Dynamic temperature calculation of a surface at solar radiation and in radiation exchange with environment

Engin Bagda, Manfred Hermann, Erkam Talha Öztürk

Introduction

Heat flow through walls depends on the surface temperature on the inside and the outside. If the temperatures of the surfaces change the heat flow gets dynamic. Dynamic heat flow through walls can be calculated numerically with the Crank-Nicolson method as explained in erkam-o/DynamicHeatFlow , where the wall is "sliced" in finite layers.

Here is explained how the temperature on and in the exterior layer *n* can be calculated depending on the environmental conditions using the code SurfaceTemp_2021_03_11.py in combination with the excel sheet Garden_2021_03_08.xlsx where are measured data for solar radiation, air temperature, surface temperature on different materials given. The measured values are compared with the calculated one and it is a good approximation detected.

Basics

The temperature in the external layer *n* is depending on the

- Heat exchange by transmission,

$$q_{transmission} = h\cdot(T_{air} - T_{surface}) \qquad [\text{W/m}^2] \qquad (1)$$

- Heat conduction between the layer *n* and the next layer *n-1*

$$q_{transmission} = 2 \cdot \lambda_n \cdot \lambda_{n-1} \cdot (T_n - T_{n-1}) \,/\, (d_n \cdot \lambda_{n-1} + d_{n-1} \cdot \lambda_n) \quad [\text{W/m}^2] \qquad (2)$$

  Equation 2 assumes that the temperature on the surface of layer *n* is the same as the temperature in the centre of the layer and the heat flow is between the centres of layers ***n*** and *n-1*. The distances between the centres and the common border between the two layers are $d_n/2$ and $d_{n-1}/2$ respectively.

- Emitted radiation from the wall to the enviroment

$$q_{emission} = \varepsilon_{surface} \cdot \sigma \cdot T_{surface}^{\;4} \qquad [\text{W/m}^2] \qquad (3)$$

- Absorbed long wave radiation from environment, like sky, clouds, ground, buildings, trees etc. according to their temperature

$$Q_{environment} = \varepsilon_{surface} \cdot \varepsilon_{environment} \cdot \sigma \cdot T_{environment}^{\;4} \qquad [\text{W/m}^2] \qquad (4)$$

- Absorbed solar radiation

  $q_{solar} = a \cdot I_{sun}$ [W/m^2] (5)

T	: Temperature	[K]
h	: Surface heat transfer coefficient	[W/(m^2·K)]
λ	: Thermal conductivity between layer n and n-1	[W/(m·K)]
d	: Thickness of the layer n and n-1	[m]
a	: Solar adsorption coefficient surface	
I_{sun}	: Solar Radiation	[W/m^2]
ε	: Emission coefficient of surface and environment	
σ	: Stefan-Boltzmann constant	[5,67·10^{-8} W/(m^2·K^4)]

The change of the heat content ΔQ_n of the layer n is the result of the heat flows in and out of the layer between time j and $j+1$ according to equation 6.

$$\Delta Q_i = (q_{convection} + q_{transmission} + q_{solar} + q_{environment} + q_{emission}) \cdot A \cdot \Delta t \qquad [J] \qquad (6)$$

$$\Delta t = t_j - t_{j+1} \qquad [s] \qquad (7)$$

ΔQ_i	: Change of the heat content of layer n	[J]
A	: Surface area of layer n	[m^2]
Δt	: Time between time t_j and t_{j+1}	[sec]

Calculation of the temperature $T_{n,j}$

The heat content $Q_{n,j}$ in the Layer n at time j is:

$$Q_{n,j} = c_V \cdot T_{n.j} \qquad [J/m^3] \qquad (8)$$

$Q_{n,j}$	: Heat content of element n at time j	[J/m^3]
c_V	: Heat capacity of the element n	[J/(m^3·K)]
$oT_{n.j}$	: Temperature of element n at time j	[K]

When the heat content ΔQ of layer n changes during time Δt, the temperature changes from $T_{n,j}$ to $T_{n,j+1}$ according to equation 9:

$$\Delta Q = Q_{n,j} - Q_{n,j+1} = \cdot c_V \cdot (T_{n.j} - T_{n.j+1}) \qquad [J/m^3] \qquad (9)$$

The combination of equations 6 and 9 leads to equation 10:

$$0 = c_V \cdot (T_{n.j} - T_{n,j+1}) \cdot d_n \cdot A$$
$$- (q_{convection} + q_{transmission} + q_{solar} + q_{surrounding} + q_{emission}) \cdot A \cdot \Delta t$$
(10)

With equation 10 the “new” temperature $T_{n,j+1}$ in the layer n could (theoretically) be calculated, because all values apart $T_{n,j+1}$ are known in equation 10.

But:
In case $T_{n,j} > T_{n,j+1}$,$T_{n,j+1}$ will be too low, because $q_{convective}$ and $q_{transmission}$ are linear proportional to T_n (equation 1 and 2) but $q_{emission}$ is proportional to the 4th power of T_n (equation 3).

From the same reason $T_{n,j+1}$ will be too high in case $T_{n,j} < T_{n,j+1}$.

To solve this problem $T_{n,j+1}$ is iterated in steps of 0.2 K in a trial-and-error function until the sum of all heat flows reach a minimum in equation 10 in absolute figures (code line 55 and 62).

Trial and Error function in code SurfaceTemp_2021_03_11.py line 45 to 66

At first the heat balance in the layer [n-1] is calculated by the function HeatBalance(): with the "old" temperature 'Temp_old'. Take into account that the layer on the surface is [n-1] in the code.

In the loop (code lines 51 to 56) Temp[n-1] is decreased until the heat balance reaches a minimum absolute value (smallest number).

In the loop (code lines 58 to 63) Temp[n-1] is increased until the heat balance reaches a minimum absolute value (smallest number).

The so reached Temp[n-1] is the value which fits equation 10 best .

Measurements

In order to validate the code 'SurfaceTemp_2021_03_11.py' two samples were used (physical properties are in table 1).

Sample **"XPS/Fibre cement"** is a model to analyse the heat congestion on a surface of a material with a low thermal effusivity. Therefore is on the surface a material with a low thermal conductivity a layer attached with a high heat capacity. This is realised with a 5 mm thick fibre cement board on 50 mm extruded polystyrene foam. With these materials can samples easily built to measure thermal effects of irradiation. The temperature sensors DS18B20 with a height and diameter of 5 mm fit perfectly in a 5 mm hole in the 5 mm thick fibre cement board and the wires to the data logger can easily be guided through the XPS. The specimens are rigid and can be exposed outdoor in different positions.

Layer	**Thickness**	**Thermal conductivity**	**Heat capacity**	**Density**	**Thermal effusivity***
Material	[m]	[W/(m·K)]	[J/(kg·K)]	[kg/m³]	J/(m²·K·s^{0,5})
Specimen "Wood"					
Wood**	0,08	0,200	1900	700	515
Specimen "Fibre cement/XPS"					
Extruded Polystyrene foam	0,050	0,035	1450	15	27
Fibre cement	0,008	0,580	960	1650	958

Table 1: Properties of the measured specimens

* Thermal effusivity is a measure of the ability of a material to exchange thermal energy with its surroundings.

$$Thermal\ effusivity = \sqrt{Thermal\ conductivity \cdot Heat\ capacity \cdot Density}$$

** Physical values for wood differ in the literature. The used values fitted the measured values best.

The coating on the surfaces of the samples has an absorption coefficient of 0.95 between 390 nm and 1100 nm. The long wave emission-coefficient is 0.92 (code line 124, 125).

The temperatures are measured with digital temperature sensors DS18B20 and the solar radiation with photodiodes TSL 2591. The results of the TSL 2591 were calibrated with a Kipp&Zonen pyranometer SP lite 2. Arduino boards are used as data logger. The timing of the measurement was 1 minute, averaged over 5 minutes and then logged. For the two samples the measured temperatures on a day with clear sky and low humidity are given in figure 1 (input code line 89, ´Garden_20212_03_08.xlxs´).

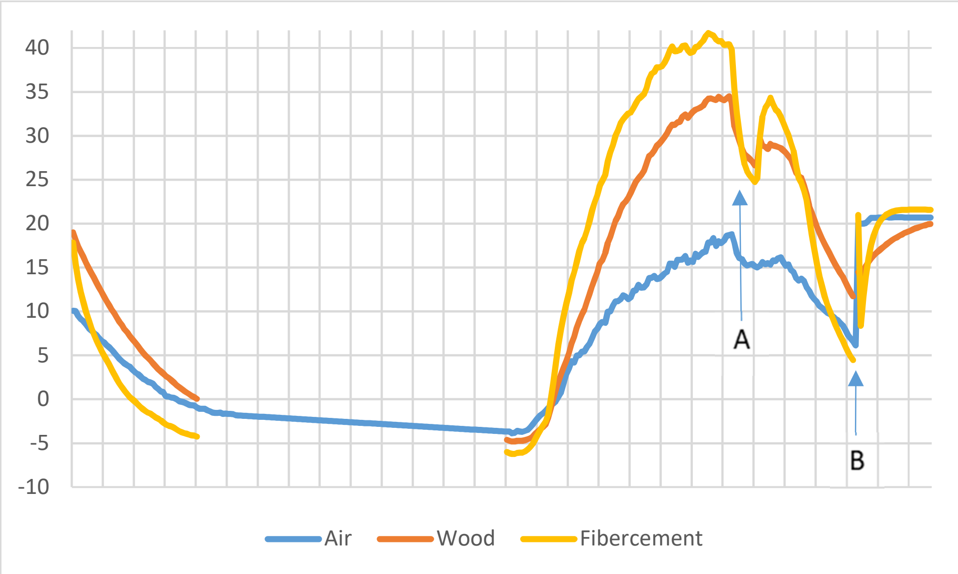

Figure 1: Measured surface and air temperatures on the samples

All samples have been conditioned at 20 °C for 24h and brought to outdoor exposition at 18:00.

The specimens where exposed in horizontal position one meter above the ground on a table (this is important for the consideration of convective heat exchange).

At 2:00 the surface temperature of the sample fibre board is 4 K less than the air temperature, because of the emission to the clear sky. In the calculation of the emission it is assumed that the temperature of the sky is 8 K less than the air temperature (code line 32).

At 07:00 the measurements start and the highest surface temperatures are measured at 13:30. According to the physical properties the surface temperature of the fibre cement/XPS sample is higher than the wood sample.

At 14:30 the samples were shaded with a marquee until 15:30 (Arrow A). In this time the long wave exchange of heat by radiation against the sky (q-emission) was interrupted. Therefore, the long wave exchange of heat (q-emission) is zeroed in the calculation (code line 38 and 39).

At 18:30 the specimens were brought in a room with 21 °C. The air temperature adapted first to the indoor conditions and the surface temperature of the wood sample last. Indoors the long wave exchange of heat (q-emission) is zeroed (code line 40 and 41).

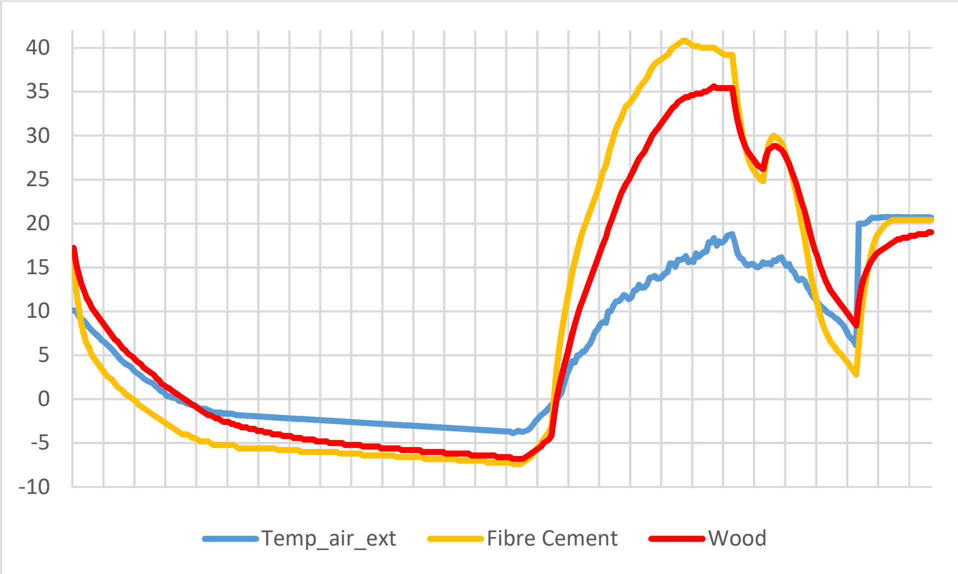

Figure 2: Calculated surface temperatures of the samples

In figure 2 are the surface temperatures for the two samples calculated with code SurfaceTemp_2021_03_11.py. The calculated values are written in to the excel file "Output_Surface_temp.xlsx" worksheet "Temperature Profile".

The comparison of the measured temperatures in figure 1 and the calculated temperatures in figure 2 shows a good agreement, taking into account all the assumptions done, especially for the heat exchange by convection with 10 W/(m²·K), the emission against the sky and the surroundings.

Conclusion

With the code 'SurfaceTemp_2021_03_11.py', surface temperatures can be calculated in a good approximation, if the sun radiation on the surfaces and air temperature are known.

In a later work we will show how the solar irradiation on walls can be calculated from the global solar irradiance, which is known for most places.