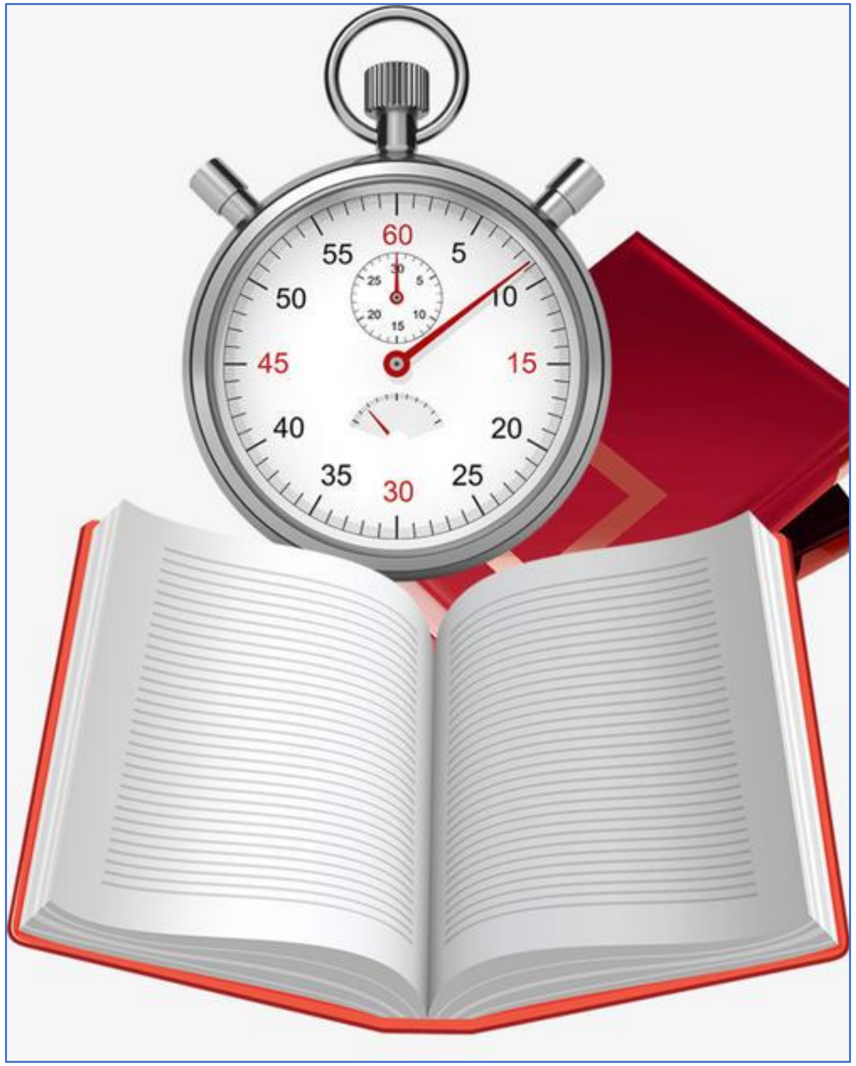
CS 103 -05

Perceptron and AI Early Day Algorithms

Jimmy Liu 刘江

2020-10-16

Lecture 04 Review



Early AI Algorithms

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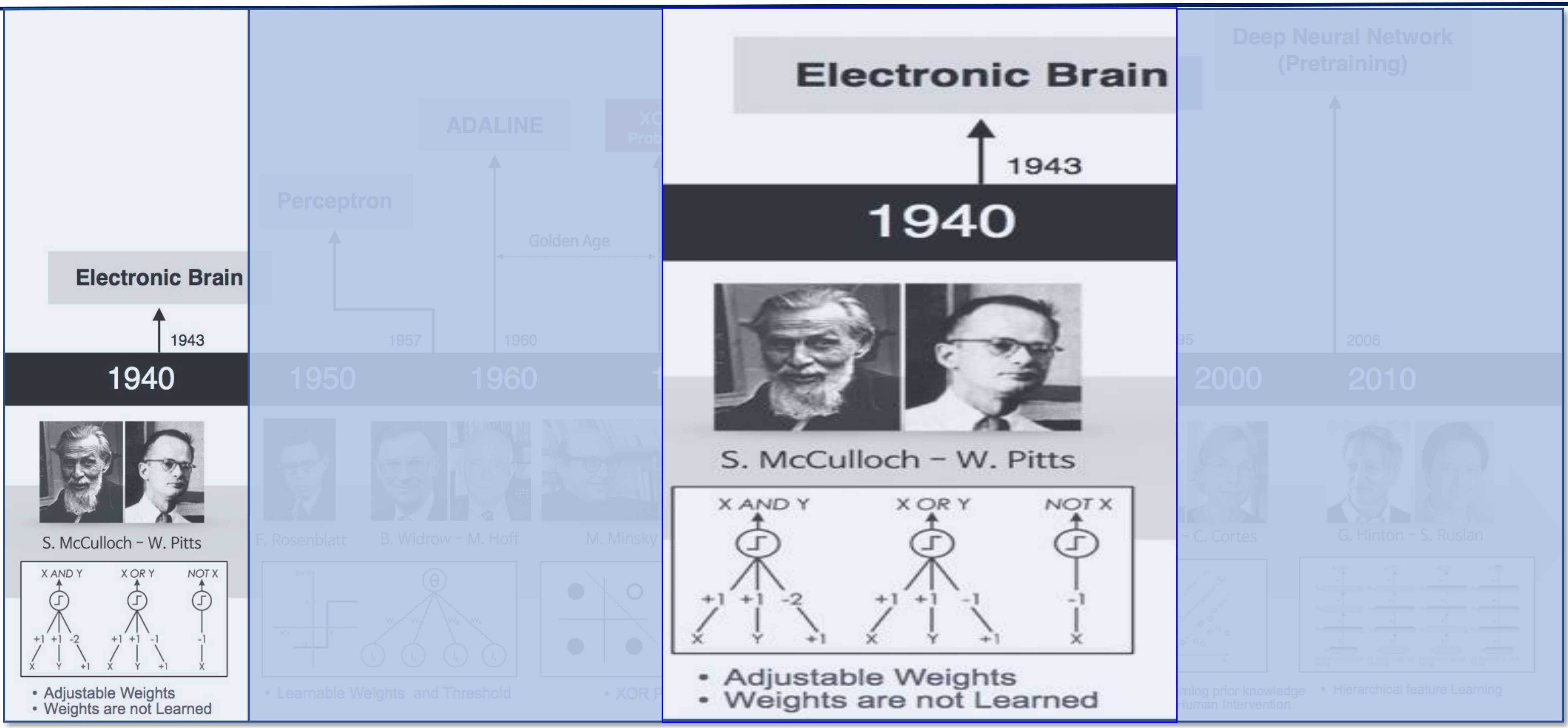
1. Early-AI Algorithms
2. Electronic Brain
3. Turing Test
4. Chinese Card Room

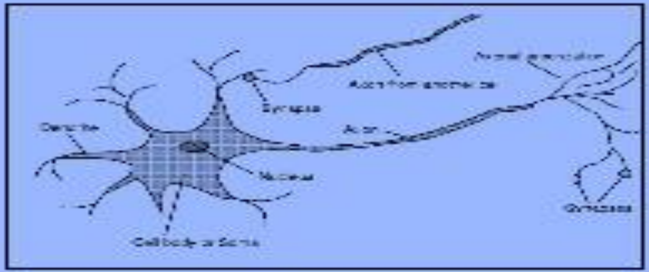
AI algorithm Developments - A Closer Look

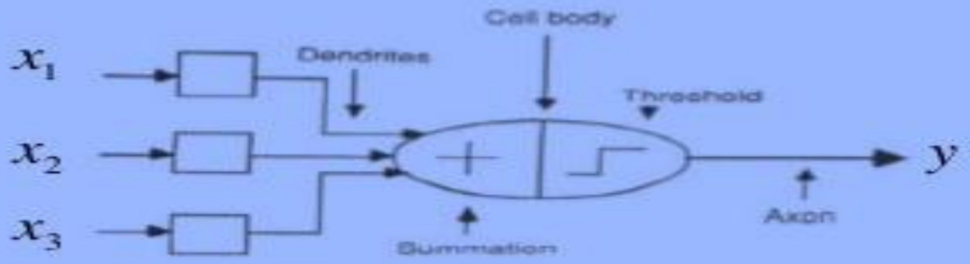


Electronic Brain from McCulloch and Pitts

Early Artificial Neurons (1943-1969)

- McCulloch- Pitts neuron

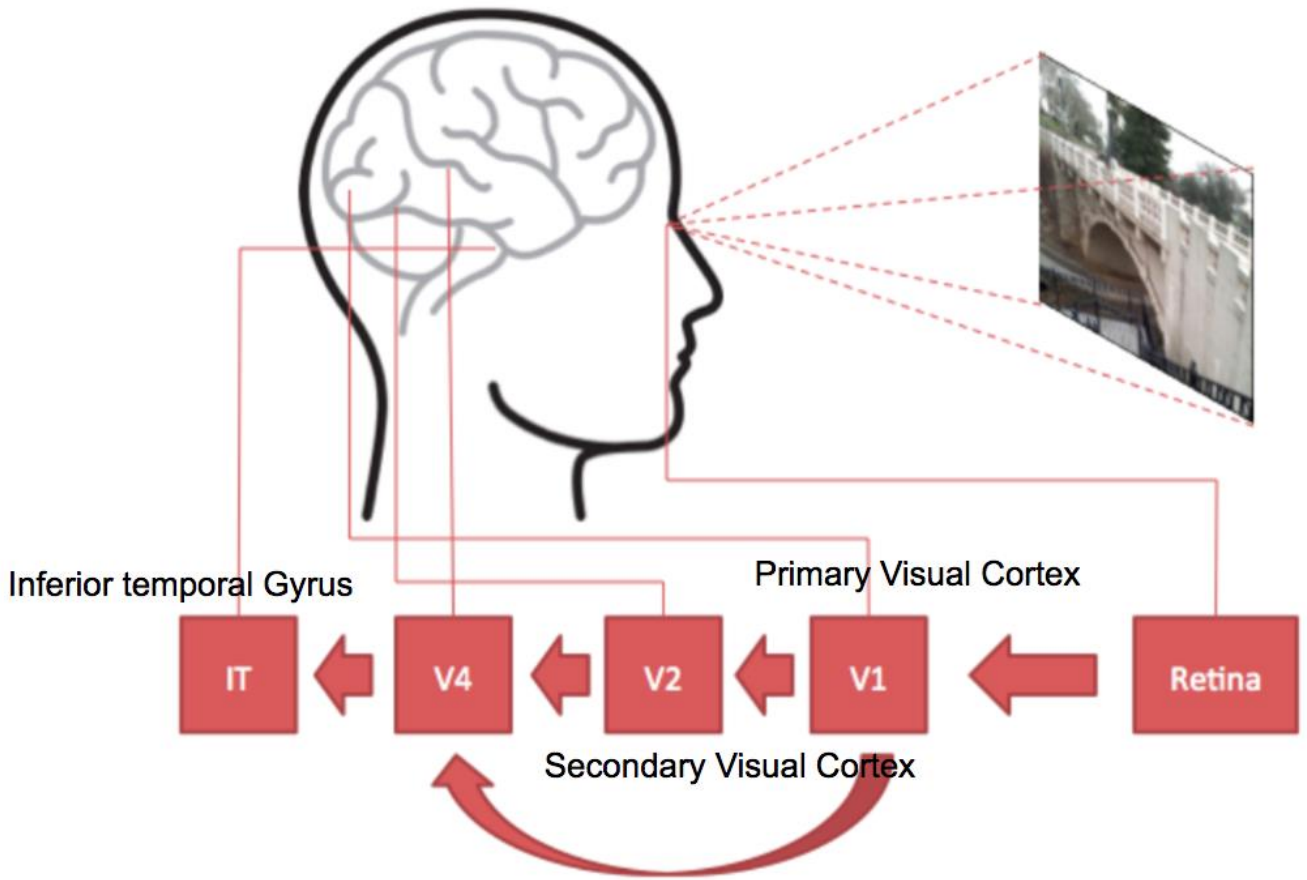




$$y = f\left(\sum_m w_m x_m - b\right) \quad f(x) = \begin{cases} 1 & \text{if } x \geq 0; \\ 0 & \text{otherwise} \end{cases}$$

- McCulloch and Pitts [1943] proposed a simple model of a neuron as computing machine
- The artificial neuron computes a weighted sum of its inputs from other neurons, and outputs a one or a zero according to whether the sum is above or below a certain threshold

Visual Input to Brain: Brain Computing



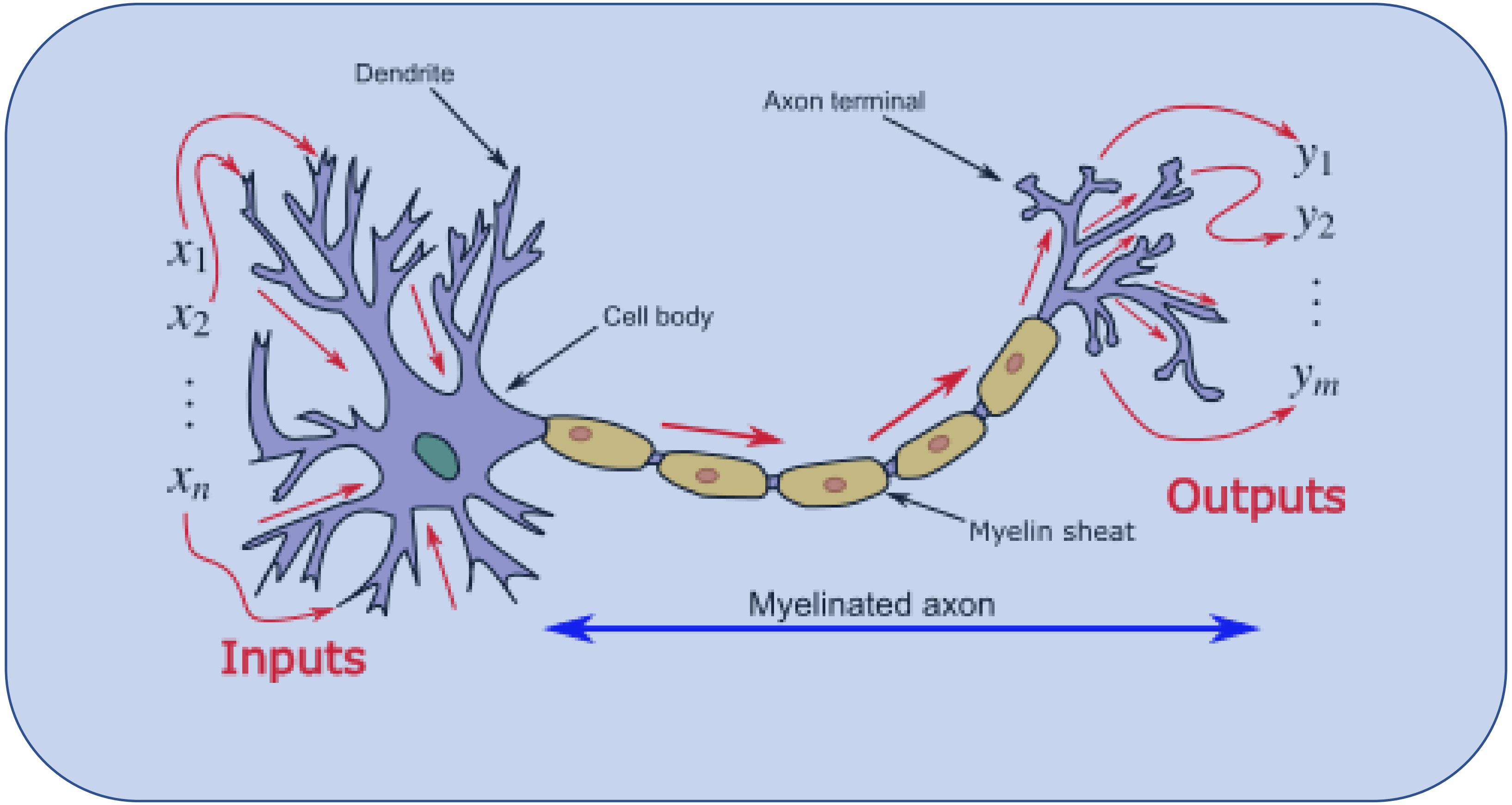
V1: Edge detection, etc.

V2: Extract simple visual properties (orientation, spatial frequency, color, etc)

V4: Detect object features of intermediate complexity

TI: Object recognition.

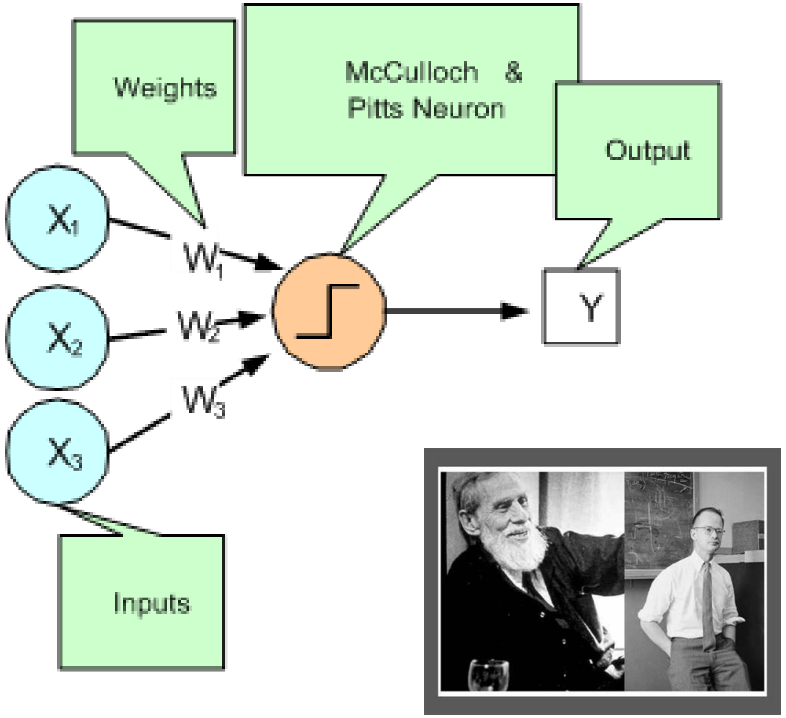
Schematic of a Biological Neuron



MCP (McCulloch and Pitts) Neuron – Weights Are Adjusted But Not Learnt

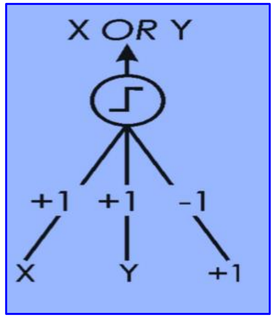
- Tried to understand how the brain could produce highly complex patterns by using many basic cells that are connected together.

- These basic brain cells are called neurons, and McCulloch and Pitts gave a highly simplified model of a MCP neuron in their paper.



Warren S. McCulloch, a neuroscientist, and Walter Pitts, a logician, "A logical calculus of the ideas immanent in nervous activity", the Bulletin of Mathematical Biophysics 5:115-133. 1943

Prove “OR” MCP (McCulloch and Pitts) Neuron



A	B	Bias	W1	W2	W3	Transfer
0	0	1	1	1	-1	G(z)
0	1	1	1	1	-1	G(z)
1	0	1	1	1	-1	G(z)
1	1	1	1	1	-1	G(z)

$Z = \sum$ = A*W1+B*W2+ Bias*W3	Output F=G(Z) (Z>=0)	A OR B
0*1 + 0*1 + 1*(-1) = (-1)	0	0
0*1 + 1*1 + 1*(-1) = 0	1	1
1*1 + 0*1 + 1*(-1) = 0	1	1
1*1 + 1*1 + 1*(-1) = 1	1	1

Transfer Function is G(z)

$$g(z) = \begin{cases} 1 \text{ if } z \geq 0 \\ 0 \text{ otherwise.} \end{cases}$$

Turing Test – Operational Definition

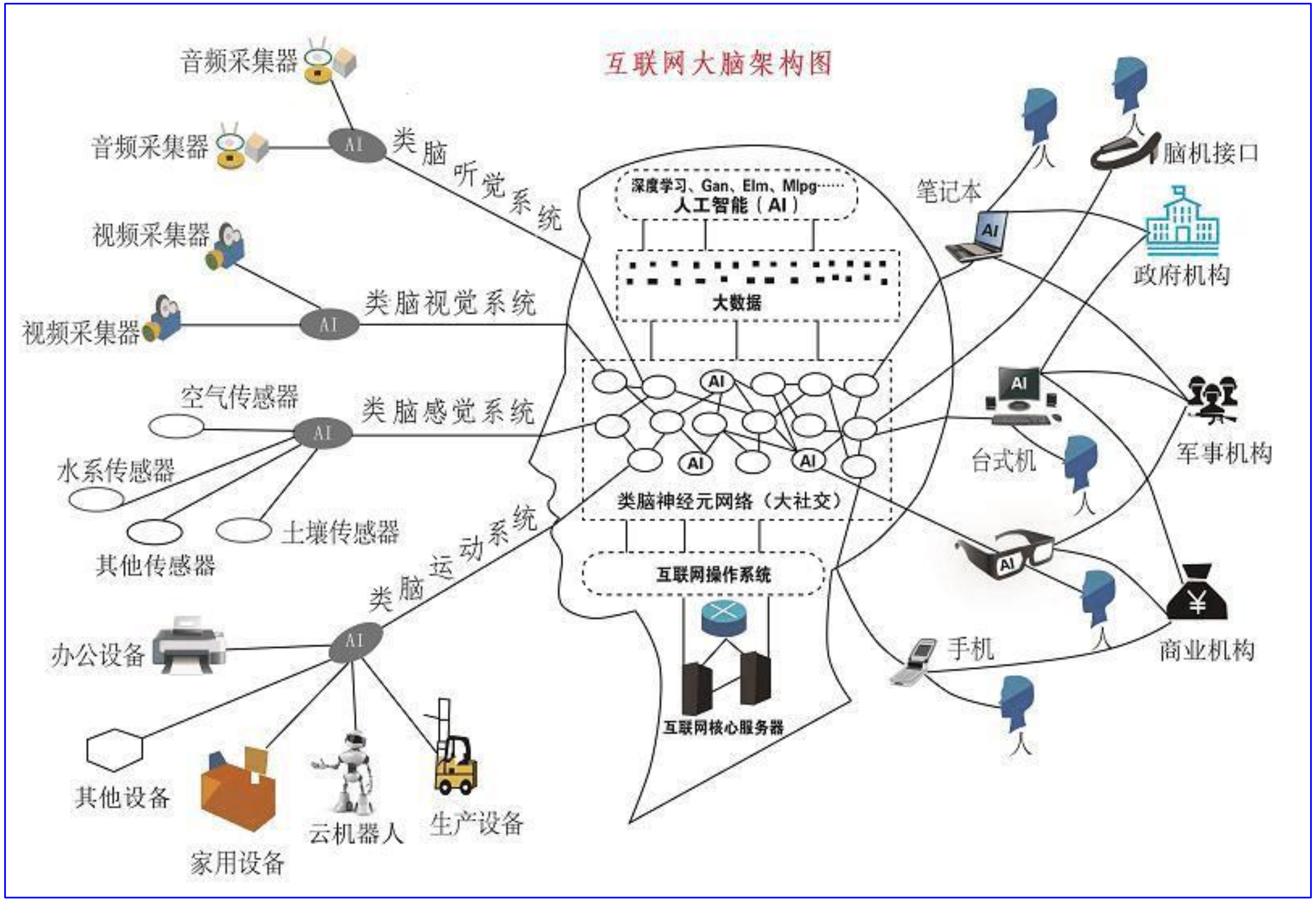
1. If the Turing Test was passed, Turing would conclude that the machine was intelligent.
2. Suggested as a way of saying when we could consider machines to be intelligent, or at least act intelligently
3. A satisfactory operational definition of intelligence

Chinese Room and Turing Test

The Chinese Room thought experiment is an analogy to artificial intelligence. A person who can't speak Chinese is sitting in a room text chatting in Chinese. They have a book that gives them an appropriate response to each series of symbols that appear in the chat. The person on the other side of the chat can't tell that they are speaking to someone who can't speak Chinese. The person in the room doesn't understand anything about the conversation and is simply looking up symbols in a book.

The Turing test, a common way to define and test artificial intelligence, involves a computer imitating a human on a chat. According to the test, if a machine can convince people that it's human, it's intelligent. The Chinese Room analogy shows that by this definition of machine intelligence that computers need not understand the conversation to pass.

类脑（Brain-Inspired Intelligence）



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3. High Score Gamer：**黄北辰**
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6. 基于MRI图像的阿尔茨海默症分类：**郑英炜**
7. AI Applications in Breast Cancer Imaging：**翟靖蕾**
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15. 人工智能在无障碍设施领域中的使用调查：**马子晗**
16. identification of handwriting elements：没有确定
17. AI虚拟主播制作计划：**张倚凡**
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21. 深度学习在自动驾驶中的应用：**王晓轩**

Wrong Answer 1

Prove "NOT X" Artificial Neuron

X	W1	Transfer	Z=-w1-x	G(z)	Not X
0	-1	G(z)	1	1	1
1	0	G(z)	-1	0	0

Wrong Answer 2

A	W^1	Transfer	$z = A * W^1$	Output = G(z)
0	-1	G(z)	0 * 1 = 0	1
1	-1	G(z)	1 * (-1) = 1	0

Wrong Answer 3

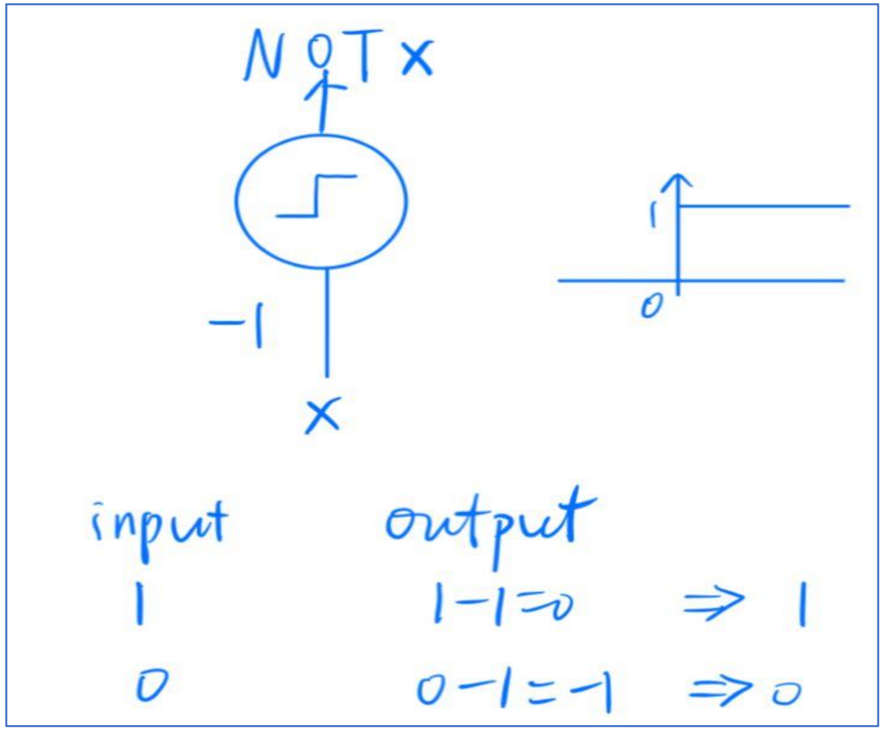
Prove not X Artifical Neuron.

A	Bias	W_1	W_2	Transfer
1	1	-1	1	$G(z)$
0	1	-1	1	$G(z)$

$Z = \Sigma = A^* W_1 + Bias^* W_2$	Output $f = G(z)$	A NOT B
$1 \times (-1) + 1 \times 1 = 0$	0	0
$0 \times (-1) + 1 \times 1 = 1$	1	1

Transfer function $G(z)$ is $g(z) = \begin{cases} 0, & \text{if } z \le 0 \\ 1, & z \text{ else} \end{cases}$

Wrong Answer 4



Wrong Answer 5

A	W1	Transfer	Output(=g(z))	NOT X
0	0	g(z)	1	1
1	-1	g(z)	0	0

$$g(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Correct Answers

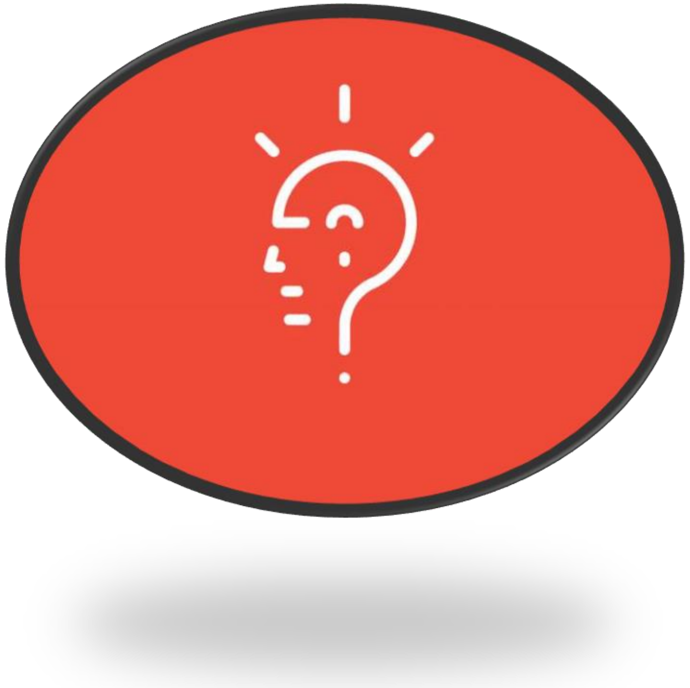
x	w	z = x*w	g(z)	NOT X
0	-1	0	1	1
1	-1	-1	0	0

A	Bias	W1	transger	Z = W1*A	Output F =G(z)	Not A
0	0	-1	G(z)	0	1	1
1	0	-1	G(z)	-1	0	0

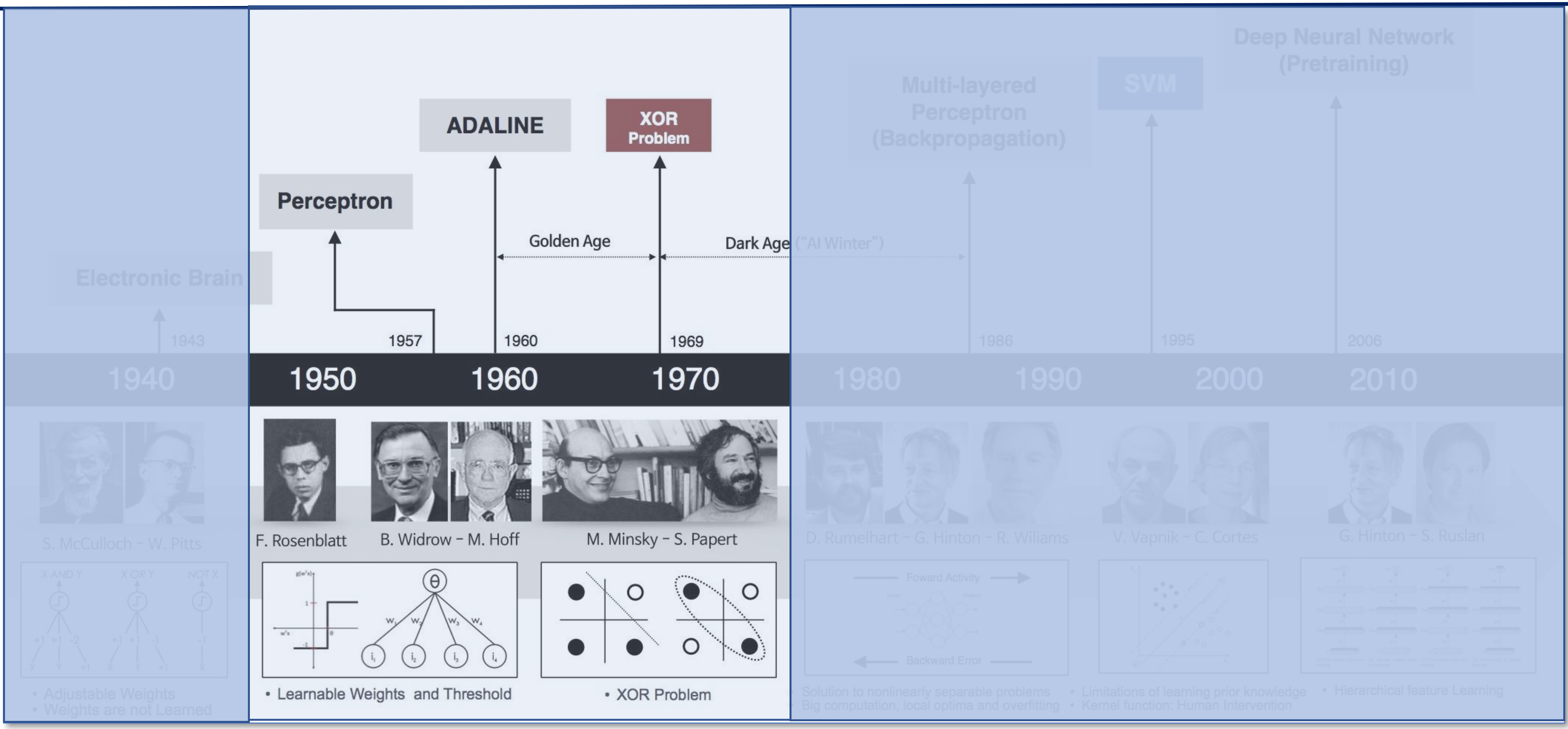
x	w	Transfer	*Z* = x*w	Output y=g（z）	Not x
1	-1	g（z）	-1	0	0
0	-1	g（z）	0	1	1

X	W	Transfer	Z=Σ=X*W	Output F=G(Z)	NOT X
0	-1	G(Z)	0*(-1)=0	1	1
1	-1	G(Z)	1*(-1)=-1	0	0

Any Question?



AI algorithm Developments - A Closer Look



Perceptron

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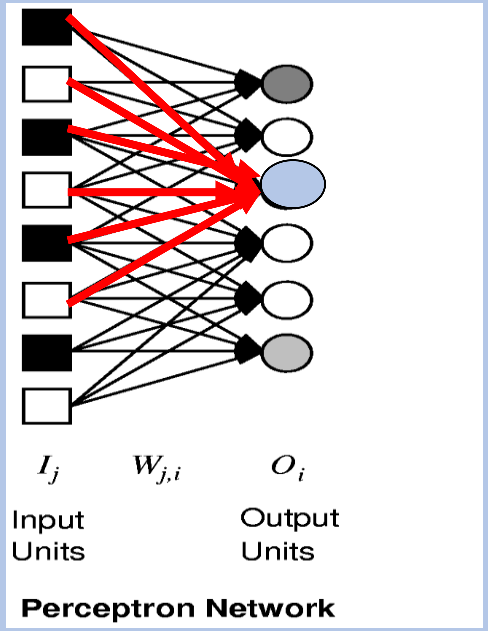
Perceptrons

•Single-layer feed forward neural network (perceptron network)

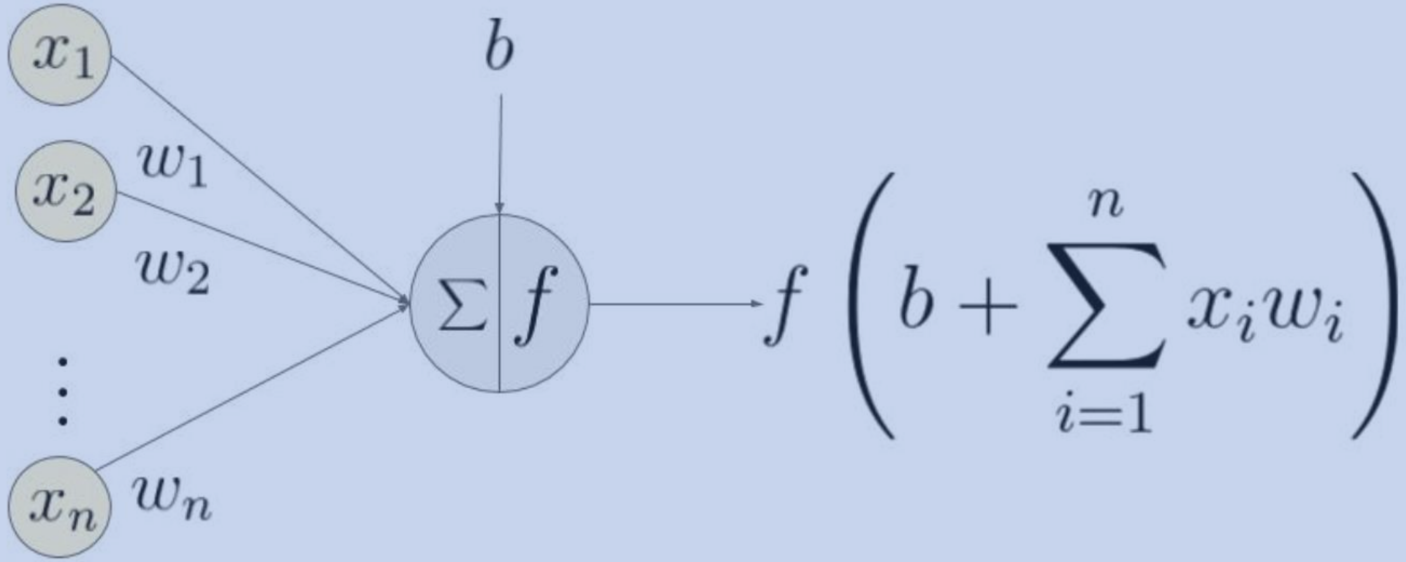
A network with all the inputs connected directly to the outputs

–Output units all operate separately: no shared weights



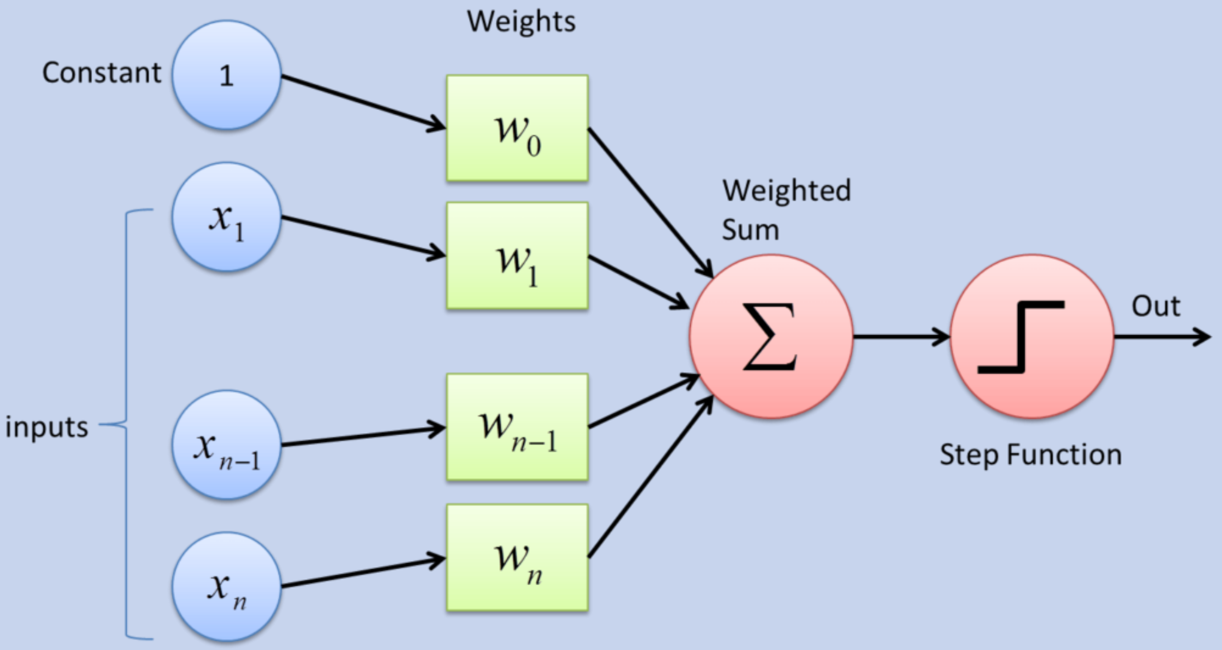
Since each output unit is independent of the others, we can limit our study to single output perceptrons.

"Artificial" Neuron



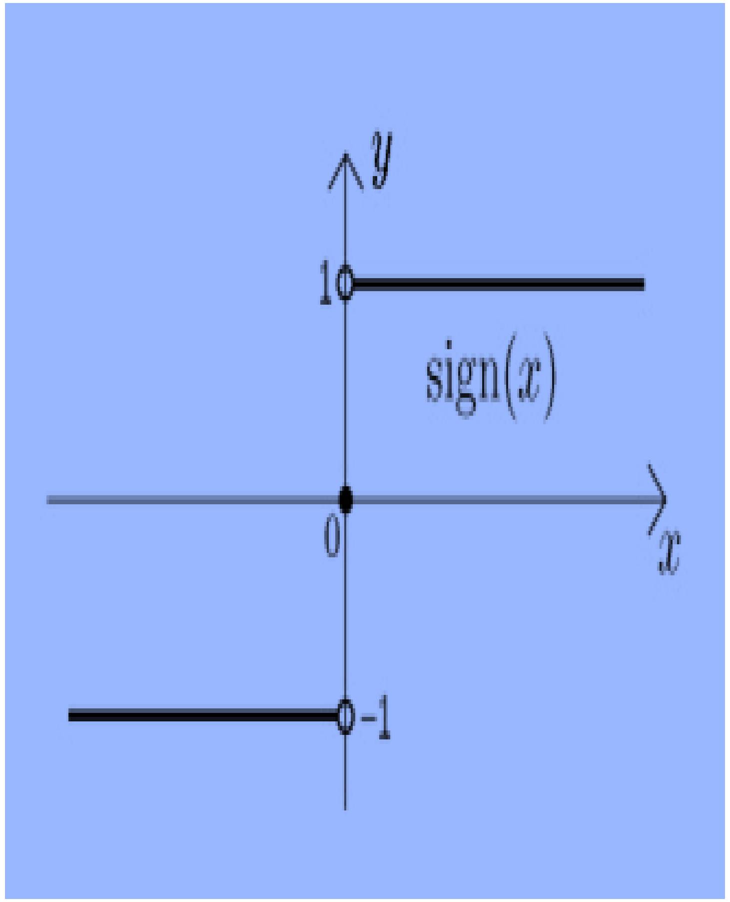
An example of a neuron showing the input (x_1 - x_n), their corresponding weights (w_1 - w_n), a bias (b) and the activation function f applied to the weighted sum of the inputs.

Traditional Perceptron



$$f(x) = sign(w \cdot x + b)$$

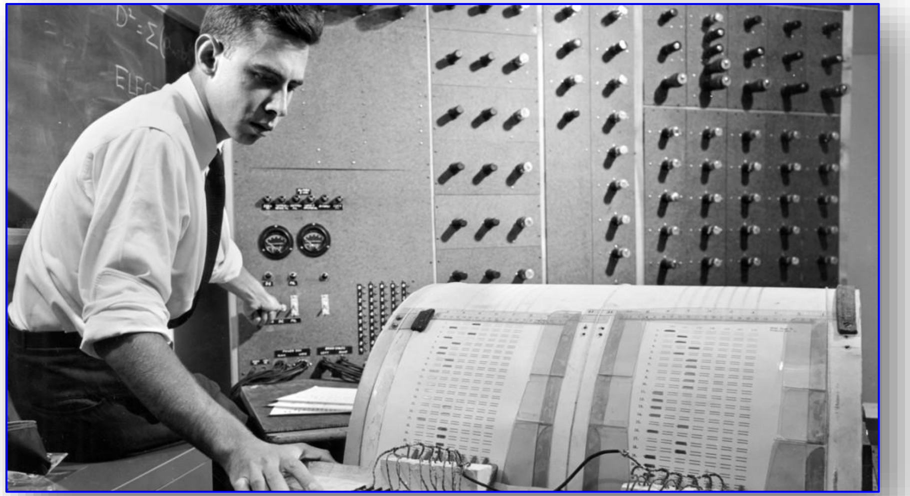
$$sign(x) = \begin{cases} +1, & x \geq 0 \\ -1, & x < 0 \end{cases}$$



Perceptron Weights are Learned Y= F (W*X)

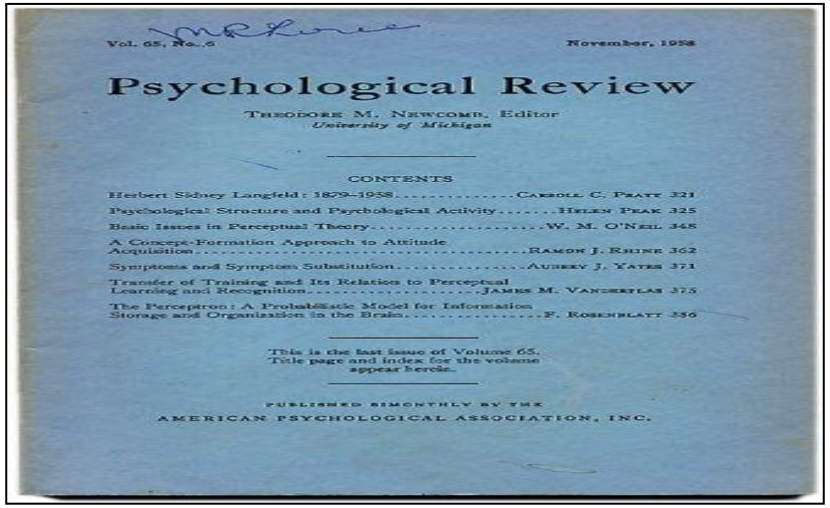
Initialize $\vec{w} = \vec{0}$	// Initialize $\vec{w}$. $\vec{w} = \vec{0}$ misclassifies everything.
while TRUE **do**	// Keep looping
$\quad m = 0$	// Count the number of misclassifications, m
$\quad$ **for** $(x_i, y_i) \in D$ **do**	// Loop over each (data, label) pair in the dataset, D
$\qquad$ **if** $y_i(\vec{w}^T \cdot \vec{x}_i) \leq 0$ **then**	// If the pair $(\vec{x}_i, y_i)$ is misclassified
$\qquad\quad \vec{w} \leftarrow \vec{w} + y\vec{x}$	// Update the weight vector $\vec{w}$
$\qquad\quad m \leftarrow m + 1$	// Counter the number of misclassification
$\qquad$ **end if**	
$\quad$ **end for**	
$\quad$ **if** $m = 0$ **then**	// If the most recent $\vec{w}$ gave 0 misclassifications
$\qquad$ break	// Break out of the while-loop
$\quad$ **end if**	
end while	// Otherwise, keep looping!

Perceptron and Rosenblatt



- In July 1958, the U.S. Office of Naval Research unveiled an invention.
- An IBM 704 – a 5-ton computer the size of a room – was fed a series of punch cards. After 50 trials, the computer taught itself to distinguish cards marked on the left from cards marked on the right.
- Frank Rosenblatt Cornell Ph.D. works on the “perceptron” – what he described as the first machine “capable of having an original idea.”

Perceptron



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Theodore M. Newcomb, Editor
University of Michigan

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ROSENBLATT, Frank. (Cornell Aeronautical Laboratory at Cornell University)

The Perceptron: A Probabilistic Model for Information Storage and Organization in the Brain.

In, Psychological Review, Vol. 65, No. 6, pp. 386-408, November, 1958.

Vol. VI, No. 2, Summer 1958

research trends

CORNELL AERONAUTICAL LABORATORY, INC., BUFFALO 21, NEW YORK

The Design of an Intelligent AUTOMATON

by FRANK ROSENBLATT

Introducing the perceptron — A machine which senses, recognizes, remembers, and responds like the human mind.

STORIES about the creation of machines having human qualities have long been a fascinating province in the realm of science fiction. Yet we are now about to witness the birth of such a machine — a machine capable of perceiving, recognizing, and identifying its surroundings without any human training or control.

Development of that machine has stemmed from a search for an understanding of the physical mechanisms which underlie human experience and intelligence. The question of the nature of these processes is at least as ancient as any other question in western science and philosophy, and, indeed, ranks as one of the greatest scientific challenges of our time.

Our understanding of this problem has gone perhaps as far as had the development of physics before Newton. We have some excellent descriptions of the phenomena to be explained, a number of interesting hypotheses, and a little detailed knowledge about events in the nervous system. But we lack agreement on any integrated set of principles by which the functioning of the nervous system can be understood.

We believe now that this ancient problem is about to yield to our theoretical investigation for three reasons:

First, in recent years our knowledge of the functioning of individual cells in the central nervous system has vastly increased.

Second, large numbers of engineers and mathematicians are, for the first time, undertaking serious study of the mathematical basis for thinking, perception, and the handling of information by the central nervous system, thus providing the hope that these problems may be within our intellectual grasp.

Third, recent developments in probability theory and in the mathematics of random processes provide new tools for the study of events in the nervous system, where only the gross statistical organization is known and the precise cell-by-cell "wiring diagram" may never be obtained.

Receives Navy Support

In July, 1957, Project PARA (Perceiving and Recognizing Automaton), an internal research program which had been in progress for over a year at Cornell Aeronautical Laboratory, received the support of the Office of Naval Research. The program had been concerned primarily with the application of probability theory to

" Introducing the Perceptron – A machine which

senses,
recognizes,
remembers,
responses

like the human minds"

Med

Perceptron From New York Times

NEW NAVY DEVICE LEARNS BY DOING

Psychologist Shows Embryo of Computer Designed to Read and Grow Wiser

able to walk, talk, see, write, reproduce itself and be conscious of its existence.

, July 7 (UPI) ...ealed the em-...onic computer ...pects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence.

The embryo—the Weather Bureau's $2,000,000 "704" computer—learned to differentiate between right and left after fifty attempts in the Navy's demonstration for newsmen.

The service said it would use this principle to build the first of its Perceptron thinking machines that will be able to read and write. It is expected to be finished in about a year at a cost of $100,000.

Dr. Frank Rosenblatt, designer of the Perceptron, conducted the demonstration. He said the machine would be the first device to think as the human brain. As do human beings, Perceptron will make mistakes at first, but will grow wiser as it gains experience, he said.

Dr. Rosenblatt, a research psychologist at the Cornell Aeronautical Laboratory, Buffalo, said Perceptrons might be fired to the planets as mechanical space explorers.

Without Human Controls

The Navy said the perceptron would be the first non-living mechanism "capable of receiving, recognizing and identifying its surroundings without any human training or control."

The "brain" is designed to remember images and information it has perceived itself. Ordinary computers remember only what is fed into them on punch cards or magnetic tape.

Later Perceptrons will be able to recognize people and call out their names and instantly translate speech in one language to speech or writing in another language, it was predicted.

Mr. Rosenblatt said in principle it would be possible to build brains that could reproduce themselves on an assembly line and which would be conscious of their existence.

falo, said Perceptrons might be fired to the planets as mechanical space explorers.

In today's demonstration, the "704" was fed two cards, one with squares marked on the left side and the other with squares on the right side.

Learns by Doing

In the first fifty trials, the machine made no distinction between them. It then started registering a "Q" for the left squares and "O" for the right squares.

Dr. Rosenblatt said he could explain why the machine learned only in highly technical te...

ciple it would be possible to build brains that could reproduce themselves on an assembly line and which would be conscious of their existence.

Perceptron and Rosenblatt

1957: Rosenblatt's perceptrons were initially simulated on an IBM 704 computer at Cornell Aeronautical Laboratory in 1957.

1958: The perceptron written by Frank Rosenblatt (his father was a medical doctor and he specilzed in psychological research) was first published in1958.

1959: In 1959 he went to Cornell's Ithaca campus as director of the Cognitive Systems Research Program and also as a lecturer in the Psychology Department.

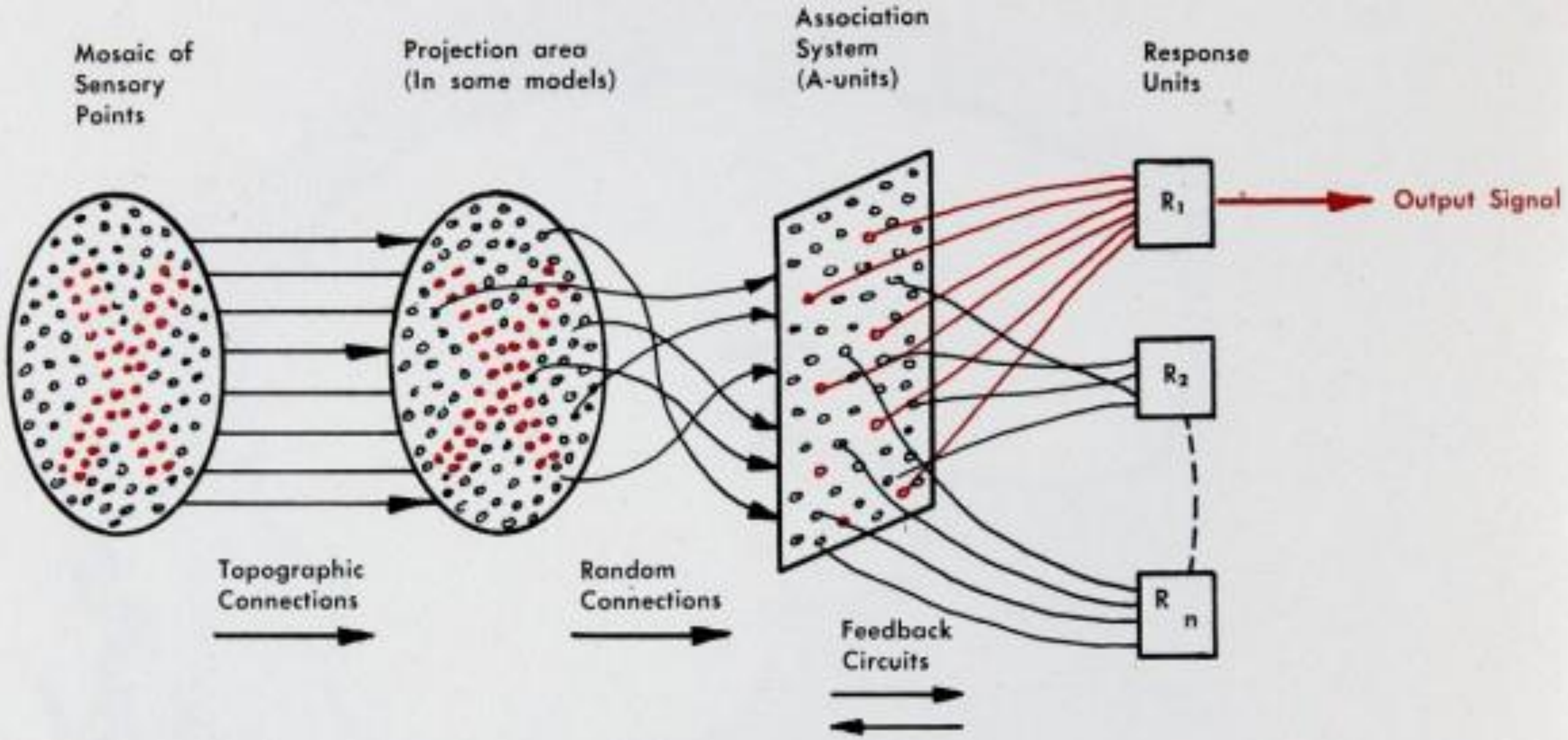
1962: Principles of neurodynamics written by Frank Rosenblatt was first published in 1962.

1966: In 1966 he joined the Section of Neurobiology and Behavior within the newly formed Division of Biological Sciences, as an associate professor.

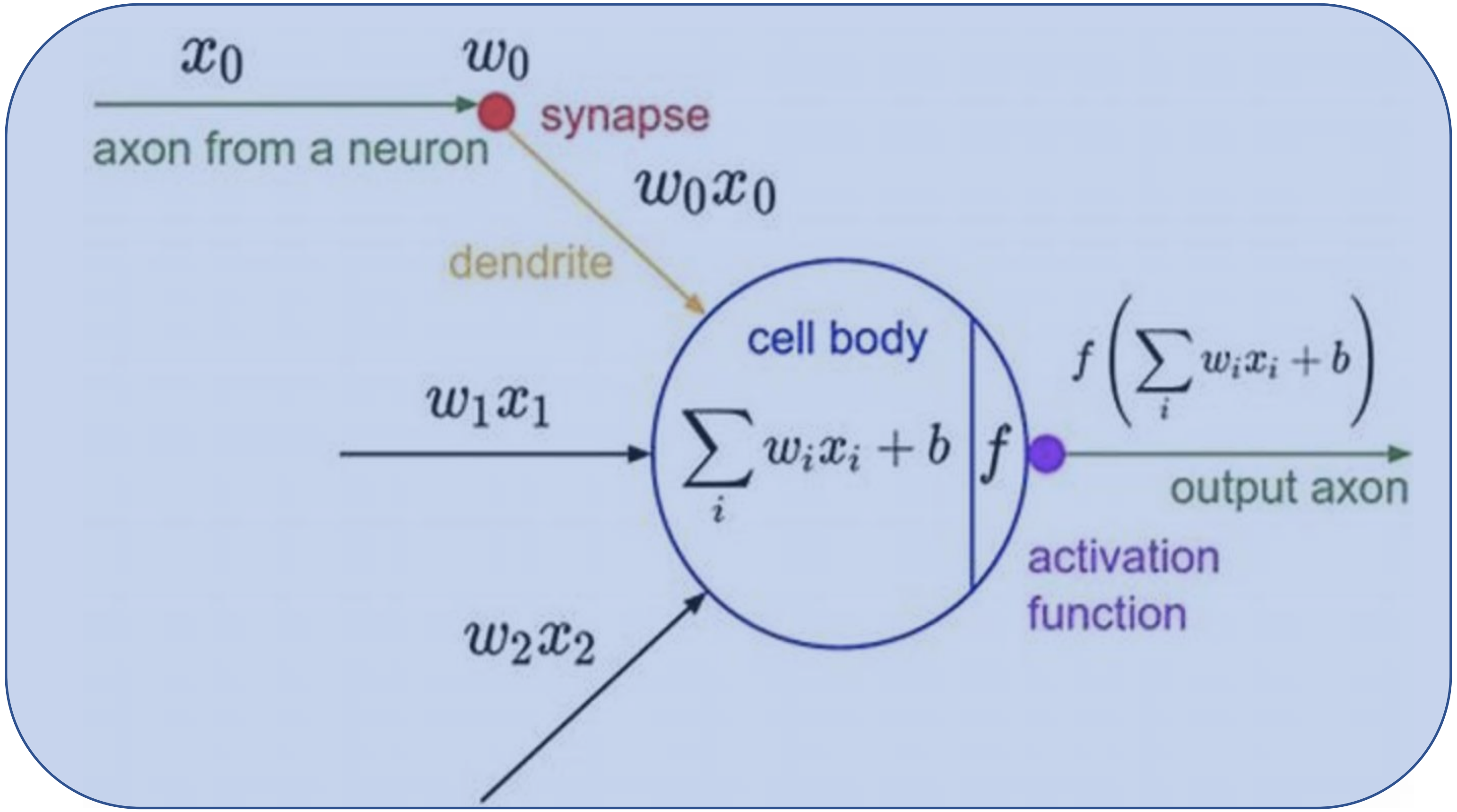
1971: Frank Rosenblatt died in July 1971 on his 43rd birthday, in a boating accident in Chesapeake Bay.

Perceptron and Human Brain

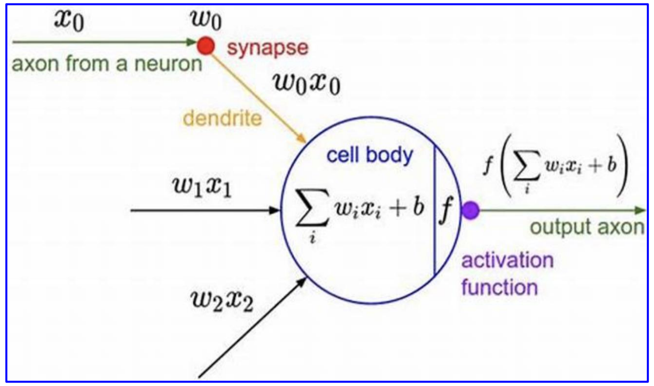
FIG. 1 — Organization of a biological brain. (Red areas indicate active cells, responding to the letter X.)



Perceptron Unit Mimics the Neuron

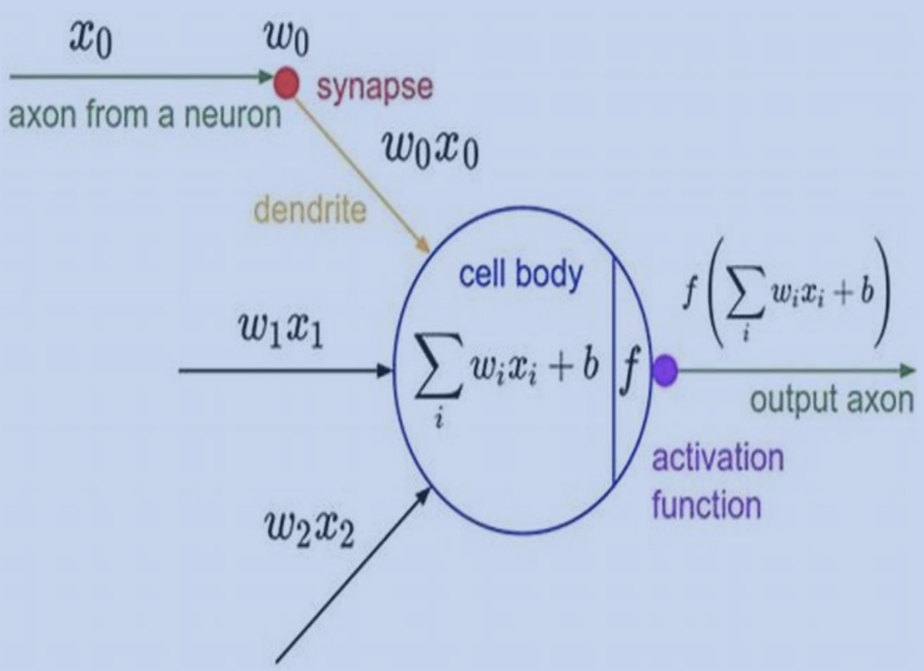


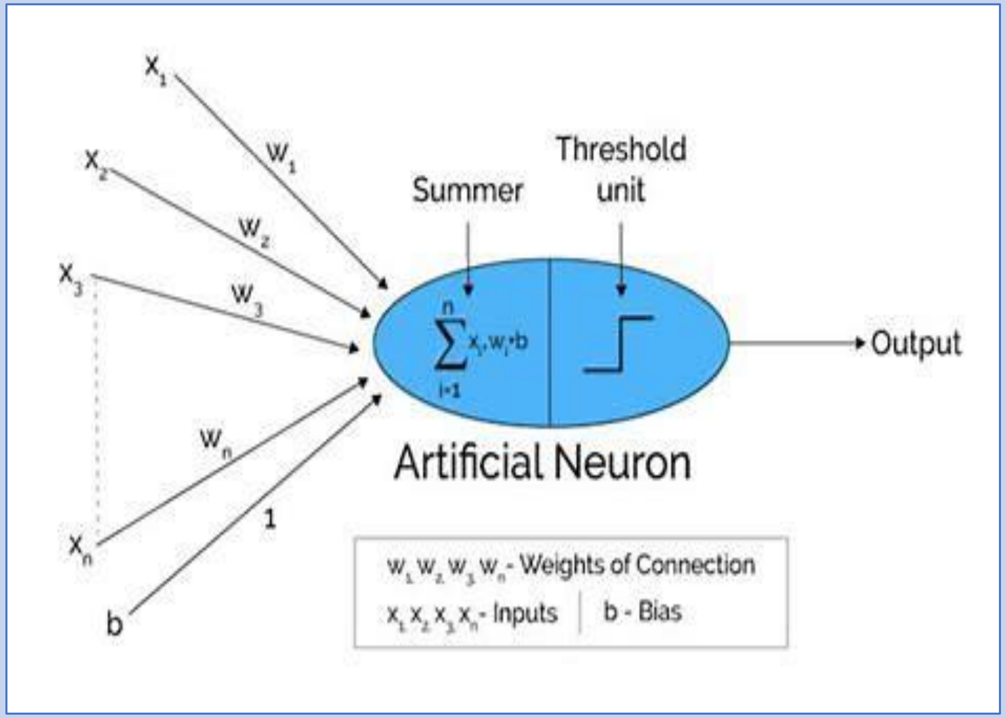
Perceptron Unit Mimics the Neuron



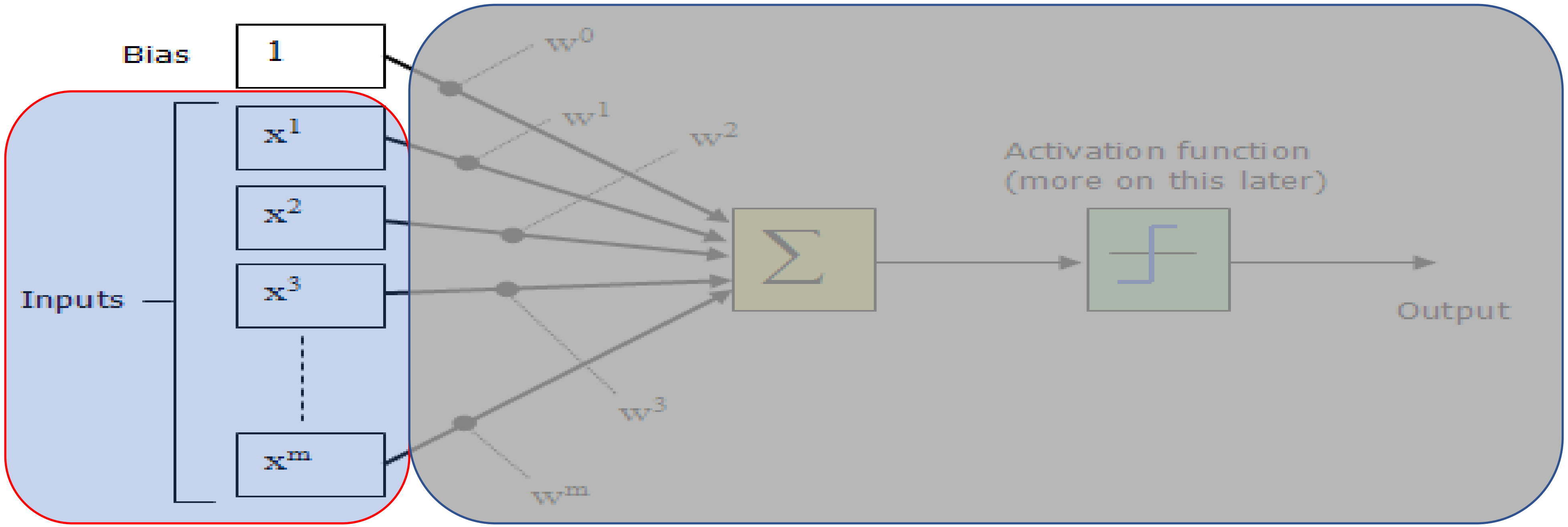
Inspired by the way neurons work together in the brain, the perceptron is a single-layer neural network – an algorithm that classifies input into two possible categories.
The neural network makes a prediction – say, right or left; or dog or cat – and if it's wrong, tweaks itself to make a more informed prediction next time. It becomes more accurate over thousands or millions of iterations.

Perceptron Unit Mimics the Neuron



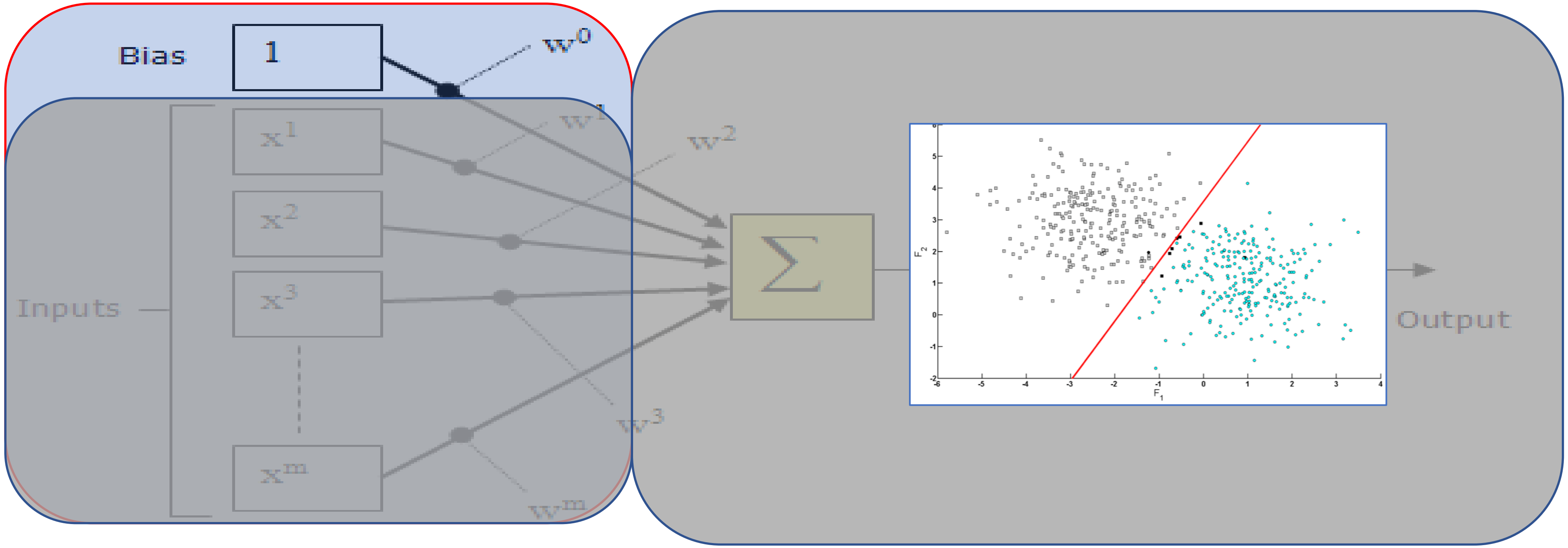


Perceptron 6 Components 1 - Input



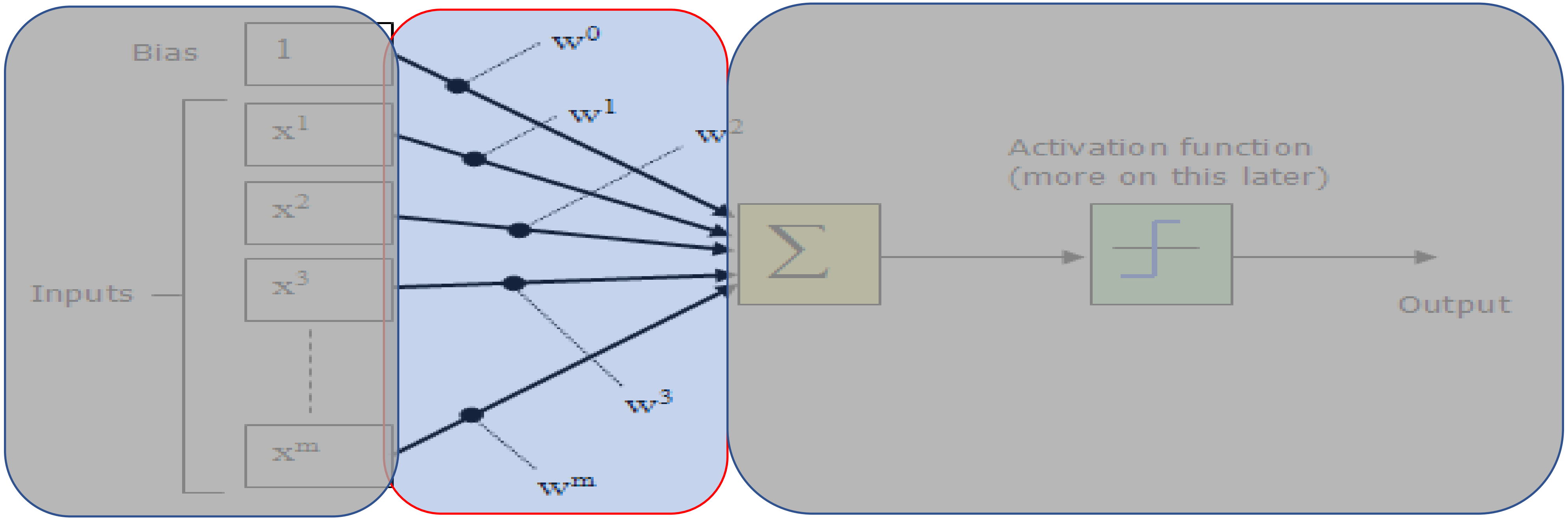
All the feature becomes the input for a perceptron. We denote the input of a perceptron by [x1, x2, x3, ..,xn], here x represent the feature value and n represent the total number of features. We also have special kind of input called the BIAS

Perceptron 6 Components 2 - Bias



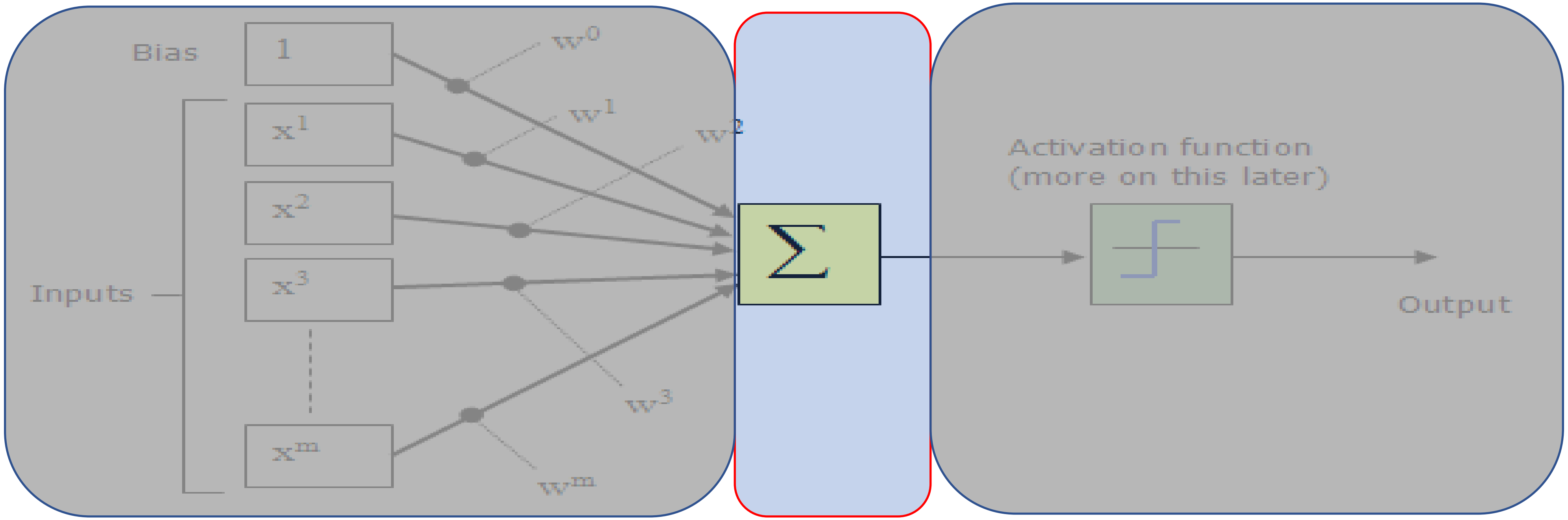
A bias neuron allows a classifier to shift the decision boundary left or right. In an algebraic term, the bias neuron allows a classifier to translate its decision boundary. To translation is to “move every point a constant distance in a specified direction”. BIAS helps to training the model faster and with better quality.

Perceptron 6 Components 3 - Weight



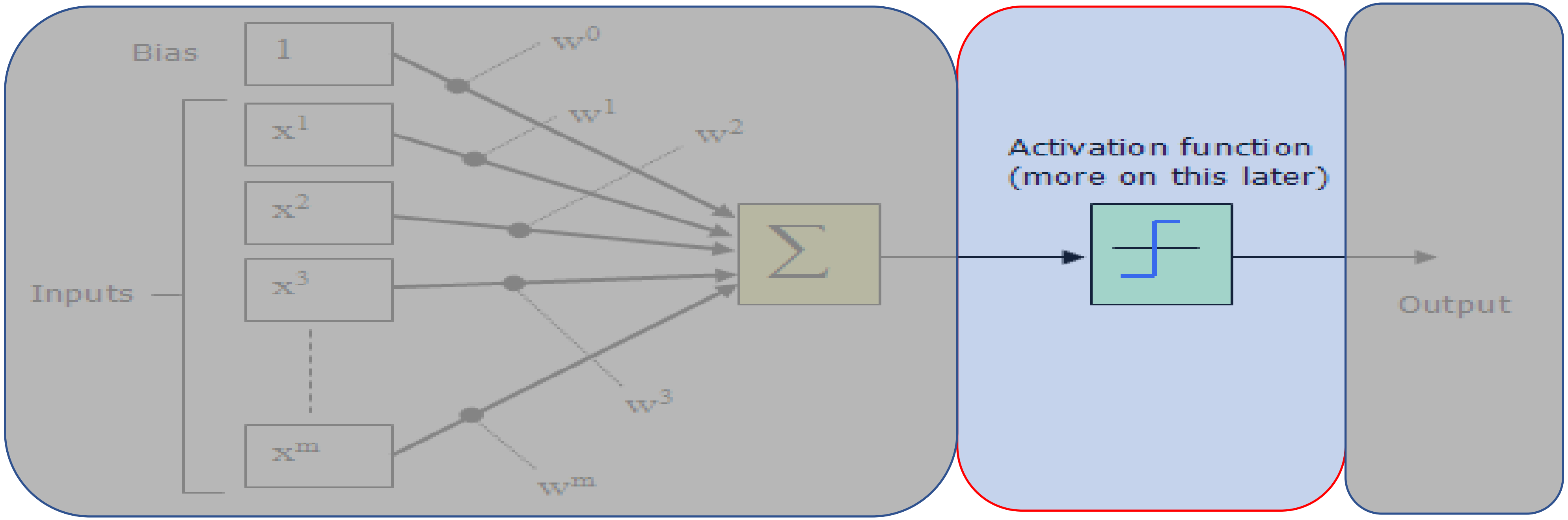
The weights offer an preliminary value in the very beginning of algorithm learning. With the occurrence of every training inaccuracy, the weights values are updated. These are mainly signified as w1, w2, w3, w4 and so on.

Perceptron 6 Components 4 – Weight Summation



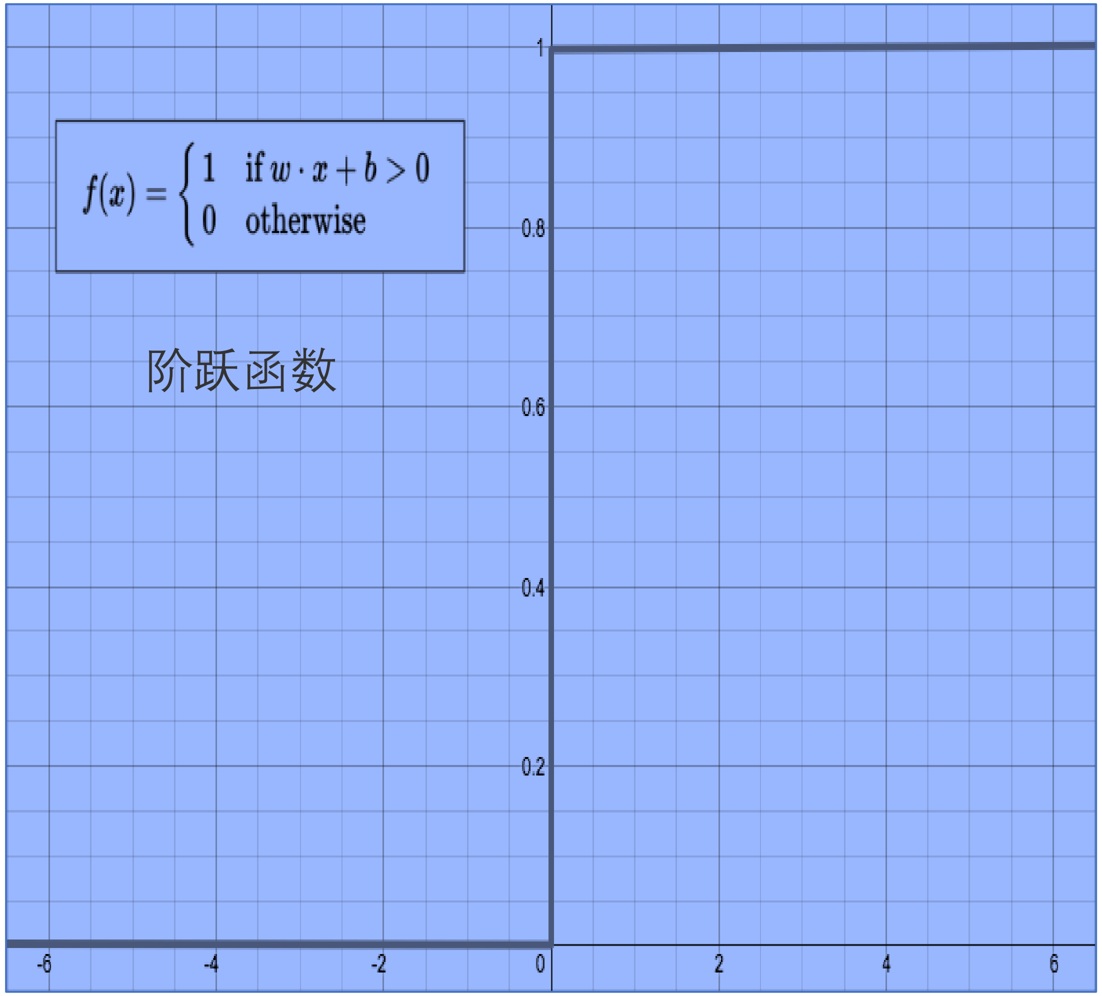
Weighted Summation is the sum of value that we get after the multiplication of each weight [wn] associated the each feature value[xn]. We represent the weighted Summation by $\sum w_i x_i$ for all **i -> [1 to n]**

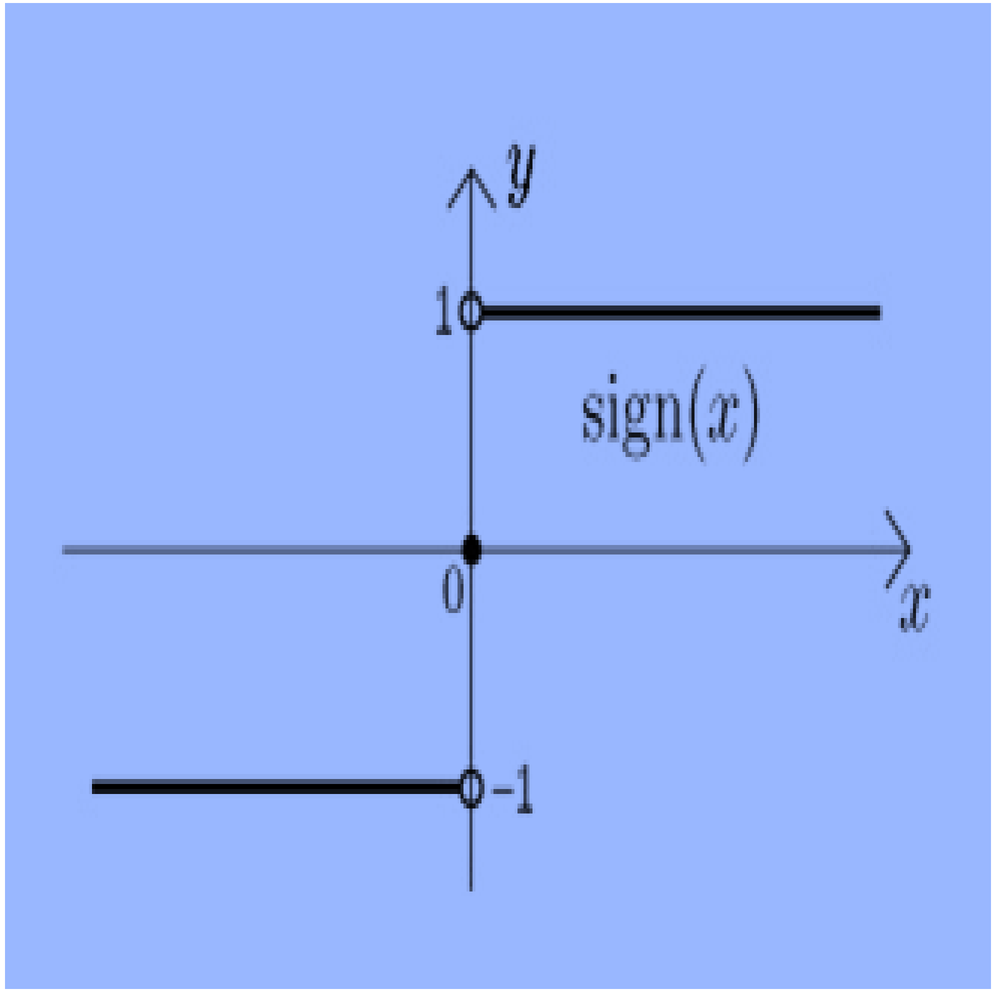
Perceptron 6 Components 5 – Transfer Function



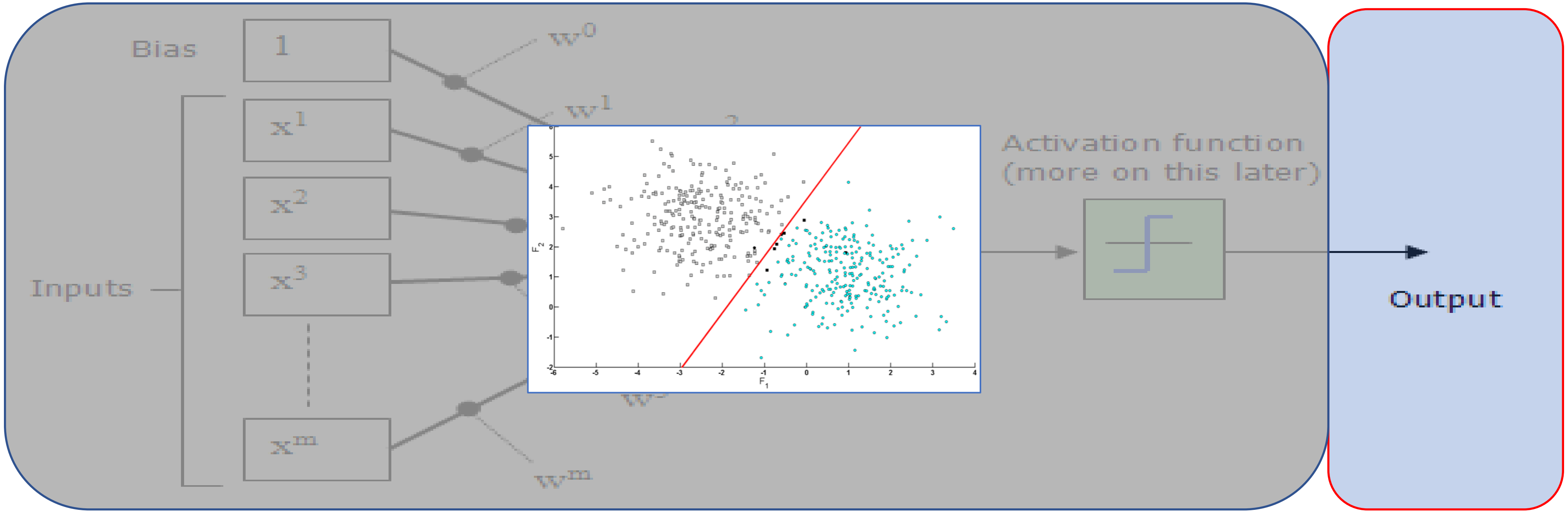
Transfer/Step/Activation Function:- the role of activation functions is make neural networks linear/non-linear. For Perceptron linearly classification of example, it typically uses Heaviside step function to make the perceptron as linear as possible.

Typical Perceptron Unit Step Transfer Function



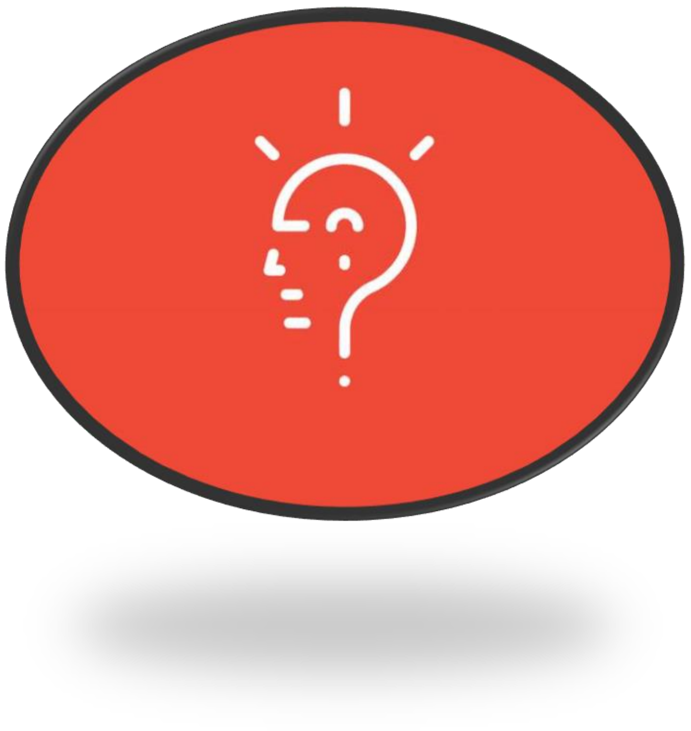


Perceptron 6 Components 6 – Output



Output:- The weighted Summation is passed to the step/activation function and whatever value we get after computation is our predicted output. (different classes, -1/1, 0/1, face/none-face, disease/non-disease, etc..)

Any Question?



Perceptron

3 4

1 Perceptron

2 Perceptron Learning

3 ADALINE

4 Limitation of Perceptron

Recall: Perceptron Weights are Learned

Initialize $\vec{w} = \vec{0}$	// Initialize $\vec{w}$. $\vec{w} = \vec{0}$ misclassifies everything.
while TRUE **do**	// Keep looping
$m = 0$	// Count the number of misclassifications, m
for $(x_i, y_i) \in D$ **do**	// Loop over each (data, label) pair in the dataset, D
if $y_i(\vec{w}^T \cdot \vec{x}_i) \leq 0$ **then**	// If the pair $(\vec{x}_i, y_i)$ is misclassified
$\vec{w} \leftarrow \vec{w} + y\vec{x}$	// Update the weight vector $\vec{w}$
$m \leftarrow m + 1$	// Counter the number of misclassification
end if	
end for	
if $m = 0$ **then**	// If the most recent $\vec{w}$ gave 0 misclassifications
break	// Break out of the while-loop
end if	
end while	// Otherwise, keep looping!

Perceptron Learning Rule 1: PLR

1. Randomly choose the weights in the range 0 and 1.

2. Training examples are presented to perceptron one by one from the beginning, and its output is observed for each training example.

3. If the output is correct then the next training example is presented to perceptron.

4. If the output is incorrect then the weights are modified as per the following Perceptron Learning Rule (PLR).

$$\textbf{New } W_i = W_i + (\eta * X_i * E).$$

Change in Weight i = Learning Rate × Current Value of Input i × E (Expected Output, Current Output).

5. A simple form of E = (Expected Output - Current Output) or

SIGN (Expected Output - Current Output).

6. In PLR, output is 1/0 (or -1), and the transfer is Threshold Step Function

Typical Perceptron Weight Updates

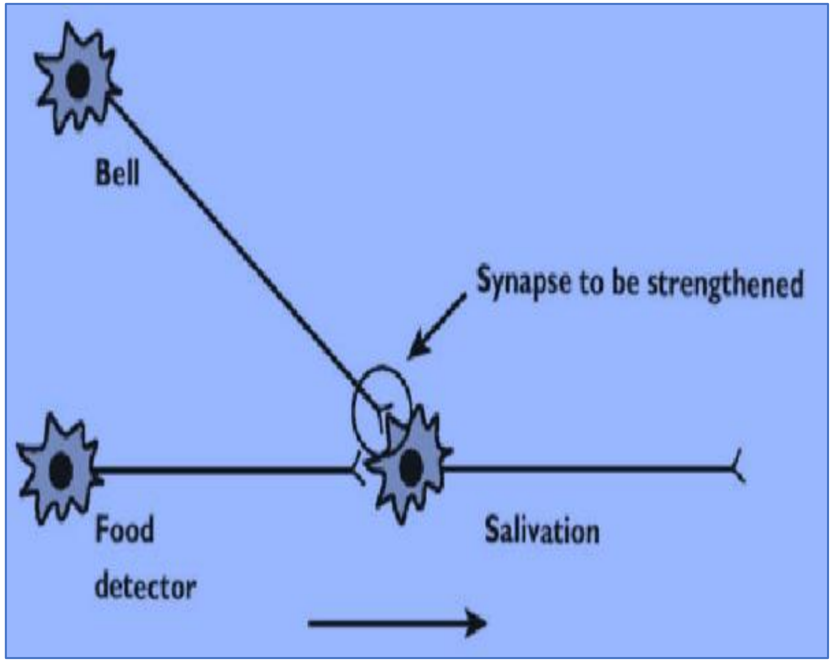
- Weights modified for each example
- Update Rule:

$$w_i \leftarrow w_i + \Delta w_i$$

where

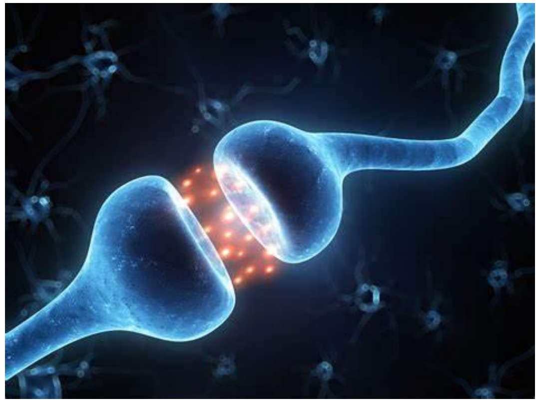
$$\Delta w_i = \underbrace{\eta}_{\text{learning rate}} (\underbrace{t}_{\text{target value}} - \underbrace{o}_{\text{perceptron output}}) \underbrace{x_i}_{\text{input value}}$$

PLR Foundation - Hebb's Law



When an axon of Neuron A is near enough to excite a Neuron B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both Neurons such that A's efficiency, as one of the Neurons firing B, is increased

Hebb



- Donald Olding Hebb (1904-1985) was a Canadian psychologist pioneer of neuropsychology (of the study of the relationship between psychology and neuroscience).
- His book ‘The Organization of Behavior’(1949) gives us a theory about behaviour, based on the physiology of the nervous system. It makes an important attempt to find the common between neurological and psychological conceptions.

Hebb Rule to Update Weights

Hebb Rule

$$w_{ij}^{new} = w_{ij}^{old} + \alpha f_i(a_{iq}) g_j(p_{jq})$$

Presynaptic Signal

Postsynaptic Signal

Simplified Form:

actual output

$$w_{ij}^{new} = w_{ij}^{old} + \alpha a_{iq} p_{jq}$$

input pattern

Supervised Form:

$$w_{ij}^{new} = w_{ij}^{old} + t_{iq} p_{jq}$$

Matrix Form:

desired output

$$\mathbf{W}^{new} = \mathbf{W}^{old} + \mathbf{t}_q \mathbf{p}_q^T$$

From Unsupervised Hebb Rule to PLR
- From "output" to "output difference"

- Feedforward unsupervised learning also known as "coincidence learning"
- The learning signal is equal to the neuron's output
- To update the weights of a neuron i, the inputs to neuron i come from a preceding neuron j (x_j)

$$w_{i,j} = w_{i,j} + \Delta w_{i,j}$$

$$\Delta w_{i,j} = \alpha * \text{output}_i * \text{input}_j$$

$$\Delta w_{i,j} = \alpha\, o_i x_j$$

- It is clear that Hebbian learning is not going to get our Perceptron to learn a set of training data, since weight changes depend only on the actual outputs and we don't have desired outputs to compare to.
- ∴ Hebb rule can be used for pattern association, pattern categorization, pattern classification and over a range of other areas

Rule 2: Perceptron Converge Theorem

The Perceptron convergence theorem states that for any data set which is linearly separable the Perceptron learning rule is guaranteed to find a solution in a finite number of steps.

In other words, the Perceptron learning rule is guaranteed to converge to a weight vector that correctly classifies the examples provided the training examples are linearly separable.

Rule 2: Perceptron Converge Theorem

Theorem 3 (Perceptron convergence). *The Perceptron Learning Algorithm makes at most $\frac{R^2}{\gamma^2}$ updates (after which it returns a separating hyperplane).*

Proof. It is immediate from the code that should the algorithm terminate and return a weight vector, then the weight vector must separate the $+$ points from the $-$ points. Thus, it suffices to show that the algorithm terminates after at most $\frac{R^2}{\gamma^2}$ updates. In other words, we need to show that k is upper-bounded by $\frac{R^2}{\gamma^2}$. Our strategy to do so is to derive both lower and upper and bounds on the length of $\mathbf{w}^{k+1}$ in terms of k, and to relate them.

Note that $\mathbf{w}^1 = \mathbf{0}$, and for $k \geqslant 1$, note that if $\mathbf{x}^j$ is the misclasssified point during iteration k, we have

$$\begin{aligned}
\mathbf{w}^{k+1} \cdot \mathbf{w}^\star &= (\mathbf{w}^k + y^j \mathbf{x}^j) \cdot \mathbf{w}^\star \\
&= \mathbf{w}^k \cdot \mathbf{w}^\star + y^j (\mathbf{x}^j \cdot \mathbf{w}^\star) \\
&> \mathbf{w}^k \cdot \mathbf{w}^\star + \gamma.
\end{aligned}$$

It follows by induction that $\mathbf{w}^{k+1} \cdot \mathbf{w}^\star > k\gamma$. Since $\mathbf{w}^{k+1} \cdot \mathbf{w}^\star \leqslant \|\mathbf{w}^{k+1}\| \|\mathbf{w}^\star\| = \|\mathbf{w}^{k+1}\|$, we get

$$\|\mathbf{w}^{k+1}\| > k\gamma. \tag{1}$$

To obtain an upper bound, we argue that

$$\begin{aligned}
\|\mathbf{w}^{k+1}\|^2 &= \|\mathbf{w}^k + y^j \mathbf{x}^j\|^2 \\
&= \|\mathbf{w}^k\|^2 + \|y^j \mathbf{x}^j\|^2 + 2(\mathbf{w}^k \cdot \mathbf{x}^j) y^j \\
&= \|\mathbf{w}^k\|^2 + \|\mathbf{x}^j\|^2 + 2(\mathbf{w}^k \cdot \mathbf{x}^j) y^j \\
&\leqslant \|\mathbf{w}^k\|^2 + \|\mathbf{x}^j\|^2 \\
&\leqslant \|\mathbf{w}^k\|^2 + R^2,
\end{aligned}$$

from which it follows by induction that

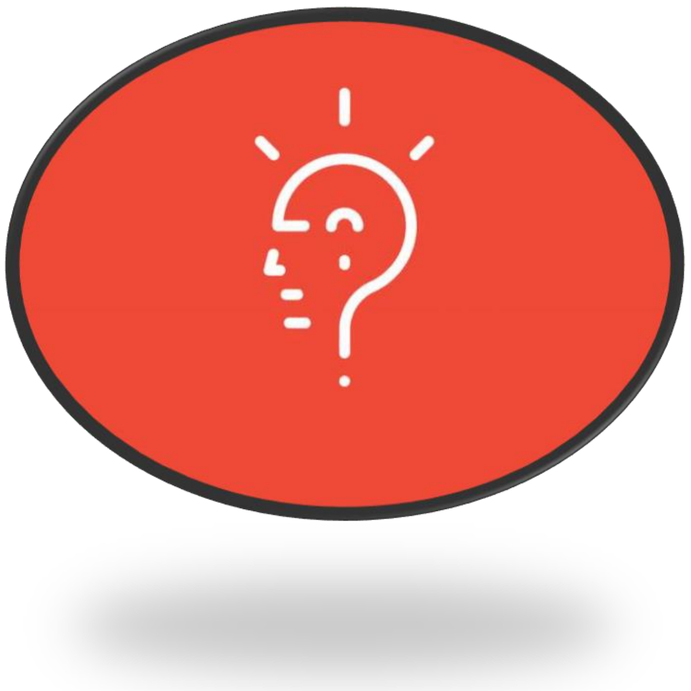
$$\|\mathbf{w}^{k+1}\|^2 \leqslant kR^2. \tag{2}$$

Together, (1) and (2) yield

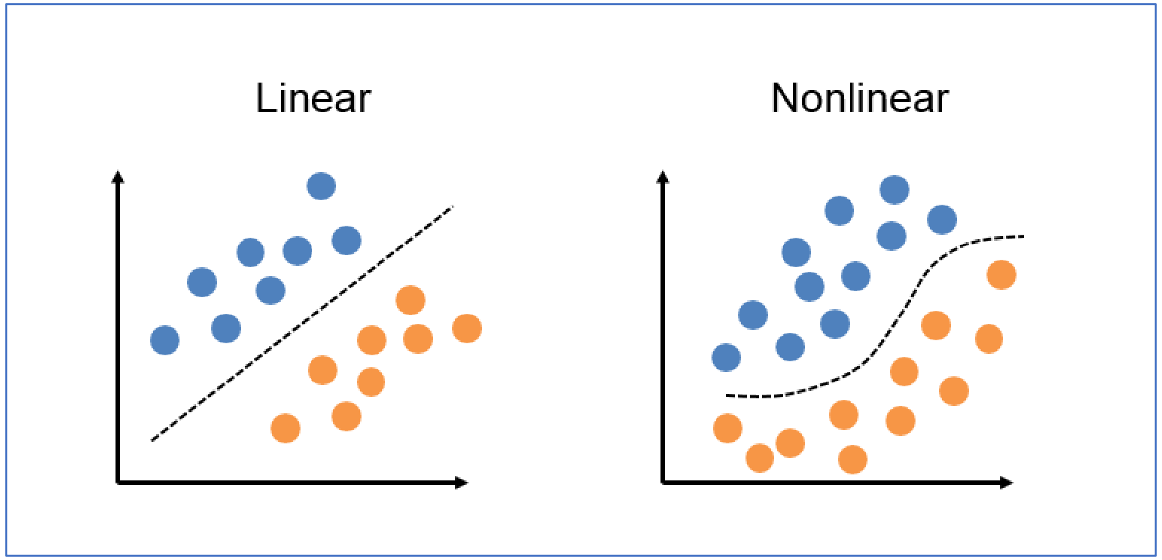
$$k^2\gamma^2 < \|\mathbf{w}^{k+1}\|^2 \leqslant kR^2,$$

which implies $k < \frac{R^2}{\gamma^2}$. Our proof is done. $\square$

Any Question?



Linearly Separable



A function is said to be linearly separable when its outputs can be discriminated by a function which is a linear combination of features, that is we can discriminate its outputs by a line or a hyperplane.

Homework 05

1. Start to Write the Introduction of Your Group Project
2. Review All the Previous Lectures and Prepare for the Mid-term Exam

CS 103 -05

Perceptron and AI Early Day Algorithms

Jimmy Liu 刘江

2020-10-16