SERIES

Sigma notation makes it easy to represent the sum of of a series of numbers.

E.g if I wanted to work out the sum of the first 5 natural numbers $(1+2+3+4+5)$, I could represent it like this:

$$\sum_{r=1}^{5} r$$

ending point of series

what we do to the r value.

This symbol is sigma, meaning 'sum of'.

starting point of series.

It can get more complex like this:

$$\sum_{r=1}^{5} 2r+1 = \underset{(2\times 1)+1}{\overset{r=1}{3}} + \underset{(2\times 2)+1}{\overset{r=2}{5}} + \overset{r=3}{7} + \overset{r=4}{9} + \overset{r=5}{11}$$

etc.

There are some standard formulae that can help us work out the answer when the series is long ie lots of terms to add.

1. If all the terms are the same:

$$\sum_{r=1}^{5} 1 = \overset{r=1}{1} + \overset{r=2}{1} + \overset{r=3}{1} + \overset{r=4}{1} + \overset{r=5}{1}$$

then we can use $\sum_{r=1}^{n} 1 = n$ ie $\sum_{r=1}^{4} 1 = 4\times 1 = \underline{\underline{4}}$

② If the series is just counting natural numbers

e.g

$$\underset{r=1}{1} + \underset{r=2}{2} + \underset{r=3}{3} + \underset{r=4}{4} + \underset{r=5}{5} = \sum_{r=1}^{5} r$$

Then we can use;

$$\boxed{\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)}$$

e.g $\sum_{r=1}^{3} r = \frac{1}{2}.3.(3+1) = \underline{\underline{6}}$

For big numbers of n this ~~value~~ really helps!

③ If the series doesnt start at 1, we can use this:

$$\boxed{\sum_{r=k}^{n} f(r) = \sum_{r=1}^{n} f(r) - \sum_{r=1}^{k-1} f(r)}$$

e.g

$$\sum_{r=3}^{7} r = \sum_{r=1}^{7} r - \sum_{r=1}^{2} r$$

n

k-1

k

just a function. f(r)

now we can use the formula:

$$\sum_{r=3}^{7} r = \sum_{r=1}^{7} r - \sum_{r=1}^{2} r$$

$$= \frac{1}{2}\times 7\times 8 - \frac{1}{2}\times 2\times 3$$

$$= 28 - 3$$

$$= \underline{\underline{25}}$$

We can also think of it like this:

$$\sum_{r=1}^{7} r = 1 + 2 + \underbrace{3 + 4 + 5 + 6 + 7}_{\sum_{r=3}^{7} r}$$

$$\sum_{r=1}^{2} r = 1 + 2$$

SERIES cont.

* We can also break up complicated functions into chunks which are easier to manage:

$$\sum_{r=1}^{n} kf(r) = k\sum_{r=1}^{n} f(r)$$

e.g

$$\sum_{r=1}^{3} 5r = 5\sum_{r=1}^{3} r \qquad \tfrac{1}{2}n(n+1)$$

$$= 5.\left[\tfrac{1}{2}.3.4.\right]$$

$$= \underline{\underline{30}}$$

This is because: $\sum_{r=1}^{3} 5r = (5\times1)+(5\times2)+(5\times3)$

$$= 5(1+2+3)$$

$$= 5\sum_{r=1}^{3} r$$

* Finally, we can break up the functions where we see the add or subtract sign:

$$\sum_{r=1}^{n}\left[f(r)+g(r)\right] = \sum_{r=1}^{n} f(r) + \sum_{r=1}^{n} g(r)$$

e.g

$$\sum_{r=1}^{3}(3r^2+r) = 3\sum_{r=1}^{3} r^2 + \sum_{r=1}^{3} r$$

$$= \overbrace{3+12+27} + \overbrace{\tfrac{1}{2}.3\times4}$$

$$= 42 + 6$$

$$= \underline{\underline{48}}$$

SERIES cont...

* If you want to find the sum of a series of square numbers, we can use:

$$\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$$

e.g

$$\sum_{r=1}^{4} r^2 = \frac{1}{6} . 4 . (4+1)((2 \times 4)+1)$$

$$= \frac{1}{6} . 4 . 5 . 9$$

$$= \underline{\underline{30}}$$

* If you want to find the sum of a series of cubic numbers, we can use:

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2$$

e.g

$$\sum_{r=1}^{3} r^3 = \frac{1}{4} 3^2 (3+1)^2$$

$$= \frac{1}{4} . 9 . 16$$

$$= \underline{\underline{36}}$$

r^2 : $\sum_{r=1}^{5} r^2$, $\sum_{r=3}^{7} r^2$

r^3 : $\sum_{r=1}^{7} r^3$, $\sum_{r=4}^{9} r^3$

Prove.

* You might also be asked to show that the sum of a series of n°s defined by a function is a general expression.

e.g. Show that $\sum_{r=1}^{n} \underbrace{r^2+r} = \frac{1}{3}n(n+1)(n+2)$

① Split up your function

$$\Rightarrow \sum_{r=1}^{n} r^2+r = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$$

② Use the formulae

$$\Rightarrow \sum_{r=1}^{n} r^2+r = \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$$

$$\Rightarrow \sum_{r=1}^{n} r^2+r = \frac{1}{6}n(n+1)\left[(2n+1)+3\right]$$

Look for things you can factorise!

$$= \frac{1}{6}n(n+1)(2n+4)$$

Factorise again!

$$= \frac{1}{6}n(n+1) \times 2(n+2)$$

$$\Rightarrow \underline{\sum_{r=1}^{n} r^2+r = \frac{1}{3}n(n+1)(n+2)}$$

Show Ex 3b together.

①

Hwk Rest of 3b