Code Challenges

1. Simple decision tree

Taken from ref. Competing cance and AIDS risks decision tree. Assume discrete time of single years. An individual starts in the `Well` state. They can transition into `Dead`, `Cancer & AIDS`, `Cancer`, `AIDS` or remain in the `Well` state.

Event probabilities are

- $\delta_0 = 1.182 \times 10^{-3}$: Die from other causes
- $\delta_c = 0.025$: Die from recurent prostate cancer
- $\delta_a = 0.080$: Die from AIDS
- $\beta_c = 0.0027$: Cancer recurs
- $\beta_a = 0.0083$: Develop AIDS

2. Markov-cycle tree

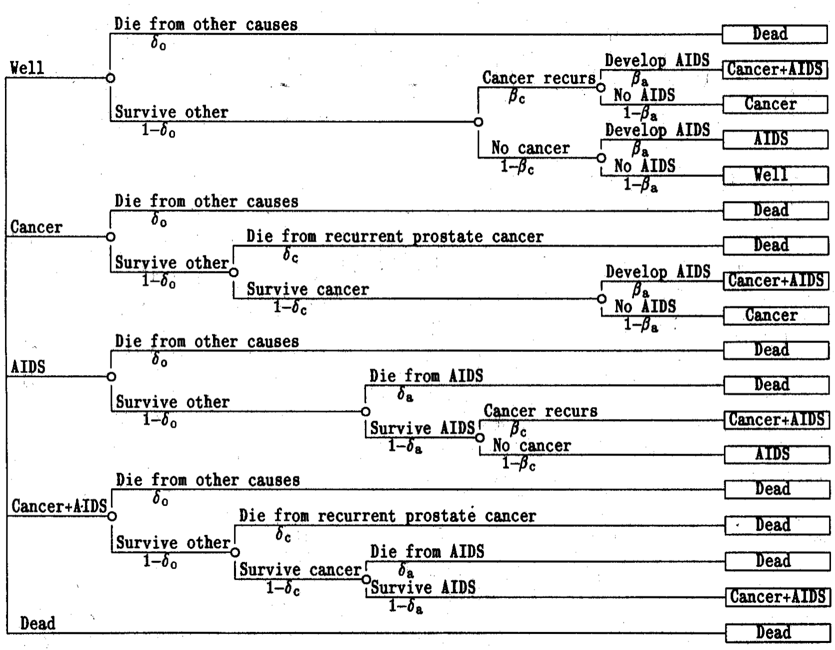
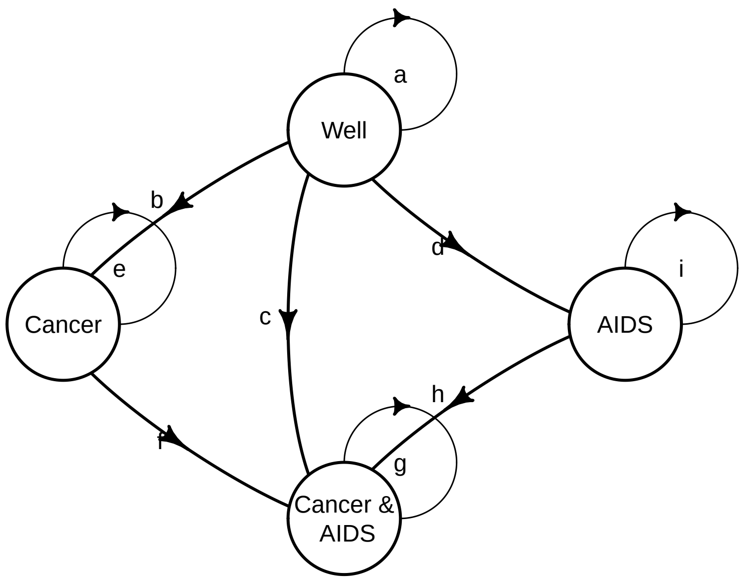


FIGURE 1. Markov-cycle tree for competing cancer and AIDS risks.

3. Discrete-time Markov model



Calculate mean QALYs

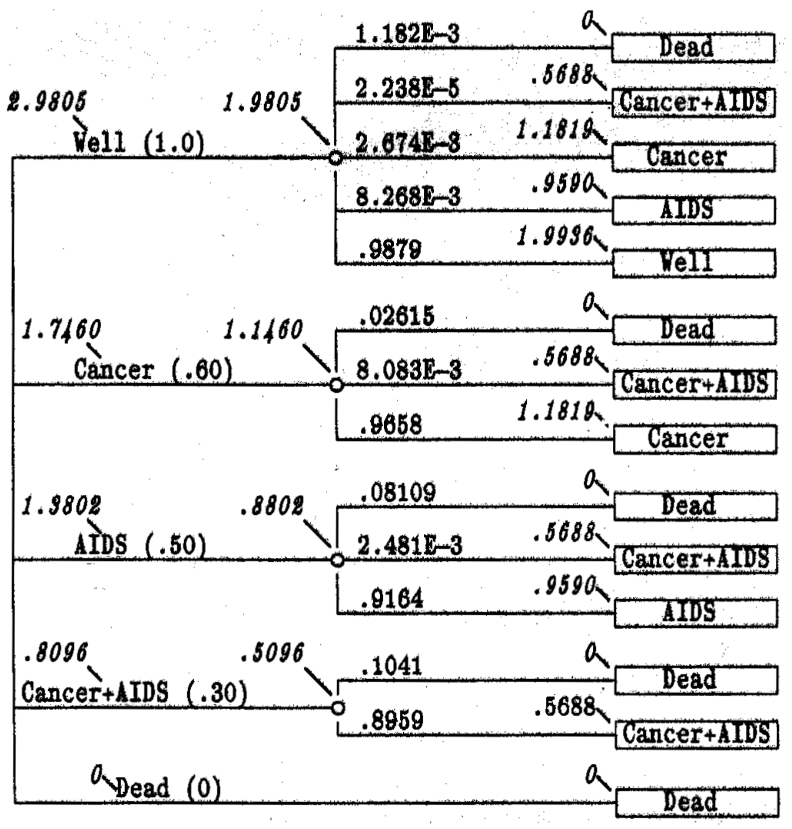
Calculate cumulative proportion of patient cycles in each state and take product with health utilities for each respectively.

Define the state utilities $R(\cdot)$:

- `Well`: 1.0
- `Cancer`: 0.60
- `AIDS`: 0.50
- `Cancer & AIDS`: 0.30
- `Dead`: 0

4. One-cycle Markov-cycle tree

We can rearrange the Markov-cycle tree to closer resemble to Markov model by collapsing the branches into a single cycle and simply combining the probabilities.



6. Roll back Markov-cycle tree

We can calculate the mean QALYs using the markov-cycle tree representation without calculating the cumulative proportion of time of patient cycles in each health state. This is done by rolling back using the recursive equation (value iteration):

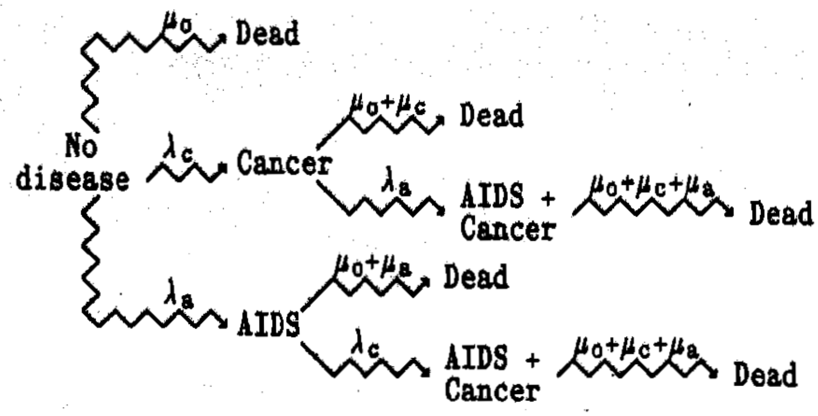
$$V_n(i) = R(i) + \sum_j p_{ij} V_{n-1}(j)$$

5. Roll back stochastic tree

So far we have only considered discrete time. The Markov-cycle tree representation can be extended to continuous time as a *stochastic tree*. Probabilities are now replaced by rates. This change is represented by zigzag lines in the diagrams. This is clearly a more compact representation.

We can calculate mean QALY in an analogous way to the discrete-time case by rolling back using the recursive equation:

$$V(S) = \frac{R(i)}{\sum_j \lambda_j} + \sum_j p_j V(S_j)$$



- `Cancer`: $\lambda_c = 0.03250/\text{year}$
- `AIDS`: $\lambda_a = 0.10/\text{year}$
- `Dead from Cancer`: $\mu_c = 0.3081/\text{year}$
- `Dead from AIDS`: $\mu_a = 0.9970/\text{year}$
- `Dead other`: $\mu_0 = 0.014191/\text{year}$