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ECE 381 Introduction to Discrete-time Signal Processing

Test 2

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Close book, close notes. Calculators allowed. Two pages of cheating sheet allowed.

Write down your name, UIN and course No. on each page.

Instructor copy.

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1. (25 points) Compute z-transform for the following sequences; don't forget ROCs.

(a). (4 points) $(1/2)^n u[n]$
(b). (5 points) $2^n (u[n] - u[n\text{-}3])$
(c). (4 points) $2^n u[-n\text{-}1]$
(d). (4 points) $2^n u[-n+1]$
(e). (8 points) $(u[n] - u[n\text{-}2]) * (\sum_{k=0}^{\infty} \delta[n-4k])$ (* denotes convolution)

(a). $\frac{z}{z-\frac{1}{2}} \quad |z| > \frac{1}{2}$

(b). $1 + 2z^{-1} + 4z^{-2} \quad |z| > 0$

(c). $-n-1 \geq 0 \Rightarrow n \leq -1$, no overlap with the z transform definition.

$X(z) = 0, \quad |z| \geq 0$

(d). $-n+1 \geq 0 \Rightarrow n \leq 1$.

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} 2^n u[-n+1] z^{-n}$$

$$\overset{-n+1 \geq 0}{\underset{n \leq 1}{=}} \sum_{n=0}^{1} 2^n z^{-n} = 1 + 2z^{-1} \quad |z| > 0.$$

(e). $\mathcal{Z}\{u[n] - u[n-2]\} = 1 + z^{-1} \quad |z| > 0$

$$\mathcal{Z}\left\{\sum_{k=0}^{\infty} \delta[n-4k]\right\} = \mathcal{Z}\{\delta[n] + \delta[n-4] + \delta[n-8] + \delta[n-12] + \dots\}$$

$$= 1 + z^{-4} + z^{-8} + z^{-12} + \dots = \sum_{n=0}^{\infty} (z^{-4})^n = \frac{1}{1-z^{-4}} \quad |z| > 1$$

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$$\Rightarrow X(z) = \left(1 + z^{-1}\right) \cdot \frac{1}{1 - z^{-4}} \qquad |z| > 1$$

$$= \frac{z^4 + z^{-3}}{z^4 - 1} \qquad |z| > 1$$

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2. (35 points) compute inverse Z-transform:

(a) (8 pints) $X(z)=\dfrac{z}{z-0.5}, |z|>0.5$

(b) (9 pints) $X(z)=\dfrac{1}{(z-1)}, |z|>1$

(c) (9 pints) $X(z)=\dfrac{z(z+1)}{z^2-5z+4}, |z|>4$

(d) (9 pints) $X(z)=e^{\frac{a}{z}}, |z|>0$

(a). $(0.5)^n u[n]$

(b). $X(z)=\dfrac{z}{z-1}\cdot z^{-1}$

$x[n]=u[n-1]$

(c). $\dfrac{X(z)}{z}=\dfrac{z+1}{(z-1)(z-4)}=\dfrac{A}{z-1}+\dfrac{B}{z-4}$

$A=(z-1)\dfrac{X(z)}{z}\Big|_{z=1}=\dfrac{z+1}{z-4}\Big|_{z=1}=-\dfrac{2}{3}$

$B=\dfrac{X(z)}{z}(z-4)\Big|_{z=4}=\dfrac{z+1}{z-1}\Big|_{z=4}=\dfrac{5}{3}$

$x(z)=\dfrac{-2/3\ z}{z-1}+\dfrac{5/3\cdot z}{z-4}$

$x[n]=-\dfrac{2}{3}u[n]+\dfrac{5}{3}(4)^n u[n]$

(d). $e^{a/z}=1+\dfrac{a}{z}+\dfrac{1}{2!}\left(\dfrac{a}{z}\right)^2+\cdots+\dfrac{1}{n!}\left(\dfrac{a}{z}\right)^n$

$x[n]=\dfrac{a^n}{n!}u[n]$.

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3. (40 points) A causal LTID system is characterized by the following difference equation:

$$y[n] - 5/6\, y[n-1] + 1/6\, y[n-2] = x[n]$$

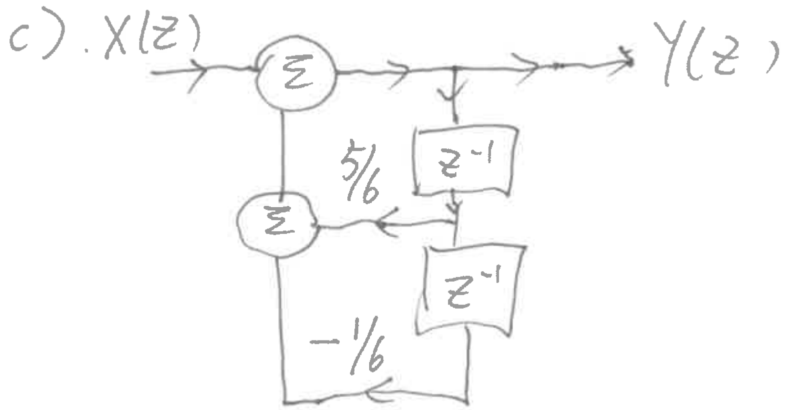
(a). (8 points) Determine the system function H(z) for the system.
(b). (8 points) Is the system stable? Justify your answer.
(c). (8 points) Realize the system with the DFII form.
(d). (8 points) Determine the impulse response $h[n]$ for the LTID system.
(e). (8 points) If $x[n] = (1/4)^n u[n]$, what is zero-state response, $y_{zs}[n]$?

a). $Y(z) - \frac{5}{6} z^{-1} Y(z) + \frac{1}{6} z^{-2} Y(z) = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}} \qquad |z| > \frac{1}{2}.$

b). Yes. Both poles are inside the unit circle.

c).



d). $H(z) = \frac{z^2}{z^2 - \frac{5}{6} z + \frac{1}{6}}$

$\frac{H(z)}{z} = \frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}}$

$A = \frac{H(z)}{z}(z-\frac{1}{2})\Big|_{z=\frac{1}{2}} = \frac{z}{z-\frac{1}{3}}\Big|_{z=\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{6}} = 3$

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$$B = \frac{H(z)}{z}\left(z-\frac{1}{3}\right)\Big|_{z=\frac{1}{3}} = \frac{z}{z-\frac{1}{2}}\Big|_{z=\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}-\frac{1}{2}} = -2$$

$$H(z) = \frac{3z}{z-\frac{1}{2}} + \frac{-2z}{z-\frac{1}{3}}$$

$$h[n] = 3\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$$

e. $y_{zs}[n] = h[n] * x[n]$

$$= 3\cdot\frac{\left(\frac{1}{2}\right)^{n+1}-\left(\frac{1}{4}\right)^{n+1}}{\frac{1}{2}-\frac{1}{4}}u[n] - 2\cdot\frac{\left(\frac{1}{3}\right)^{n+1}-\left(\frac{1}{4}\right)^{n+1}}{\frac{1}{3}-\frac{1}{4}}u[n]$$

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