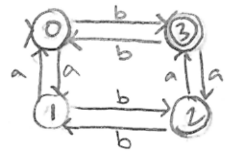
1. The language of the DFA M in ex. 6.3.4 consists of all strings over $\{a,b\}$ with an even number of a's and an odd number of b's. Use Algorithm 6.2.2 to construct a regular expression for $L(M)$.

G_1

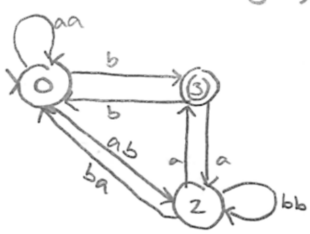


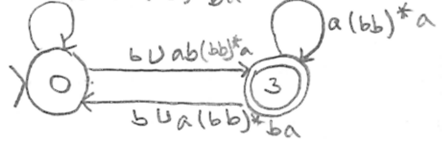
$S \to aA \mid bB$
$A \to aS \mid bC$
$B \to bS \mid aC \mid \lambda$
$C \to aB \mid bA$

ω eliminate q_1

j	k	i		
q_0	q_0	q_1	$w^a_{0,1}\ w^\emptyset_{1,1}\ w^a_{1,0} = aa$	$w_{0,0}$
q_0	q_2	q_1	$w^a_{0,1}\ w^\emptyset_{1,1}\ w^b_{1,2} = ab$	$w_{0,2}$
q_0	q_3	q_1	$w^a_{0,1}\ w^\lambda_{1,1}\ w^a_{1,3} = \emptyset$	$w_{0,3}$
q_2	q_2	q_1	$w^b_{2,1}\ w^\lambda_{1,1}\ w^b_{1,2} = bb$	$w_{2,2}$
q_2	q_0	q_1	$w^b_{2,1}\ w^\lambda_{1,1}\ w^a_{1,0} = ba$	$w_{2,0}$
q_2	q_3	q_1	$w^b_{2,1}\ w^\lambda_{1,1}\ w^\lambda_{1,3} = \emptyset$	$w_{2,3}$
q_3	q_3	q_1	$w^\lambda_{3,1}\ w_{1,1}\ w_{1,3} = \emptyset$	$w_{3,3}$
q_3	q_0	q_1	$w^\lambda_{3,1}\ w_{1,1}\ w_{1,0} = \emptyset$	$w_{3,0}$
q_3	q_2	q_1	$w^\lambda_{3,1}\ w_{1,1}\ w_{1,2} = \emptyset$	$w_{3,2}$

G_2





ω eliminate q_2

j	k	i		
q_0	q_0	q_2	$w^{ab}_{0,2}\ w^{(bb)^*}_{2,2}\ w^{ba}_{2,0}$	$w_{0,0} = ab(bb)^*ba$
q_0	q_3	q_2	$w^{ab}_{0,2}\ w^{(bb)^*}_{2,2}\ w^{a}_{2,3}$	$w_{0,3} = ab(bb)^*a$
q_3	q_3	q_2	$w^{a}_{3,2}\ w^{(bb)^*}_{2,2}\ w^{a}_{2,3}$	$w_{3,3} = a(bb)^*a$
q_3	q_0	q_2	$w^{a}_{3,2}\ w^{(bb)^*}_{2,2}\ w^{ba}_{2,0}$	$w_{3,0} = a(bb)^*ba$

$$\left[((aa)\cup(ab(bb)^*ba))^*\, b \cup ab(bb)^*a\,(a(bb)^*a)^*\right](b \cup (a(bb)^*ba)\ \text{part1})^*$$

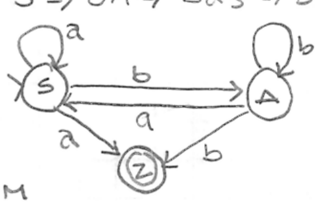
Let part1 = the regex delimited by square brackets.

2. Let G be the grammar $S \to aS \mid bA \mid a$
$A \to aS \mid bA \mid b$

(a) Use Theorem 6.3.1 to build an NFA M that accepts $L(G)$.
(b) Using the result of part (a), build a DFA M' that accepts $L(G)$.
(c) Construct a regular grammar from M that generates $L(M)$.
(d) Construct a regular grammar from M' that generates $L(M')$
(e) Give a regular expression for $L(G)$.

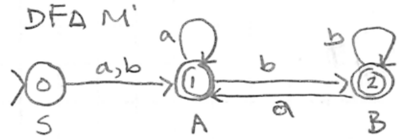
$S \Rightarrow aS \Rightarrow aaS \Rightarrow aabA \Rightarrow aabaS \Rightarrow aababA \Rightarrow aababb$
$S \Rightarrow bA \Rightarrow baS \Rightarrow babA \Rightarrow baba$

a)



NFA M

b) DFA M'



c) G': $S \to aS \mid bA \mid aZ$
$A \to aS \mid bA \mid bZ$
$Z \to \lambda$

d) G'': $S \to aA \mid bA$
$A \to aA \mid bB \mid \lambda$
$B \to bB \mid aA \mid \lambda$

e) $a \cup b(a^*(b^+)^*(a^+(b^+)^*))^*$

3. Use the pumping lemma to show that the set of strings over $\{a,b\}$ in which the number of a's is a perfect cube is not regular.

$L = \{a^i \mid i \text{ is a perfect cube}\}$ Assume L is regular. According to the pumping lemma, there exists a number i such that for every string w where $|w| \geq i$, $w = xyz$

$* \ |y| > \emptyset$
$* \ |xy| \leq n$ } for all $k \geq \emptyset$, $w = xy^k z \in L$

I choose $w = a^c$ where $c = n^3$ ← perfect cube

Let $w = ..aaa...aaa = xyz$ $\Rightarrow$ $|xyz| = |xz| + |y| = (n^3 - k) + k$
$w = \underset{x}{aaa}\ \underset{y}{a^k}\ \underset{z}{aaa..}$ $\quad |xy^2z| = |xz| + 2|y| = (n^3 - k) + 2k = n^3 + k$

$n^3 + k \leq n^3 + n < n^3 + 3n^2 + 3n + 1 = (n+1)^3$

$n^3 + k > n^3$

$\Rightarrow n^3 < n^3 + k < (n+1)^3$

$n^3 + k$ is not a perfect cube, so xy^2z is not in L

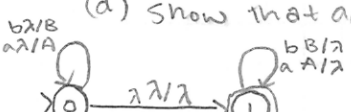
$\{a^i \mid i \text{ is a perfect cube}\}$ is not regular

4. A context free grammar $G = (V, \Sigma, P, S)'$ is called left linear if each rule is of the form: $A \to u$, $A \to Bu$ where $A, B \in V$ and $u \in \Sigma^*$. Show that the left linear grammars generate precisely the regular sets.

we can produce rules $\quad A \to u$
$u = a_1 a_2 a_3 \ldots a_n$ and $A_1 \to A_1 u_1$
$A_2 \to A_1 u_2 u_1 \ldots$
$A_{n-1} \to u_n$

$A \to Bu$
$A_{n-1} \to Bu_n$
$B \to$ repeat until terminal, next non term, or lambda

5. Let M be the PDA in Example 7.1.3.
(a) Give the transition table of M.
(b) Trace all computations of the strings ab, abb, abbb in M.
(c) Show that aaaa, baab $\in L(M)$
(d) Show that aaa, ab $\notin L(M)$.



$\{ww^R \mid w \in \{a,b\}^*\}$

a)
$\delta(q_0, a, \lambda) = \{[q_0, A]\}$
$\delta(q_0, b, \lambda) = \{[q_0, B]\}$
$\delta(q_0, \lambda, \lambda) = \{[q_1, \lambda]\}$
$\delta(q_1, a, A) = \{[q_1, \lambda]\}$
$\delta(q_1, b, B) = \{[q_1, \lambda]\}$

b)
$[q_0, ab, \lambda]$
$\vdash [q_1, ab, \lambda]$ ×

$[q_0, ab, \lambda]$
$\vdash [q_0, b, A]$
$\vdash [q_0, \lambda, BA]$
$\vdash [q_1, \lambda, BA]$ ×

$[q_0, ab, \lambda]$
$\vdash [q_0, b, A]$
$\vdash [q_1, b, A]$ ×

$[q_0, abb, \lambda]$
$\vdash [q_1, abb, \lambda]$ ×

$[q_0, abb, \lambda]$
$\vdash [q_0, bb, A]$
$\vdash [q_0, b, BA]$
$\vdash [q_1, b, BA]$ ×

$[q_0, abb, \lambda]$
$\vdash [q_0, bb, A]$
$\vdash [q_0, b, BA]$
$\vdash [q_0, \lambda, BBA]$
$\vdash [q_1, \lambda, BBA]$ ×

$[q_0, abbb, \lambda]$
$\vdash [q_1, abbb, \lambda]$ ×

$[q_0, abbb, \lambda]$
$\vdash [q_0, bbb, A]$
$\vdash [q_1, bbb, A]$ ×

$[q_0, abbb, \lambda]$
$\vdash [q_0, bbb, A]$
$\vdash [q_0, bb, BA]$
$\vdash [q_1, bb, BA]$ ×

$[q_0, abbb, \lambda]$
$\vdash [q_0, bbb, A]$
$\vdash [q_0, bb, BA]$
$\vdash [q_0, b, BBA]$
$\vdash [q_1, b, BBA]$ ×

$[q_0, abbb, \lambda]$
$\vdash [q_0, bbb, A]$
$\vdash [q_0, bb, BA]$
$\vdash [q_0, b, BBA]$
$\vdash [q_0, \lambda, BBBA]$
$\vdash [q_1, \lambda, BBBA]$ ×

5c. $[q_0, aaaa, \lambda] \in L(M)$
$\vdash [q_0, aaa, A]$
$\vdash [q_0, aa, AA]$
$\vdash [q_1, aa, AA]$
$\vdash [q_1, a, A]$
$\vdash [q_1, \lambda, \lambda]$ ✓

$[q_0, baab, \lambda] \in L(M)$
$\vdash [q_0, aab, B]$
$\vdash [q_0, ab, AB]$
$\vdash [q_1, ab, AB]$
$\vdash [q_1, b, B]$
$\vdash [q_1, \lambda, \lambda]$ ✓

d) $[q_0, aaa, \lambda] \notin L(M)$
$\vdash [q_0, aa, A]$
$\vdash [q_1, aa, A]$
$\vdash [q_1, a, \lambda]$ ×

$[q_0, aaa, \lambda]$
$\vdash [q_0, aa, A]$
$\vdash [q_0, a, AA]$
$\vdash [q_1, a, AA]$
$\vdash [q_1, \lambda, A]$ ×

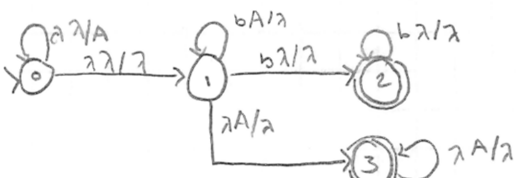
$[q_0, aaa, \lambda]$
$\vdash [q_0, aa, A]$
$\vdash [q_0, a, AA]$
$\vdash [q_0, \lambda, AAA]$
$\vdash [q_1, \lambda, AAA]$ ×

$[q_0, ab, \lambda]$
$\vdash [q_1, ab, \lambda]$ ×

$[q_0, ab, \lambda]$
$\vdash [q_0, b, A]$
$\vdash [q_1, b, A]$ ×

$[q_0, ab, \lambda]$
$\vdash [q_0, b, A]$
$\vdash [q_0, \lambda, BA]$
$\vdash [q_1, \lambda, BA]$

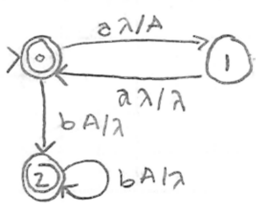
6. Construct a PDA that accepts the language $\{a^i b^j \mid i \neq j\}$

M:


Q: $\{q_0 \ldots 3\}$
Γ: $\{A\}$
Σ: $\{a, b\}$
F: $\{q_2, 3\}$

$\delta(q_0, a, \lambda) = \{[q_0, A]\}$
$\delta(q_0, \lambda, \lambda) = \{[q_1, \lambda]\}$
$\delta(q_1, b, A) = \{[q_1, \lambda]\}$
$\delta(q_1, \lambda, A) = \{[q_3, \lambda]\}$
$\delta(q_1, b, \lambda) = \{[q_2, \lambda]\}$
$\delta(q_2, b, \lambda) = \{[q_2, \lambda]\}$
$\delta(q_3, \lambda, A) = \{[q_3, \lambda]\}$

7. Let $L = \{a^{2i} b^i \mid i \geq 0\}$
(a) construct a PDA M_1 with $L(M_1) = L$
(b) construct an atomic PDA M_2 with $L(M_2) = L$
(c) construct an extended PDA M_3 with $L(M_3) = L$ that has fewer transitions than M_1
(d) Trace the computation that accepts the string aab in each of the PDA's that you constructed.

a)


Q: $\{q_0 \ldots 2\}$
Γ: $\{A\}$
Σ: $\{a, b\}$
F: $\{q_0, 2\}$

$\delta(q_0, a, \lambda) = \{[q_1, A]\}$ ①
$\delta(q_0, b, A) = \{[q_2, \lambda]\}$ ②
$\delta(q_2, b, A) = \{[q_2, \lambda]\}$ ③
$\delta(q_1, a, \lambda) = \{[q_0, \lambda]\}$ ④ ✓ i

b) Transitions in an atomic PDA have the form
i) $\delta(q_i, a, \lambda) = \{[q_j, \lambda]\}$
ii) $\delta(q_i, \lambda, A) = \{[q_j, \lambda]\}$
iii) $\delta(q_i, \lambda, \lambda) = \{[q_j, A]\}$

$\boxed{\delta(q_i, a, A) = \{[q_j, B]\}}$

$\delta(q_i, a, \lambda) = \{[p_1, \lambda]\}$
$\delta(p_1, \lambda, A) = \{[p_2, \lambda]\}$
$\delta(p_2, \lambda, \lambda) = \{[q_j, B]\}$

① $\boxed{\delta(q_0, a, \lambda) = \{[q_1, A]\}}$

0 $\delta(q_0, a, \lambda) = \{[p_1, \lambda]\}$
1 $\delta(p_1, \lambda, \lambda) = \{[q_1, A]\}$

② $\boxed{\delta(q_0, b, A) = \{[q_2, \lambda]\}}$

2 $\delta(q_0, b, \lambda) = \{[p_2, \lambda]\}$
3 $\delta(p_2, \lambda, A) = \{[q_2, \lambda]\}$

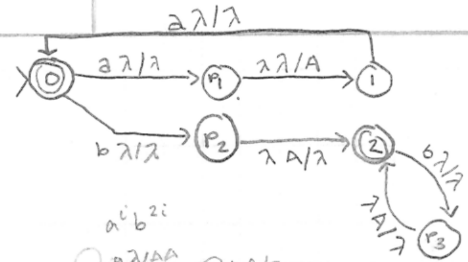
③ $\boxed{\delta(q_2, b, A) = \{[q_2, \lambda]\}}$

4 $\delta(q_2, b, \lambda) = \{[p_3, \lambda]\}$
5 $\delta(p_3, \lambda, A) = \{[q_2, \lambda]\}$

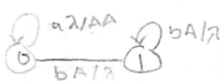
④

6 $\delta(q_1, a, \lambda) = \{[q_0, \lambda]\}$

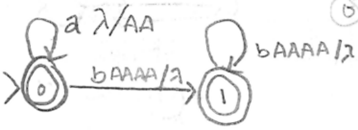
b) Q: {$q_{0..6}$}
Π: {A}
Σ: {a, b}
F: {q_0, q_2}



$a^i b^{2i}$



c)



AAAAAAAA
aaaabb
AAAA λ

Q: {$q_{0,1}$}
Π: {A}
Σ: {a, b}
F: {$q_{0,1}$}

$\delta(q_0, a, \lambda) = \{[q_0, A^2]\}$
$\delta(q_0, b, A^4) = \{[q_1, \lambda]\}$
$\delta(q_1, b, A^4) = \{[q_1, \lambda]\}$

d) PDA M_1 $[q_0, aab, \lambda]$
$\vdash [q_1, ab, A]$
$\vdash [q_0, b, A]$
$\vdash [q_2, \lambda, \lambda]$ ✓

PDA M_2 $[q_0, aab, \lambda]$
$\vdash [p_1, ab, \lambda]$
$\vdash [q_1, ab, A]$
$\vdash [q_0, b, A]$
$\vdash [p_2, \lambda, A]$
$\vdash [q_2, \lambda, \lambda]$ ✓

PDA M_3 $[q_0, aab, \lambda]$
$\vdash [q_0, ab, A^2]$
$\vdash [q_0, b, A^4]$
$\vdash [q_1, \lambda, \lambda]$ ✓

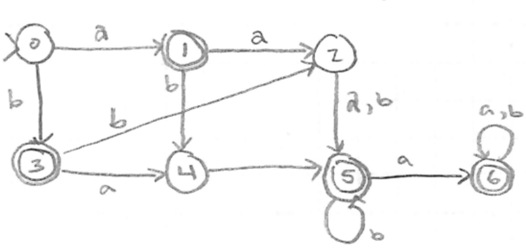
8. Use the pumping lemma to prove that $\{ww^Rw \mid w \in \{a,b\}^*\}$ is not context free

Assume that $\{ww^Rw \mid w \in \{a,b\}^*\}$ is context free.
i) $|vwx| \le k$
ii) v and x not null
iii) $uv^iwx^iy \in L$ for all $i \ge 0$

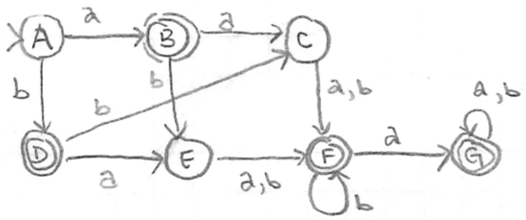
$z = (a^kb^k)(a^kb^k)^R(a^kb^k)$
$= (a^kb^k)(b^ka^k)(a^kb^k)$
$= a^kb^{2k}a^{2k}b^k$ → $u\underline{v}w\underline{x}y$ → ii) v/x must be at least 1 terminal.
↳ either a or b
i) $|vwx| \le k$, so z cannot contain a's from both sides of b^{2k}

If the a's in the in the substring vwx of z are before b^{2k}, then uv^2wx^2y adds to the a's before b^{2k}, while keeping the # of a's after b^{2k} the same as 2k. Therefore uv^2wx^2 is no longer in L and L is not context free.

9. For the DFA on pg. 190, part (c)
(a) Trace the actions of Algorithm 5.7.2 to determine the equivalent states of M. Give the values of D[i,j] and S[i,j] computed by the algorithm.
(b) Give the equivalence classes of states
(c) Give the state diagram of the minimal state DFA that accepts L(M).



DFA renamed



Distinguishable: given pair of states, 1 is final, 1 is not

$$\left[\begin{array}{cccc} D,A\checkmark & B,A\checkmark & F,A\checkmark & G,A\checkmark \\ D,E\checkmark & B,E\checkmark & F,E\checkmark & G,E\checkmark \\ D,C\checkmark & B,C\checkmark & F,C\checkmark & G,C\checkmark \end{array}\right] \times$$

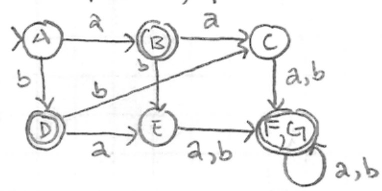
Inspecting each input symbol for each pair marked by the table.

Table 1

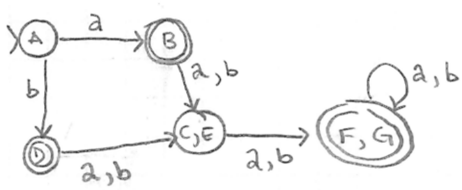
	G	F	E	D	C	B
A	X	X	X	X	X	X
B	X	A,E A,C X	X	✓	X	
C	X	X	B,D ✓	X		
D	X	A,E A,C X	X			
E	X	X				
F	✓					

Pair	IS*	Depends	Mark?
A, E	a	B, F	
A, E	b	D, F	
A, C	a	B, F	
A, C	b	D, F	
B, G	a	C, G	✓
B, G	b	E, G	
B, F	a	C, G	✓
B, F	b	E, F	
B, D	a	C, E	
B, D	b	E, C	
C, E	a	F, F	
C, E	b	F, F	
D, G	a	E, G	✓
D, G	b	C, G	
D, F	a	E, G	✓
D, F	b	C, F	
F, G	a	G, G	
F, G	b	F, G	

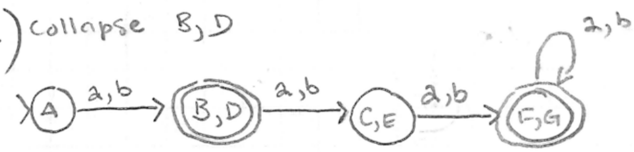
collapse F, G



collapse C, E



c) collapse B, D



b) The equiv classes following Table 1 are

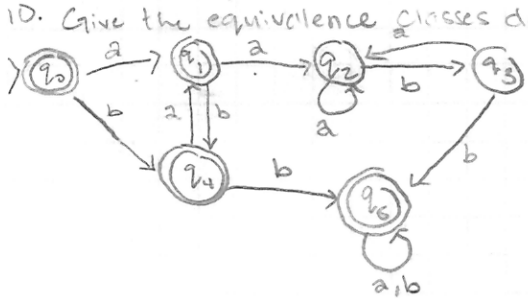
$[e_1]$ = A

$[e_2]$ = B, D

$[e_3]$ = C, E

$[e_4]$ = F, G

10. Give the equivalence classes defined by the relation $\equiv_M$ for the example 5.3.3



$[a]$ $aa^+ \cup aa^+b \cup aa^+ba^+ \cup aba(ba)^*a^+ \cup aba(ba)^*(a^+b)^* \cup aba(ba)^*a^+ba$

$[b]$ $b(ab)^*a^+ \cup b(ab)^*(a^+b)^* \cup b(ab)^*a^+ba^+ \cup baa^+ \cup baa^+b(a^+b)^*$