an additional contribution to these and we will assume it can be well modelled by the form:

$$\Phi + \Theta_{\rm free}(\mathbf{k}, \mathbf{k}', \mathbf{k}_L) \to \Phi + \Theta_{\rm free}(\mathbf{k}, \mathbf{k}', \mathbf{k}_L) + \Psi_g, \tag{4}$$

where

$$\Psi_g = \psi_g(\mathbf{k} - 2\mathbf{k}_L) + \psi_g(\mathbf{k}' - 2\mathbf{k}_L) - \psi_g(\mathbf{k}) - \psi_g(\mathbf{k}') \tag{5}$$

where $\Psi_g(\mathbf{k})$ is the contribution of the phase Ψ [Eq. (1)] that arises as a correction due to gravity, for a particle initially created with momentum $\mathbf{k}$. We eliminate other contributions as we assume they will already be captured by, e.g., $\Theta_{\rm free}(\mathbf{k}, \mathbf{k}', \mathbf{k}_L)$ and our previous careful treatment of the phase of the anomalous moment. Moreover, we expect incorporating gravity as a simple shift to the phase-settings is a reasonable approximation in the limit of weak scattering, where at most one atom-pair is present in the two coupled pairs of momentum modes in one shot of the experiment.

The phase $\Psi_g(\mathbf{k})$ is determined by computing the classical trajectories of particles traversing the Rarity-Tapster interferometer. It is straightforward to obtain the velocity and position of a classical particle created with velocity $\mathbf{v}_i = \hbar \mathbf{k}_i / m$ as,

$$v_z(t) = \begin{cases} v_{i,z} + g(t - t_i) & \text{if } t < t_c \\ v_{i,z} + g(t - t_i) \pm 2v_L & \text{if } t \geq t_c \end{cases}, \tag{6}$$

and

$$z(t) = \begin{cases} z_i + v_i(t - t_i) + \frac{g}{2}(t - t_i)^2 & \text{if } t < t_c \\ z(t_c) + v(t_c)(t - t_c) + \frac{g}{2}(t - t_c)^2 & \text{if } t \geq t_c \end{cases}, \tag{7}$$

with v_x, v_y constants, $x(t) = v_x t$ and $y(t) = v_y t$. The particle is assumed to be born at $t_{\rm birth} = t_i$. The momentum kick imparted by the Bragg pulse is accounted for by the correction $\pm v_L$ to the velocity for $t \geq t_c$. In the following we will typically ignore the motion along x and y as less relevant.

I. CLASSICAL CALCULATIONS

A. Benchmark scenario

To help setup our basic approach, we will start with a simple preliminary example where we assume that all particles are created at $t = 0$ and $z_i = 0$, and the interferometer is timed such that $\Delta t_{\text{free}} = t_c$. We will compute the expected gravitational phase for each of the ANU and UQ configurations and compare the results.

ANU configuration: We consider two pairs of particles defined by initial momenta $k_{1,z} \simeq k_0/2$, $k_{2,z} \simeq k_0/2$, $k_{3,z} \simeq k_{1,z} - 2k_L$ and $k_{4,z} \simeq k_{2,z} - 2k_L$. We do *not* assume the momenta are precisely aligned (i.e., $\mathbf{k}_{1,z} \neq \mathbf{k}_{2,z}$ in general). Solving the classical trajectory yields,

$$\psi(\mathbf{k}_1) = \frac{\hbar k_1^2}{m} t_c + 2\frac{\hbar}{m}\left(k_L^2 - k_1 k_L\right) t_c - 2g k_L t_c^2, \qquad (8)$$

or focusing on the gravitational contribution, $\Psi_g(\mathbf{k}_1) = -2gk_L t_c^2$.

Repeating the calculation for the other momenta leads to a total phase shift,

$$\Psi_g = -8gk_L t_c^2 = -gk_0 T^2, \qquad (9)$$

where $T = \Delta t_{\text{free}} + t_c = 2t_c$ and we took $2k_L = k_0$. This result is consistent with the semi-classical result obtained for a typical matter-wave interferometer. An important point to note is that the result *does not* depend on the precise values of the targeted pairs of momenta, i.e., the phase shift is uniform across any relevant integration volume and depends only on the magnitude of the imparted momentum kick.

UQ configuration: We repeat the calculation assuming instead the pairs have initial momenta $k_{1,z} \simeq k_0/\sqrt{2}$, $k_{2,z} \simeq -k_0/\sqrt{2}$, $k_{3,z} \simeq k_{1,z} - 2k_L$ and $k_{4,z} \simeq k_{2,z} + 2\mathbf{k}_L$. In this case, while we find a similar form,

$$\psi(\mathbf{k}_1) = \frac{\hbar k_1^2}{m} t_c + 2\frac{\hbar}{m}\left(k_L^2 - k_1 k_L\right) t_c - 2g k_L t_c^2, \qquad (10)$$

the distinct (flipped) configuration of $\mathbf{k}_2$ and $\mathbf{k}_4$ means that the total gravitational phase shift actually cancels,

$$\Psi_g = 0. \qquad (11)$$

The different contribution of gravity to the phase-sensitive correlations in the Rarity-Tapster scheme for the UQ and ANU configurations may appear confusing and concerning at first glance. First, let us highlight that the reason the two configurations have such disparate final results is because it is the phase shift of the *pair* of scattered particles that matters. In the UQ configuration, each partner of a single scattered pair experiences an opposite momentum kick (e.g., $k_{1,z} \to k_{1,z} - 2k_L$ and $k_{2,z} + 2k_L$) and thus the accrued phase collectively cancels out. On the other hand, the partners are identically kicked in the ANU configuration and thus each pair accrues a well defined phase shift.

A second important point is that the size of the phase-shift in the ANU configuration is not yet concerning. In particular, note that the ANU result is a *uniform* shift and as such would only appear as a constant offset in the CHSH phase-settings.

B. Pairs created at different times

Let us build on the previous result and make our model a little more realistic by incorporating the fact that particles are created at different times. As discussed in previous notes, the initial collision process occurs on a timescale t_{sep} that reflects the time it takes for the BECs to split, and pairs are generated during this time with a probability $\propto e^{-(t/t_{\text{sep}})^2}$.

Nevertheless, for the simplicity of the calculation we will continue to assume that the particles are always created at $z_i = 0$.

We thus setup our calculation by assuming that the pair $(\mathbf{k}_1, \mathbf{k}_2)$ is created at $t_{\text{birth}} = t_{12}$ and similarly the pair $(\mathbf{k}_3, \mathbf{k}_4)$ at $t_{\text{birth}} = t_{34}$. For simplicity we will continue to assume that the particles are always created at $z_i = 0$. As previously, we also take $\Delta t_{\text{free}} = t_c$ (although this is not pivotal to the calculation).

ANU configuration: Under these assumptions, solving the classical trajectories yields the phase,

$$\psi(\mathbf{k}_1) = \frac{\hbar k_1^2}{2m}(t_{12} - 2t_c) + 2\frac{\hbar}{m}\left(k_L^2 - k_1 k_L\right) t_c - 2gk_L t_c (t_c - t_{12}), \qquad (12)$$

which has the gravitational contribution $\Psi_g(\mathbf{k}_1) = -2gk_L t_c(t_c - t_{12})$.

Repeating the calculation for the other momenta leads to a total phase shift,

$$\Psi_g = -4g(2t_c - t_{12} - t_{34}) t_c k_L = -2gk_0 t_c (2t_c - t_{12} - t_{34}), \qquad (13)$$

which trivially reduces to our previous result (9) for $t_{12} = t_{34} = 0$.

To incorporate the random distribution of times at which pairs are created, we compute

$$\overline{e^{i\Psi_g}} \propto \int_0^\infty dt_{12} \int_0^\infty dt_{34} \left[e^{-i2gk_0 t_c(2t_c - t_{12} - t_{34})} \times e^{-\left(\frac{t_{12}}{t_{\text{sep}}}\right)^2} e^{-\left(\frac{t_{34}}{t_{\text{sep}}}\right)^2} \right], \qquad (14)$$

which yields,

$$\overline{e^{i\Psi_g}} = e^{-2g^2 k_0^2 t_c^2 t_{\text{sep}}^2 - 4igk_0 t_c^2} \left[1 + \text{erf}\left(igk_0 t_c t_{\text{sep}}\right)\right]^2. \qquad (15)$$

In the limit of $gk_0 t_c t_{\text{sep}} \gg 1$ (relevant for the ANU experiment), the slowest decaying contribution is

$$\overline{e^{i\Psi_g}} \simeq -\frac{e^{-4igk_0 t_c^2}}{\pi g^2 k_0^2 t_c^2 t_{\text{sep}}^2}. \qquad (16)$$

For the experimental parameters corresponding to the weak trap, this latter expression is relevant and predicts that phase sensitive correlations will be strongly washed out ($[\pi g^2 k_0^2 t_c^2 t_{\mathrm{sep}}^2]^{-1} \sim 10^{-4}$). On the other hand, the tight trap seems to suffer far less and we expect gravity would only slightly reduce the amplitude of any phase-sensitive correlations by a factor ~ 0.7.

UQ configuration: Performing the same calculation with the alternate UQ configuration produces a starkly different result. It is straightforward to obtain that the gravitational phase is always $\Psi_g = 0$. As discussed in the previous subsection, this is simply a consequence of the fact that the correlated pairs undergo opposite momentum kicks, and thus no gravitational phase accumulates.