# 1 Two-switch Forward Converter

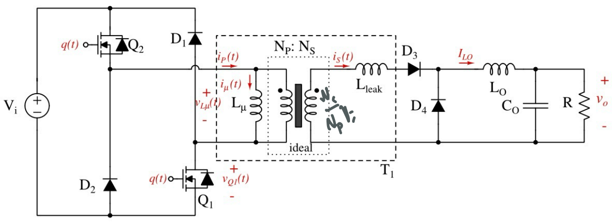
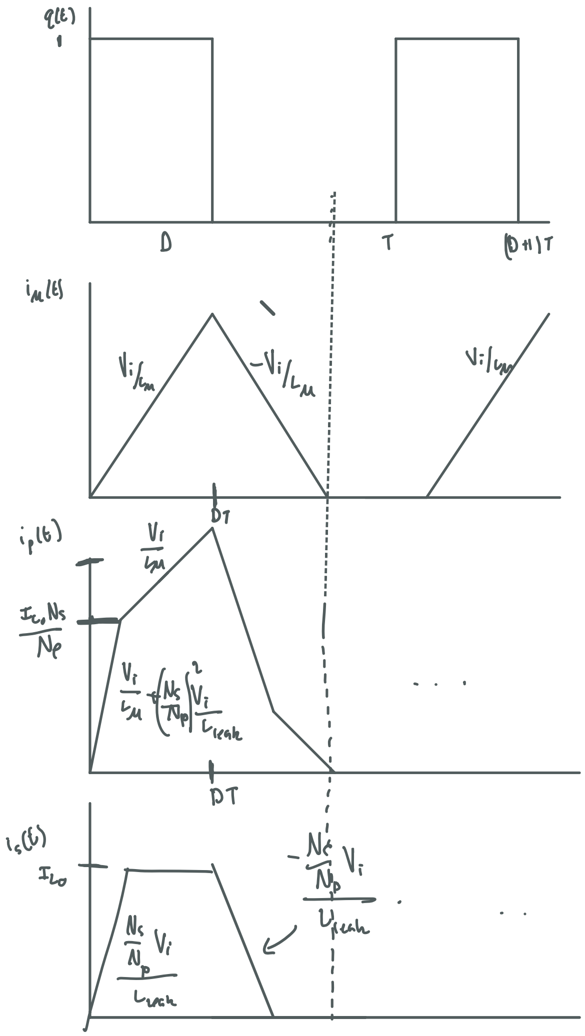



Figure 1.1: Two-switch forward converter.

$$V_p = \frac{I_p}{L_p}$$

Consider the two-switch forward converter in Figure 1.1. The two switches $Q_1$ and $Q_2$ are turned on-off at the same time. You may assume the converter operates in CCM and that $L_O$ is large enough to make the current $I_{LO}$ constant.

- [x] Sketch the waveforms of $q(t)$ $i_\mu(t)$, $i_p(t)$, and $i_s(t)$ of the figure. $L_{leak}$ models the leakage inductance in the transformer. Mark and label key features of the waveforms.




$$L\frac{di}{dt} = V$$

$i_s + i_p = i_\mu$

$i_p(t)$

$i_p(t) = i_\mu + i_p'$

T

$\frac{i_s n_s}{n_p}$

$i_p n_p + i_s n_s = 0$

How does leakage inductance impact circuit?

$$V\frac{dr}{dt} = L$$

- [ ] Simulate the circuit of Figure 1.1 in LTspice assuming $V_i = 50$ V, $f_s = 100$ kHz, $D = 0.25$, $N_P = 14$, $N_S = 7$, $L_\mu = 125\ \mu$H, and $I_{LO} = 10$ A.
  - You can consider all the devices (except the transformer) as ideal.

$$NV_i\,(DT)$$

$$(NV_i - V_o)DT + V_o(D')T = 0$$

$$1-D-D$$

$$NV_i D = V_o$$

$$ND = \frac{V_o}{V_i}$$

$$N = \frac{n_s}{n_p} = \frac{1}{2}$$

$$\frac{1}{2}\left(\frac{1}{4}\right) = \frac{V_o}{V_i}$$

$$\frac{1}{8} = \frac{V_o}{V_i}$$

$$\frac{1}{8} = \frac{V_o}{50}$$

$$6.25 = V_o$$

$$\langle I_{Lo}\rangle + \cancel{\langle I_C\rangle} = \langle I_o\rangle$$

$$\langle I_{Lo}\rangle = \langle I_o\rangle$$

$$\frac{V_o}{I_o} = R_o = \frac{6.25}{10} = 0.625\,\Omega$$

No voltage ripple specs, so just use large $C_o$

No current ripple specs " " large $L_o$

$$\frac{V_p}{14} = \frac{V_s}{7}$$

$$V_p = 2V_s$$

- [ ] In one page, plot 4 cycles of $i_\mu(t)$, $i_p(t)$, and $i_s(t)$ and $v_{L\mu}(t)$, $v_{Q1}(t)$ and $V_{D4}$ when $L_{leak} = 0$.

See bottom

- [ ] What is the maximum voltage across $v_{Q1}(t)$?

50V (see bottom)

- [ ] What is the maximum and minimum of the magnetizing current $i_\mu(t)$? (only one trace per plot please).

min: 0A (theoretical

max: $\frac{V_i}{L_\mu}DT = 1$ A (theoretical)

Actual min: -0.1mA

Actual max: 998.5 mA

See bottom

- [ ] In one page, plot 4 cycles of $i_\mu(t)$, $i_p(t)$, and $i_s(t)$ and $v_{L\mu}(t)$, $v_{Q1}(t)$ and $V_{D4}$ when $L_{leak} = 500$ nH.

see bottom

# 2 Push-pull converter

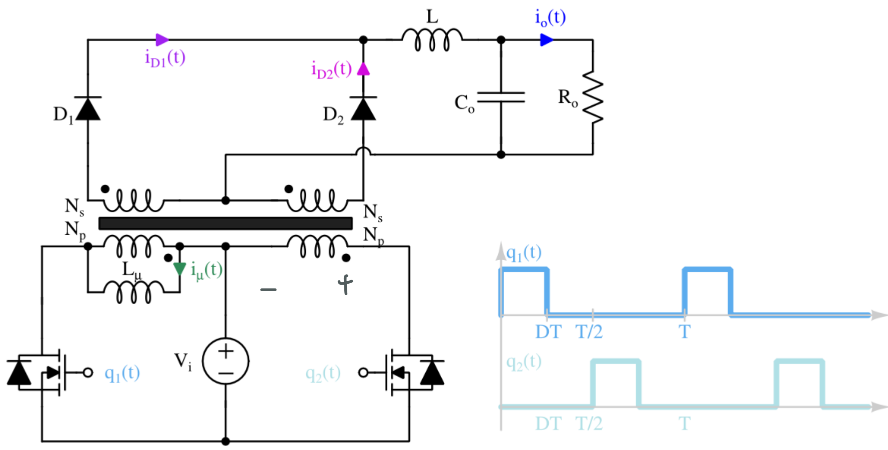
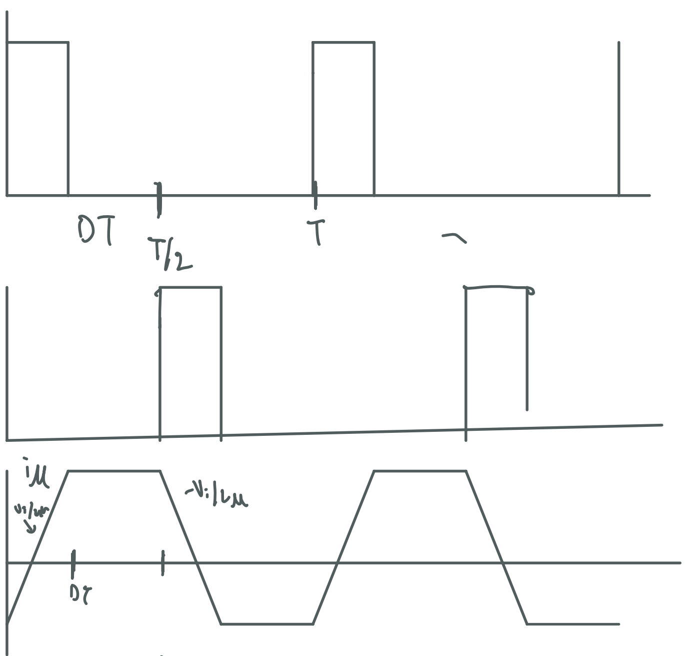



Figure 2.1: Push-pull converter.

Consider the push-pull converter of Figure 2.1. Assume loss-less components and an duty ration of $D = 0.25$. The transformer has a finite magnetizing inductance $L_\mu$.

- [ ] Draw the waveforms of $i_\mu(t)$, $i_{D1}(t)$ and $i_{D2}(t)$ for the large load case where $\frac{N_s}{N_p} i_o \gg i_\mu$.



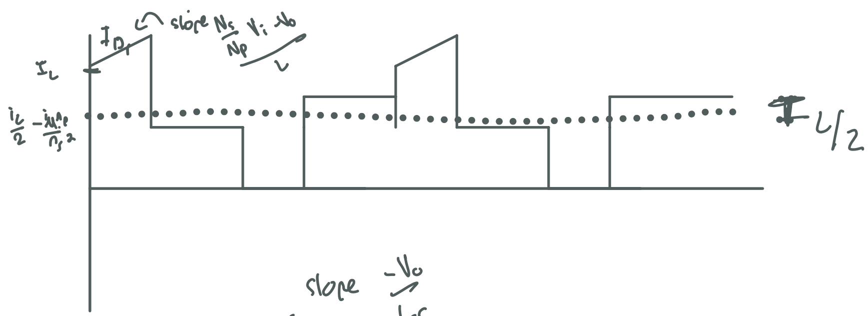

$\frac{d\phi}{dt}$ $\Phi c$: V

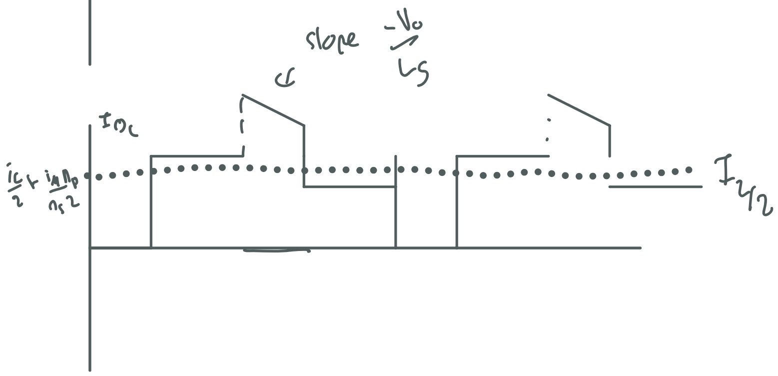
$\int L I$ Vt

Why start at neg? Need current flow when all switcus are off. First creates positive, then creates negative. Symmetrical or

If you can ramp up the $i_\mu$ in neg directin then can become neg

neg portion because negative current flow through the transformer




When $q_1$ & $q_2$ are off

$i_p = i_\mu + i_p' = 0$

$n_p i_p' - n_s i_{D_1} + n_s i_{D_2} = 0$

$0 = -i_\mu n_p - n_s i_{D_1} + n_s i_{D2}$

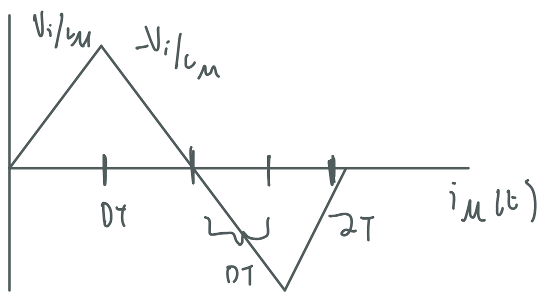
$i_L = i_{D1} + i_{D2}$ to split equally

$i_L = \frac{i_\mu n_p + n_s i_{D_1}}{n_s} + i_{D_1}$

$$\boxed{\begin{aligned} \frac{i_L}{2} - \frac{i_\mu n_p}{n_s 2} &= i_{D_1} \\ \frac{i_L}{2} + \frac{i_\mu n_p}{n_s 2} &= i_{D_2} \end{aligned}}$$

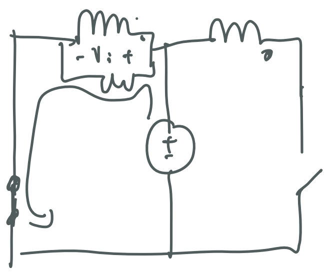
$\frac{N_s}{n_p} i_o >> i_\mu$

- [ ] At light load as $i_o \Rightarrow 0$ draw the magnetizing current $i_\mu(t)$. How does $i_\mu(t)$ at light load compares to the magnetizing current at heavy load?, which one is larger? Explain.

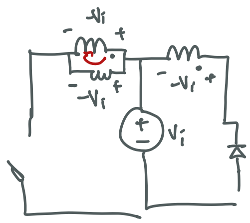



left w/ bottom of circuit because top basically doesn't turn on.

$i_\mu(t)$ at light loads is 2x as large because it starts at 0 & goes up $\frac{V_i DT}{L}$.
$i_\mu(t)$ at heavy loads starts at $-\frac{V_i DT}{L}$

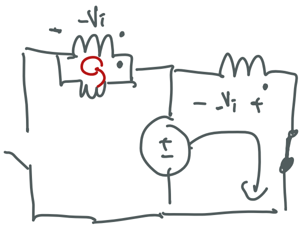



The current goes through Lm, is not reflected to the transformer bc the MOSFET does not allow current to flow initially.

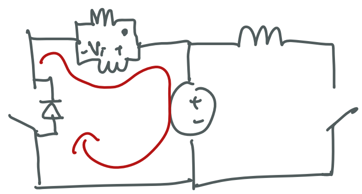



The mag current reflects to the other primary winding. The current flows across the winding There is $-V_i$ reflected to second primary and the second loop is therefore $-V_i + V_i$ allowing current to flow in the diode

$-V_i - V_p = 0$
$-V_i = V_p$




The current goes through the secondary winding. It induces a voltage $-Vi$ on the inductor. The inductor current continues to [illegible]




Current in Lm can't change immediately. Therefore, it continues to flow. $V_i$ in $L_M$ and $V_{L_M} - V_i = 0$

# 3 Forward Converter with Active Clamp

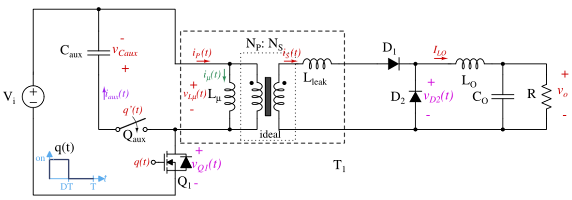



Figure 3.1: Forward converter with active clamp.

Figure 3.1 shows a forward converter with active clamp. The main switch $Q$ is modulated *ON/OFF* with a duty cycle $D$.
The auxiliary switch $Q_{aux}$ is turned $ON/OFF$ oppositely to the main switch (while avoiding *shoot-through* of course). The function of the clamp circuit (comprising $Q_{aux}$ and $C_{aux}$ ) is to reset the transformer core.

- ☐ Assuming operation in PSS and CCM, what is the voltage $C_{aux}$ as function of $V_i$ and $D$?

When q(t)=1

$V_{L_\mu} = V_i$ $\quad \frac{V_i}{L_\mu} = \frac{di}{dt}$

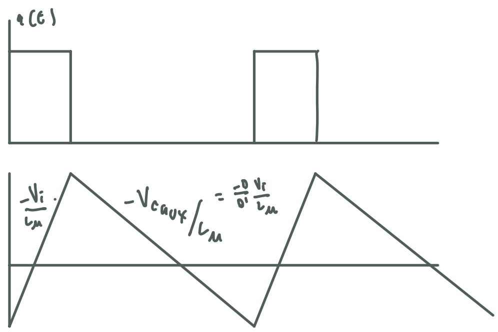
When q(t)=0

$V_{L_\mu} = -V_{caux}$ $\quad \frac{-V_{caux}}{L_\mu} = \frac{di}{dt}$

$V_i D + -V_{caux}(1-D) = 0$

$$\boxed{\frac{V_i D}{1-D} = V_{caux}}$$

- ☐ Sketch the magnetizing inductance current $i_{L\mu}$ when $V_i = 50$ V, $D = 0.25$, $f_s = 100$ kHz, $N_p = 14$, $N_s = 7$, $L_\mu = 125\ \mu$H, $I_{LO} = 10$ A.
  - For this part, you can ignore $L_{leak}$, and assume $C_{aux}$ is large.




goes to neg so cap can fully discharge (PSS)
Cic >=0

- ☐ Simulate the converter in Figure 3.1 in LTspice and plot assuming $V_i = 50$ V, $f_s = 100$ kHz, $D = 0.25$, $N_P = 14$, $N_S = 7$, $L_\mu = 125\ \mu$H, and $I_{LO} = 10$ A. Make $C_{aux} = 10\ \mu$F and assume it has an $ESR = 100$ m$\Omega$. - You can consider all the devices (except the transformer and $C_{aux}$) as ideal.

Look at bottom

same calcs for R as top

- In one page, plot 4 cycles of $i_{aux}(t)$, $i_\mu(t)$, $i_p(t)$, and $i_s(t)$ and $v_{L\mu}(t)$, $v_{Q1}(t)$ and $v_{D2}(t)$ when $L_leak = 0$. What is the maximum voltage across $v_{Q1}(t)$? What is the maximum and minimum magnetizing current $i_\mu(t)$? (only one trace per plot please)

Look at word doc

(sim)

max voltage across $V_{Q_1}(t) = 66.85$

maximum $i_M = 500.58$ mA

minimum $i_M = -500.05$ mA

Also on bottom

calcs

$V_i + V_{C\,aux} = \frac{V_i}{D'} = 66.667$ V

Max $i_M$ $\frac{\Delta i_M}{2} = 500$ mA

Min $i_M$ $-\frac{\Delta i_M}{2} = 500$ mA

$\Delta i_M = \frac{V_i}{L_M} DT = 1$ A

- In one page, plot 4 cycles of $i_\mu(t)$, $i_p(t)$, and $i_s(t)$ and $v_{L\mu}(t)$, $v_{Q1}(t)$ and $v_{D2}(t)$ when $L_{leak} = 500$ nH. What is the maximum voltage across $v_{Q1}(t)$? What is the maximum and minimum of the magnetizing current $i_\mu(t)$? (only one trace per plot please)

max voltage across $V_{Q_1}(t) : 66.48$ V

max $i_M$ : 330.67 mA

min $i_M$ -669.17 mA

Also on bottom

- ☐ Compare your plots with the plots you got for the two-switch forward converter of Figure 1.1. Comment on the differences and how each circuit handles the leakage inductance of the transformer.

On the bottom

# 4 Multi-output Flyback converter (E&M 6.10)

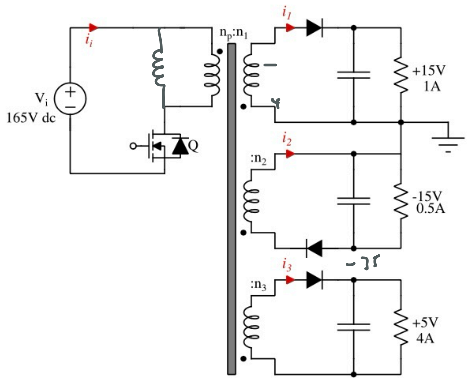



Figure 4.1: Three-output flyback converter design.

Design of a multiple-output dc-dc flyback converter. For this problem, you may neglect all losses and transformer leakage inductances. It is desired that the three-output flyback converter shown in Figure 4.1 operates in the discontinuous conduction mode, switch a switching frequency of $f_s = 100$ kHz. The nominal operating conditions re given in the diagram, and you may assume that there are no variations in the input voltage or the load currents. Select $D_3 = 0.1$ (the duty cycle of sub-interval 3, in which all semiconductors are off). The objective of this problem is to find a good steady-state design, in which the semiconductor peak blocking voltages and peak currents are reasonably low.

- [ ] It is possible to find a design in which the transistor peak blocking voltage is less than 300 V and the peak diode blocking voltages are all less than 35 V, under stady-state conditions.

  - Design the converter such that this is true.
  - Specify: *(i)* the transistor duty cycle D, *(ii)* the magnetizing inductance $L_M$, referred to the primary, *(iii)* the turns ratios $\frac{n_1}{n_p}$ and $\frac{n_3}{n_p}$.

- [ ] For your design of the previous part, determine the rms currents of the four windings.
  - Note that they don't simply scale by the turns ratios.

MOSFET Peak Voltage

$V_{in} + V_{L_{max}} < 300$

$V_{L_{max}} < 135$

MOSFET Peak Voltage

$V_i + V_o \left(\frac{n_p}{n_s}\right) < 300$

Circuit 1 & circuit 2

$15/(n_p) < 135$

35 V peak diode block

Circuit one & two

$15 - 35 = -20$

biggest voltage

35 V peak diode block

Circuit one & two

$$\left|\frac{n_s V_{in}}{n_p}\right| < 20$$

$$\frac{n_s}{n_p} < \frac{4}{33}$$

$15-35=-20$

$V_{in,max} = 135$

$$\frac{n_p}{n_s} < 9$$

$$\frac{1}{9} < \frac{n_s}{n_p}$$

Circuit 3

$$5\left(\frac{n_p}{n_s}\right) < 135$$

$$\frac{n_p}{n_s} < 27$$

$$\frac{1}{27} < \frac{n_s}{n}$$

$$\left|\frac{V_{in} n_s}{n_p}\right| < 20$$

$$\frac{n_s}{n_p} < \frac{4}{33}$$

~~Circuit 3~~

~~$\frac{V_{in}}{n_p} < 30$~~

~~$\frac{n_s}{n_p} < \frac{6}{33}$~~

drop comes from here

$5-35=-30$

Not impacted

$$\boxed{\frac{1}{9} \le \frac{n_1}{n_p} \le 4/33 \qquad \frac{1}{27} \le \frac{n_3}{n_p} \le \cancel{6}/33}$$

can choose values by calculating being more or other first.

Circuit 3

$$\frac{n_s V_{in}}{n_p} < 30$$

$$\frac{n_s}{n_p} < \frac{6}{33}$$

For same D, $\frac{n_3}{n_p} = \frac{n_1}{n_p} \cdot 3$ because $\frac{n_3}{n_1} = 1$ &

Voltage balance Inductor m

$$V_{in} D + (.9 - D)\left(\frac{-V_o}{N}\right) = 0$$

$$\frac{V_{in} D}{(.9-D)}\left(\frac{n_1}{n_p}\right) = V_o = 15$$

$$\frac{V_{in} D}{.9-D}\left(\frac{n_3}{n_p}\right) = V_o = 5$$

if $\boxed{\frac{n_1}{n_p} = \frac{3}{25}, \text{ then } \frac{n_3}{n_p} = \frac{1}{25}}$

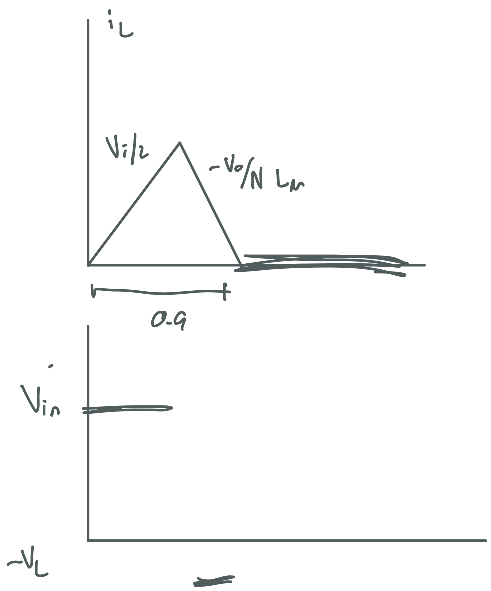
$$\frac{165(D)}{.9-D}\left(\frac{3}{25}\right) = 15$$

$$\frac{D}{.9-D} = 0.757\overline{5}$$

$$D = .68\overline{1} - .757\overline{5}\, D$$

$$1.75\, D = .68\overline{1}$$

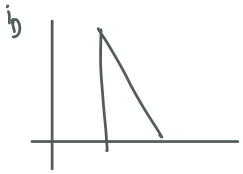
$$\boxed{D = 0.388}$$




$$i_{pk} = \frac{V_i}{L_M} DT$$

$$0 = n_p i_M + n_1 i_{D_1 pk} + n_2 i_{D_2 pk} + n_3 i_{D_3 pk}$$

$i_D$



$\frac{1}{2} D_2 \, i_{d,peak} = \langle I_o \rangle$

$i_{d_1 peak} = \frac{\langle I_o \rangle 2}{.9-D} = \frac{2}{0.9-D} = 3.90625$

$i_{d_2 peak} = \frac{\langle I_o \rangle 2}{.9-D} = \frac{1}{0.9-D} = 1.9531$

$i_{d_3 peak} = \frac{(4)2)}{.9-D} = 15.625$

$0 = i_M + \frac{3}{25}(3.906) + \frac{3}{25}(1.9531) + \frac{1}{25}[15.625] = 0$

$i_M = 1.328$

$\frac{-D V_i}{L_M f_s} = i_M \Rightarrow \boxed{i_M = 482 \mu H}$

- For your design of the previous part, determine the rms currents of the four windings.
  - Note that they don't simply scale by the turns ratios.

$R_{ms} = \sqrt{\sum x^2 / N}$

$\sqrt{\frac{\int_0^{x_0} (ax)^2 dx}{x_0}}$ ] Full $\Delta$

$\sqrt{\frac{\int_0^{D_2 x_0} (\frac{ax}{D_2})^2 dx}{x_0}}$ ] partial $\Delta$

$\frac{a_0}{D_2}\sqrt{\frac{x^3}{3x_0}\Big|_0^{D_2 x_0}} = \frac{a_0 x_0}{D_2}\sqrt{\frac{D_2^3}{3}} = I_{peak}\sqrt{\frac{D_2}{3}}$

$a_0 x_0$ = peak value so $I_{peak}$

From above

$I_{D_1 peak} = 3.90625$

$I_{D_2 peak} = 1.9531$

$I_{D_3 peak} = 15.625$

$I_{M peak} = 1.328$

$I_{1rms} = 3.90625\sqrt{\frac{.9-.388}{3}} = \boxed{1.614 A}$

$I_{2rms} = 0.8068 A$

$I_{3rms} = 6.455 A$

$I_{M peak} = 0.548 A$

# 5 Average circuit modeling of an inderect boost-buck converter

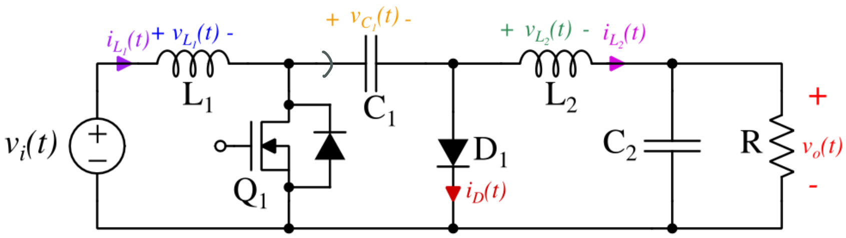


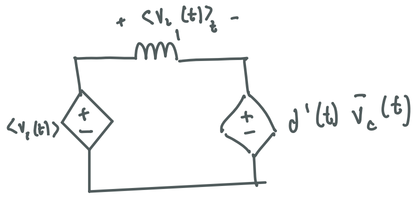

Figure 5.1: Indirect boost-buck dc-dc converter.

Consider the indirect boost-buck converter of Figure 5.1. Assume the converter is operating in CCM and that all components are ideal.

- [ ] Obtain the non-linear averaged circuit model of the converter

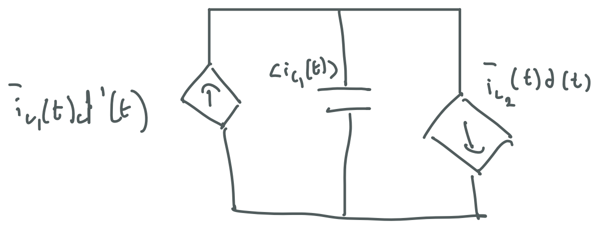
$L_1$

$$\langle v_{L_1}(t)\rangle_{T_s} = \bar{v}_i(t)\, d(t) + d'(t)\left(\bar{v}_i(t) - \bar{v}_c(t)\right)$$




$C_1$

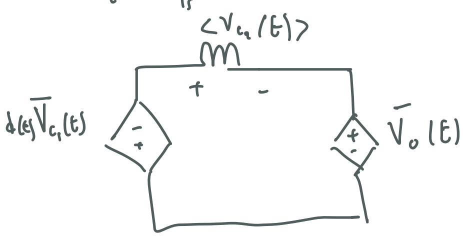
$$-\bar{i}_{L_2}(t)\, d(t) + \bar{i}_{L_1}(t)\, d'(t) = \langle i_{C_1}(t)\rangle$$




$L_2$

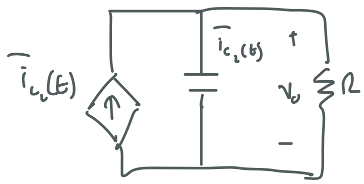
$$\langle V_{L_2}(t)\rangle_{T_s} = \left(-\bar{V}_{C_1}(t) - V_o(t)\right) d(t) + -\bar{V}_o(t)\, d'(t)$$

$$\langle V_{L_2}(t)\rangle_{T_s} = -\bar{V}_{C_1}(t)\, d(t) + -\bar{V}_o(t)$$



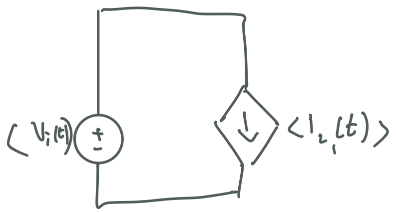

$C_2$

$$\left(\bar{i}_{L_2}(t) - \frac{\bar{V}_o}{R}(t)\right) = \langle \bar{i}_{C_2}(t)\rangle$$

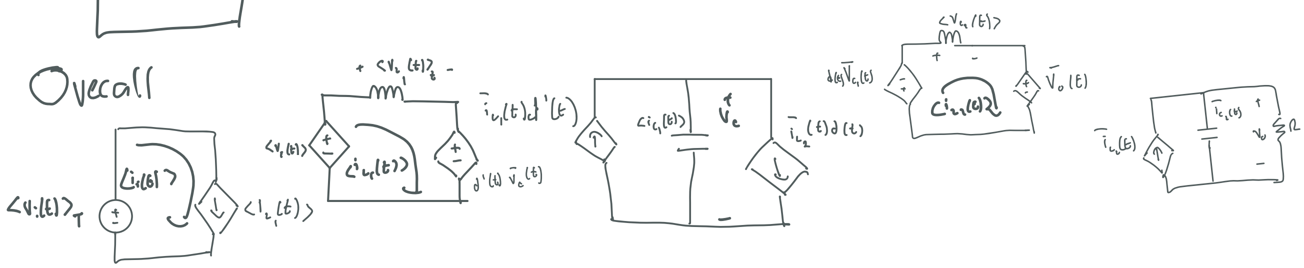


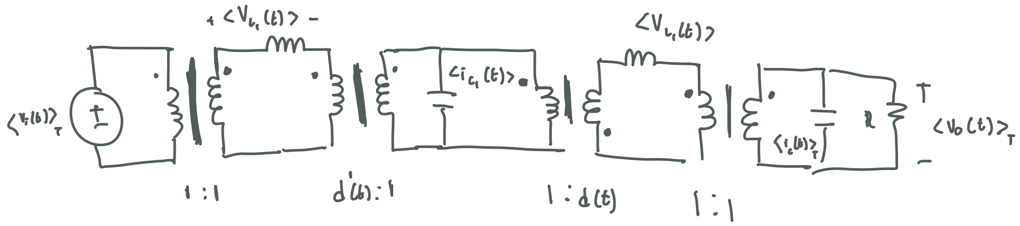
input

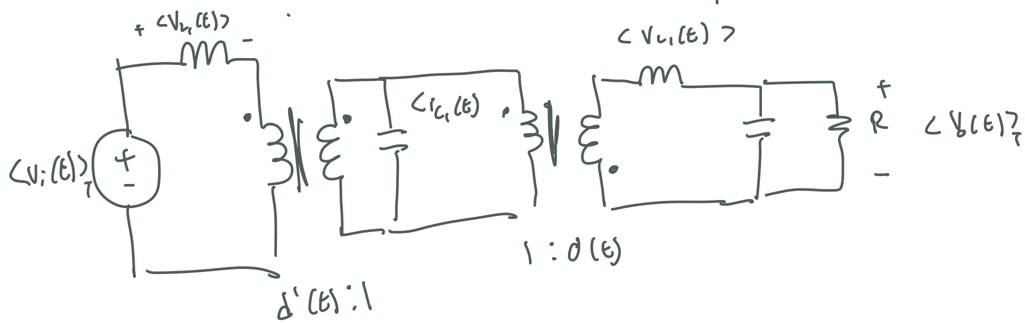
$\bar{i}_2(t) = i_1(t)$



Overall







☐ Perturb and linearize the non-linear averaged circuit model and obtain the ac linerized equivalent circuit model for the converter.

$$\langle v_L(t)\rangle_{T_s} = \bar{v}_i(t)\, d(t) + d'(t)\left(\bar{v}_i(t) - \bar{v}_c(t)\right)$$

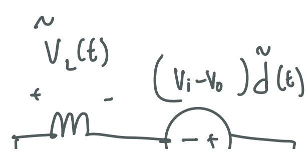
$$\langle v_L(t)\rangle_{T_s} = v_i(t) - d'(t)\,\tilde{v}_c(t)$$

$$d'(t) = 1 - D + d'(t)$$

deriv of constant = 0

$$L\left(\frac{dI_L}{dt} + \frac{d\tilde{i}_L(t)}{dt}\right) = \left(V_i + \tilde{v}_i(t)\right) - \left(D' - \tilde{d}(t)\right)\left(V_c + \tilde{V}_c(t)\right)$$

$$= V_i - D'V_c \quad \text{constants cancel derivative} = 0$$



$$+\ \tilde{v}_i(t)\ +\ \tilde{d}(t)V_c - D'\tilde{v}_c(t)$$

$$+\ \tilde{d}(t)\tilde{v}_c(t)$$ 2nd order (Very small)


$$\tilde{v}_L(t) = L\frac{d\tilde{i}_L(t)}{d(t)} = \tilde{v}_i(t) + \underbrace{\tilde{d}(t)V_c}_{V_c = V_i - V_o =} - D'\tilde{v}_c(t)$$

$L_2$

$$\langle v_{L_2}(t)\rangle_{T_s} = -\bar{v}_{c_1}(t)\,d(t) + -\bar{v}_o(t)$$

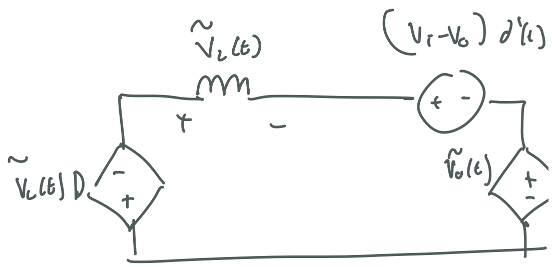
$$L\left(\frac{dI_L}{dt} + \frac{d\tilde{i}_L(t)}{d(t)}\right) = -(V_c + \tilde{v}_c(t))(D + \tilde{d}(t)) - (V_o + \tilde{v}_o(t))$$

$$= -DV_c - V_o \quad \cancel{\ }$$

$$+\ -\tilde{v}_c(t)D - \tilde{d}(t)V_c - \tilde{v}_o(t)$$

$$+\ -\tilde{v}_c(t)\tilde{d}'(t) \quad \times$$

$$\tilde{v}_L(t) = -\tilde{v}_c(t)D - \tilde{d}'(t)V_c - \tilde{v}_o(t)$$




$C_1$

$$-i_{L_2}(t)d(t) + \bar{i}_{L_1}(t)d'(t) = \langle i_{c_1}(t)\rangle$$

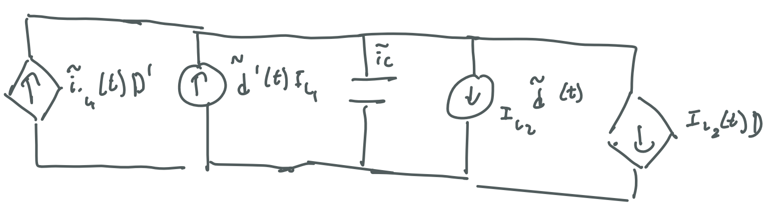
$$-(I_{L_2} + \tilde{i}_{L_2}(t))(D + \tilde{d}(t)) + (I_{L_1} + \tilde{i}_{L_1}(t))(D' - \tilde{d}'(t)) = \langle i_{c_1}(t)\rangle$$

$$= -I_{L_2}D + I_{L_1}D' \quad \cancel{\ }$$

$$+\ -I_{L_2}\tilde{d}(t) - \tilde{i}_{L_2}(t)D + \tilde{i}_{L_1}(t)D' - \tilde{d}'(t)I_{L_1}$$

$$+\ \tilde{i}_{L_2}(t)\tilde{d}(t) - \tilde{i}_{L_1}(t)\tilde{d}'(t) \quad \times$$

$$\tilde{i}_c = -I_{L_2}\tilde{d}(t) - \tilde{i}_{L_2}(t)D + \tilde{i}_{L_1}(t)D' - \tilde{d}'(t)I_{L_1}$$



$C_2$

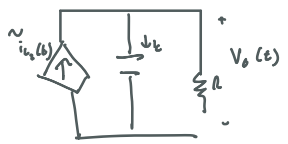
$$\left(\bar{i}_{c_2}(t) - \frac{\bar{V}_0}{R}(t)\right) = \langle \bar{i}_{c_2}(t) \rangle$$

$$I_{c_2} + \tilde{i}_{c_2}(t) - \frac{V_0 + \tilde{v}_0(t)}{R} = \langle i_{c_2}(t) \rangle$$

$$= I_{L_2} - \frac{V_0}{R} \times$$

$$+ \tilde{i}_{L_2} - \frac{\tilde{v}_0(t)}{R}$$

$$\tilde{i}_{L_2}(t) - \frac{\tilde{v}_0}{R}(t) = i_c$$




input current

$$\bar{i}_{L_1}(t) = i_i(t)$$

$$I_L + \tilde{i}_L(t) = I_i + \tilde{i}_i(t)$$

$I_i$ ignore

$$\tilde{i}_L(t) = i_i(t)$$