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Signals and Systems

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Exam #1

1. We know:

$$\delta[n-k]x[n] = x[k]$$
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$$
$$g[n] = u[n] - u[n-4]$$

   a. For $x[n] = \delta[n-1]$:

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-1]g[n-2k]$$

   Since k only exists at 1, $y[n] = g[n-2] = u[n-2] - u[n-6]$

   b. For $x[n] = \delta[n-2]$:

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-2]g[n-2k] = g[n-4] = u[n-4] - u[n-8]$$

   c. Let's suppose that for $x[n] = \delta[n]$:

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k]g[n-2k] = g[n]$$

   For this system to be time invariant, it would mean that:
   $x[n] \leftrightarrow g[n]$, then $x[n-1] \leftrightarrow g[n-1]$. But since $x[n-1] \leftrightarrow g[n-2]$, the system must be time variant.

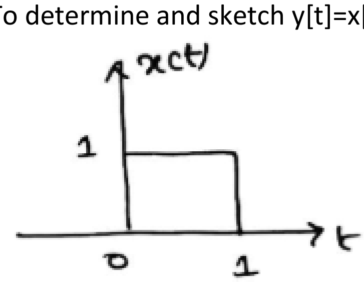
2. Here, $y[n] = z[n] - z[n-1]$, where $z[n] = 2x[n] - y[n-1]$
   Substituting, we get:

$$y[n] = 2x[n] - y[n-1] - 2x[n-1] - 2y[n-2]$$

   That is,

$$y[n] + y[n-1] + 2y[n-2] = 2(x[n] - x[n-1])$$

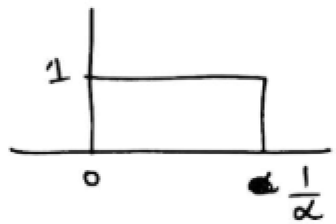
3. To determine and sketch y[t]=x[t]*h[t], let's first sketch x[t] and h[t], respectively:



= x[t]

$h(t) = x(\frac{t}{\alpha})$

$=$

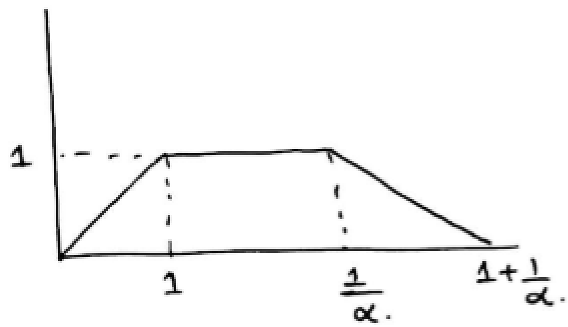


$0 \leq \alpha < 1$

= h[t]

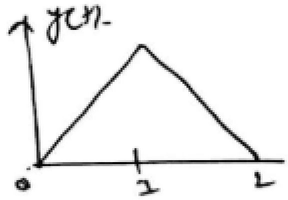
a. Now, we can sketch y[t]=x[t]*h[t]:

$x(t) * h(t)$



= y[t]

b. From our sketch above, we can see that if $\frac{dy[t]}{dt}$ contains only 3 discontinuities, then $\alpha = 1$ because our sketch would look like below:



= y[t], where $2 = 1 + \frac{1}{\alpha}$

4. x[-1] = -1, x[0] = 0, x[1] = 1
   v[0] = 1, v[1] = 1, v[2] = 1
   This can be rewritten as:

$$x[n] = \delta[n-1] - \delta[n+1]$$
$$v[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

So we can write the convolution y[n] = x[n]*v[n]:

$$y[n] = x[n] * v[n] = \sum_{k=-\infty}^{\infty} x[k]v[n-k]$$

Substituting the values we receive from the original discrete time signals, we can solve for the convolution sum:

$$y[n] = \delta[n] * \delta[n-1] - \delta[n+1] * \delta[n] + \delta[n-1] * \delta[n-1] - \delta[n+1] * \delta[n-1] + \delta[n-1] * \delta[n+2] - \delta[n+1] * \delta[n+2]$$
$$y[n] = \delta[n-1] - \delta[n+1] + \delta[n-2] - \delta[n] + \delta[n+1] - \delta[n-3]$$

$$y[n] = -\delta[n-3] - \delta[n] + \delta[n-1] + \delta[n-2]$$

Finally, we can plot the result below: