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Signals and Systems

23 November 2019

Project 2

1. **Problem 4.33 in Textbook (all parts)**
Given:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

a. Let's take the Fourier Transform on both sides of this equation:

$$(j\omega)^2 Y(j\omega) + 6(j\omega)Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$$

We then get:

$$\frac{Y(j\omega)}{X(j\omega)} = H(j\omega) = \frac{2}{-\omega^2 + 6j\omega + 8}$$

We can use the Matlab reside function to perform a partial fraction expansion:

$$H(j\omega) = \frac{1}{j\omega + 2} - \frac{1}{j\omega + 4}$$

Finally, we take Inverse Fourier Transform of above expression:

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

```
>> n = [2]
>> d = [-1 6 8]
>> rpk = residue(n, d)

r =
1.0000
1.0000

p =
-2.0000
-4.0000

k =
[]
```

b. Given:

$$x(t) = te^{-t}u(t)$$

Taking the Fourier Transform of x(t):

$$X(j\omega) = \frac{1}{(j\omega + 2)^2}$$

From part a), we know

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{(j\omega+2)(j\omega+4)}$$
$$\rightarrow Y(j\omega) = \frac{2}{(j\omega+2)(j\omega+4)}X(j\omega)$$
$$\rightarrow Y(j\omega) = \frac{2}{(j\omega+2)^3(j\omega+4)}$$

We can use the Matlab residue function to perform a partial fraction expansion:

$$Y(j\omega) = \frac{1/4}{j\omega+2} - \frac{1/2}{(j\omega+2)^2} + \frac{1}{(j\omega+2)^3} - \frac{1/4}{j\omega+4}$$

Finally, we take the Inverse Fourier Transform of the above expression:

$$y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{t}{2}e^{-2t}u(t) + \frac{t^2}{2}e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t)$$

```
>> n = [2]
>> d = [1 1 1 1 18]
>> rpk = residue(n, d)

r =
0.2500
-0.5000
1.0000
0.2500

p =
-2.0000
-4.0000
-8.0000
-4.0000

k =
[]
```

c. Let’s take the Fourier Transform of the given differential equation:

$$Y(j\omega)\left[-\omega^2 + \sqrt{2}j\omega + 1\right] = X(j\omega)[-2\omega^2 - 2]$$
$$\rightarrow H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2(-\omega^2-1)}{-\omega^2+\sqrt{2}j\omega+1}$$

We can use the Matlab residue function to perform a partial fraction expansion:

$$H(j\omega) = 2 + \frac{-\sqrt{2}+\sqrt{2}j}{j\omega + \frac{1-j}{\sqrt{2}}} + \frac{-\sqrt{2}-\sqrt{2}j}{j\omega + \frac{1+j}{\sqrt{2}}}$$

Finally, we take the Inverse Fourier Transform of the above expression:

$$h(t) = 2\delta(t) - \sqrt{2}(1-j)e^{-\frac{(1-j)}{\sqrt{2}}t}u(t) - \sqrt{2}(1+j)e^{-\frac{(1+j)}{\sqrt{2}}t}u(t)$$

2. **Problem 4.34 in Textbook (part b)**
    b. Given the frequency response of LTI system S:
    $$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$
    We can split the denominator above into 2 factors:
    $$H(j\omega) = \frac{j\omega + 4}{(j\omega + 2)(j\omega + 3)}$$
    We can use the Matlab residue to perform a partial fraction expansion:
    $$H(j\omega) = \frac{2}{j\omega + 2} - \frac{1}{j\omega + 3}$$
    Finally, we take the Inverse Fourier Transform of the above expression:
    $$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)$$

3. **Problem 9.22 in Textbook (part e)**
    e.
```
>> n=[1 1]; %n=s+1
>> d=[1 5 6]; %d=s^2(s+2)=s^2+5s+6
>> [r p k]=residue(n d);

r =
2
-1

p =
-3
-2

k =
[]
```

    This shows us that our partial fraction expansion is:
    $$F(s) = \frac{2}{s + 3} - \frac{1}{s + 2}$$

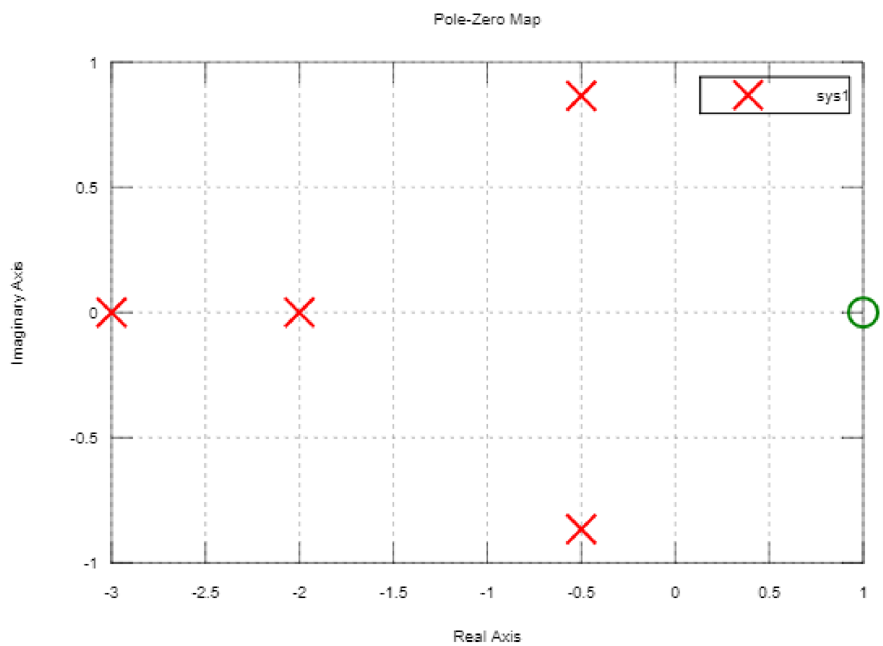
    Next, we take the inverse Laplace transform:

```
>>syms s t
>> F = 2/(s+3) – 1/(s+2);
>> ilaplace(F, t)
```

answer = $2e^{-3t}u(t) - e^{-2t}u(t)$

4. **Problem 9.7 in Textbook**

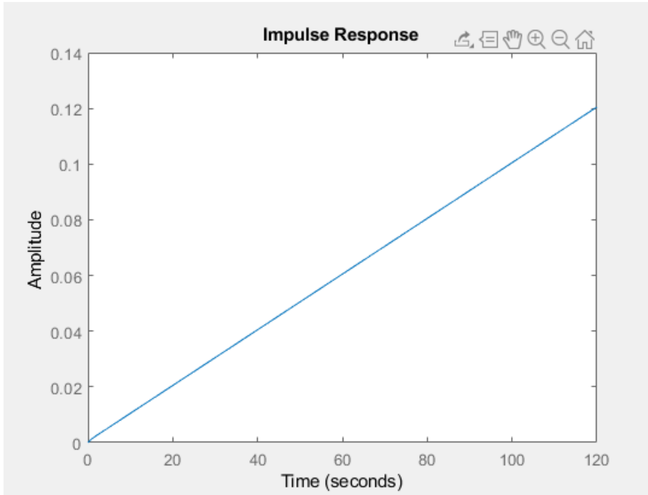
```
>> n=[1 -1];
>> d=[1 6 12 11 6];
>> sys1=tf(n,d)
>> pzmap(sys1)
```



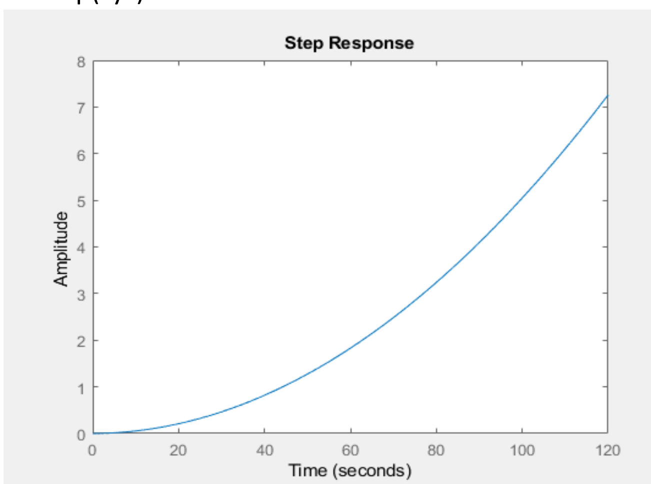
5.

a.
```
>> n1 = [1 1]
>> d1 = [1 2]
>> G1 = tf(n1, d1)

>> n2 = [1]
>> d2 = [500 0 0]
>> G2 = tf(n2, d2)

>> G3 = G1*G2 %forward path transfer function

>> [n d] = series(n1, d1, n2, d2)
>> printsys(n, d)
>> sys = tf(n, d)
```

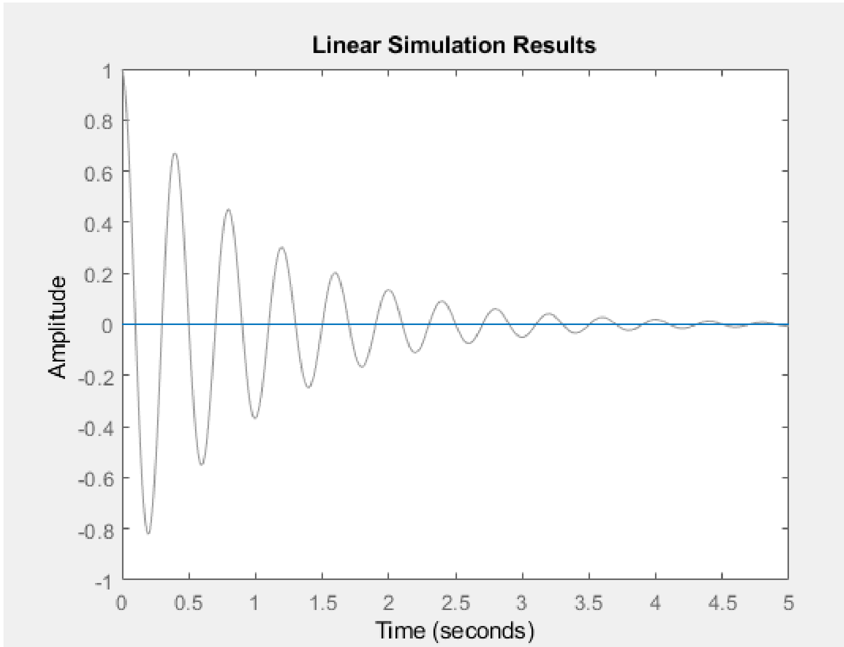
b.
```
>> impulse(G3) %plots impulse response of G(s) found in part a
```



c. >> step(sys)



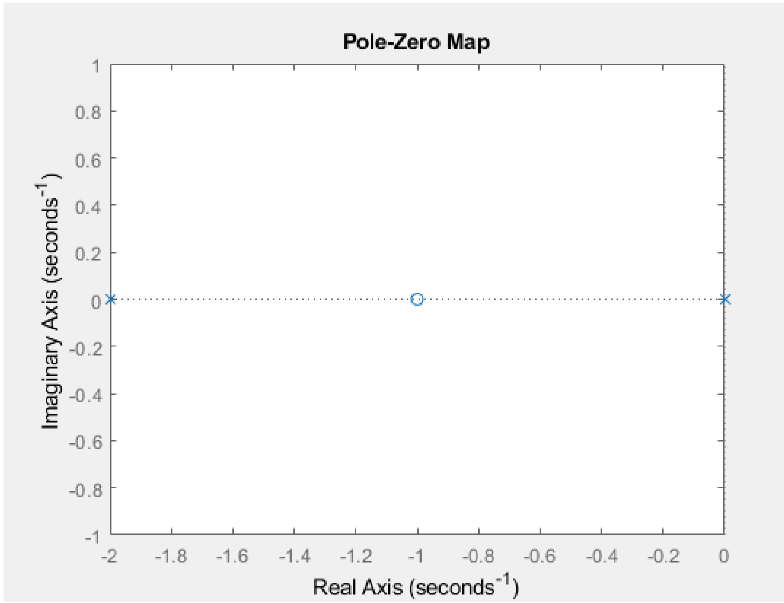
d.
```
>> T=0:0.002:5;
>> U=cos(15.7*T) .* exp(-T);
>> lsim(sys, U, T)
```



e. >> [p z] = pzmap(G3) %since all poles lie on left side of s-plane, system is stable

```
p =
0
0
-2

z =
-1
```

f. pzmap(G3)

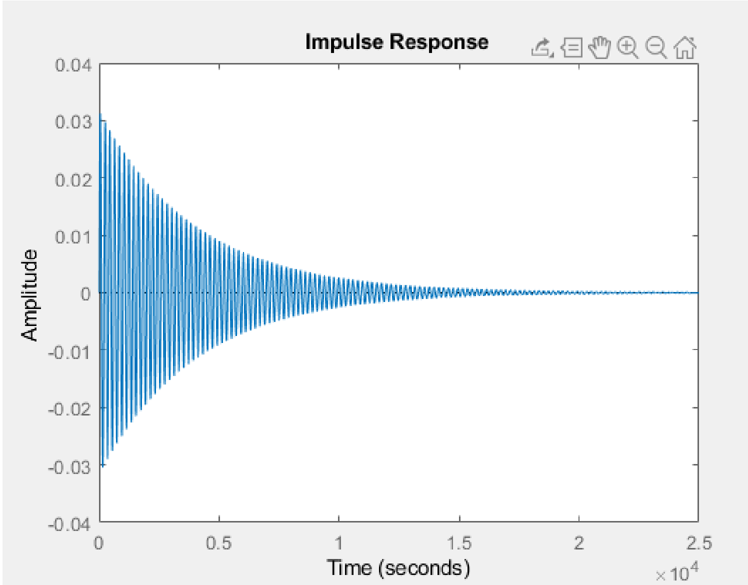


6.

a.
```
>> n1 = [1 1]
>> d1 = [1 2]
>> G1 = tf(n1, d1)

>> n2 = [1]
>> d2 = [500 0 0]
>> G2 = tf(n2, d2)

>> G3 = G1*G2 %forward path transfer function
>> G = feedback(G3, 1) % closed loop transfer function with % gain of feedback path =
unity
```

b.
```
>> [n d] = series(n1, d1, n2, d2)
>> printsys(n, d)
>> sys = tf(n, d)
```

c. >> impulse(G) %plots impulse response of G(s) found in part a



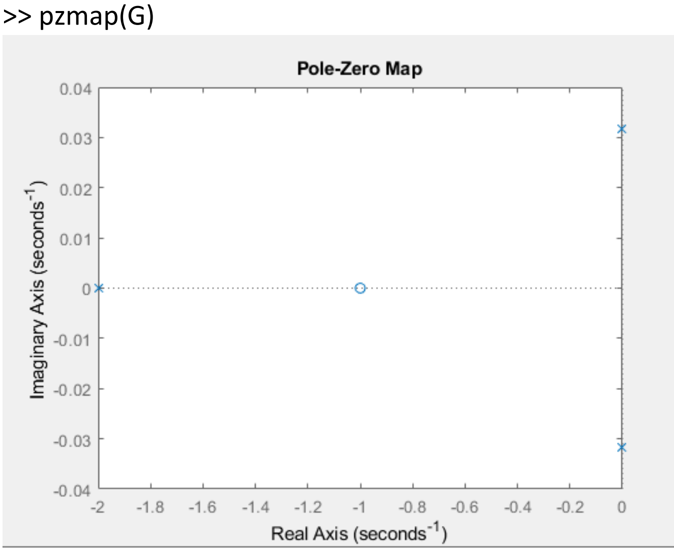
Adding feedback to this system reduces the overall gain of the system with the degree of reduction being related to the systems open-loop gain.

d. >> [p z] = pzmap(G) %since all poles lie on left side of s-plane, system is stable

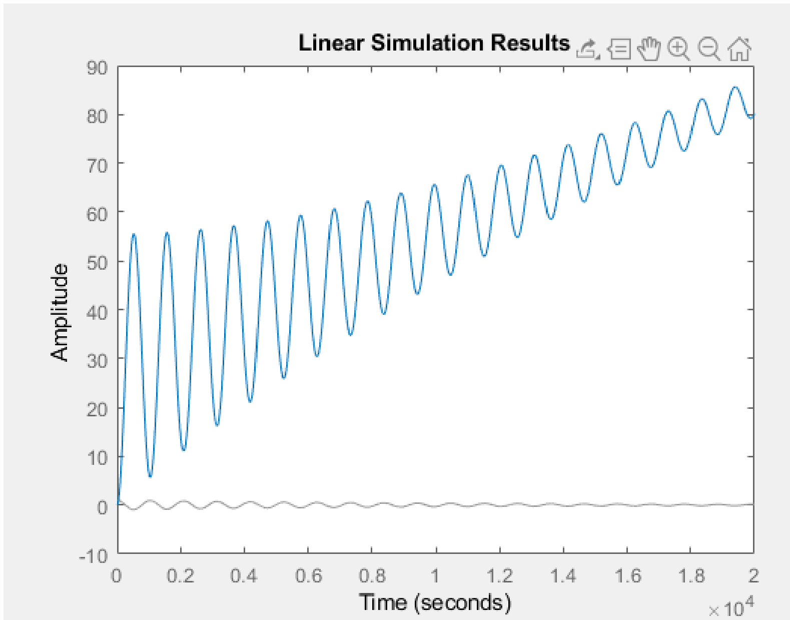
p =
-1.9995 + 0.0000i
-0.0002 + 0.0316i
-0.0002 - 0.0316i

z =
-1

e. >> pzmap(G)



f.
```
>> T=0:10:20000;
>> U=cos(0.006*T) .* exp(-T/10000);
```

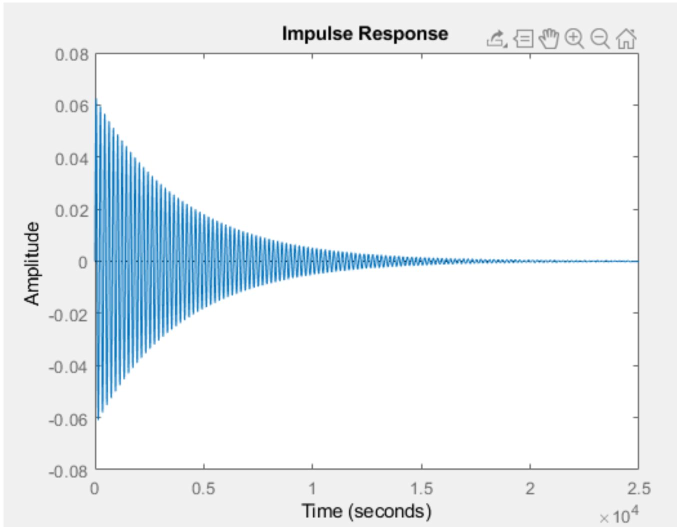


7.

a.
```
>> n1 = [1]
>> d1 = [500 0 0]
>> G1 = tf(n1, d1)

>> n2 = [1 1]
>> d2 = [1 2]
>> H1 = tf(n2, d2)

>> G = feedback(G1, H1)
```

b.
```
>> [n d] = series(n1, d1, n2, d2)
>> printsys(n, d)
>> sys=tf(n, d)
```

c. >> impulse(G) %plots impulse response of G(s) found in part a

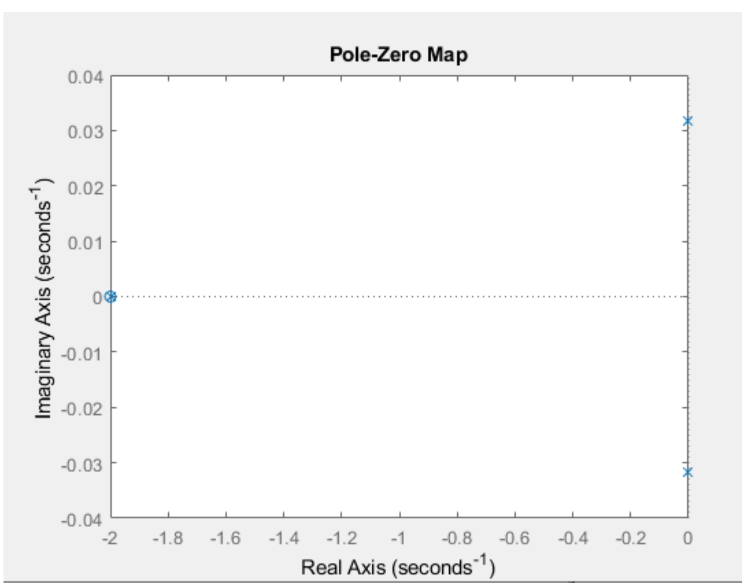


d. >> [p z] = pzmap(G) %since all poles lie on left side of s-plane, system is stable

p =
-1.9995 + 0.0000i
-0.0002 + 0.0316i
-0.0002 - 0.0316i

z =
-2

e. >>pzmap(G)



8.

a. Given:

$$y(n) = 0.6y(n-1) + x(n)$$

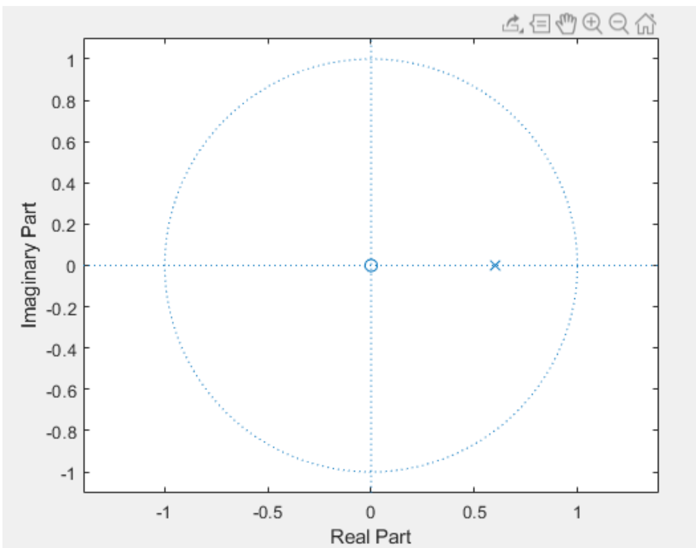
Taking Z transform in above difference equation:

$$Y(z) = 0.6z^{-1}Y(z) + X(z)$$

$$\rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.6z^{-1}}$$

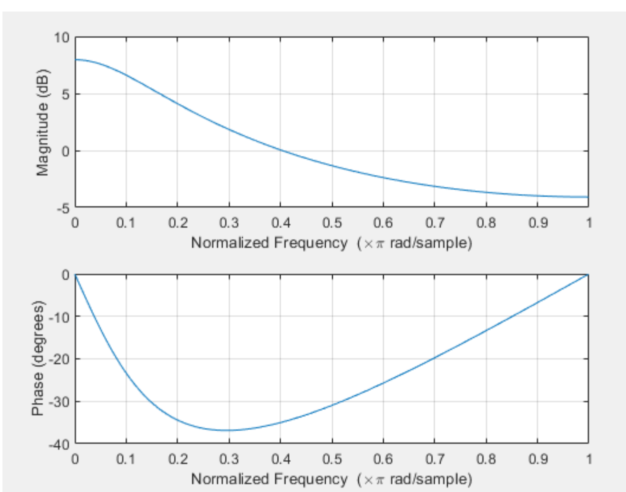
```
>> n = [1]
>> d = [1 -0.6]
```

```
>> zplane(n, d)
```

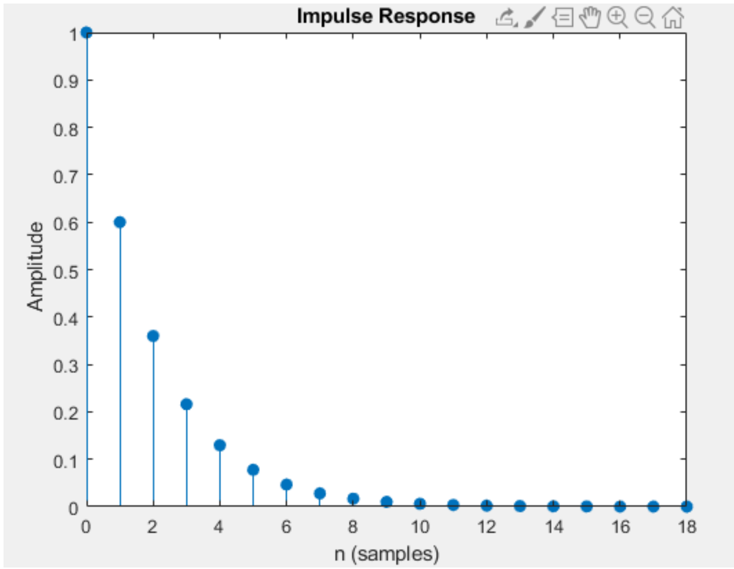


Because the pole lies inside the unit circle, the system is stable.

b.
```
>> freqz(n, d, 1024)
```



c. >> impz(n, d)


Impulse Response
Amplitude
n (samples)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
16
18