Decision-based Design: Utility Theory

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ME 615 Spring 2020

Issues with Traditional RRBDO

- No rationale for selecting weights w_1 and w_2
- Limited to normal distributions
- If you use the FOSM approximation of the std dev term, you are then taking the gradient and Hessian of a sum of gradients squared:
  - $f(x) = \nabla f^T d + \frac{1}{2} d^T H d$
- The reliability assessment requires a double loop optimization
  - *One can get around this using a method such as SORA by creating a pseudo "equivalent" deterministic constraint (paper on CANVAS)*

Solution

- We can look to utility theory to address the first three issues.
- To use utility theory, we will need to take a closer look at the concept of utility…

Three Types of Decision Analysis

- Normative Decision Analysis identifies logically compelling properties with which decision behavior should conform.
  - *What people should do (in theory)*
- Prescriptive Decision Analysis is a model which can and should be used by a real decision maker, tuned to both the specific situation and needs of the decision maker.
  - *What people should and can do*
- Descriptive Decision Analysis identifies the way people actually make choices.
  - *What people actually do, or have done*

Dillon, Stuart M. "Descriptive decision making: Comparing theory with practice."
Unpublished manuscript, Department of Management Systems, University of Waikato, New Zealand (1998).

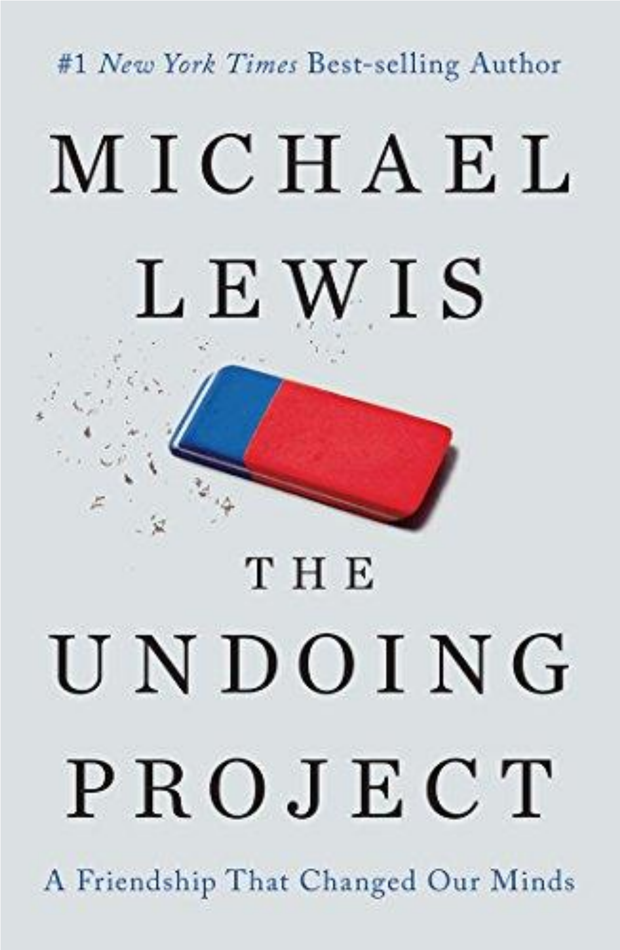
Some Issues in Decision Making

Specific to uncertainty (from Wikipedia):

- *Loss aversion: People's tendency to prefer avoiding losses to acquiring equivalent gains: it's better to not lose $5 than to find $5. Some studies have suggested that losses are twice as powerful, psychologically, as gains*
- *Underestimating uncertainty and the illusion of control: People tend to underestimate future uncertainty because of a tendency to believe they have more control over events than they really do.*
- *Optimism bias: A tendency to overestimate the likelihood of positive events occurring in the future and underestimate the likelihood of negative events.*

Issues in Decision Making

- People have modeled how people "actually" make decisions, e.g. Prospect Theory.
- We are more interested in making optimal decisions, optimal with respect to the project goals.



Notions on Decision Analysis

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- Utility analysis is a *normative* modeling tool.
- The goal is not to mimic the choices of decision makers, but to help them make better decisions.
- Unaided human decision making often exhibits inconsistencies, irrationality and suboptimal choices, particularly when complex tradeoffs under uncertainty must be made.
- "Utility" refers to a preference function built on the axiomatic basis originally developed by von Neumann and Morgenstern (vN-M)

Utility Function

- To fulfill our desirable attributes of a design selection method, we are using **utility theory**.
- A **Utility Function** is composed of three part:
  - ***Quantified Uncertainty*** + ***Risk Attitude*** + ***Design Objective(s)***

What is Utility?

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- Value is a standardized measure of benefit that one can gain from either a good or service, generally measured relative to units of currency.
- Utility is a representation of preferences over some set of goods and services.
  - *Preferences have a (continuous) utility representation so long as they are transitive, complete, and continuous.*
  - *Implies that uncertainty is present*
- We can talk about Expected Value (person invariant) and Expected Utility (person specific)

Scales for utility

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- Ordinal Scale:
  - Only provides an ordinal ranking ("10 is better than 5")
- Interval Scale:
  - Differences can be measured ("the difference between 10 and 5 = the difference between 5 and 0")
- Ratio Scale:
  - Ratios can be quantified ("10 is twice as much as 5")

Four Types of Utility Functions

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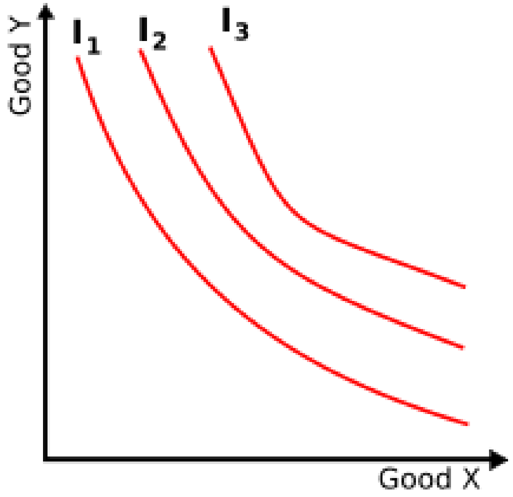
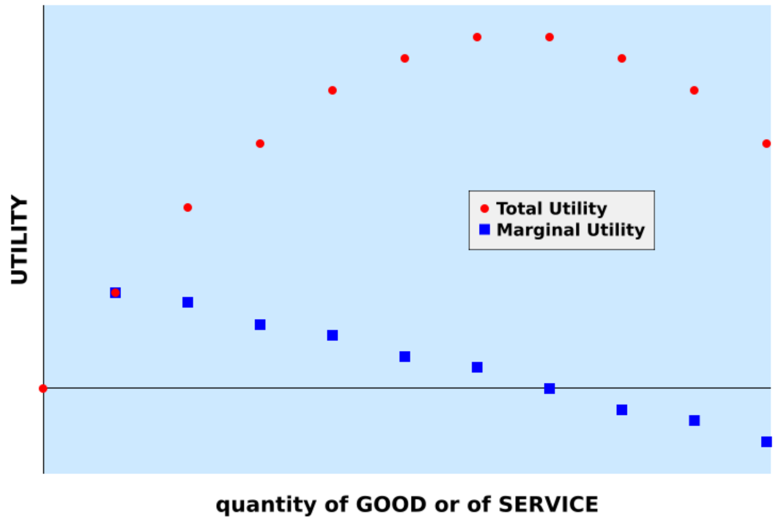
- **Ordinal utility** (*ordinal scale*):
  - A function in which goods are ordered such that one is considered by an individual to be worse than, equal to, or better than the other.
  - The Pugh matrix for design selection is based upon Ordinal Utility
  - Indifference curves can be created for multiple goods, such as **X** and **Y**
  - Used in optimization to create Pareto frontiers for multiobjective optimization



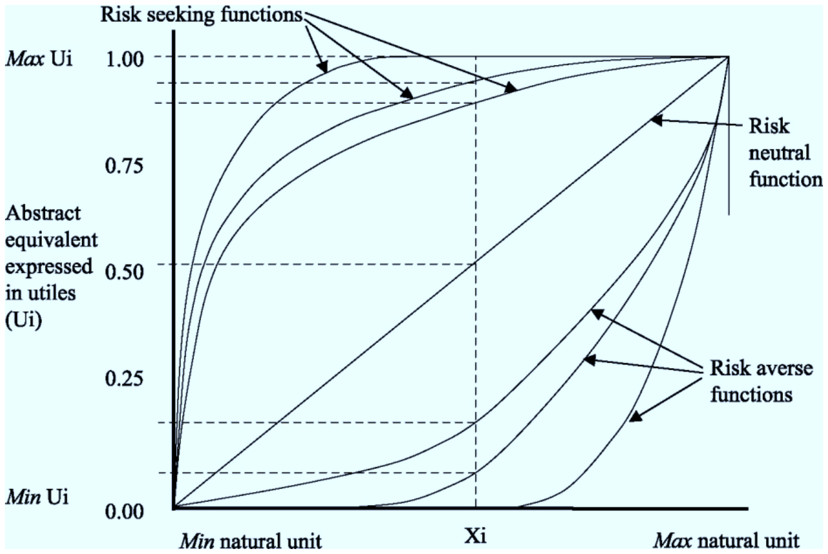
Four Types of Utility Functions

- **Marginal Utility Theory** (*interval scale*):
  - Also known as the law of diminishing marginal utility, it states that the first unit of consumption of a good or service yields more utility than the second & subsequent units, with a continuing reduction for greater amounts each.
  - Useful in cases in which we want to value additional goods—limited application in engineering design.



Four Types of Utility Functions

- **Expected Utility Theory** (*interval scale*):
  - Refers to the analysis of choices among risky alternatives with (possibly multidimensional) uncertain outcomes (i.e. gambles).
  - Requires one to assign probabilities to the possible outcomes
  - This theory states that if certain axioms are satisfied, the subjective value associated with a gamble by an individual is the statistical expectation of that individual's valuations (utility) of the outcomes of that gamble.



Four Types of Utility Functions

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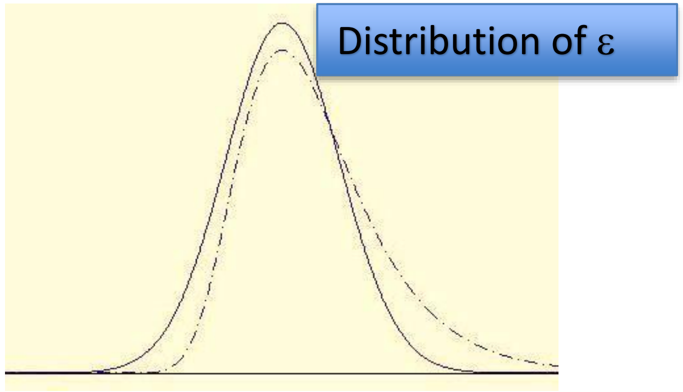
- **Random Utility Theory** (*interval scale*):
  - Utility derived by looking at choices agents make but not necessarily in risky situations (i.e. **inferred utility**).
  - Even if we assume that choices are made rationally, in general we cannot measure utility (predict choices) exactly because, for example, we may not be able to observe or measure every characteristic of the individual, product, or choice situation that affects choice behavior.
  - The utility function is the sum of the observed components of the choice process ***W*** and the unobserved components ***ε***.

$$U_{in} = W_{in} + \varepsilon_{in}$$



How will use these?

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- **Ordinal utility**
  - *Used in the Design Automation course (ME 617) and Intro to Design (ME 382)*
- **Marginal Utility Theory**
  - *Will not be explicitly used in our theory development*
- **(Expected) Utility Theory**
  - *Will be used to form our normative decision making theory.*
  - *i.e. Model of **our design decision***
- **Random Utility**:
  - *Will be used to infer how users (or consumers) choose a product alternative.*
  - *i.e. Model of the **users' product selection decision***

Introduction to Expected Utility Theory

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- **Utility function**
  - Quantifies how we “value” outcomes, i.e., it reflects our preferences
  - Can be also applied to “value” outcomes other than money and gains (e.g. utility of a patient being healthy, or ill)
- **Utility theory**
  - Defines axioms on preferences that involve **uncertainty** and ways to manipulate them.
  - Uncertainty is modeled through **lotteries**

Lottery: [p: A ; (1-p): C]

Outcome A with probability p

Outcome C with probability (1-p)

Definition of Symbols

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- $\succ$ or $\prec$ represents "is preferred to" (similar to the > < symbols)
  - For example, for motor weight 20kg > 10kg but I can have the preference 10kg $\succ$ 20kg
- ~ represents "indifferent" (similar to the = symbol)
  - For example, 1 apple ≠ 2 bananas but I can have the preference 1 apple ~ 2 bananas.

Expected Utility Theory Axioms (von Neumann & Morgenstern)

- **vNM 1 Completeness** assumes that an individual has well defined preferences and can always decide between any two alternatives.

$$A \succ B \text{ or } A \sim B \text{ or } B \succ A$$

- **vNM 2 Transitivity** assumes that, as an individual decides according to the completeness axiom, the individual also decides consistently.
  - Given three states, if an agent prefers *A* to *B* and prefers *B* to C, then he must prefer *A* to C.

$$\text{If } A \succ B \text{ and } B \succ C\text{, then } A \succ C$$

Expected Utility Theory Axioms (von Neumann & Morgenstern)

- **vNM 3 Independence** also pertains to well-defined preferences and assumes that two gambles mixed with a third one (with some probability p) maintain the same preference order as when the two are presented independently of the third one.

$$A \succ B \text{ if and only if } pA + (1-p)C \succ pB + (1-p)C$$

- **vNM 4 Continuity** assumes that when there are three states (A, B and C) and the individual prefers A to B and B to C, then there should be a possible combination of A and C in which the individual is indifferent between this mix and B.

$$\text{If } A \succ B \succ C \text{ then there is some } p \text{ such that } B \sim pA + (1-p)C$$

Expected Utility Properties

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- If we adhere to the four vNM axioms, we can formulate the three properties of Expected Utility Theory:
  1. A $\succ$ B if and only if $u(A) > u(B)$
  2. $u[pA + (1 - p)B] = pu(A) + (1 - p)u(B)$
  3. For every other function u' satisfying 1. and 2., there are numbers $c > 0$ and d such that $u' = c \cdot u + d$
- What do these properties mean for us?

Expected Utility Properties

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- Property 1: A $\succ$ B if and only if $u(A) > u(B)$
  - Translation: the utility function u assigns higher numbers to more preferred lotteries.
  - Also, we can state that A $\sim$ B if and only if $u(A) = u(B)$
- Property 2: $u[pA + (1 - p)B] = pu(A) + (1 - p)u(B)$
  - Translation: the utility of a compound lottery equals the expected utility of its components.
  - This means that someone who obeys the 4 axioms acts in accordance with the principle of maximizing expected utility.
- Property 3: $u' = c \cdot u + d$
  - Translation: all utility functions satisfying property 1 and 2 are positive linear transforms of each other.
  - This means the vNM utility is measured on an interval scale.

What can we do with the properties?

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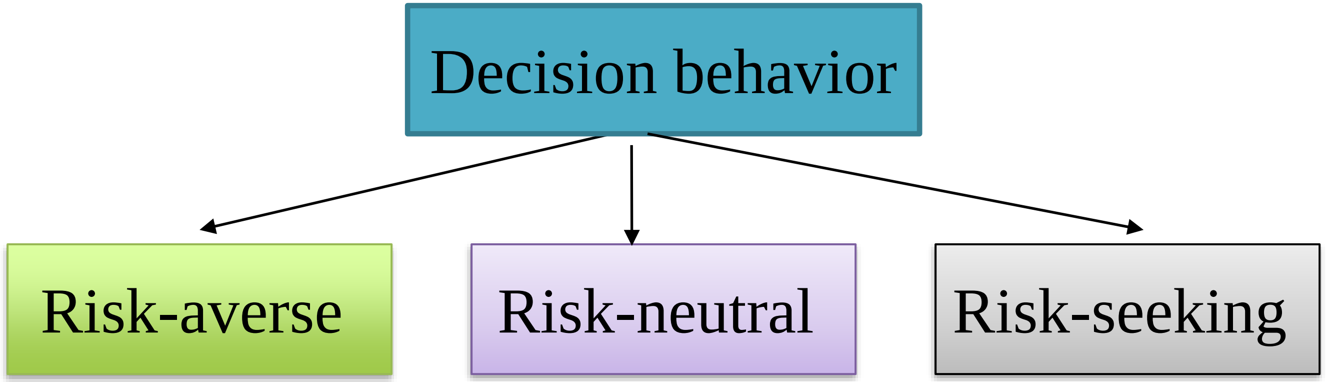
- Property 1:
  - *We can create a utility function which turns preferences into quantified numbers.*
- Property 2:
  - *We can compute the expected utility of a lottery by adding up the expected utility of its components*
- Property 3:
  - *We only need to develop a utility function on an interval scale—we can choose the scale on which utility is measured (e.g. 0-1)*

What are we going to measure: Risk-Preference Attitude

- Not all people select the same action when faced with the same decision situation.
  - One important reason for making different choices is that the decision makers have different attitudes about taking a risk.
  - Their risk preferences influence their view on the potential outcomes.



Risk-preference Attitude (contd.)

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- Decision makers who base their **decision solely on the highest expected payoff** or the lowest expected cost, i.e. on expected-value, are **risk-neutral**.
- Decision makers who are willing to take on additional **risk for higher payoffs** are **risk-seeking (risk prone)**.
- Decision makers who are **not willing to take on additional risk** for higher payoffs are **risk-averse**.

Lottery Method for Utility Function Formulation

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- Using properties 1-3, we can now create a methodology for estimating utility functions.
- We will use the "lottery" method for measuring risk preference attitude.

1. Ask a series of questions in which the user expresses his/her "indifference" point for a lottery vs. a certain sum of money. *The indifference point is a measure of utility (property 1)*
2. From the answers, we can create a plot of utility vs. value. We can choose the scale of the utility (*property 3*)
3. With this plot, we can compute the utility for lotteries not assessed in the original interview (*property 2*)

Lottery Method for Utility Function

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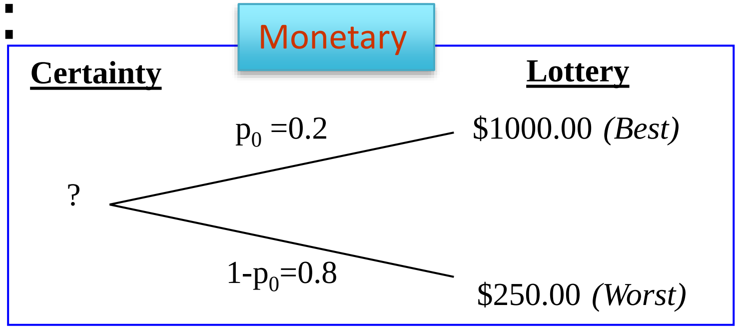
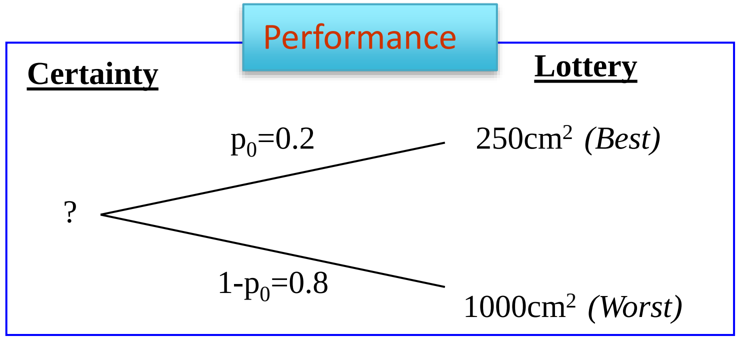
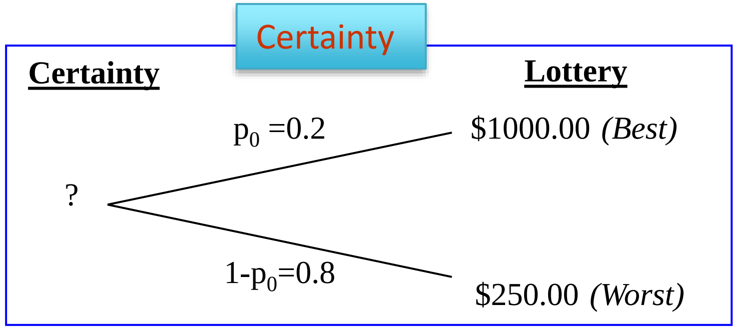
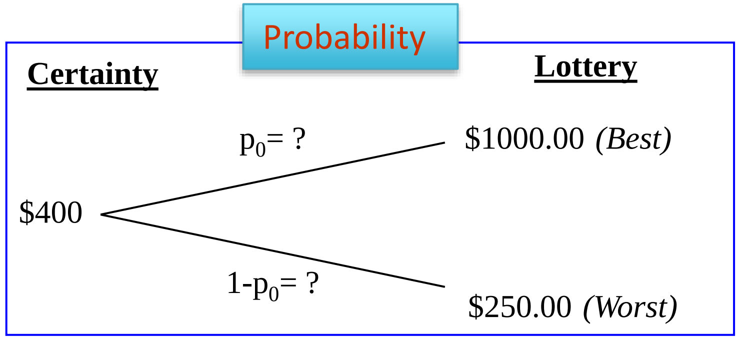
- Can conduct the lottery for both monetary and performance measures:





- Can ask the lottery questions both in terms of certain equivalent or probability





Krishnamurty (2007)