CS208 · Algorithm Design and Analysis

Assignment 3

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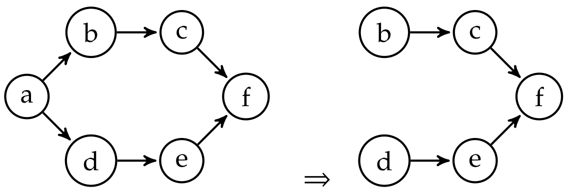
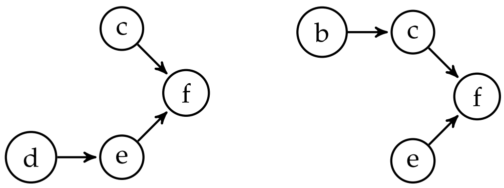
Ch.3 - Ex.1

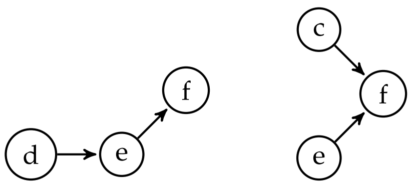
We start from the only node with a zero in degree, a. Remove it and add it to the first place of the topological ordering.



Let's remove another node in the second and third step. Now we have $a - b -?$ or $a - d -?$.



We take the first case as an example. We can either delete c first, then the left three can be only ordered as $d - e - f$; we can also first delete d, then use $c - e - f$ or $e - c - f$. There are three ordering for this subgraph, similarly, we find the other case has three ordering.



In summary, we have $2 \times 3 = 6$ possible topological orderings, listed as below.

$$a - b - c - d - e - f$$

$$a - b - d - c - e - f$$

$$a - b - d - e - c - f$$

$$a - d - b - c - e - f$$

$$a - d - b - e - c - f$$

$$a - d - e - b - c - f$$

Ch.3 - Ex.3

Algorithm 1 Extend Topological Sort

1: List<Node> topologicalOrdered
2: **procedure** EXTTOPOLOGICAL(GRAPH G)
3:     **if** G is empty **then** ▷ prior recursions have cleared the DAG
4:         Return the topological order (list in line 1)
5:     Find a node v with $\deg^{-}(v) = 0$ from G
6:     **if** v exists **then**
7:         Add v to list topologicalOrdered
8:         Delete v from G
9:         Recursively compute a topological ordering of $G - \{v\}$
10:     **else** ▷ all nodes in G have income edge, G must contains cycle *(Algorithm Design, pp. 100)*
11:         Create a list to store the cycle Randomly pick a node u from G
12:         **while** u is not visited **do** ▷ $O(1)$ each loop
13:             Mark u as visited
14:             Add u to the list (stated in line 9)
15:             $u \leftarrow$ the first node in this node's income nodes' list
16:         We've find a whole cycle and store in the list, now return the answer

The above algorithm is modified base on the topological sort algorithm. If G is a DAG, then certainly it will never enter the *else* branch and runs as the normal topological sort algorithm, taking $O(m + n)$. If G is not a DAG, it may first run this algorithm as the normal condition, when it first meets a cycle (any node in a cycle), it takes $O(1)$ to check each node in the cycle, since the cycle is not larger than n nodes, the total cost to find the cycle is $O(n)$. With a prior part using $O(m + n)$, the total time complexity is still $O(m + n)$.