南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

CS201 DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

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Graph Concepts

- $G = (V, E)$, *simple* graph, *multigraph*, *pseudograph*
- *Undirected*, *directed* graph
- Special graphs

  K_n, C_n, W_n, Q_n, $K_{m,n}$

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  Hall's Marriage Theorem on *bipartite* graphs
- Representation of graphs

  adjacency list, *adjacency matrix*, *incidence matrix*

Isomorphism of Graphs

- **Definition** The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there is a one-to-one and onto function from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function is called an *isomorphism*.

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- Useful graph invariants include the number of vertices, number of edges, degree sequence, etc.

Cut Vertices and Cut Edges

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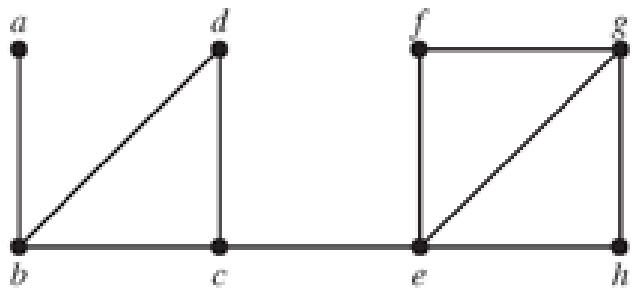

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Paths and Isomorphism

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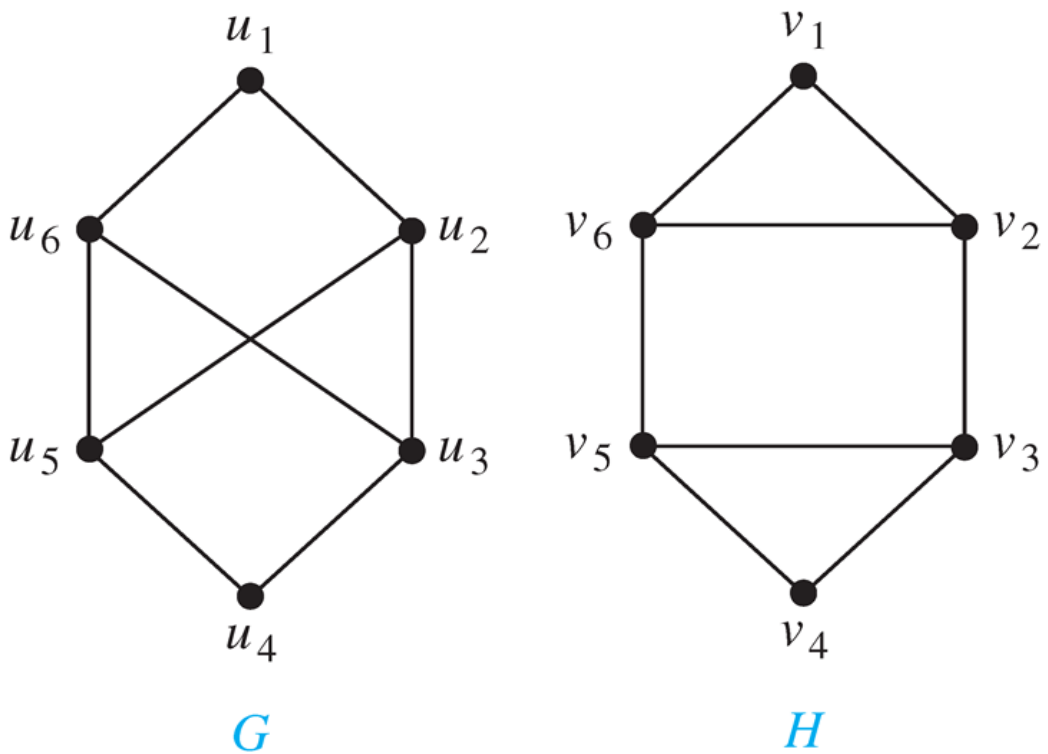

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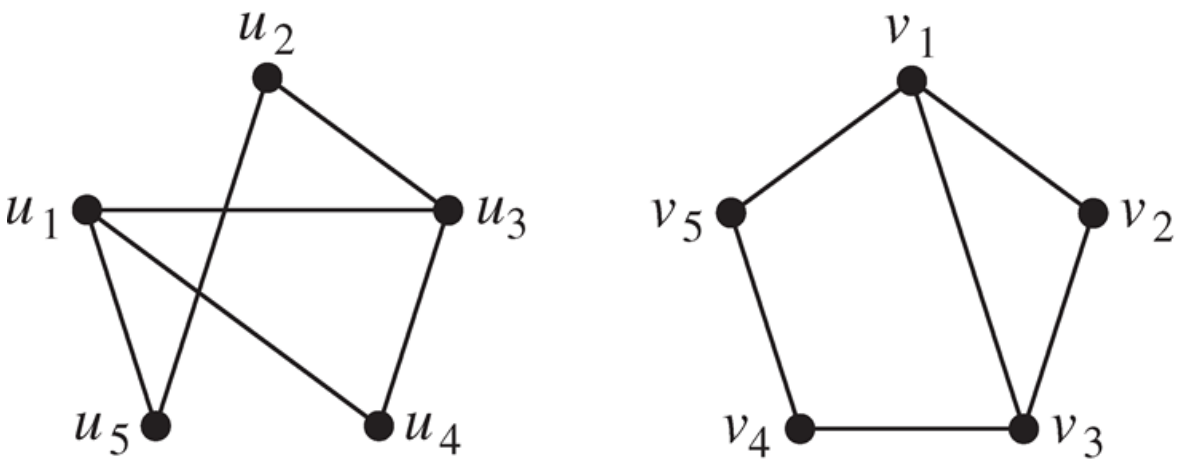

Counting Paths between Vertices

- **Theorem** Let G be a graph with adjacency matrix $\mathbf{A}$ with respect to the ordering $v_1, v_2, \ldots, v_n$ of vertices. The number of different paths of length r from v_i to v_j, where $r > 0$ is positive, equals the (i, j)-th entry of $\mathbf{A}^r$.

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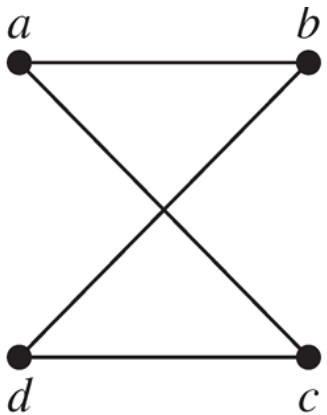
  $\mathbf{A}^{r+1} = \mathbf{A}^r\mathbf{A}$, the (i, j)-th entry of $\mathbf{A}^{r+1}$ equals $b_{i1}a_{1j} + b_{i2}a_{2j} + \cdots + b_{in}a_{nj}$, where b_{ik} is the (i, k)-th entry of $\mathbf{A}^r$.

Counting Paths between Vertices

- **Example** How many paths of length 4 are there from a to d in the graph G?



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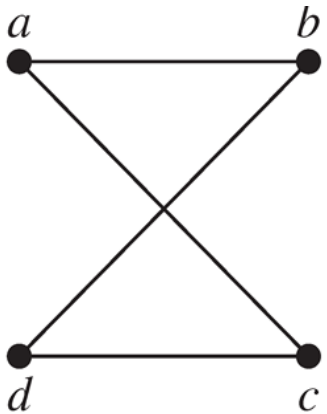

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

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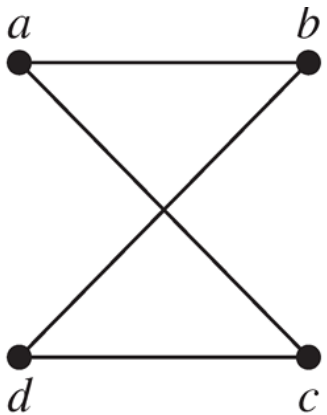

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Euler Paths

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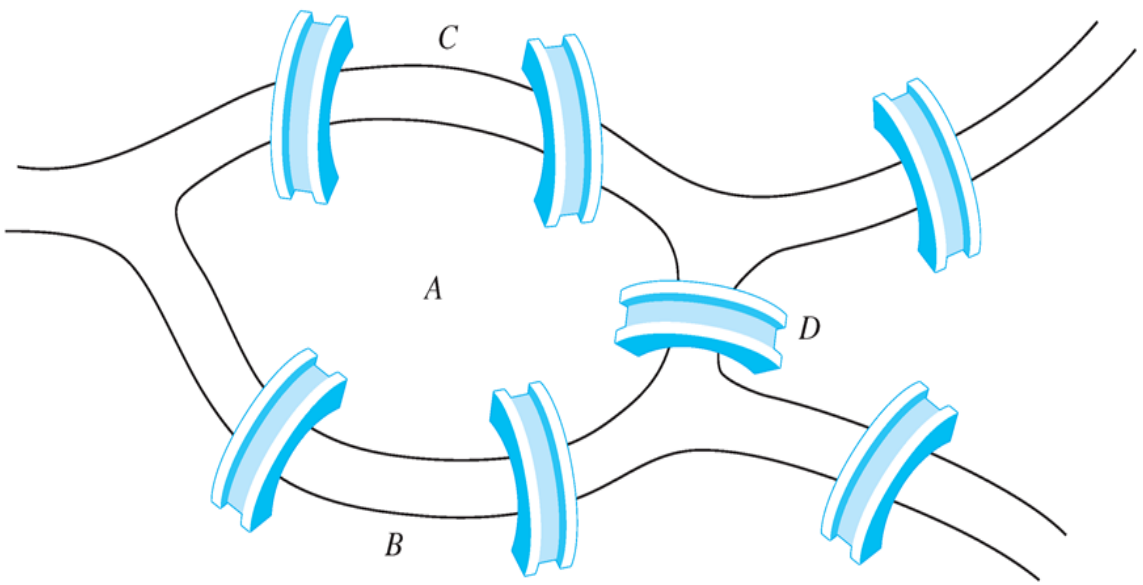

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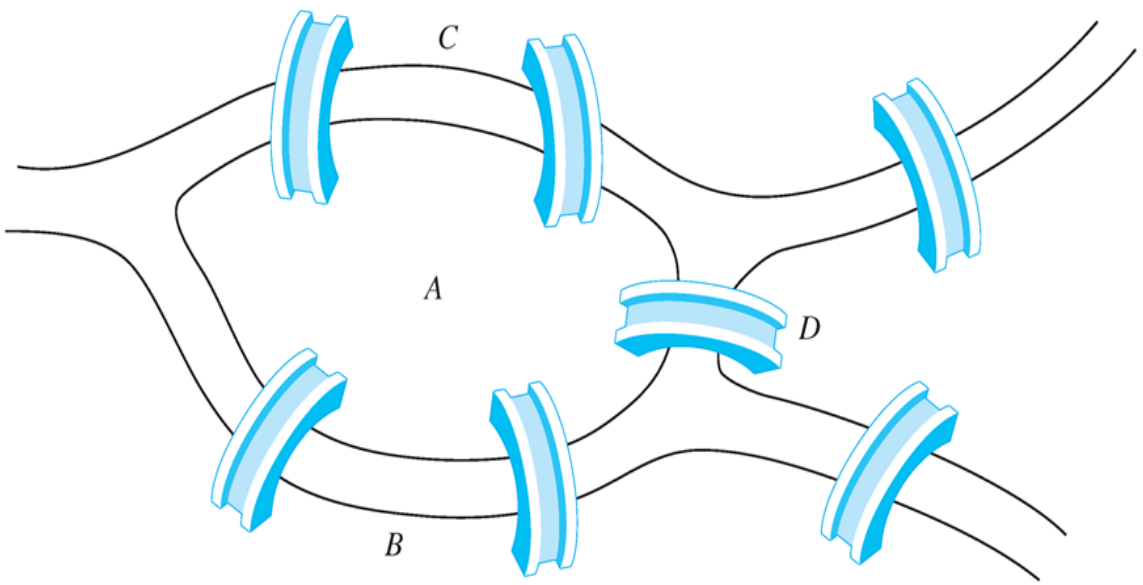

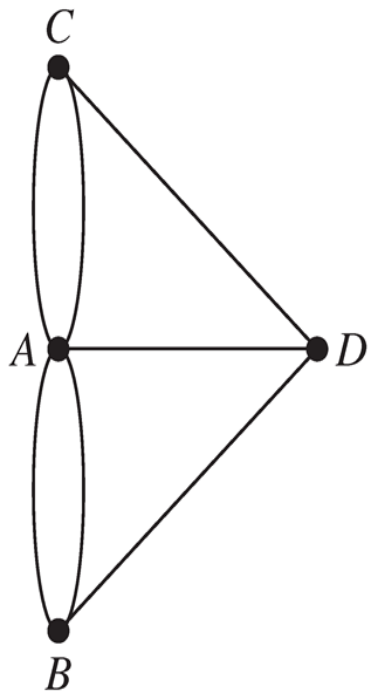

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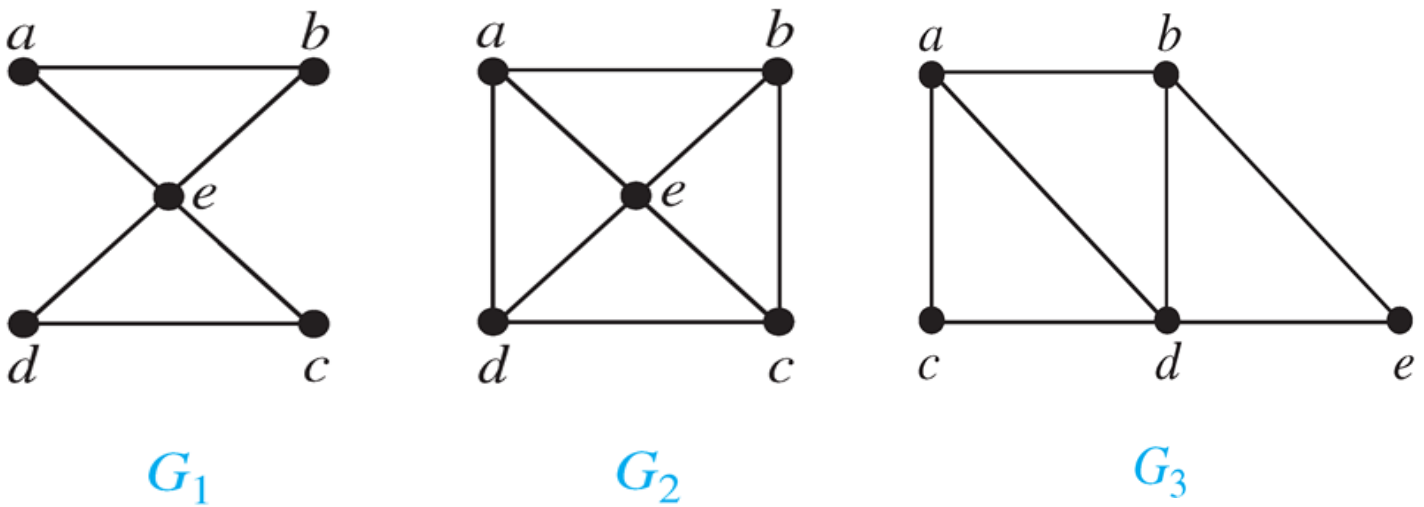

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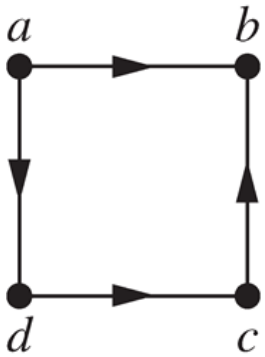
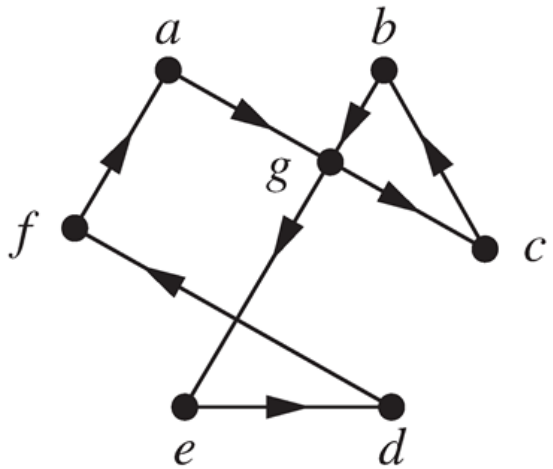


H_1



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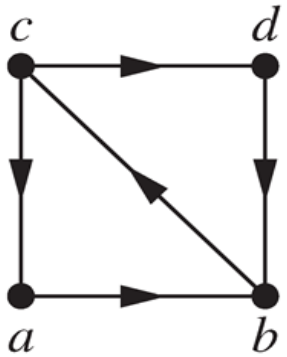

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- ◇ The initial vertex and the final vertex of an Euler path have odd degree.

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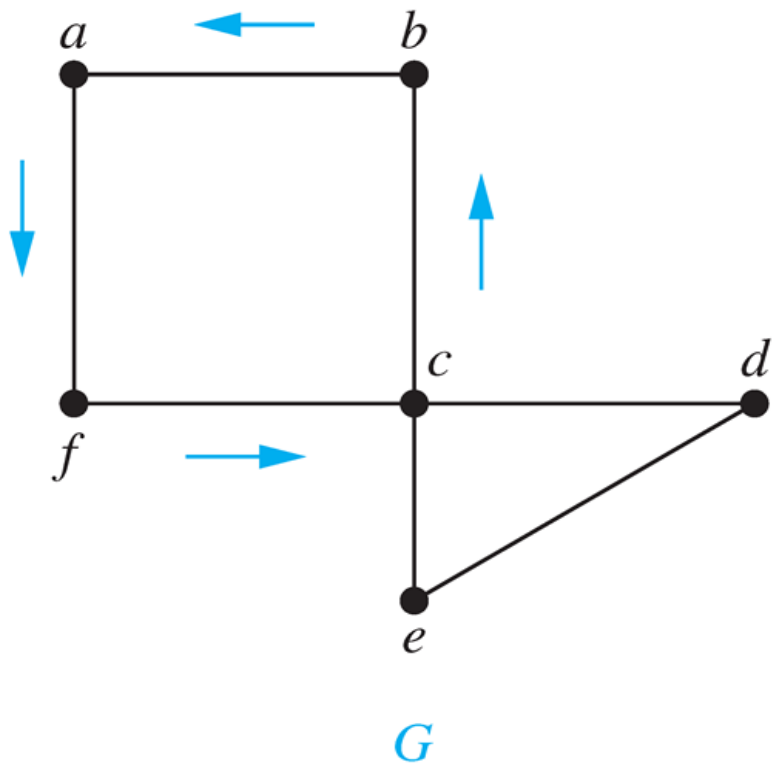

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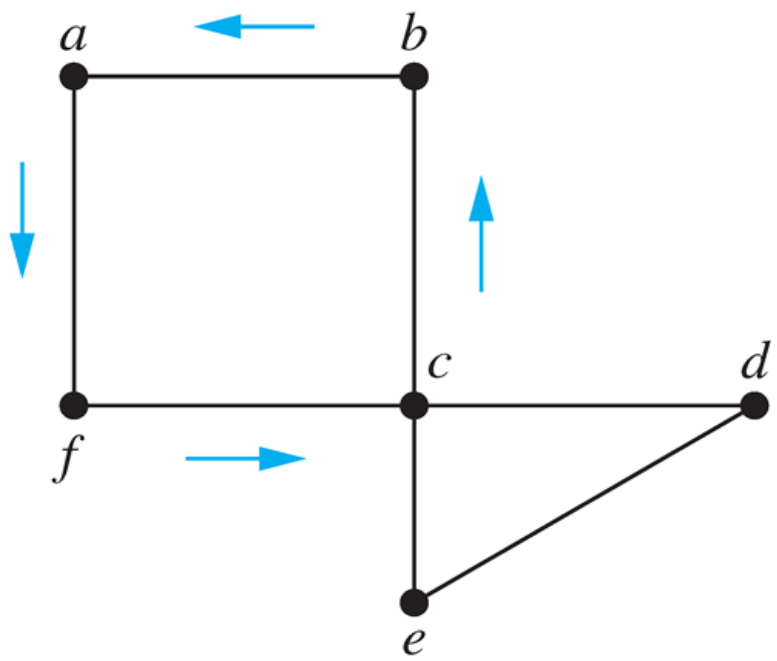

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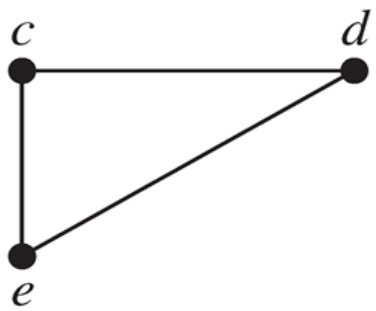

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procedure *Euler*(*G*: connected multigraph with all vertices of even degree)
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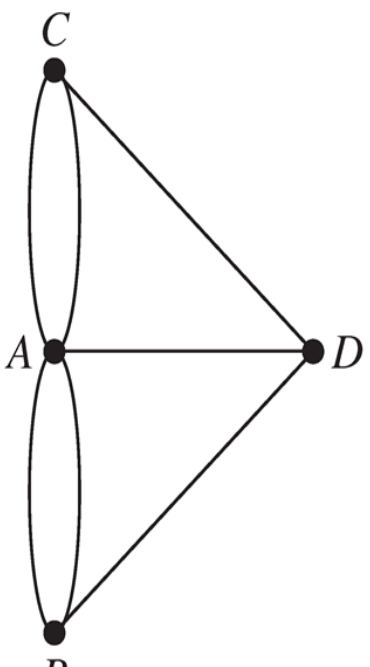

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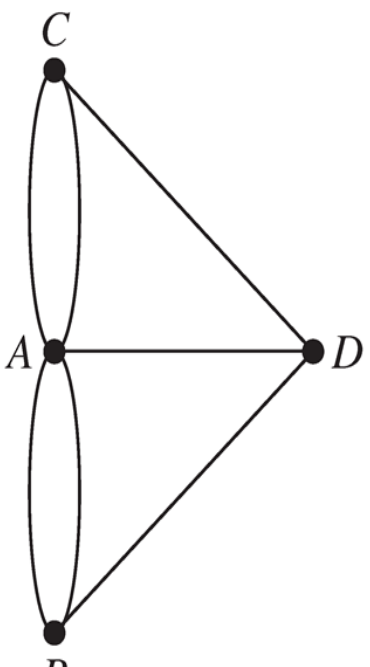

No Euler circuit

Euler Circuits and Paths

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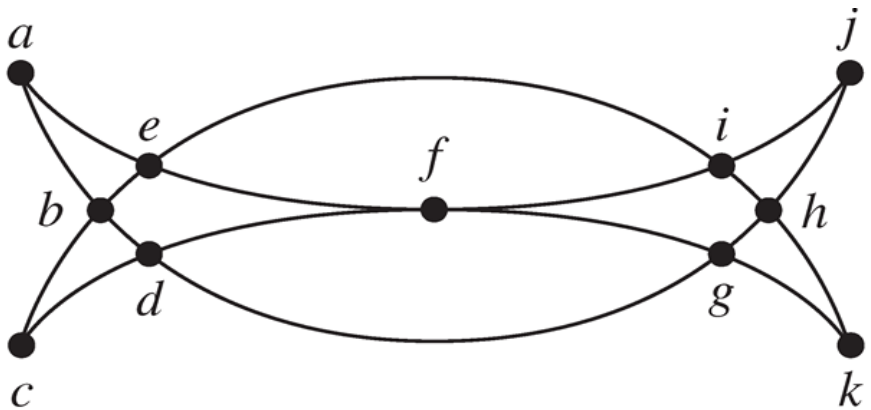

G

FIGURE 6 **Mohammed's Scimitars.**

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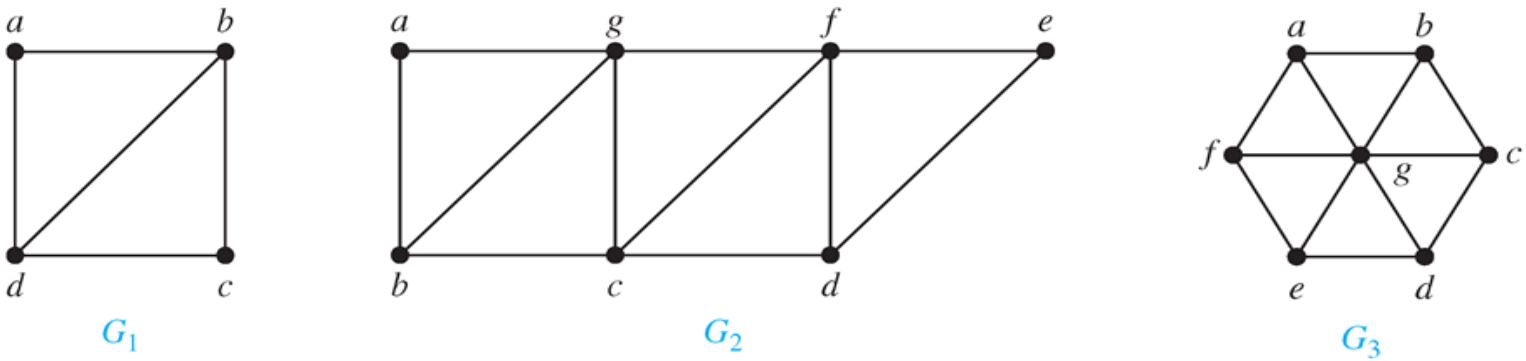

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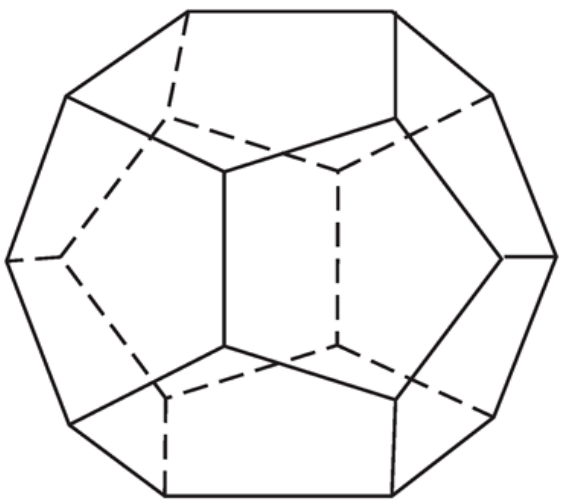

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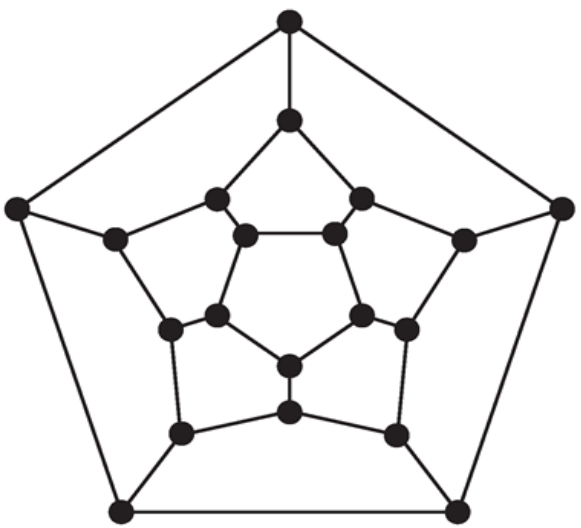

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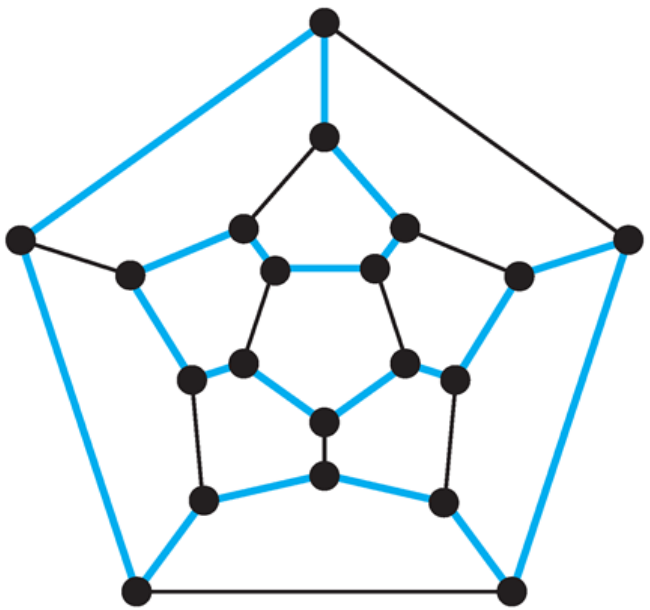

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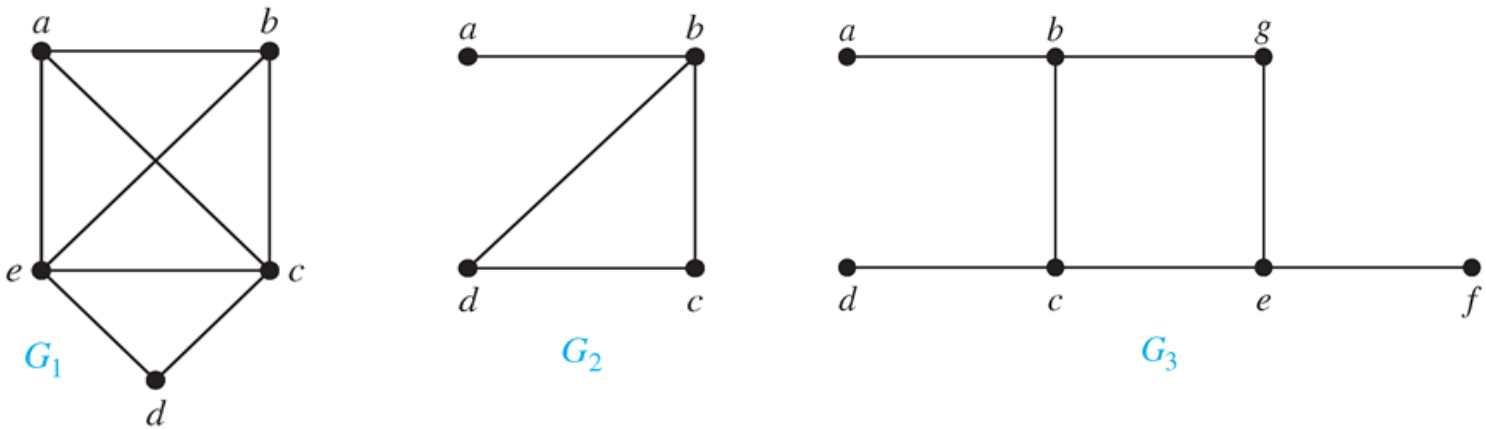

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  Hamilton path problem $\in$ NPC

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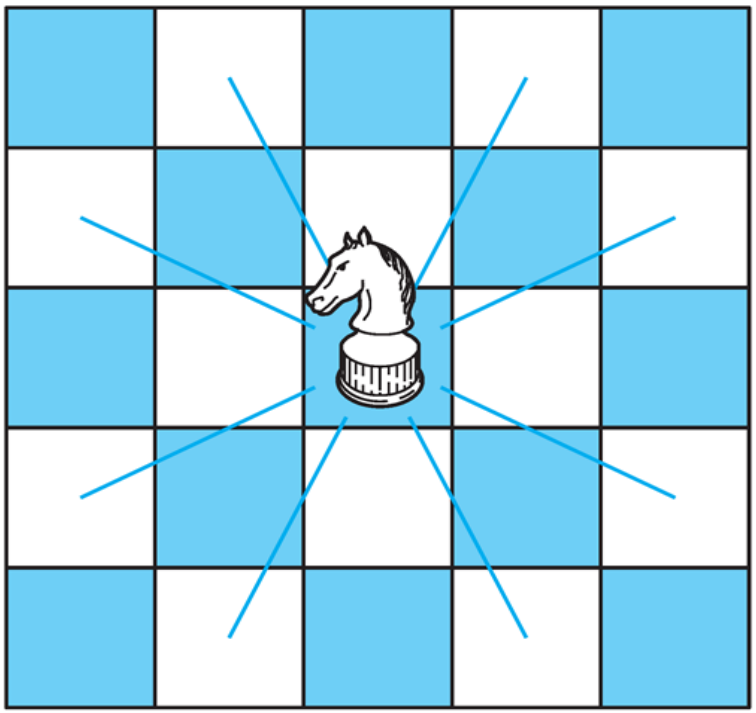
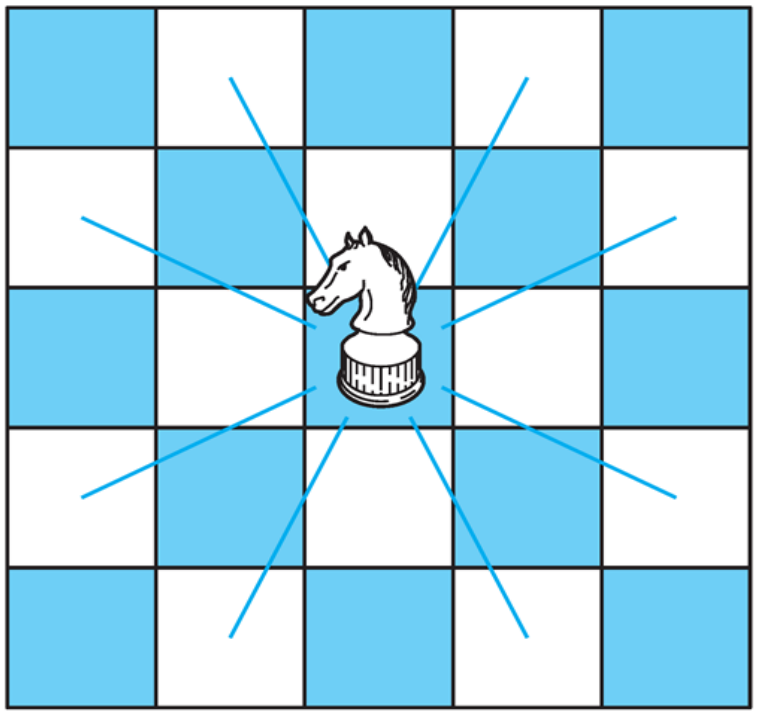


Applications of Hamilton Paths and Circuits

- Can we traverse every space (and come back) in the 5×5 chessboard?



What about in 6×6 chessboard?

Shortest Path Problems

- Using graphs with weights assigned to their edges

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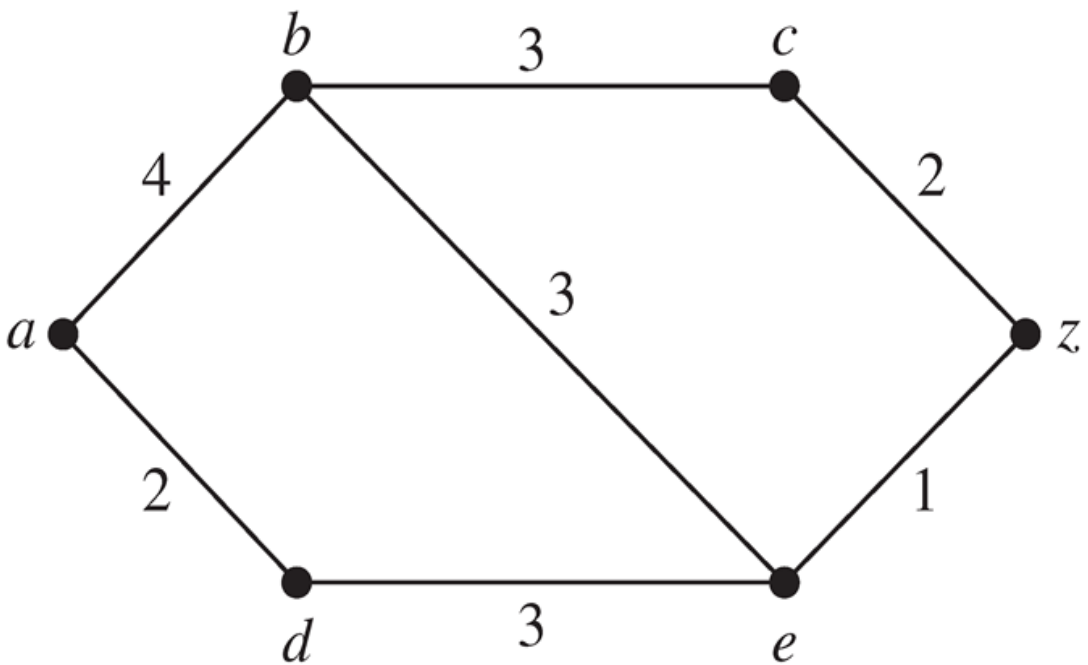

Shortest Path Problems

- **Definition** Let G^{α} be an weighted graph, with a weight function $\alpha : E \to \mathbf{R}^{+}$ on its edges. If $P = e_1 e_2 \cdots e_k$ is a path, then its weight is $\alpha(P) = \sum_{i=1}^{k} \alpha(e_i)$. The *minimum weighted distance* between two vertices is

$$d(u, v) = \min\{\alpha(P) | P : u \to v\}$$

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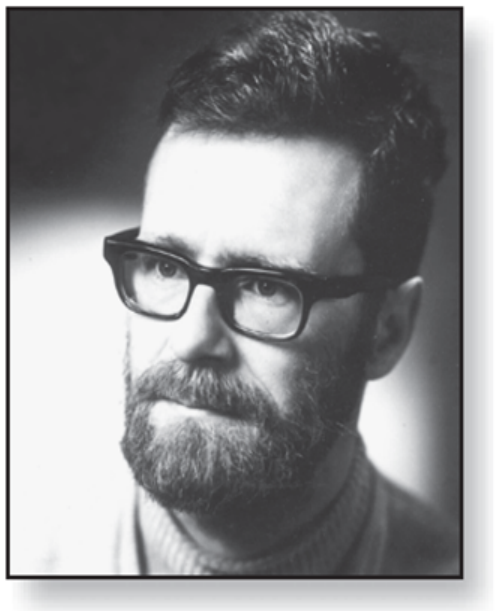

Edsger Wybe Dijkstra

Dijkstra's Algorithm

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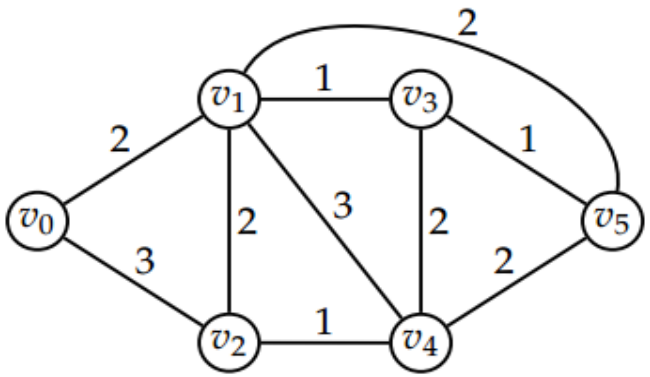

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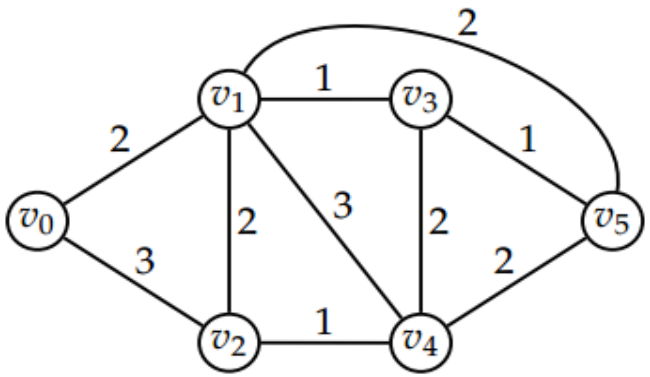

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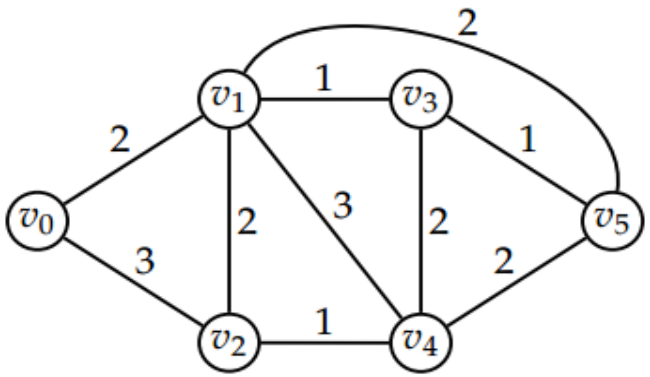

v_0	v_1	v_2	v_3	v_4	v_5
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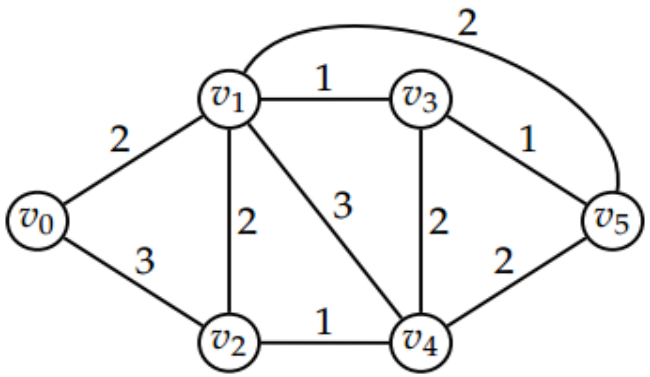

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0	∞	∞	∞	∞	∞

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$d(v_1) = \min\{\infty, 2\} = 2$, $d(v_2) = \min\{\infty, 3\} = 3$

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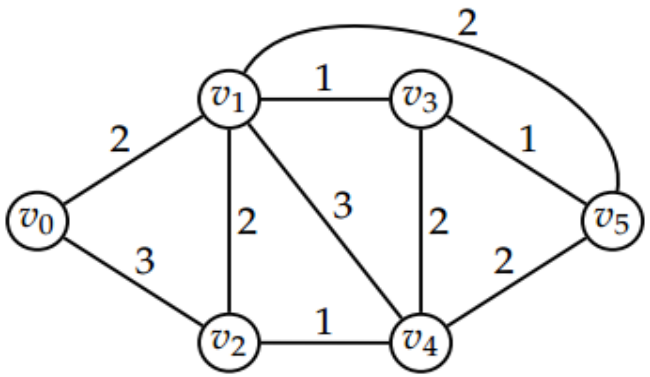

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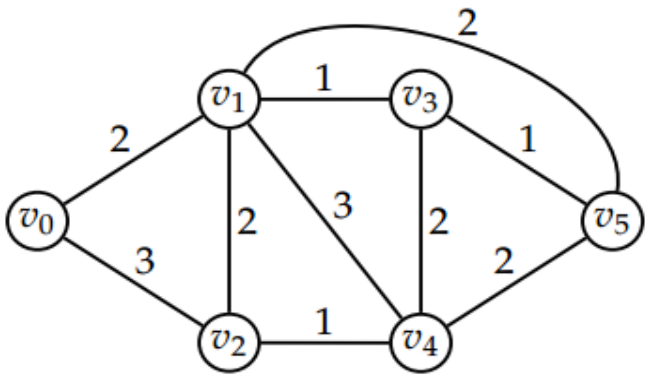

v_0	v_1	v_2	v_3	v_4	v_5
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$d(v_2) = \min\{3, d(v_1) + \alpha(v_1 v_2)\} = \min\{3, 4\} = 3$,

$d(v_3) = 2 + 1 = 3$, $d(v_4) = 2 + 3 = 5$,

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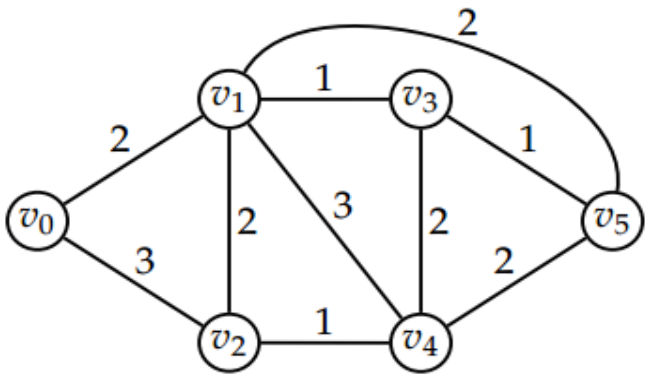

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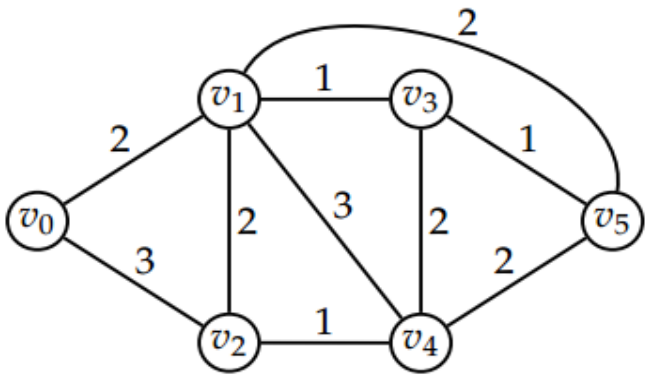

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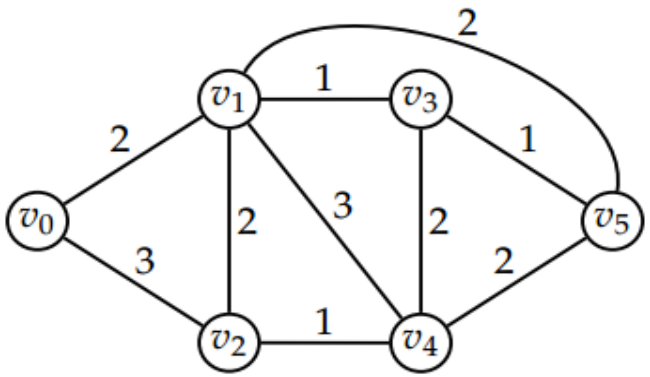

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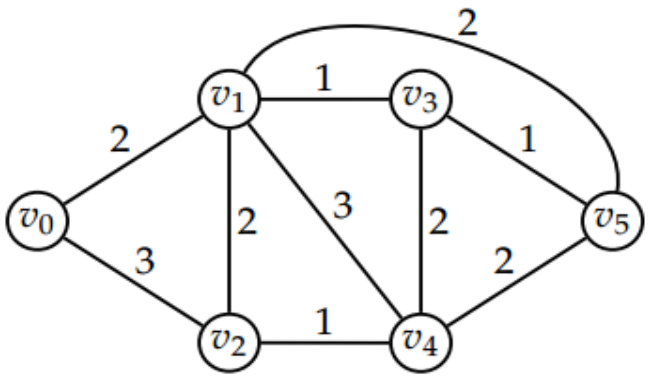

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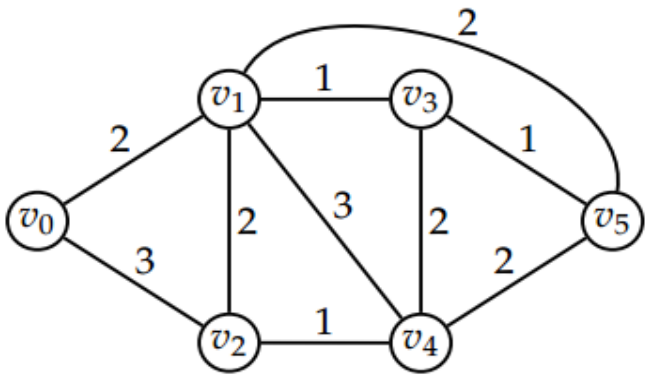

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$i = 4$

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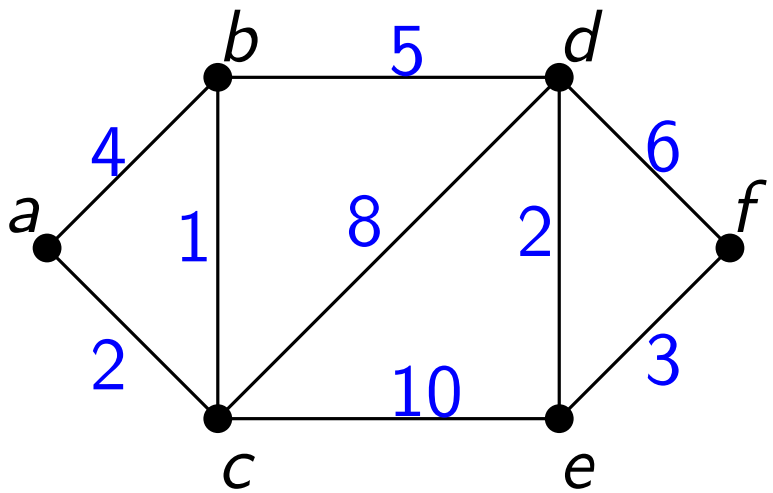
  read the Textbook p.712 – p.714

Another Example



Next Lecture

- Graph theory III ...