University of BRISTOL

COMS 30115

Raytracing

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Introduction

Today

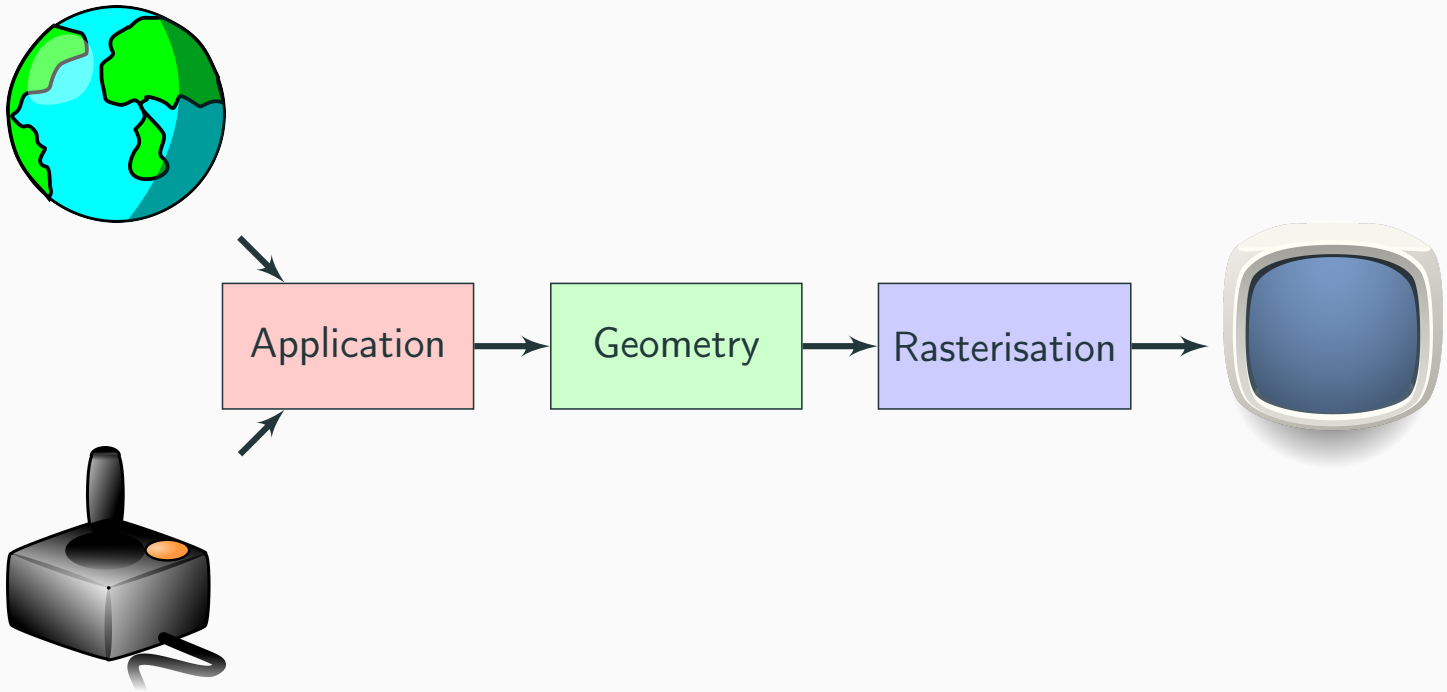
- First rendering method
  - Raytracing/Raycasting
- Cameras
- Intersection calculations
- After today you should be able to start with Lab 1

Scratchapixel

- Introduction to Ray Tracing: a Simple Method for Creating 3D Images
- An Overview of the Ray-Tracing Rendering Technique

Raytracing

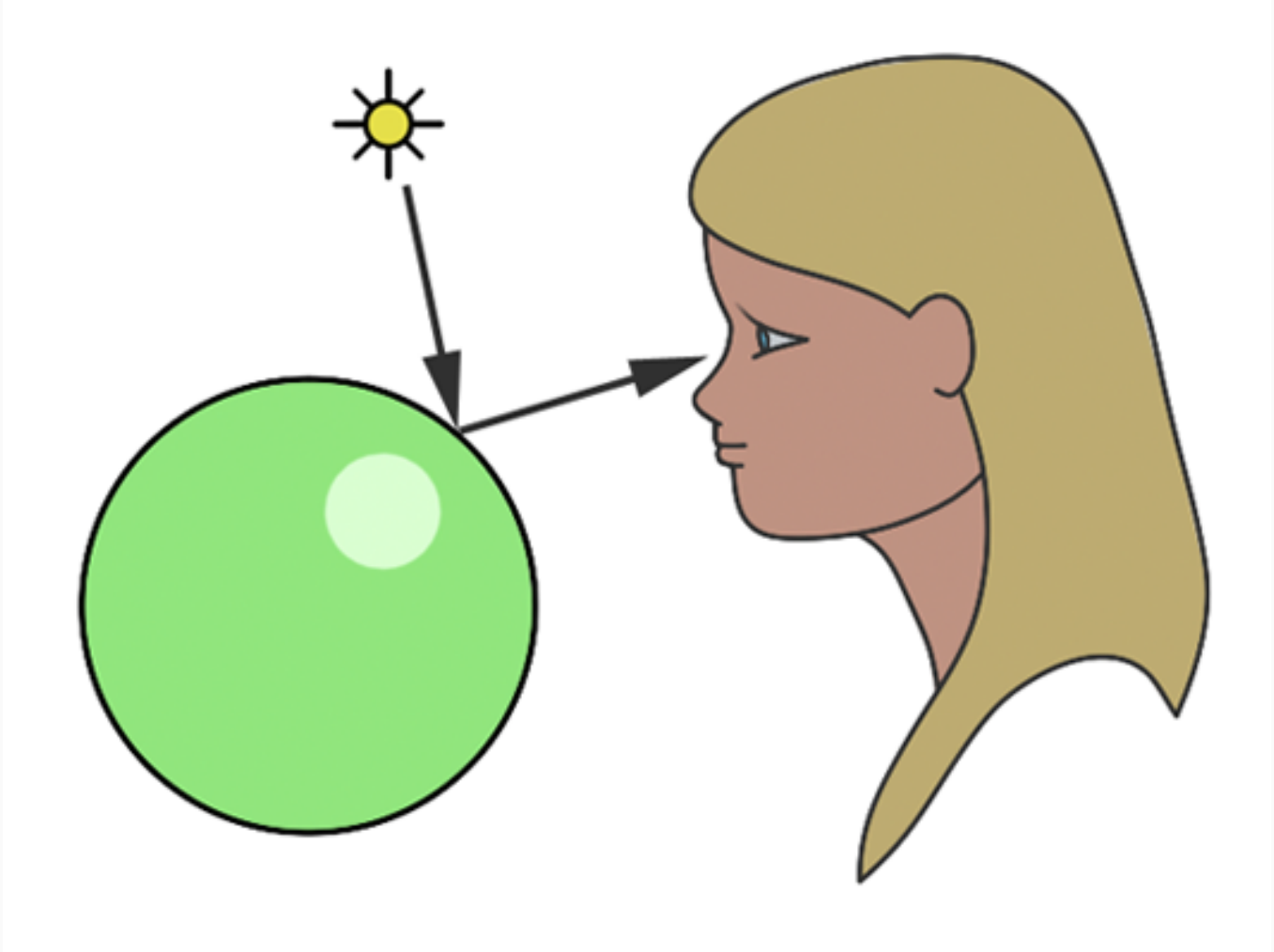
Pipeline



Pipeline


Geometry
View Transformation
Lighting
Projection
Clipping
Screen Mapping
Primitives

Raytracing



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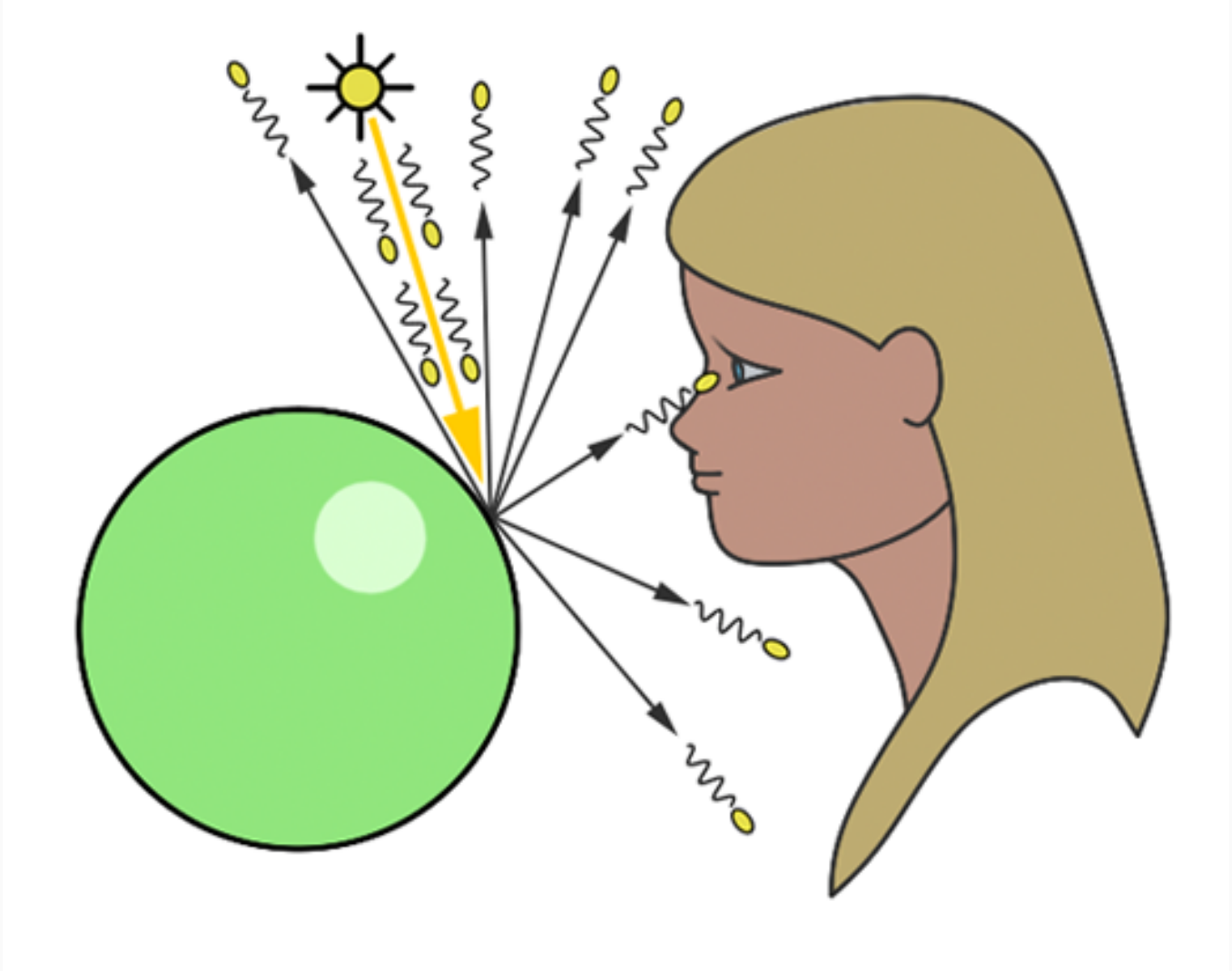

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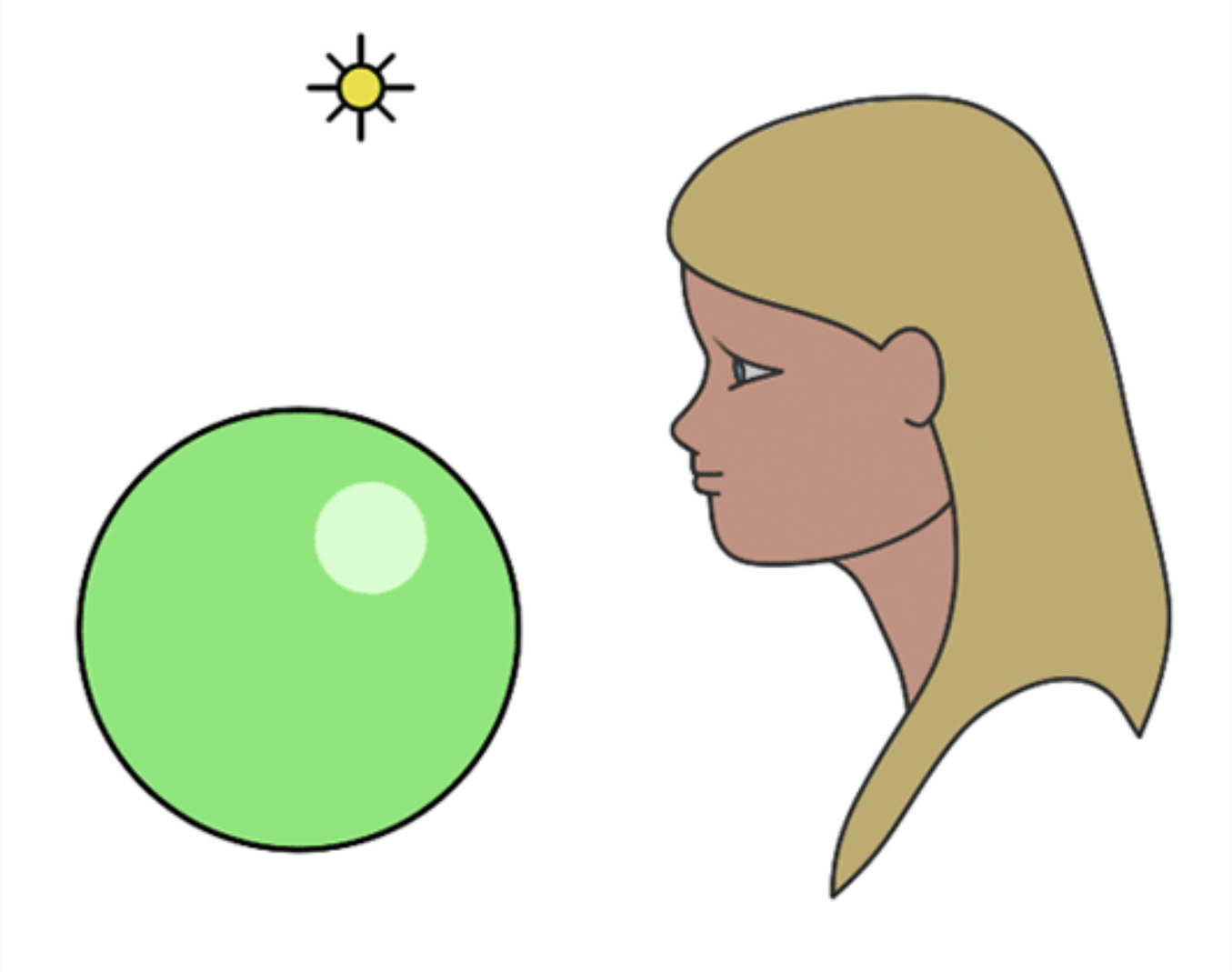

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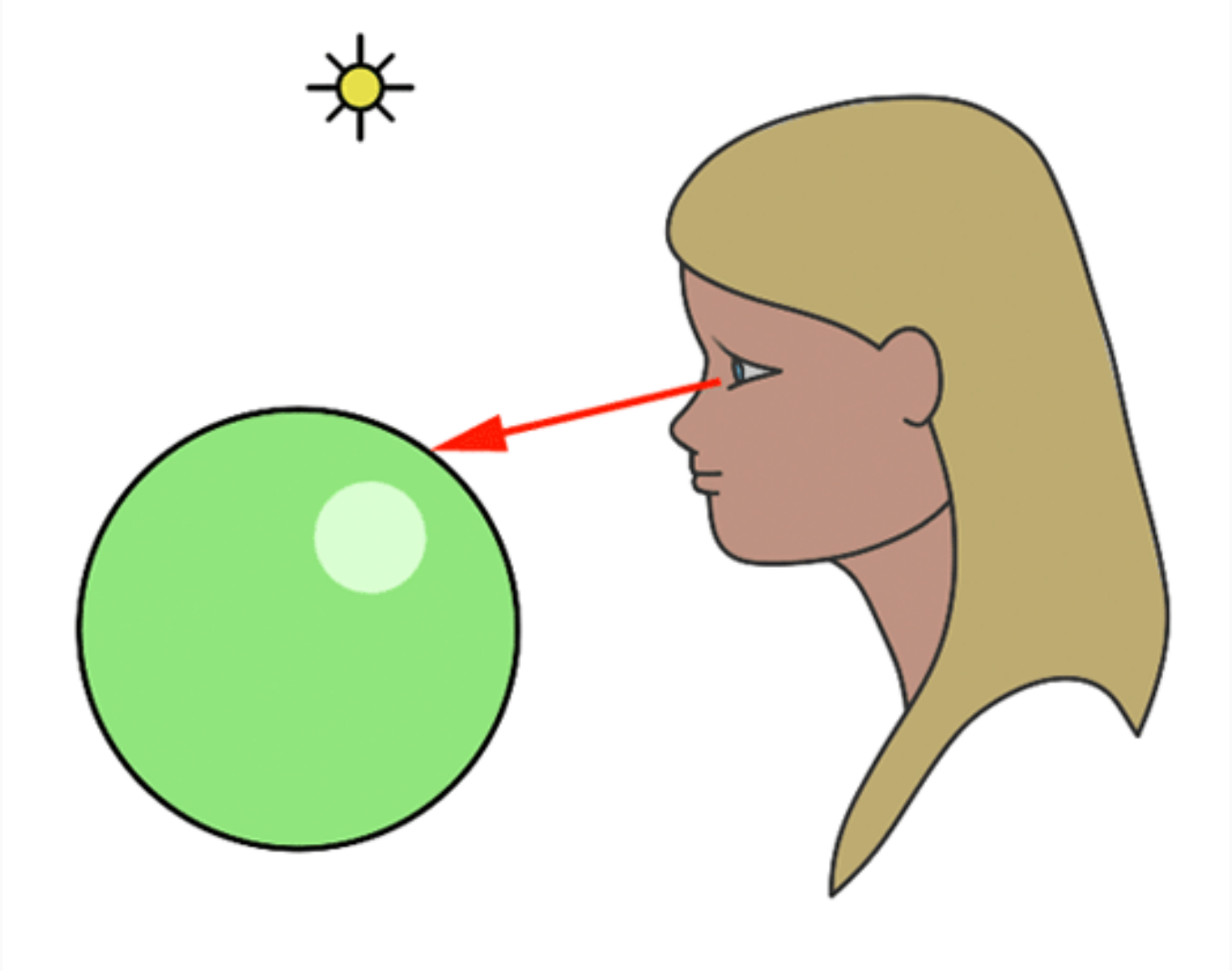

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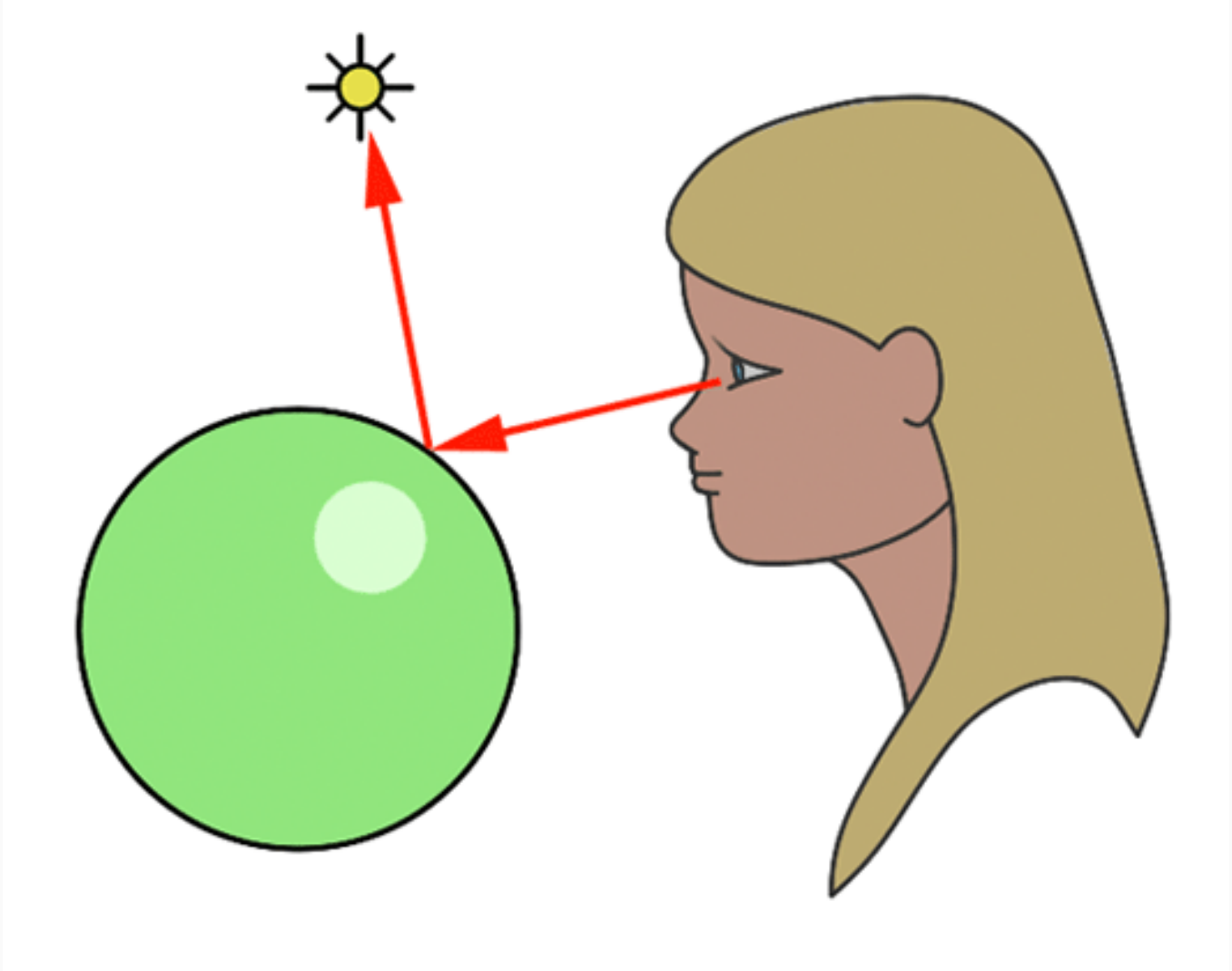

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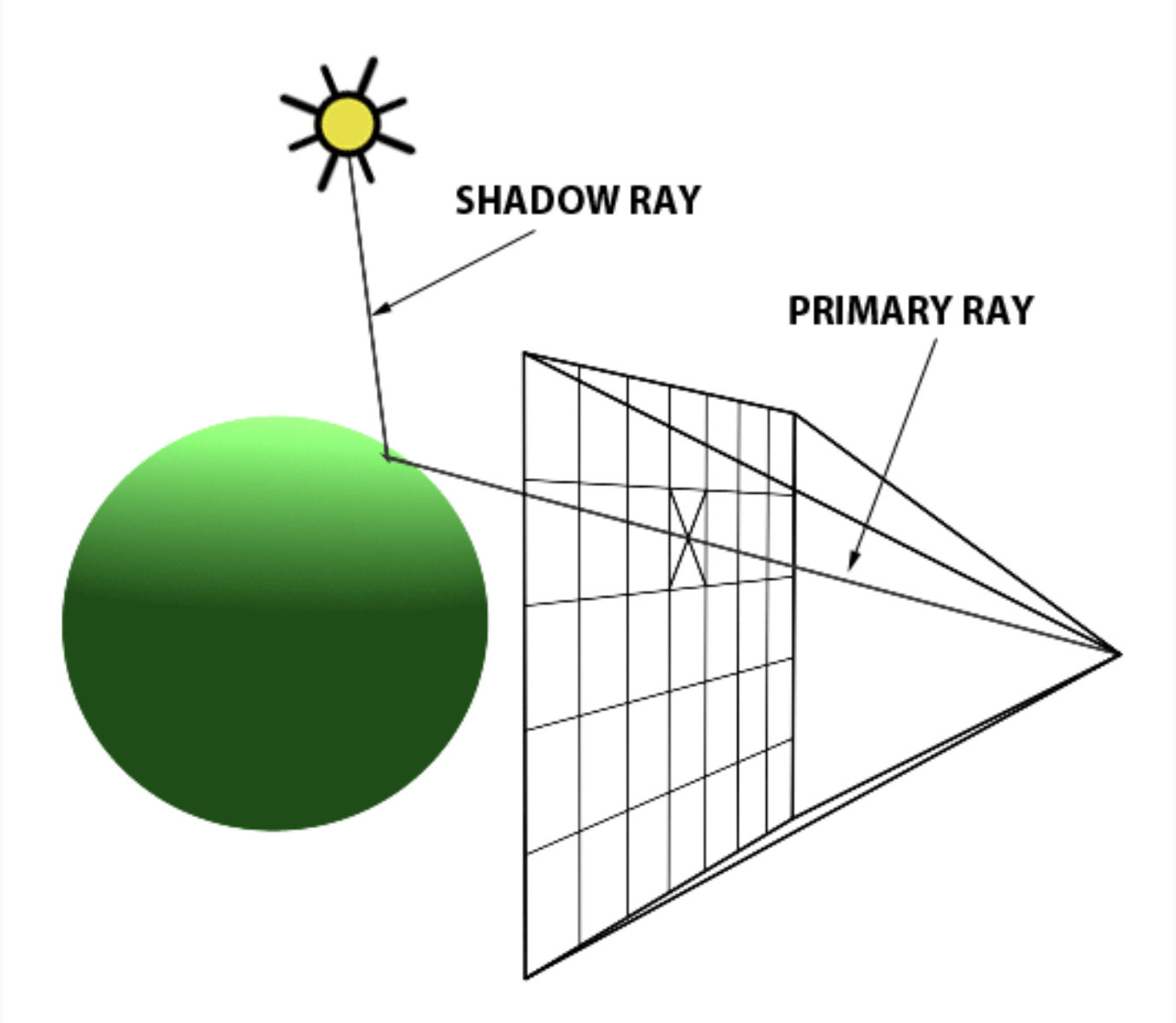

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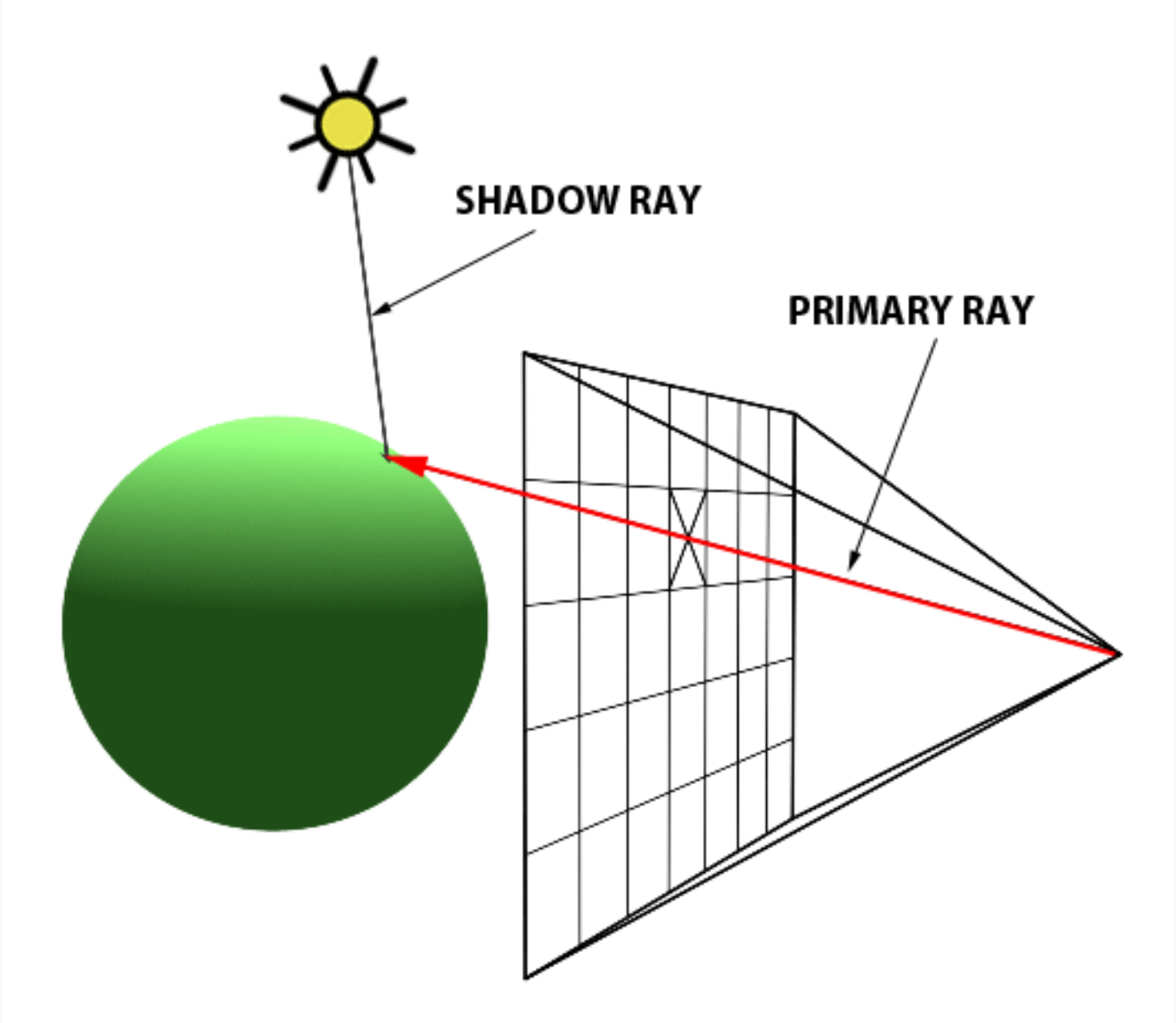

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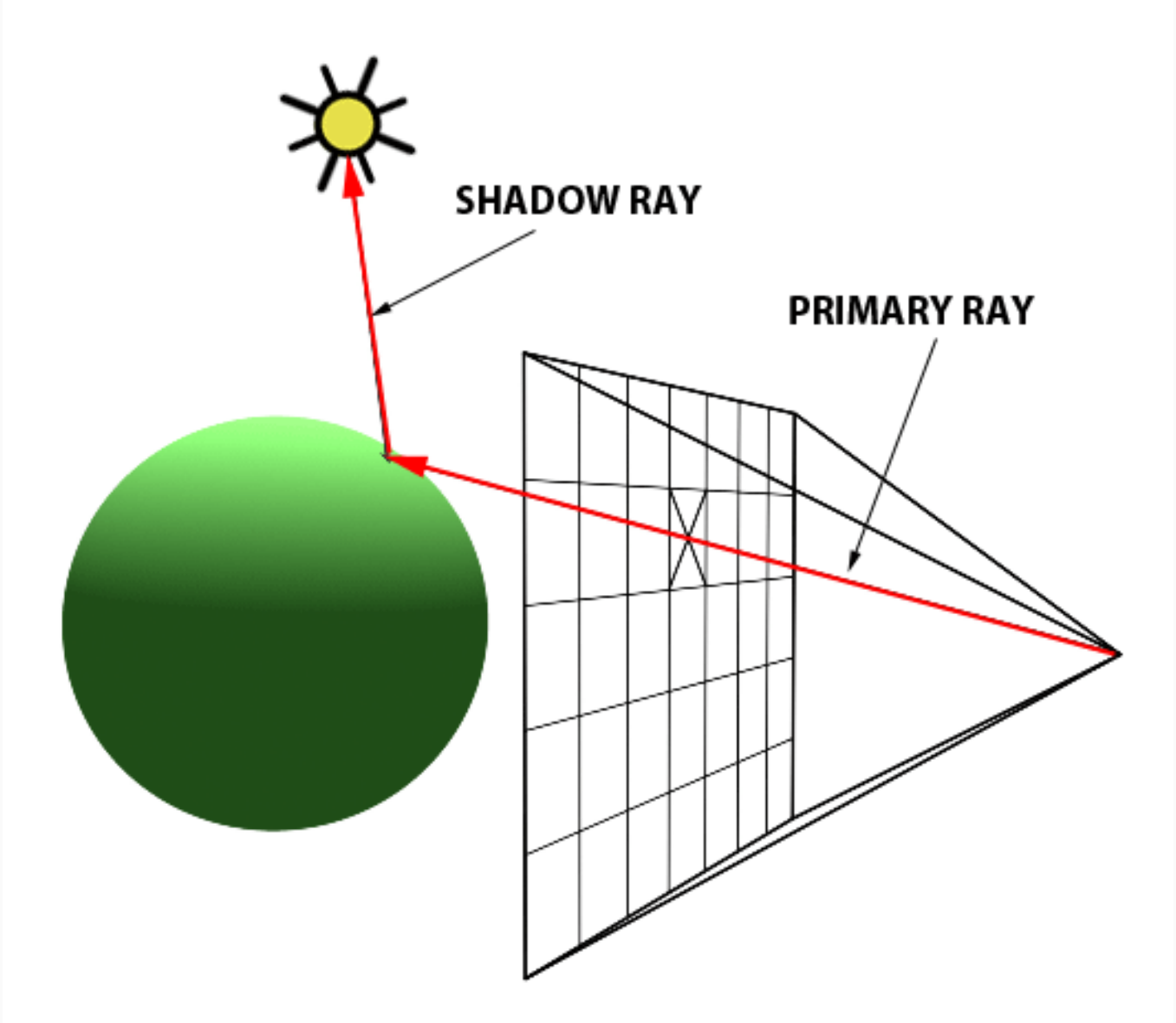

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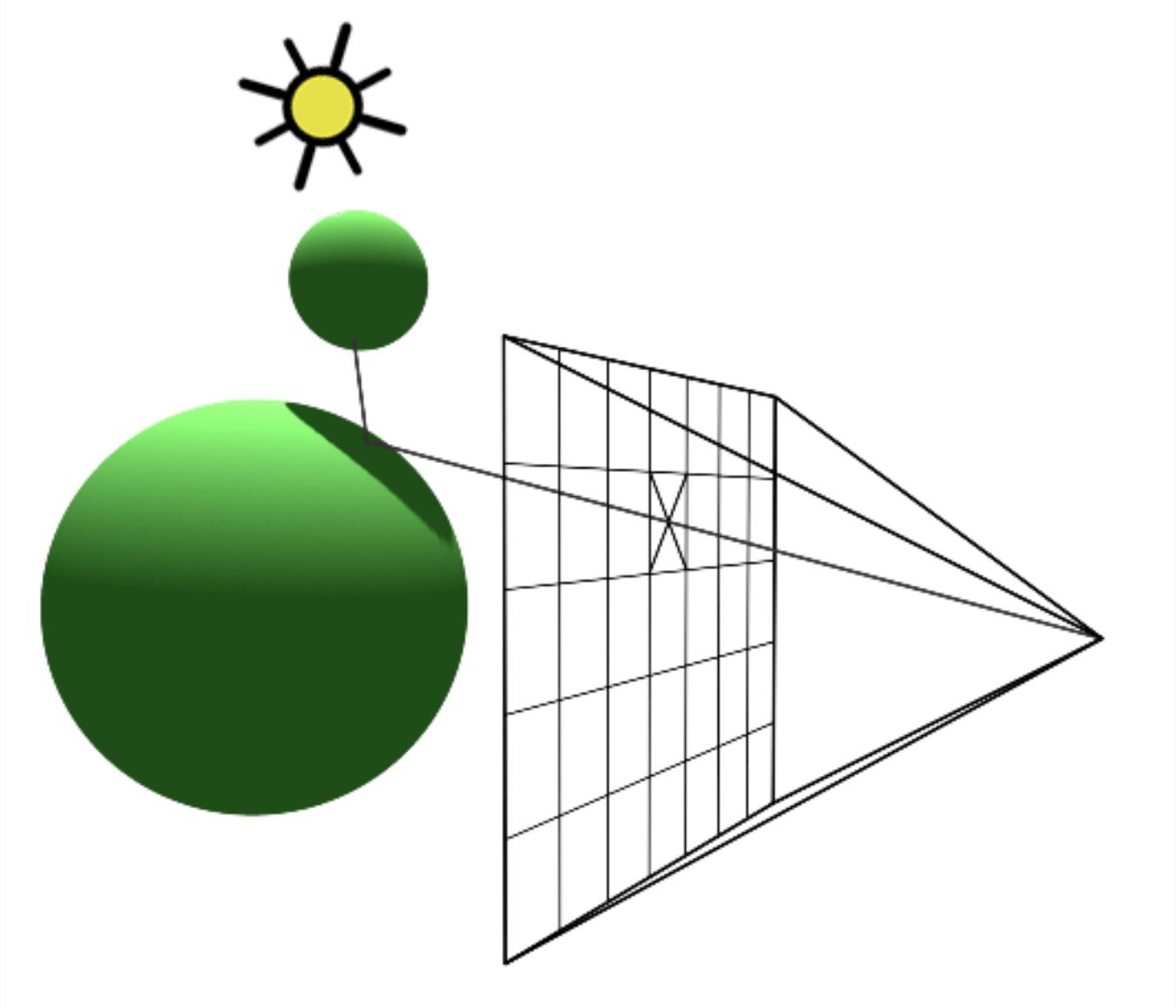

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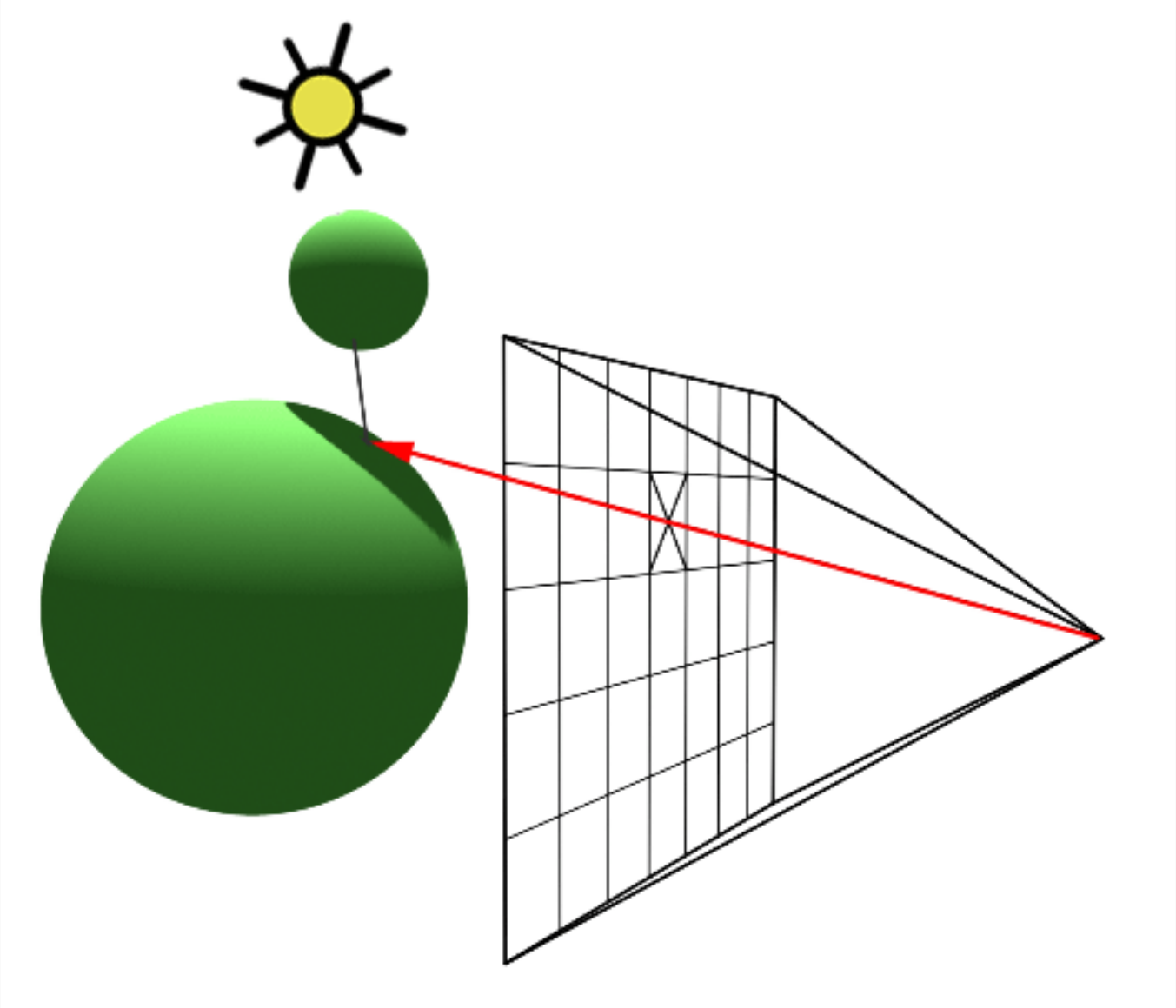

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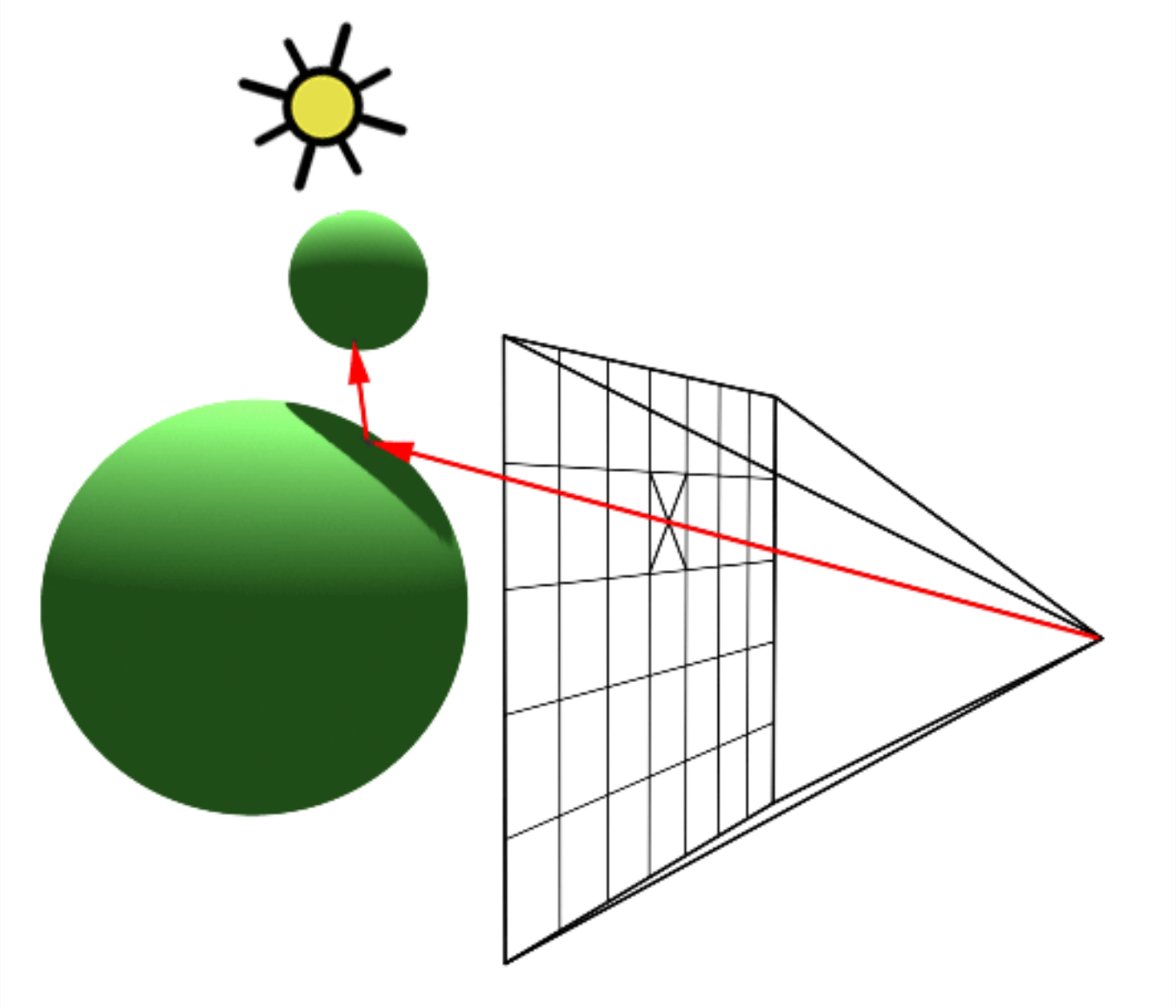

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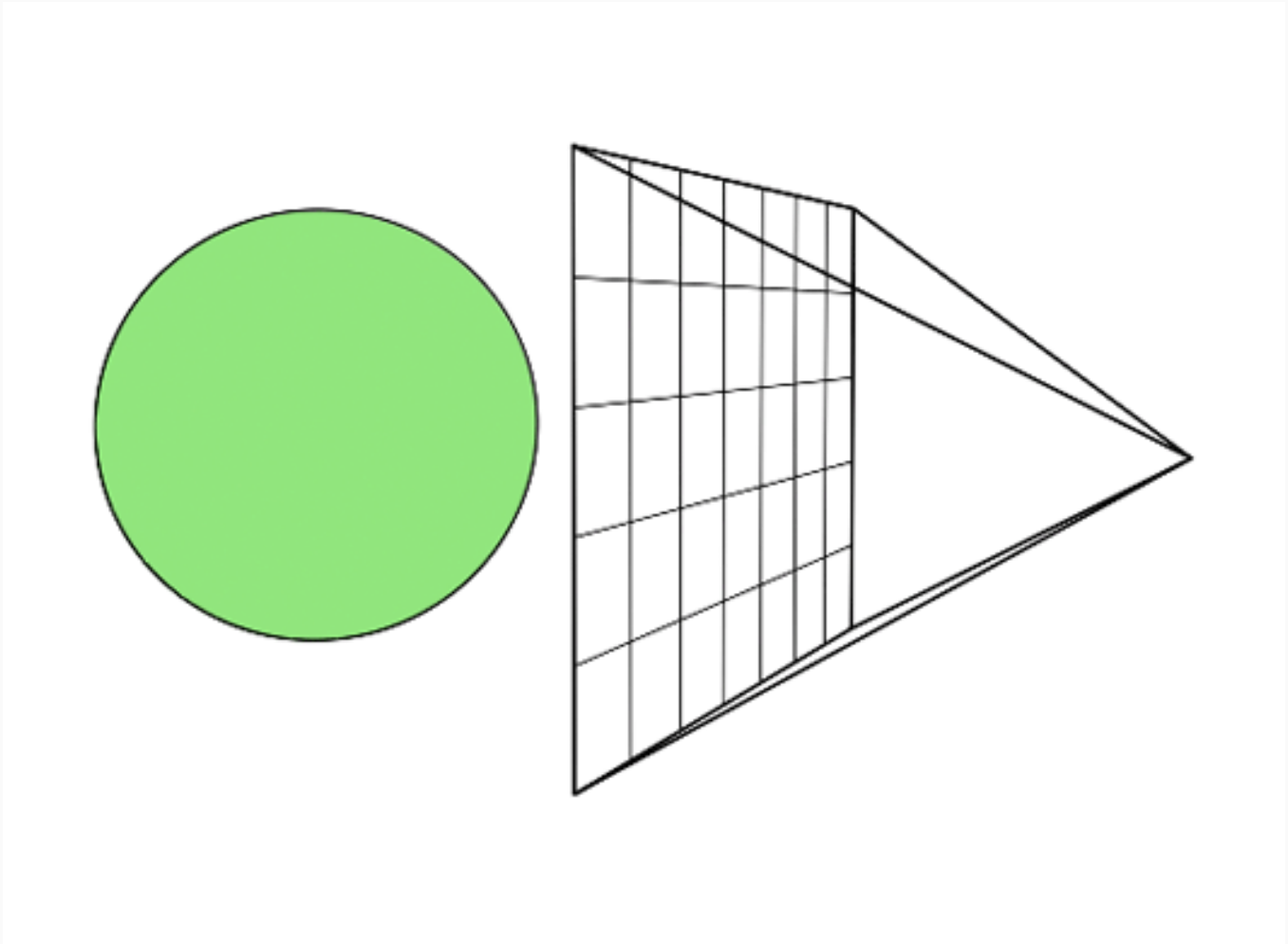

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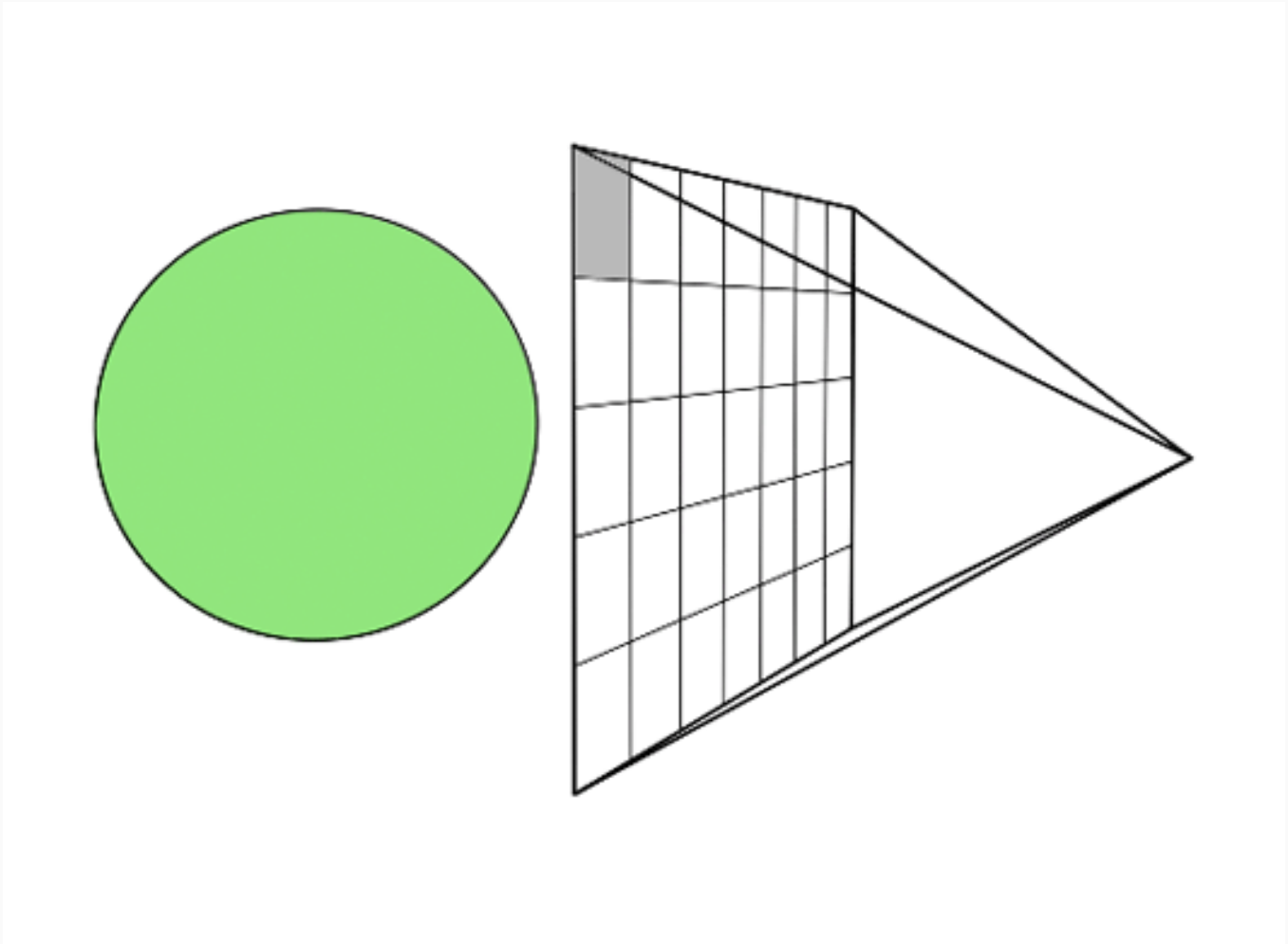

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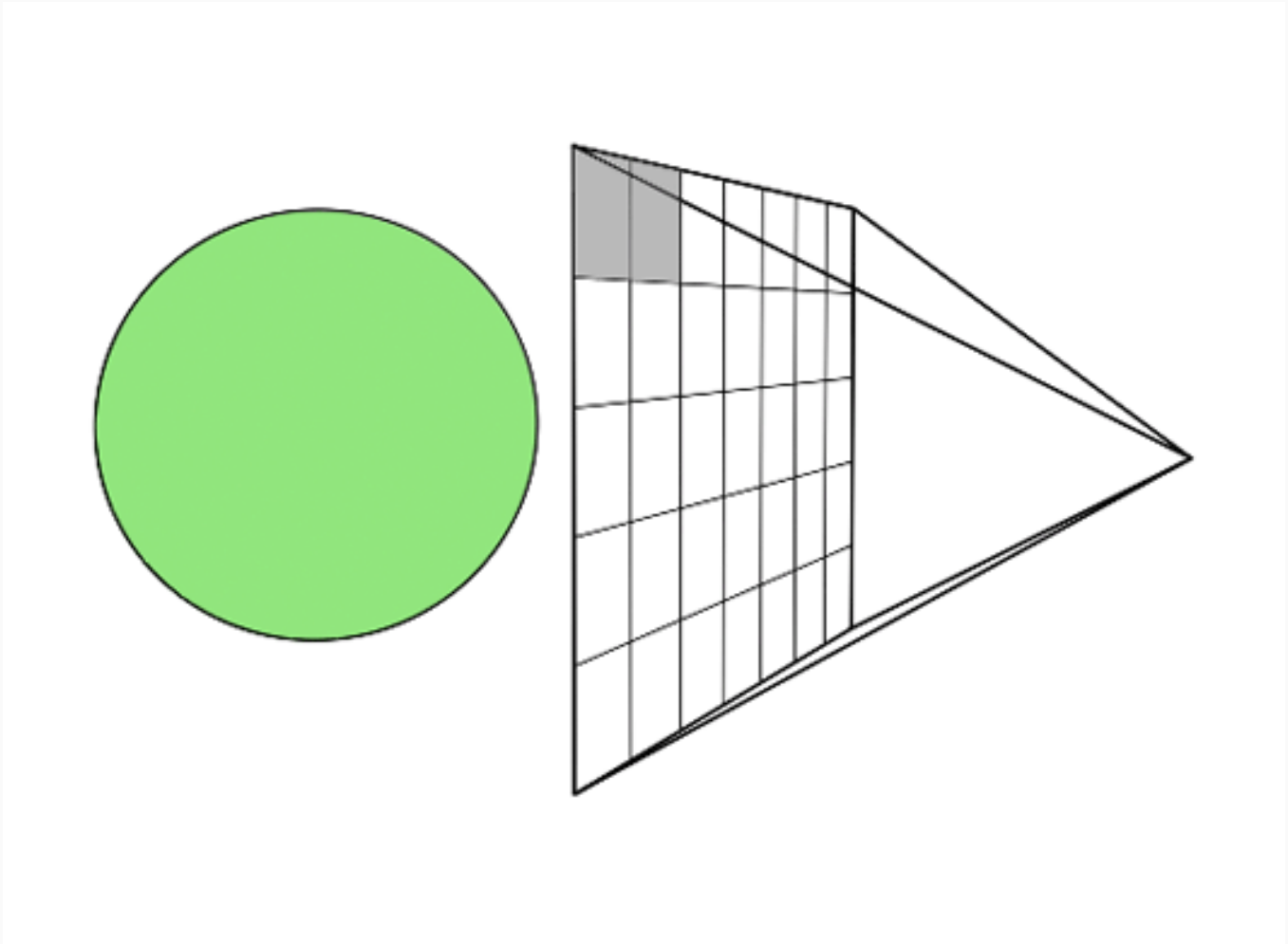

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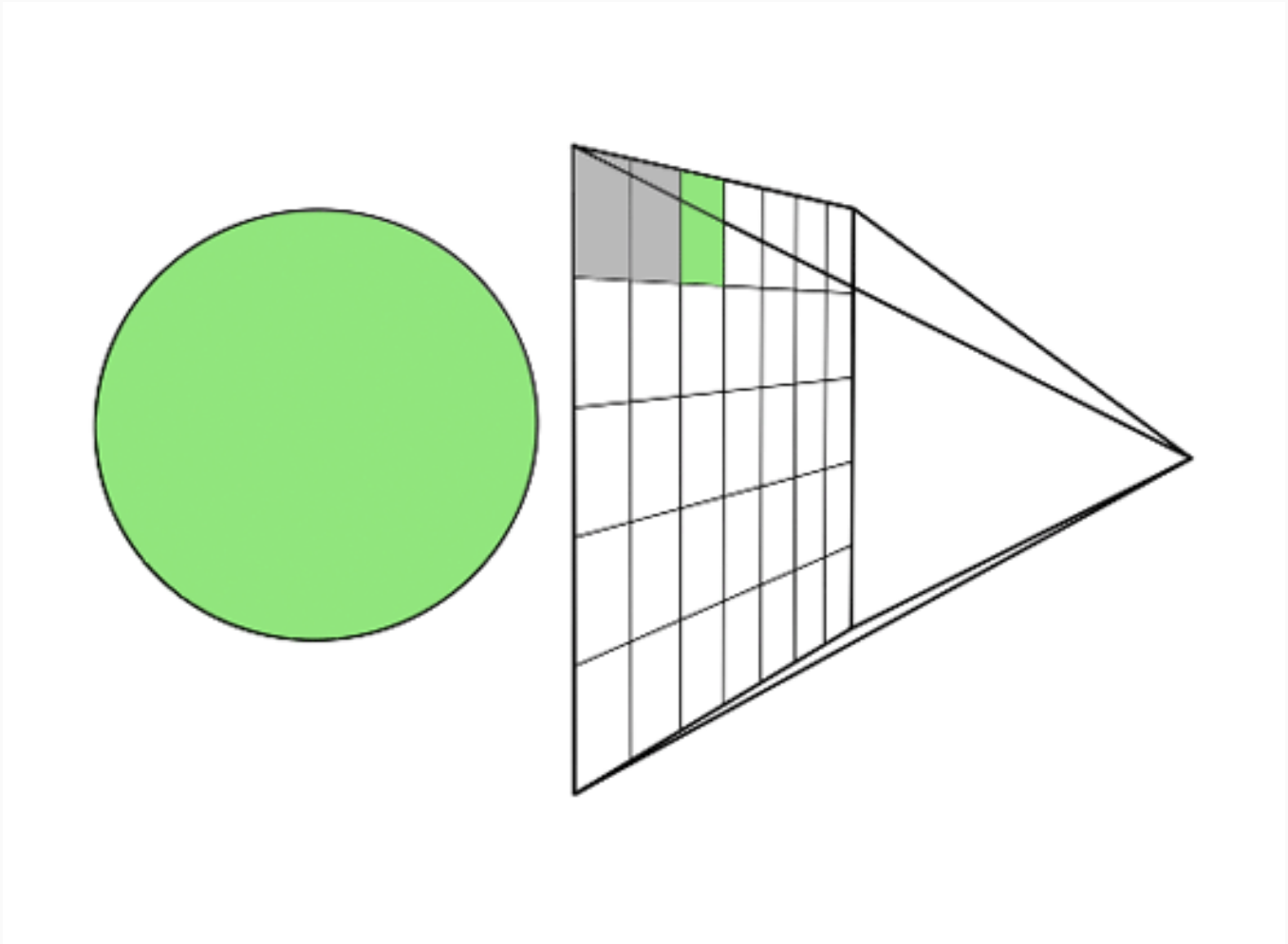

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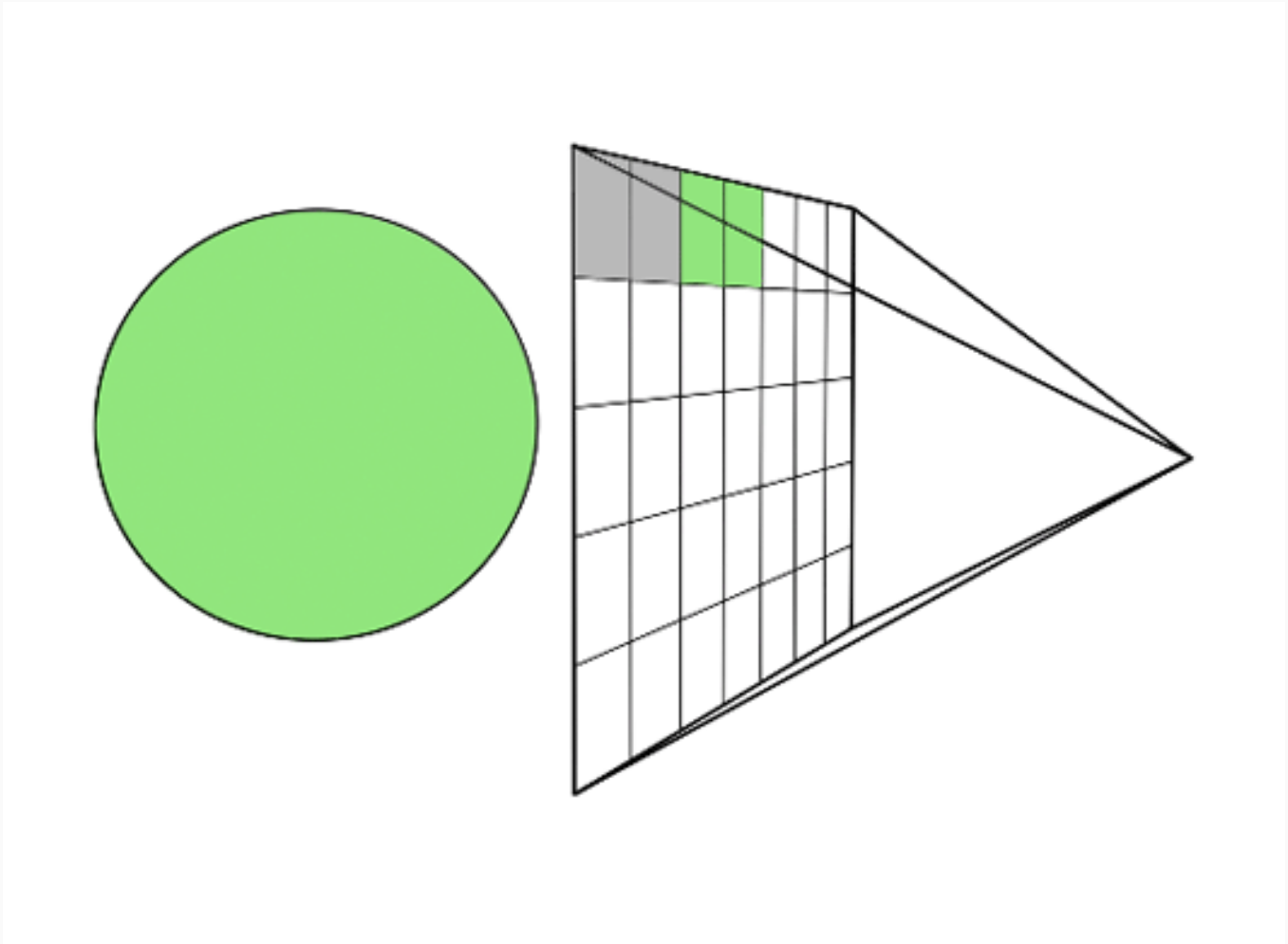

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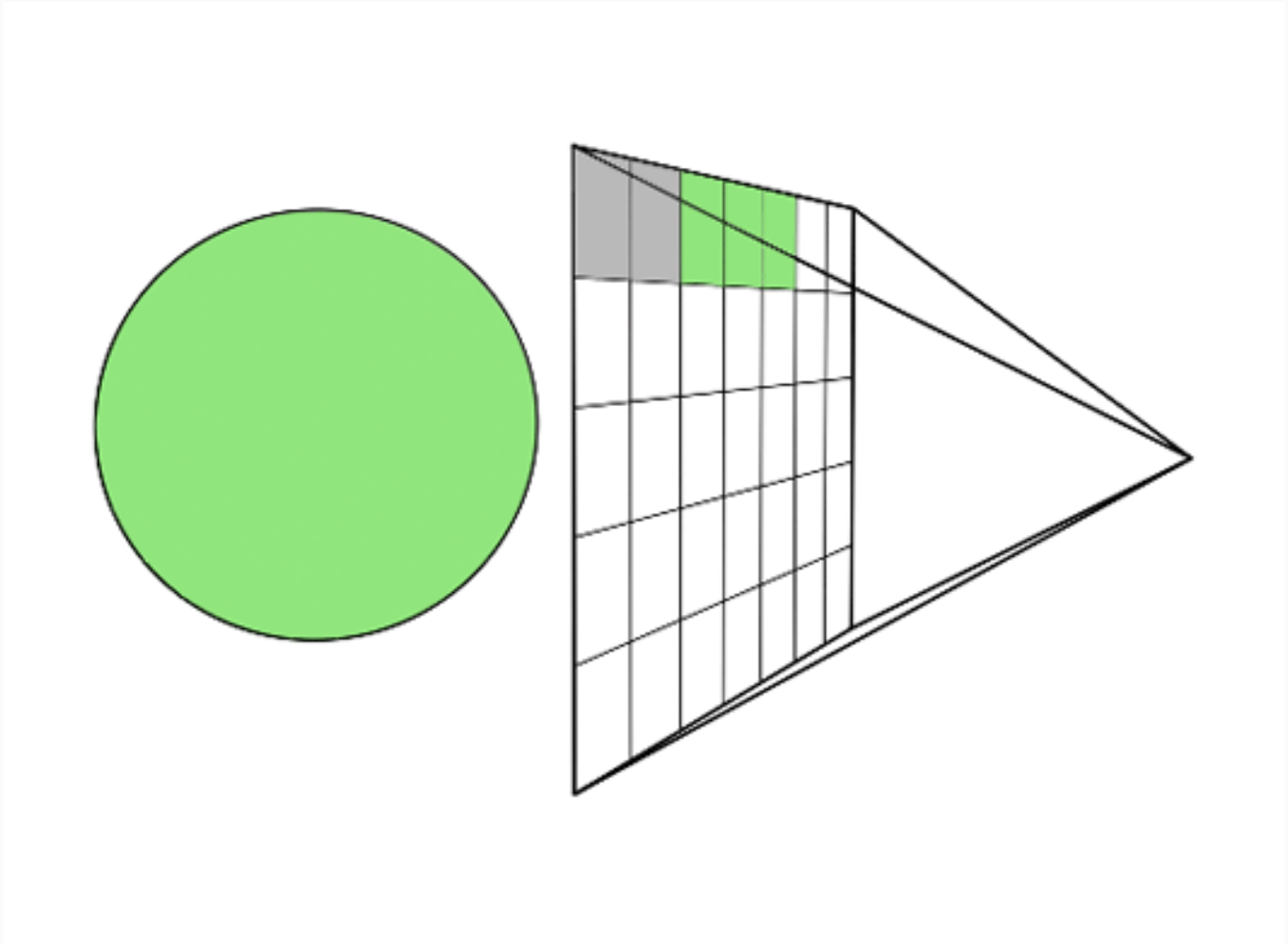

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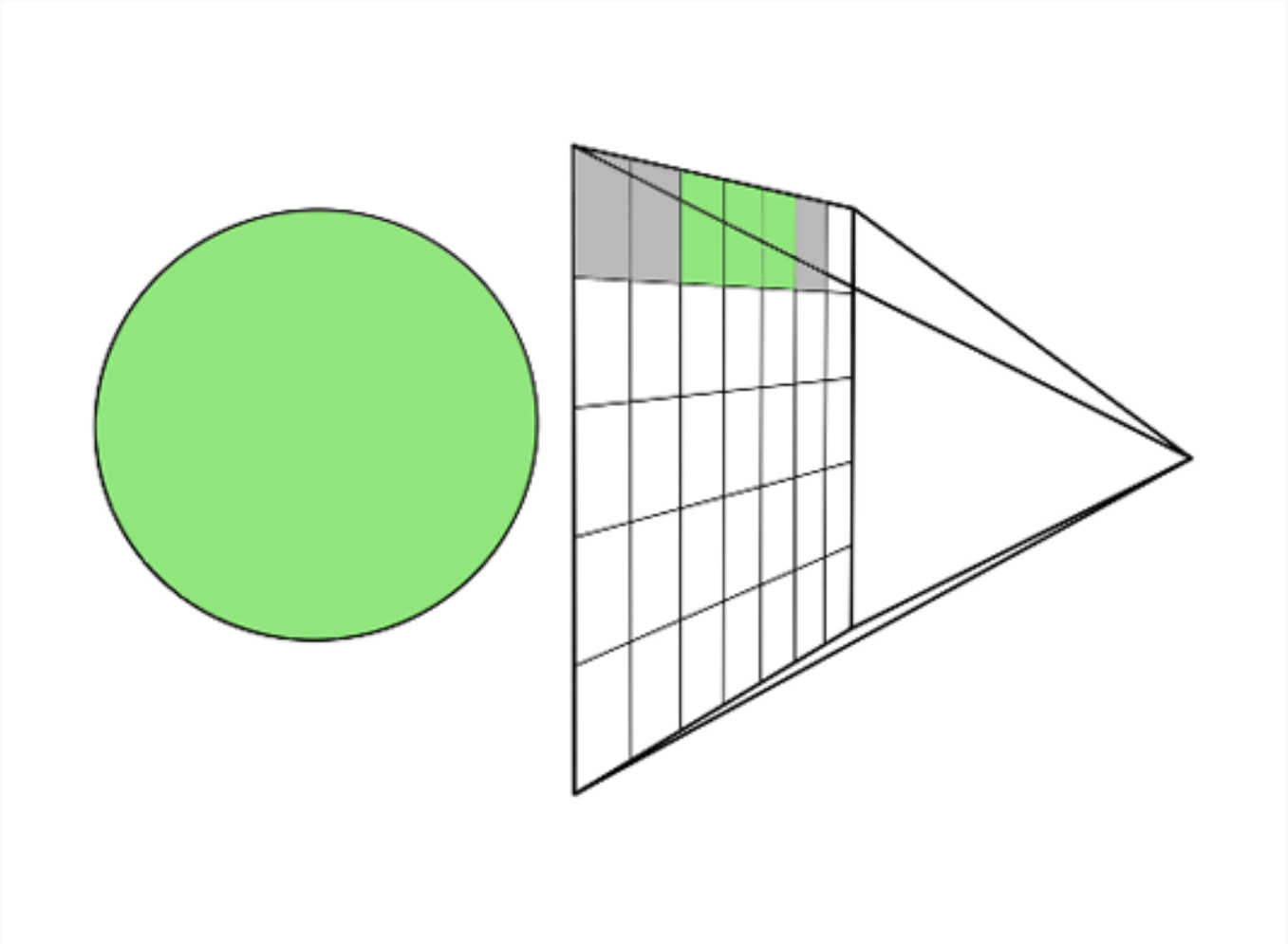

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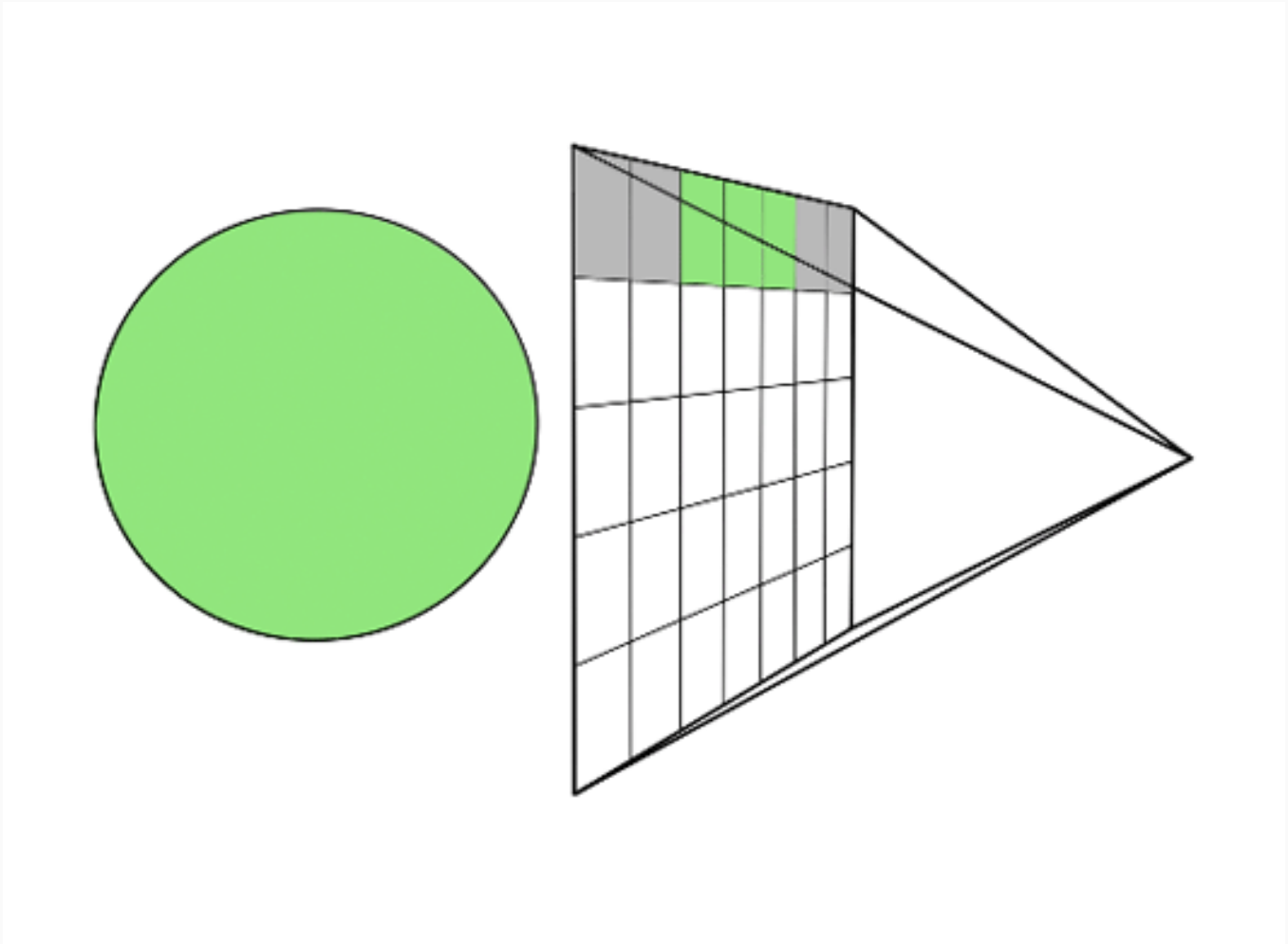

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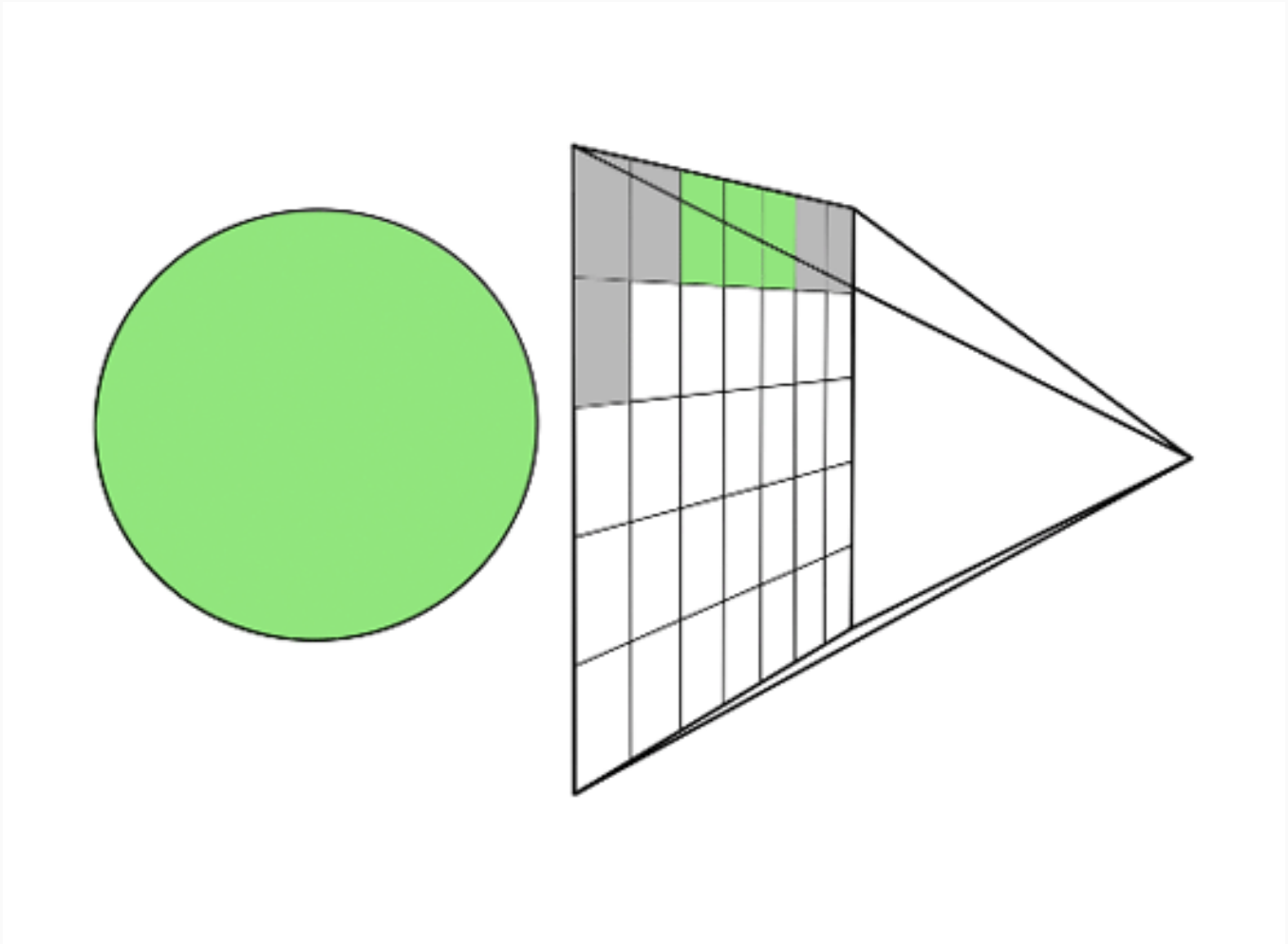

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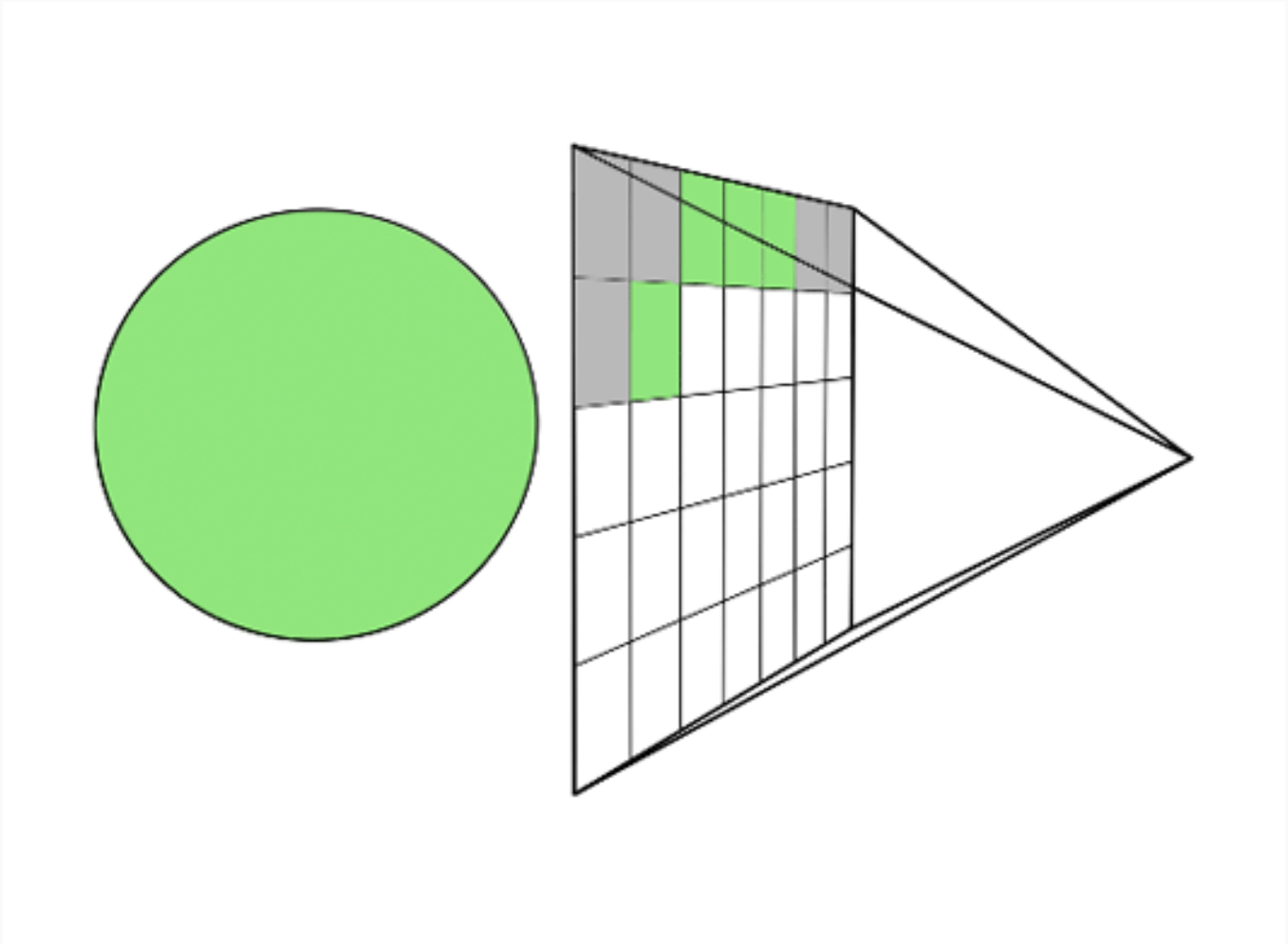

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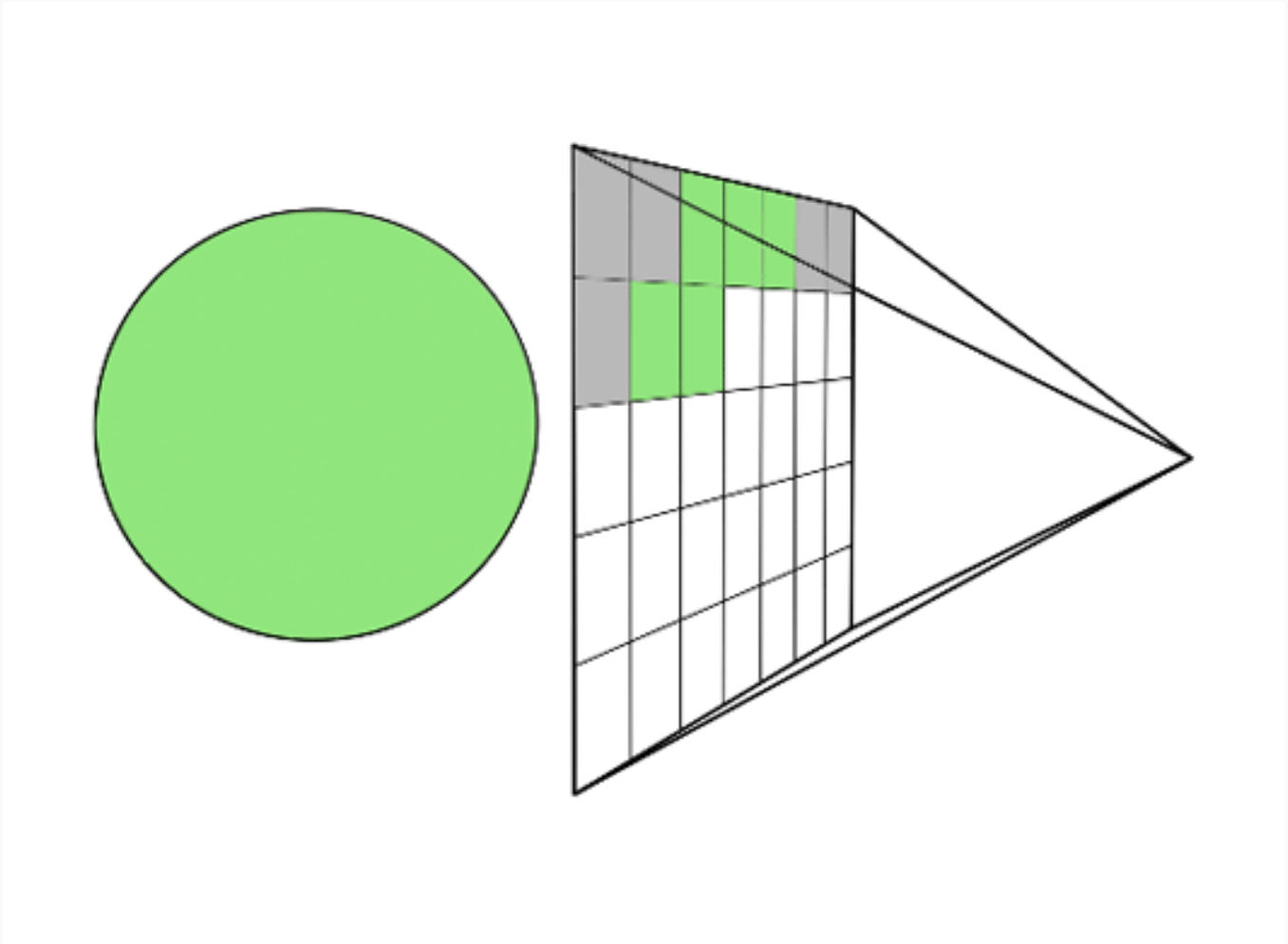

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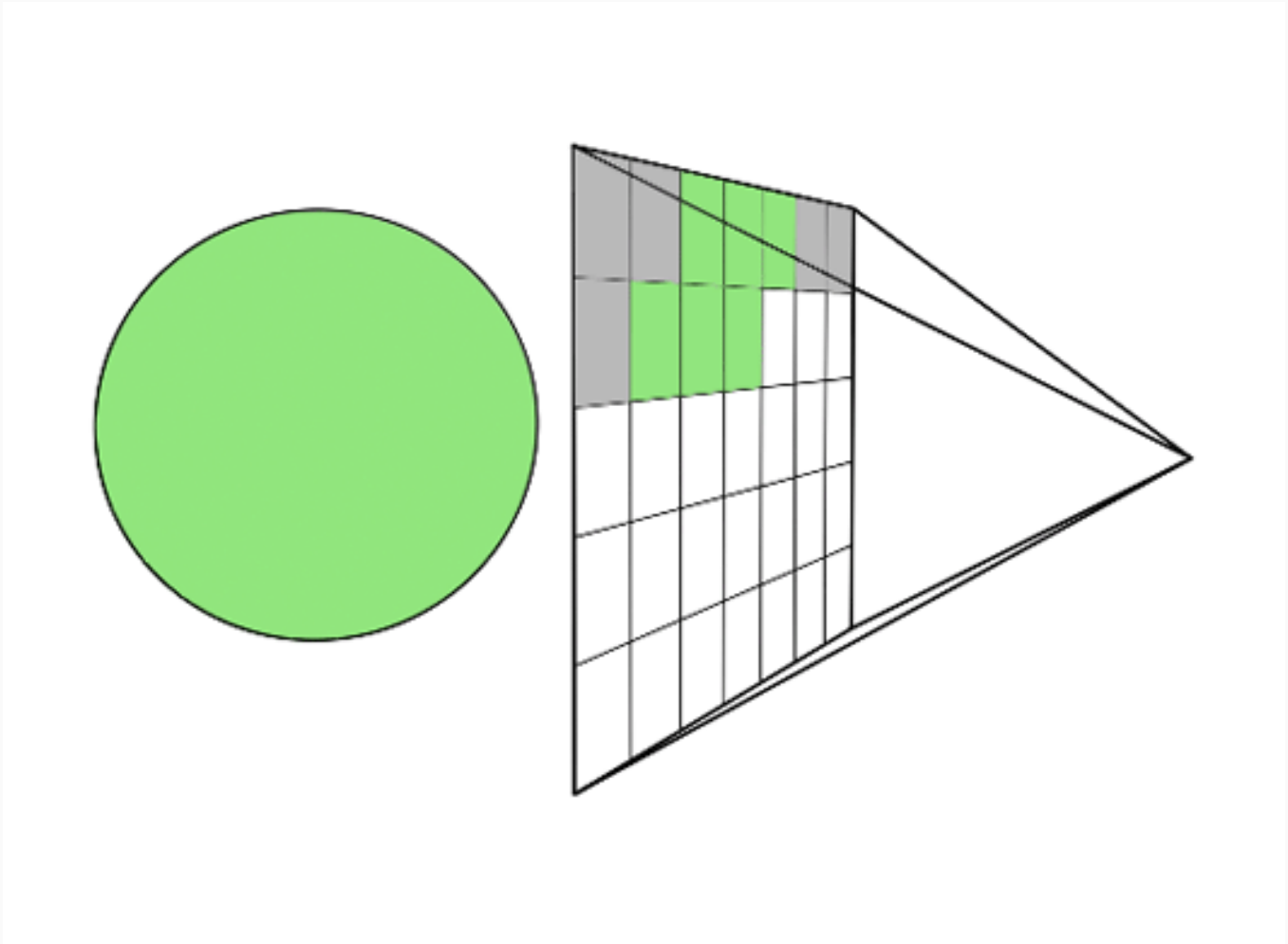

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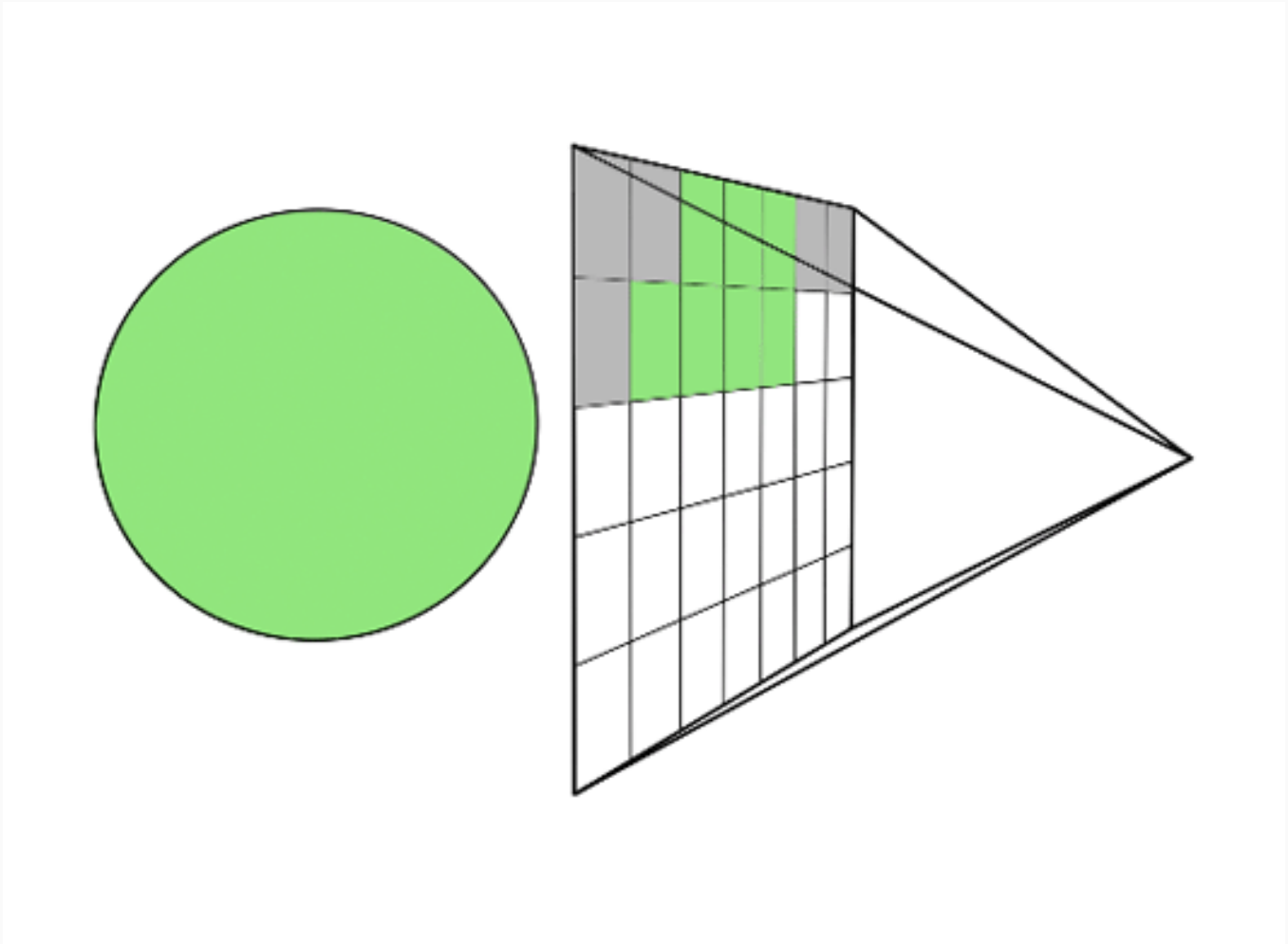

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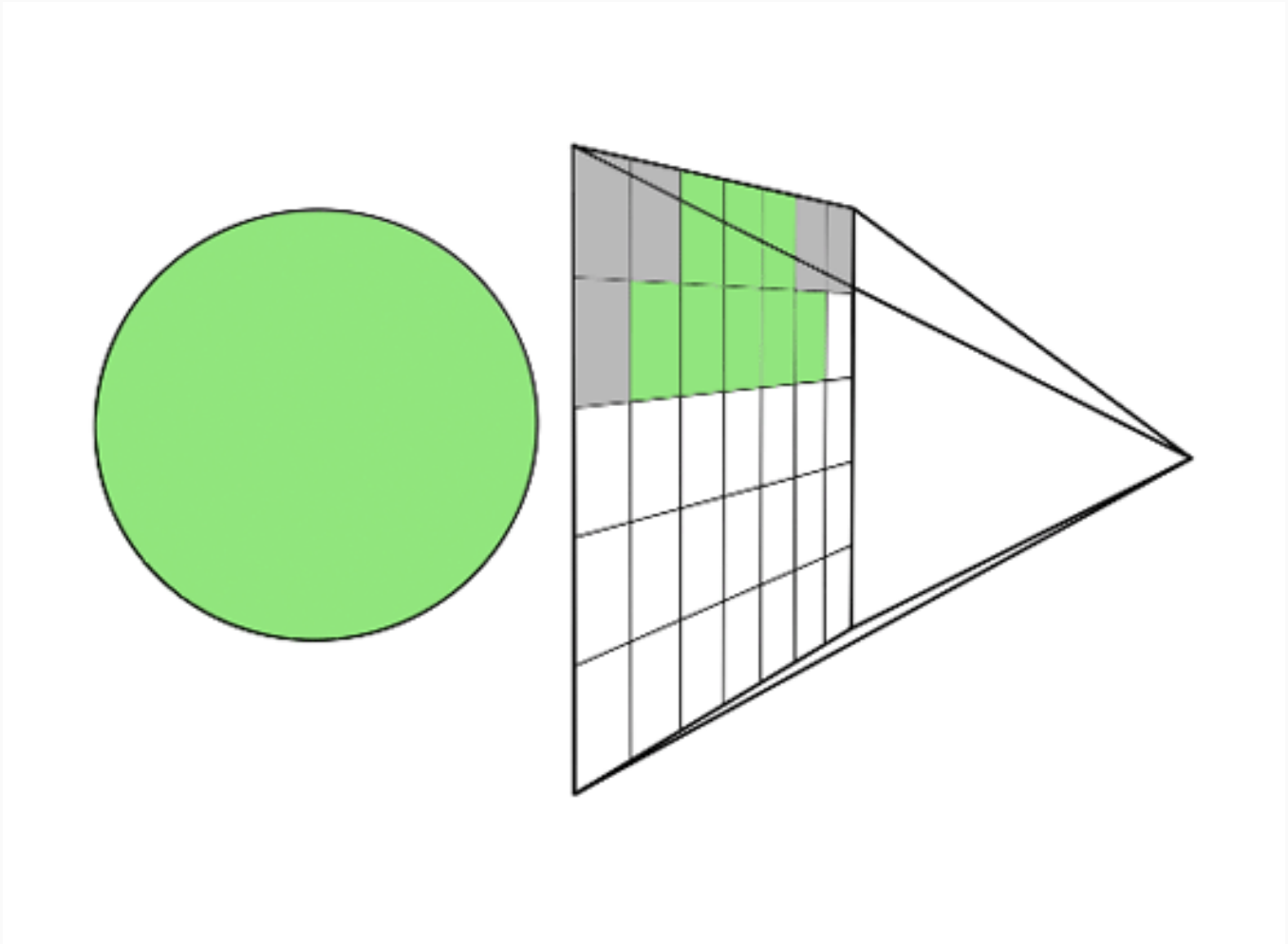

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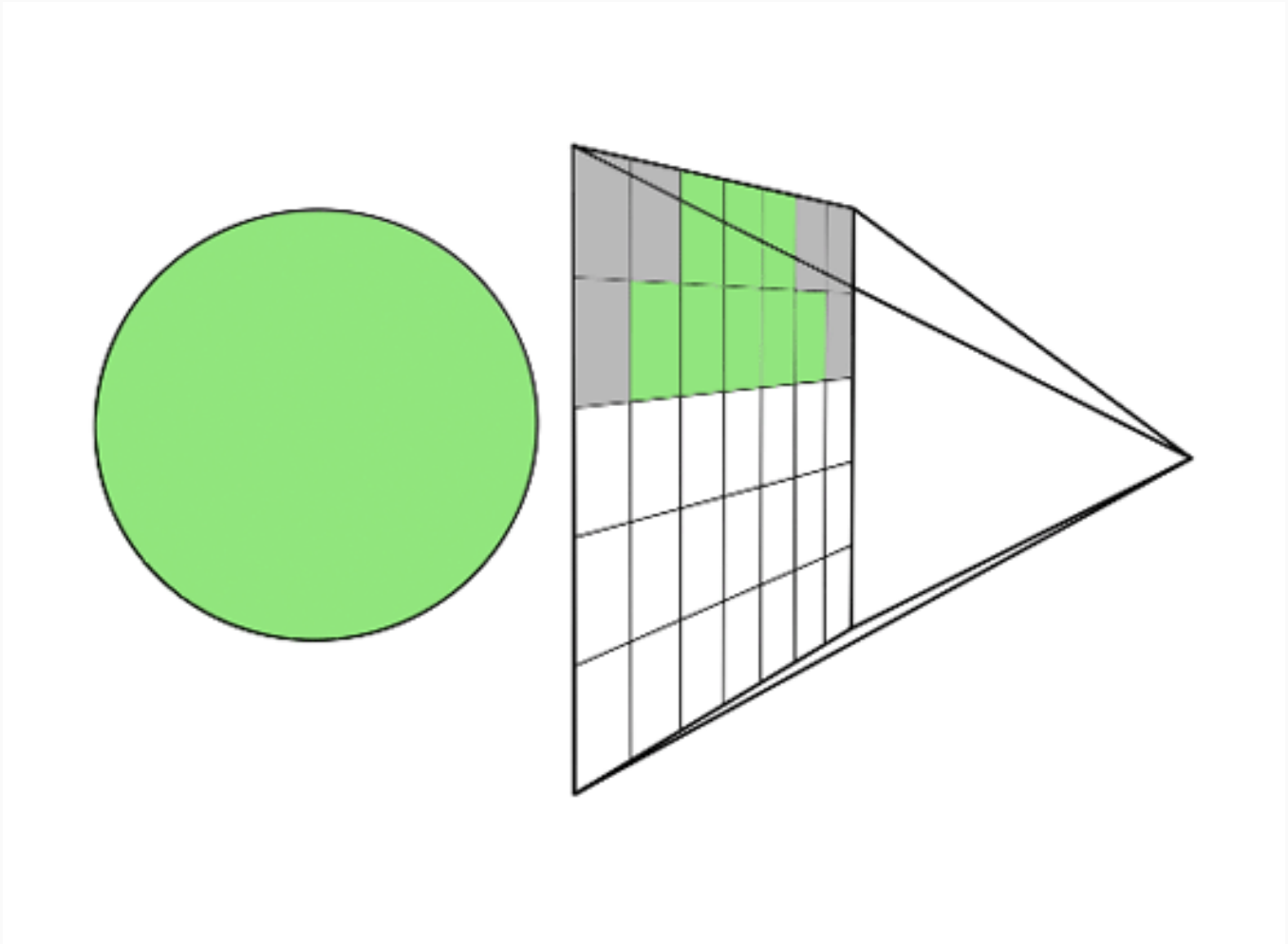

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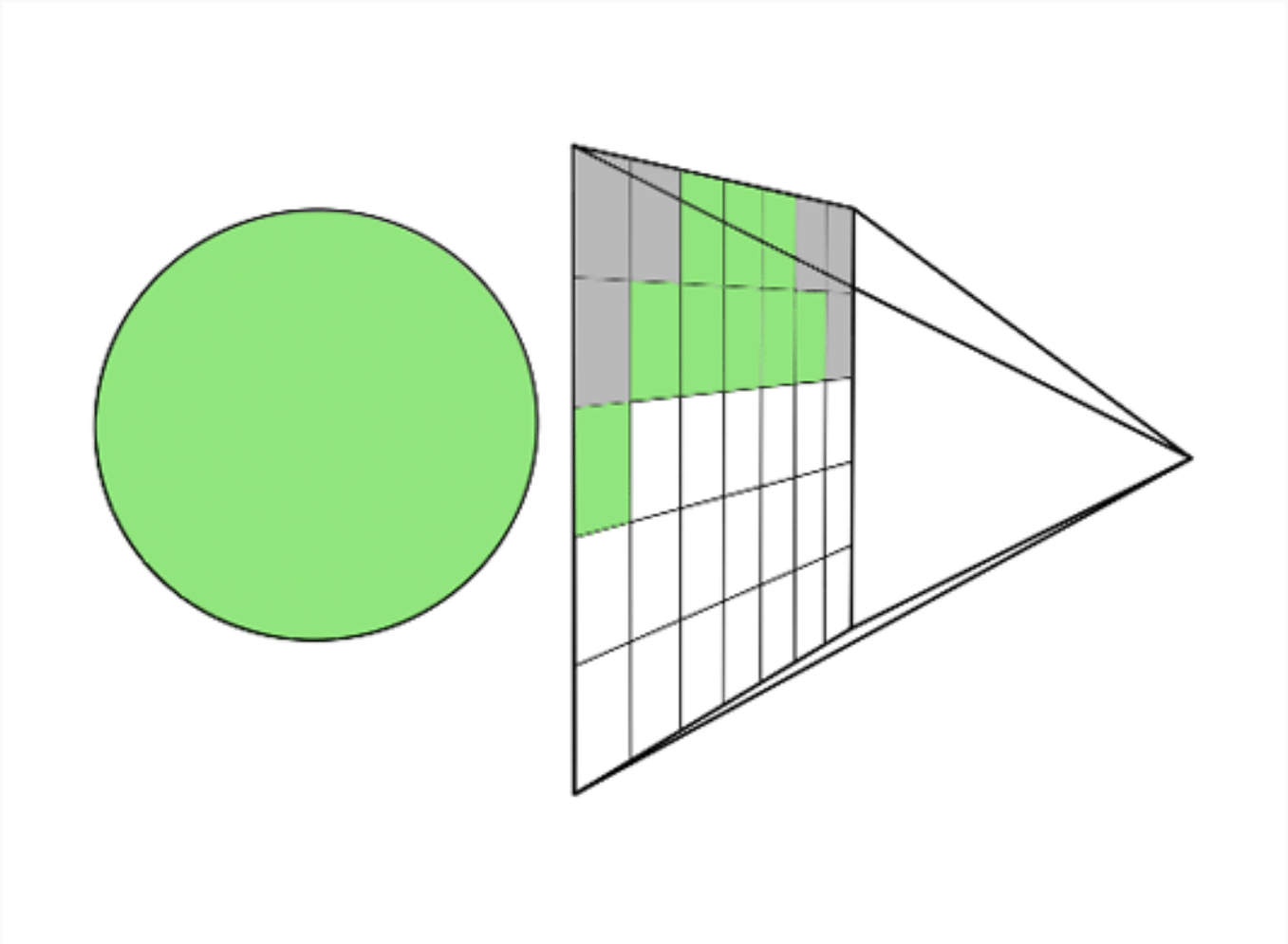

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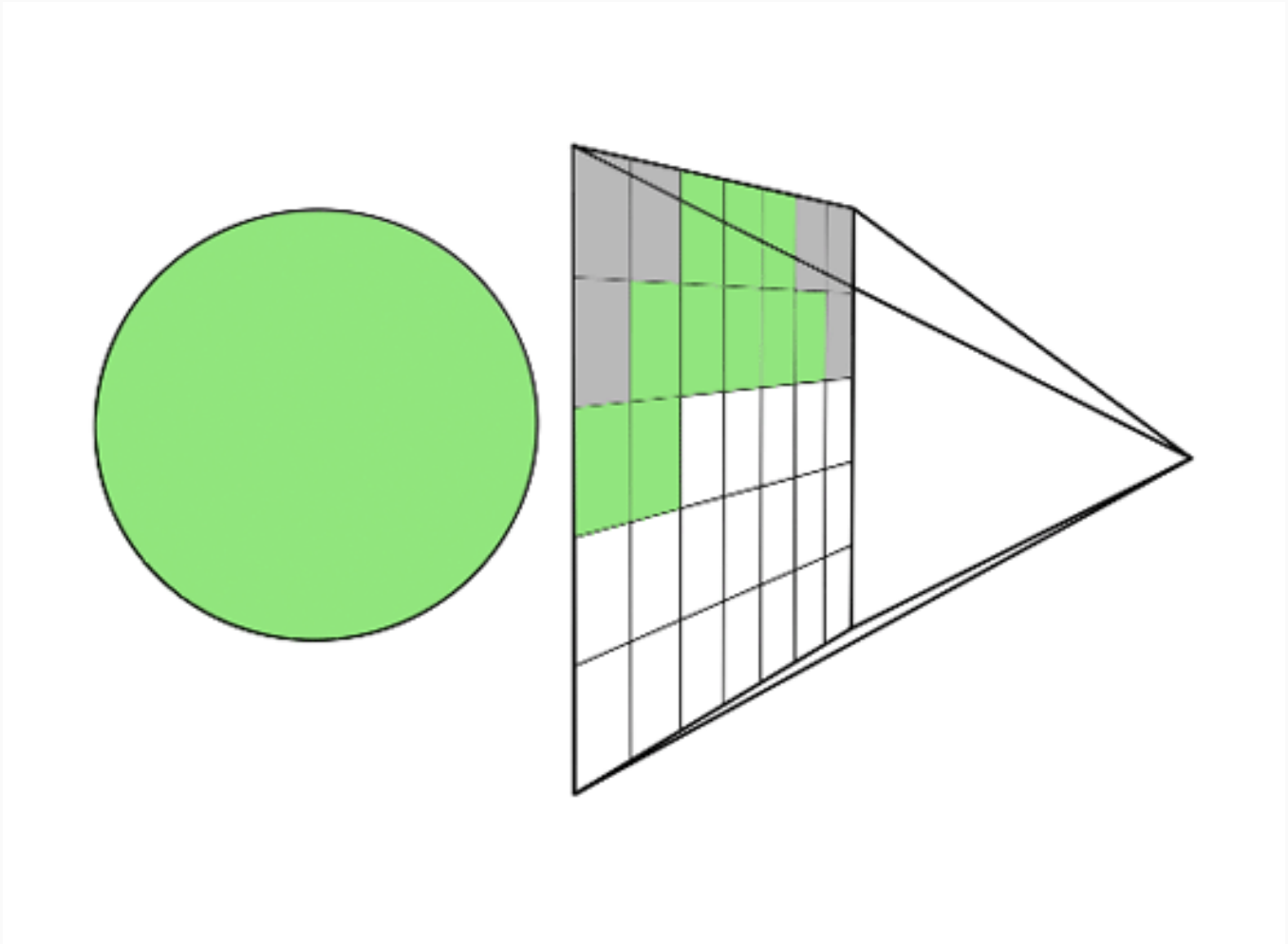

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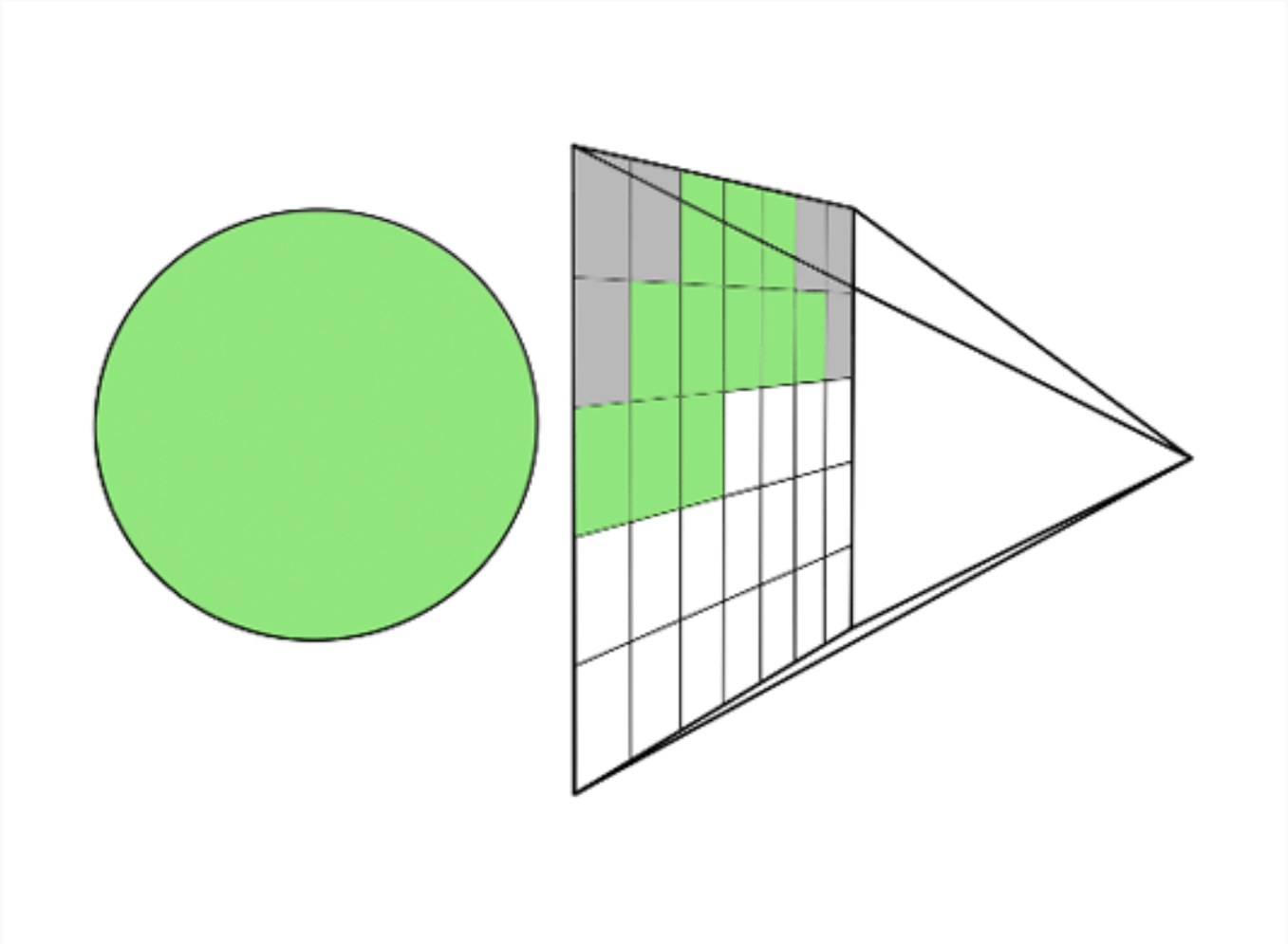

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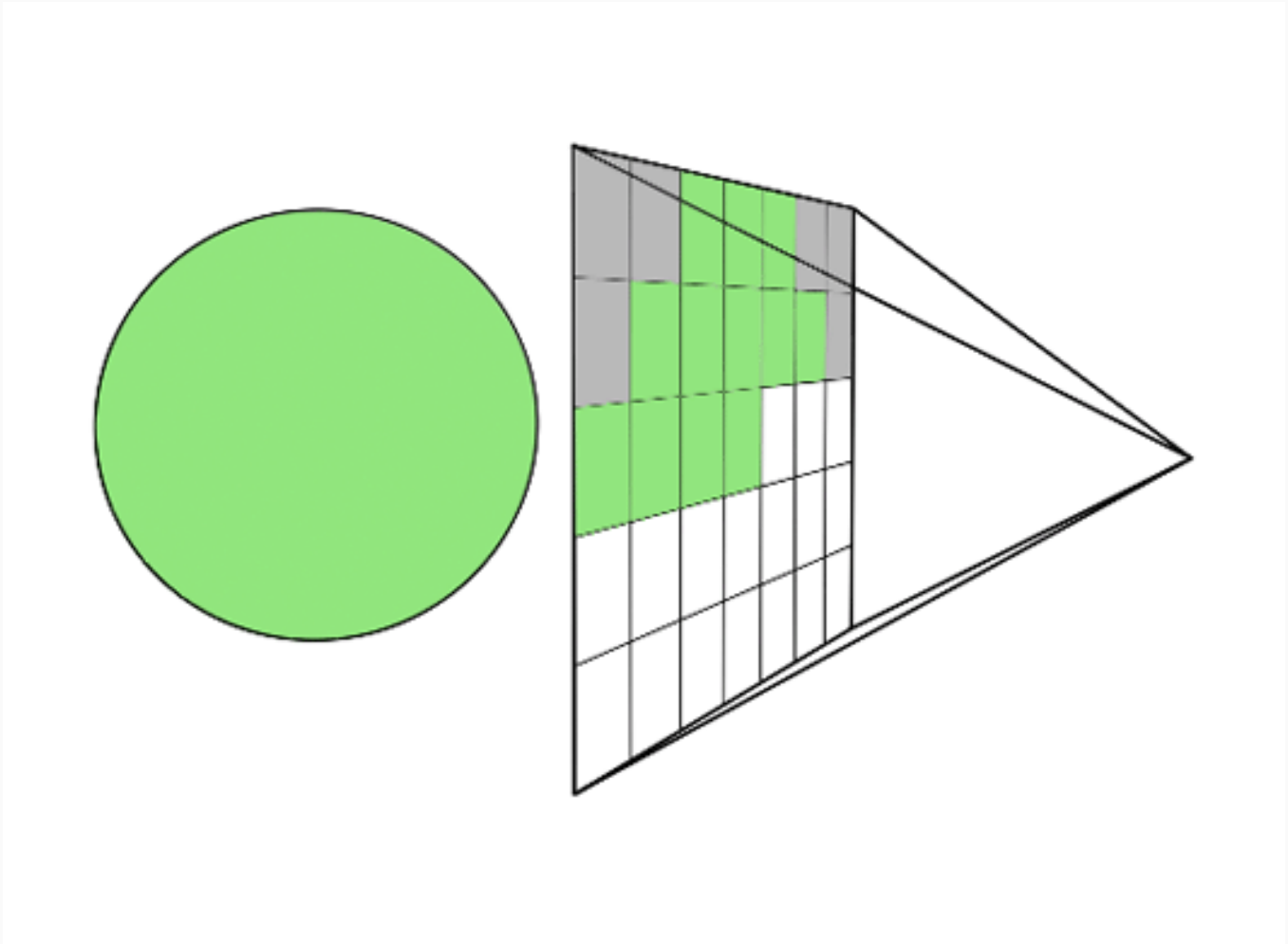

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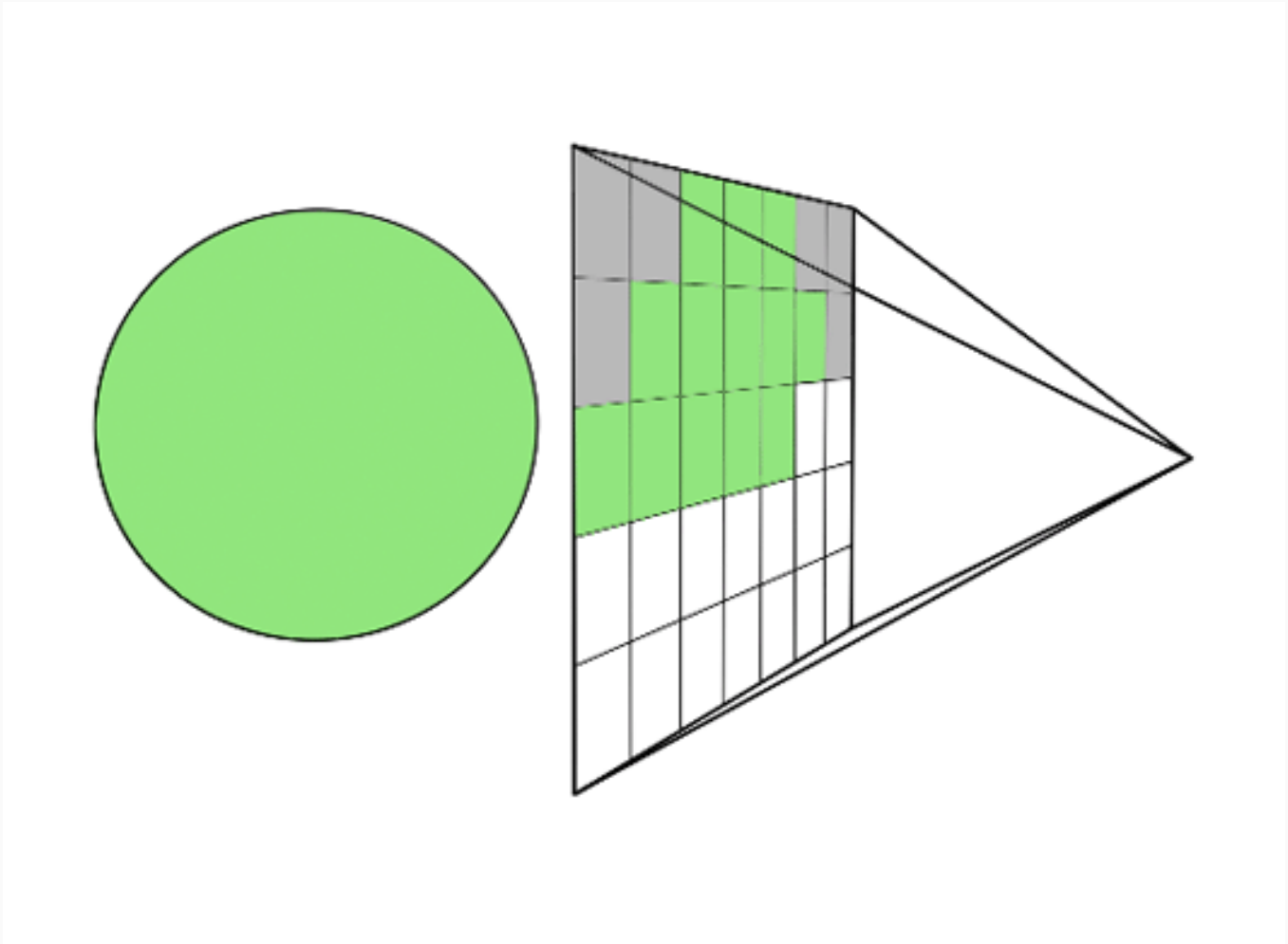

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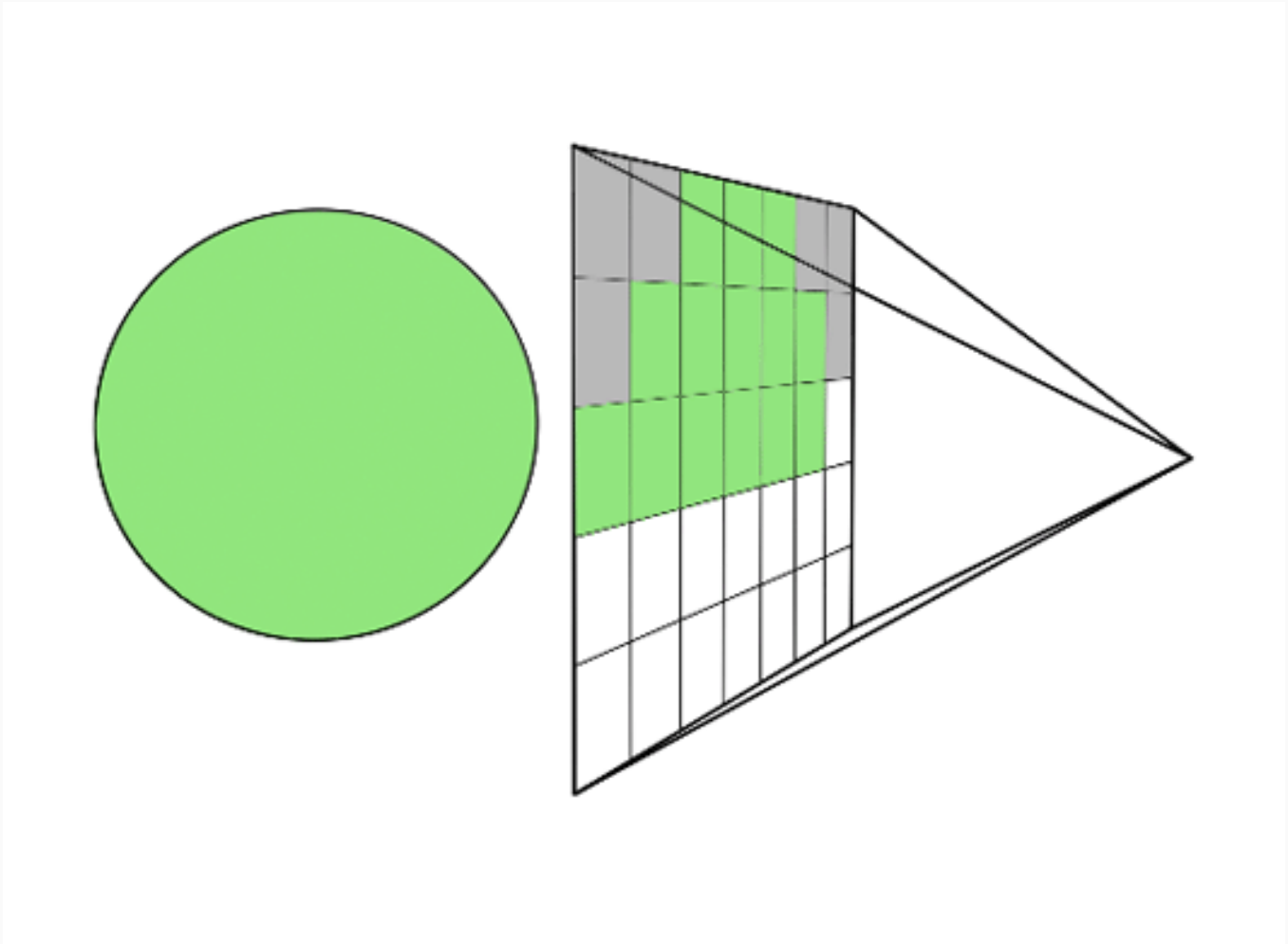

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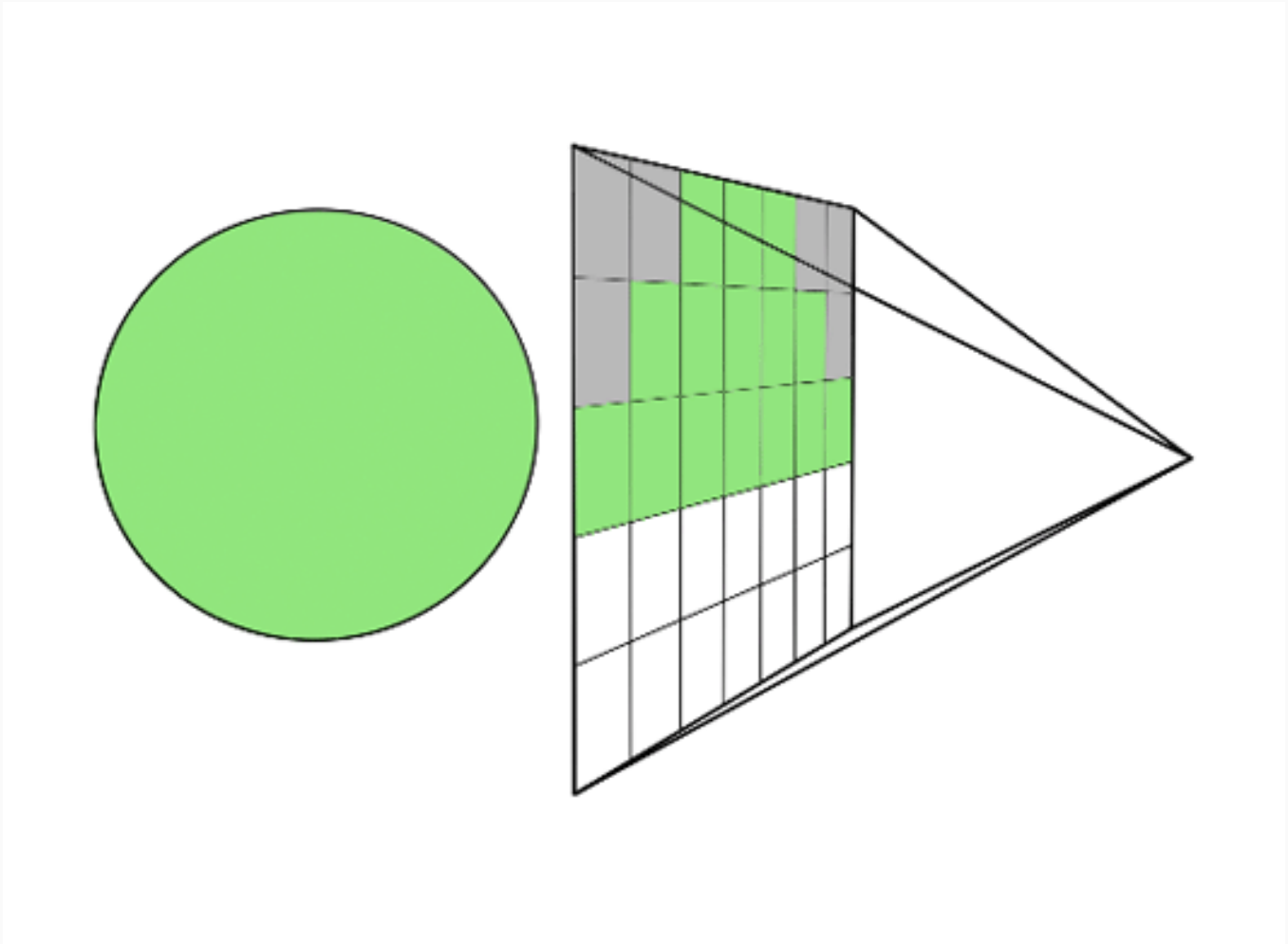

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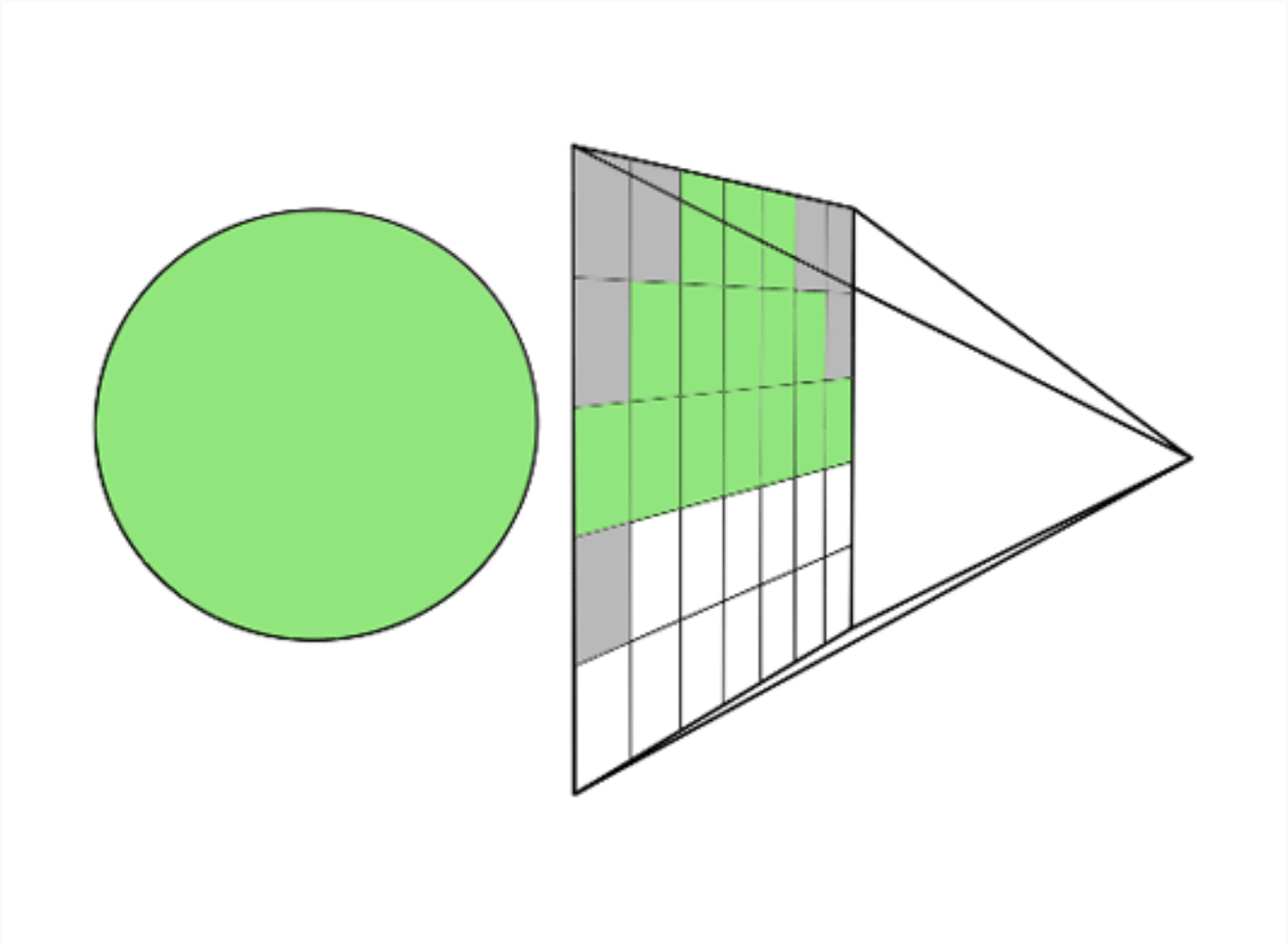

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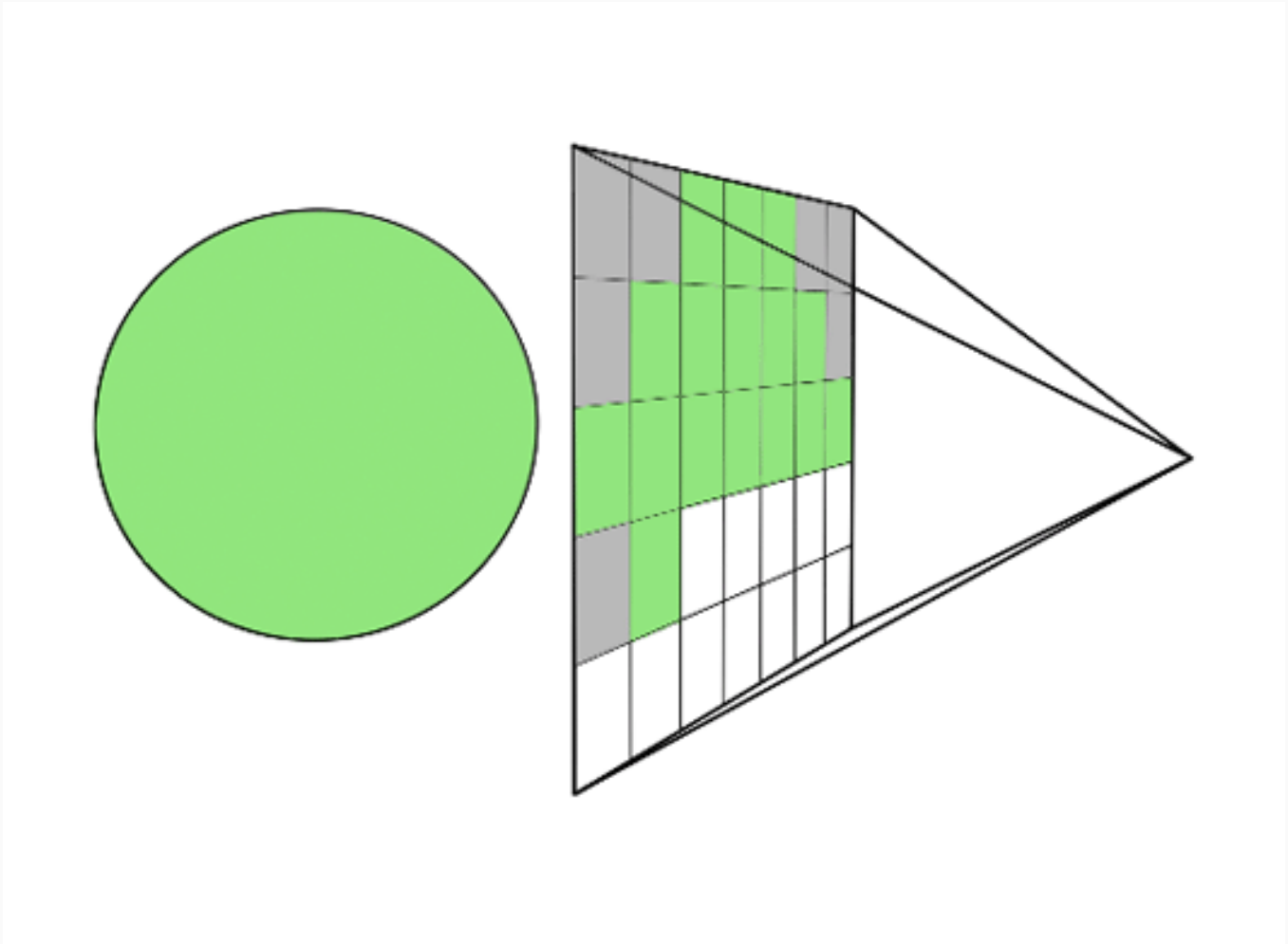

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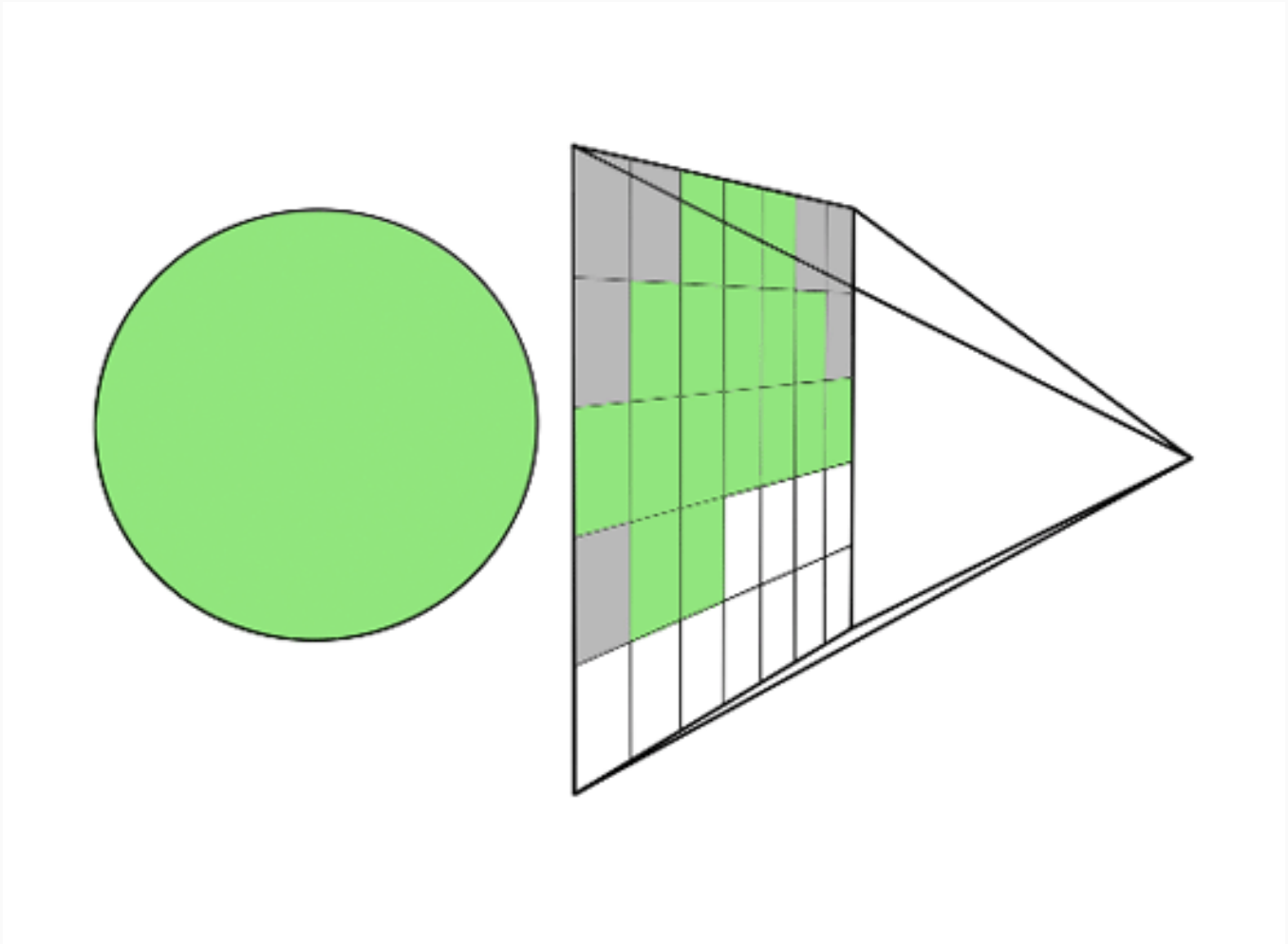

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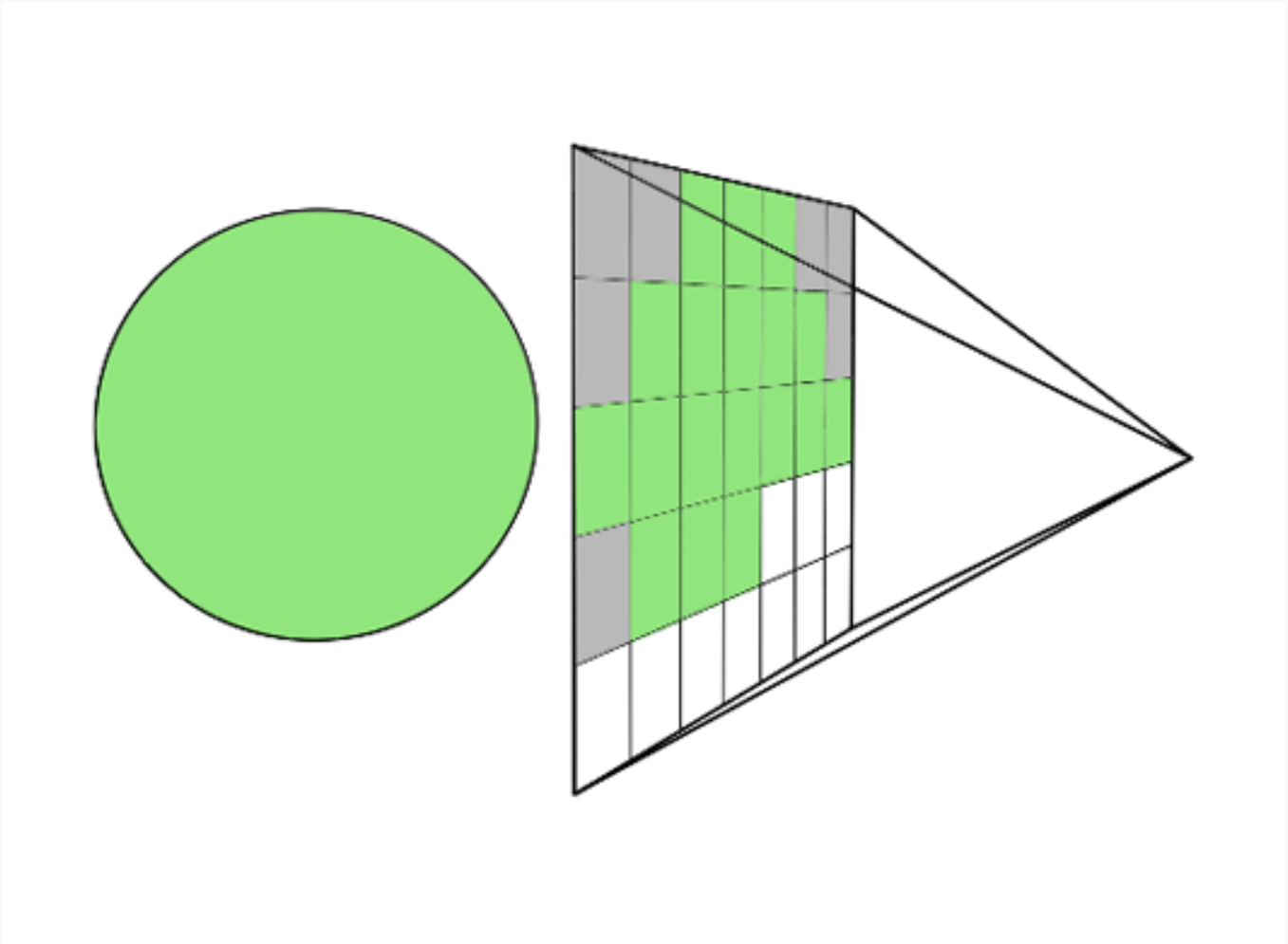

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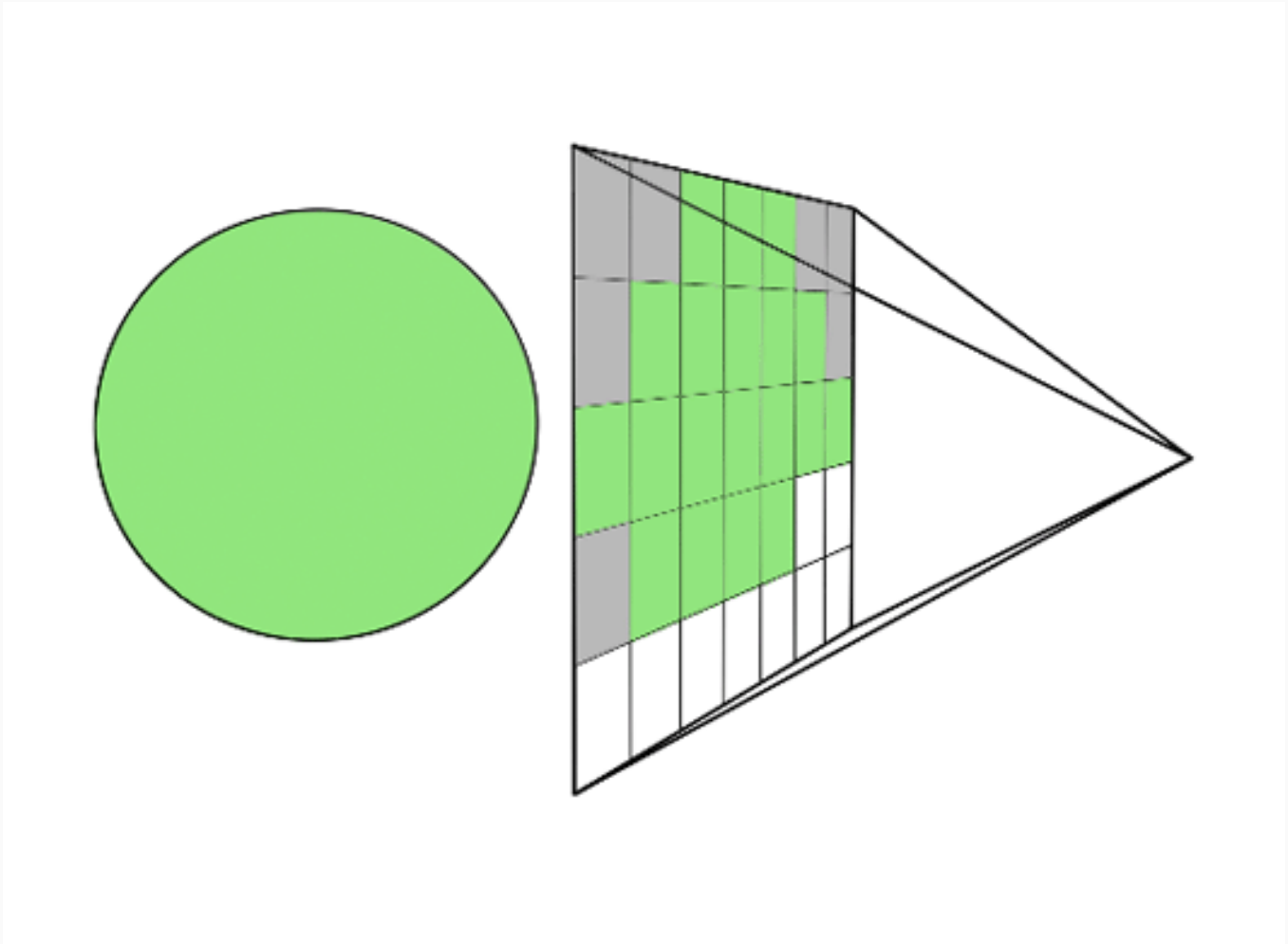

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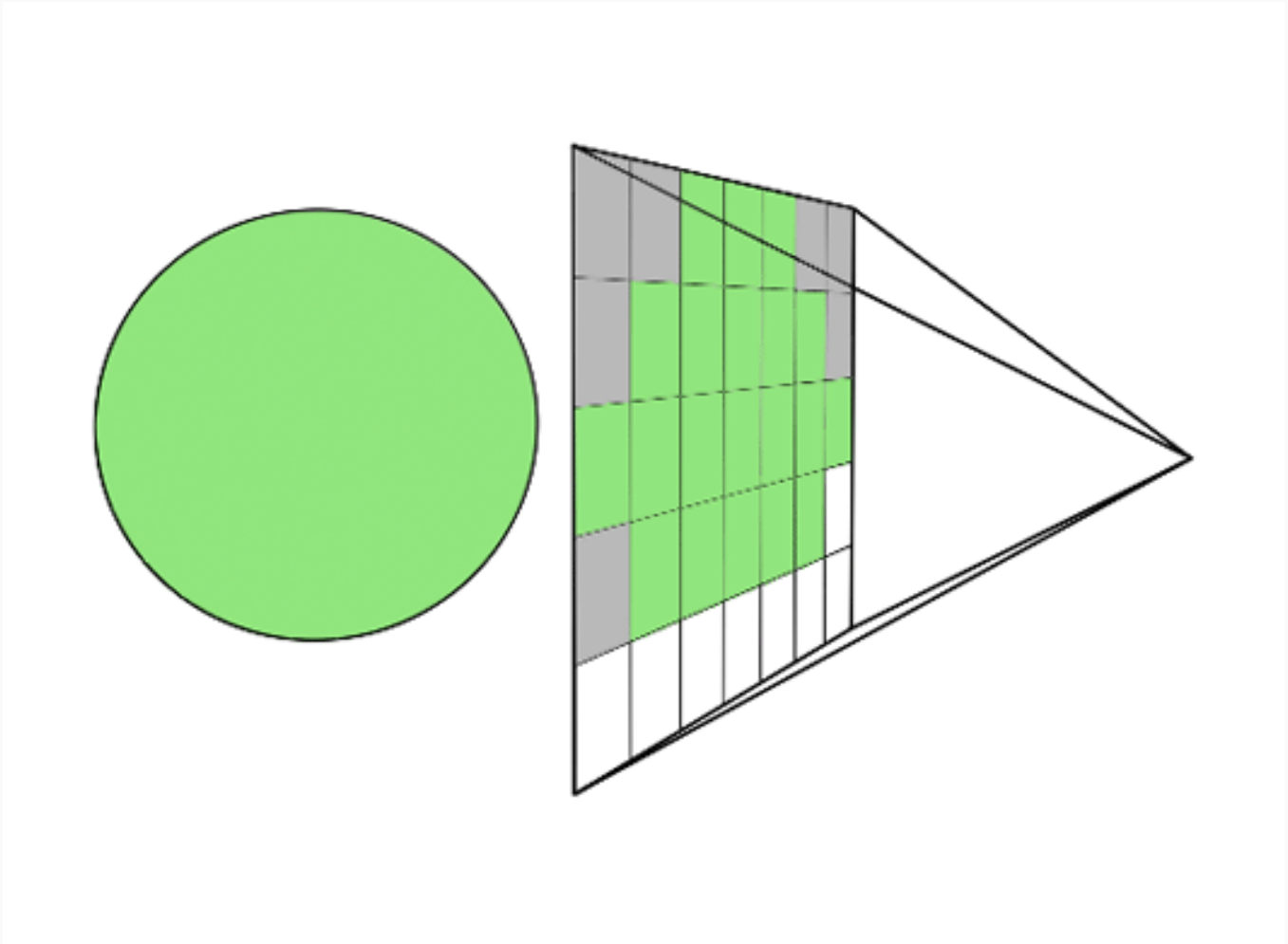

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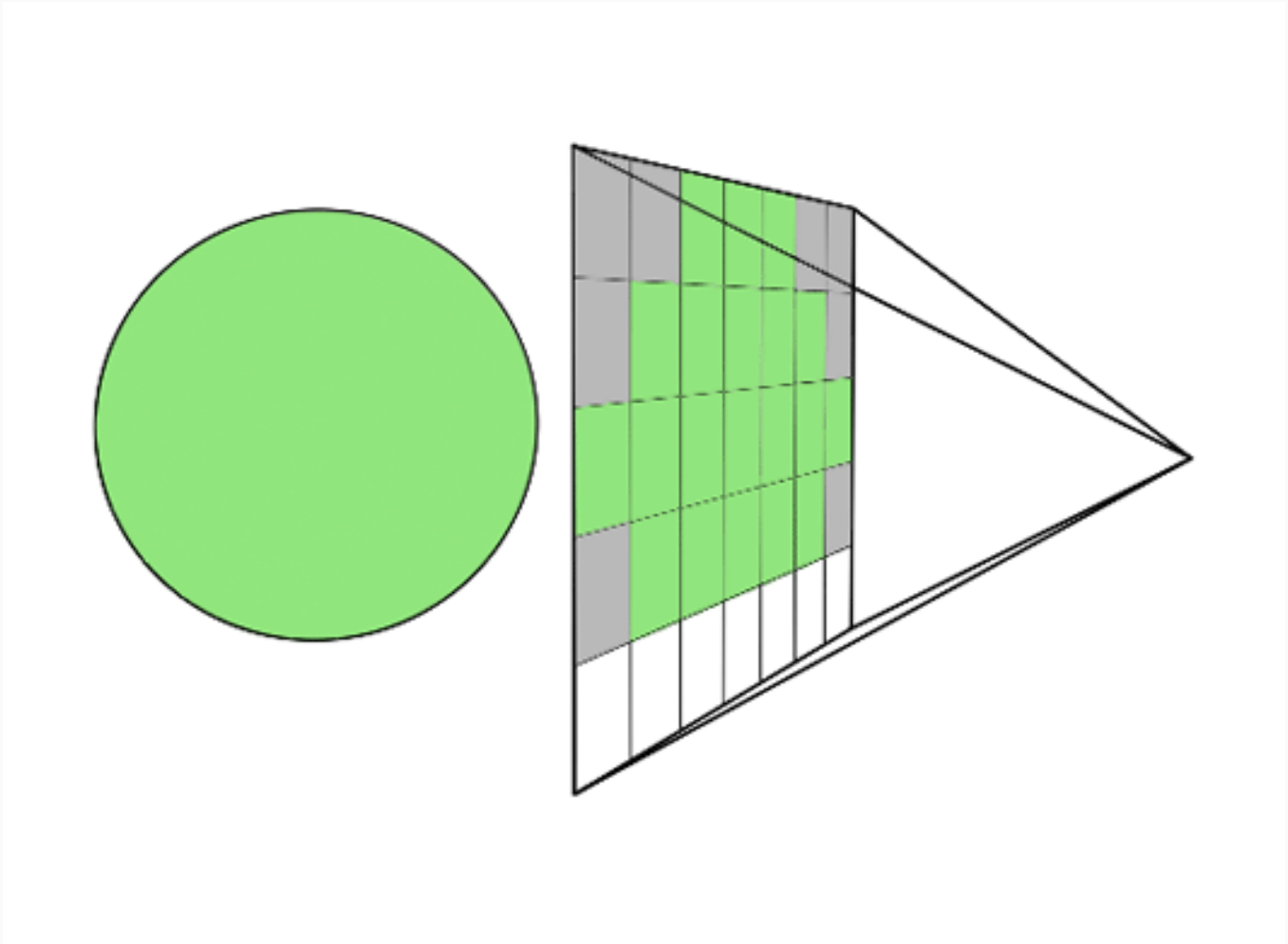

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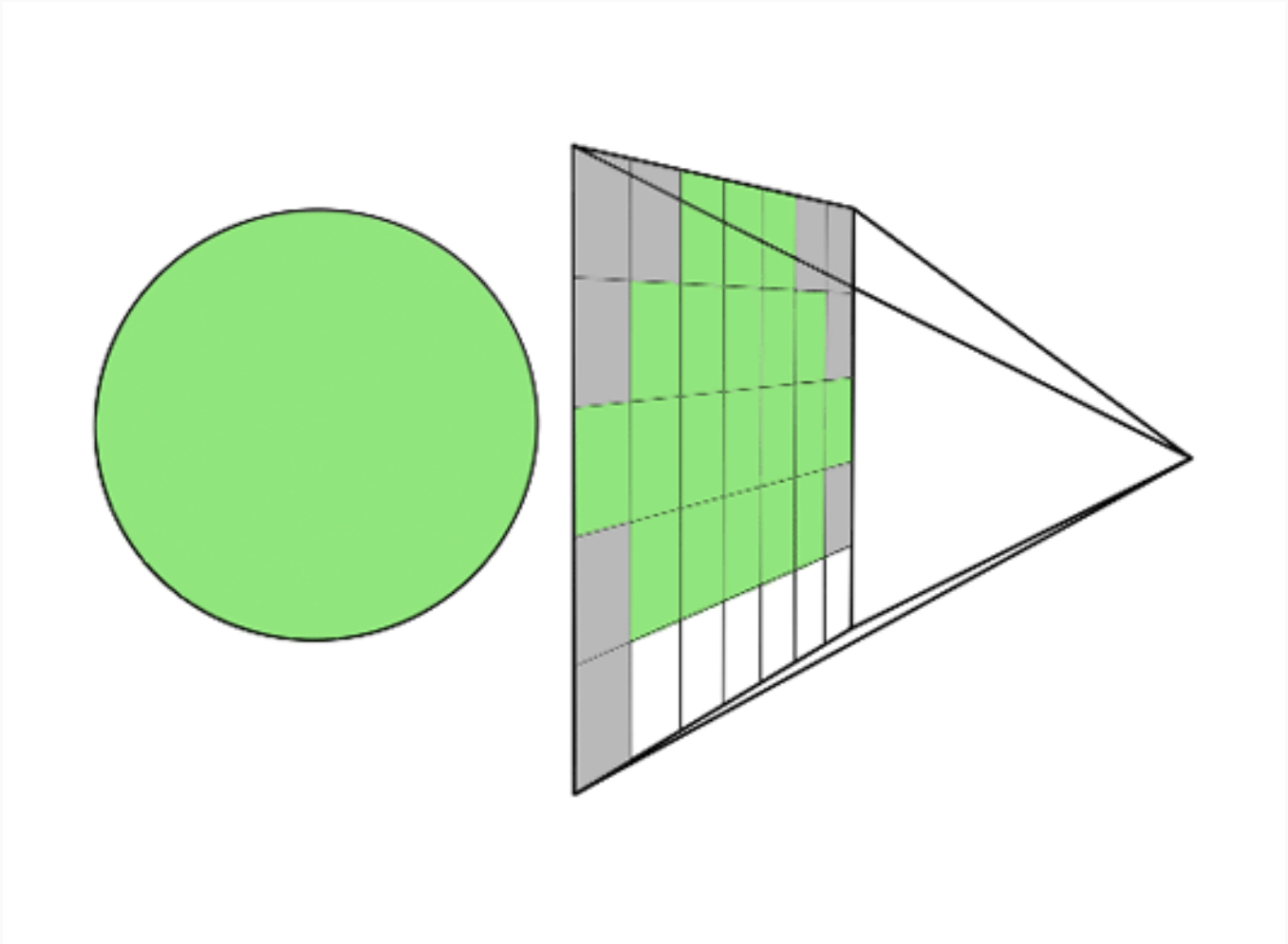

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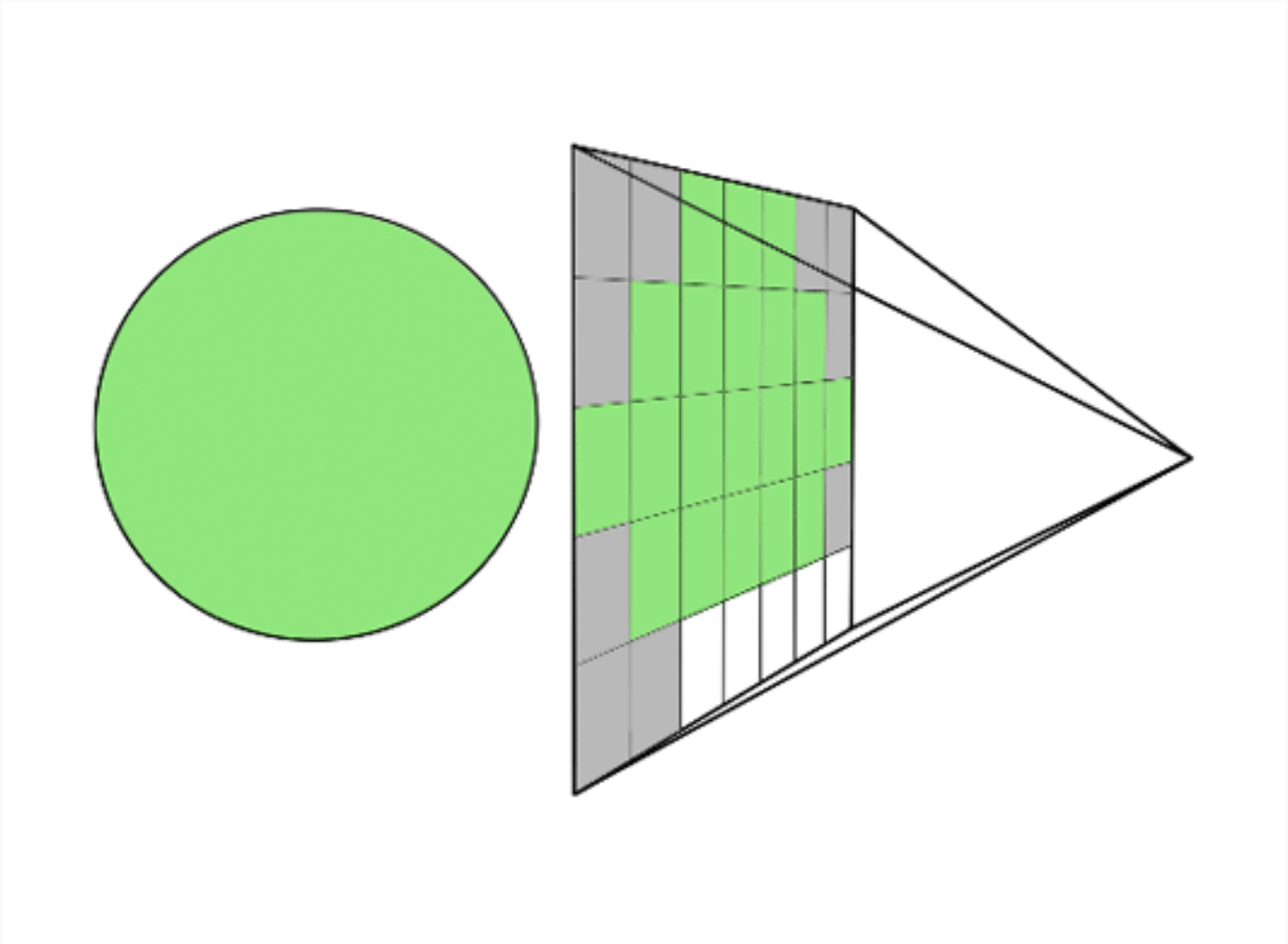

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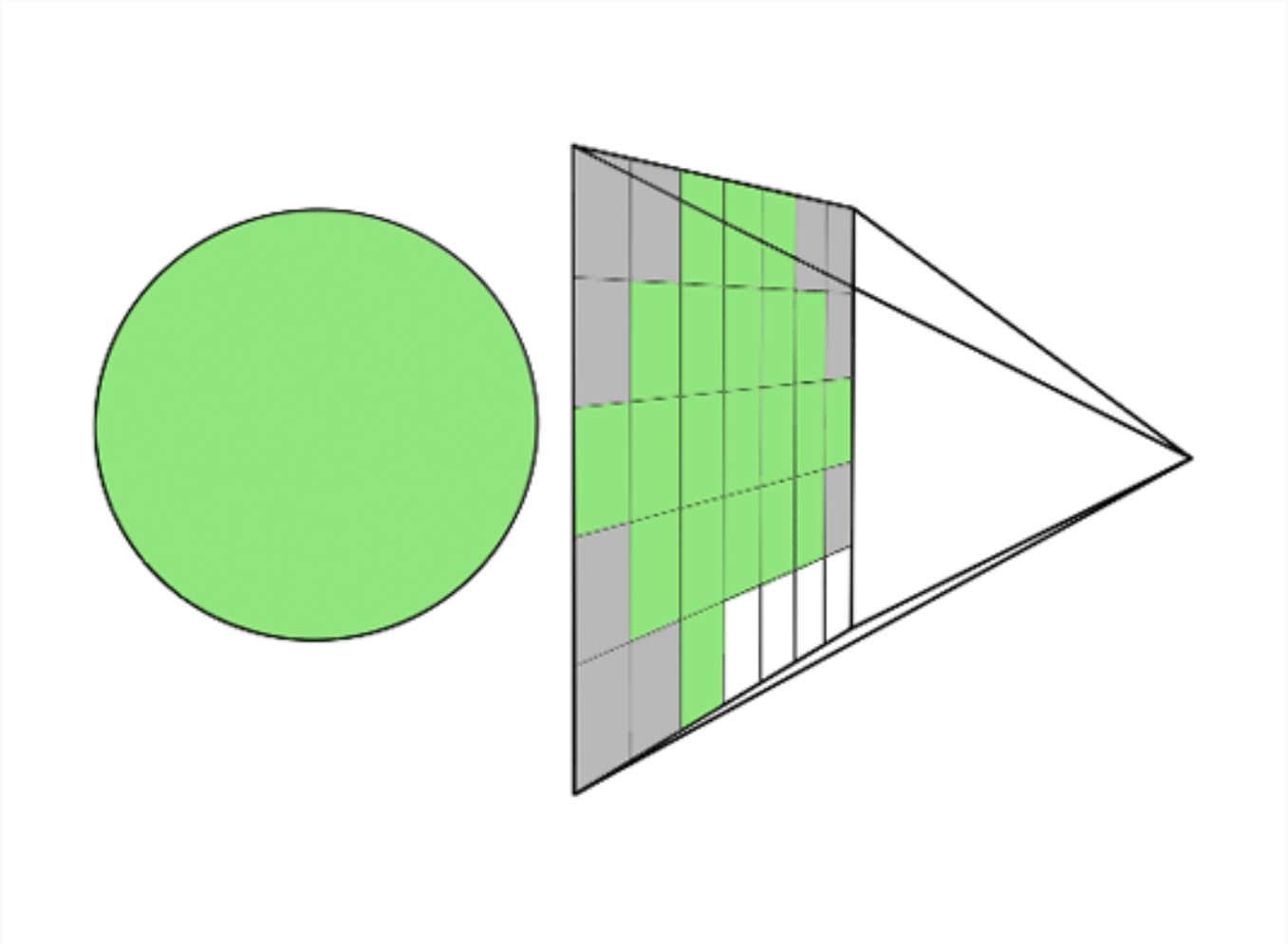

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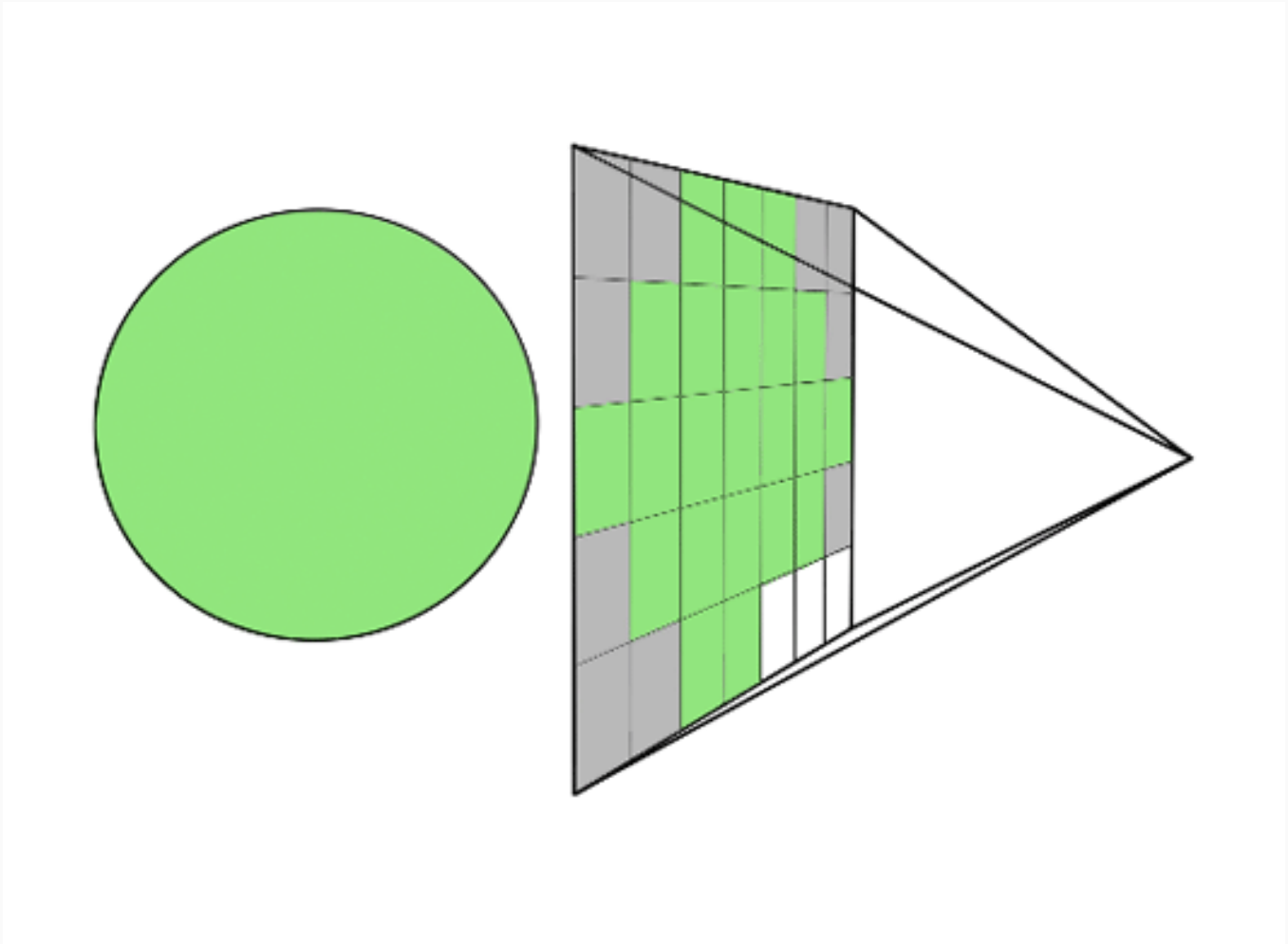

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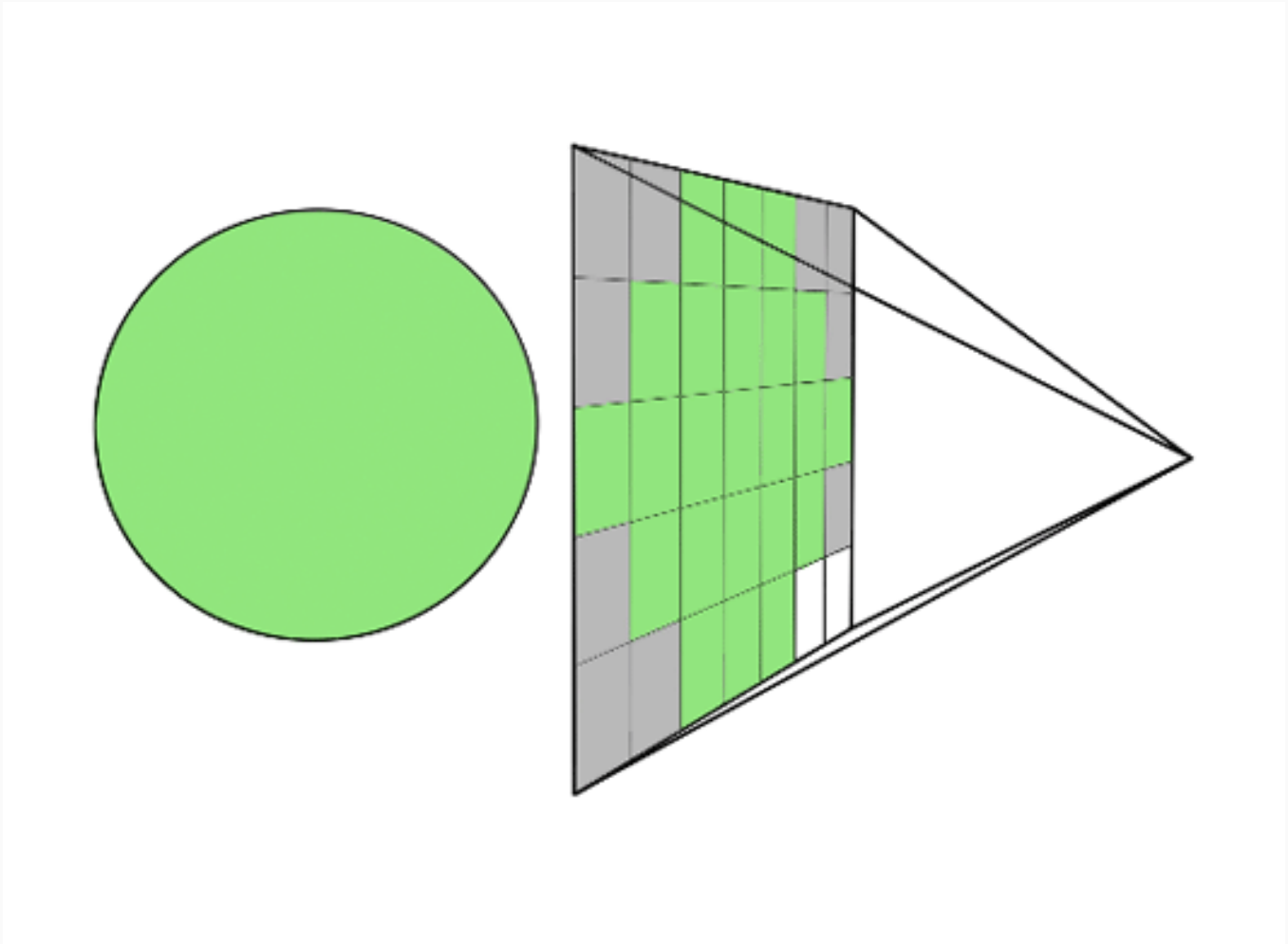

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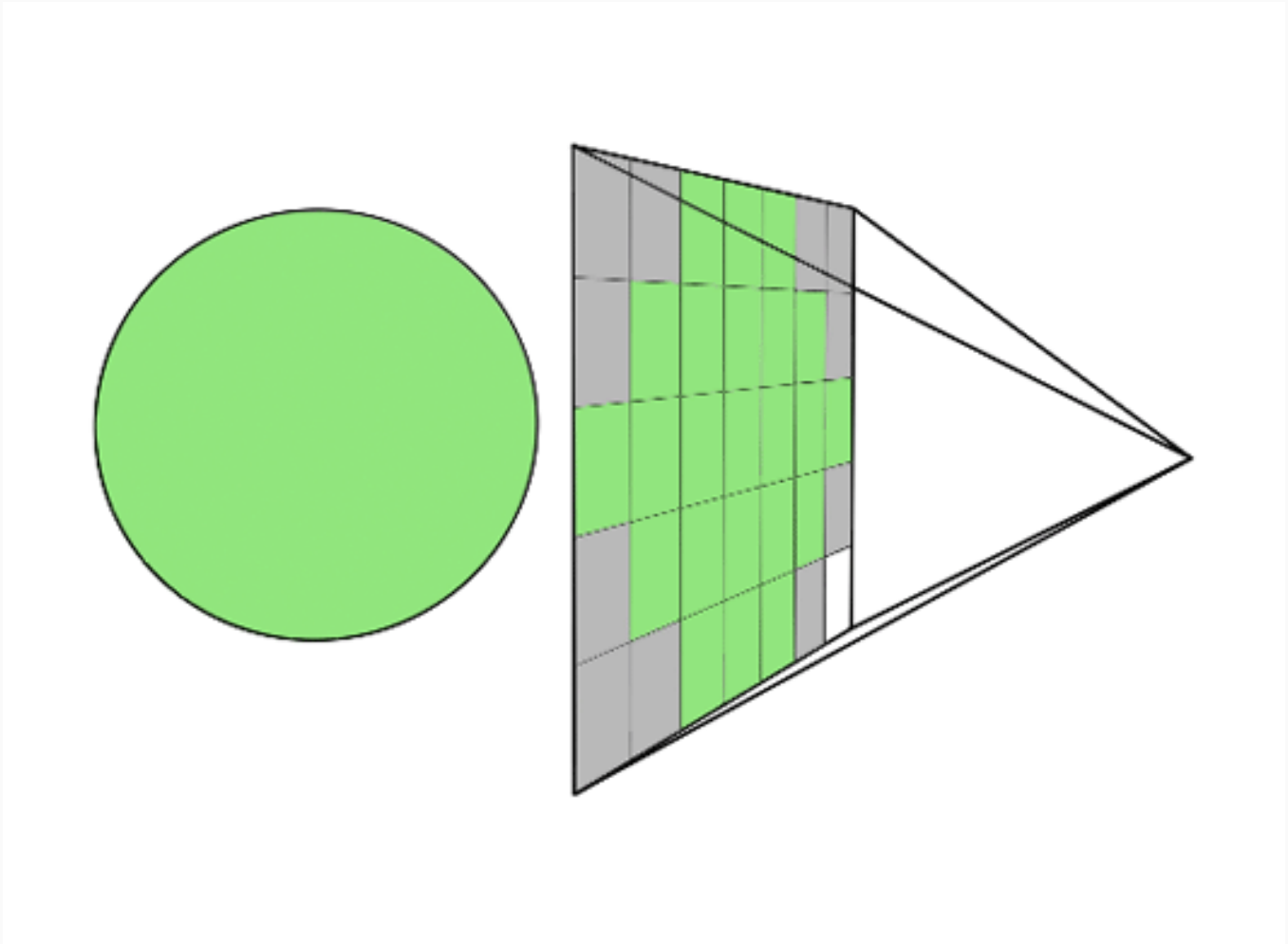

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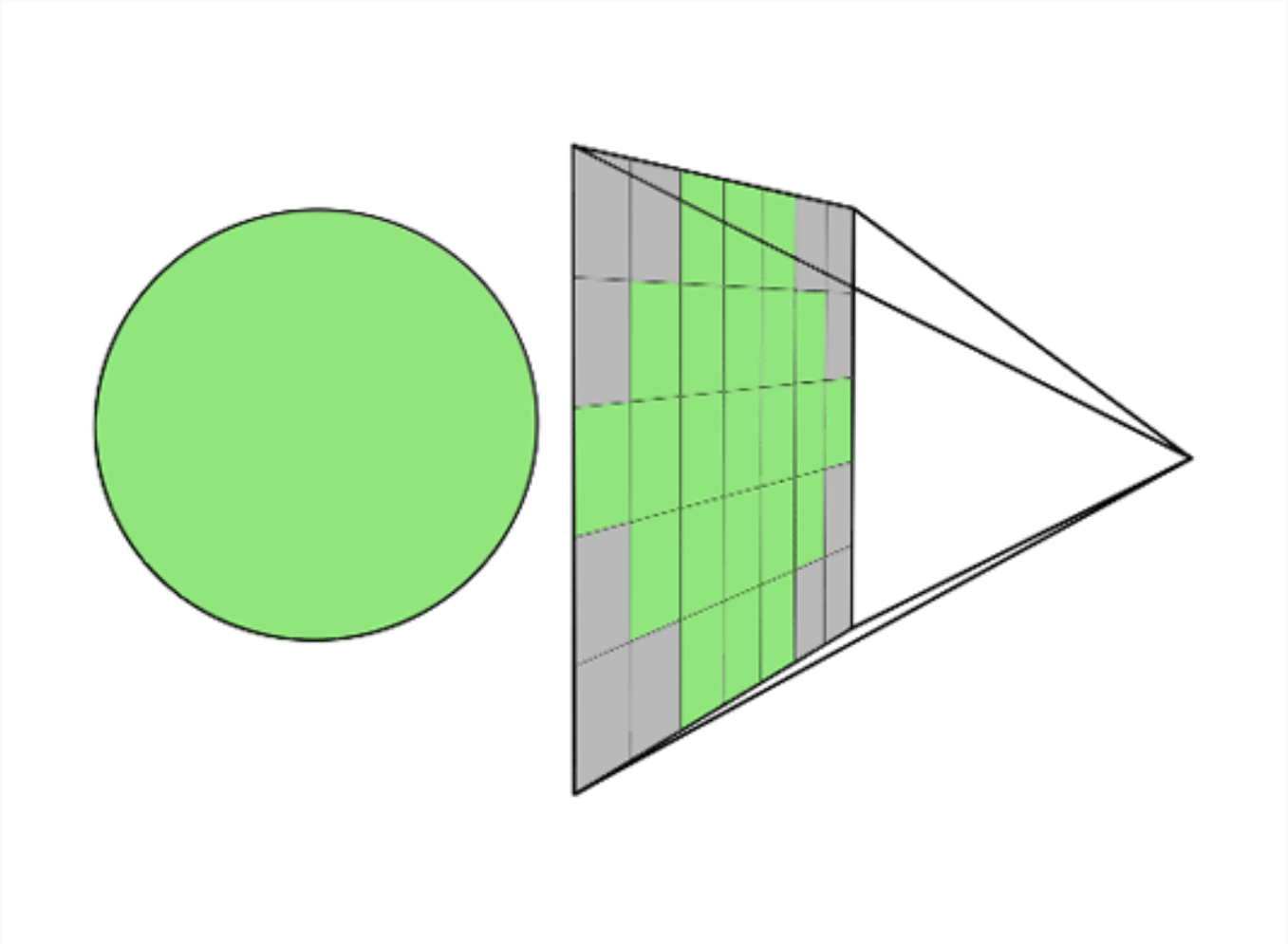


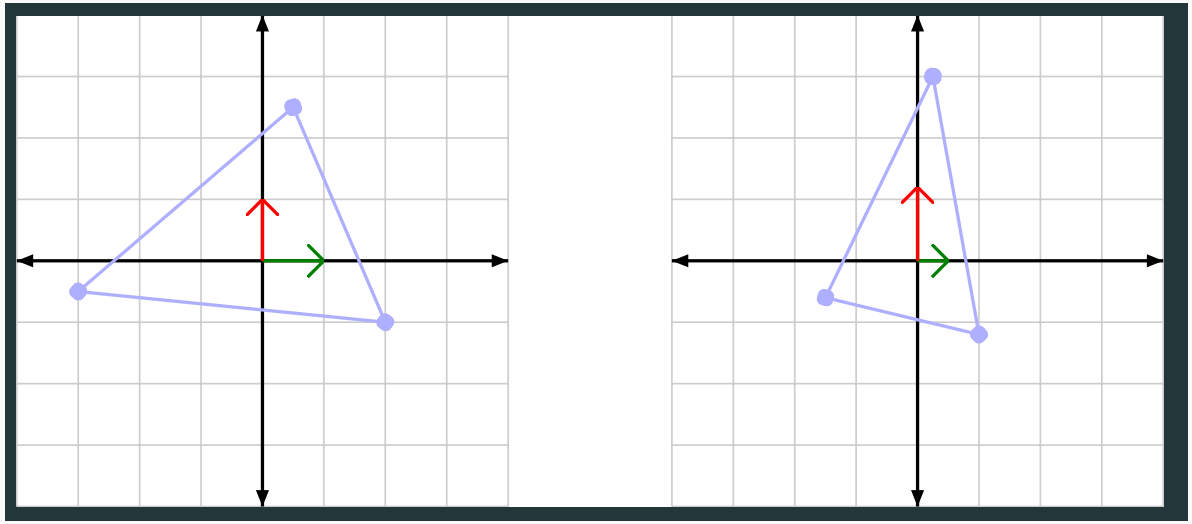
Linear Algebra

Vector Spaces

Point in vector space does not make sense without basis

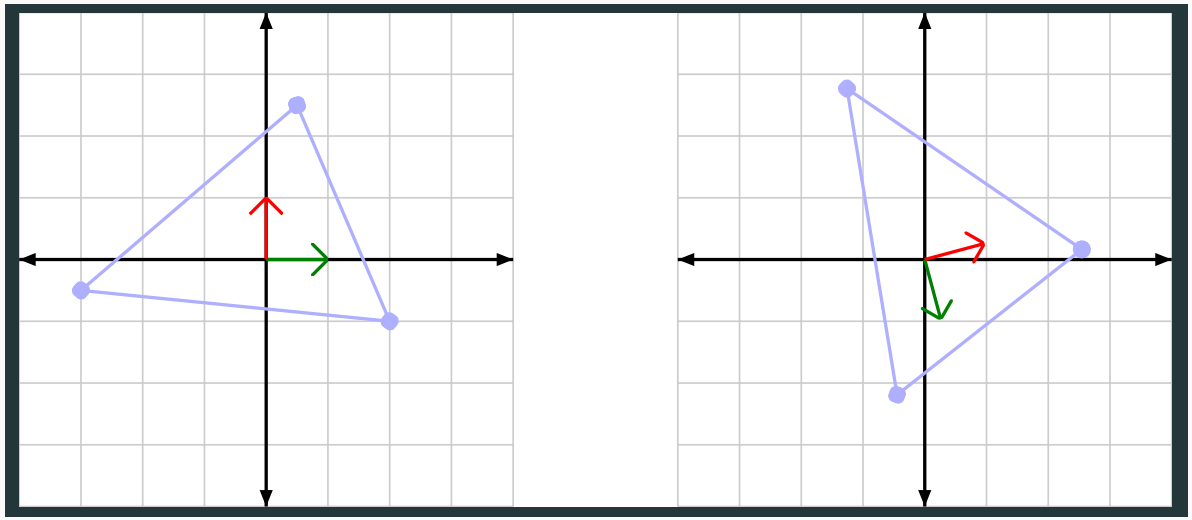
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Scaling



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [v_x \mathbf{e}_1, v_y \mathbf{e}_2, v_z \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} v_x & 0 & 0 \\ 0 & v_y & 0 \\ 0 & 0 & v_z \end{bmatrix} [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rotation



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \mathbf{R}\,[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rotation

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

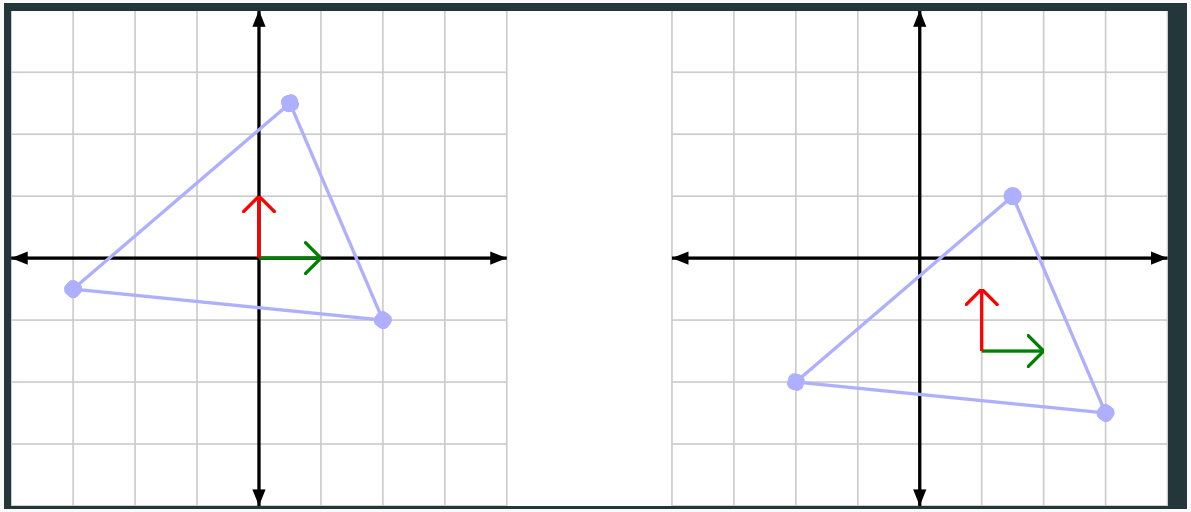
- Matrix multiplication is not commutative
  - $\mathbf{AB} \neq \mathbf{BA}$
  - Need to determine order convention
- Pick and order and stick with it

Rotation

$$R_{xyz} = \begin{bmatrix} \cos\theta_y \cos\theta_z & -\cos\theta_x \sin\theta_z + \sin\theta_x \sin\theta_y \cos\theta_z & \sin\theta_x \sin\theta_z + \cos\theta_x \sin\theta_y \cos\theta_z \\ \cos\theta_y \sin\theta_z & \cos\theta_x \cos\theta_z + \sin\theta_x \sin\theta_y \sin\theta_z & -\sin\theta_x \cos\theta_z + \cos\theta_x \sin\theta_y \sin\theta_z \\ -\sin\theta_y & \sin\theta_x \cos\theta_y & \cos\theta_x \cos\theta_y \end{bmatrix}$$

- Matrix multiplication is not commutative
  - $\mathbf{AB} \neq \mathbf{BA}$
  - Need to determine order convention
- Pick and order and stick with it

Translation



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x + dx \\ y + dy \\ z + dz \end{bmatrix}$$

Homogenous Coordinates[1]

- We do not want to deal with translation differently compared to other transformations
- We can work in *projective* or *homogenous* coordinate representations instead
- We will append the space with one dimension and specify an equivivalence class of all projections

[1] https://en.wikipedia.org/wiki/Homogeneous_coordinates

Homogenous Coordinates

- Perspective Projection

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \end{bmatrix}$$

- Specify equivivalence class between all projected points

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \sim \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

- Homogenous coordinates

$$\begin{bmatrix} \frac{2}{4} \\ \frac{3}{4} \\ \frac{4}{4} \end{bmatrix} = \begin{bmatrix} \frac{4}{8} \\ \frac{6}{8} \\ \frac{8}{8} \end{bmatrix}$$

Homogenous Coordinates

- Cartesian 3D space

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Allows for representing points in infinity

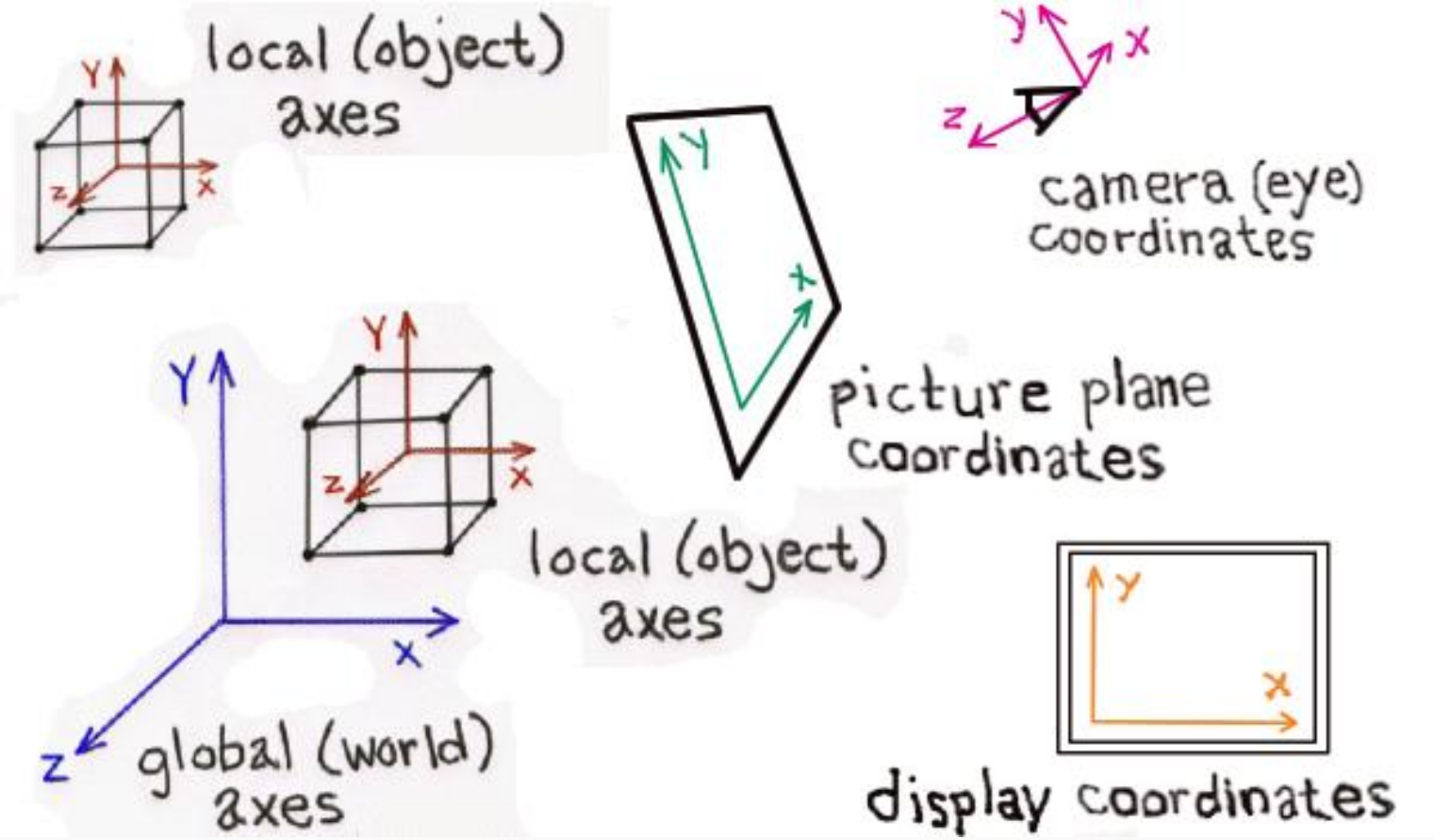
$$\lim_{d \to \infty} \begin{bmatrix} x + d \\ y + d \\ z + d \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

- For example, under projection two parallell lines will meet

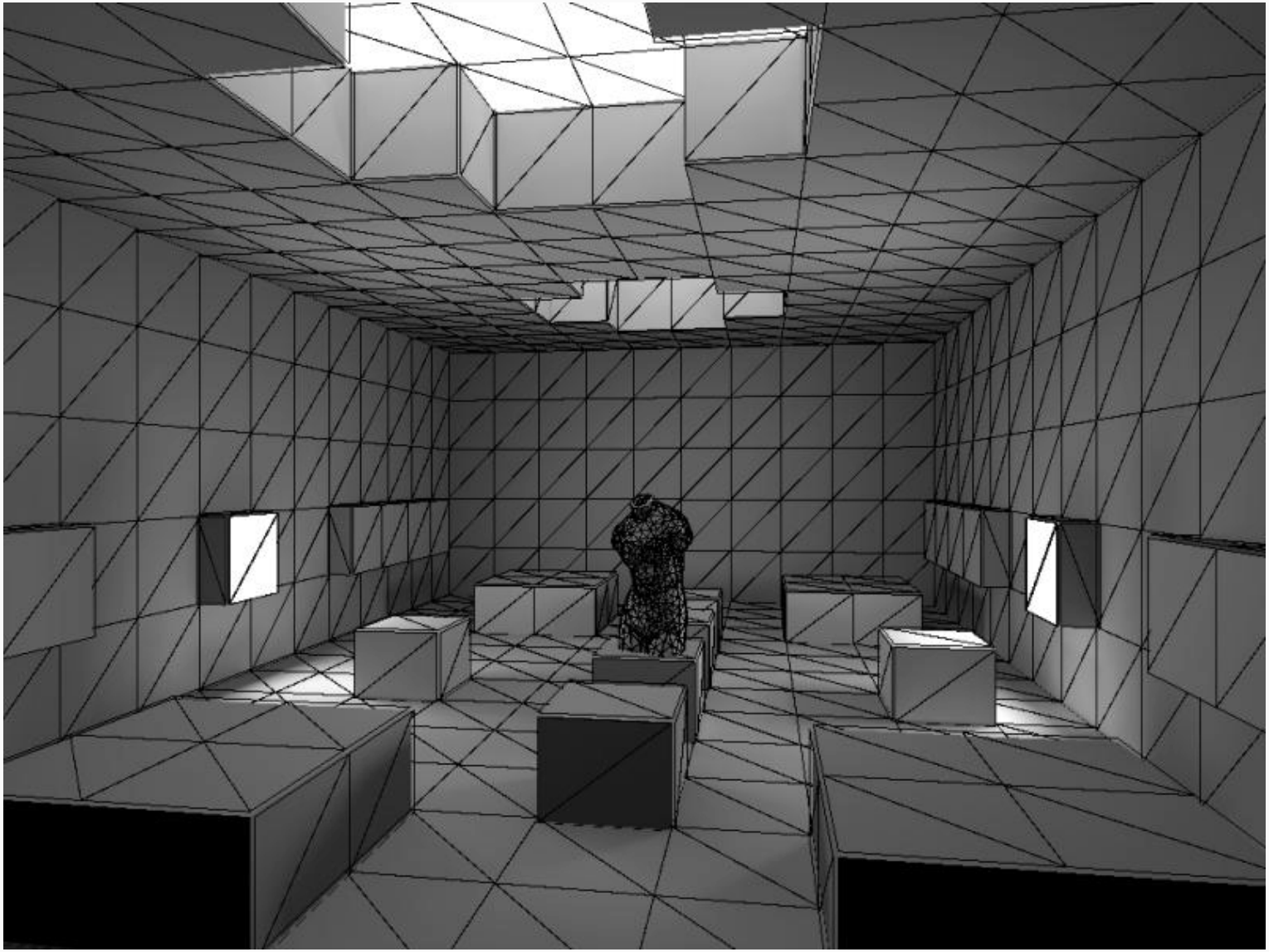
Homogenous Coordinates

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} v_x \cdot R_{11} & R_{12} & R_{13} & dx \\ R_{21} & v_y \cdot R_{22} & R_{23} & dx \\ R_{31} & R_{32} & v_z \cdot R_{33} & dx \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

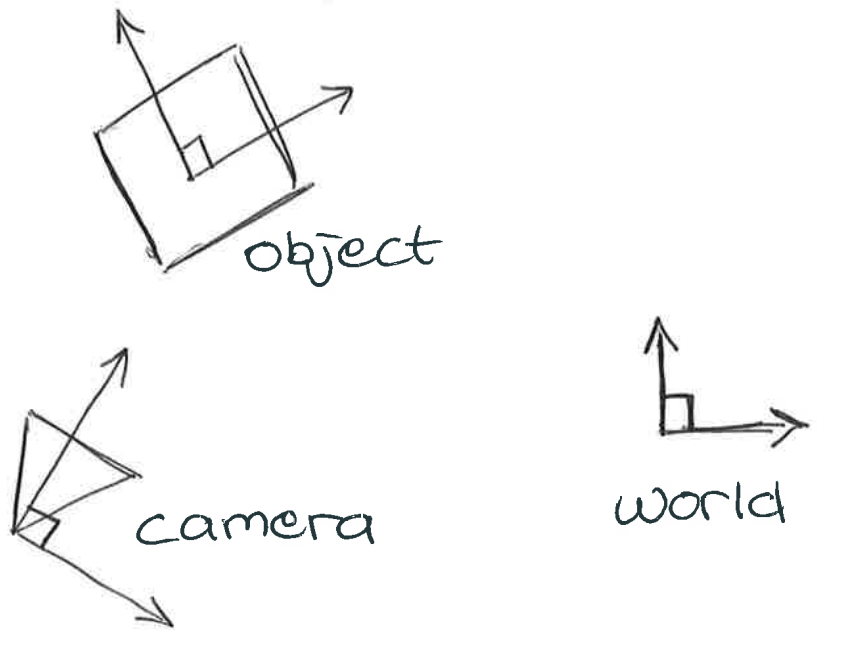
Cordinate Transforms



Scene



View Transformations



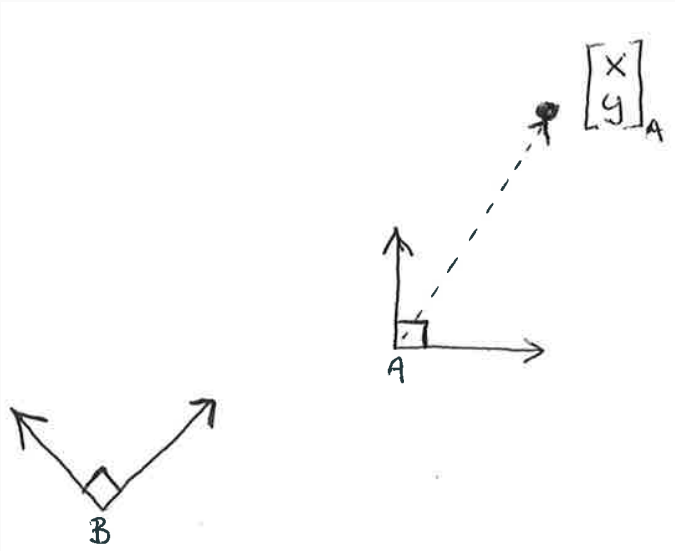
View Transformations

1. Translate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}_A - \begin{bmatrix} x \\ y \end{bmatrix}_{O_B}$$

2. Rotate (negative)

$$\begin{bmatrix} x \\ y \end{bmatrix}_B = \begin{bmatrix} cos(-\theta) & -sin(-\theta) \\ sin(-\theta) & cos(-\theta) \end{bmatrix}_B \begin{bmatrix} x' \\ y' \end{bmatrix}$$



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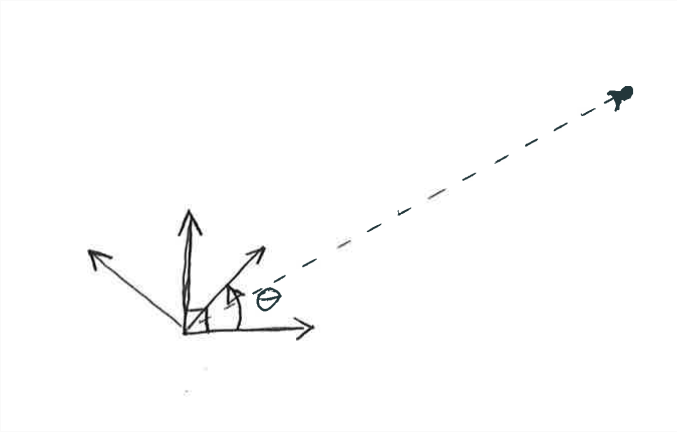

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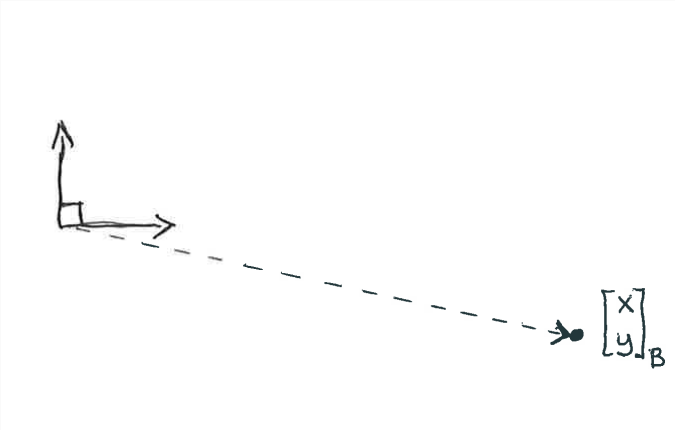

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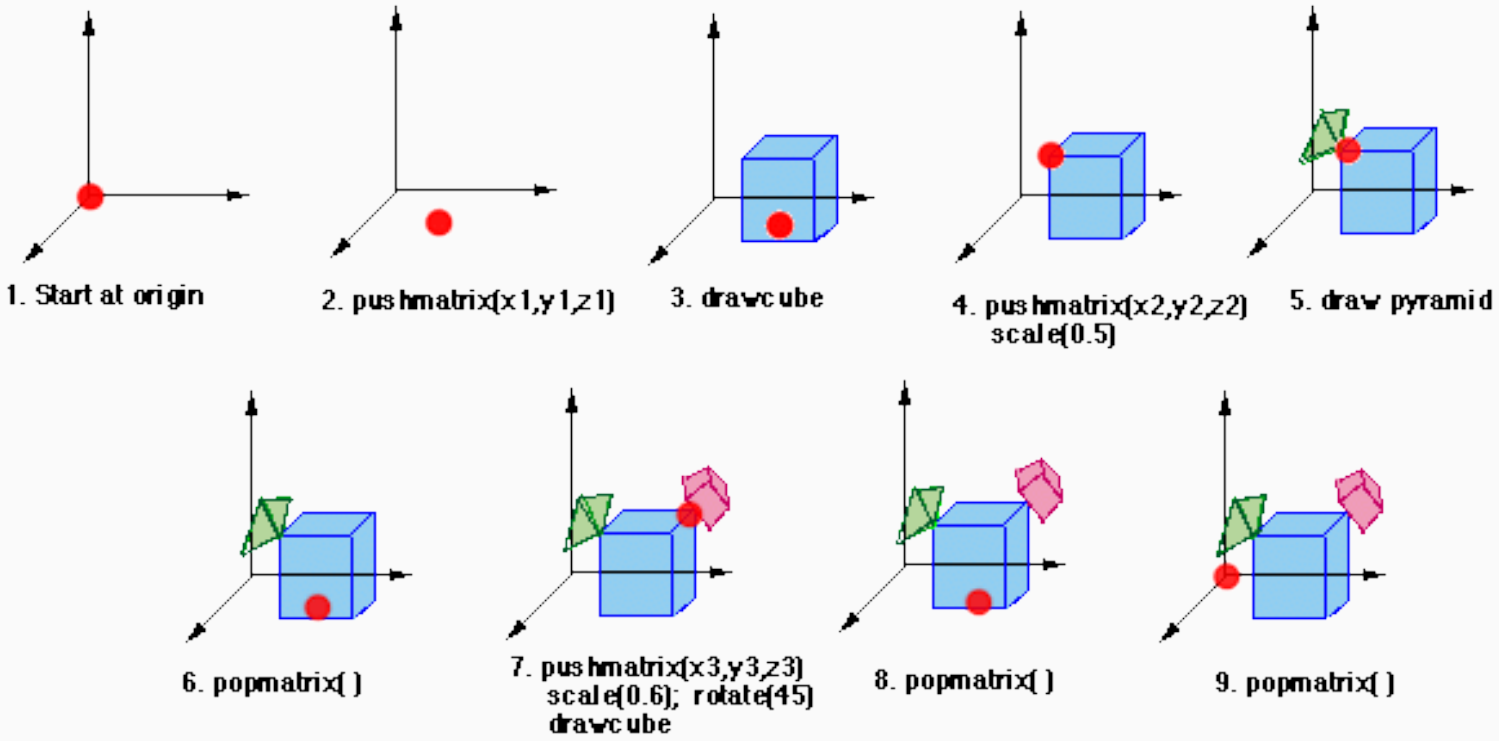
View Transformations

Transformation

Always the same order for every transformation of coordinate systems

1. Translate
2. Rotate
3. Scale

Transformation Stack

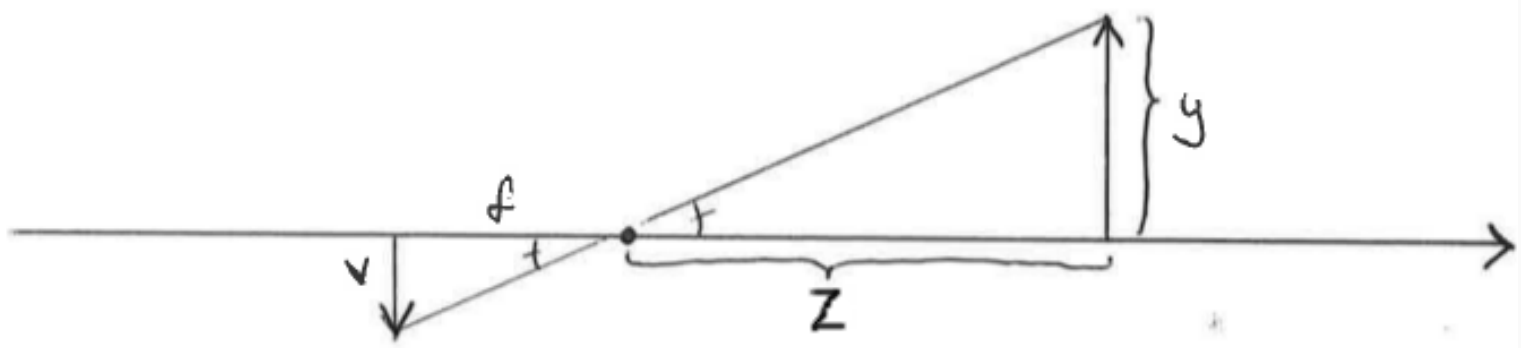


Raytracing

Camera

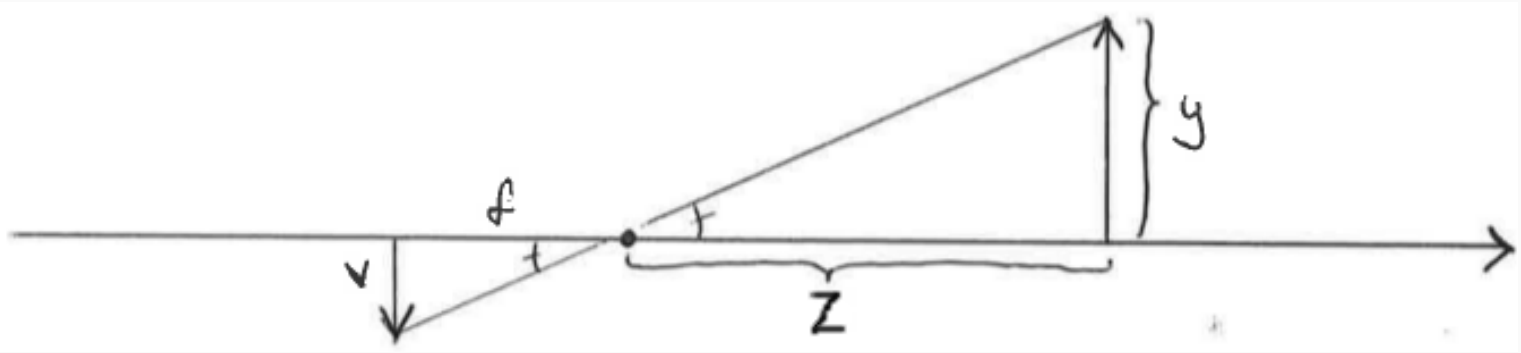
- Cameras create images
- Lens
  - captures rays
- Focal lenth
- Shutter opening

Pinhole camera



- Simplest form camera, where one ray of light lits up each pixel
- Defined by a *location*, *focal length*, *view-direction* and *up-direction*
- is this a realistic camera?

Pinhole camera

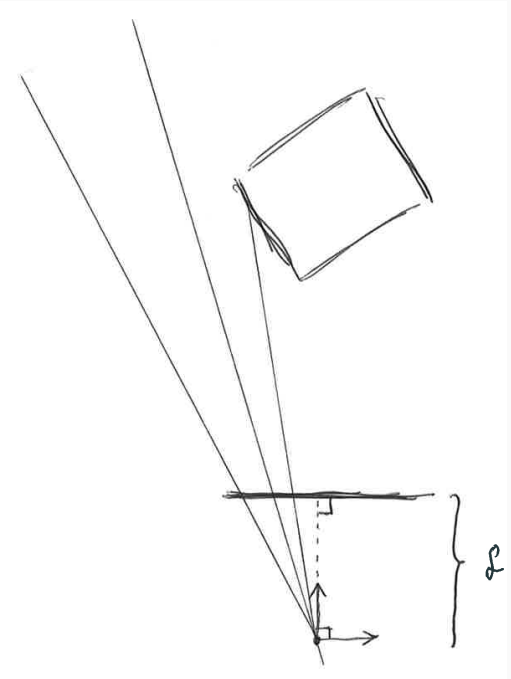


$$\frac{v}{f} = \frac{y}{z}$$

$$\Rightarrow v = \frac{f}{z} y$$

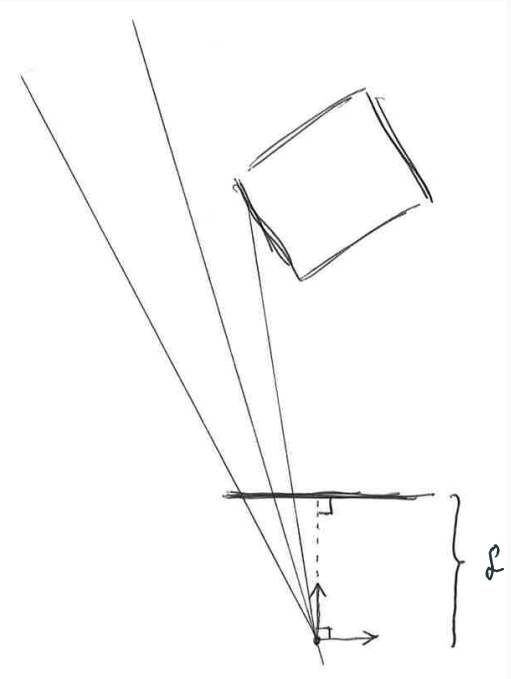
Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$



Raycasting

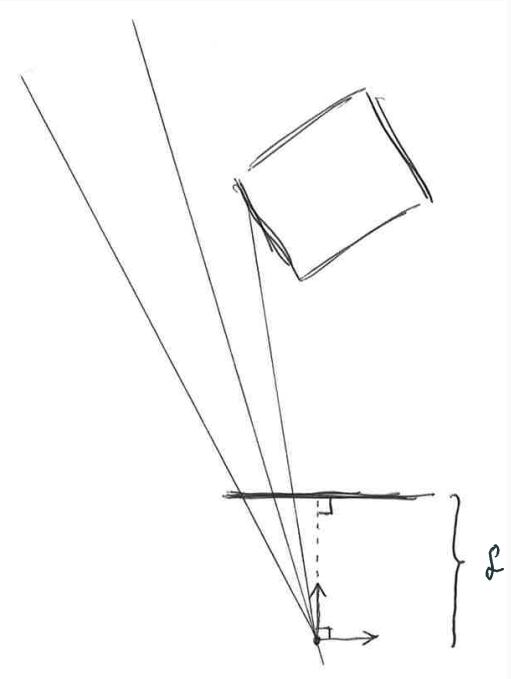
1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$



Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

$$\{[u, v, f]^{\mathrm{T}} | -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f\}$$



Raycasting

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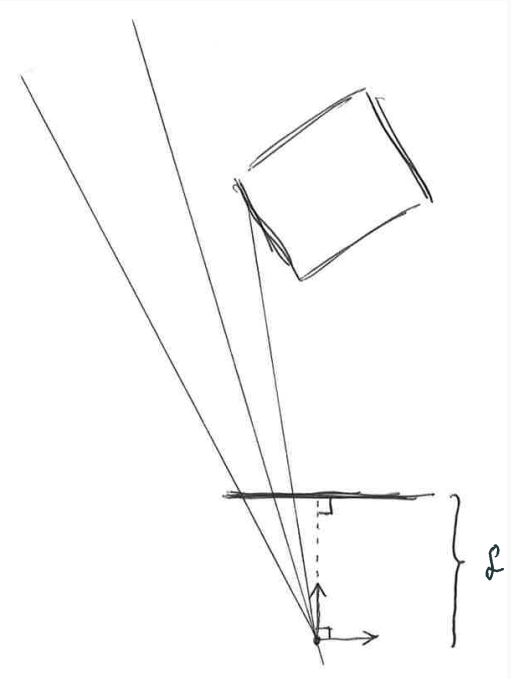
$$\{[u, v, f]^{\mathrm{T}} | -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f\}$$

4. Compute normalised vector from origin to image-plane

$$\mathbf{d}_{ij} = \frac{1}{\sqrt{[u_i, v_j, f][u_i, v_j, f]^{\mathrm{T}}}} [u, v, f]^{\mathrm{T}}$$



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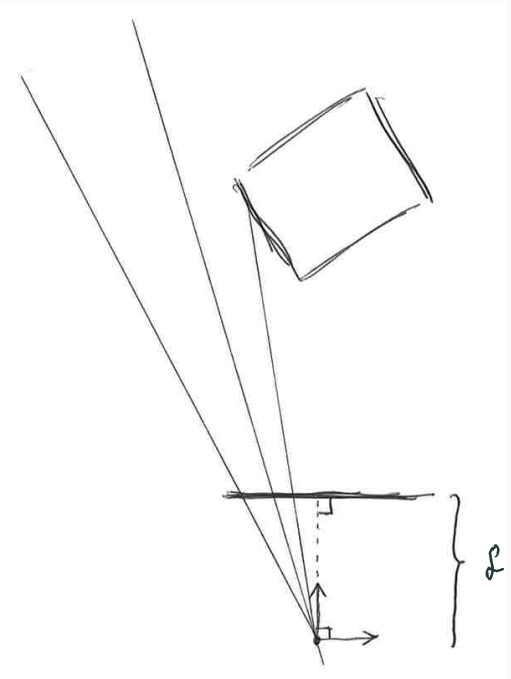
5. Parmatrise Ray

$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$



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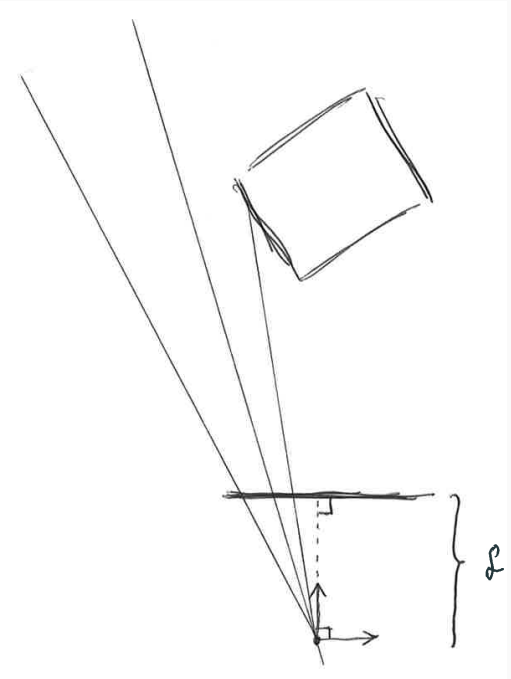
5. Parmatrise Ray

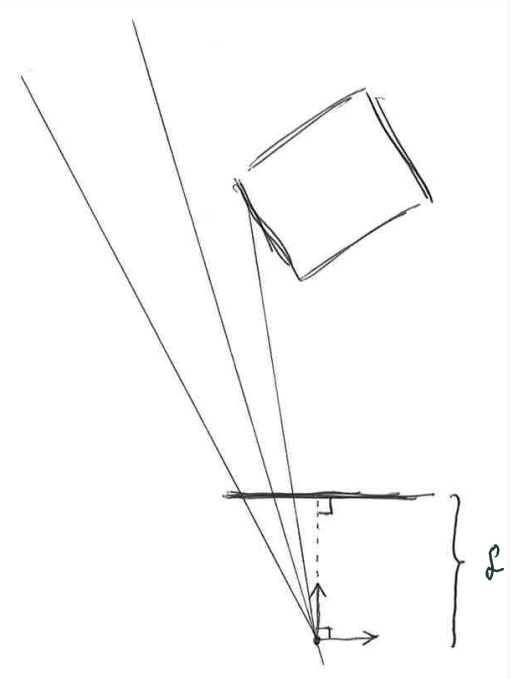
$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$

6. One degree-of-freedom

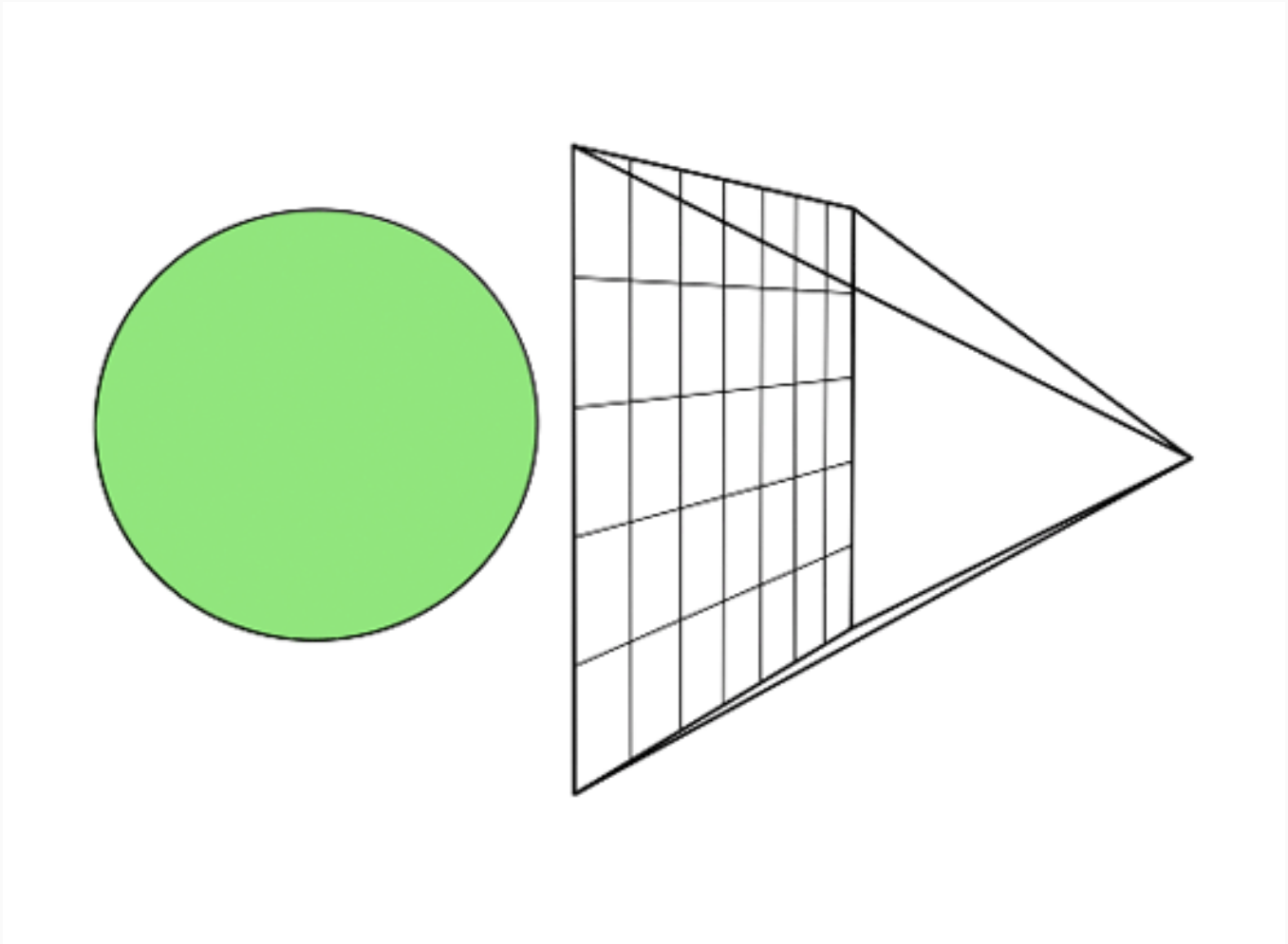


Raycasting

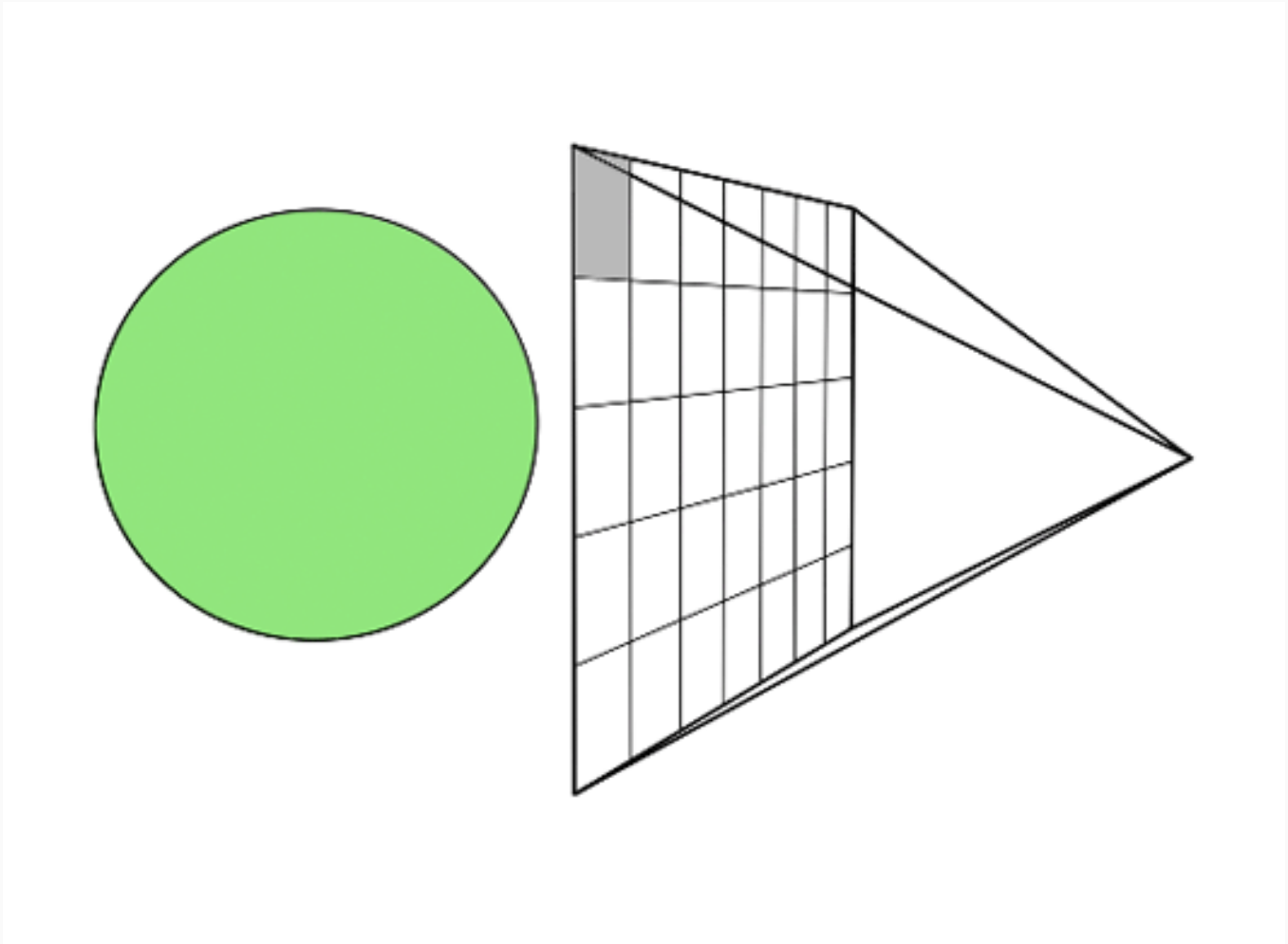
- Trace a ray for each pixel
- Compute intersection with all objects in world
- Plot colour of intersecting surface
- No "lighting" often called Raycaster



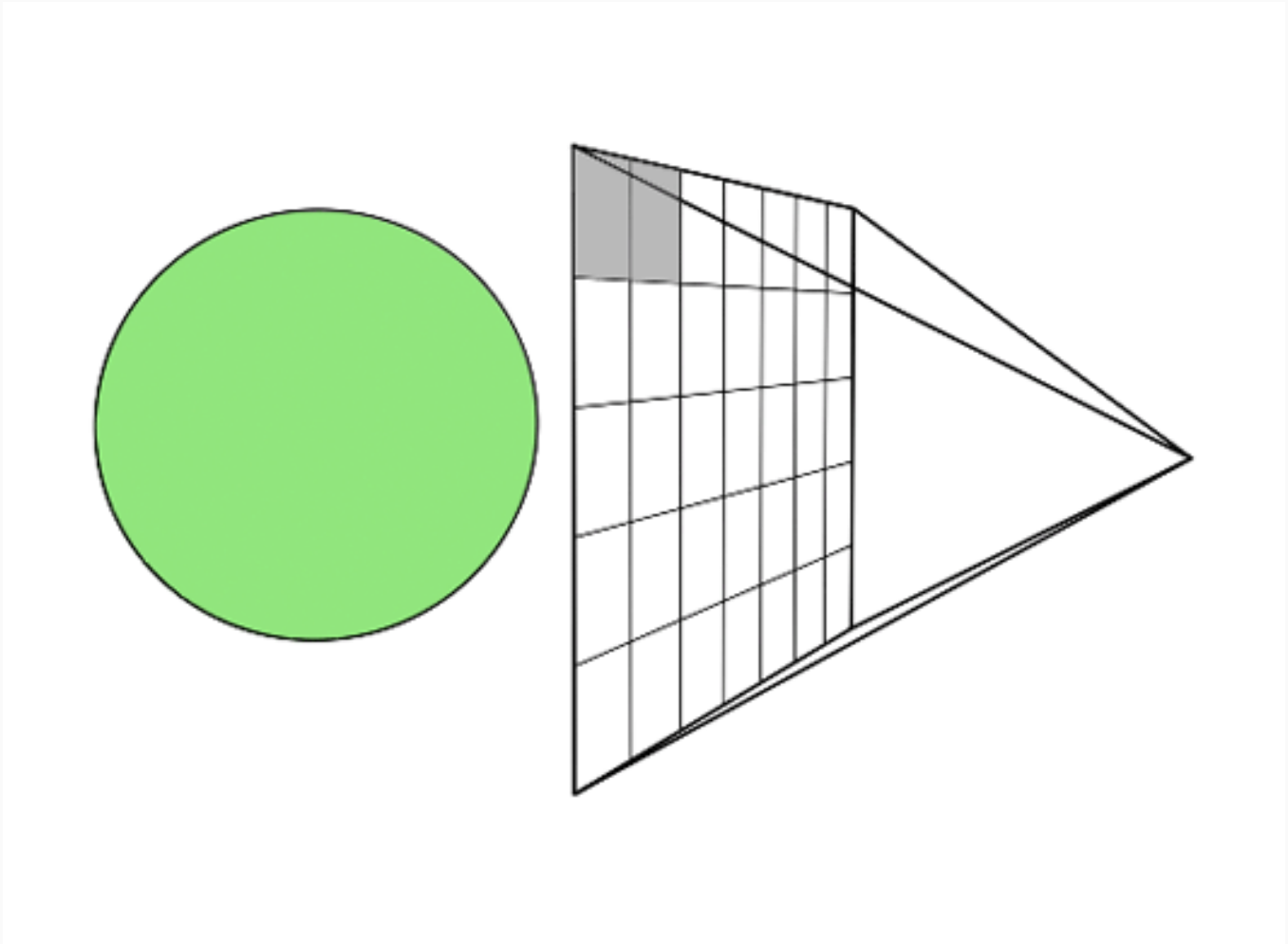
Raycasting Anim



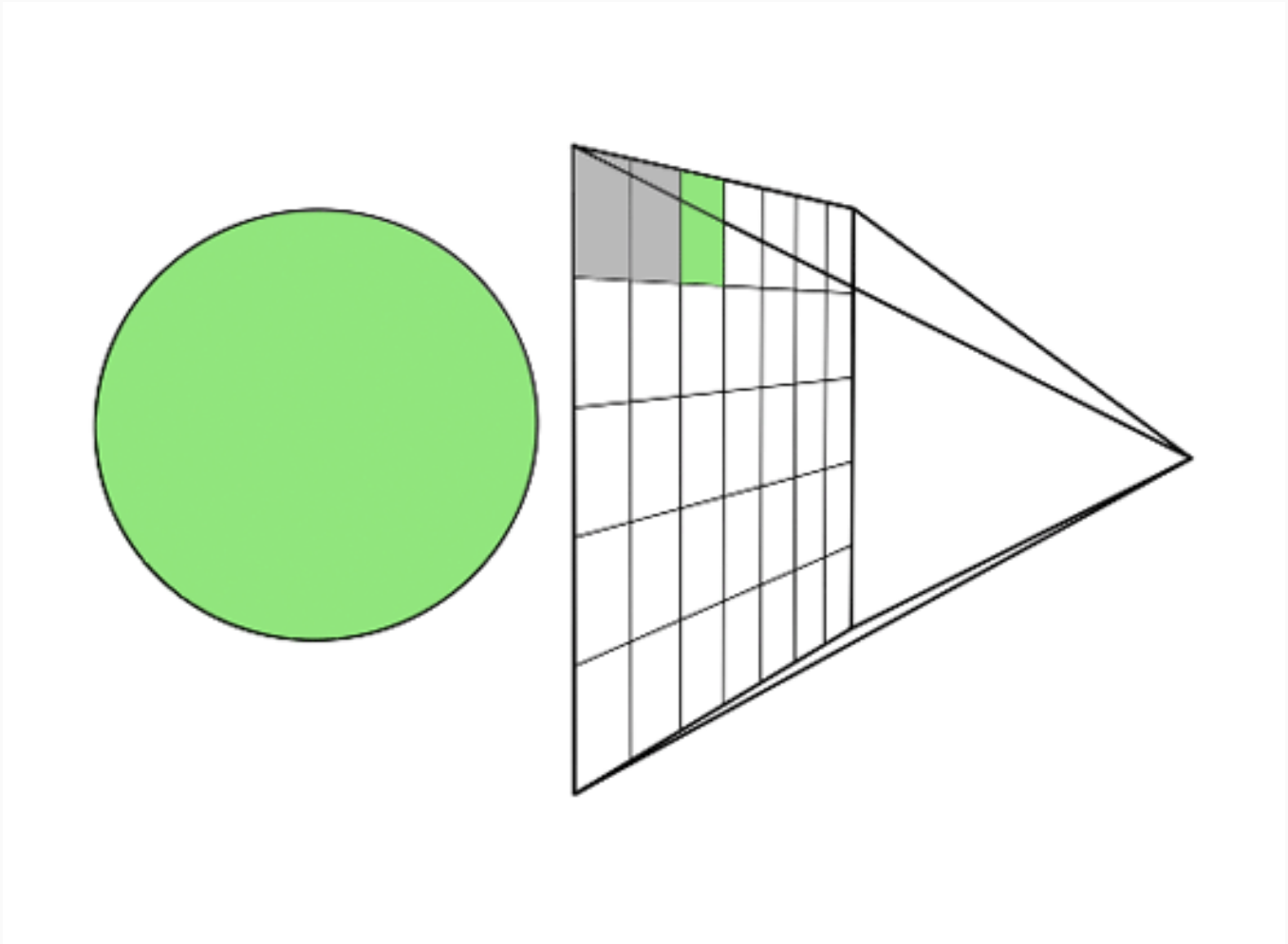
Raycasting Anim



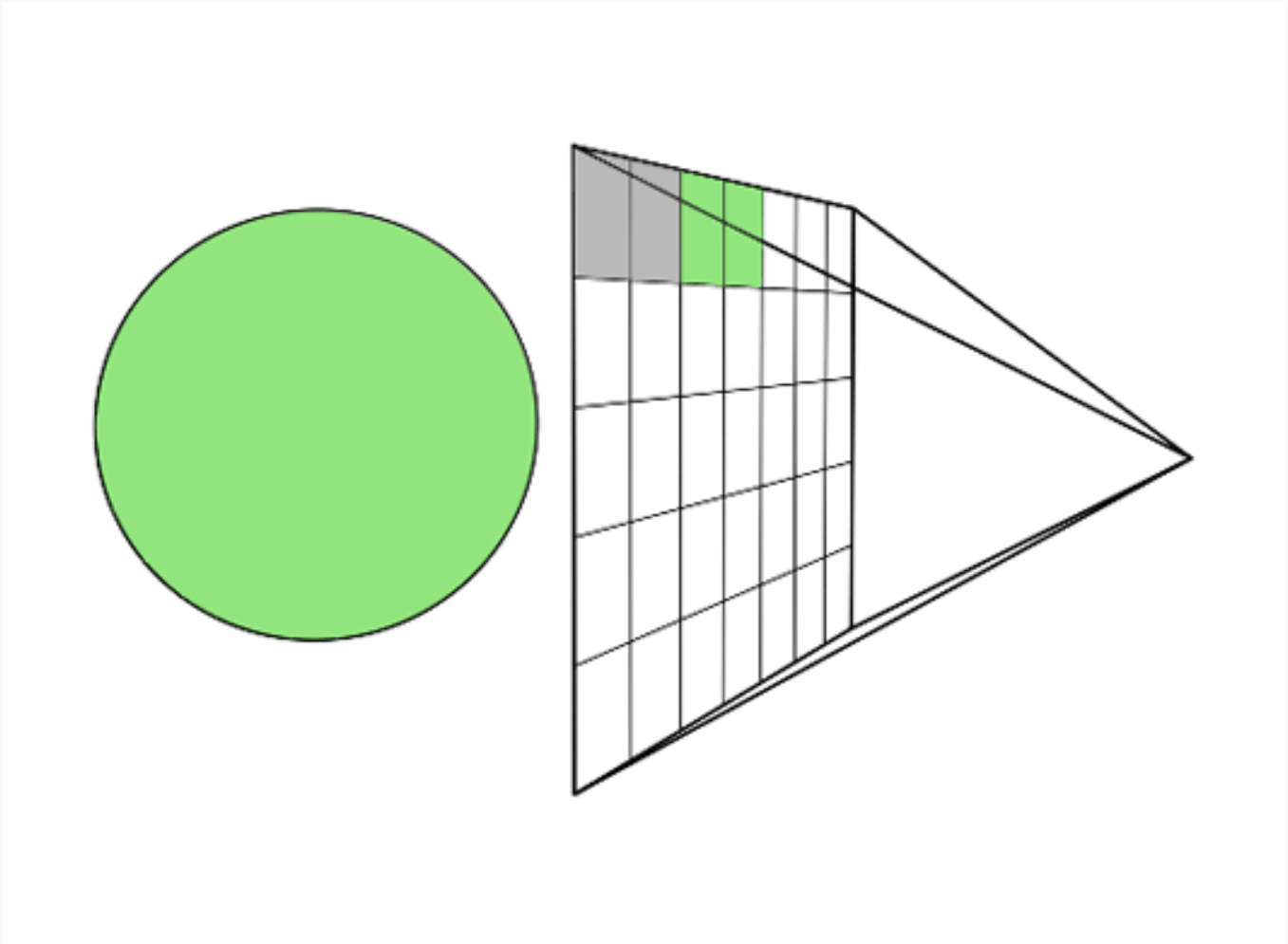
Raycasting Anim



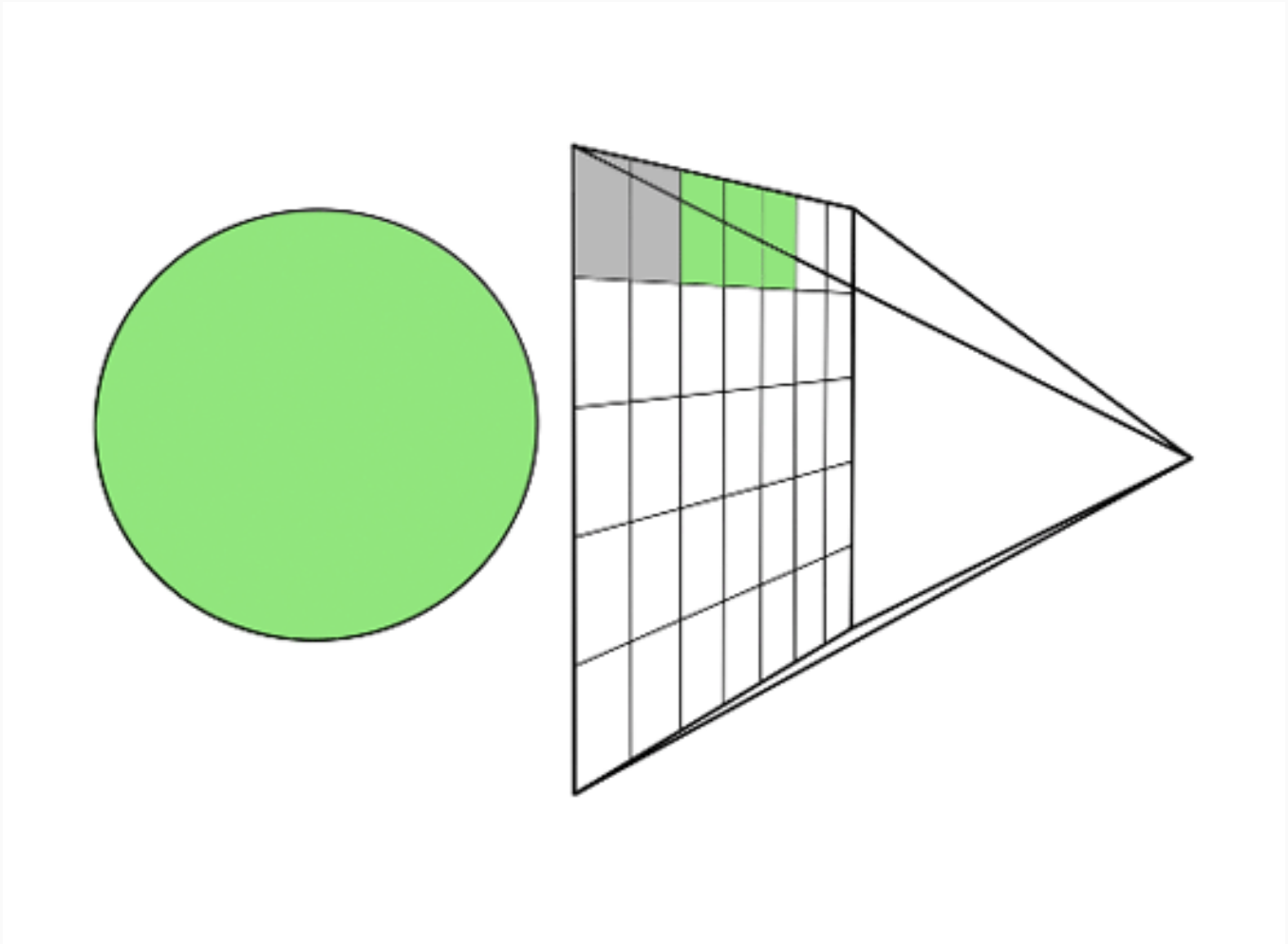
Raycasting Anim



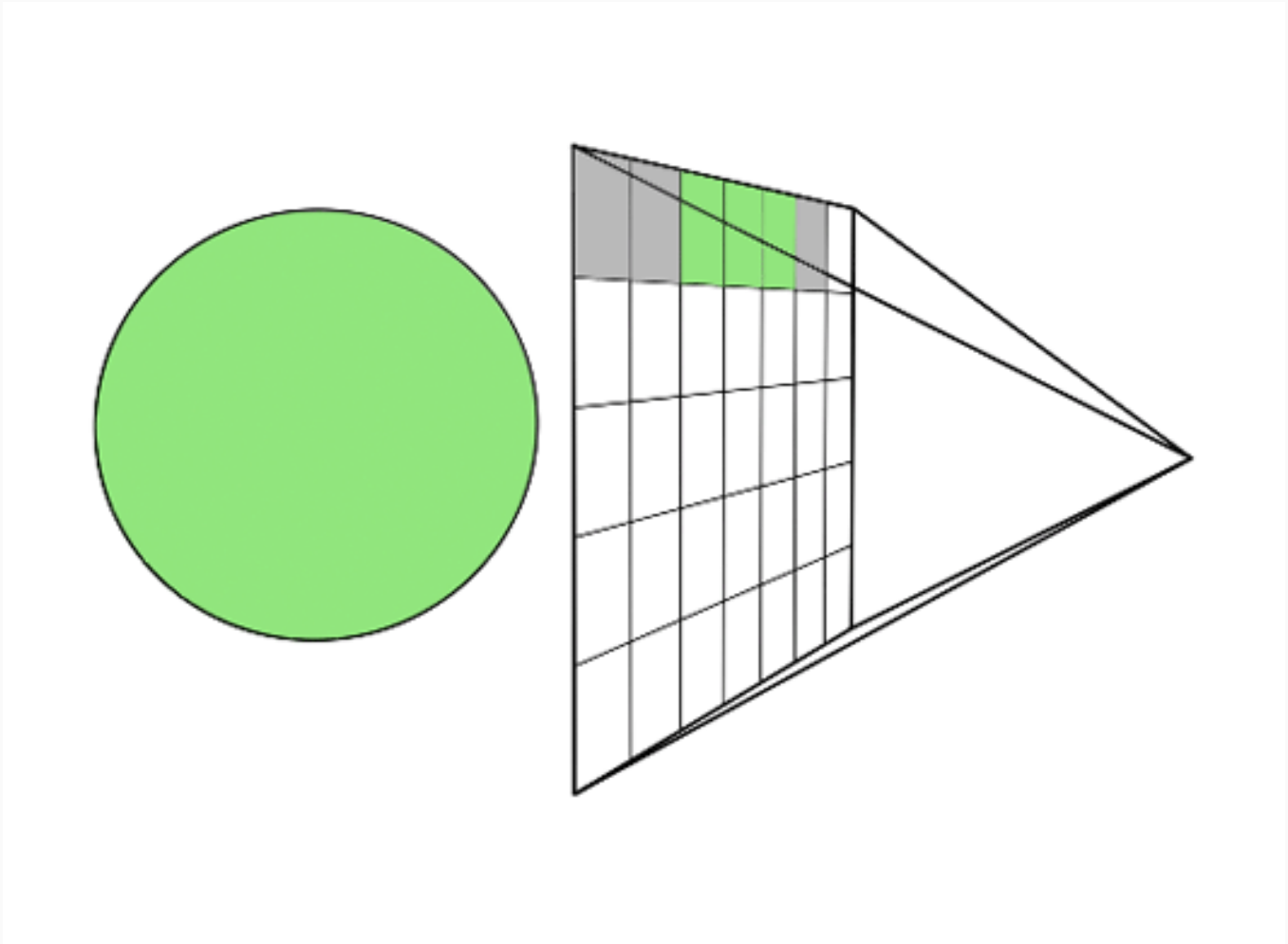
Raycasting Anim



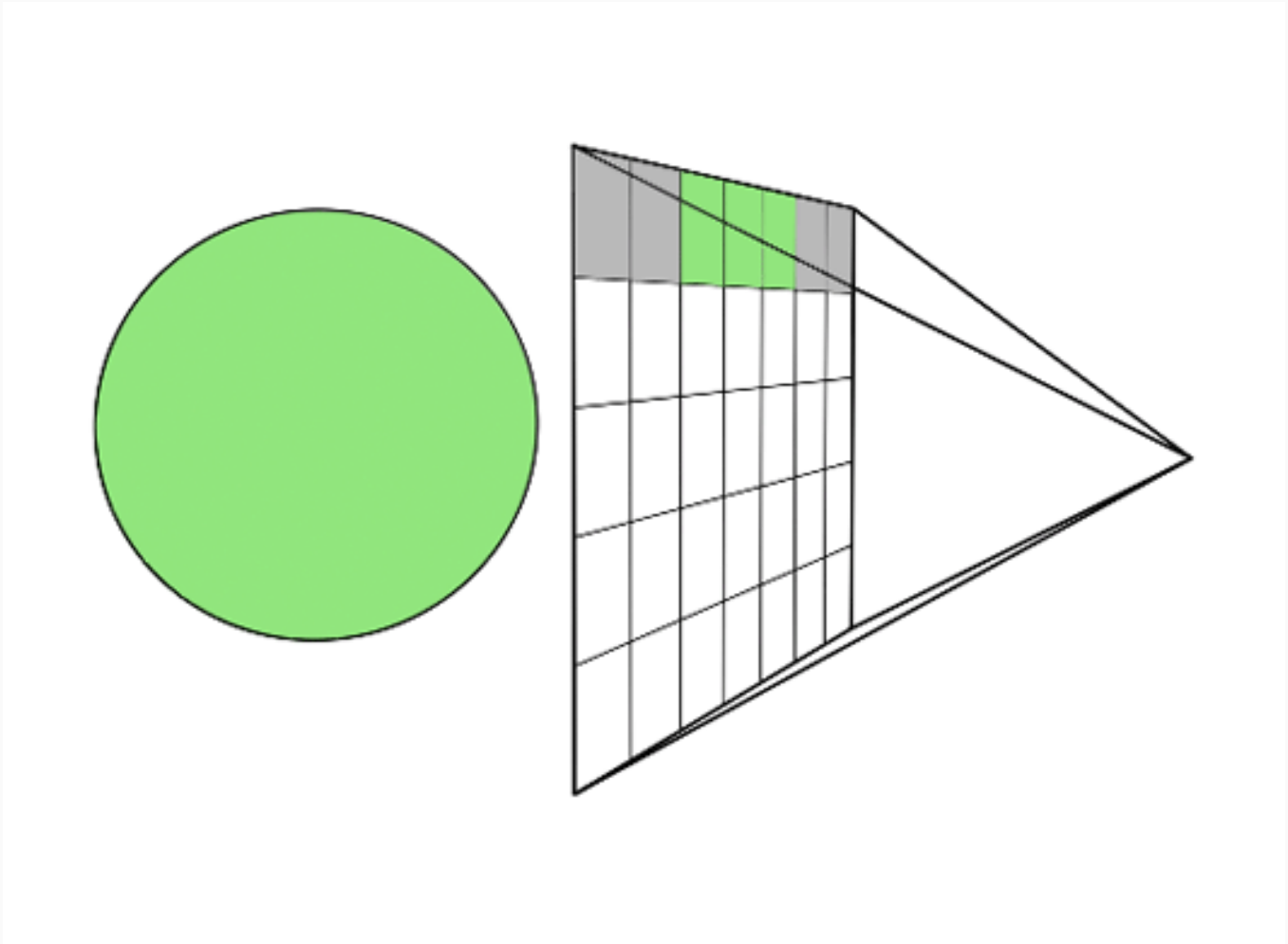
Raycasting Anim



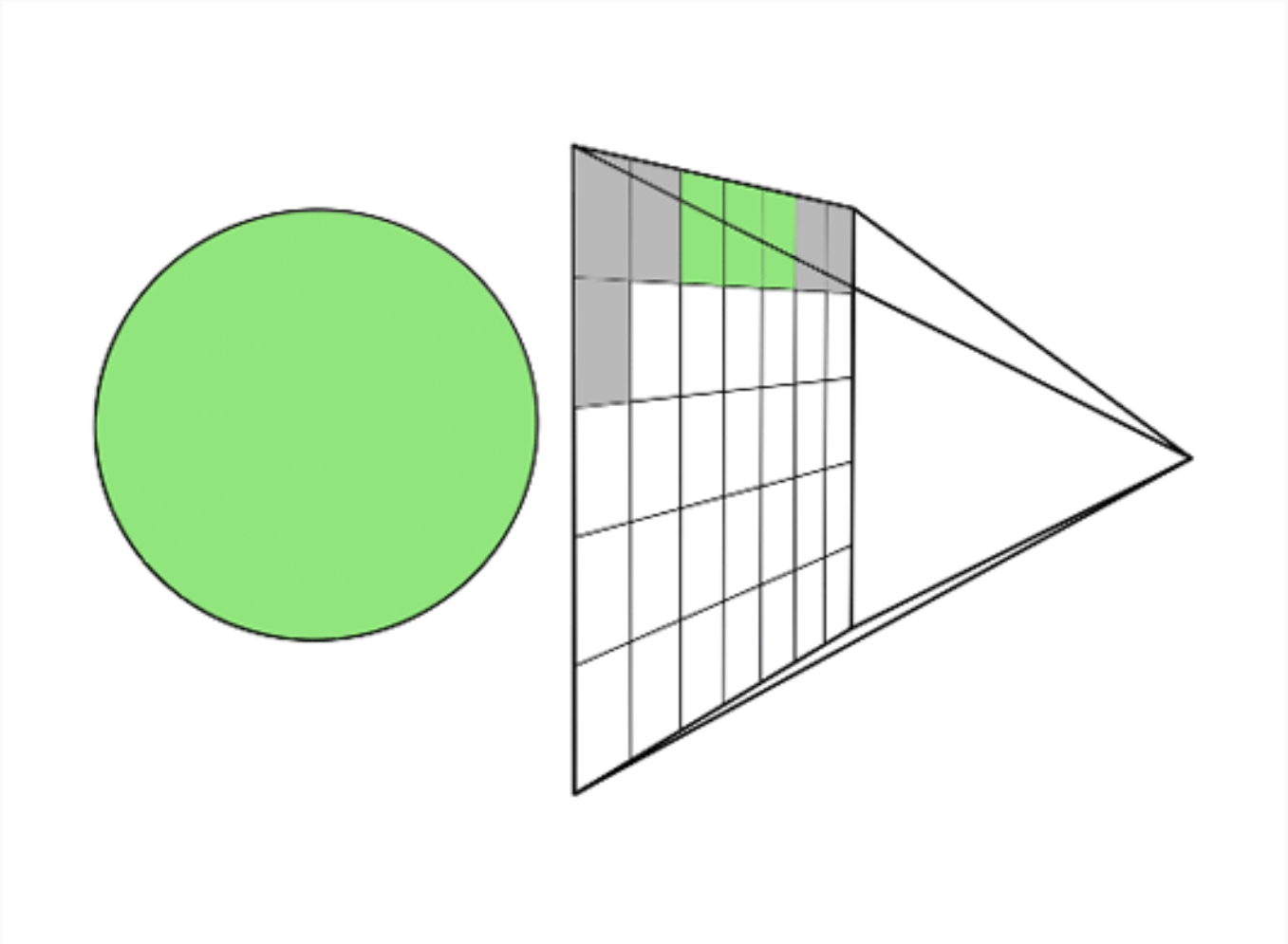
Raycasting Anim



Raycasting Anim



Raycasting Anim



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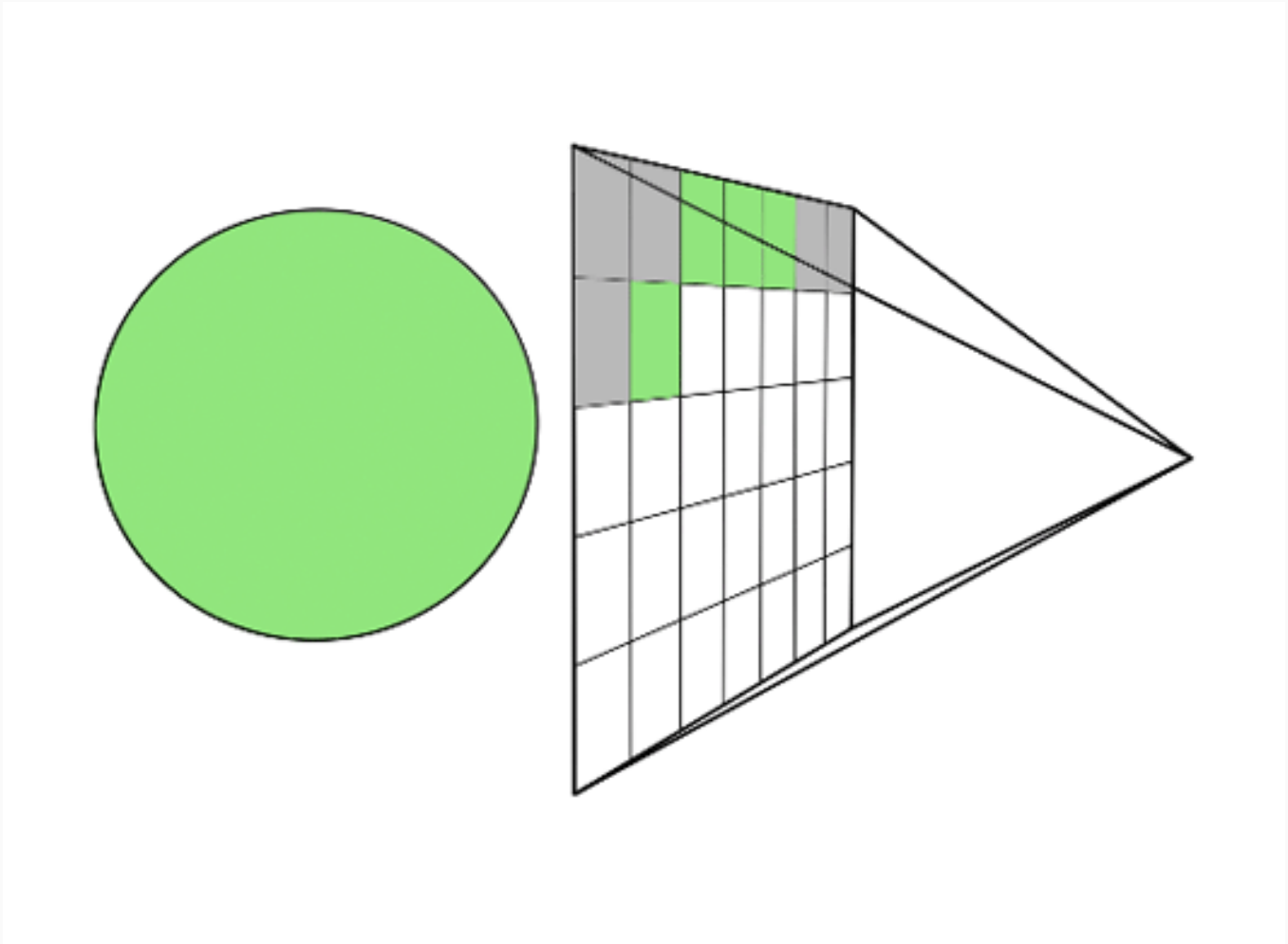

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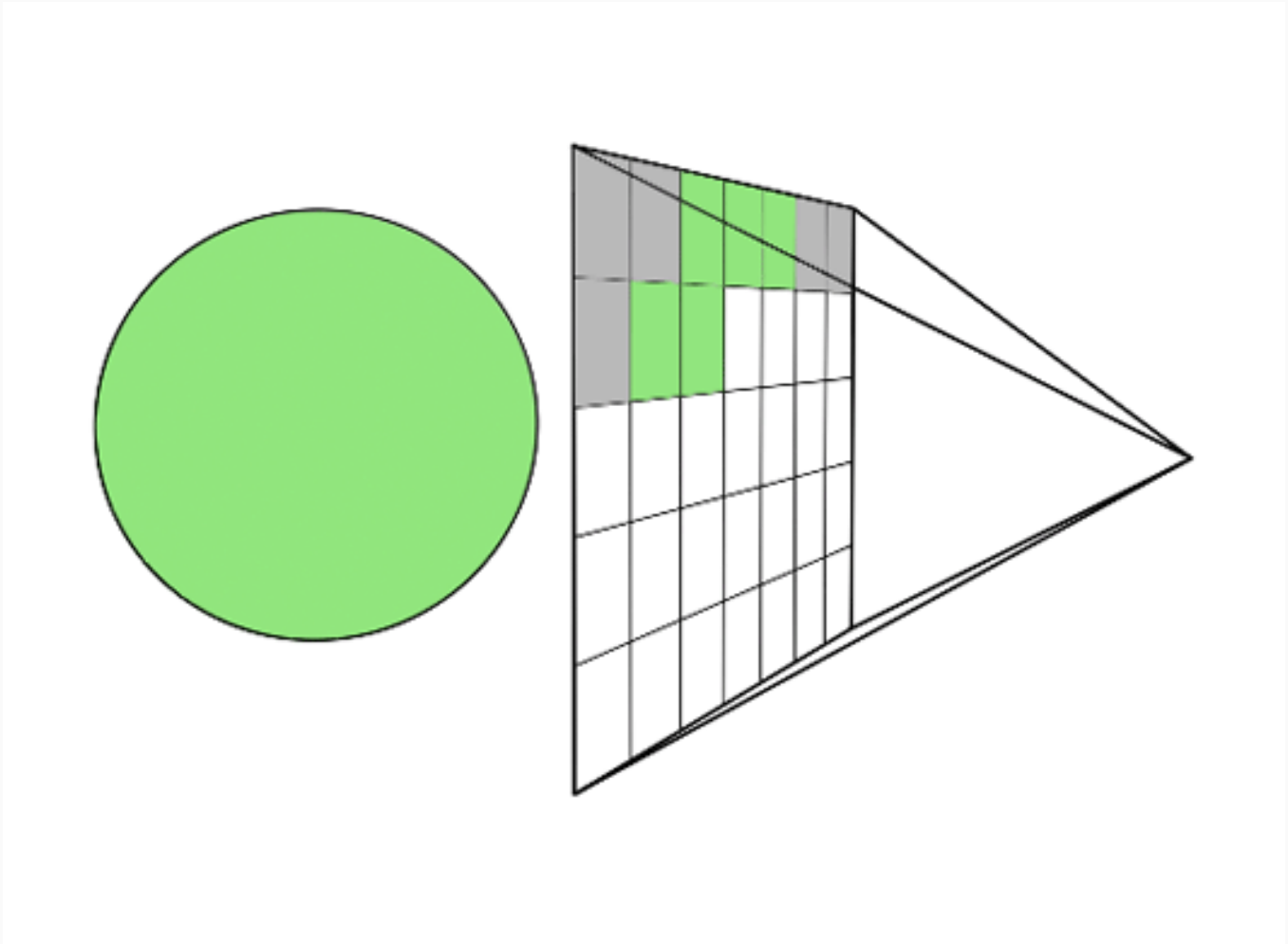
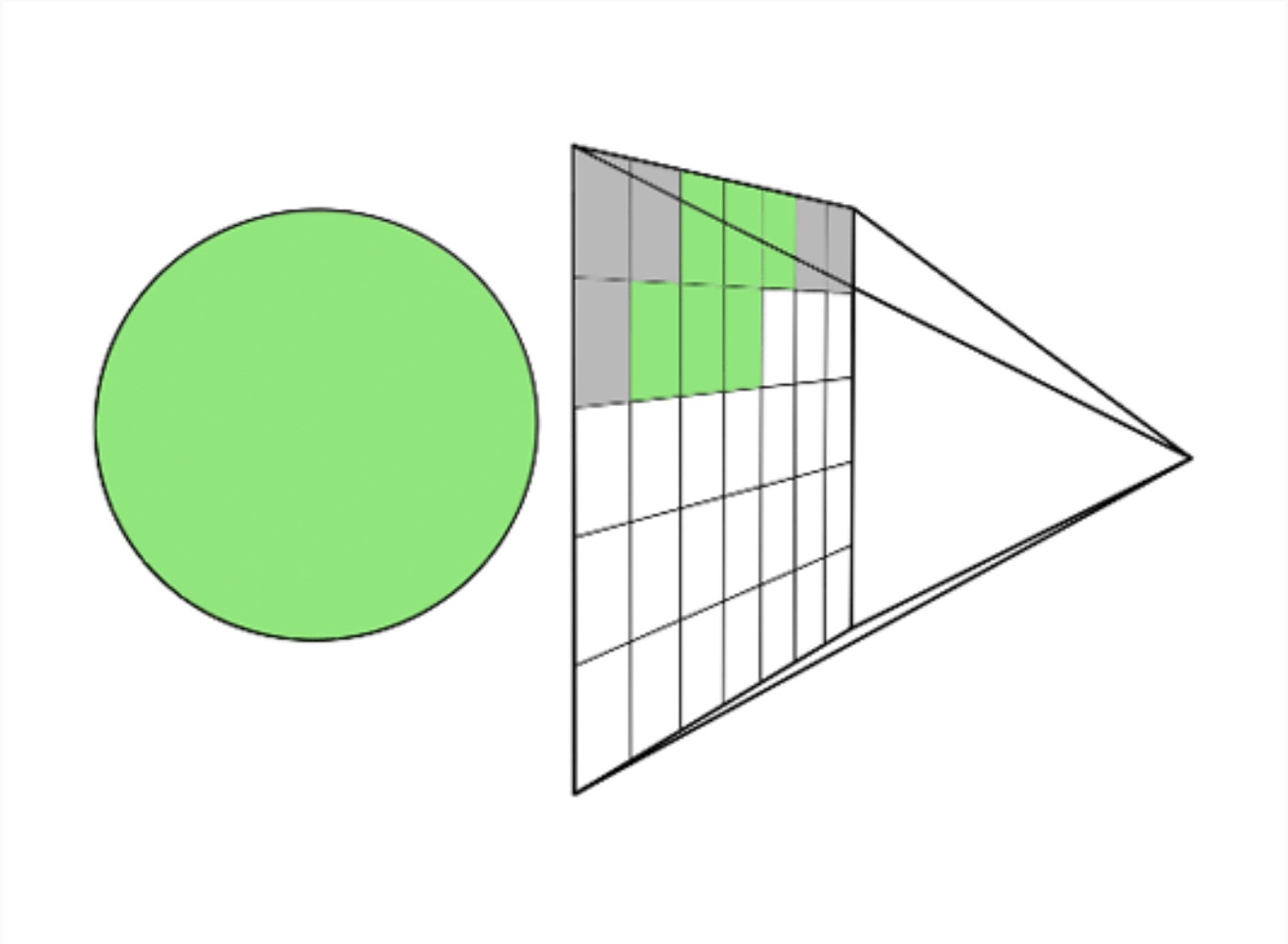
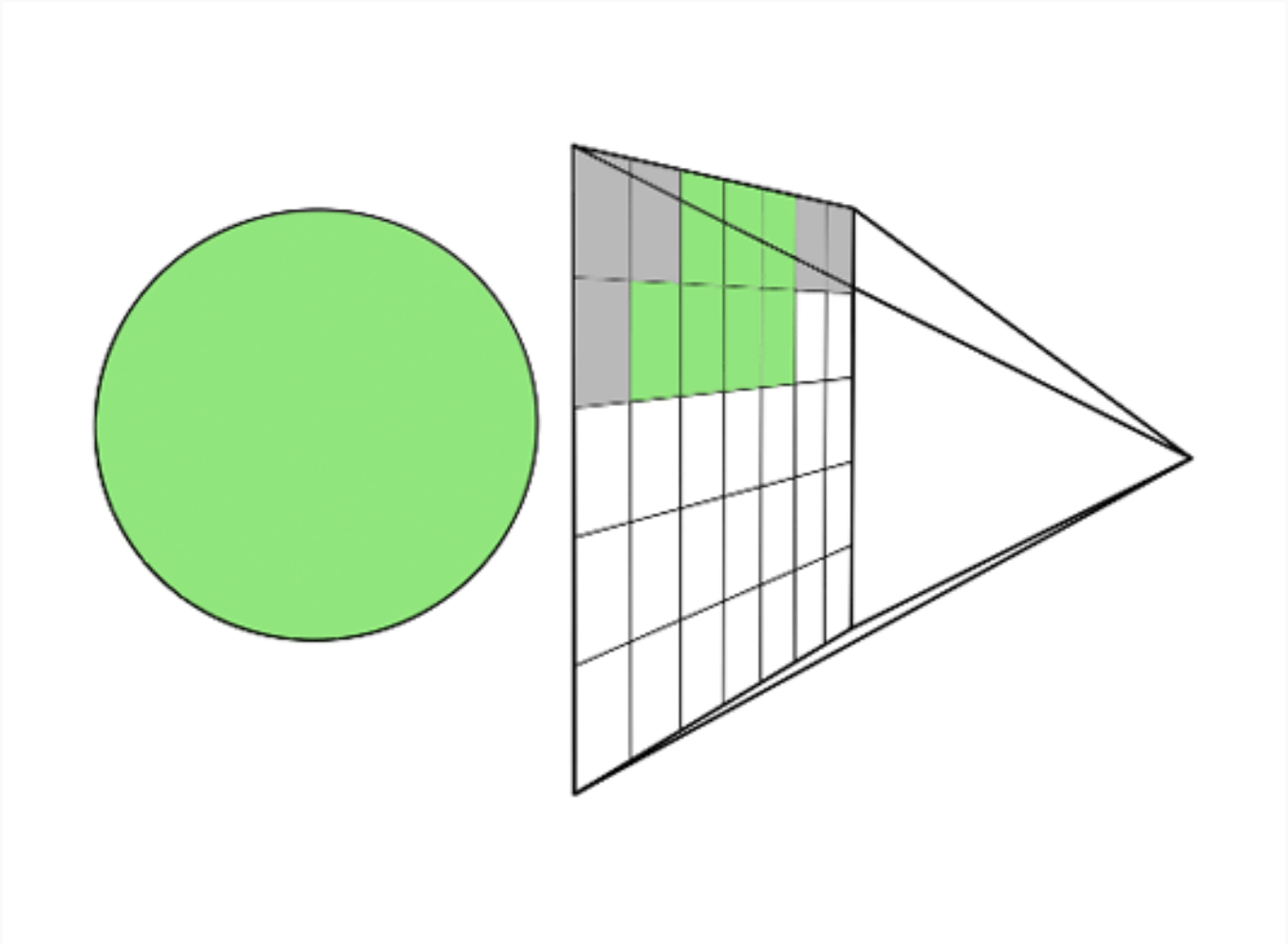


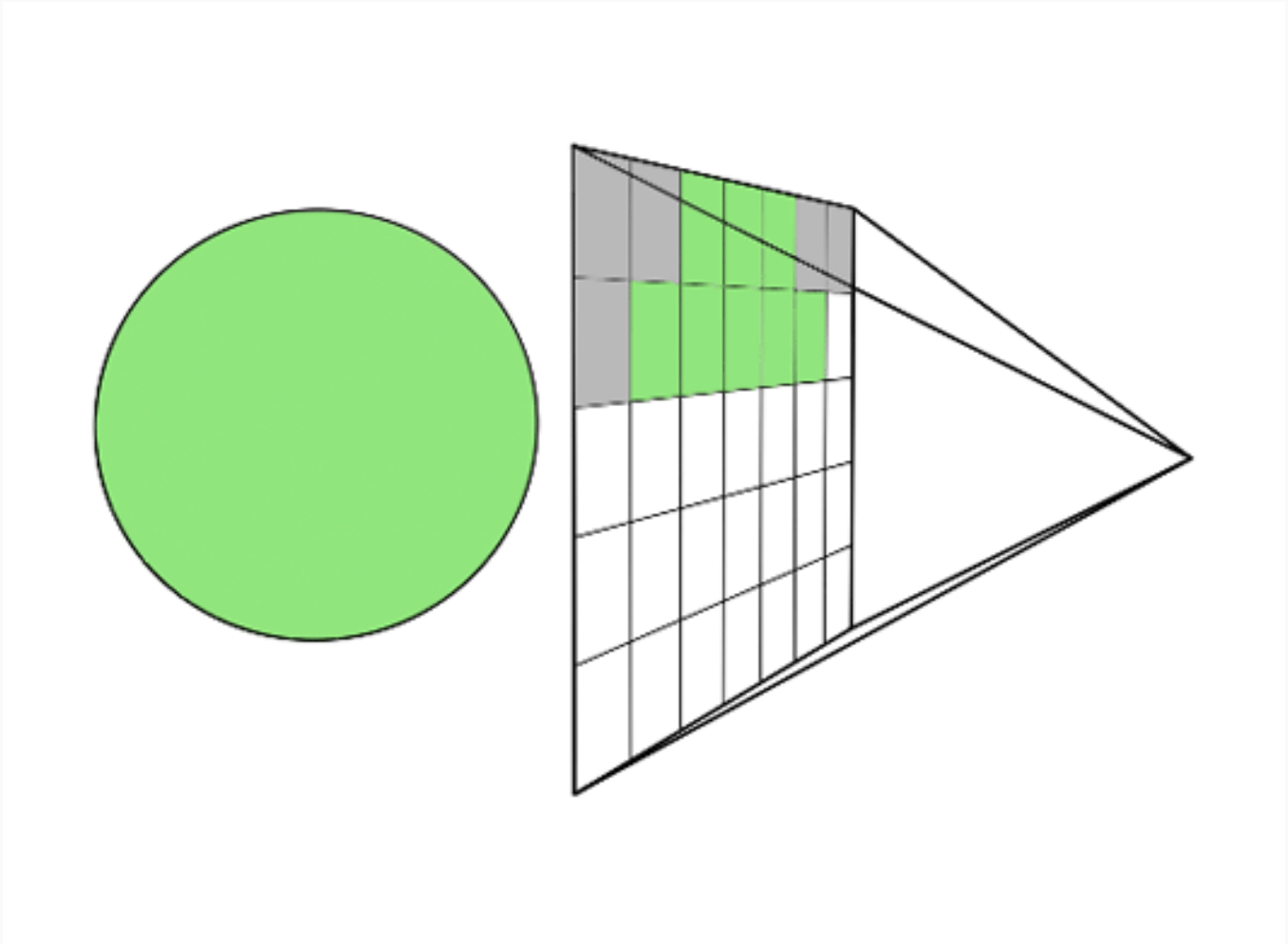
Raycasting Anim



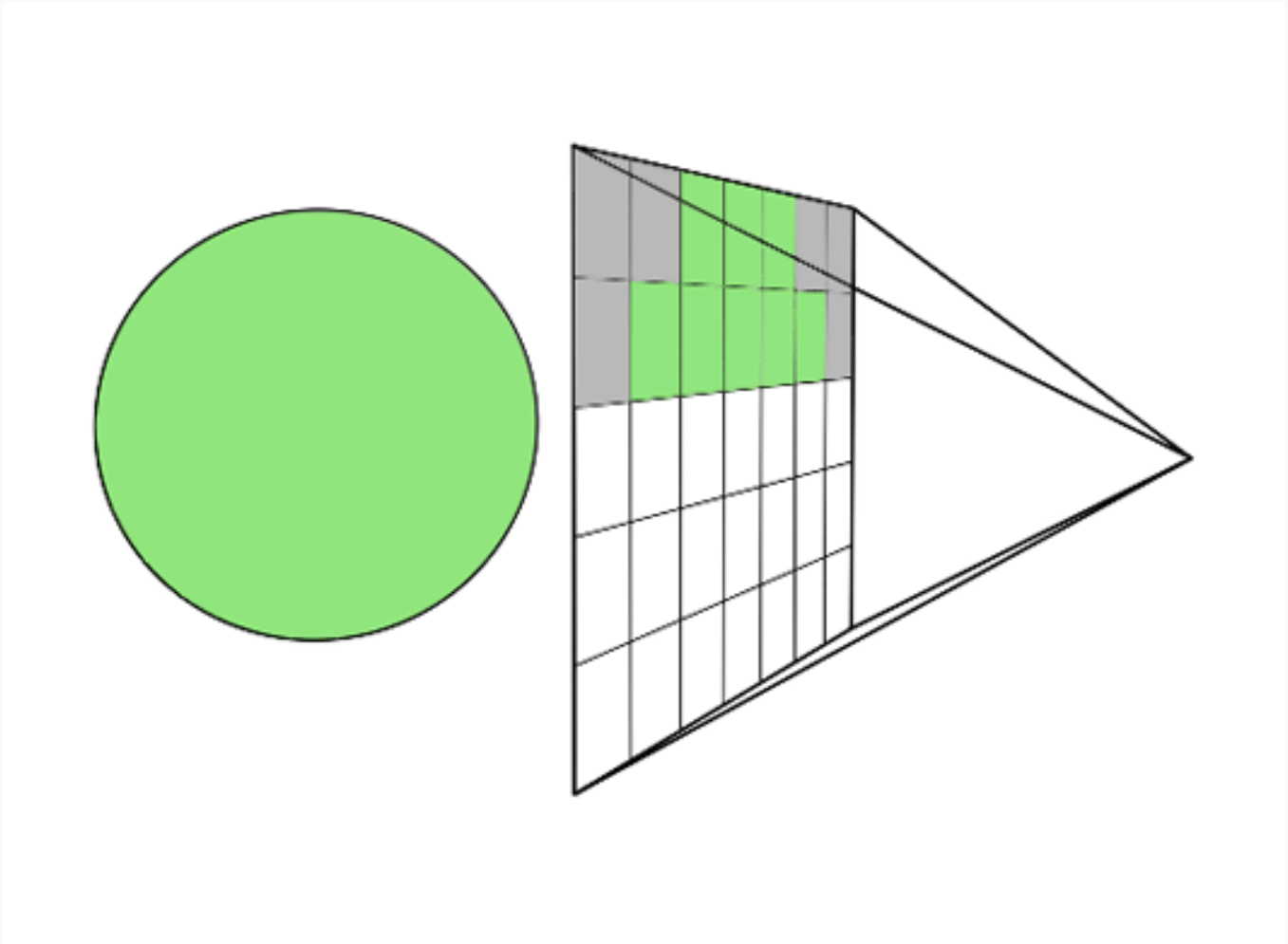
Raycasting Anim



Raycasting Anim



Raycasting Anim



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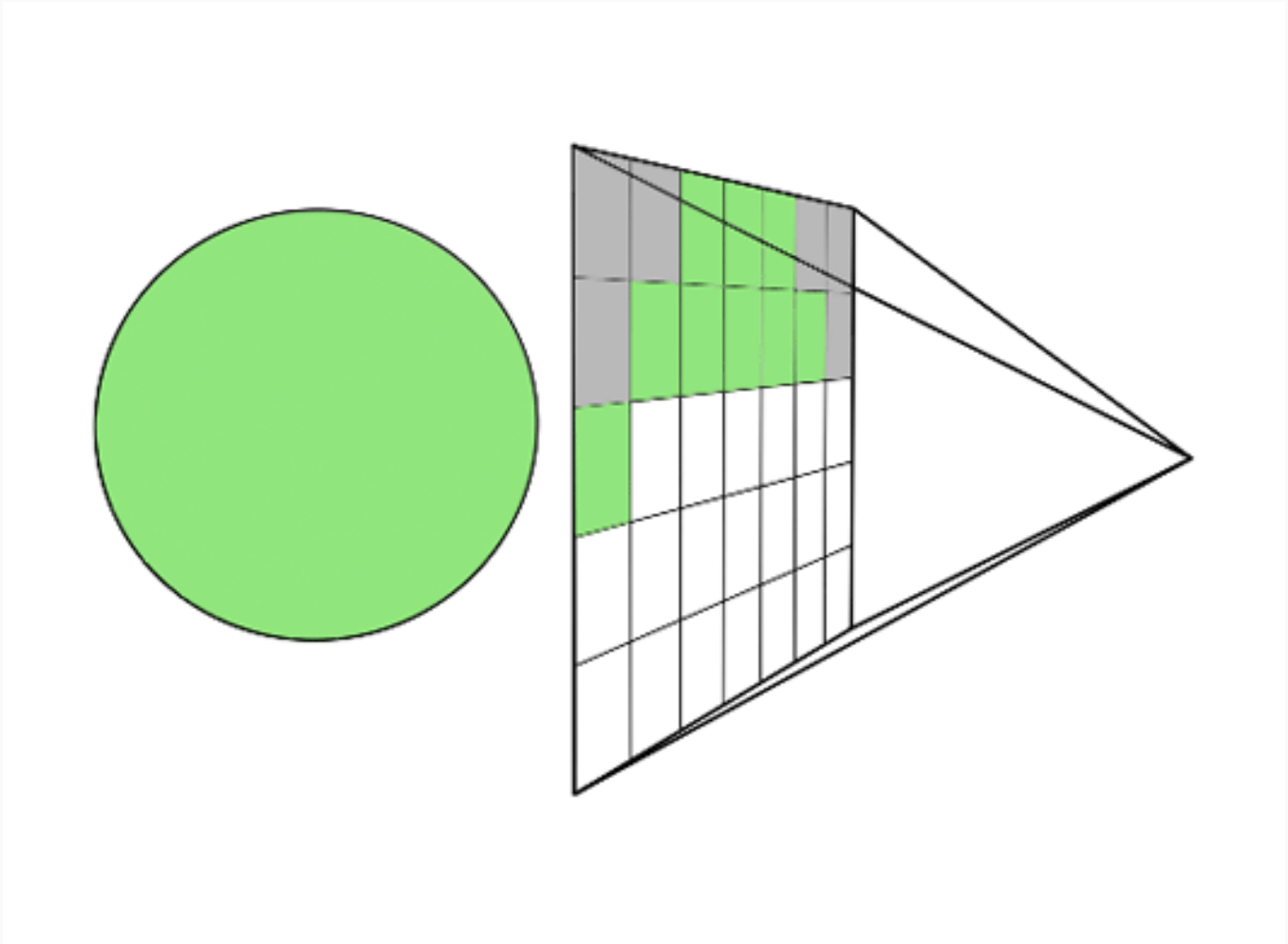
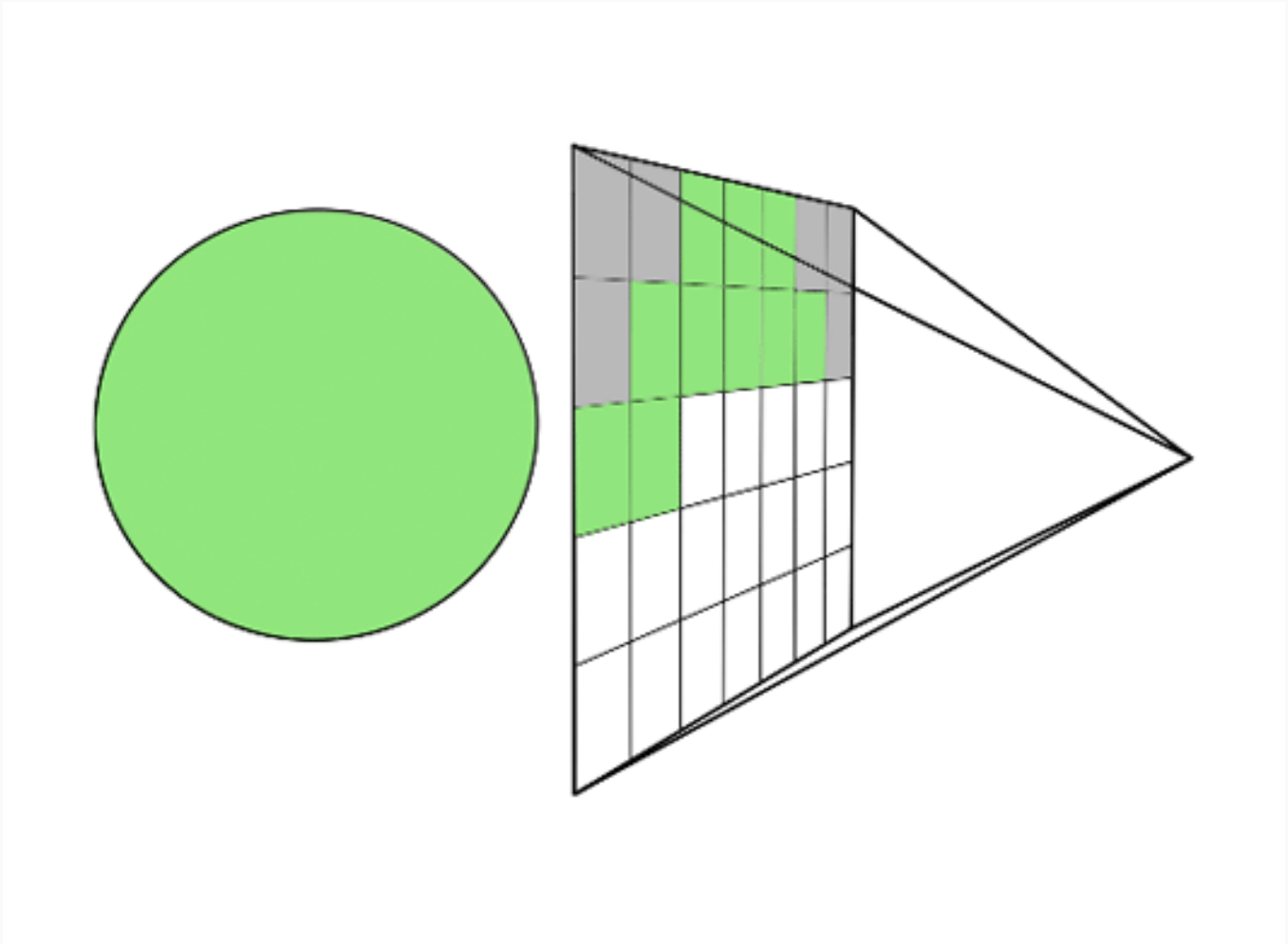
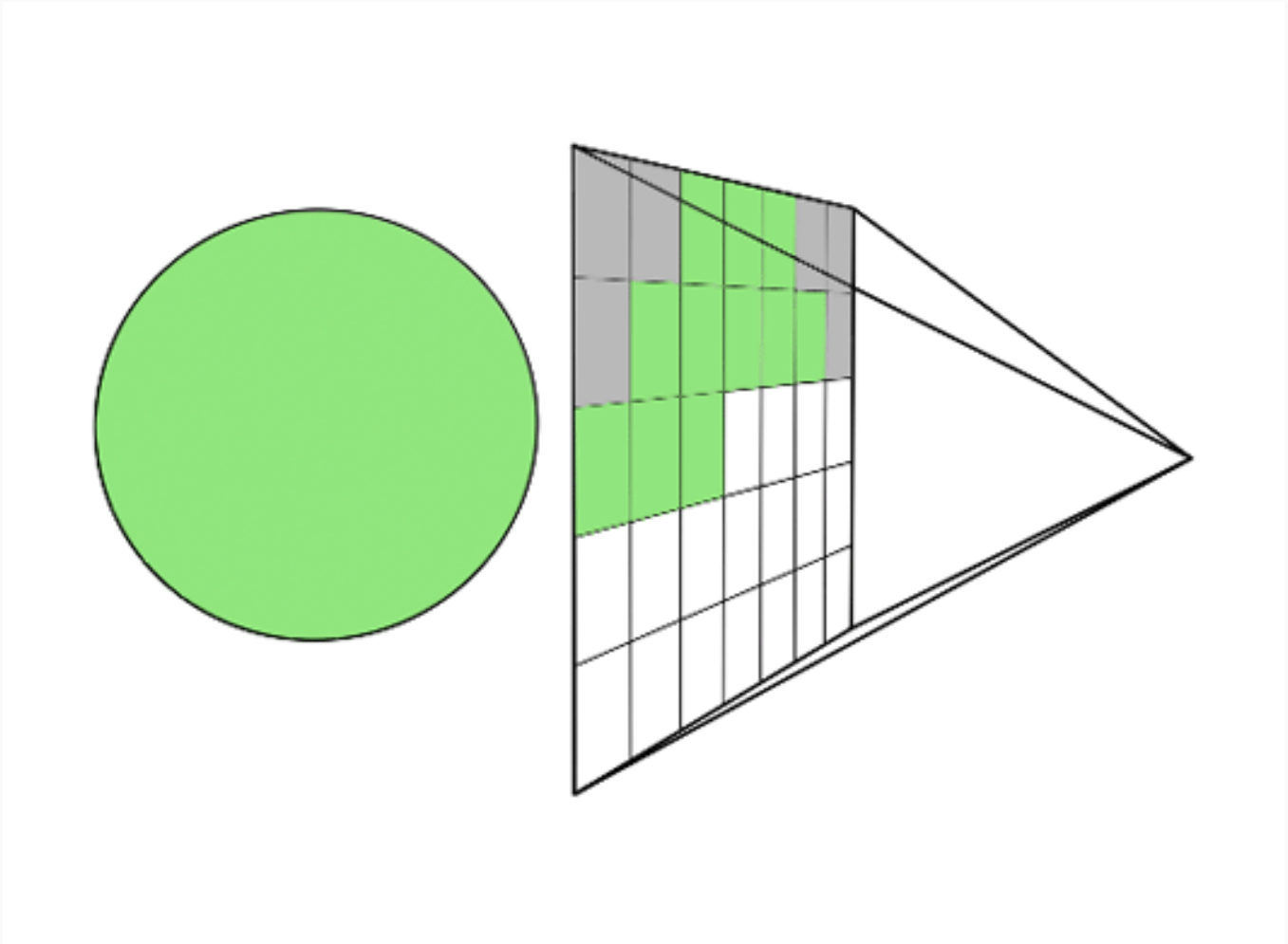


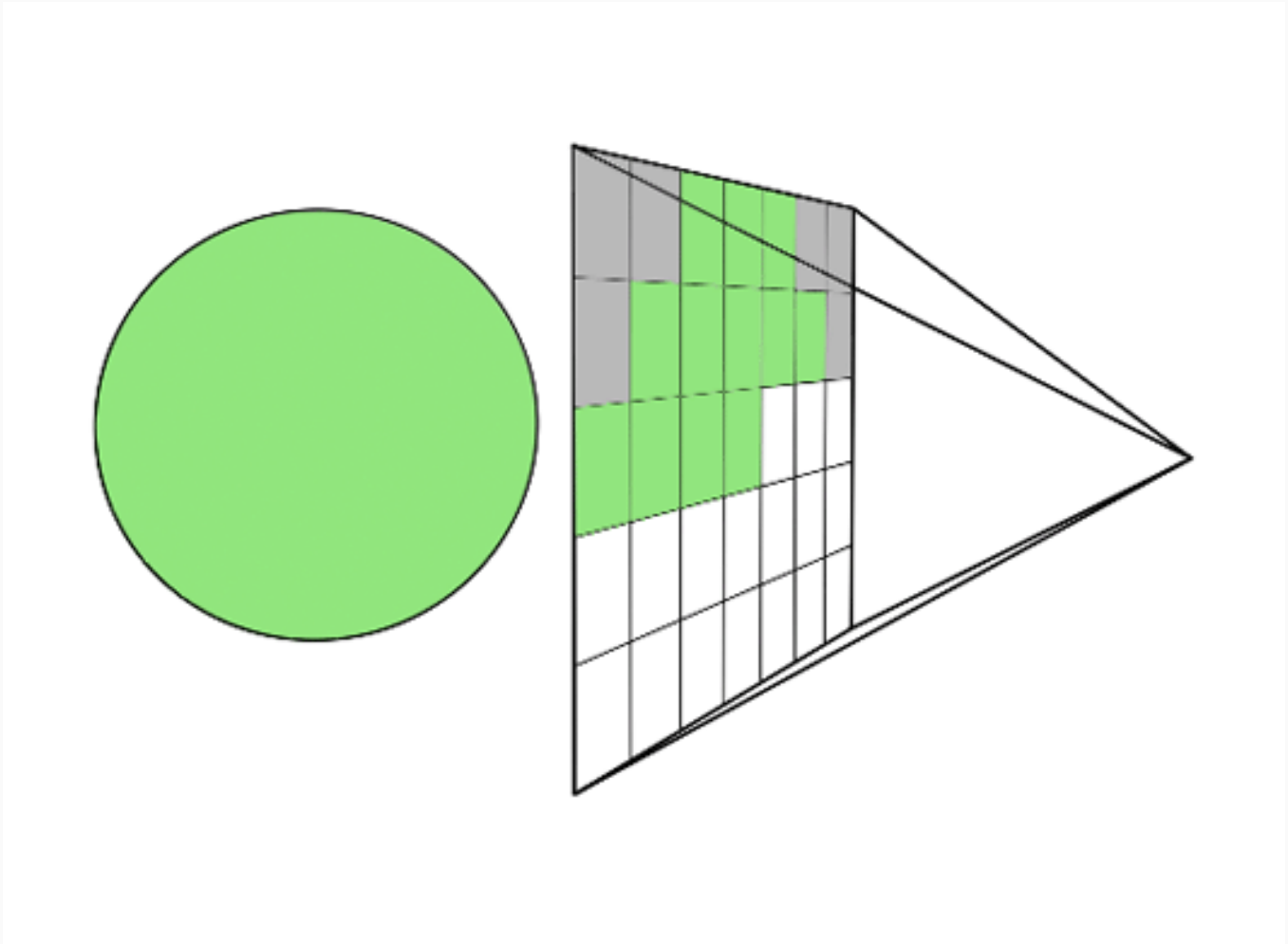
Raycasting Anim



Raycasting Anim



Raycasting Anim



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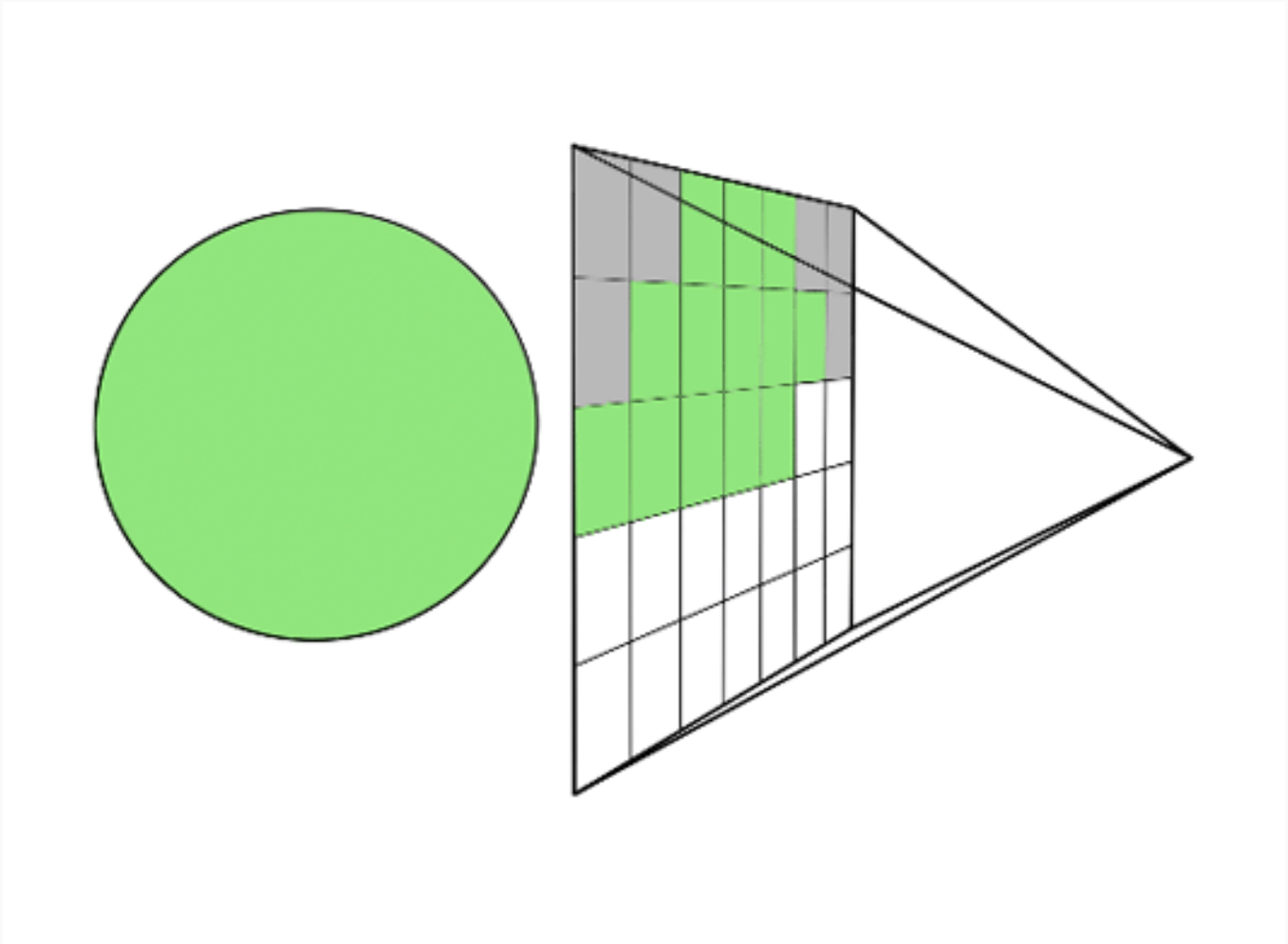
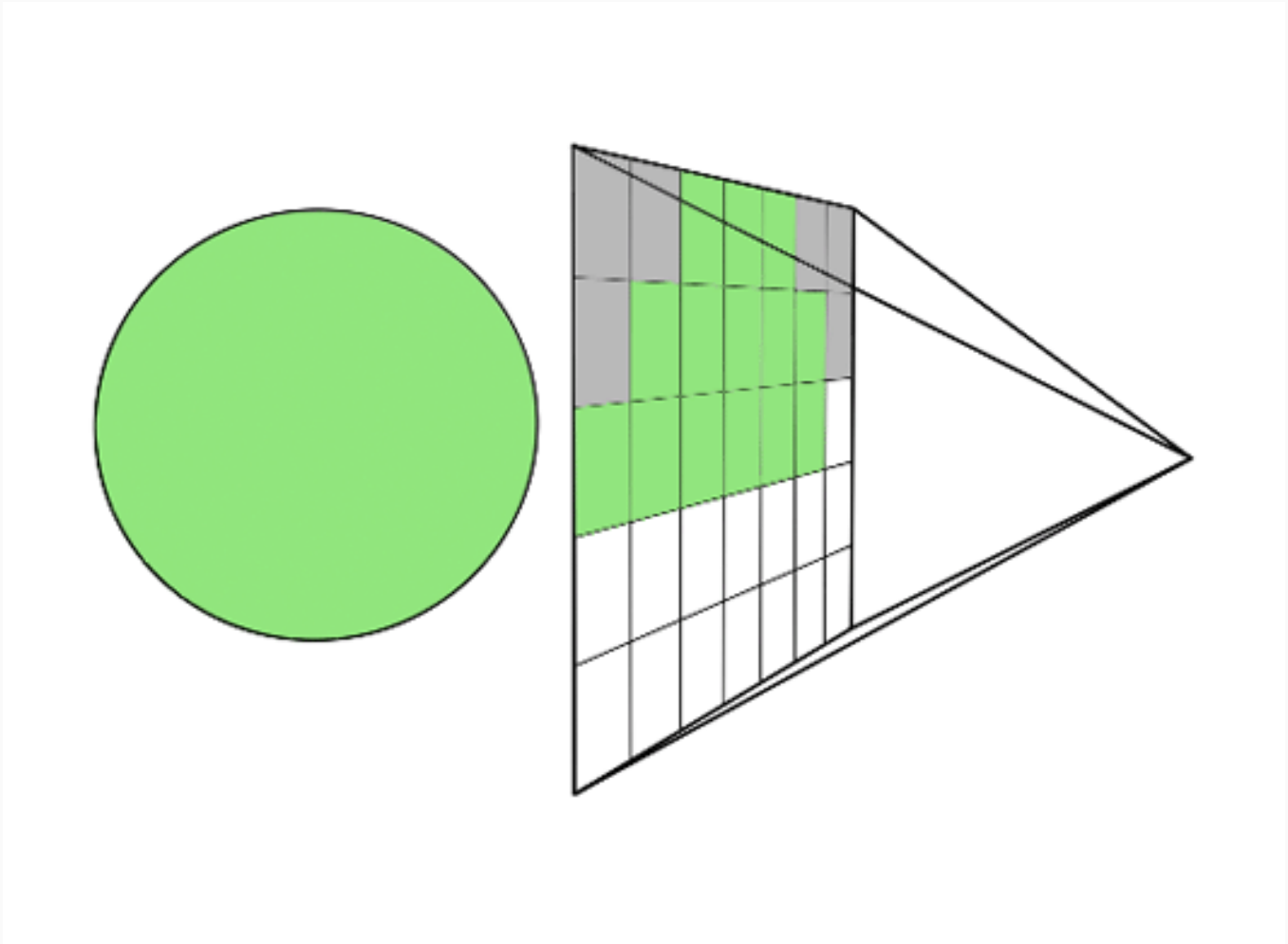
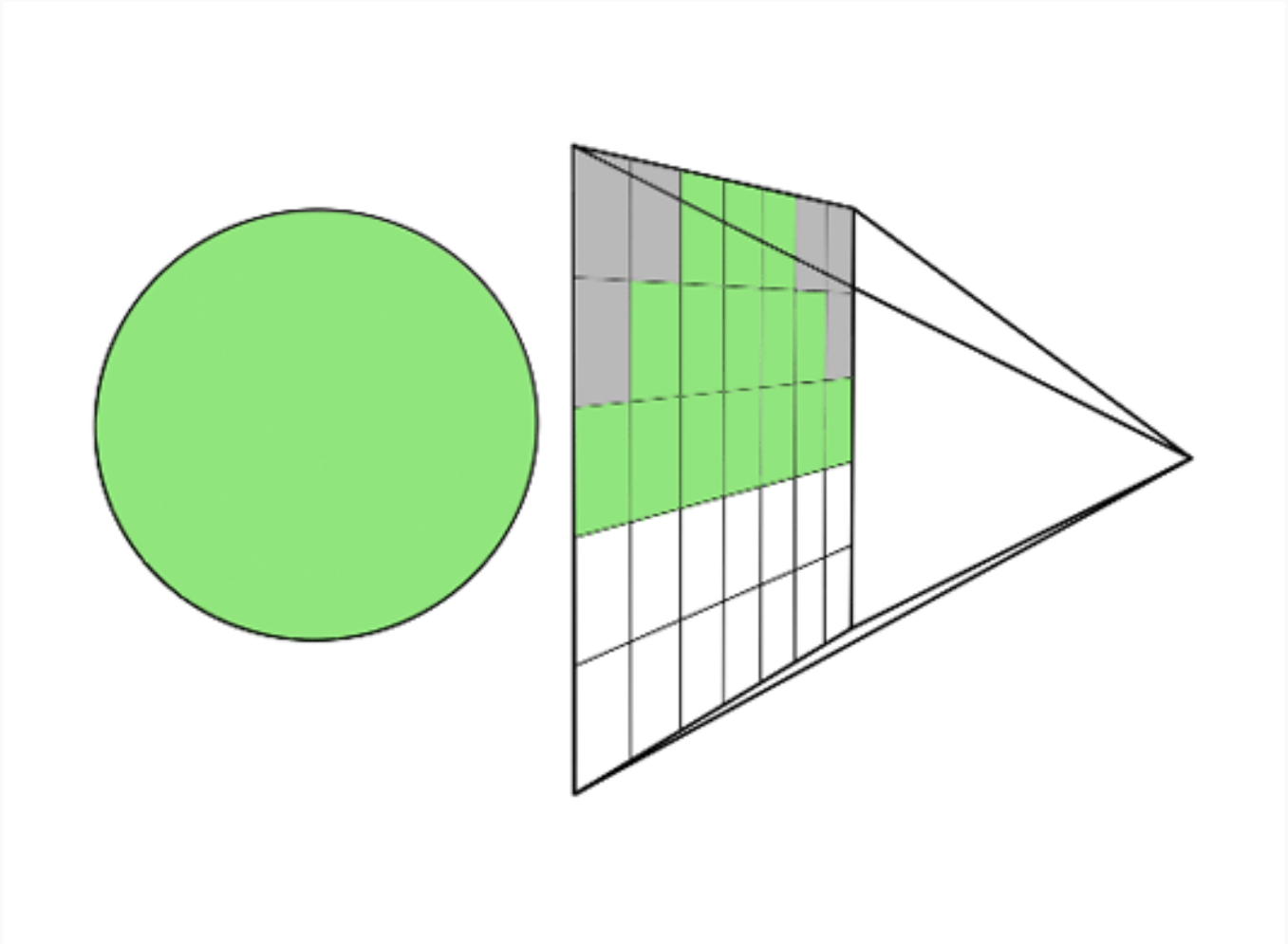


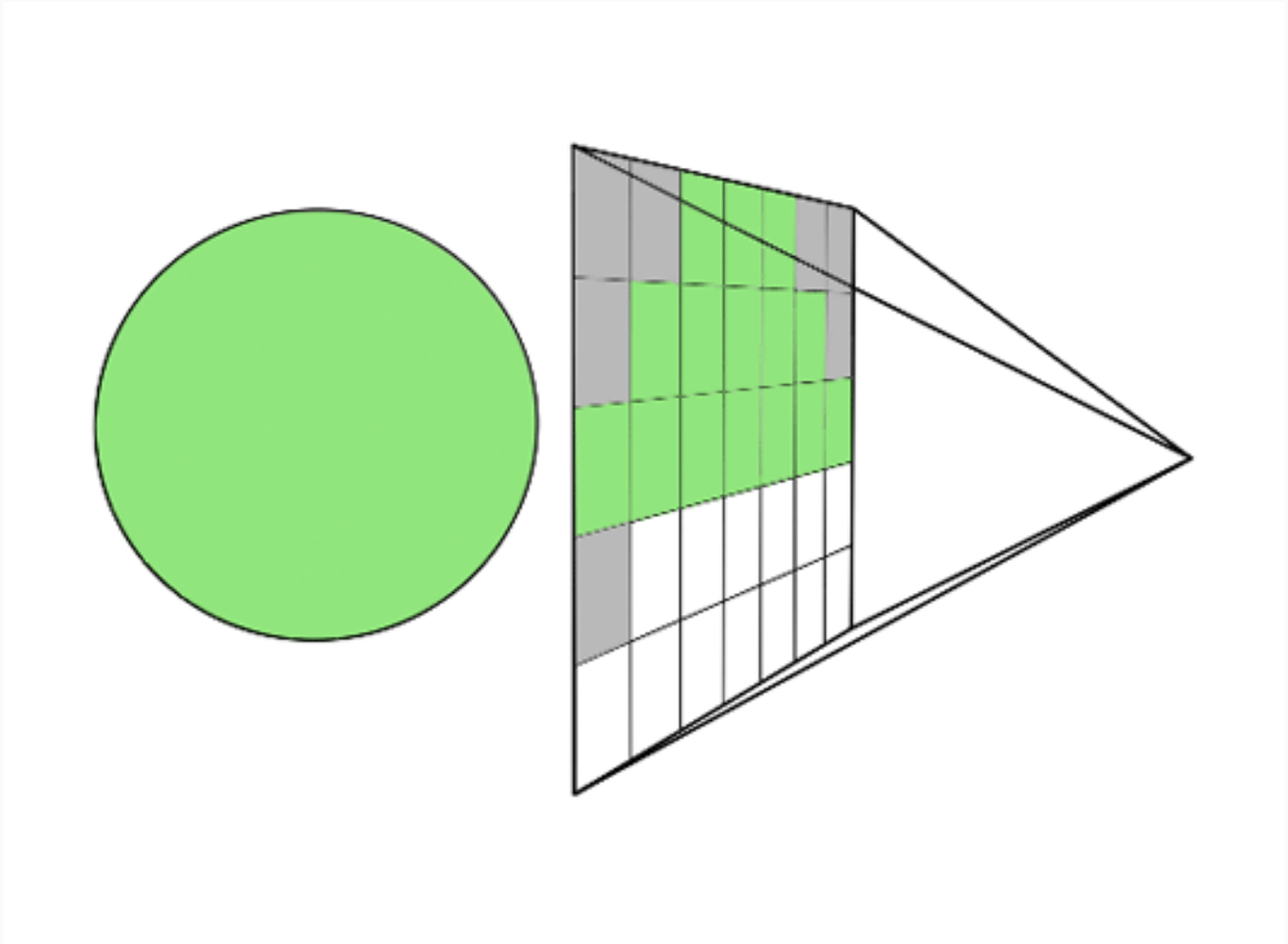
Raycasting Anim



Raycasting Anim



Raycasting Anim



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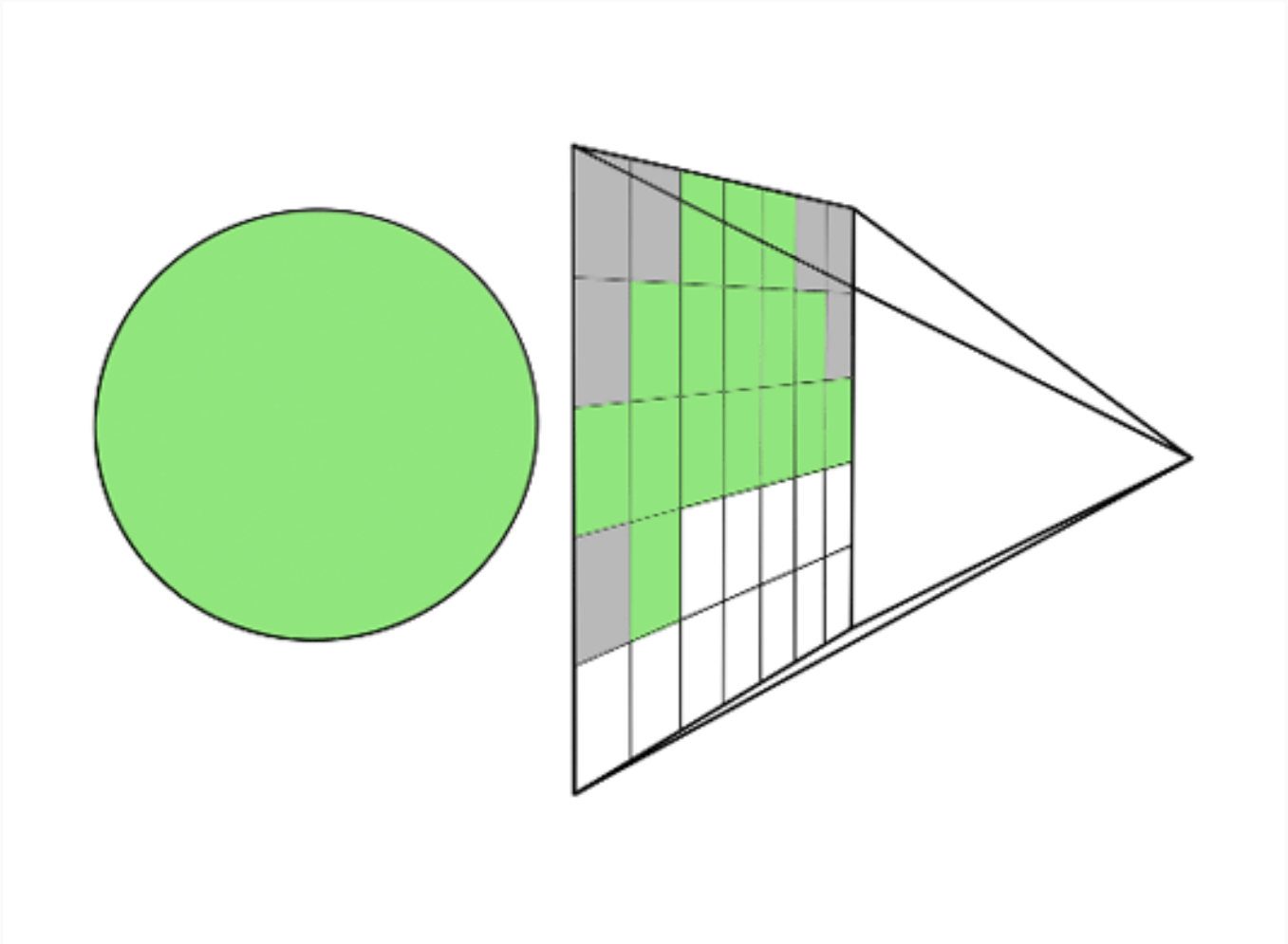
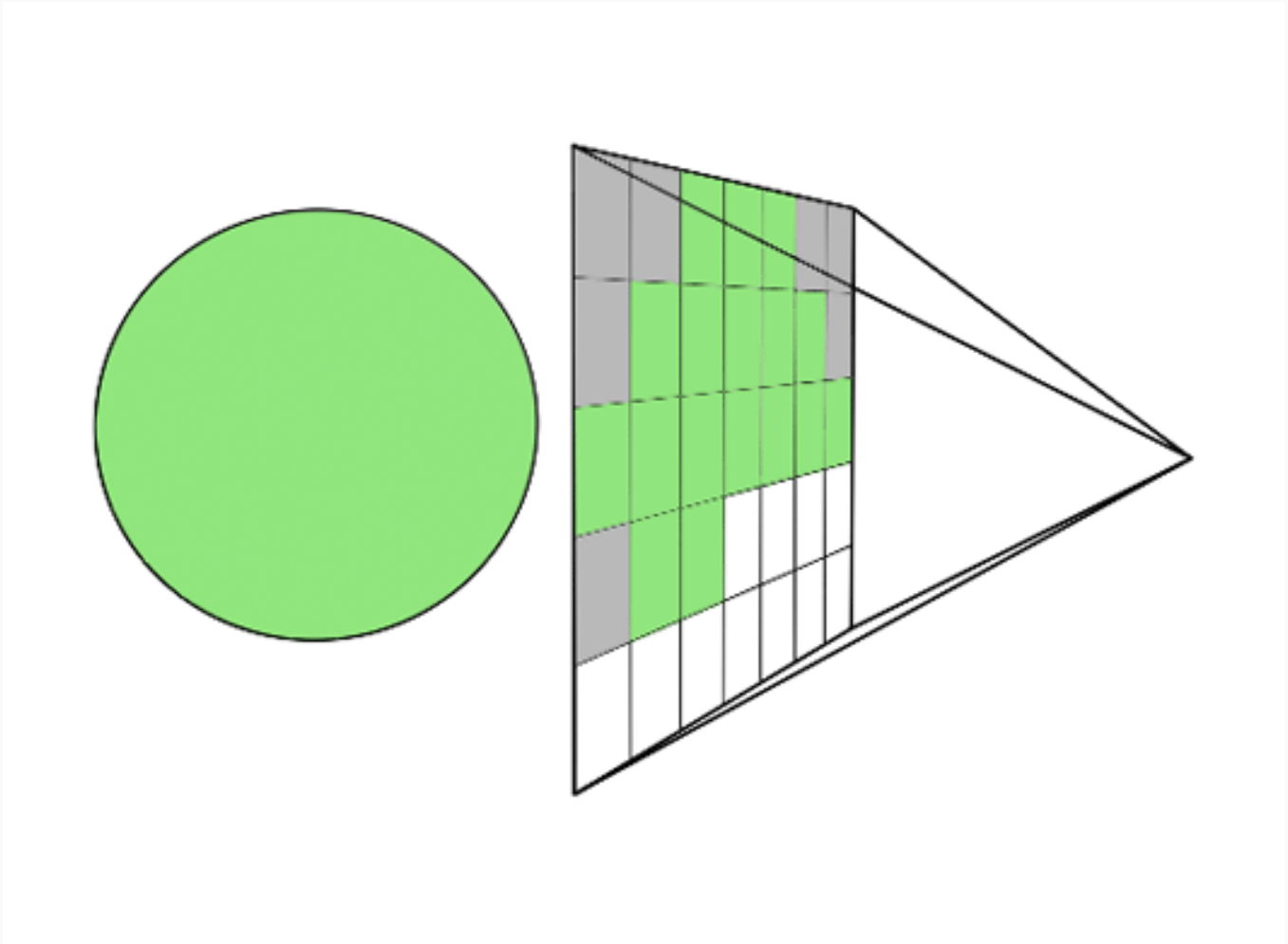
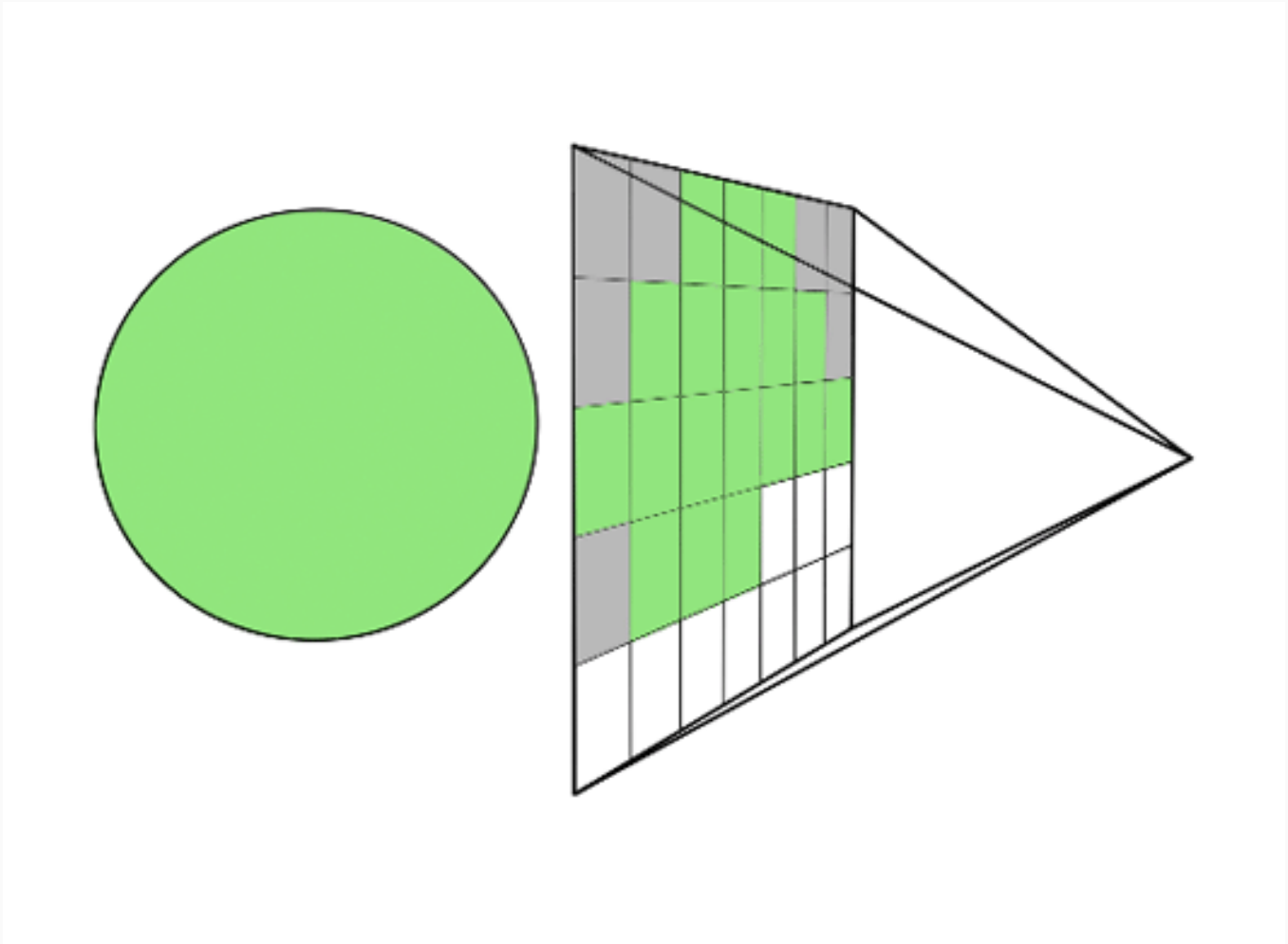


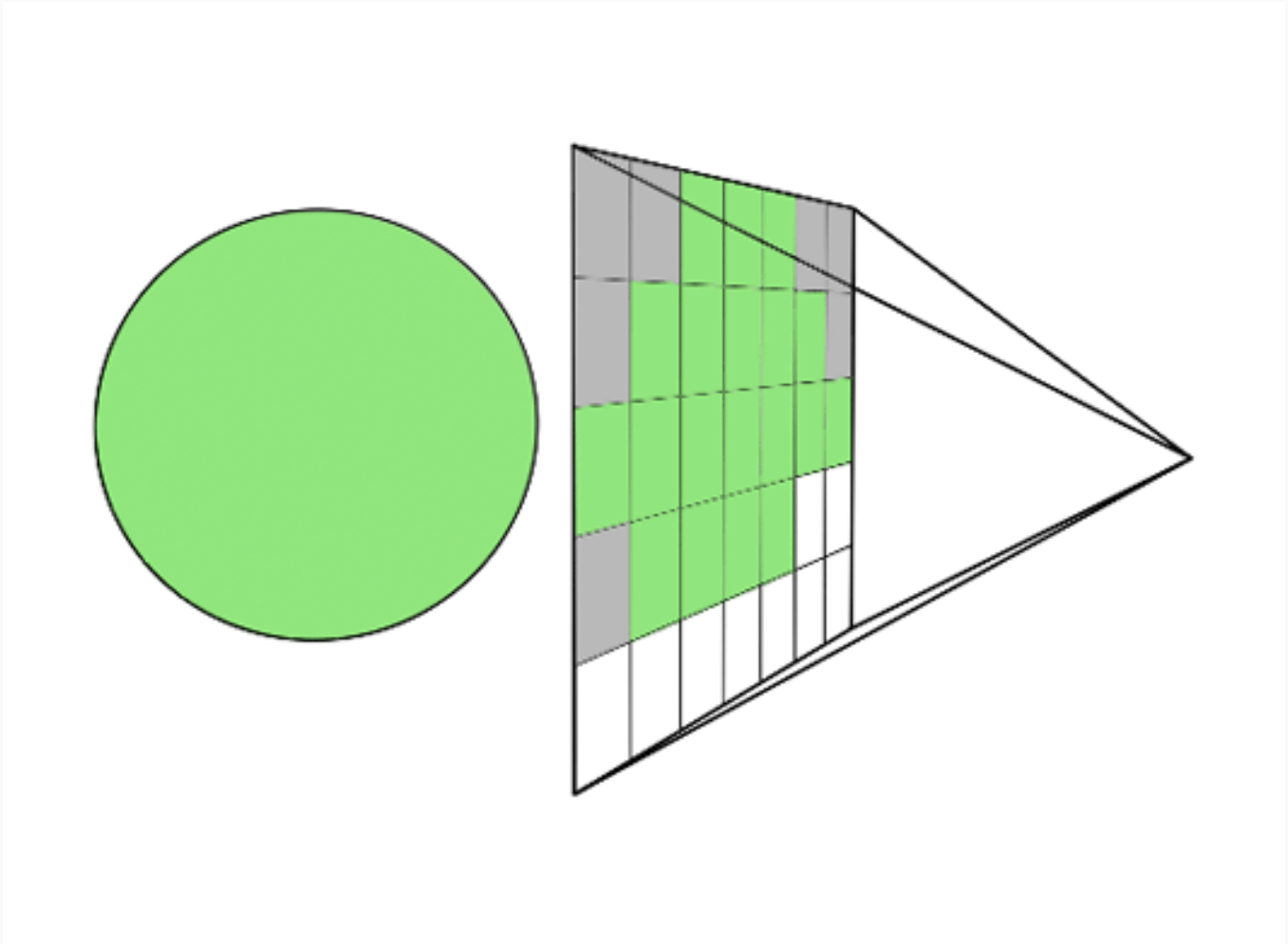
Raycasting Anim



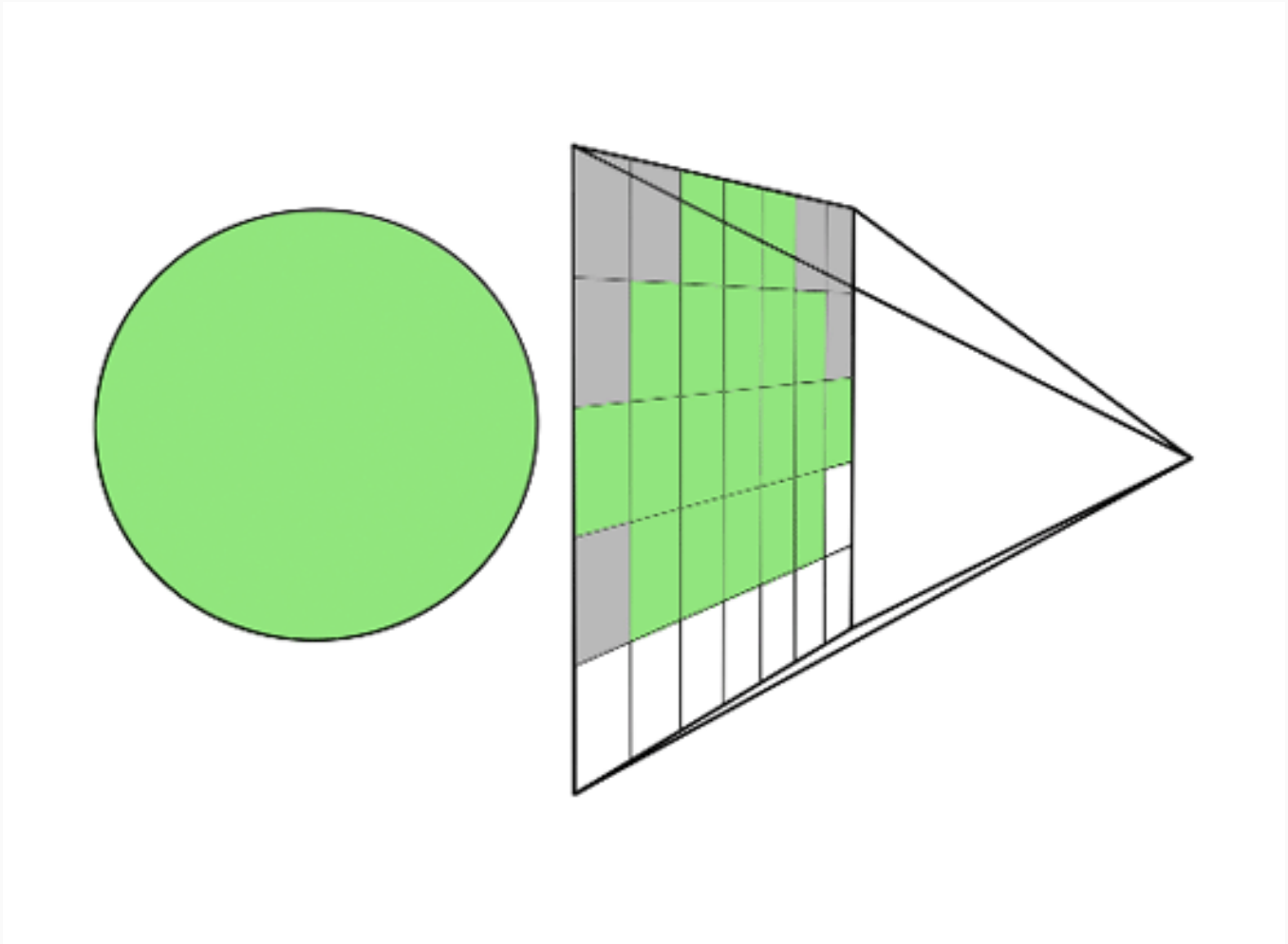
Raycasting Anim



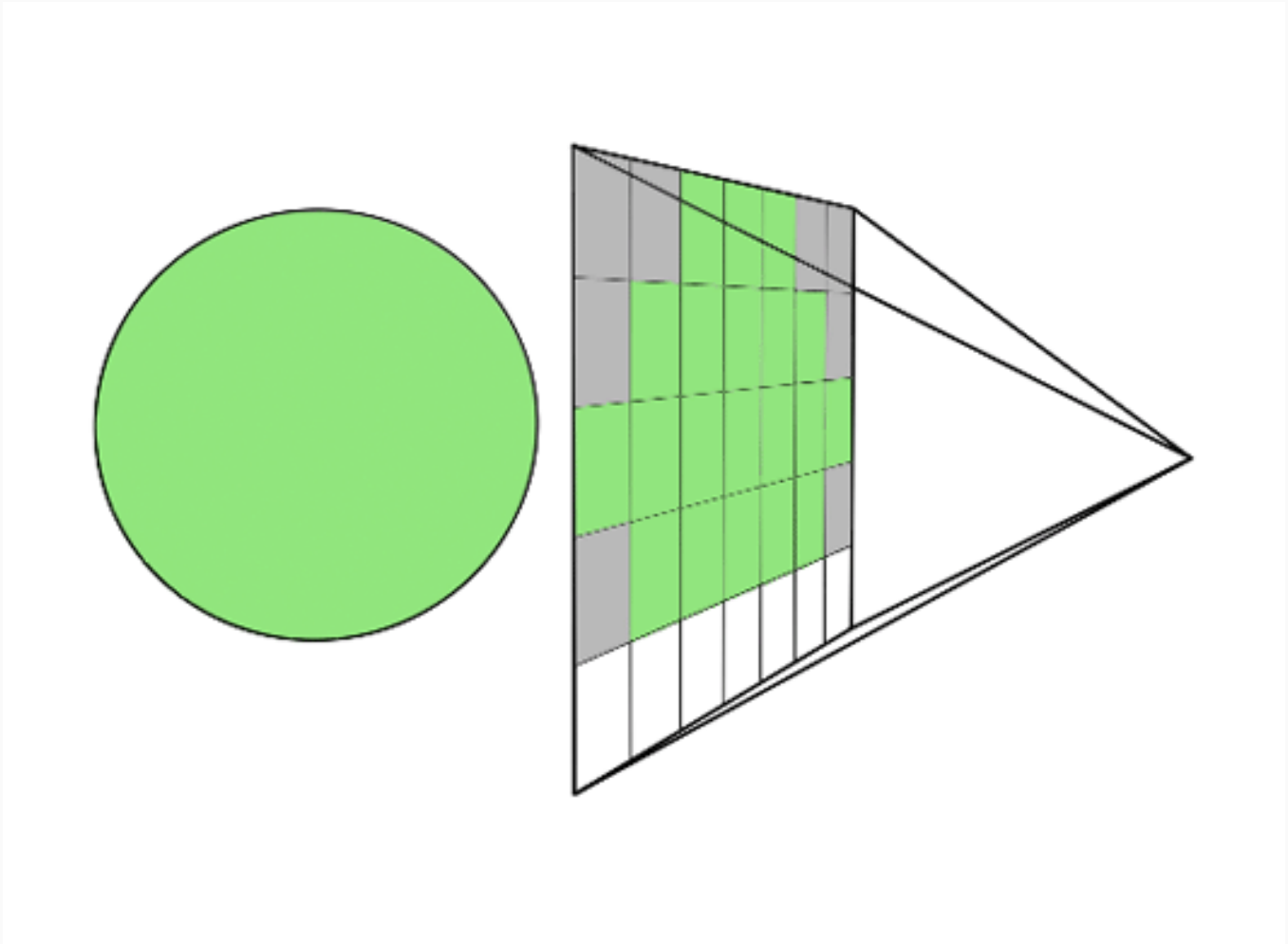
Raycasting Anim



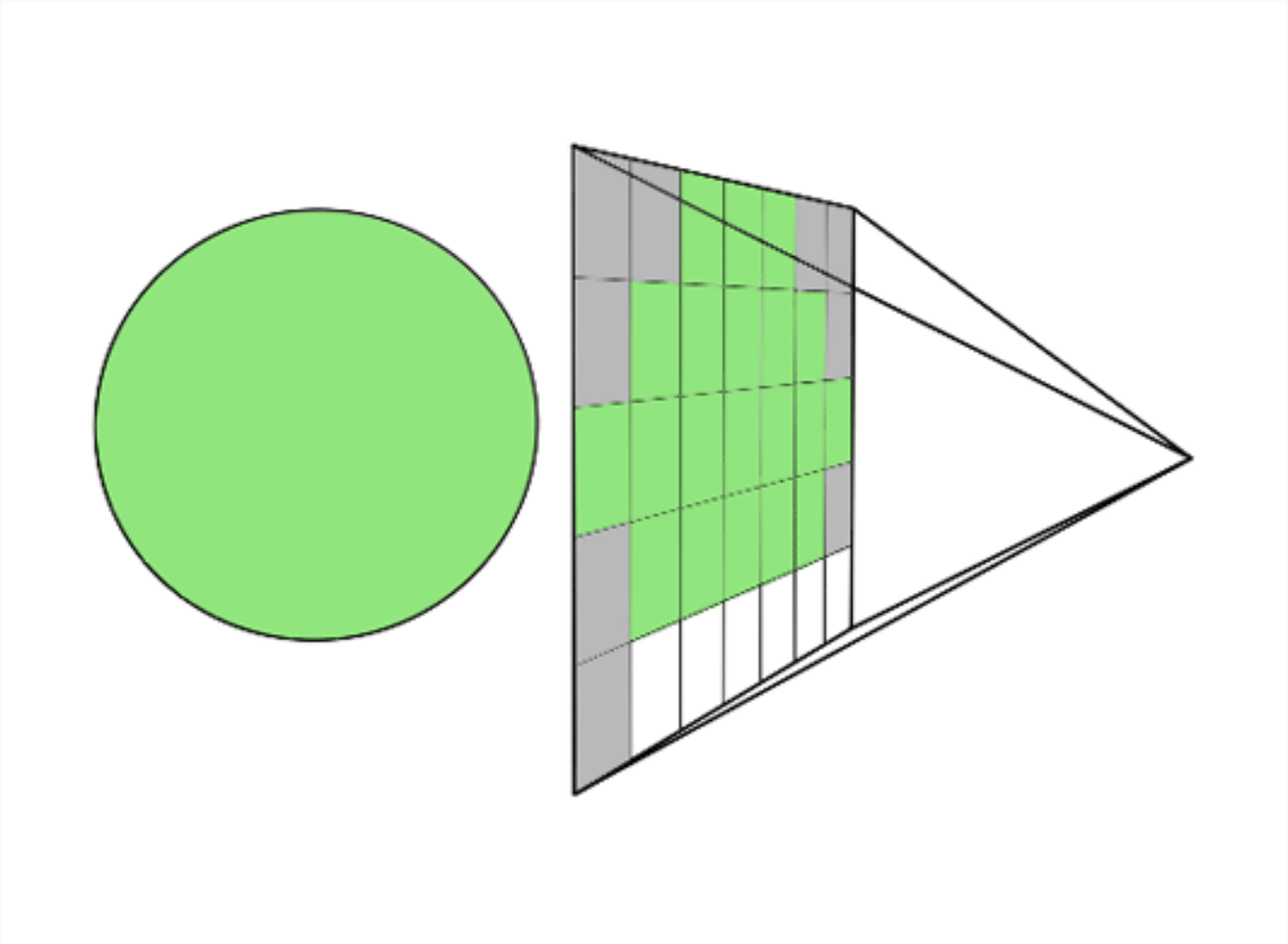
Raycasting Anim



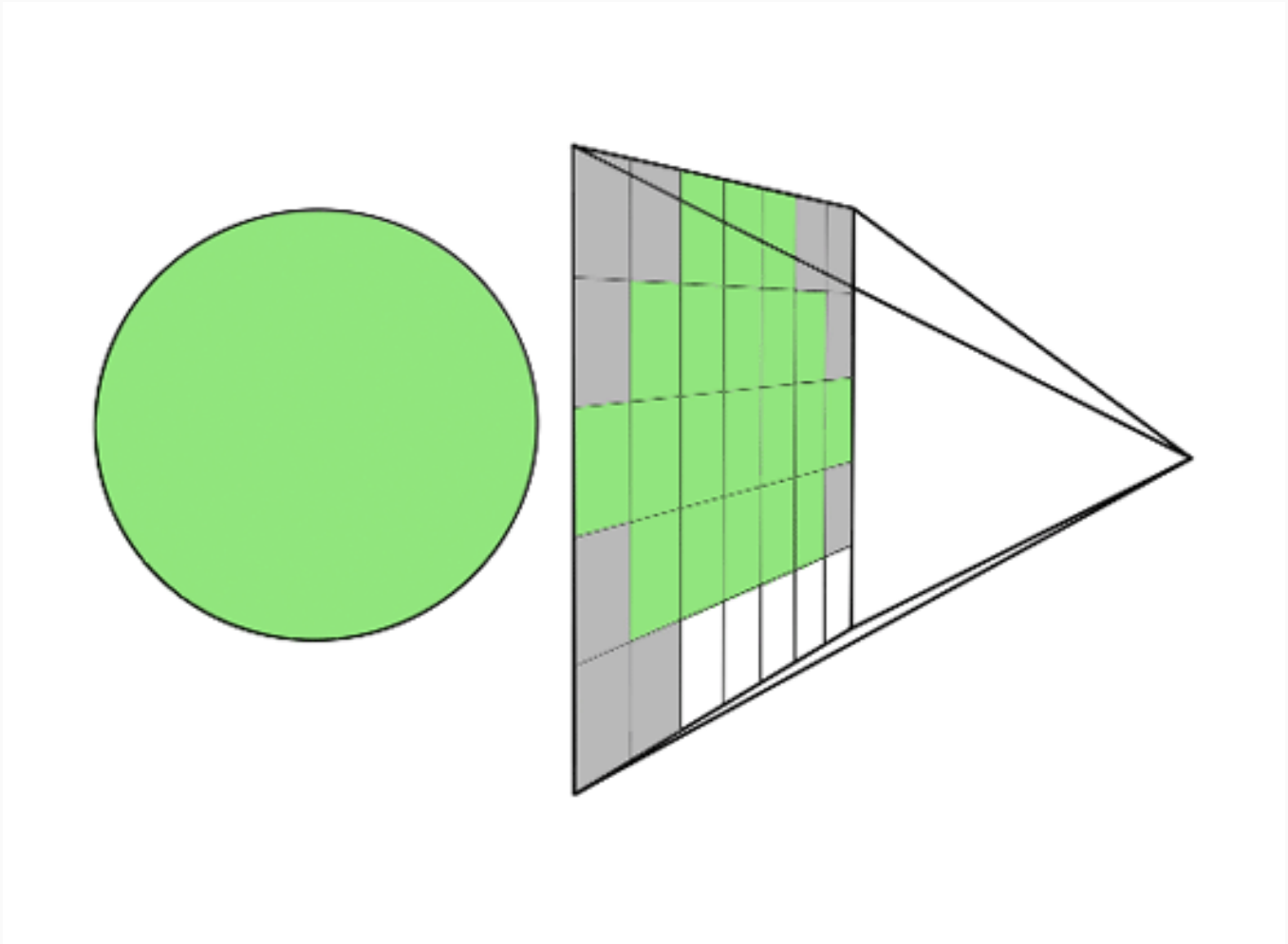
Raycasting Anim



Raycasting Anim



Raycasting Anim



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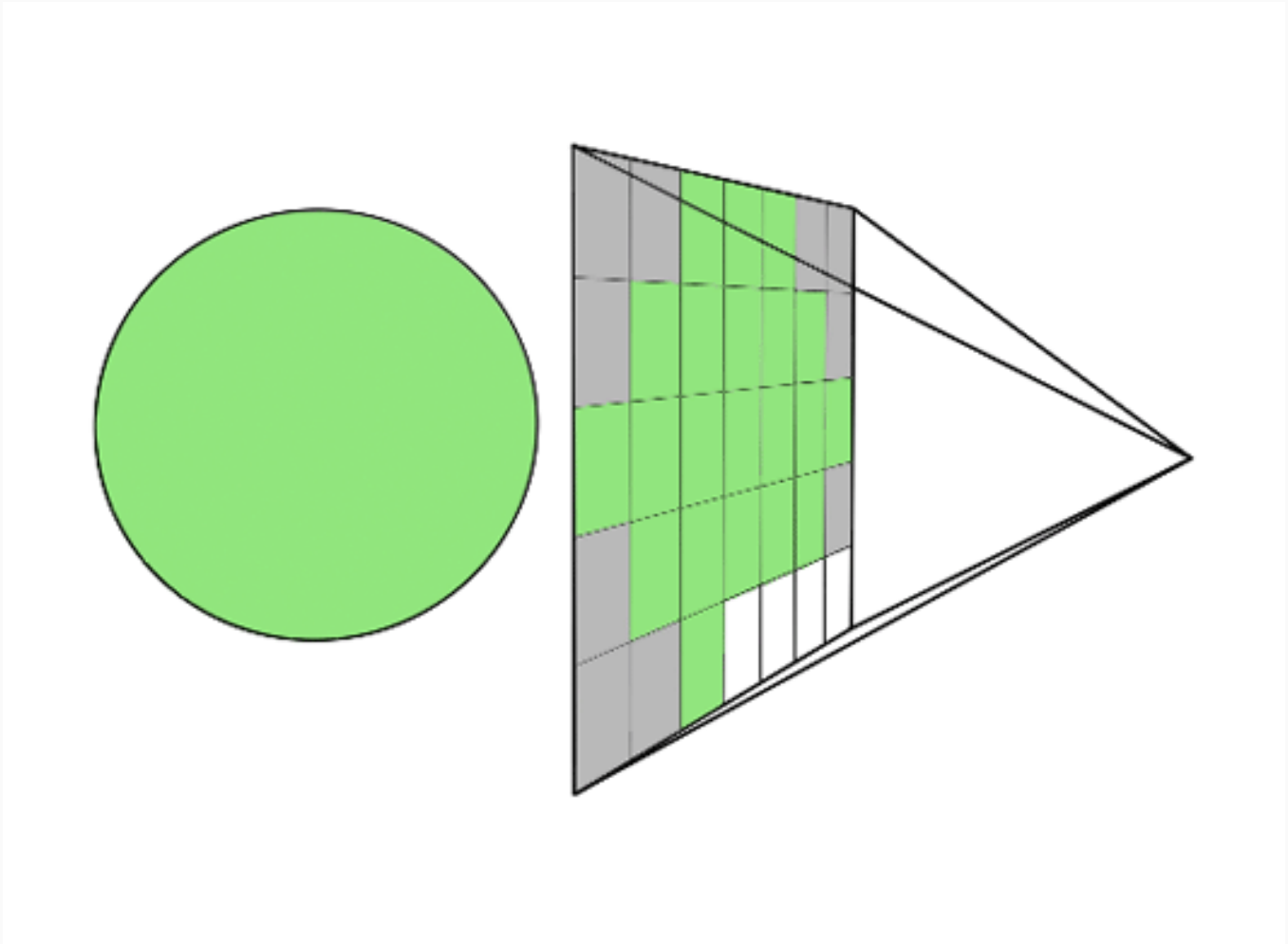
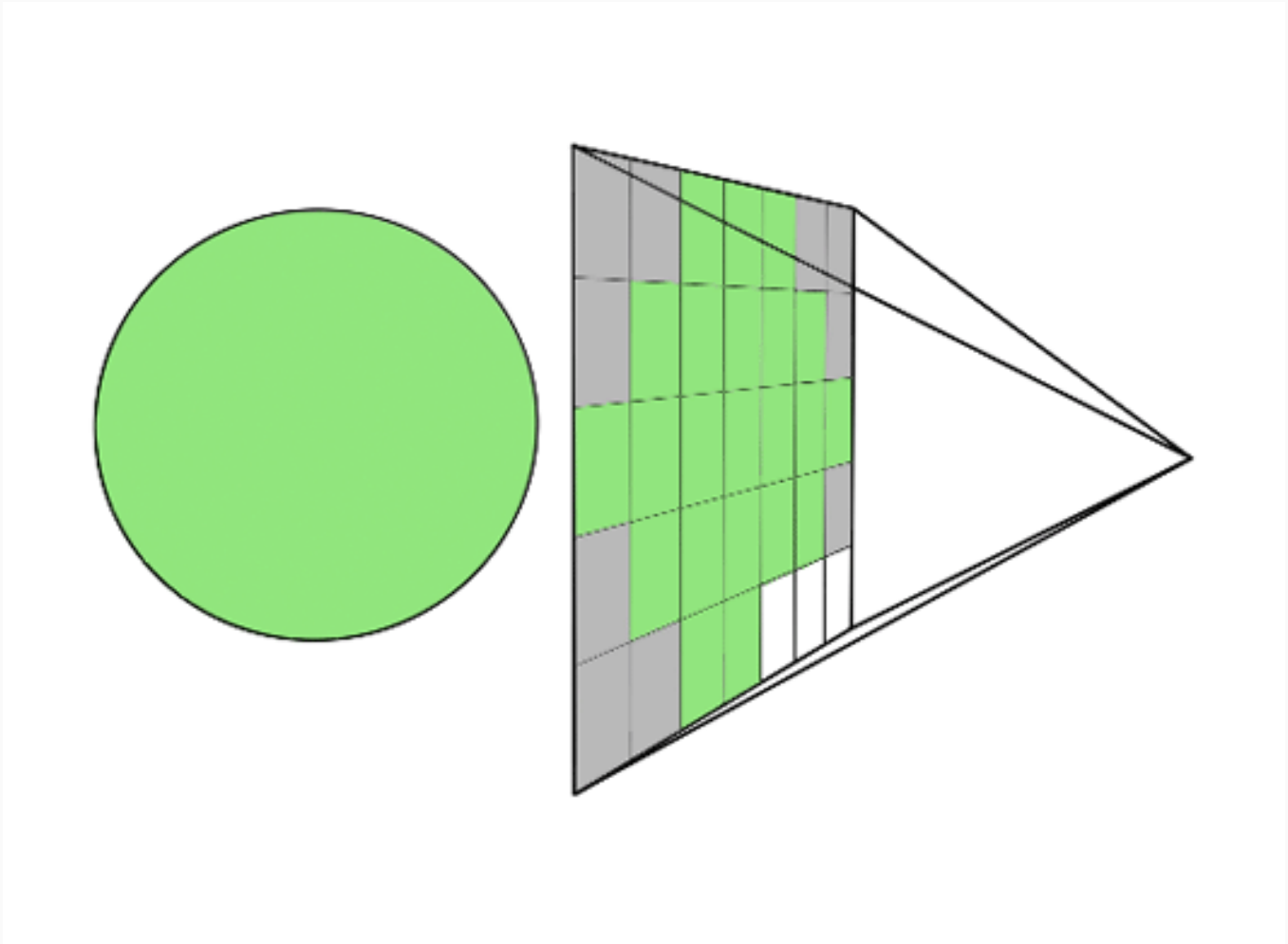
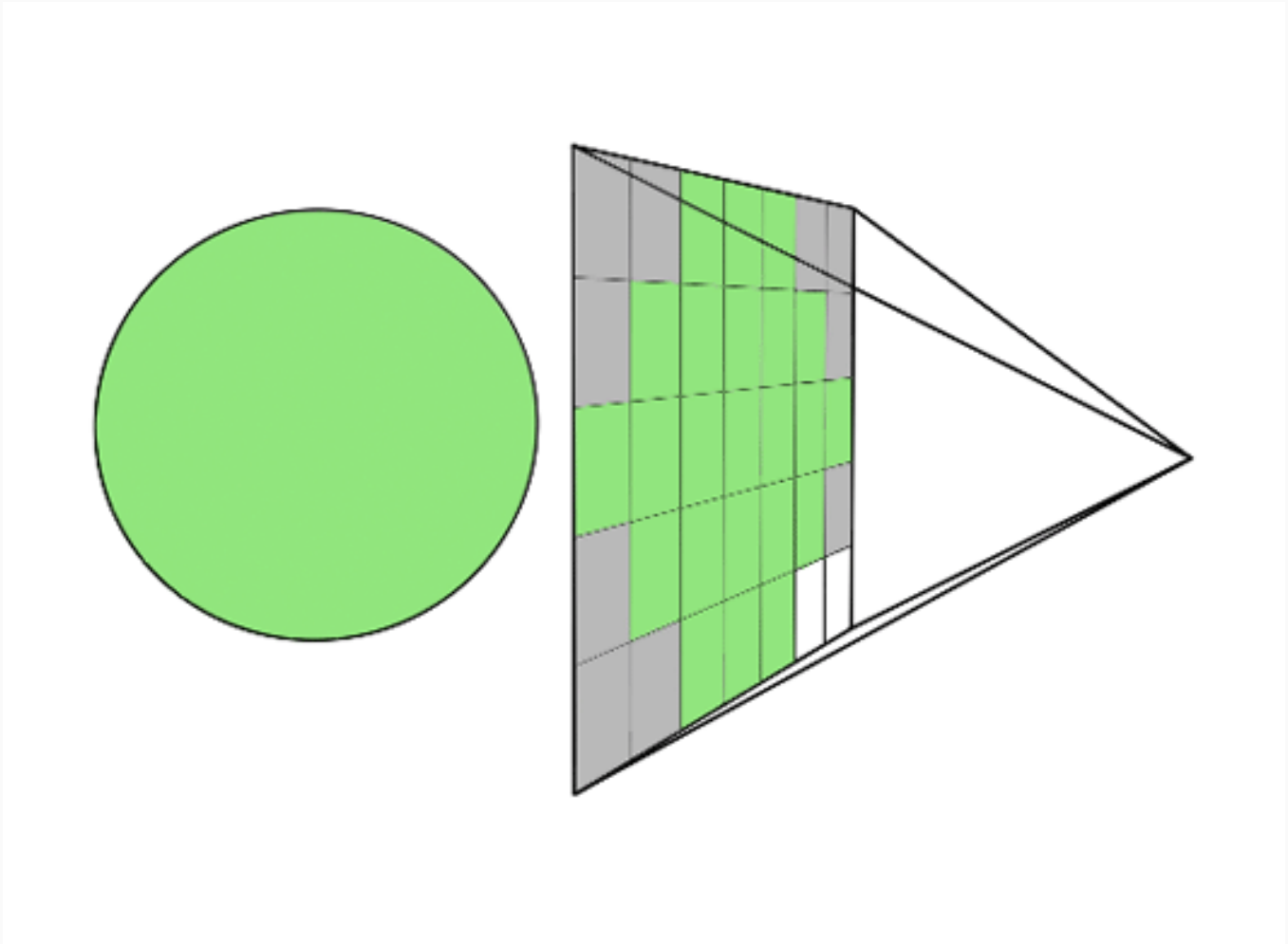


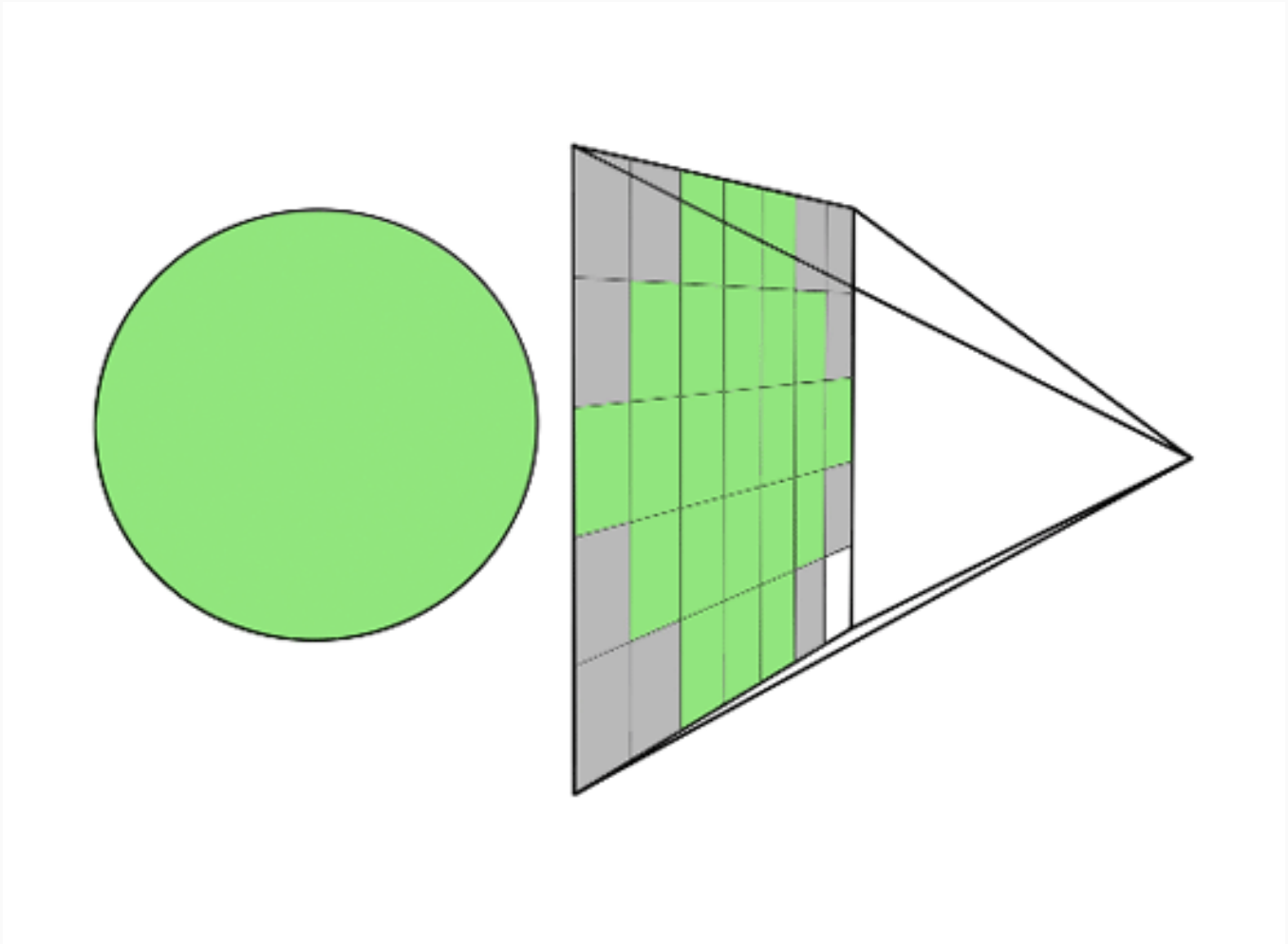
Raycasting Anim



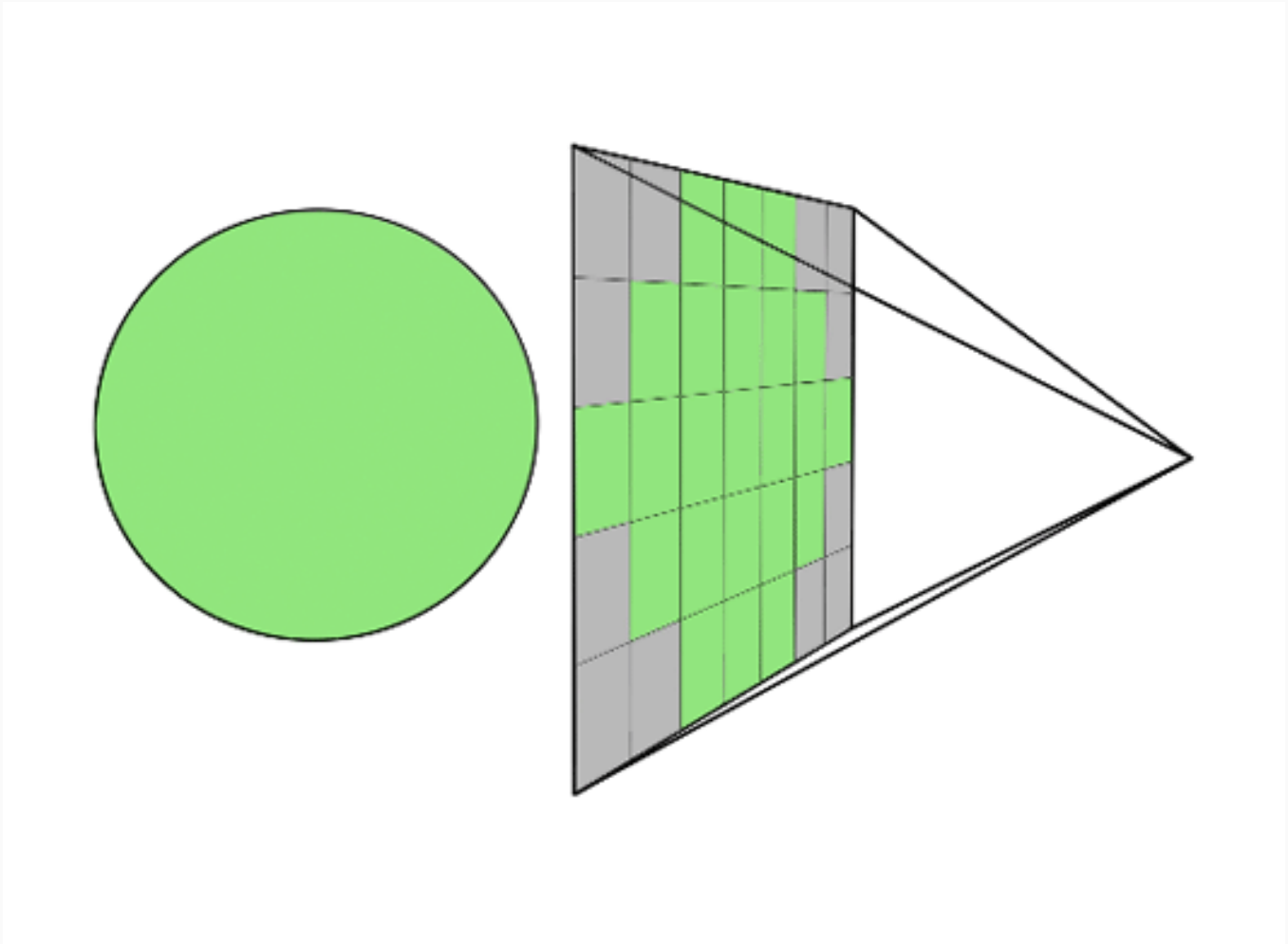
Raycasting Anim



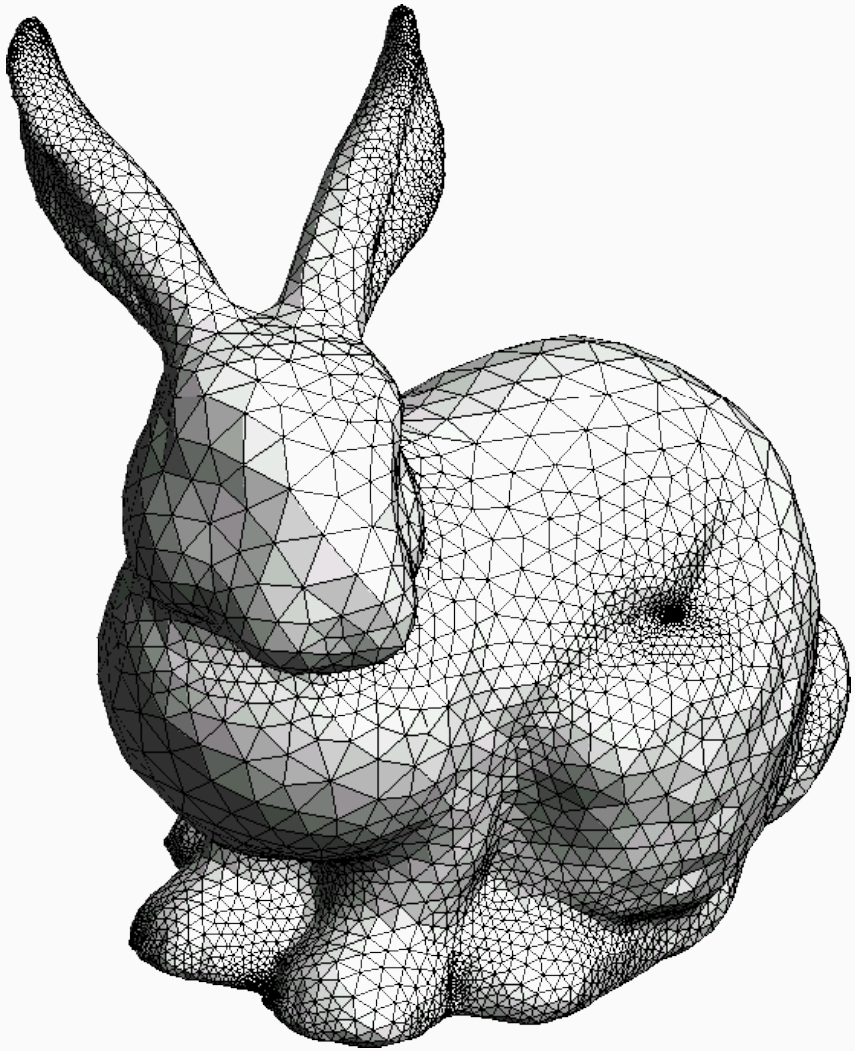
Raycasting Anim



Raycasting Anim



Intersections

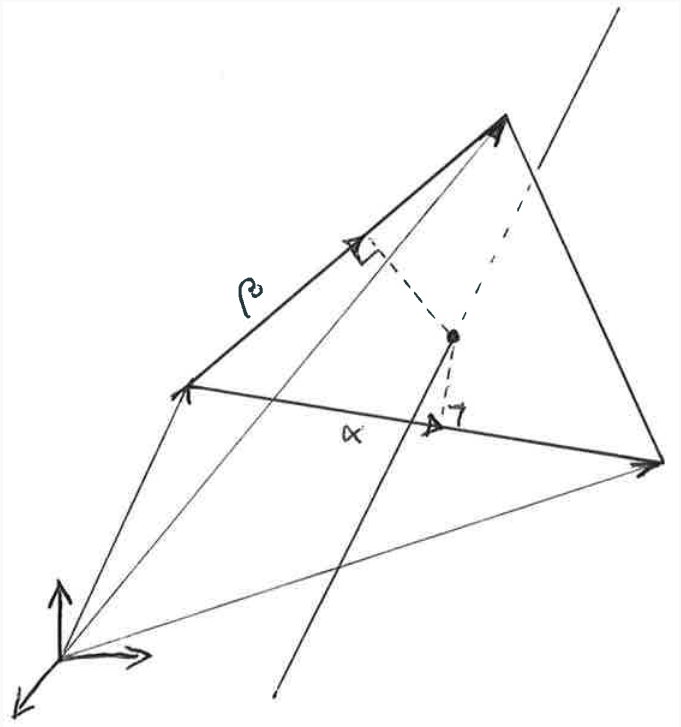


Ray-Triangel Intersection

- Compute two vectors in plane

$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$

$$\mathbf{e}_2 = \mathbf{v}_2 - \mathbf{v}_0$$



Ray-Triangel Intersection

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$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$

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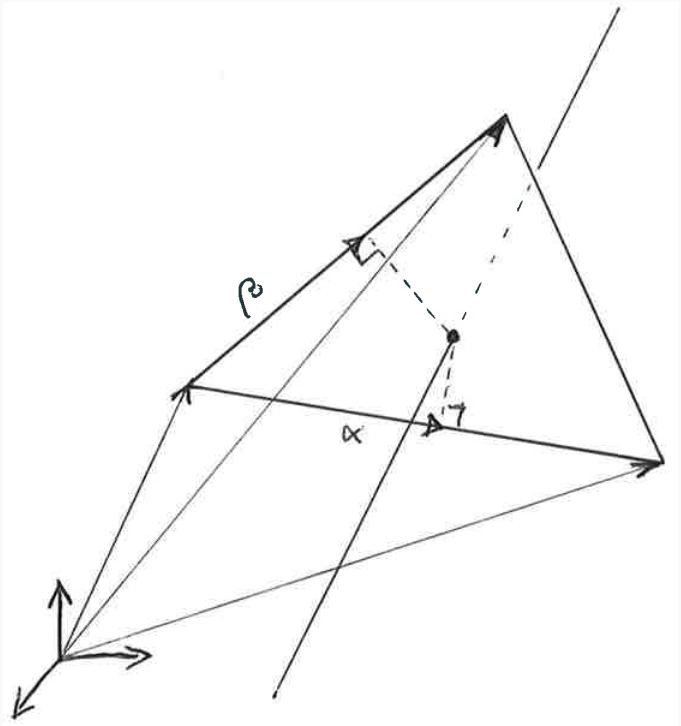
- All points on plane that triangle lies in

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$



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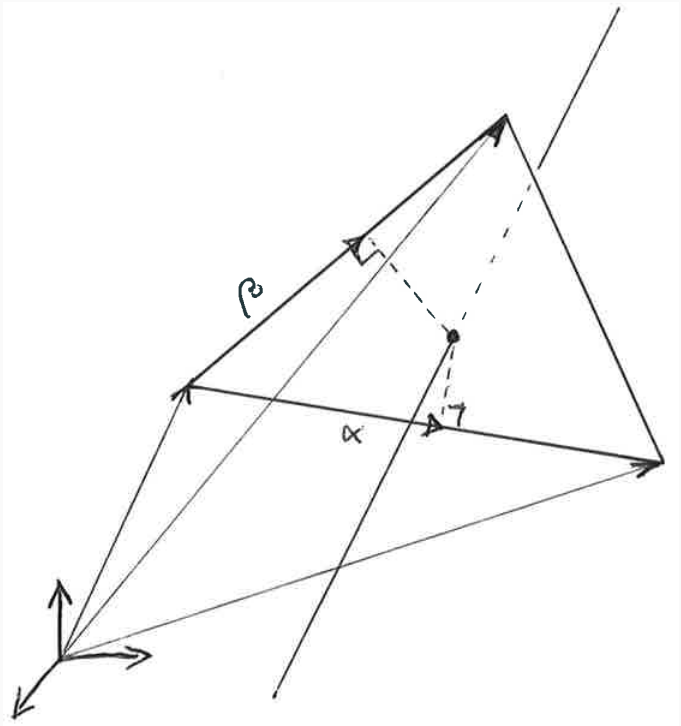
- Two degrees-of-freedom



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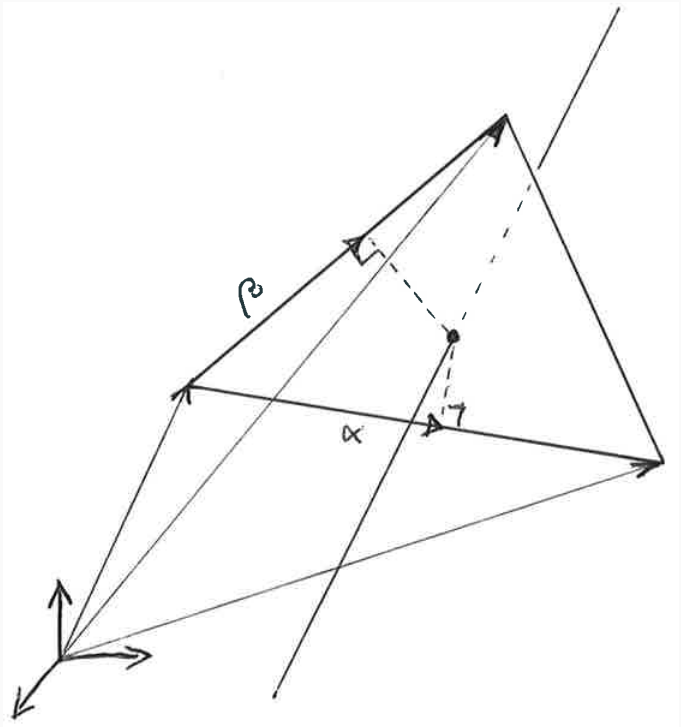
- All points on plane that triangle lies in

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

- Two degrees-of-freedom
- Inside triangle

$$\{u, v | u \geq 0, v \geq 0, v+u \leq 1\}$$



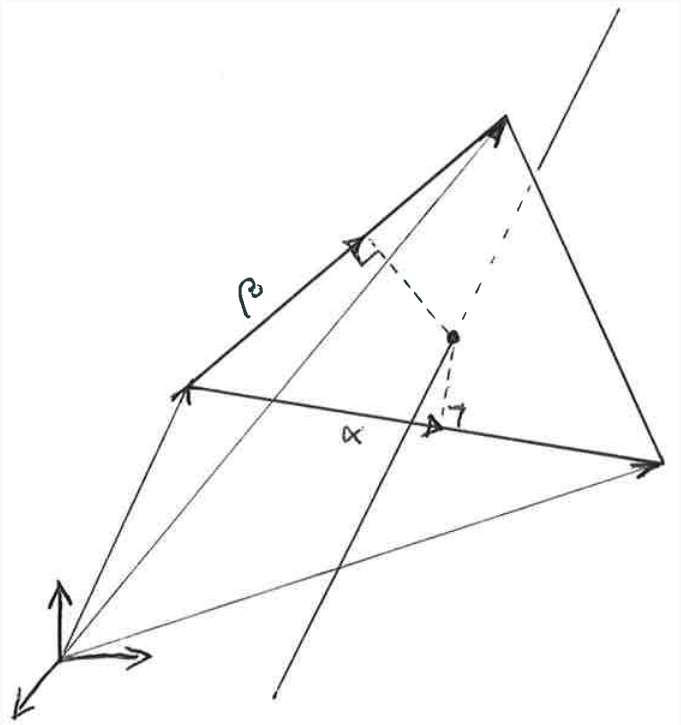
Ray-Triangel Intersection

- Solve for intersection

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$

$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$



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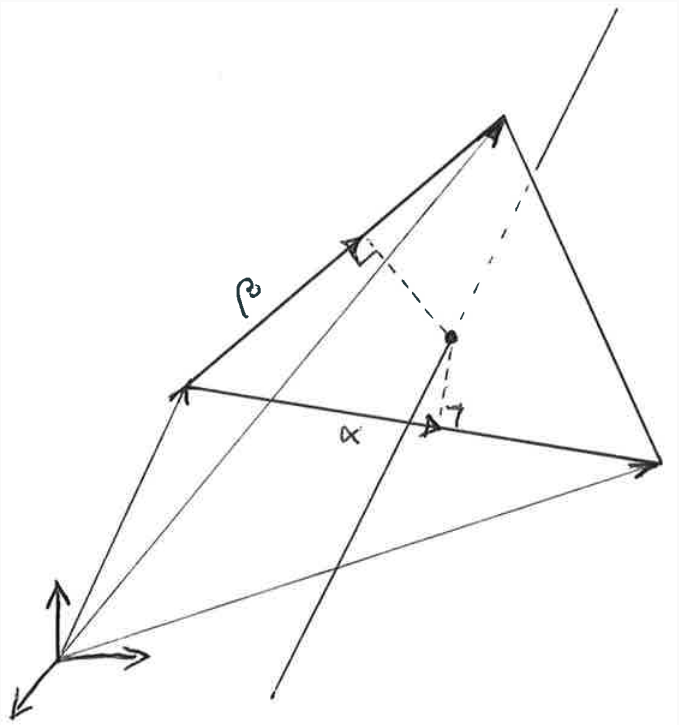
- Unknown: $[u, v, t]^{\mathrm{T}} \in \mathbb{R}^3$



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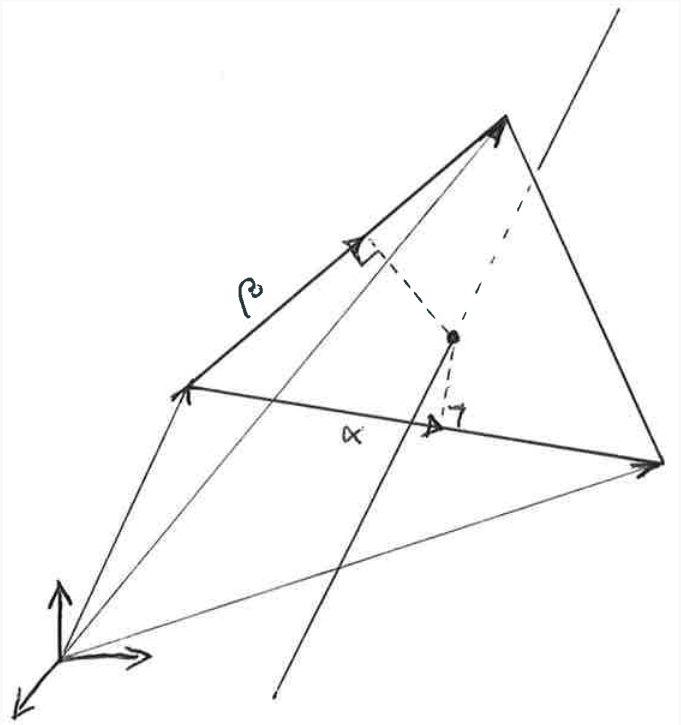
- Three equations three unknowns!



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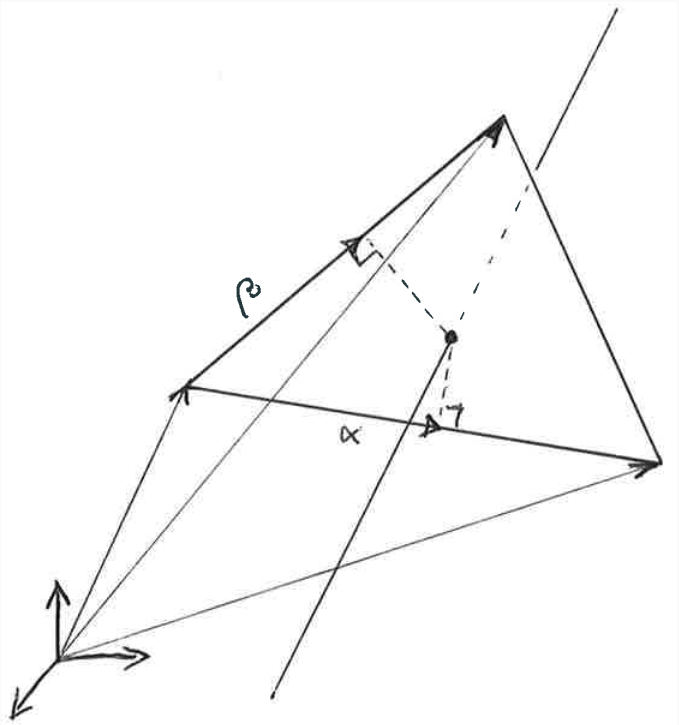
- Intersection of plane



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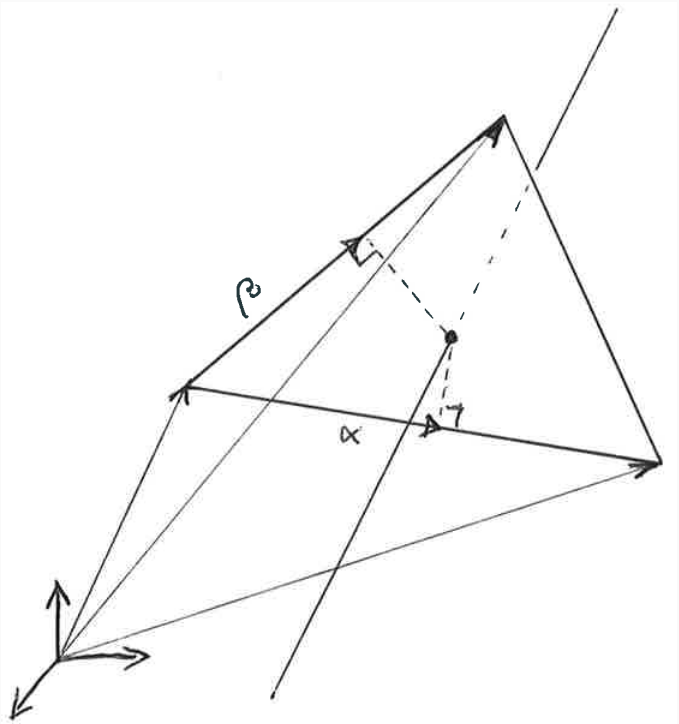
- Unknown: $[u, v, t]^{\mathrm{T}} \in \mathbb{R}^3$
- Three equations three unknowns!
- Intersection of plane
- Check boundary conditions of triangle



Ray-Triangle Intersection

- Ray in plane equation

$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

$$s - \mathbf{v}_0 = -t \cdot d_{ij} + u\mathbf{e}_0 + v\mathbf{e}_1$$

Ray-Triangle Intersection

- Ray in plane equation

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- Write on matrix form

$$\begin{bmatrix} d_{ij}^x & e_0^x & e_1^x \\ d_{ij}^y & e_0^y & e_1^y \\ d_{ij}^z & e_0^z & e_1^z \end{bmatrix} \cdot \begin{bmatrix} t \\ u \\ v \end{bmatrix} = \begin{bmatrix} s^x - v_0^x \\ s^y - v_0^y \\ s^z - v_0^z \end{bmatrix}$$

Ray-Triangle Intersection

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- This we know from basic math as

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

Ray-Triangle Intersection

- Ray in plane equation

$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

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- Write on matrix form

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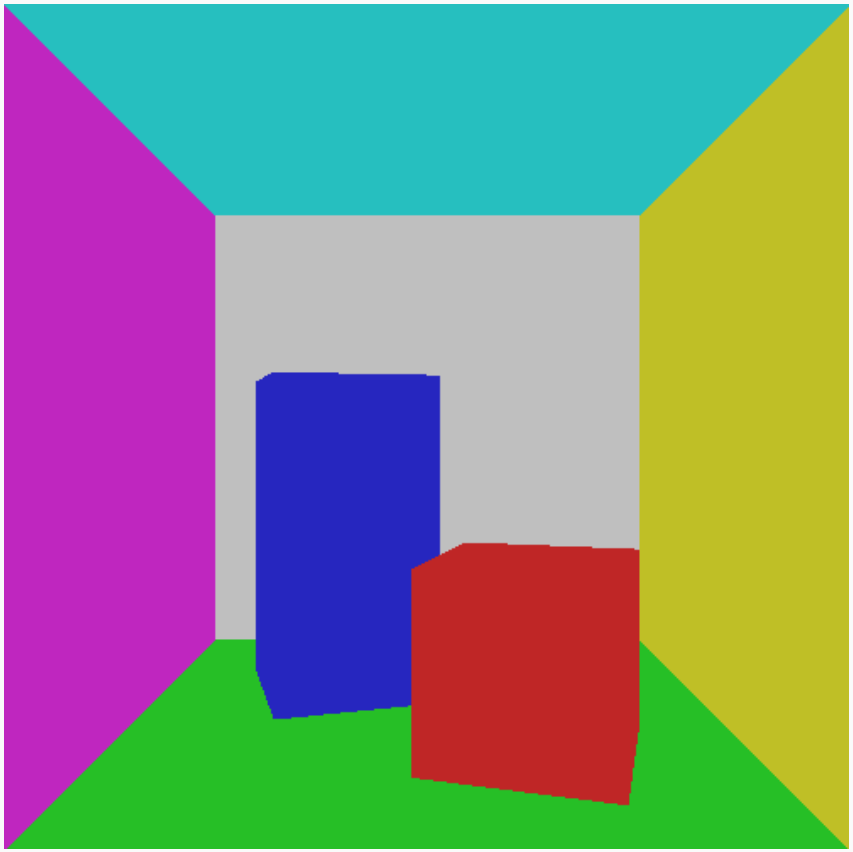
- This we know from basic math as

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

- *silliest solution*

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

Intersections



Ray-Sphere Intersection

- A point $\mathbf{p}$ lies the surface of a sphere with radius r and center $\mathbf{c}$ iff

$$\sqrt{(\mathbf{p}-\mathbf{c})^{\mathrm{T}}(\mathbf{p}-\mathbf{c})}-r=0$$

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- Insert ray-equation into sphere

$$\sqrt{(\mathbf{s}+t\mathbf{d}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}+t\mathbf{d}-\mathbf{c})}-r=0$$

$$(\mathbf{s}+t\mathbf{d}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}+t\mathbf{d}-\mathbf{c})=r^2$$

$$t^2+2t(\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c}))+(\mathbf{s}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}-\mathbf{c})-r^2=0$$

Ray-Sphere Intersection

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- Insert ray-equation into sphere

$$\sqrt{(\mathbf{s}+t\mathbf{d}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}+t\mathbf{d}-\mathbf{c})}-r=0$$

$$(\mathbf{s}+t\mathbf{d}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}+t\mathbf{d}-\mathbf{c})=r^2$$

$$t^2+2t(\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c}))+(\mathbf{s}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}-\mathbf{c})-r^2=0$$

- Quadratic expression i t

$$t=-\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c})\pm\sqrt{(\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c}))^2-(\mathbf{s}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}-\mathbf{c})-r^2}$$

Ray Intersections

- A Raytracer spends most its time doing intersection computations (profile your code)
- Rules of Thumb
  - Can we trivially reject something?
  - Can we re-use computations?
- If we need to do several tests do them in order of computational cost
- Realtime

Ray Intersections

- A Raytracer spends most its time doing intersection computations (profile your code)
- Rules of Thumb
  - Can we trivially reject something?
  - Can we re-use computations?
- If we need to do several tests do them in order of computational cost
- Realtime
- *Its really fun optimisations as the code is often very small and the solutions are really cute*

Ray-Sphere Intersection

- Sphere intersection

$$t = -\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c}) \pm \sqrt{(\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c}))^2 - (\mathbf{s}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}-\mathbf{c}) - r^2}$$

- Look at square-root
  - $= 0$ Only one solution, no need to compute square-root
  - < 0 No real solution, no intersection
  - > 0 We have to compute square root (two intersections)

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  - $= 0$ Only one solution, no need to compute square-root
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  - > 0 We have to compute square root (two intersections)
- *Which of the above cases do we have most of the time?*

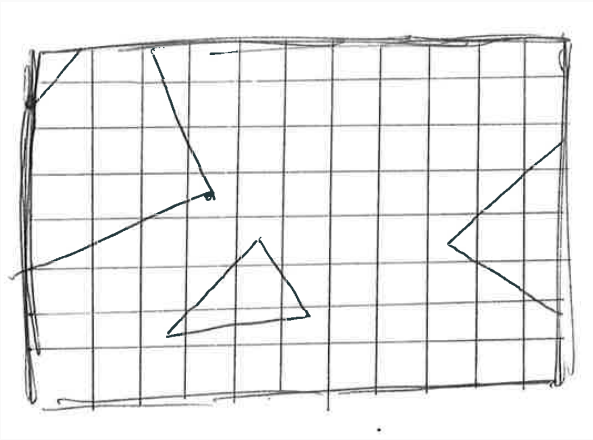
Raytracer Pipeline

Pipeline


Geometry
View Transformation
Lighting
Projection
Clipping
Screen Mapping
Primitives

Clipping and Screen Mapping

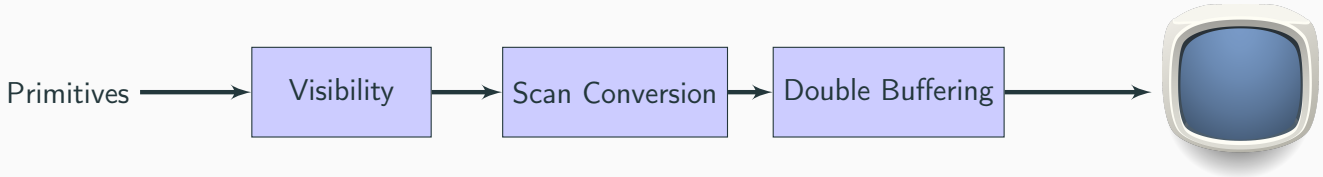
- Clip primitives to fit onto image
- Discretise visible primitives to pixels
- Our primitives are discrete pixels
- Trivial for a Raytracer

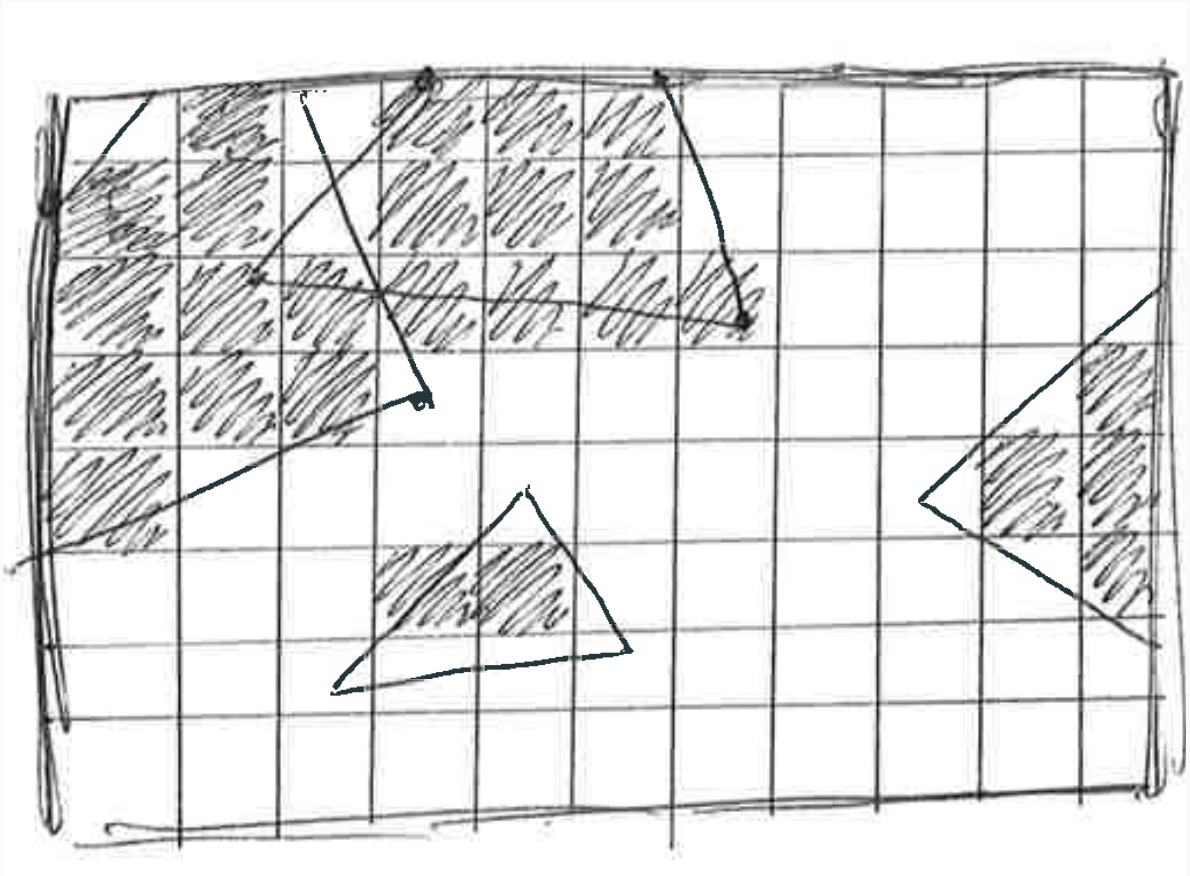


Pipeline


Geometry
View Transformation
Lighting
Projection
Clipping
Screen Mapping
Primitives

Rasterisation





Raycaster

```
for (int y=-h/2;y<h/2;y++)
  {
    for(int x=-w/2;x<w/2;x++)
   {
     for(int i=0;i<N_primitives;i++)
       {
         /*1. compute intersection
           2. check closest*/
       }
   }
  }
```

Next Time

Next Time

Lab Start with Lab 1

- Implement a camera structure and infrastructure to deal with transformations

Lecture Thursday 2nd of February

- 11-12 WMB 3.31
- Illumination: Light I
- How to decide colour of pixel

END

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