Test 2:

1. ⓐ. $x[n] = (a^{n-1}u[n-1]) * (b^{n}u[n+1])$

$= (a^{n-1}u[n-1]) * \frac{1}{b}(b^{n+1}u[n+1])$

$= \frac{1}{b} \cdot (a^{n-1}u[n-1]) * (b^{n+1}u[n+1]) = \frac{1}{b}(a^{n}u[n]) * (b^{n}u[n])$

$Z\{x[n]\} = \frac{1}{b} \cdot \frac{z}{z-a} \cdot \frac{z}{z-b}, \quad |z| > |a|$

ⓑ. $x[n] = (a^{n}u[n]) * (\sum_{k=0}^{n} \delta[k])$

$= (a^{n}u[n]) * u[n]$

$Z\{x[n]\} = \frac{z}{z-a} \cdot \frac{z}{z-1}, \quad |z| > \max\{|a|, 1\}.$

ⓒ. $x[n] = \sum_{k=0}^{\infty} \delta[n-3k].$

$Z\{x[n]\} = \sum_{n=0}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} (\sum_{k=0}^{\infty} \delta[n-3k]) z^{-n}$

$= \sum_{n=0}^{\infty} (\delta[n] + \delta[n-3] + \delta[n-6] + \cdots \delta[n-3k]) \cdot z^{-n}$

$= z^{0} + z^{-3} + z^{-6} + \cdots z^{-3k} + \cdots$

$= \sum_{k=0}^{\infty} z^{-3k} = \sum_{k=0}^{\infty} (z^{-3})^{k} = \frac{1-(z^{-3})^{\infty+1}}{1-z^{-3}} = \frac{1}{1-z^{-3}} \quad |z| > 1$

2. (a). $x(z) = \frac{1}{z-\frac{1}{2}} \quad |z| > \frac{1}{2}$

$$X(z) = \frac{z}{z-\frac{1}{2}} \cdot z^{-1} \Rightarrow x[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1]$$

(b). $X(z) = \frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{3})} \quad |z| > \frac{1}{2}.$

$$\frac{X(z)}{z} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}} = \frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$A = \frac{X(z)}{z}(z-\tfrac{1}{2})\Big|_{z=\frac{1}{2}} = \frac{z}{z-\frac{1}{3}}\Big|_{z=\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}-\frac{1}{3}} = 3$$

$$B = \frac{X(z)}{z}(z-\tfrac{1}{3})\Big|_{z=\frac{1}{3}} = \frac{z}{z-\frac{1}{2}}\Big|_{z=\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}-\frac{1}{2}} = -2$$

$$X(z) = \frac{3z}{z-\frac{1}{2}} + \frac{-2z}{z-\frac{1}{3}}$$

$$x[n] = 3\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$$

(c). $X(z) = 1 + z^{-2} + z^{-4}, \quad |z| > 0$

$$x[n] = \delta[n] + \delta[n-2] + \delta[n-4].$$

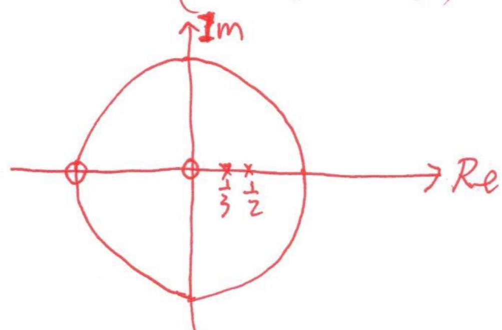
3). (a). $Y[n+2] - 5/6\, Y[n+1] + 1/6\, Y[n] = X[n+2] + X[n+1]$

$$z^2 Y(z) - \tfrac{5}{6} z Y(z) + \tfrac{1}{6} Y(z) = z^2 X(z) + z X(z)$$

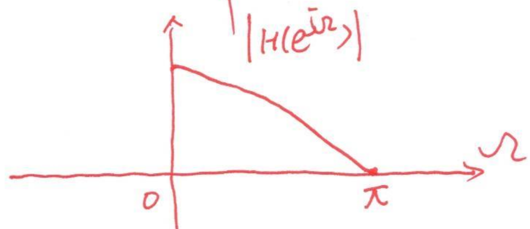
$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2 + z}{z^2 - \frac{5}{6} z + \frac{1}{6}} = \frac{1 + z^{-1}}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}} \quad |z| > \frac{1}{2}.$$

(b). The system is stable because all poles are inside the unit circle.

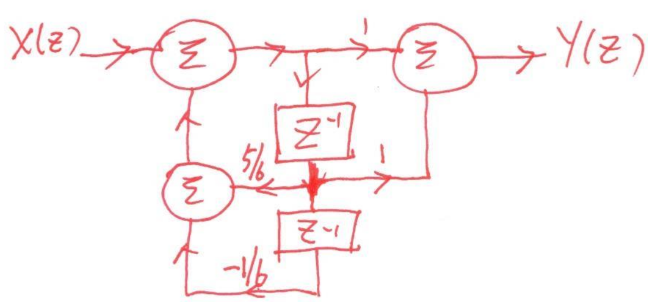
(c). $$H(z) = \frac{z(1+z)}{(z-\frac{1}{2})(z-\frac{1}{3})}, \quad |z| > \frac{1}{2}$$



(d).



(e).



(8). $H(z) = \dfrac{z^2+z}{(z-\frac{1}{2})(z-\frac{1}{3})}$ $|z| > \frac{1}{2}$

$$\frac{H(z)}{z} = \frac{z+1}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}}$$

$$A = \frac{H(z)}{z}(z-\tfrac{1}{2})\Big|_{z=\frac{1}{2}} = \frac{z+1}{z-\frac{1}{3}}\Big|_{z=\frac{1}{2}} = \frac{\frac{1}{2}+1}{\frac{1}{2}-\frac{1}{3}} = 9$$

$$B = \frac{H(z)}{z}(z-\tfrac{1}{3})\Big|_{z=\frac{1}{3}} = \frac{z+1}{z-\frac{1}{2}}\Big|_{z=\frac{1}{3}} = \frac{\frac{1}{3}+1}{\frac{1}{3}-\frac{1}{2}} = -8$$

$$H(z) = \frac{9z}{z-\frac{1}{2}} + \frac{-8z}{z-\frac{1}{3}} \quad |z| > \tfrac{1}{2}$$

$$h[n] = 9\left(\tfrac{1}{2}\right)^n u[n] - 8\left(\tfrac{1}{3}\right)^n u[n]$$

for $x[n] = (\frac{1}{4})^n u[n] \Rightarrow y_{zs}[n] = h[n] * u[n] = 9 \cdot \dfrac{(\frac{1}{2})^{n+1} - (\frac{1}{4})^{n+1}}{\frac{1}{2}-\frac{1}{4}} u[n] - 8\, \dfrac{(\frac{1}{3})^{n+1} - (\frac{1}{4})^{n+1}}{\frac{1}{3}-\frac{1}{4}} u[n]$

(9). $H(e^{j\Omega}) = H(z)\Big|_{z=e^{j\Omega}} = \dfrac{e^{2j\Omega} + e^{j\Omega}}{(e^{j\Omega}-\frac{1}{2})(e^{j\Omega}-\frac{1}{3})}$

$$H(e^{j\pi}) = H(e^{j\Omega})\Big|_{\Omega=\pi} = \frac{e^{j2\pi}+e^{j\pi}}{(e^{j\pi}-\frac{1}{2})(e^{j\pi}-\frac{1}{3})} = 0$$

So for $x[n] = \cos(\pi n + \pi/3)$

$$\Rightarrow y[n] = |H(e^{j\Omega})| \cos(\pi n + \tfrac{\pi}{3} + \angle H(e^{j\Omega}))$$

$$= |H(e^{j\pi})| \cos(\pi n + \tfrac{\pi}{3} + \angle H(e^{j\pi})) = 0$$ for all n.