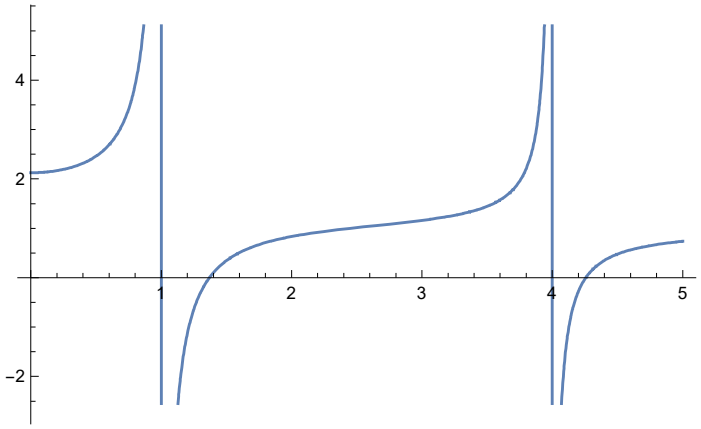
In[1]:=
$$\text{polzMdl} = A1 + \frac{f1}{E1^2 - h^2\,\omega^2} + \frac{f2}{E2^2 - h^2\,\omega^2}$$

```
Plot[polzMdl /. {A1 → 1, f1 → 1, f2 → 2, E1 → 1, E2 → 4, h → 1}, {ω, 0, 5}]
```

Out[1]=
$$A1 + \frac{f1}{E1^2 - h^2\,\omega^2} + \frac{f2}{E2^2 - h^2\,\omega^2}$$

Out[2]=



2 component model

In[3]:=
```
Real[polzMdl /. {A1 → 1, f1 → 1, f2 → 2, E1 → 1, E2 → 4, ω → 1.5, h → 1}]
```

Out[3]=
```
Real[0.345455]
```

Get the tune out freq from the model

In[4]:=
```
ToFreqTwoLevel = Solve[0 == polzMdl /. {A1 → 0}, ω]
ToFreqTwoLevel = ω /. ToFreqTwoLevel[[2]]
FullSimplify[ToFreqTwoLevel, Assumptions → {h > 0}]
```

Out[4]=
$$\left\{\left\{\omega \to -\frac{\sqrt{E2^2\,f1 + E1^2\,f2}}{\sqrt{f1\,h^2 + f2\,h^2}}\right\}, \left\{\omega \to \frac{\sqrt{E2^2\,f1 + E1^2\,f2}}{\sqrt{f1\,h^2 + f2\,h^2}}\right\}\right\}$$

Out[5]=
$$\frac{\sqrt{E2^2\,f1 + E1^2\,f2}}{\sqrt{f1\,h^2 + f2\,h^2}}$$

Out[6]=
$$\frac{\sqrt{E2^2\,f1 + E1^2\,f2}}{\sqrt{f1 + f2}\;h}$$

replace one of the osc strengths with X1 times f1 so that we can express the answer in terms of the osc strength ratio

In[7]:=
```
ToFreqTwoLevel = Simplify[ToFreqTwoLevel /. f2 → X1 f1]
```

Out[7]=
$$\frac{\sqrt{f1\,\left(E2^2 + E1^2\,X1\right)}}{\sqrt{f1\,h^2\,(1 + X1)}}$$

Get the ratio from the tune out and the transition freqs

In[8]:=
```
RatioVal = Solve[ωto == ToFreqTwoLevel, X1]
RatioVal = FullSimplify[RatioVal, Assumptions →
    {h ∈ ℝ, E1 ∈ ℝ, E2 ∈ ℝ, ωto ∈ ℝ, X1 ∈ ℝ, X2 ∈ ℝ, f1 ∈ ℝ, h > 0, E1 > 0, E2 > 0, ωto > 0}]
```

Out[8]=
$$\left\{\left\{X1 \to \frac{-E2^2 + h^2\,\omega to^2}{E1^2 - h^2\,\omega to^2}\right\}\right\}$$

Out[9]=
$$\left\{\left\{X1 \to \frac{-E2^2 + h^2\,\omega to^2}{E1^2 - h^2\,\omega to^2}\right\}\right\}$$

Get the derivative of this ratio with the tune out freq

In[10]:=

In[11]:=
```
DerivTOWithX = D[ToFreqTwoLevel, X1]
DerivTOWithX =
 FullSimplify[DerivTOWithX, Assumptions → {h ∈ ℝ, E1 ∈ ℝ, E2 ∈ ℝ, A1 ∈ ℝ, X1 ∈ ℝ,
    X2 ∈ ℝ, f1 ∈ ℝ, h > 0, E1 > 0, E2 > 0, A1 > 0, f1 > 0}]
```

Out[11]=
$$\frac{E1^2\,f1}{2\sqrt{f1\,h^2\,(1+X1)}\,\sqrt{f1\left(E2^2+E1^2\,X1\right)}} - \frac{f1\,h^2\,\sqrt{f1\left(E2^2+E1^2\,X1\right)}}{2\left(f1\,h^2\,(1+X1)\right)^{3/2}}$$

Out[12]=
$$\frac{(E1-E2)\,(E1+E2)}{2\,h\,(1+X1)^{3/2}\,\sqrt{E2^2+E1^2\,X1}}$$

Use this to find the fractional uncert of X given some uncert in the tune out freq

In[13]:=
```
FracUncX = (1 / DerivTOWithX) δωTo / X1 /. RatioVal[[1]]
Simplify[FracUncX, Assumptions →
   {h ∈ ℝ, E1 ∈ ℝ, E2 ∈ ℝ, ωto ∈ ℝ, X1 ∈ ℝ, X2 ∈ ℝ, f1 ∈ ℝ, h > 0, E1 > 0, E2 > 0, ωto > 0}]
```

Out[13]=
$$\frac{2\,h\,\delta\omega To\left(E1^2 - h^2\,\omega to^2\right)\left(1+\frac{-E2^2+h^2\,\omega to^2}{E1^2-h^2\,\omega to^2}\right)^{3/2}\sqrt{E2^2+\frac{E1^2\left(-E2^2+h^2\,\omega to^2\right)}{E1^2-h^2\,\omega to^2}}}{(E1-E2)\,(E1+E2)\left(-E2^2+h^2\,\omega to^2\right)}$$

Out[14]=
$$\frac{2\left(E1^2-E2^2\right)h^2\,\delta\omega To\,\omega to}{\left(E1^2-h^2\,\omega to^2\right)\left(-E2^2+h^2\,\omega to^2\right)}$$

2 component model for R

What are our number like if we calulate the ratio of the dipole matrix elements instead of the oscillator strengths

In[*]:=
```
polzMdl = A1 + 1/ħ 2 T1 D1/(T1^2 - ω^2) + 1/ħ 2 T2 D2/(T2^2 - ω^2)
```

Out[*]=
$$A1 + \frac{2\,D1\,T1}{\left(T1^2-\omega^2\right)\hbar} + \frac{2\,D2\,T2}{\left(T2^2-\omega^2\right)\hbar}$$

In[•]:=
```
Real[polzMdl /. {A1 → 1, D1 → 1, D2 → 2, T1 → 1, T2 → 4, ω → 1.5, ℏ → 1}]
```

Out[•]=
```
Real[0.563636]
```

In[•]:=
```
ToFreqTwoLevel = Solve[0 == polzMdl /. {A1 → 0}, ω]
ToFreqTwoLevel = ω /. ToFreqTwoLevel[[2]]
FullSimplify[ToFreqTwoLevel, Assumptions → {ℏ > 0}]
```

Out[•]=
$$\left\{\left\{\omega \to -\frac{\sqrt{D2\ T1^2\ T2 + D1\ T1\ T2^2}}{\sqrt{D1\ T1 + D2\ T2}}\right\}, \left\{\omega \to \frac{\sqrt{D2\ T1^2\ T2 + D1\ T1\ T2^2}}{\sqrt{D1\ T1 + D2\ T2}}\right\}\right\}$$

Out[•]=
$$\frac{\sqrt{D2\ T1^2\ T2 + D1\ T1\ T2^2}}{\sqrt{D1\ T1 + D2\ T2}}$$

Out[•]=
$$\frac{\sqrt{T1\ T2\ (D2\ T1 + D1\ T2)}}{\sqrt{D1\ T1 + D2\ T2}}$$

In[•]:=
```
ToFreqTwoLevel = Simplify[ToFreqTwoLevel /. D2 → R D1]
```

Out[•]=
$$\frac{\sqrt{D1\ T1\ T2\ (R\ T1 + T2)}}{\sqrt{D1\ (T1 + R\ T2)}}$$

In[•]:=
```
RatioVal = Solve[ωto == ToFreqTwoLevel, R]
RatioVal = FullSimplify[RatioVal, Assumptions →
    {ℏ ∈ ℝ, T1 ∈ ℝ, T2 ∈ ℝ, ωto ∈ ℝ, D1 ∈ ℝ, D2 ∈ ℝ, R ∈ ℝ, ℏ > 0, T1 > 0, T2 > 0, ωto > 0}]
```

Out[•]=
$$\left\{\left\{R \to \frac{-T1\ T2^2 + T1\ \omega to^2}{T2\ \left(T1^2 - \omega to^2\right)}\right\}\right\}$$

Out[•]=
$$\left\{\left\{R \to \frac{T1\ \left(-T2^2 + \omega to^2\right)}{T2\ \left(T1^2 - \omega to^2\right)}\right\}\right\}$$

In[•]:=

In[•]:=
```
DerivTOWithR = D[ToFreqTwoLevel, R]
DerivTOWithR =
 FullSimplify[DerivTOWithR, Assumptions → {ℏ ∈ ℝ, T1 ∈ ℝ, T2 ∈ ℝ, ωto ∈ ℝ, D1 ∈ ℝ,
    D2 ∈ ℝ, R ∈ ℝ, ℏ > 0, D1 > 0, T2 > 0, T1 > 0, ωto > 0}]
```

Out[•]=
$$-\frac{D1\ T2\ \sqrt{D1\ T1\ T2\ (R\ T1 + T2)}}{2\ (D1\ (T1 + R\ T2))^{3/2}} + \frac{D1\ T1^2\ T2}{2\ \sqrt{D1\ T1\ T2\ (R\ T1 + T2)}\ \sqrt{D1\ (T1 + R\ T2)}}$$

Out[•]=
$$\frac{(T1 - T2)\ (T1 + T2)}{2\ \sqrt{\frac{1}{T1} + \frac{R}{T2}}\ (T1 + R\ T2)^{3/2}}$$

In[•]:=
```
FracUncX = (1 / DerivTOWithR) δωTo / R /. RatioVal[[1]]
Simplify[FracUncX, Assumptions → {ℏ ∈ ℝ, T1 ∈ ℝ, T2 ∈ ℝ,
    ωto ∈ ℝ, D1 ∈ ℝ, D2 ∈ ℝ, R ∈ ℝ, ℏ > 0, D1 > 0, T2 > 0, T1 > 0, ωto > 0}]
```

Out[•]=
$$\frac{2\,T2\,\delta\omega To\,\left(T1^2-\omega to^2\right)\,\left(T1+\frac{T1\,\left(-T2^2+\omega to^2\right)}{T1^2-\omega to^2}\right)^{3/2}\,\sqrt{\frac{1}{T1}+\frac{T1\,\left(-T2^2+\omega to^2\right)}{T2^2\,\left(T1^2-\omega to^2\right)}}}{T1\,(T1-T2)\,(T1+T2)\,\left(-T2^2+\omega to^2\right)}$$

Out[•]=
$$\frac{2\,\left(T1^2-T2^2\right)\,\delta\omega To\,\omega to}{\left(T1^2-\omega to^2\right)\,\left(-T2^2+\omega to^2\right)}$$

3 component system

Now lets try this again but with the offset

In[•]:=
```
ToFreqTwoLevel = Solve[0 == polzMdl, ω]
```

Out[•]=
$$\left\{\left\{\omega\to -\frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{E1^2}{h^2}+\frac{E2^2}{h^2}+\frac{f1}{A1\,h^2}+\frac{f2}{A1\,h^2}-\frac{1}{A1\,h^4}\left(\sqrt{\left(\left(A1^2\,E1^4-2\,A1^2\,E1^2\,E2^2+A1^2\,E2^4+2\,A1\,E1^2\,f1-2\,A1\,E2^2\,f1+f1^2-2\,A1\,E1^2\,f2+2\,A1\,E2^2\,f2+2\,f1\,f2+f2^2\right)\,h^4\right)}\right)\right)}\right)\right\},\right.$$

$$\left\{\omega\to \frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{E1^2}{h^2}+\frac{E2^2}{h^2}+\frac{f1}{A1\,h^2}+\frac{f2}{A1\,h^2}-\frac{1}{A1\,h^4}\left(\sqrt{\left(\left(A1^2\,E1^4-2\,A1^2\,E1^2\,E2^2+A1^2\,E2^4+2\,A1\,E1^2\,f1-2\,A1\,E2^2\,f1+f1^2-2\,A1\,E1^2\,f2+2\,A1\,E2^2\,f2+2\,f1\,f2+f2^2\right)\,h^4\right)}\right)\right)}\right)\right\},$$

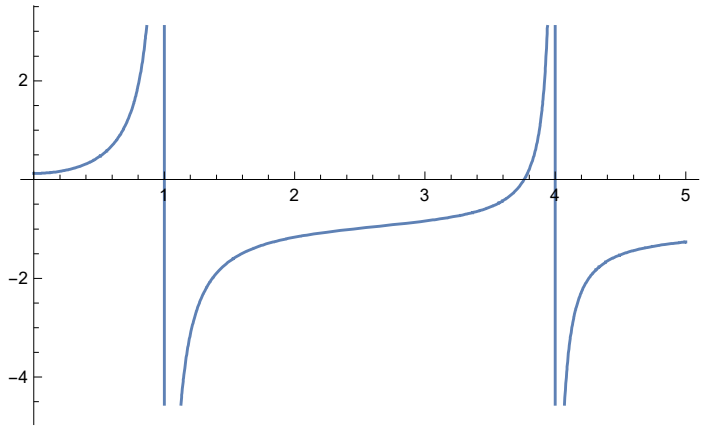
$$\left\{\omega\to -\frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{E1^2}{h^2}+\frac{E2^2}{h^2}+\frac{f1}{A1\,h^2}+\frac{f2}{A1\,h^2}+\frac{1}{A1\,h^4}\left(\sqrt{\left(\left(A1^2\,E1^4-2\,A1^2\,E1^2\,E2^2+A1^2\,E2^4+2\,A1\,E1^2\,f1-2\,A1\,E2^2\,f1+f1^2-2\,A1\,E1^2\,f2+2\,A1\,E2^2\,f2+2\,f1\,f2+f2^2\right)\,h^4\right)}\right)\right)}\right)\right\},$$

$$\left.\left\{\omega\to \frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{E1^2}{h^2}+\frac{E2^2}{h^2}+\frac{f1}{A1\,h^2}+\frac{f2}{A1\,h^2}+\frac{1}{A1\,h^4}\left(\sqrt{\left(\left(A1^2\,E1^4-2\,A1^2\,E1^2\,E2^2+A1^2\,E2^4+2\,A1\,E1^2\,f1-2\,A1\,E2^2\,f1+f1^2-2\,A1\,E1^2\,f2+2\,A1\,E2^2\,f2+2\,f1\,f2+f2^2\right)\,h^4\right)}\right)\right)}\right)\right\}\right\}$$

Which of these roots do i want??
ok looks like the second

In[•]:=
```
Plot[polzMdl /. {A1 → -1, f1 → 1, f2 → 2, E1 → 1, E2 → 4, h → 1}, {ω, 0, 5}]
N[ToFreqTwoLevel /. {A1 → -1, f1 → 1, f2 → 2, E1 → 1, E2 → 4, h → 1}]
```

Out[•]=



Out[•]= $\{\{\omega \to -3.76051\}, \{\omega \to 3.76051\}, \{\omega \to 0. - 0.37607\,i\}, \{\omega \to 0. + 0.37607\,i\}\}$

In[•]:=
```
ToFreqThreeComp = ω /. ToFreqTwoLevel[[2]]
ToFreqThreeComp = ToFreqThreeComp /. {f2 → X1 f1, A1 → X2 f1}
```

Out[•]=
$$\frac{\sqrt{\frac{E1^2}{h^2}+\frac{E2^2}{h^2}+\frac{f1}{A1\,h^2}+\frac{f2}{A1\,h^2}-\frac{\sqrt{\left(A1^2\,E1^4-2\,A1^2\,E1^2\,E2^2+A1^2\,E2^4+2\,A1\,E1^2\,f1-2\,A1\,E2^2\,f1+f1^2-2\,A1\,E1^2\,f2+2\,A1\,E2^2\,f2+2\,f1\,f2+f2^2\right)\,h^4}}{A1\,h^4}}}{\sqrt{2}}$$

Out[•]=
$$\frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{E1^2}{h^2}+\frac{E2^2}{h^2}+\frac{1}{h^2\,X2}+\frac{X1}{h^2\,X2}-\frac{1}{f1\,h^4\,X2}\left(\sqrt{\left(h^4\,\left(f1^2+2\,f1^2\,X1+f1^2\,X1^2+2\,E1^2\,f1^2\,X2-2\,E2^2\,f1^2\,X2-2\,E1^2\,f1^2\,X1\,X2+2\,E2^2\,f1^2\,X1\,X2+E1^4\,f1^2\,X2^2-2\,E1^2\,E2^2\,f1^2\,X2^2+E2^4\,f1^2\,X2^2\right)\right)\right)\right)\right)}$$

In[•]:=
```
Solve[ωto == ToFreqThreeComp, X1]
```

Out[•]=
$$\left\{\left\{X1 \to -\frac{\left(E2^2-h^2\,\omega to^2\right)\,\left(-1-E1^2\,X2+h^2\,X2\,\omega to^2\right)}{-E1^2+h^2\,\omega to^2}\right\}\right\}$$

In[•]:=
```
ToFreqThreeComp =
 FullSimplify[ToFreqThreeComp, Assumptions → {h ∈ ℝ, E1 ∈ ℝ, E2 ∈ ℝ, A1 ∈ ℝ,
    X1 ∈ ℝ, X2 ∈ ℝ, f1 ∈ ℝ, h > 0, E1 > 0, E2 > 0, A1 > 0, f1 > 0}]
```

Out[•]=
$$\frac{\sqrt{\frac{1+X1+\left(E1^2+E2^2\right)\,X2-\sqrt{X1^2+X1\,\left(2-2\,E1^2\,X2+2\,E2^2\,X2\right)+\left(1+\left(E1-E2\right)\,\left(E1+E2\right)\,X2\right)^2}}{h^2\,X2}}}{\sqrt{2}}$$

In[•]:=
```
DerivTOWithX =
  FullSimplify[D[ToFreqThreeComp, X1], Assumptions → {h ∈ ℝ, E1 ∈ ℝ, E2 ∈ ℝ, A1 ∈ ℝ,
    X1 ∈ ℝ, X2 ∈ ℝ, f1 ∈ ℝ, h > 0, E1 > 0, E2 > 0, A1 > 0, f1 > 0}]
```

Out[•]=

$$\frac{1-\frac{1+X1-E1^2\,X2+E2^2\,X2}{\sqrt{X1^2+X1\,\left(2-2\,E1^2\,X2+2\,E2^2\,X2\right)+\left(1+\left(E1-E2\right)\,\left(E1+E2\right)\,X2\right)^2}}}{2\,\sqrt{2}\,h\,X2\,\sqrt{\frac{1+X1+\left(E1^2+E2^2\right)\,X2-\sqrt{X1^2+X1\,\left(2-2\,E1^2\,X2+2\,E2^2\,X2\right)+\left(1+\left(E1-E2\right)\,\left(E1+E2\right)\,X2\right)^2}}{X2}}}$$