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Computer Algorithms

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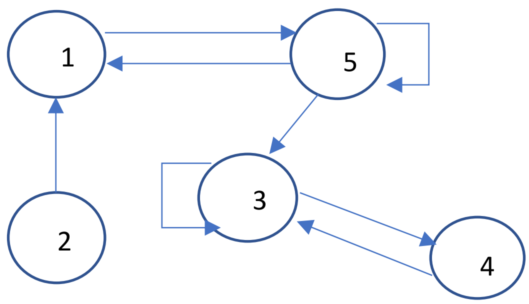
Homework 6

1. In the given matrix, there are 5 rows and 5 columns. Thus, we can say that our graph has 5 vertices. According to the given matrix, our graph is as follows:



   a. Reflexive closure matrix at k = 1. The intermediate vertex between 2 and 5 is vertex 1

$$\begin{bmatrix} 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \\ 0&0&1&0&0 \\ 1&0&1&0&1 \end{bmatrix}$$

   b. Reflexive closure matrix at k = 2. There is no intermediate vertex, thus our reflexive closure matrix for this step is the same as above in part a.

$$\begin{bmatrix} 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \\ 0&0&1&0&0 \\ 1&0&1&0&1 \end{bmatrix}$$

   c. Reflexive closure matrix at k = 3. The intermediate vertices between paths (5, 4) and (4, 4) is 3. Thus,

$$\begin{bmatrix} 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \\ 0&0&1&1&0 \\ 1&0&1&1&1 \end{bmatrix}$$

   d. Reflexive closure matrix at k = 4. There is no intermediate vertex, thus our reflexive closure matrix for this step is the same as above in part c.

$$\begin{bmatrix} 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \\ 0&0&1&1&0 \\ 1&0&1&1&1 \end{bmatrix}$$

e. Reflexive closure matrix at k = 5. The intermediate vertex between paths (1, 1), (1, 3), (1, 4), (2, 3), and (2, 4) is 5. Thus, our final reflexive closure matrix as well as our transitive closure is as follows:

$$\begin{bmatrix} 1\,0\,1\,1\,1 \\ 1\,0\,1\,1\,1 \\ 0\,0\,1\,1\,0 \\ 0\,0\,1\,1\,0 \\ 1\,0\,1\,1\,1 \end{bmatrix}$$

2. Given matrix:

$$D_0 = \begin{bmatrix} 0\,0\,0\,0\,1 \\ 1\,0\,0\,0\,0 \\ 0\,0\,1\,1\,0 \\ 0\,0\,1\,0\,0 \\ 1\,0\,1\,0\,1 \end{bmatrix}$$

Suppose ${d_{ij}}^k$ is the value in the k^th iteration in path (i, j) of matrix D. We can solve for this value using:

$${d_{ij}}^k = min\left({d_{ik}}^{k-1} + {d_{kj}}^{k-1}, {d_{ij}}^{k-1}\right)$$

Now, by iteration of D matrix, we get:

$$D_1 = \begin{bmatrix} 0\,0\,0\,0\,1 \\ 1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 1\,0\,1\,0\,1 \end{bmatrix}$$

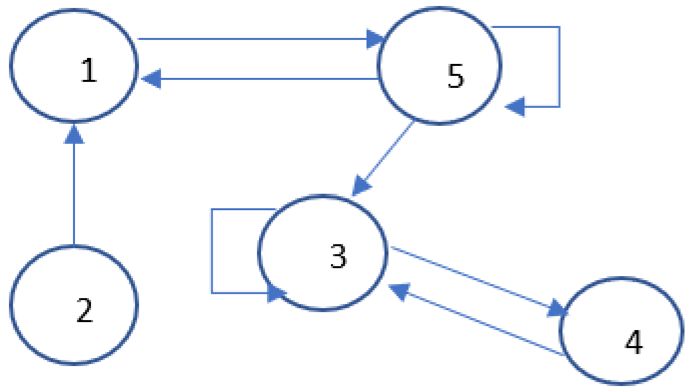
$$D_2 = \begin{bmatrix} 0\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \end{bmatrix}$$

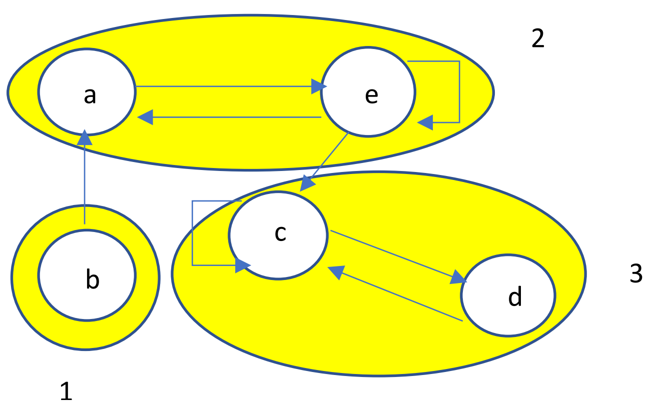
Now, because all values are 0 in our 3rd iteration of D matrix, there is no need to iterate further until 5. Thus, the final matrix with path lengths is given by:

$$\begin{bmatrix} 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \end{bmatrix}$$

3. As was solved in problem 1, the graph that corresponds to the given matrix is as follows:



The three strongly connected components of our graph above are as follows:
1 -> (2) = (b)
2 -> (1, 5) = (a, e)
3 -> (3, 4) = (c, d)
Thus, our condensation graph with renamed vertices is as follows:



The topological order of the graph above is components 1, 2, then 3. In the above graph, the yellow circles represent our new vertices and the labels 1, 2, and 3 represent the new vertices' names, respectively. So there are 3 vertices in our condensation graph: 1, 2, and 3.

The adjacency matrix of our condensation graph would be given as follows:

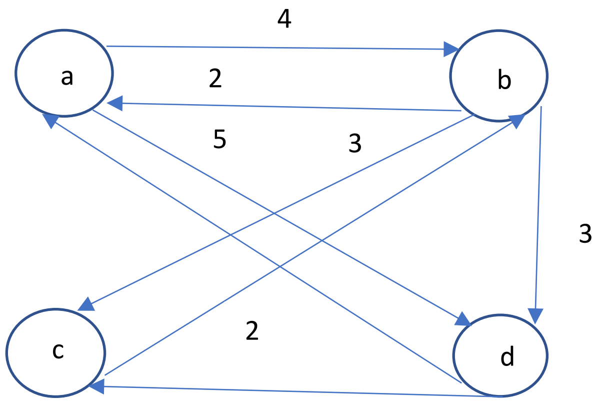
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Using this adjacency matrix, we can calculate the reflexive transitive closure. For vertices x and y in a graph G, we set the position (x, x) as 1, and if there exists a path from vertex x to vertex y, then set the position (x, y) as 1, otherwise 0. Thus, our transitive reflexive closure is as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Because this matrix is a right-diagonal matrix, we can say that this indicates a total order.

4. From the given matrix, we can draw our weighted directed graph:



a. Iteration 1:

$$\begin{bmatrix} 0 & 4 & \infty & 5 \\ 2 & 0 & 3 & 3 \\ \infty & 2 & 0 & \infty \\ -2 & 2 & -4 & 0 \end{bmatrix}$$

b. Iteration 2:

$$\begin{bmatrix} 0 & 4 & 7 & 5 \\ 2 & 0 & 3 & 3 \\ 4 & 2 & 0 & 5 \\ -2 & 2 & -4 & 0 \end{bmatrix}$$

c. Iteration 3:

$$\begin{bmatrix} 0 & 4 & 7 & 5 \\ 2 & 0 & 3 & 3 \\ 4 & 2 & 0 & 5 \\ -2 & 2 & -4 & 0 \end{bmatrix}$$

d. Iteration 4:

$$\begin{bmatrix} 0 & 3 & 1 & 5 \\ 1 & 0 & -1 & 3 \\ 3 & 2 & 0 & 5 \\ -2 & -2 & -4 & 0 \end{bmatrix}$$

Therefore, this is our resultant distance matrix of the given weighted directed graph.