Ex 1:

a). $(\gamma-\frac{1}{2})(\gamma-\frac{1}{3})=0 \Rightarrow \gamma_1=\frac{1}{2}, \gamma_2=\frac{1}{3}$

$h[n]=C_1(\frac{1}{2})^n+C_2(\frac{1}{3})^n \quad n\geq 0.$

$y[n]=\frac{5}{6}y[n-1]-\frac{1}{6}y[n-2]+x[n]$

$h[n]=\frac{5}{6}h[n-1]-\frac{1}{6}h[n-2]+\delta[n]$

$h[-2]=h[-1]=0$

$h[0]=\frac{5}{6}h[-1]-\frac{1}{6}h[-2]+\delta[0]=1$

$h[1]=\frac{5}{6}h[0]-\frac{1}{6}\cancel{h[-1]}^{0}+\cancel{\delta[1]}^{0}=\frac{5}{6}$

$$\left.\begin{array}{l} n=0 \Rightarrow h[0]=C_1(\frac{1}{2})^0+C_2(\frac{1}{3})^0=1 \Rightarrow C_1+C_2=1 \\ n=1 \Rightarrow h[1]=C_1(\frac{1}{2})^1+C_2(\frac{1}{3})^1=\frac{5}{6} \Rightarrow \frac{1}{2}C_1+\frac{1}{3}C_2=\frac{5}{6} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} C_1=3 \\ C_2=-2 \end{array}\right.$$

$\Rightarrow h[n]=3(\frac{1}{2})^n u[n]-2(\frac{1}{3})^n u[n]$

b). $\sum_{n=-\infty}^{\infty}|h[n]|=\sum_{n=0}^{\infty}\left|3(\frac{1}{2})^n-2(\frac{1}{3})^n\right|<\sum_{n=0}^{\infty}3(\frac{1}{2})^n+\sum_{n=0}^{\infty}2(\frac{1}{3})^n$

$=3\cdot\frac{1}{1-\frac{1}{2}}+2\cdot\frac{1}{1-\frac{1}{3}}=6+3=9<\infty \Rightarrow$ stable.

$h[n]=0$ for $n<0$, $\Rightarrow$ causal.

c). $y[n]=x[n]*h[n]=(\frac{1}{5})^n u[n]*\left(3(\frac{1}{2})^n u[n]-2(\frac{1}{3})^n u[n]\right)$

$$=3\cdot\frac{(\frac{1}{5})^{n+1}-(\frac{1}{2})^{n+1}}{\frac{1}{5}-\frac{1}{2}}u[n]-2\frac{(\frac{1}{5})^{n+1}-(\frac{1}{3})^{n+1}}{\frac{1}{5}-\frac{1}{3}}u[n]$$

Ex 2:

a). $u[n] * u[n] = \sum_{k=-\infty}^{\infty} u[k]\,u[n-k] = \sum_{k=0}^{n} 1 = (n+1)u[n]$

$k \geq 0 \quad n-k \geq 0$

$k \leq n$

b). $x[n] * \delta[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \overset{k=n}{=} x[n]$

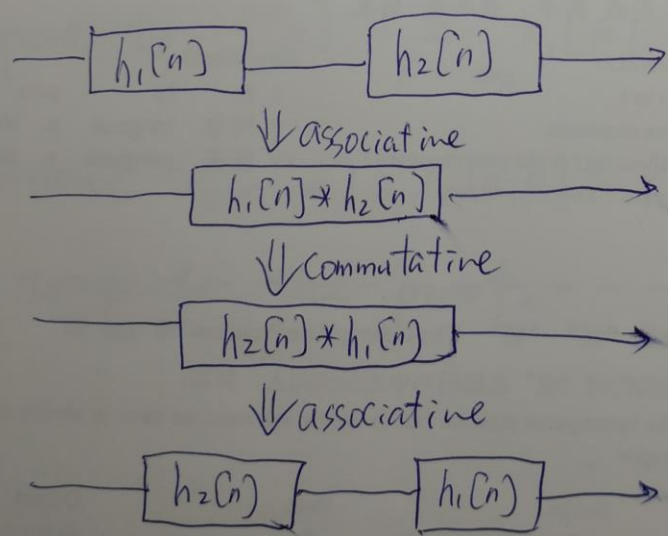
$k \geq 0 \quad n=k$

c). $(u[n]-u[n-10]) * (u[n]-u[n-3])$

$= \{\underset{\uparrow}{1}\ 2\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 2\ 1\}$

d). $(\frac{1}{2})^n u[n] * (\frac{1}{3})^n u[n] = \dfrac{(\frac{1}{2})^{n+1} - (\frac{1}{3})^{n+1}}{\frac{1}{2}-\frac{1}{3}} u[n]$

Ex 3:



~~Ex4~~:

EX 4. Given $C[n] = x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k]$

$x_1[n-m] * x_2[n-p]$

$$= \sum_{k=-\infty}^{\infty} x_1[k-m] \, x_2[n-k-p]$$

Let $k-m = u$, then we have $k = u+m$

and we sum over u:

$$= \sum_{u=-\infty}^{\infty} x_1[u] \cdot x_2[n-(u+m)-p]$$

$$= \sum_{u=-\infty}^{\infty} x_1[u] \, x_2[(n-m-p)-u]$$

Compare to $C[n] = \sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k]$

We know $x_1[n-m] * x_2[n-p] = C[n-m-p]$.

We complete the proof.