CS201: Discrete Math for Computer Science
2021 Fall Semester   Written Assignment #1
Due: Sep. 29th, 2021, please submit at the beginning of class

Q.1 Let p, q and r be the propositions
p: You get an A on the final exam.
q: You do every exercise in this book.
r: You get an A in this class.
Write these propositions using p, q, and r and logical connectives (including negations).

(a) You get an A in this class, but you do not do every exercise in this book.

(b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

(c) To get an A in this class, it is necessary for you to get an A on the final.

(d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

(e) Getting an A on the final and doing every exercise in this book is suffcient for getting an A in this class.

(f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Q.2 Construct a truth table for each of these compound propositions.

(a) $p \oplus \neg p$

(b) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

(c) $(p \oplus q) \rightarrow (p \oplus \neg q)$

Q.3 Use truth tables to decide whether or not the following two propositions are equivalent.

(a) $p \oplus q$ and $\neg p \vee \neg q$

(b) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

(c) $(p \to q) \to r$ and $p \to (q \to r)$

(d) $(\neg q \wedge \neg(p \to q))$ and $\neg p$

Q.4 Use logical equivalences to prove the following statements.

(a) $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are equivalent.

(b) $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology.

(c) $\neg(p \to q) \to \neg q$ is a tautology.

Q.5

Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true, when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

Q.6 Determine whether or not the following two are logically equivalent, and explain your answer.

(a) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

(b) $(p \to q) \to r$ and $p \to (q \to r)$

(c) $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$

(d) $(p \vee q) \to r$ and $(p \to r) \vee (q \to r)$.

Q.7 Prove that if $p \to q$, $\neg p \to r$, $r \to s$, then $\neg q \to s$.

Q.8 Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++". Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

(a) There is a student at your school who can speak Russian and who knows C++.

(b) There is a student at your school who can speak Russian but who doesn't know C++.

(c) Every student at your school either can speak Russian or knows C++.

(d) No student at your school can speak Russian or knows C++.

Q.9 Let $L(x, y)$ be the statement "x loves y", where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statement.

(a) Everybody loves Jerry.

(b) Everybody loves somebody.

(c) There is somebody whom everybody loves.

(d) Nobody loves everybody.

(e) There is somebody whom Lydia does not love.

(f) There is somebody whom no one loves.

(g) There is exactly one person whom every body loves.

(h) There are exactly two people whom Lynn loves.

(i) Everyone loves himself or herself.

(j) There is someone who loves no one besides himself or herself.

Q.10 Suppose that variables x and y represent real numbers, and $L(x, y)$: $x < y$, $Q(x, y)$: $x = y$, $E(x)$: x is even, $I(x)$: x is an integer. Write the following statements using these predicates and any needed quantifiers.

(1) Every integer is even.

(2) If $x < y$, then x is not equal to y.

(3) There is no largest real number.

Q.11 Express the negations of each of these statements so that all negation symbols immediately precede predicates.

(a) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

(b) $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$

(c) $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$

(d) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$

Q.12

Let $P(x, y)$ be a propositional function. Prove or disprove that $\exists x \forall y P(x, y) \rightarrow \forall y \exists x P(x, y)$ is a tautology.

Q.13 Each of the two below contains a pair of statements, (i) and (ii). For each pair, say whether (i) is equivalent to (ii), i.e., for all $P(x)$ and $Q(x)$, (i) is true if and only if (ii) is true. Here $\mathbb{R}$ denotes the set of all *real numbers*.

If they are equivalent, *all you have to do is to say that they are equivalent.* If they are not equivalent, give a counterexample. A counterexample should involve a specification of $P(x)$ and $Q(x)$ and an explanation as to why the resulting statement is false.

(1) (i) $(\forall x \in \mathbb{R}\ P(x)) \vee (\forall x \in \mathbb{R}\ Q(x))$
(ii) $\forall x \in \mathbb{R}\,(P(x) \vee Q(x))$

(2) (i) $(\forall x \in \mathbb{R}\ P(x)) \wedge (\forall x \in \mathbb{R}\ Q(x))$
(ii) $\forall x \in \mathbb{R}\,(P(x) \wedge Q(x))$

(3) (i) $(\forall x \in \mathbb{R}\ P(x)) \wedge (\exists y \in \mathbb{R}\ Q(y))$
(ii) $\forall x \in \mathbb{R}\,(\exists y \in \mathbb{R}\ (P(x) \wedge Q(y)))$

Q.14 For the following argument, explain which rules of inference are used for each step.

"Each of five roommates, A, B, C, D, and E, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year."

Q.15 Prove the **triangle inequality**, which states that if x and y are real numbers, then $|x| + |y| \geq |x + y|$ (where $|x|$ represents the absolute value of x, which equals x if $x \geq 0$ and equals $-x$ if $x < 0$.

Q.16

Prove that between every two rational numbers there is an irrational number.

Q.17

Prove that between every rational number and every irrational number there is an irrational number.

Q.18 Prove that $\sqrt[3]{2}$ is irrational.

Q.19 Suppose that we have a theorem: "$\sqrt{n}$ is irrational whenever n is a positive integer that is *not* a perfect square." Use this theorem to prove that $\sqrt{2} + \sqrt{3}$ is irrational.