1 Indirect Boost-buck Converter

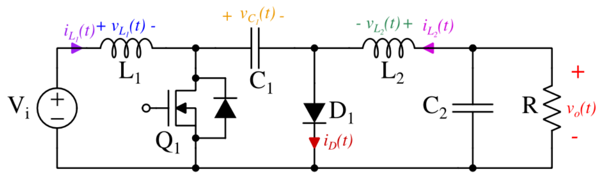
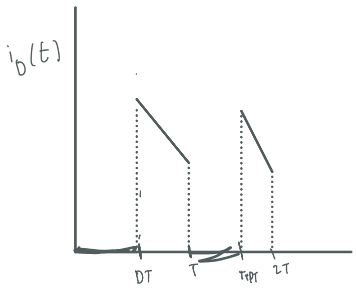


Figure 1.1: Indirect Boost-buck Converter.

Figure 1.1 shows an Indirect Boost-buck Converter converter. For this problem, the capacitors values are large resulting in a small voltage ripple across across their terminals.

- ☐ Sketch $i_D(t)$ when the converter operates in Continuous Conduction Mode (CCM).
  - ☐ Find the peak value of $i_D(t)$ in terms of the ripple magnitude Δi_{L1} and Δi_{L2} and the I_{L1} and I_{L2} (Note: I_{L1} is the average of $i_{L1}(t)$)



$$\boxed{i_D(t) = I_{L_1} + \frac{\Delta i_{L_1}}{2} + \frac{\Delta i_{L_2}}{2} + I_{L_2}}$$

- ☐ Find the conditions at which the converter operate in discontinuous conduction mode (DCM) in the from $K < K_{crit}(D)$

$\Delta i_{L_1} = \frac{V_i D}{L_1 f_s}$

from hw I we know that

$\Delta i_{L_2} = -V_o D' / L_2 f_s$

$I_{L_1}(1-D) + I_{L_2} D = 0$

$-\frac{V_o D' \cancel{D}}{\cancel{D}\,(L_1 f_s)} = \Delta i_{L_1}$

$I_{in} = I_{L_1};\ I_{L_2} = I_o$

$I_{in} = -\frac{I_{L_2} D}{1-D}$

$\frac{D' V_o}{L f_s} = \Delta i_{L_1}$

$\Delta i_{L_1} + \Delta i_{L_2} = \frac{-V_o D'}{L_1 f_s} + \frac{-V_o D'}{L_2 f_s}.$

$$I_{L_1} + I_{L_2} < \frac{\Delta i_{L_1} + \Delta i_{L_2}}{2}$$

$$\frac{-I_o D}{D'} \qquad -I_o$$

$$\frac{-V_o D}{D' R} + \frac{-V_o}{R} < \frac{-V_o D'}{2 f_s}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

$$\frac{D}{D'} + 1 < \frac{D' R}{2 f_s (L_1 \| L_2)}$$

$$\frac{D + 1 - D}{(1-D)^2} < \frac{R}{2 f_s L_1 \| L_2}$$

$$\boxed{\underbrace{\frac{2(L_1 \| L_2)}{R T}}_{K} < \underbrace{(D')^2}_{K_{crit}\, D}}$$

- [ ] Simulate the converter of [Figure 1.1]() in LTspice.
  - [ ] Using **.step** plot 5 complete cycles of the diode current $i_D(t)$ when $50\Omega \leq R \leq 250\Omega$.

> **Warning**
>
> Please ensure that the converter has reached a periodic steady state by observing the converter waveforms over multiple switching cycles.
>
> If necessary, you can set up the initial conditions in the capacitors to expedite the simulation. Refer to the LTspice help manual for more information.

- [ ] In your simulation, determine the value of R that will result in the operation at the boundary between CCM and DCM for the current $i_D(t)$.
- [ ] Plot 5 complete cycles of the currents $i_{L1}(t)$ and i_{L2} when the converter is operating at the boundary between CCM and DCM, and when the converter has reached periodic steady state.

See graphs on bottom

- [ ] Assuming that the converter of [Figure 1.1]() operates in DCM, find an **analytical** expression for the conversion ratio $\frac{V_o}{Vi}$ of the converter as a function of the duty ratio and K.

Charge balance

$$-ri_{L_2} D + i_{L_1} D_2 T + i_{L_1} + i_{L_2} (D_3 T) = 0$$

We know that $i_{L_1} + i_{L_2} = 0$

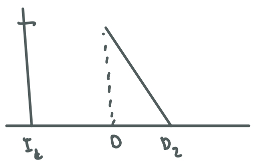
Therefore

$$-D i_{L_2} T + i_{L_1} D_2 T = 0$$

$$\frac{D_2}{D} = \frac{i_{L_2}}{i_{L_1}}$$

$$\frac{V_o}{V_{in}} = \frac{I_{in}}{I_o} = \frac{-I_{L_1}}{I_{L_2}} = \frac{-D}{D_2}$$

$$\langle I_{d_1} \rangle + \underbrace{\cancel{\langle I_{c_1} \rangle}}_{0} = \langle I_{L_2} \rangle$$



$$\langle I_{d_1} \rangle = -\langle I_o \rangle$$

$$\frac{1}{T}\int_0^T I_{L_1 pk} + I_{L_2 pk}\, dt = \frac{V_o}{R}$$

$$\frac{1}{T}\frac{1}{2}\left(\frac{V_i D T}{L_1} + \frac{V_i D T}{L_2}\right) D_2 T = \frac{V_o}{R}$$

$$\frac{V_i D T}{2 \; L_1 \| L_2} (D_2) = \frac{V_o}{R}$$

$$D_2 = \frac{V_o \; 2(L_1 \| L_2)}{R \; V_i D T}$$

$$\frac{-D V_{in}}{V_o} = \frac{V_o \; 2(L_1 \| L_2)}{V_i \cdot R V_i D T}$$

$$\sqrt{\frac{T R D^2}{(L_1 \| L_2)(2)}} = \sqrt{\left(\frac{V_o}{V_{in}}\right)^2}$$

$$\frac{2L}{RT} = K$$

$$\boxed{-\frac{D}{\sqrt{K}} = \frac{V_o}{V_{in}}}$$

$$i_{C_1 peak} = V_o + V_{C_1} = V_o + V_i - V_o$$

$$\frac{V_i}{L} D T$$

$$I_{L_1 pk} = \frac{V_i D T}{L_1}$$

$$I_{L_2 pk} = \frac{-V_o D_2 T}{L_2}$$ from voltage ratio

$$I_{L_2 pk} = \frac{V_i D T}{L_2}$$

2 Constant ON-time Boost Converter (E&M 5-15)

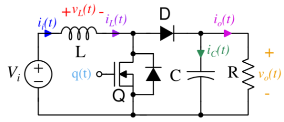
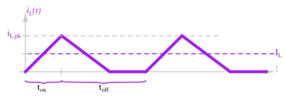


Figure 2.1: Boost converter

In battery-powered portable devices, dc-dc converters need to efficiently regulate the load voltage even when the load is in low-power *"sleep"* mode. The power required by the transistor gate-drive circuitry, as well as much of the switching loss, depends on the switching frequency but not on the load current. To achieve high efficiency at very low load powers, a *variable-frequency* control scheme is used, where the switching frequency is reduced in proportion to the load current. Let's consider the boost converter of Figure 2.1.

The input consists of two nickel-cadmium battery cells, which produce a voltage of $V_i = 2.4 \pm 0.4$ V. The converter boosts this voltage to a regulated 5 V. As shown in Figure 2.2, the converter operates in DCM with a *constant transistor on-time* ($\mathbf{t_{on}}$). The transistor off-time ($\mathbf{t_{of}}$) is varied by the controller to regulate the output voltage.



- [ ] Write the conditions that the converter needs to meet to operate at the CCM-DCM boundary, and of the voltage conversion ratio $\frac{V_o}{V_i}$ in terms of t_{on}, t_{off}, L and R.

CCM

$$\frac{V_i(t_{on}) + (V_i - V_o)(t_{off})}{t_{on}+t_{off}} = 0$$

$$\frac{V_i(t_{on}+t_{off})}{t_{off}} = V_o$$

$$\boxed{1+\frac{t_{on}}{t_{off}} = \frac{V_o}{V_i}}$$

$$\frac{V_o}{V_i} = \frac{I_i}{I_o} = \frac{I_{L1}}{I_o} = 1+\frac{t_{on}}{t_{off}}$$

$$\left(1+\frac{t_{on}}{t_{off}}\right)\frac{V_o}{R}$$

$$I_{L1} < \frac{\Delta i_{L1}}{2}$$

$$\left(1+\frac{t_{on}}{t_{off}}\right)\frac{V_o}{R} < \frac{V_i}{L}\frac{t_{on}}{2}$$

$$\frac{\cancel{V_o}}{R}\left(1+\frac{t_{on}}{t_{off}}\right) < \frac{\cancel{V_o}}{\left(1+\frac{t_{on}}{t_{off}}\right)}\frac{t_{on}}{L2}$$

$$\left(1+\frac{t_{on}}{t_{off}}\right)^2 < \frac{t_{on}R}{2L}$$

To act in DCM

$$\boxed{\left(1+\frac{t_{on}}{t_{off}}\right)^2 < \frac{t_{on}R}{2L}}$$

Voltage conversion of DCM

$$V_i t_{on} + (V_i - V_o)(t_2) = 0$$

$$V_i (t_{on} + t_2) = V_o t_2$$

$$\frac{t_{on} + t_2}{t_2} = \frac{V_o}{V_i}$$

$$\frac{t_{on}}{t_2} + 1 = \frac{V_o}{V_i}$$

$$\langle I_D \rangle = \langle I_C \rangle + I_o$$

$$\frac{1}{T}\int_0^T I_D \, dt = I_o$$

$$\frac{V_{in} \; t_{on} \; (t_2)}{2(t_{on} + t_{off}) L} = \frac{V_o}{R}$$

$$\frac{R(t_{on})(t_2)}{2(t_{on} + t_{off}) L} = \frac{V_o}{V_{in}}$$

$$t_2 = \frac{V_o}{V_i} \frac{(2(t_{on} + t_{off}))L}{R(t_{on})}$$

$$\frac{t_{on} \cdot V_i R t_{on}}{V_o 2 (t_{on} + t_{off}) L} + 1 = \frac{V_o}{V_i}$$

$$0 = \frac{V_o^2}{V_i^2} - \frac{V_o}{V_i} - \frac{t_{on}^2 R}{2(t_{on} + t_{off})L}$$

$$\frac{1 + \sqrt{1 + \frac{2(t_{on}^2)R}{L(t_{on} + t_{off})}}}{2}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2(t_{on}^2)R}{L(t_{on} + t_{off})}}}$$

To conclude:

$$\boxed{\frac{V_o}{V_i} = \begin{cases} \dfrac{t_{on}+t_{off}}{t_{off}} & \text{for CCM} \\ \dfrac{1}{2}+\dfrac{1}{2}\sqrt{\dfrac{1+2(t_{on})^2 R}{L(t_{on}+t_{off})}} & \text{for DCM} \end{cases}}$$

For DCM

$$\boxed{\left(1+\frac{t_{on}}{t_{off}}\right)^2 < \frac{t_{on} R}{2L}}$$

For the remainder of the problem, let's assume that the load current can vary between 100 μA and 1 A. The transistor's *on-time* is fixed at $t_{on} = 10\ \mu$s.

- [ ] Choose values for L and C so that:
    - The peak ripple in the output voltage is no greater than 50 mV.
    - The converter always operates in DCM, and
    - The peak inductor current is minimized as much as possible.

$100\mu \leq I_o < 1$

$\Delta V_o = 50m$

$V_o = 5$

$2.0 \leq V_i \leq 2.8$

To operate at BCM use 2.0 V

$$\frac{5}{2} = \frac{t_{on}+t_{off}}{t_{off}} \Rightarrow t_{off} = 6.6667\mu$$

$$\langle I_o \rangle + \langle \cancel{I_c} \rangle = \langle I_D \rangle$$

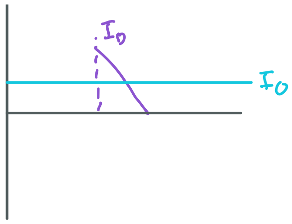
$$\frac{1}{T}\int_0^T i_D(t)\,dt = \underbrace{\frac{V_i}{2L} t_{on}}_{\text{peak}} \cdot t_{off} \frac{1}{t_{on}+t_{off}} = I_{o\,max}$$

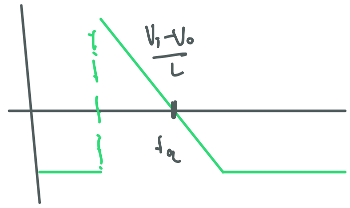
$$\boxed{L = 4\mu H}$$

$I_c + I_o = I_D$

$I_c = I_D - I_O$

|





For max voltage ripple,

We know that Due to capacitor charge balance. Take the positive area

$$\Delta V = \frac{\int I\,dt}{C}$$

Use $V_i = 2.8$ & $I_o = 1m$ to get biggest possible peak

$$I_{peak} = \frac{V_i\, t_{on}}{L} - I_o \approx 7A$$

$$0 = 7 - \frac{(2.8-5)\, t_q}{4E-6}$$

$$t_q = 1.273 \cdot 10^{-5}\, s$$

$$C\,\Delta V = \tfrac{1}{2} t_q\, (i_{max})$$

$$\Delta V = \frac{7\,(1.27E-5)}{2\,C} \Rightarrow C = \boxed{891\,mF}$$

- [ ] For your design from the previous section, what are the maximum and minimum values of the switching frequency?

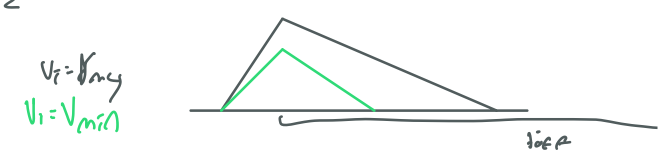
Max switching frequency occurs at BCM

where $V_i = 2$ and $I_o = I_{o\,max}$

$t_{on} = 10\mu$

$t_{off} = 6.6667\,\mu$

$$\frac{1}{t_{on} + t_{off}} = 60kHz$$



Min switching frequency occurs when

$V_i = 2.8$ $\quad I_o = I_{o\,min}$

to minimize switching frequency we use smallest current bc. larger t_{on} will give larger t_{off}.

We use higher V_i because want the triangle to be so that t_{off} has to be larger to create load avg.

$$\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1+2(t_{on})^2 R}{L(t_{on}+t_{off})}}$$

$$\langle I_D\rangle = \langle I_C\rangle + I_0$$

$$\frac{1}{T}\int_0^T I_d\,dt = I_c$$

$$\frac{V_{in}\ t_{on}\ (t_2)}{2(t_{on}+t_{off})\ L} = I_0$$

$$V_i\ t_{on} + (V_i - V_0)(t_2) = 0$$

$$-\frac{V_i}{V_i - V_0}\,t_{on} = t_2$$

$$\frac{V_{in}\,t_{on}\,(-V_i)}{2(t_{on}+t_{off})L)(V_i - V_0)} = I_0$$

- .

$$t_{off} = 44543.43$$

$$f_{max} = 2.244$$

$$\frac{t_{on}\,V_i^{\,2}}{2(L)(V_i - V_0)\,I_0} - t_{on} = t_{off}$$

$$t_{off} = 44543$$

$$\frac{1}{0.44548} = 2.24\quad Hz$$

3 Sine PWM inverter

- [ ] Use LTSpice to simulate the inverter of Figure 3.1 using a sine-triangle intercept PWM modulation.
    - You can use a circuit similar to the one shown in Figure 3.2 to obtain the PWM signals of the fast switching devices (S_1 and S_2).
    - We define k as the *modulation depth*, and it is a parameter that you vary to change the amplitude of the output's fundamental.
    - The parameter values are: $V_i = 125$ V, $L = 27$ μH, $C = 10$ μF, $R = 3$ Ω, and $f_{ac} = 60$ Hz.

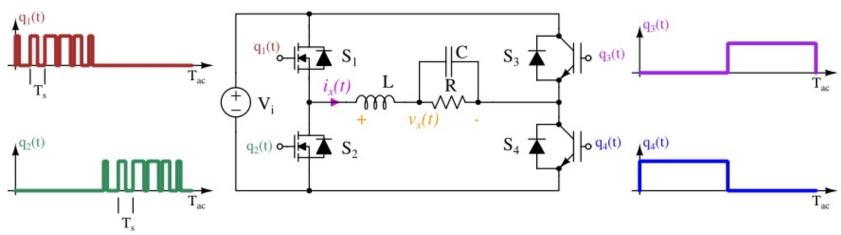


Figure 3.1: Inverter.

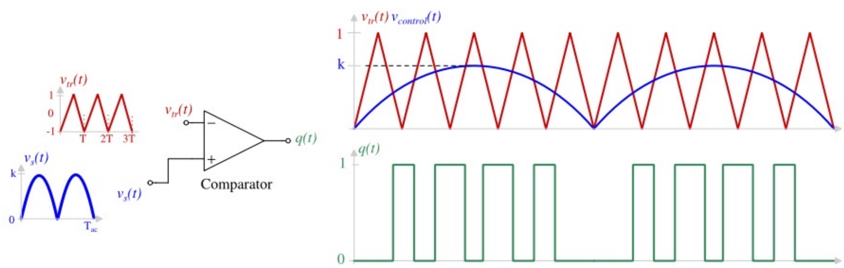


Figure 3.2: Sine-triangle intercept modulation.

For the simulation:

- [ ] Set the frequency of the carrier triangle signal, v_{tr}, to 225 kHz and set $k = 0.75$.
    - [ ] Plot the inductor current $i_x(t)$, $v_x(t)$, and the voltage across the resistive load.

- [ ] Set the frequency of the carrier triangle signal, v_{tr}, to 225 kHz and set $k = 0.75$.
  - [ ] Plot the inductor current $i_x(t)$, $v_x(t)$, and the voltage across the resistive load.
  - [ ] Also, plot the FFT of these signals **using at least three full ac cycles** of the corresponding waveforms to obtain the FFT.
- [ ] In addition to the plots you'll submit to gradescope, please upload your simulation file (or netlist) to Canvas in a Zip file that includes the following: Simulation file (not the simulation output).
  - Sub-circuit libraries (if you use any).
  - A *"plot-setting"* file that displays the requested time domain waveforms automatically after running the simulation.
  - Use your last name to name the file followed by **_EE254PWM.zip**.

See bottom

4 Grid-tied Inverter

- [ ] Use LTspice to simulate an inverter supplying power to the ac line, similar to the one shown in Figure 4.1. For this problem, make $V_{ac} = 120\ \mathrm{V_{RMS}}$, 60 Hz
- [ ] Implement a hysteretic controller to regulate the inductor current, as demonstrated in Figure 4.2. The inductor current should be a sine wave in phase with the ac line, with an amplitude of $I_{ac} = 7\ \mathrm{A_{RMS}}$, and with a *hysteresis band* of $\Delta i = 0.5$ A. The input voltage, V_i, should be set to 200 $\mathrm{V_{dc}}$. *One way to establish the hysteresis band is by connecting a couple of resistors with positive feedback between the output and the input of a comparator.*

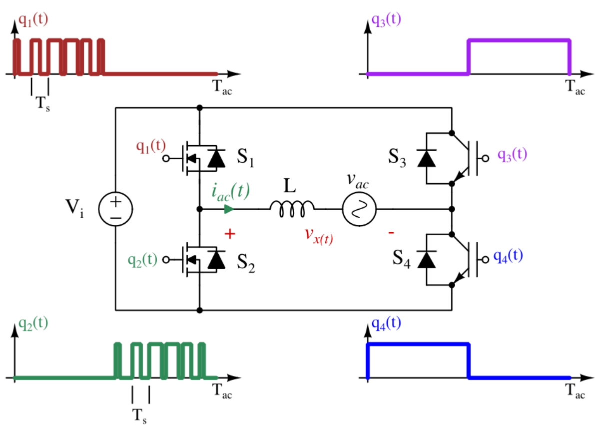


Figure 4.1: Grid-tied Inverter.

Figure 4.1: Grid-tied Inverter.

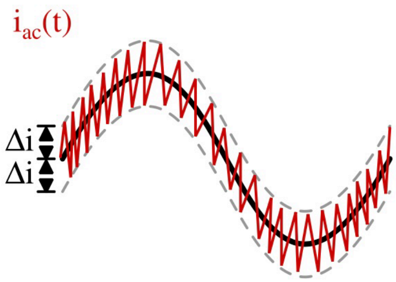
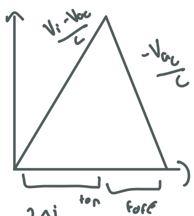


Figure 4.2: Sine-triangle intercept modulation.

- [ ] Calculate the inductor value required to achieve a maximum switching frequency of $f_{s,max} = 100\,\mathrm{kHz}$ KHz.

Calculating L @ $f_{s_{max}} = 100\mathrm{kHz}$



slope

$m = \frac{\Delta y}{\Delta x}$

$$\frac{2\Delta i_L}{\frac{V_i - V_{ac}}{L}} + \frac{2\Delta i_L}{\frac{V_{ac}}{L}} = t_{on} + t_{off}$$

$$\frac{1}{V_i - V_{ac}} + \frac{1}{V_{ac}} = \frac{t_{on}+t_{off}}{2\Delta i_L L}$$

$$\frac{V_{ac} + V_i - V_{ac}}{(V_i - V_{ac})V_{ac}} = \frac{V_i}{(V_i - V_{ac})V_{ac}} = \frac{t_{on}+t_{off}}{2\Delta i_L L}$$

$$① \quad \frac{(V_i - V_{ac})V_{ac}}{2V_i \Delta i_L L} = \frac{1}{t_{on}+t_{off}}$$

$$f_s = \frac{1 \quad V_i}{2\Delta i \; L}$$

to find where this is highest take derivate equals 0

$$V_i - 2V_{ac} = 0$$

$$V_{ac} = \frac{V_i}{2}$$

Using ①

$$f_{max} = \frac{(\frac{\cancel{V_i}}{2})(\frac{V_i}{2})}{2\cancel{V_i}\Delta i_L L}$$

$\Delta i_L = 0.5$ from the problem

$$\boxed{L = 500\,\mu H}$$

Figure 4.1: Grid-tied Inverter.

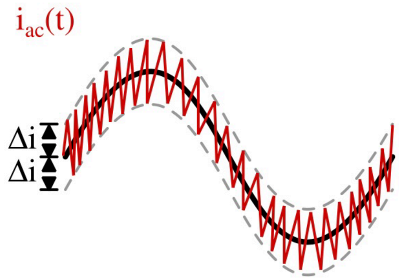


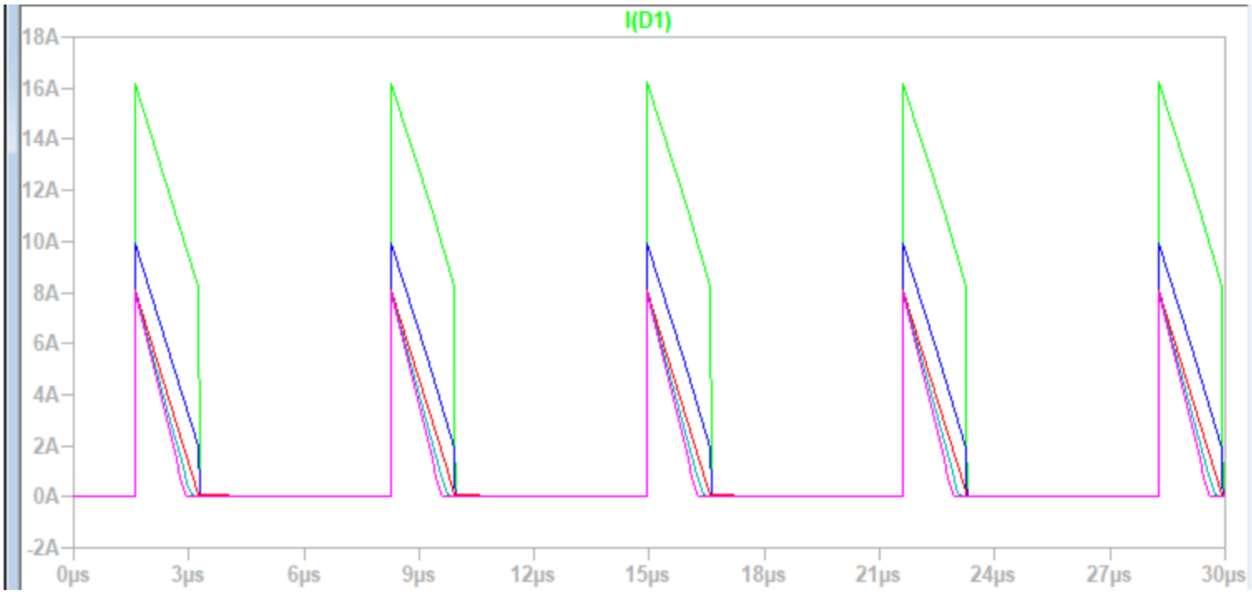
Figure 4.2: Sine-triangle intercept modulation.

- [ ] Calculate the inductor value required to achieve a maximum switching frequency of $f_{s,max} = 100\,\text{kHz}$ KHz.
- [ ] Plot the time domain waveform of the output of the full bridge ($v_x(t)$) and the inductor current ($i_{ac}(t)$).
  - [ ] Additionally, plot the FFT of the waveforms. Ensure that at least three full AC cycles are used to obtain the FFT.
- [ ] In addition to the plots you'll submit to gradescope, please upload your simulation file (or netlist) to Canvas in a Zip file that includes the following: Simulation file (not the simulation output).
  - Sub-circuit libraries (if you use any).
  - A *"plot-setting"* file that displays the requested time domain waveforms automatically after running the simulation.
  - Use your last name to name the file followed by **_EE254PWM.zip**.

Problem 1

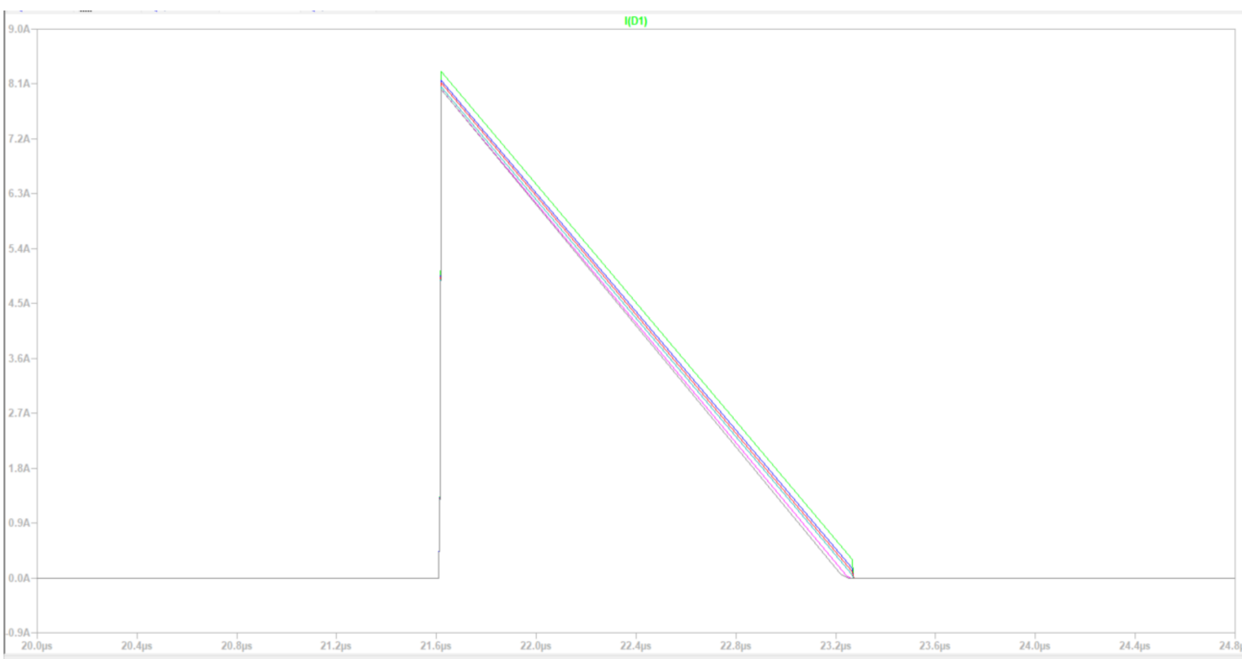
a. Using **.step** plot 5 complete cycles of the diode current iD(t) when 50Ω≤R≤250Ω .

Diode Current when Ro = 50, 100, 150, 200, 250



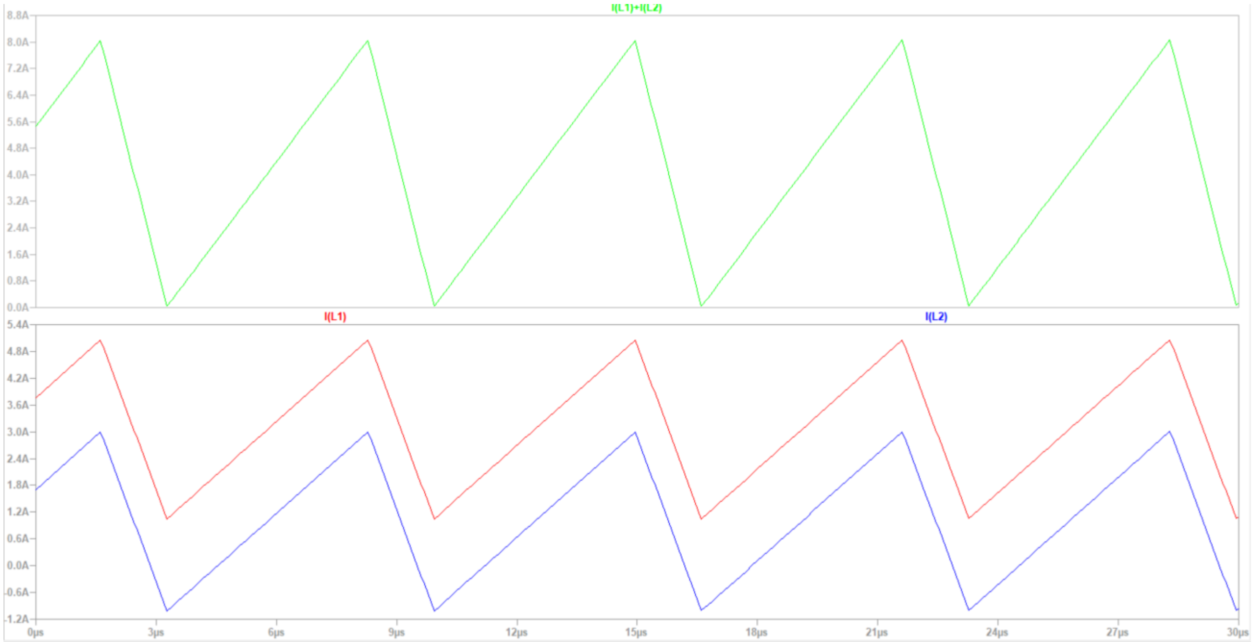
b. In your simulation, determine the value of R that will result in the operation at the boundary between CCM and DCM for the current iD(t).

Ro with sweep through 140, 144, 148, 152, 156, 160



Based on these pictures, Ro =152 is the boundary between CCM and DCM.

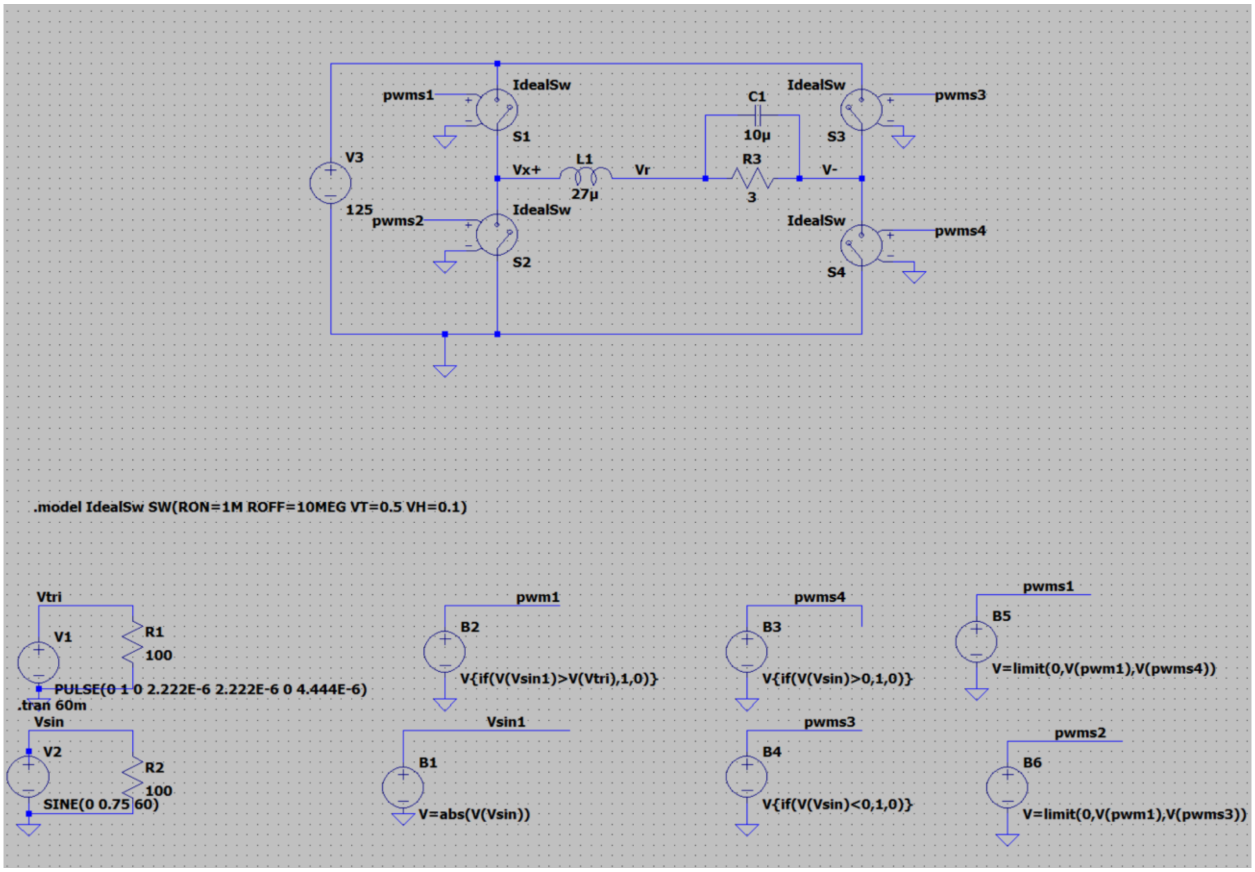
c. Plot 5 complete cycles of the currents iL1(t) and iL2 when the converter is operating at the boundary between CCM and DCM, and when the converter has reached periodic steady state.



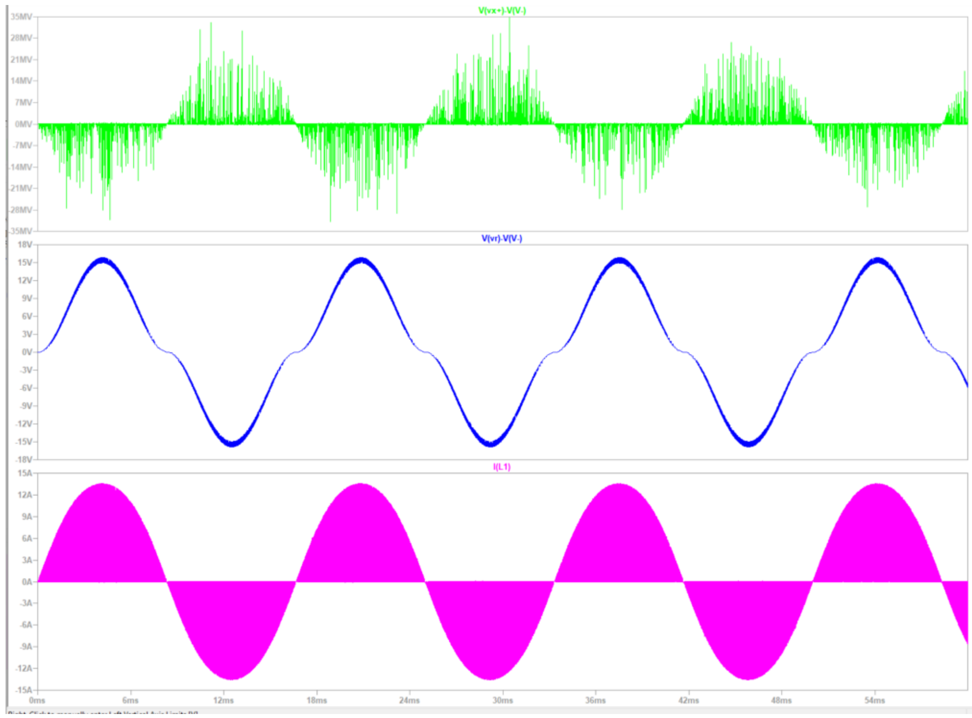
Problem 3

Use LTSpice to simulate the inverter of Figure 3.1 using a sine-triangle intercept PWM modulation.

- You can use a circuit similar to the one shown in Figure 3.2 to obtain the PWM signals of the fast switching devices (S1 and S2).
- We define k as the *modulation depth*, and it is a parameter that you vary to change the amplitude of the output's fundamental.
- The parameter values are: Vi=125 V, L=27 μH, C=10 μF, R=3 Ω, and fac=60 Hz.

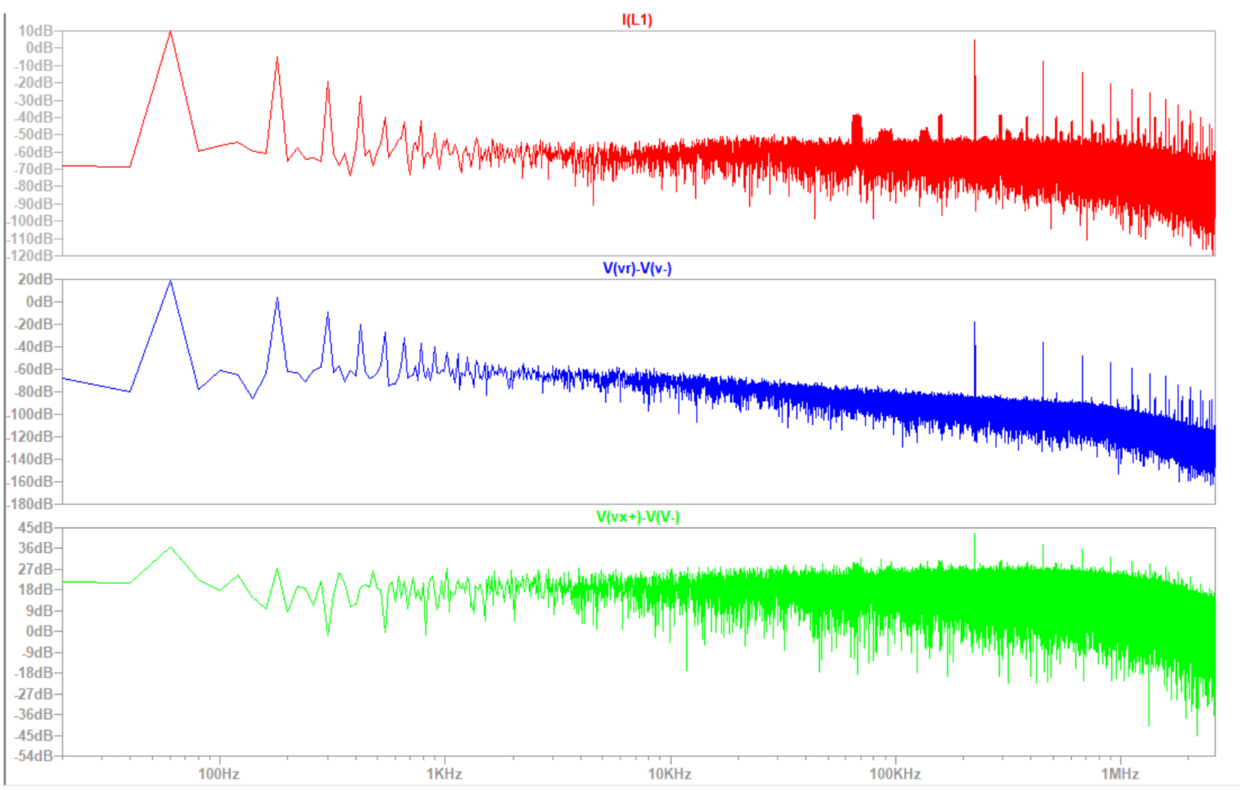


- Set the frequency of the carrier triangle signal, vtr, to 225 kHz and set k=0.75.
    - Plot the inductor current ix(t), vx(t), and the voltage across the resistive load.



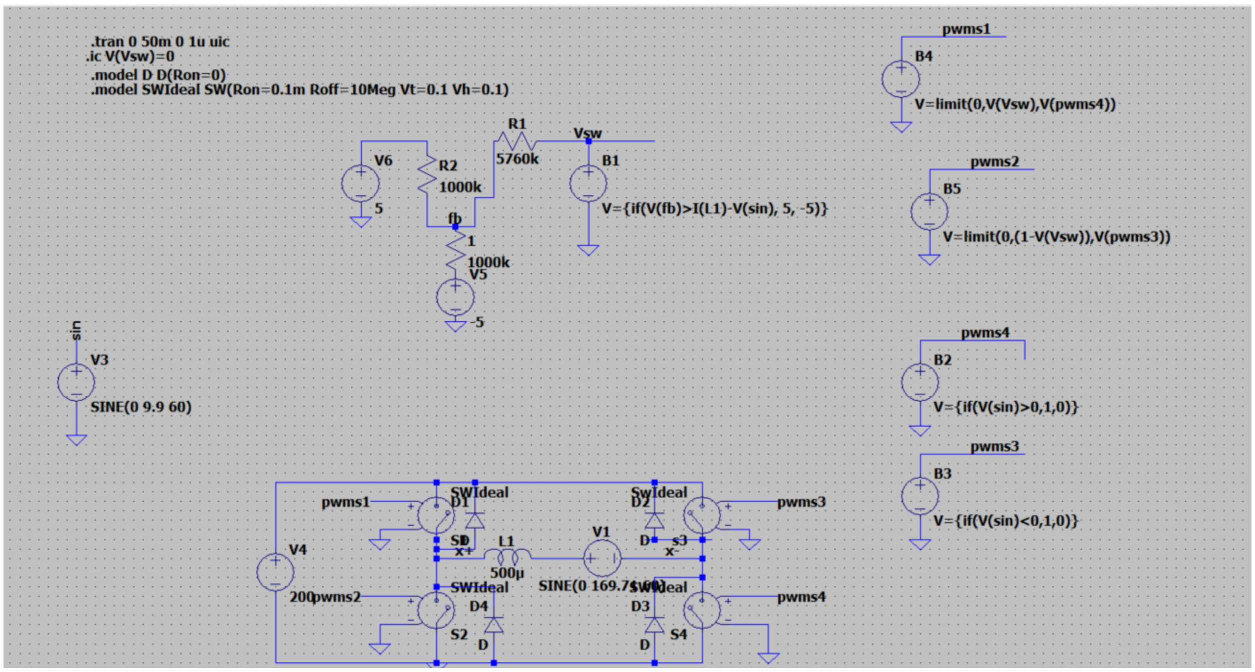
- Also, plot the FFT of these signals **using at least three full ac cycles** of the corresponding waveforms to obtain the FFT.

As to be expected, Vx has one major peak at 60 hz, and the next major peak is at 225 kHz.



Problem 4

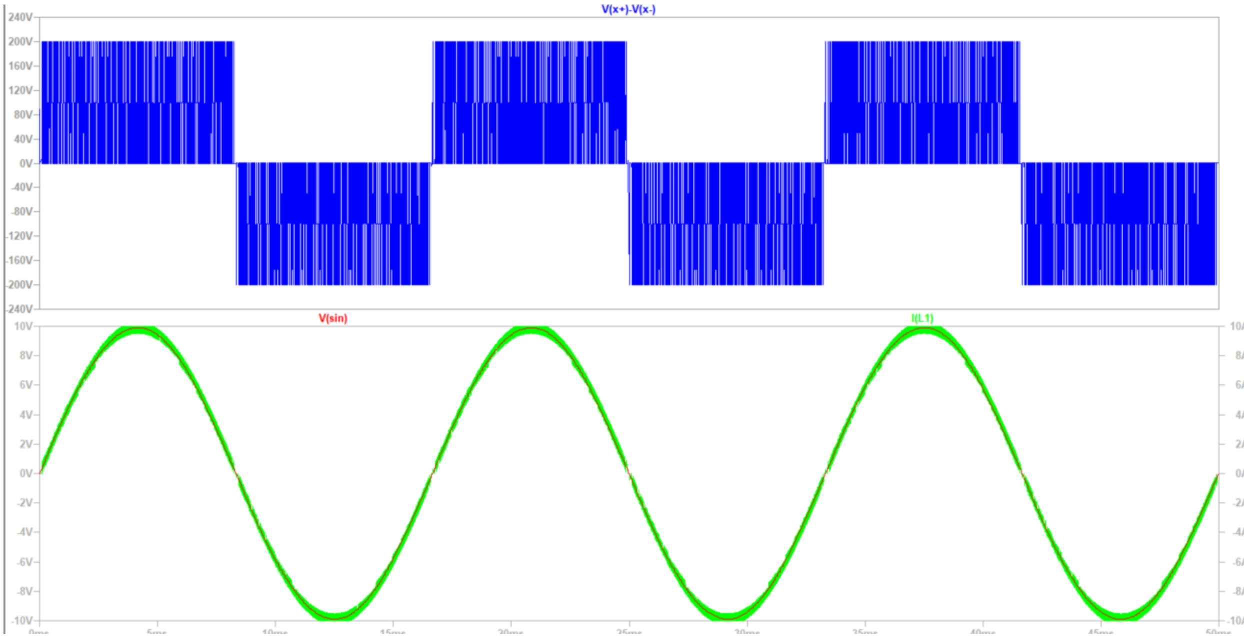
- Use LTspice to simulate an inverter supplying power to the ac line, similar to the one shown in Figure 4.1. For this problem, make Vac=120 VRMS, 60 Hz
- Implement a hysteretic controller to regulate the inductor current, as demonstrated in Figure 4.2. The inductor current should be a sine wave in phase with the ac line, with an amplitude of Iac=7 ARMS, and with a *hysteresis band* of Δi=0.5 A. The input voltage, Vi, should be set to 200 Vdc. *One way to establish the hysteresis band is by connecting a couple of resistors with positive feedback between the output and the input of a comparator.*



- Calculate the inductor value required to achieve a maximum switching frequency of fs,max=100 kHz KHz.

In the written part of the hw

Plot the time domain waveform of the output of the full bridge (vx(t)) and the inductor current (iac(t)).



Additionally, plot the FFT of the waveforms. Ensure that at least three full AC cycles are used to obtain the FFT.