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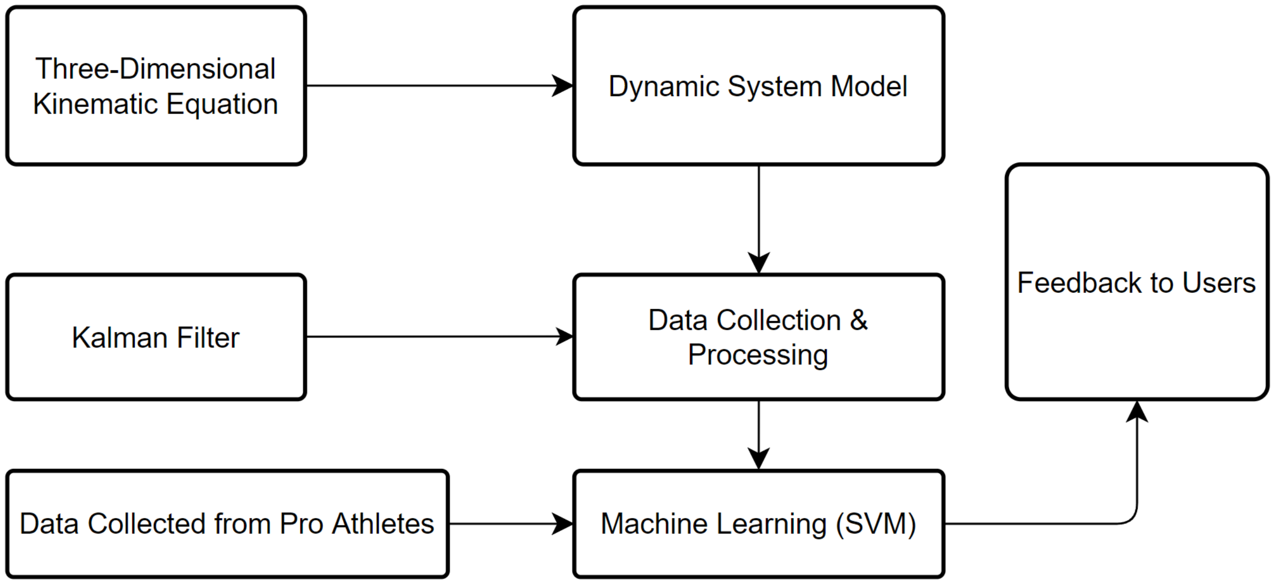
Contents

1 Introduction

1.1 Project Detail

This article is used for ENGG1320 group project: AI Coaching System. We introduced a method to calculate the limb actions of basketball players with wearable device (bracelet integrated with gyroscope, accelerometer and velocity detector and action-tracing gloves).

1.2 Algorithm Framework



2 Kalman Filter

Our model requires Bayesian model to build up the detection. Thus we apply Kalman filter on the dataset. The following gives a brief derivation.

2.1 Derivation

Two possible noise are the white noise $w_k \sim N(0, \sigma^2_{w_k})$ and measurement noise v_k. Then for real data and data measured,

$$x_{k+1} = \Phi x_k + w_k \quad z_k = Hx_k + v_k \tag{1}$$

Assume for data estimation, the prior prediction is $\hat{x}'_k$, and actual estimation is $\hat{x}_k$. Estimation bias is $e_k = x_k - \hat{x}_k$, and one assumption is that prior model bias is proportional to measurement bias, i.e.

$$\hat{x}_k - \hat{x}'_k = K_k(z_k - H\hat{x}'_k) \tag{2}$$

where K_k is the Kalman gain. The filter tends to maximize the sum of variance of estimation bias. We first consider bias covariance matrix

$$\begin{aligned} E(e_k e_k^T) = P_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] & \\ E[(x_k - \hat{x}'_k - K_k z_k + K_k H\hat{x}_k)(x_k - \hat{x}'_k - K_k z_k + K_k H\hat{x}_k)^T] & \\ = (I - K_k H)E[(x_k - \hat{x}'_k)(x_k - \hat{x}'_k)^T](I - K_k H)^T + K_k E(v_k v_k^T)K_k^T & \end{aligned} \tag{3}$$

Define $P'_k = E[(x_k - \hat{x}'_k)(x_k - \hat{x}'_k)^T] = E(e'_k e'^T_k), E(v_k v_k^T) = R$ as the prior covariance matrix. Then

$$P_k = (I - K_k H)P'_k(I - K_k H)^T + K_k R K_k^T \tag{4}$$

The model tends to minimize the variance of estimation bias (trace of covariance matrix), according to which coefficient K_k is determined, that is

$$\begin{aligned} &\arg\min_{K_k} \boldsymbol{tr}(P_k) \to \frac{\partial}{\partial K_k}\boldsymbol{tr}(P_k) = 0 \\ &\longrightarrow -2(HP'_k)^T + 2K_k(HP'_kH^T + R) = 0 \\ &\longrightarrow K_k = P'_kH^T(HP'_kH^T + R)^{-1} = K_k^* \end{aligned} \tag{5}$$

then

$$\begin{aligned} \min \boldsymbol{tr}(P_k) &= \boldsymbol{tr}(P'_k) - 2\boldsymbol{tr}[K_kHP'_k] + T\left[K_k(HP'_kH + R)K_k^T\right]\Big|_{K_k=K_k^*} \\ &= (I - K_k^*H)P'_k \end{aligned} \tag{6}$$

Then the prior prediction bias $e'_{k+1} = x_{k+1} - \hat{x}'_{k+1}$. Based on previous data, prior prediction can be predicted as

$$\hat{x}'_{k+1} = \Phi\hat{x}_k \tag{7}$$

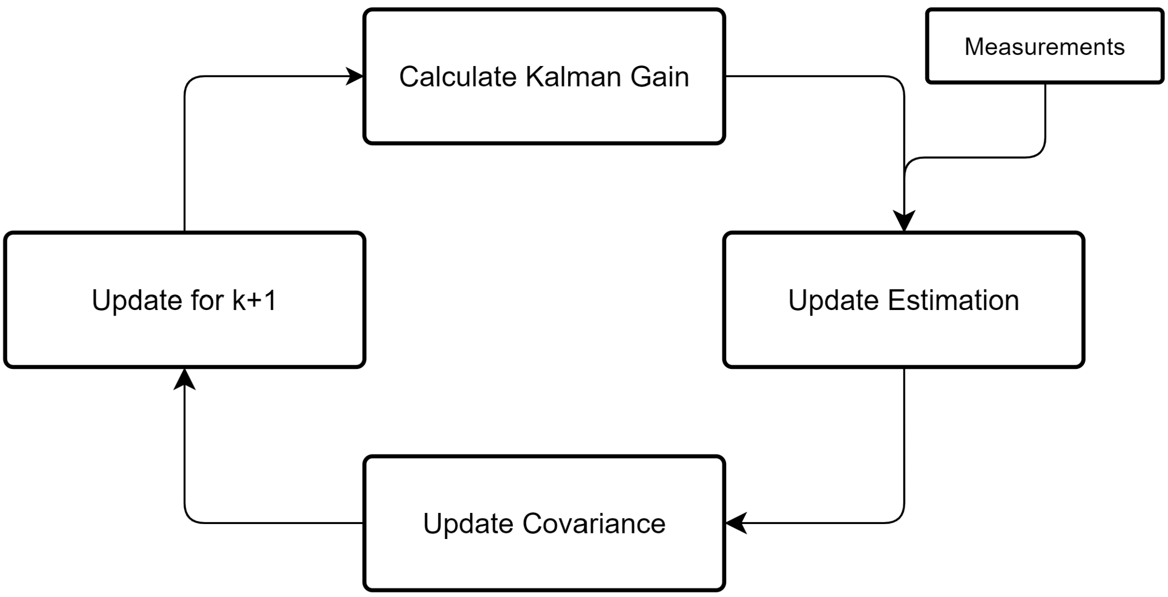
Thus the prior prediction bias can be calculated as follows

$$\begin{aligned} e'_{k+1} &= x_{k+1} - \hat{x}'_{k+1} \\ &= (\Phi x_k + w_k) - \Phi\hat{x}_k \\ &= \Phi e_k + w_k \end{aligned} \tag{8}$$

Notice that the bias and noise are mutually independent, hence the prior prediction bias matrix

$$\begin{aligned} P'_{k+1} &= E(e'_{k+1}e^T_{k+1}) \\ &= E\left[(\Phi e_k + w_k)(\Phi e_k + w_k)^T\right] \\ &= \Phi E(e_k e_k^T)\Phi^T + E(w_k w_k^T) = \Phi P_k \Phi^T + Q_w \end{aligned} \tag{9}$$

2.2 Loop Procedure



Algorithm 1 Kalman Filter

Require: sampling period T_s, initial state P'_0

while $k \in T_s$ **do**

Kalman Gain: $K_k = P'_k H^T (H P'_k H^T + R)^{-1}$

Update Estimate: $\hat{x}_k = \hat{x}'_k + K_k(z_k - H\hat{x}'_k)$

Update Covariance: $P_k = (I - K_k H) P'_k$

Project to Next Loop:

$$\hat{x}'_{k+1} = \Phi \hat{x}_k$$
$$P_{k+1} = \Phi P_k \Phi^T + Q_w$$

end while

3 Physical Model

The following is the physical model to be applied in the motion detection algorithm. Physical quantities we require include the body velocity v_b, acceleration a_b, limb angular velocity ω_b and attitude angles (roll angle γ, pitch angle θ).

3.1 Kinematic Equation

We first consider kinematic equation in limb movement. Acceleration a_b gives

$$\begin{aligned} a_b &= \frac{\partial}{\partial t} v_b = \frac{\partial}{\partial t}(v_\theta \hat{\theta} + v_r \hat{r}) \\ &= (\dot{v_\theta} + \dot{v_r}) + (v_\theta \frac{\partial \hat{\theta}}{\partial \theta}\frac{\partial \theta}{\partial t} + v_r \frac{\partial \hat{r}}{\partial \theta}\frac{\partial \theta}{\partial t}) = (\dot{v_\theta} + \dot{v_r}) + \omega(-v_\theta \hat{r} + v_r \hat{\theta}) = \dot{v_b} + \omega \times v_b \end{aligned} \tag{10}$$

Accelerometer measurement f_b and body acceleration a_b has relationship

$$f_b = g_b + a_b \tag{11}$$

Assume in accelerometer measurement, acceleration a_b has relationship

$$\dot{a_b} = \eta a_b + w_1 \tag{12}$$

where Gaussian noise $w_1 \sim N(0, \sigma^2_{w_1})$, and rational coefficient η is a constant.

3.2 Stochastic Model

Assume that we combine the limb motion together to form a matrix X_1, where

$$X_1 = \begin{bmatrix} v_b \\ a_b \end{bmatrix} \tag{13}$$

and it holds for time-shifting relationship as follows, where gyroscope zero shift w_g has standard deviation σ_{w_g}.

$$\dot{X_1} = \begin{bmatrix} \dot{v_b} \\ \dot{a_b} \end{bmatrix} = \begin{bmatrix} a_b - \omega_b \times v_b + w_g \\ \eta a_b + w_1 \end{bmatrix} = \begin{bmatrix} -[\omega_b]_\times & I_3 \\ 0_3 & \eta I \end{bmatrix} X_1 + \begin{bmatrix} w_g \\ w_1 \end{bmatrix} = A_1 X_1 + W_1 \tag{14}$$

During sampling period $T_s \in \mathcal{T}$, we have state transition equation

$$\begin{aligned} v_b^{(k+1)} &\approx v_b^{(k)} + \dot{v}_b{}^{(k)}\Delta t \to v_b^{(k+1)} \approx \left(I - [\omega_b]_\times T_s + R_a^v\right) v_b^{(k)} + w_g T_s \\ a_b^{(k+1)} &\approx a_b^{(k)} + \dot{a}_b{}^{(k)}\Delta t \to a_b^{(k+1)} \approx (I + \eta I T_s) a_b^{(k)} + w_1 T_s \end{aligned} \tag{15}$$

Then the noise covariance matrix, given that the measurement and noise have no correlation, is

$$Q_1^{(k)} = \begin{bmatrix} Q_{w_g}^{(k)} & 0_3 \\ 0_3 & Q_{w_1}^{(k)} \end{bmatrix} = \begin{bmatrix} \sigma_{w_g}^2 T_s^2 [v_b]_\times^{(k)} \left([v_b]_\times^{(k)}\right)^T & 0_3 \\ 0_3 & \sigma_{w_1}^2 T_s^2 I_3 \end{bmatrix} \tag{16}$$

Assume the noise can be neglected in the iteration of dynamic system, then the state equation in respect of time for X_1 is

$$\dot{X}_1 = A_1(T_s) X_1 \to X_1^{(k+1)} = \exp\left(A_1^{(k)} T_s\right) X_1^{(k)} + W_1^{(k)} \tag{17}$$

Zhaoying Zhou et al. (2004) developed equation

$$\begin{bmatrix} \dot{g}_x \\ \dot{g}_y \\ \dot{g}_z \end{bmatrix} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} \longrightarrow \dot{g}_b = [\omega_b]_\times g_b \tag{18}$$

Like wise, the recursive relationship and covariance matrix can be derived as

$$g_b^{(k+1)} = \exp\left([\omega_b]_\times^{(k)} T_s\right) g_b^{(k)} + W_2^{(k)} \tag{19}$$

where W_2 is corresponding process noise. Likewise,

$$\begin{aligned} g_b^{(k+1)} \approx g_b^{(k)} + \dot{g}_b^{(k)}\Delta t &= g_b^{(k)} + [\omega_b]_\times \left(g_{b\text{ real}}^{(k)} + g_{b\text{ bias}}^{(k)} + g_{b\text{ noise}}\right) T_s \\ &\approx (I + [\omega_b]_\times T_s) g_b^{(k)} + [\omega_b]_\times g_{b\text{ noise}} T_s \end{aligned} \tag{20}$$

Then the noise covariance equation

$$Q_2 = \sigma_g^2 T_s^2 [g_b]_\times \left([g_b]_\times\right)^T \tag{21}$$

3.3 Measurement Model

Assume that the data (tri-axis velocity v_m and body total acceleration f_m) we collect from wearable device have Gaussian noise $\nu_1 \sim N(0, \sigma_{\nu_1}), \nu_2 \sim N(0, \sigma_{\nu_2})$ respectively. Then denote the state matrix as Y.

$$Y = \begin{bmatrix} v_m \\ f_m \end{bmatrix} = \begin{bmatrix} v_b + \nu_1 \\ a_b + g_b + \nu_2 \end{bmatrix} = \begin{bmatrix} I_3 & 0_3 & 0_3 \\ 0_3 & I_3 & I_3 \end{bmatrix} \begin{bmatrix} v_b \\ a_b \\ g_b \end{bmatrix} = H \begin{bmatrix} X_1 \\ g_b \end{bmatrix} + \nu \tag{22}$$

Take the augmented state matrix $X = \begin{bmatrix} X_1 \\ g_b \end{bmatrix}$ and augmented noise matrix $W = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$, then

$$Y = HX + \nu \to Y^{(k+1)} = H^{(k)} X^{(k)} + \nu^{(k)} \tag{23}$$

$$X^{(k+1)} = \begin{bmatrix} \exp\left(A_1^{(k)} T_s\right) & 0_3 \\ 0_3 & \exp\left([\omega_b]_\times^{(k)} T_s\right) \end{bmatrix} X^{(k)} + W^{(k)} = \Phi^{(k)} X^{(k)} + W^{(k)} \tag{24}$$

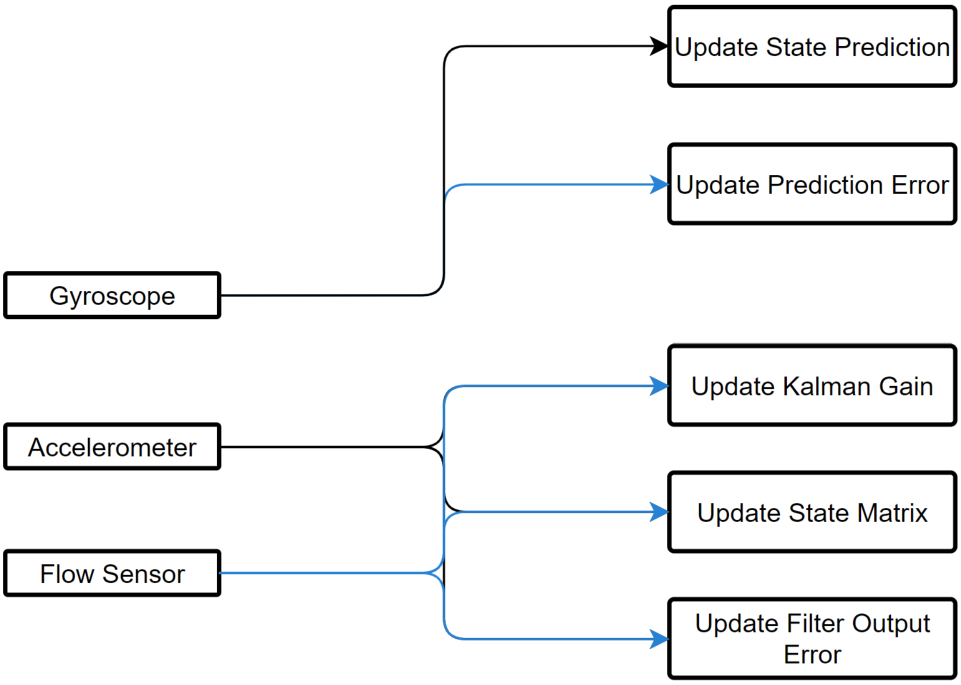
$$Q^{(k)} = \begin{bmatrix} Q_1^{(k)} & 0_{6\times 3} \\ 0_{3\times 6} & Q_2^{(k)} \end{bmatrix} \tag{25}$$

Measurement noise covariance R can be calculated as

$$R^{(k)} = \nu^{(k)} \left(\nu^{(k)}\right)^T \tag{26}$$

3.4 Filtering Process

Assume that P is the state prediction error, then the filtering process is shown as



Algorithm 2 Data Measurement Filtering

Require: sampling period T_s, initial state P_0'

while $k \in T_s$ **do**

Update state prediction: $(X^{(k+1)})' = \Phi^{(k)} X^{(k)}$

Update prediction error: $(P^{(k+1)})' = \Phi^{(k)} P^{(k)} (\Phi^{(k)})^T + Q^{(k)}$

Update Kalman gain: $K^{(k+1)} = (P^{(k+1)})'(H^{(k)})^T \left(H^{(k)}(P^{(k+1)})'(H^{(k)})^T + R^{(k)}\right)^{-1}$

Update state matrix: $X^{(k+1)} = (X^{(k+1)})' + K^{(k+1)}(Y^{(k+1)} - H^{(k)}(X^{(k+1)})')$

Update covariance: $P^{(k+1)} = (I - K^{(k+1)} H^{(k)})(P^{(k+1)})'$

end while

3.5 From Kinematic Quantities to Attitude

We use Euler's angle to depict attitude of limbs. Roll and pitch angles (γ, θ) can be measured by the gravitational acceleration g_b we obtained from the model. Assume no yaw angle occurs, then

$$\begin{aligned} \boldsymbol{g}_b = \boldsymbol{Z}_\gamma \boldsymbol{Z}_\theta \boldsymbol{g} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \\ &= \begin{bmatrix} -\sin\theta \\ \sin\gamma\cos\theta \\ -\cos\gamma\cos\theta \end{bmatrix} g \end{aligned} \tag{27}$$

Then we can come up with the attitude from the body reference frame gravitational acceleration, where

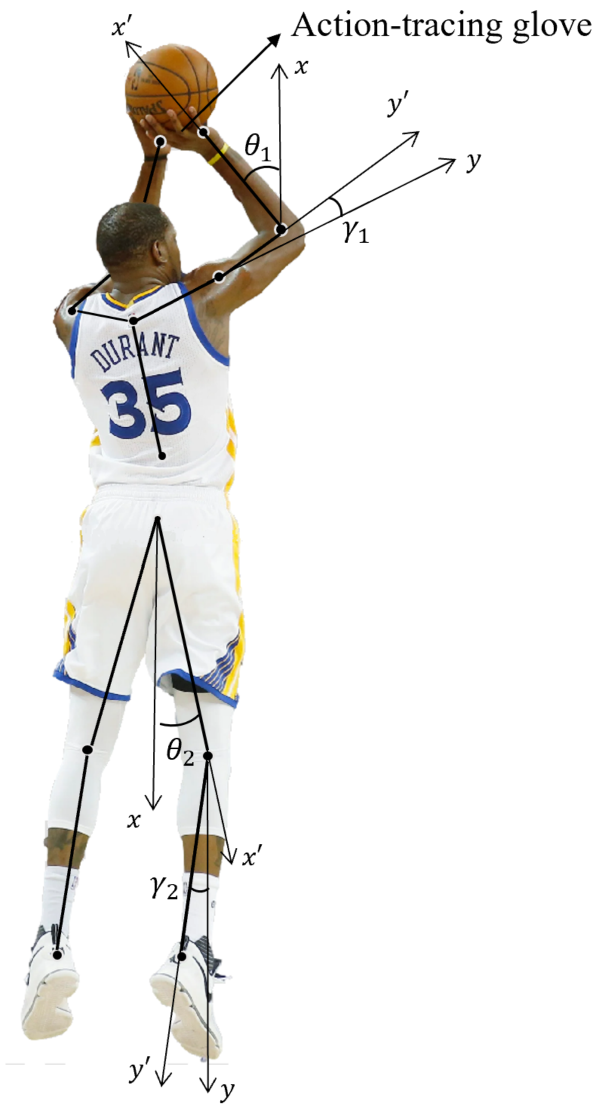
$$\begin{cases} \gamma = -\arctan \dfrac{g_{by}}{g_{bz}} \\ \theta = \arctan \left(\dfrac{g_{bx}}{\cos \gamma g_{bz}} \right) \end{cases} \tag{28}$$

4 Data Analysis

4.1 Model of Users' Motion

We establish the state model $\boldsymbol{S}$ for motion of players. Data considered in analysis section includes the roll angle and pitch angle of players' limbs. Upper limb angles are denoted as $(\boldsymbol{\theta}_1, \boldsymbol{\gamma}_1)$, and thigh angles are denoted as $(\boldsymbol{\theta}_2, \boldsymbol{\gamma}_2)$. Wearable devices are adjusted to two independent coordinate respectively. Average angular velocity $\bar{\boldsymbol{\omega}}$ can be measured by

$$\bar{\boldsymbol{\omega}} = \frac{1}{\sum_{T_s} \Delta t} \sum_{t \in T_s} \boldsymbol{\theta}(t) \tag{29}$$



Assume during time period (t_1, t_2), positive total acceleration is detected, then the maximum height h from the ground can be measured by

$$h = \frac{1}{2} g \left(\sum_{t_1}^{t_2} \Delta t \right)^2 \tag{30}$$

Under polar coordinate, we depict the comparative position of user with the basket with (β, r). Then the complete characteristic vector $\boldsymbol{S}$ is

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{\theta}_1 & \boldsymbol{\gamma}_1 & \boldsymbol{\theta}_2 & \boldsymbol{\gamma}_2 & \bar{\boldsymbol{\omega}}_1 & h & \beta & r \end{bmatrix}^T \tag{31}$$

where

$$\boldsymbol{\theta}_1 = \begin{bmatrix} \theta_{\text{left upperlimb}} & \theta_{\text{right upperlimb}} \end{bmatrix} \tag{32}$$

$$\boldsymbol{\gamma}_1 = \begin{bmatrix} \gamma_{\text{left upperlimb}} & \gamma_{\text{right upperlimb}} \end{bmatrix} \tag{33}$$

$$\boldsymbol{\theta}_2 = \begin{bmatrix} \theta_{\text{left thigh}} & \theta_{\text{right thigh}} \end{bmatrix} \tag{34}$$

$$\boldsymbol{\gamma}_1 = \begin{bmatrix} \gamma_{\text{left thigh}} & \gamma_{\text{right thigh}} \end{bmatrix} \tag{35}$$

4.2 Data Analysis Method: Support Vector Machine

The user shooting outcome can be mapped into label space $\mathcal{H}$, where

$$\mathcal{H} = \begin{cases} -1 & \text{field goal in} \\ 1 & \text{field goal made} \end{cases} \tag{36}$$