HW #5 Due on Apr. 3rd, 2020.

5.3-5

Solution 5.3-5

(a) Because part (b) requires us to separate the response into zero-input and zero-state components, we shall start with the delay operator form of the equations, as

$$y[n] + 2y[n-1] = x[n].$$

To determine the initial condition $y[-1]$, we set $n = 0$ in this equation and substitute $y[0] = 1$ to obtain

$$1 + 2y[-1] = x[0] = e \quad \Longrightarrow y[-1] = (e-1)/2.$$

The z-transform of the delay form of equation yields

$$Y[z] + 2\left[\frac{1}{z}Y[z] + \frac{e-1}{2}\right] = \frac{ez}{z - e^{-1}}.$$

Rearranging the terms yields

$$\frac{Y[z]}{z} = \frac{1}{z+2}\left[(1-e) + \frac{ez}{z-e^{-1}}\right].$$

The term $(1-e)$ on the right-hand side is due to the initial condition, and hence represents the zero-input component. The second term on the right-hand side represents the zero-state component of the response. Thus,

$$\begin{aligned}\frac{Y[z]}{z} &= \frac{1-e}{z+2} + \left[\frac{ez}{(z-e^{-1})(z+2)}\right]\\ &= \frac{1-e}{z+2} + \frac{2e^2}{(2e+1)(z+2)} + \frac{e}{(2e+1)(z-e^{-1})}\end{aligned}$$

and

$$Y[z] = (1-e)\frac{z}{z+2} + \frac{2e^2}{2e+1}\,\frac{z}{z+2} + \frac{e}{2e+1}\,\frac{z}{z-e^{-1}}.$$

The first term on the right-hand side is the zero-input component and the remaining two terms represent the zero-state component. Thus,

$$y[n] = \underbrace{(1-e)(-2)^n u[n]}_{\text{zir}} + \underbrace{\frac{2e^2}{2e+1}(-2)^n u[n] + \frac{e}{2e+1}e^{-n}u[n]}_{\text{zsr}}.$$

The total response is

$$y[n] = \frac{1}{2e+1}\left[(e+1)(-2)^n + e^{-(n-1)}\right]u[n].$$

(b) Referring to part (a), we see that

$$y_{\text{zir}}[n] = (1-e)(-2)^n u[n]$$

and

$$y_{\text{zsr}}[n] = \frac{2e^2}{2e+1}(-2)^n u[n] + \frac{e}{2e+1}e^{-n}u[n].$$

5.4-2

Solution 5.4-2

(a) We want to realize the system

$$H[z] = \frac{z(3z-1.8)}{z^2 - z + 0.16} = \frac{3 - 1.8z^{-1}}{1 - z^{-1} + 0.16z^{-2}}.$$

Figure S5.4-2a shows the canonical direct form (DFII) of the system.

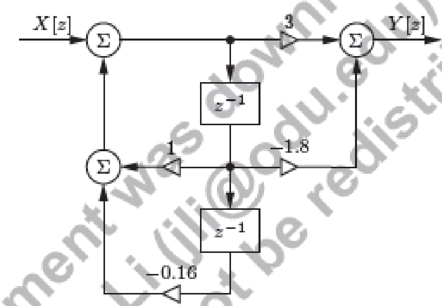

Figure S5.4-2a

To construct a parallel realization, we use partial fractions. To begin, notice that

$$\frac{H[z]}{z} = \frac{3z - 1.8}{z^2 - z + 0.16}.$$

We use MATLAB to compute the necessary partial fraction expansion.

```
>>  [r,p,k] = residue([3 -1.8],[1 -1 0.16])
    r =  1.0000  2.0000
    p =  0.8000  0.2000
    k = []
```

Thus,

$$H[z] = \frac{2z}{z-0.2} + \frac{z}{z-0.8}.$$

Figure S5.4-2b shows a parallel realization based on this expression for $H[z]$.

To construct a series realization, we simply factor $H[z]$ as

$$H[z] = \left(\frac{3z}{z-0.2}\right)\left(\frac{z-0.6}{z-0.8}\right).$$

Figure S5.4-2c shows a series realization based on this expression for $H[z]$. Notice that the parallel and series representations of the system are not unique.

(b) We can obtain the transpose of a block diagram by the following operations:

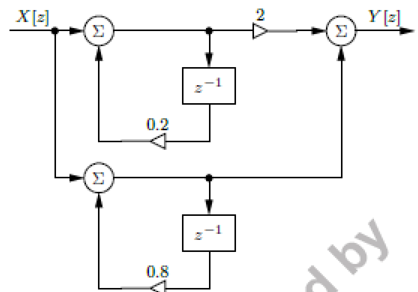

Figure S5.4-2b

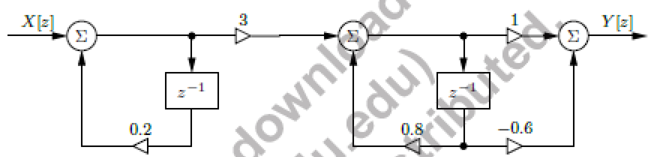

Figure S5.4-2c

1. Reverse the directions of all paths.
2. Replace summing nodes with pick-off nodes and pick-off nodes with summing nodes.
3. Interchange the input $x[n]$ and the output $y[n]$.

Figures S5.4-2d, S5.4-2e, and S5.4-2f are the transpose realizations of Figs. S5.4-2a, S5.4-2b, and S5.4-2c, respectively.

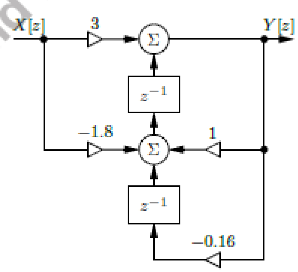

Figure S5.4-2d

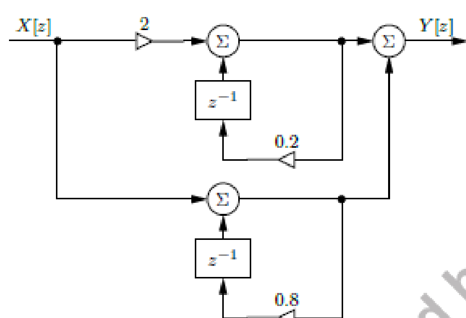

Figure S5.4-2e

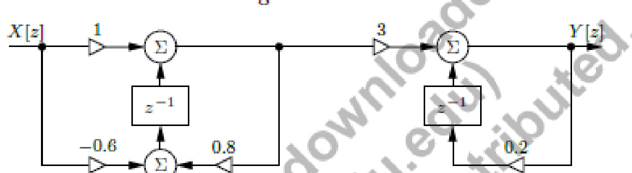

Figure S5.4-2f

5.4-8

Solution 5.4-8

We want to realize the system

$$H[z] = \frac{2z^4 + z^3 + 0.8z^2 + 2z + 8}{z^4} = 2 + z^{-1} + 0.8z^{-2} + 2z^{-3} + 8z^{-4}.$$

Since there is no feedback, we see that $H[z]$ represents a finite impulse response (FIR) system. We can simply realize the system using the direct form structure shown in Fig. S5.4-8.

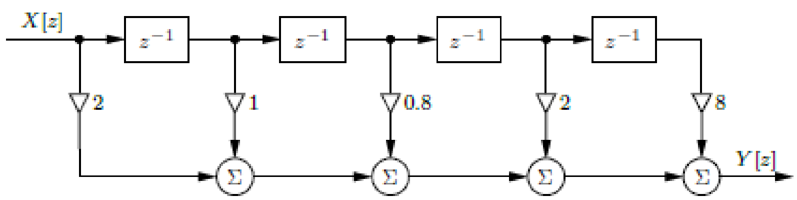

Figure S5.4-8

5.6-1

Solution 5.6-1

In general, pole-zero plots do not provide the overall gain b_0 of a system. For each of the two cases, we therefore normalize the magnitude response by $|b_0|$ and adjust the phase response by $-\angle b_0$.

(a) Figure S5.6-1a shows sketches of the filter's magnitude and phase responses. The magnitude response is relatively high at frequencies $\Omega = \pm\pi/4$, where the poles are closest to the unit circle. The gain is smallest at $\Omega = \pm\pi$, where the poles are farthest away. The zero at the origin does not affect the magnitude response.

The phase of the zero is zero and the phases of the two poles are equal and opposite at $\Omega = 0$. Thus, the (adjusted) phase response is 0 at $\Omega = 0$. As Ω increases, the phases of the zero and both poles increase toward π. At $\Omega = \pi$, the phase response is therefore $\pi - (\pi + \pi) = -\pi$. The phase response changes most quickly near $\Omega = \pm\pi/4$, where the phase of nearby poles are likewise rapidly changing.

By inspection of the pole-zero plot, we see that the system transfer function is, at least approximately, given by

$$H_{\mathrm{a}}[z] = b_0 \frac{z}{(z - 0.75e^{j\pi/4})(z - 0.75e^{-j\pi/4})}.$$

With this expression, we can use MATLAB to readily confirm the system's frequency response characteristics.

```
>> Omega = linspace(-pi,pi,1001);
>> Ha = @(z) z./((z-0.75*exp(j*pi/4)).*(z-0.75*exp(-j*pi/4)));
>> subplot(121); plot(Omega,abs(Ha(exp(1j*Omega)))); grid on;
>> axis([-pi pi 0 3.5]); xlabel('\Omega'); ylabel('|H_a[e^{j\Omega}]/b_0|');
>> set(gca,'xtick',-pi:pi/2:pi,'ytick',0:.5:3.5);
>> subplot(122); plot(Omega,angle(Ha(exp(1j*Omega))));
>> grid on; axis([-pi pi -pi pi]);
>> xlabel('\Omega'); ylabel('\angle H_a[e^{j\Omega}]-\angle b_0');
>> set(gca,'xtick',-pi:pi/2:pi,'ytick',-pi:pi/2:pi);
```

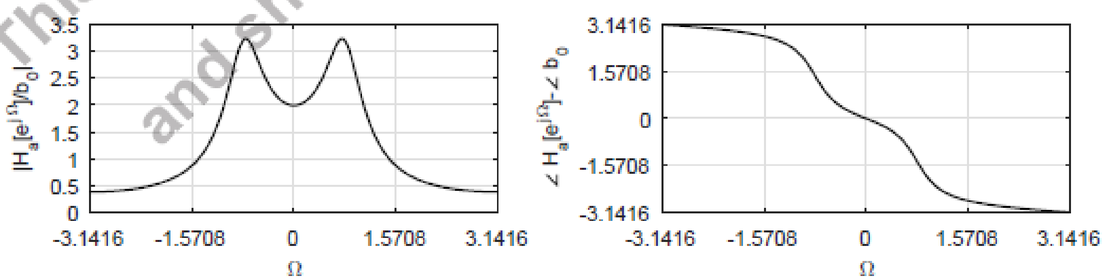

Figure S5.6-1a

(b) Figure S5.6-1b shows sketches of the filter's magnitude and phase responses. The magnitude response is relatively high at frequencies $\Omega = \pm 7\pi/8$, where the poles are closest to the unit circle. The gain is smallest at $\Omega = 0$, where the poles are farthest away. The zeros at the origin do not affect the magnitude response.

The phases of the zero are zero and the phases of the two poles are equal and opposite at $\Omega = 0$. Thus, the (adjusted) phase response is 0 at $\Omega = 0$. At $\Omega = \pi$, the phase response is $2\pi - (\pi + \theta + \pi - \theta) = 0$. As Ω moves between 0 and π, the overall phase bumps up before returning to zero. The phase response changes most quickly near $\Omega = \pm 7\pi/8$, where the phase of nearby poles are likewise rapidly changing.

By inspection of the pole-zero plot, we see that the system transfer function is, at least approximately, given by

$$H_{\mathrm{b}}[z] = b_0 \frac{z^2}{(z - 0.825e^{j7\pi/8})(z - 0.825e^{-j7\pi/8})}.$$

With this expression, we can use MATLAB to readily confirm the system's frequency response characteristics.

```
>>  Omega = linspace(-pi,pi,1001);
>>  Hb = @(z) z.^2./((z-0.825*exp(j*7*pi/8)).*(z-0.825*exp(-j*7*pi/8)));
>>  subplot(121); plot(Omega,abs(Hb(exp(1j*Omega)))); grid on;
>>  axis([-pi pi 0 8.5]); xlabel('\Omega'); ylabel('|H_b[e^{j\Omega}]/b_0|');
>>  set(gca,'xtick',-pi:pi/2:pi,'ytick',0:1:8.5);
>>  subplot(122); plot(Omega,angle(Hb(exp(1j*Omega))));
>>  grid on; axis([-pi pi -pi pi]);
>>  xlabel('\Omega'); ylabel('\angle H_b[e^{j\Omega}]-\angle b_0');
>>  set(gca,'xtick',-pi:pi/2:pi,'ytick',-pi:pi/2:pi);
```

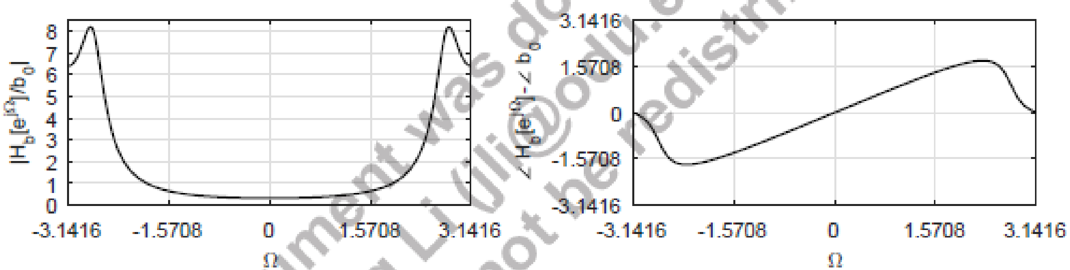

Figure S5.6-1b

5.6-9 a) and b)

Solution 5.6-9

(a) By direct substitution, we see that

$$b_0 \frac{e^{-j2\pi}+1}{e^{-j2\pi}-\frac{9}{16}} = b_0 \frac{2}{\frac{7}{16}} = -1.$$

Thus,

$$b_0 = -\frac{7}{32}.$$

(b) As shown in Fig. S5.6-9b, the system has

poles at $z = \frac{3}{4}$ and $z = -\frac{3}{4}$ and zeros at $z = j$ and $z = -j$.

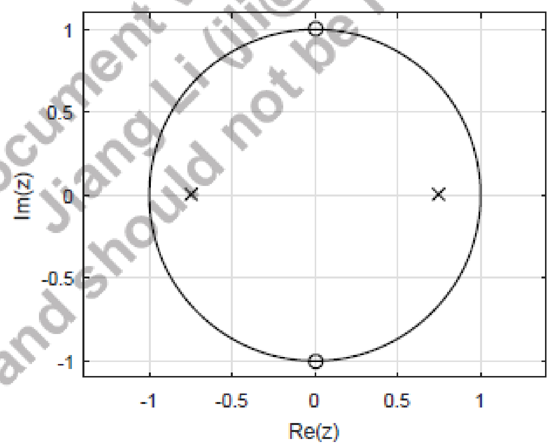

Figure S5.6-9b

5.6-12 a) and b)

Solution 5.6-12

For this problem, we have $H[z] = K\frac{z+1}{z-a}$.

(a) Figure S5.6-12a illustrates a TDFII implementation of the system.

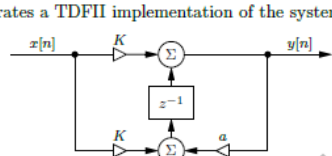

Figure S5.6-12a

(b) This system has a zero at $z = -1$ and a pole at $z = a$. For $|a| < 1$, the pole is closer to $\Omega = 0$ than is the system zero. Hence there is highest gain at dc, and the system is lowpass in nature. Figure S5.6-12b shows the K-normalized magnitude response for $a = \frac{1}{2}$, 0, and $-\frac{1}{2}$.

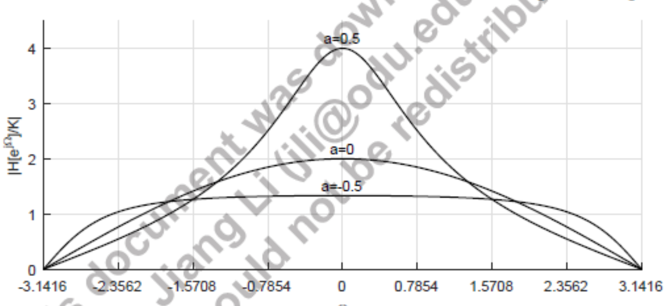

Figure S5.6-12b