CS208 · Algorithm Design and Analysis

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Assignment 1

Ch.1 - Ex.1

False Consider such counter-example:

man\pref	#1	#2
m_1	w_1	w_2
m_2	w_2	w_1

(a) Men's preference list

woman\pref	#1	#2
w_1	m_2	m_1
w_2	m_1	m_2

(b) Women's preference list

Run the G-S algorithm to get a stable matching $\{m_1 - w_1, m_2 - w_2\}$. It's trivial that there's no such pair (m, w) that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

Ch.1 - Ex.2

True Suppose that, there exists a "stable matching" S for this instance, in which m is not pairing with w. By the fact that *m is ranked first for w and w is ranked first for m*, we know that m is pairing with someone he prefers less than w, similar for w. By the definition of *unstable pair*, the pair $m - w$ is unstable and thus let S cannot be a stable matching.

By the above contradiction, there does not exist any "stable matching" for this instance s.t. m is not pairing with w. A.k.a. in every stable matching S for this instance, the pair (m, w) belongs to S.

Ch.1 - Ex.3

(b) No stable pair of schedules Let the n in this example be 2. Consider that Network $\mathcal{A}$ has two shows: A_1 and A_2, while Network $\mathcal{B}$ has two shows as B_1 and B_2.

Assume that the ratings are $A_1 > B_1 > A_2 > B_2$. Let's check all the 4 possible arrangements.

Slot	$\mathcal{A}$	$\mathcal{B}$
1	A_1	B_1
2	A_2	B_2

(a)

Slot	$\mathcal{A}$	$\mathcal{B}$
1	A_1	B_2
2	A_2	B_1

(b)

Slot	$\mathcal{A}$	$\mathcal{B}$
1	A_2	B_1
2	A_1	B_2

(c)

Slot	$\mathcal{A}$	$\mathcal{B}$
1	A_2	B_2
2	A_1	B_1

(d)

For (a), Network $\mathcal{B}$ can reorder its schedule to (b) s.t. it wins $0 \rightarrow 1$ slot.

For (b), Network $\mathcal{A}$ can reorder its schedule to (d) s.t. it wins $1 \rightarrow 2$ slots.

For (c), Network $\mathcal{A}$ can reorder its schedule to (a) s.t. it wins $1 \to 2$ slots.

For (d), Network $\mathcal{B}$ can reorder its schedule to (c) s.t. it wins $0 \to 1$ slot.

All the possible arrangements for this example are therefore not stable. Q.E.D.

Ch.1 - Ex.8

(b) Could improve the partner of a woman Let w_1 be the lying woman, assume that her actual preference sequence is $m_1 - m_2 - m_3$ but her lies that $m_1 - m_3 - m_2$. We then construct others preference list as below:

man\pref	#1	#2	#3
m_1	w_3	w_1	w_2
m_2	w_1	w_3	w_2
m_3	w_1	w_3	w_2

(a) Men's preference list

woman\pref	#1	#2	#3
w_1 (fake)	m_1	m_3	m_2
w_2	m_2	m_1	m_3
w_3	m_2	m_1	m_3

(b) Women's preference list

Base on the fake preference list, we run the G-S algorithm and will get a stable matching $\{m_1 - w_1, m_2 - w_3, m_3 - w_2\}$. But when we run the G-S algorithm on the real preference list we will get $\{m_1 - w_3, m_2 - w_1, m_3 - w_2\}$. By lying, w_1 successfully got married with the man m_1, who is actually more preferred by w_1 than the other two men.