ROOTS OF POLYNOMIALS - QUADRATICS

{LHS → left hand side

* A quadratic $(ax^2+bx+c=0)$ has two roots. These roots are values of x for which the LHS evaluates to 0.

* We can label these roots α and β and find a relationship between these roots and the coefficients in the quadratic a, b and c.

e.g. $x^2+3x+2=0$. coefficients $a=1$, $b=3$, $c=2$.

$\Rightarrow (x+1)(x+2)=0$ roots $\alpha=-1$, $\beta=-2$

* The relationships between the product or sum of the roots and the coefficients are:

SUM → $\boxed{\alpha+\beta = -\dfrac{b}{a} \quad \text{AND} \quad \alpha\beta = \dfrac{c}{a}}$ ← PRODUCT

* SO, if we know the coefficients, we know something about the roots.

<u>AND</u>, if we know the roots, we know something about the coefficients.

* We can derive these relationships thus:

GENERAL QUADRATIC EQ^{n} → $ax^2+bx+c=0$

$\div a$

$\Rightarrow x^2+\dfrac{b}{a}x+\dfrac{c}{a}=0$.

factorizing LHS.

$\Rightarrow x^2+\dfrac{b}{a}x+\dfrac{c}{a}=(x-\alpha)(x-\beta)$

expand RHS

$\Rightarrow x^2+\dfrac{b}{a}x+\dfrac{c}{a}=x^2-(\alpha+\beta)x+\alpha\beta$

By comparison of LHS and RHS.

$\Rightarrow \alpha+\beta=-\dfrac{b}{a}$ AND $\alpha\beta=\dfrac{c}{a}$

* We can also prove these relationships using the quadratic formula:

for a quadratic $ax^2 + bx + c = 0$.
there are two roots:

There are conjugate pairs

$$\alpha = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{AND} \quad \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

<u>SO</u> $\alpha + \beta = \frac{-b - \sqrt{b^2-4ac}}{2a} + \left(\frac{-b + \sqrt{b^2-4ac}}{2a}\right)$

$$\Rightarrow \alpha + \beta = \frac{-b - b}{2a}$$

$$= -\frac{b}{a}.$$

<u>AND</u> $\alpha\beta = \frac{(-b - \sqrt{b^2-4ac})(-b + \sqrt{b^2-4ac})}{2a \times 2a}$.

Using difference of two squares

$$\alpha\beta = \frac{b^2 - (b^2 - 4ac)}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$= \frac{c}{a}$$