Name: UIN: Course No. 381

ECE 381 Introduction to Discrete-time Signal Processing

Test 1

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Close book, close notes. Calculators allowed.

Write down your name, UIN and course No. on each page.

Instructor copy

Name: UIN: Course No.

1. (10 points) Sketch the signal $x[n] = (-1)^n u[n]$. Is the signal a power signal or an energy signal? Find the energy or power of the signal.

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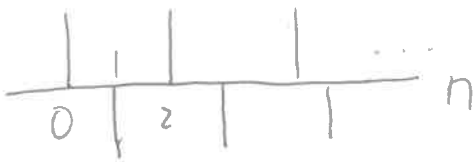

Power signal.

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$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 = \lim_{N \to \infty} \frac{N+1}{2N+1} = \frac{1}{2}$$

Name: UIN: Course No.

2. (10 pints). Given the accumulator system

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

a) (5 points) What is the impulse response of the system?
b) (5 points) if the input for the system is $x[n] = \delta[n] - \delta[n-1]$, Find the zero-state response for this input.

a). $h[n] = \sum_{k=-\infty}^{n} \delta[k] = \delta[-\infty] + \delta[-\infty+1]$

$+ \cdots + \delta[-1] + \delta[0] + \delta[1] + \cdots \delta[n]$

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$= \begin{cases} 1, & n \geq 0 \\ 0, & \text{o.w} \end{cases} = u[n]$

b). $y_{zs}[n] = h[n] * x[n] = u[n] * (\delta[n] - \delta[n-1])$

$= u[n] - u[n-1] = \delta[n]$

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Name: UIN: Course No.

3. (30 points, 6 points each) For the discrete-time system $y[n] = (n - 1)x[n]\, u[n+1]$, is the system a) linear? b) BIBO stable? c) time invariant? d) memory less? and e) causal? Prove or justify your answer.

a). $T\{a x_1[n] + b x_2[n]\} \overset{?}{=} a T\{x_1[n]\} + b T\{x_2[n]\}$

Left $= (n-1)(a x_1[n] + b x_2[n])\, u[n+1]$

$= a(n-1)\, x_1[n]\, u[n+1] + b\,(n-1)\, x_2[n]\, u[n+1]$

right $= a \cdot (n-1)\, x_1[n]\, u[n+1] + b\,(n-1)\, x_2[n]\, u[n+1]$

Left = right $\Rightarrow$ Linear

b). if $n \to \infty$, $y[n] \to \infty$ $\Rightarrow$ not BIBO stable.

c). $y[n-n_0] \overset{?}{=} T\{x[n-n_0]\}$

Left $= (n-n_0-1)\, x[n-n_0]\, u[n-n_0+1]$

right $= (n-1)\, x[n-n_0]\, u[n+1]$

Left $\neq$ right $\Rightarrow$ not T.I.

d). Yes. $Y[n]$ only depends on $x[n]$. no memory.

e). Yes. $Y[n]$ does not depend on future inputs.

4. (30 points) Consider the following difference equation:

$$y[n] - 1/4\, y[n-1] = x[n]$$

(a). (5 points). State the definition of impulse response $h[n]$.
(b). (5 points). Determine the impulse response $h[n]$ for the LTID system.
(c). (10 points). Is the system stable (give your proof)? Is the system causal (give your justification)?

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(d). (10 points). Determine the zero state response $y_{zs}[n]$ if $x[n] = (1/5)^n u[n]$.

a). $h[n]$ is the zero-state response of a LTIP system when input is $\delta[n]$.

b). $h[n] = (\frac{1}{4})^n u[n]$

c). $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} (\frac{1}{4})^n u[n] = \sum_{n=0}^{\infty} (\frac{1}{4})^n = \frac{1}{1-\frac{1}{4}} < \infty$

stable.

$h[n] = 0$ for $n < 0$, $\Rightarrow$ causal.

d). $y_{zs}[n] = (\frac{1}{4})^n u[n] * (\frac{1}{5})^n u[n] = \dfrac{(\frac{1}{4})^{n+1} - (\frac{1}{5})^{n+1}}{\frac{1}{4} - \frac{1}{5}} u[n]$.

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Name: UIN: Course No.

Name: UIN: Course No.

5. (20 points) Compute the convolutions of the following sequences.
(a). $(2)^n u[n] * (3)^n u[n]$ (5 points)
(b). Show graphically the convolution of the following two sequences: (show all steps, 15 points)

$x_1[n] = u[n] - u[n-3]$
$x_2[n] = u[n] - u[n-3]$

a). $\dfrac{2^{n+1} - 3^{n+1}}{2-3} u[n]$

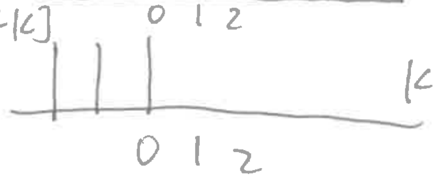
b). $x_1[n]$:

$x_2[n]$

$y[n] = x_1[n] * x_2[n]$. $n<0$, $y[n]=0$

$n=0$: $x_1[k]$

$x_2[-k]$



$y[0] = 1$

$n=1$: $x_1[k]$

$x_2[1-k]$:

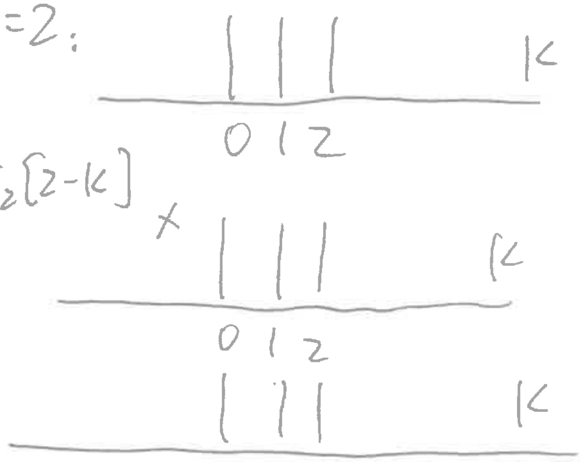
$y[1] = 2$

Name: UIN: Course No.

$x_1[k]$

$n=2$:

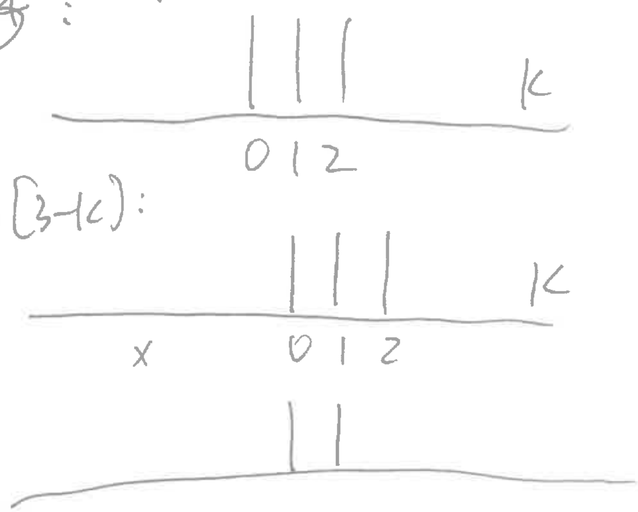
$x_2[2-k]$



$Y[2]=3$

$n=3$: $x_1[k]$

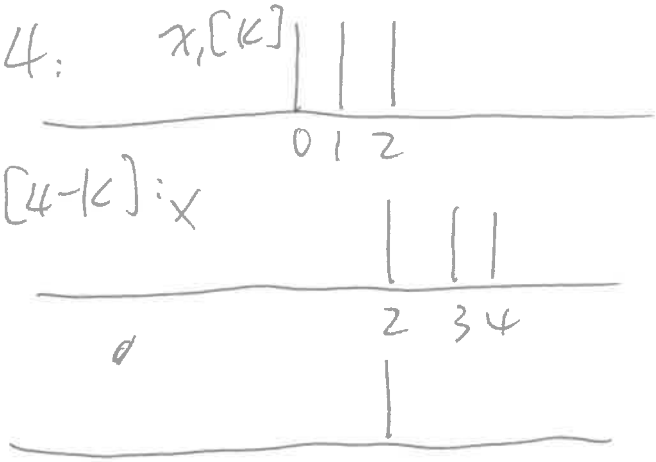
$x_2[3-k]$:



$Y[3]=2$.

$n=4$: $x_1[k]$

$x_2[4-k]$: x



$Y[4]=1$

$Y[n]=0, \; n \geq 5$.

$Y[n] = \{\underset{\uparrow}{1}, 2, 3, 2, 1\}$