1. Let $X = \{1,2,3,4\}$ and $Y = \{0,2,4,6\}$
   a. $X \cup Y = \{0,1,2,3,4,6\}$
   b. $X \cap Y = \{2,4\}$
   c. $X - Y = \{1,4\}$
   d. $Y - X = \{0,6\}$
   e. $P(X) = \{\{\},\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},$
   $\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\}$

4. Let $X = \{n^3+3n^2+3n \mid n \geq 0\}$ and $Y = \{n^3-1 \mid n > 0\}$. Show $X=Y$

Show that every element of Y is also an element of X. Let $y \in Y$
$y = n^3 - 1$ for some number $n > 0$. Let n_0 be that number
$y = (n_0)^3 - 1$ $\quad x \in X$
$x = (n_0)^3 + 3(n_0)^2 + 3(n_0)$ $\quad n^3+3n^2+3n$
$= n_0((n_0)^2 + 3(n_0) + 3)$ $\quad n(n^2+3n+3)$
irreducible factorization

$n+1$
$n+1$
$n+1$
n^2+n
n^2+2n+1
$n+1$
n^2+2n+1
n^3+2n^2+n

n^3+3n^2+3n+1

$y = (n_0+1)^3 - 1 = n^3+3n^2+3n$
$Y \subseteq X$

$x = n_0((n_0)^2 + 3(n_0) + 3)$
$x = 1\ ((1) + 3(1) + 3) \rightarrow 7 \quad X \subseteq Y$

6. Give functions $f: \mathbb{N} \to \mathbb{N}$ that satisfy the following
   a) f is total and 1-1 but not onto
   $f(n) = n+2$; the range is a distinct element in the set
   $f(n) = f(m)$; $n+2 = m+2 \rightarrow n = m$ ✓ 1-1
   $f(1) = 1+2 = 3$; the range of f does not include every element in the set
   b) f is total and onto but not one-to-one
   $f(n) = \begin{cases} n+2 & \text{if } n=0 \\ n-1 & \text{otherwise} \end{cases}$; function range is the entire set $\mathbb{N}$, $f(0)$ is mapped to 2, so its not 1-1
   $f(0) = 0+2 = 2$
   $f(1) = 1-1 = 0$
   $f(2) = 2-1 = 1$
   c) f is total, 1-1, and onto but not the identity
   $f(n) = 2n$; the range is defined for all in the set, and is not an identity $f(n)=n$
   $f(0) = 2(0) = 0$
   $f(1) = 2(1) = 2$
   d) f is not total but is onto
   $f(n) = n \bmod 2$; this function maps every element in the set with either 0 or 1, which are contained in the set $\mathbb{N}$, thus preserving onto; yet, the output can never be anything >1, thus is not total

10. Let $\equiv$ be the binary relation on $\mathbb{N}$ defined by $n \equiv m$ iff $n = m$. Prove that $\equiv$ is an equivalence relation. Describe the equivalence classes of $\equiv$.
   $\mathbb{N} \times \mathbb{N}$ $\quad LT = \{[i,j] \mid i < j\}$ $\quad$ Let $EQ = \{[n,m] \mid n = m\}$
   i) For every $\mathbb{N}$ n, $n \equiv n$
   ii) if $n \equiv m$, then $m \equiv n$
   iii) if $n \equiv m$, $m \equiv k$, then $m \equiv k$

   $[EQ]_{\equiv} = \{\mathbb{N}\}$ describes the infinite elements in this equivalence relation.

22. A total function f from $\mathbb{N}$ to $\mathbb{N}$ is monotone increasing if $f(n) < f(n+1)$ for all $n \in \mathbb{N}$. Prove that there are an uncountable number of monotone increasing functions $f_0 < f_1 < f_2 < f_3 < f_4 \ldots$

Assume that the set of monotone increasing functions is countable.

$f_0, f_1, f_2, f_3, \ldots f_n$

$f(0) = f_0(0) + 1$ for $i > 0$. $f(i) = f_i(i) + f(i) > f(i-1)$

$f_1(1) = f_1(1) + f(1) > f(1-1) \Rightarrow$ monotone increasing

this contradiction proves that the statement $f_0 \ldots f_n$ is not an enumerable set.

29. Give a recursive def. of the relation is equal to on $\mathbb{N} \times \mathbb{N}$ using the operator s.

$EQ = \{[i,j] \mid i = j\}$

i) Basis: $[0,0] \in EQ$

ii) Recursive step: If $[i,j]$ is $\in EQ$, then $[s(i), s(j)]$ is $\in EQ$

iii) closure: $[i,j]$ is $\in EQ$ if it can be obtained from $[0,0]$ by a finite # of the operations in the recursive step.

34. Give a recursive definition of the predecessor operation

$$pred(n) = \begin{cases} 0 & \text{if } n = 0 \\ n-1 & \text{otherwise} \end{cases}$$

i) Basis: $0 \in pred$

ii) Recursive step: if $n \in pred$, then $s(n-2) \in pred$

$s(n) = n+1$
$s(0) = 0+1 = 1$
$s(1) = 1+1 = 2$
$s(2) = 2+1 = 3$

$pred(0) = 0$
$pred(1) = (s(1) - 2) = 0$ ✓
$pred(2) = (s(2) - 2) = 1$

38. Prove $2 + 5 + 8 + \ldots + (3n-1) = n\frac{(3n+1)}{2}$ for all $n > 0$.

Basis step; $n = 1$

$3(1) - 1 = 2$

$2 \stackrel{?}{=} \frac{1}{2}(3(1)+1)$

$\stackrel{?}{=} 2$ ✓

Induction step; Assume true for $n = k$, show true for $n = k+1$

Assume: $2 + 5 + 8 + \ldots + (3k-1) = \frac{k}{2}(3k+1)$

show :

$\underbrace{2 + 5 + 8 + \ldots + (3k-1)} + (3(k+1)-1) = \frac{k+1}{2}(3(k+1)+1)$

$\frac{k}{2}(3k+1) + (3(k+1)-1) \stackrel{?}{=} \frac{k+1}{2}(3(k+1)+1)$

$\frac{k}{2}(3k+1) + 3k + 2 = \frac{k+1}{2}(3k+4)$

$\frac{3k^2}{2} + \frac{k}{2} + 3k + 2 = \frac{k+1}{2}(3k+4)$

$* \frac{2}{2}$

$\frac{3k^2}{2} + \frac{k}{2} + \frac{6k}{2} + 2 = \frac{3k(k+1)}{2} + \frac{4(k+1)}{2}$

$\frac{3k^2}{2} + \frac{7k}{2} + 2 \stackrel{?}{=} \frac{3k^2}{2} + \frac{3k}{2} + \frac{4k}{2} + \left[\frac{4}{2}\right] = 2$

$\frac{3k^2}{2} + \frac{7k}{2} + 2 \stackrel{\checkmark}{=} \frac{3k^2}{2} + \frac{7k}{2} + 2$

42. Let $P = \{A, B\}$ be a set of two proposition letters (Boolean vars.). The set E of well formed conjunctive and disjunctive Boolean expressions over P is defined recursively as follows:

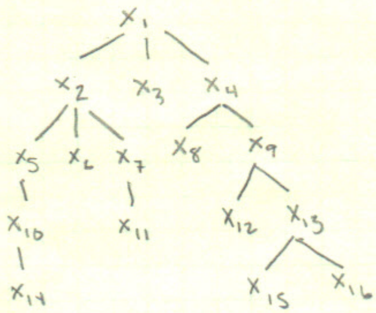
i) Basis $A, B \in E$

ii) Recursive step: if $u, v \in E$, then $(u \vee v) \in E$ and $(U \wedge v) \in E$

iii) Closure: An expression is in E only if it is obtained from the basis by a finite number of iterations of the recursive step.

a) $E_0 =$

46.



a) the depth of the tree = 4

b) the ancestors of $x_{11} = \{x_{11}, x_7, x_2, x_1\}$

c) MCA of $x_{14} \wedge x_{11} = x_2$

MCA of $x_{15} \wedge x_{11} = x_1$

d) subtree generated by x_2

$[x_2, x_5], [x_2, x_6], [x_2, x_7], [x_5, x_{10}], [x_7, x_{11}], [x_{10}, x_{14}]$

e) frontier $\{x_{14}, x_6, x_{11}, x_3, x_8, x_{12}, x_{15}, x_{16}\}$