19. Convert the grammar

$G: S \to aAbB \mid ABC \mid a$
$A \to aA \mid a$
$B \to bBcC \mid b$
$C \to abc$

to chomsky normal form. G already satisfies the conditions on the start symbol S, λ-rules, useless symbols, and chain rules.

removing terminals from the RHS structure

$S \to A'AB'B$, $A' \to a$, $B' \to b$ $\xrightarrow{\text{for } (aAbB)}$ $S \to A'T_1$, $T_1 \to AT_2$, $T_2 \to B'B$, $A' \to a$, $B' \to b$ $\xrightarrow{\text{for } (ABC)}$ $S \to A'T_1 \mid AT_3 \mid a$, $T_1 \to AT_2$, $T_2 \to B'B$, $T_3 \to BC$ $\xrightarrow{\text{for } (bBcC)}$ $B \to B'BC'C$, $C' \to c$ $\longrightarrow$

$B \to B'T_4$, $T_4 \to BT_5$, $T_5 \to C'C$ $\longrightarrow$ $B \to B'T_4 \mid b$, $T_4 \to BT_5$, $T_5 \to C'C$, $C' \to c$ $\xrightarrow{\text{consolidate}}$

$S \to A'T_1 \mid AT_3 \mid a$
$T_1 \to AT_2$
$T_2 \to B'B$
$T_3 \to BC$
$A \to A'A \mid a$
$B \to B'T_4 \mid b$
$T_4 \to BT_5$
$T_5 \to C'C$
$C \to A'T_6$
$T_6 \to B'C'$
$A' \to a$
$B' \to b$
$C' \to c$

add C and terminals: $C \to A'T_6$, $T_6 \to B'C'$

27. Let G be the grammar

$G: S \to A \mid B$
$A \to aaB \mid Aab \mid Aba$
$B \to bB \mid Bb \mid aba$

a) Give a regular expression for L(G)
b) construct a grammar G' that contains no left-recursive rules and is equivalent to G.

a. $\{(ab \cup ba)^* aa \{b^* aba\}\} \cup \{b^* aba\}$

b. $S \to A \mid B$

$A \to A\underbrace{ab}_{u_1} \mid A\underbrace{ba}_{u_2} \mid \underbrace{aaB}_{v_1}$

$A \to aaBZ \mid aaB$
$Z \to abZ \mid baZ \mid ab \mid ba$

$B \to B\underbrace{b}_{u_1} \mid \underbrace{bB}_{v_1} \mid \underbrace{aba}_{v_2}$

$B \to bBZ_1 \mid abaZ_1 \mid bB \mid aba$
$Z_1 \to bZ_1 \mid b$

abaZ represents a derived string where aba terminates the sequence of nonterminal B's

30. Construct a Greibach normal form grammar equivalent to

$S \to aAb \mid a$ — remove indirect left recursion

$A \to SS \mid b$

$A \to aAbS \mid b$

$S \to aAb \to a(SS)b \to a((aAb)S)b \to aa(b)b(aAb)b \to aabbabbb$

$S = 1,\ A = 2$ $\quad S \to aAb \mid a$, $\ A \to aAbS \mid b$ $\quad \to \quad A \to aABS \mid b$, $\ B \to b$

$\Rightarrow$

$S \to aAB \mid a$

$A \to aABS \mid b$

$B \to b$

33. Convert the following grammar in CNF to GNF. Process the variables according to the order S, A, B

$S \to BA \mid AB \mid \lambda$

$A \to BB \mid AA \mid a$ — Remove left recursion: $A \to v_1Z \mid v_2Z \mid v_1 \mid v_2$, $\ Z \to AZ \mid Z$ $\quad \to \quad A \to BBZ \mid aZ \mid BB \mid a$, $\ Z \to AZ \mid Z$

$B \to AA \mid b$

$S = 1,\ A = 2,\ B = 3$

$B \to AA \to BBZA \to bBZA$

$B \to AA \to aZA$

$B \to AA \to BBA \to bBA$

$B \to AA \to aA$

Revised rules:

$S \to BA \mid AB \mid \lambda$

$A \to BBZ \mid aZ \mid BB \mid a$

$B \to bBZA \mid aZA \mid bBA \mid aA$

$Z \to AZ \mid A$

For S rules

$S \to BA \to bBZAA$	$S \to bBZAA$
$S \to BA \to aZAA$	$S \to aZAA$
$S \to BA \to bBAA$	$S \to bBAA$
$S \to BA \to aAA$	$S \to aAA$
$S \to AB \to BBZ \to bBZABZ$	$S \to bBZABZ$
$S \to AB \to BBZ \to aZABZ$	$S \to aZABZ$
$S \to AB \to BBZ \to bBABZ$	$S \to bBABZ$
$S \to AB \to BBZ \to aABZ$	$S \to aABZ$

for A rules

$A \to BBZ \to bBZABZ$	$A \to bBZABZ$
$A \to BBZ \to aZABZ$	$A \to aZABZ$
$A \to BBZ \to bBABZ$	$A \to bBABZ$
$A \to BBZ \to aABZ$	$A \to aABZ$
$A \to BB \to bBZAB$	$A \to bBZAB$
$A \to BB \to aZAB$	$A \to aZAB$
$A \to BB \to bBAB$	$A \to bBAB$
$A \to BB \to aAB$	$A \to aAB$

$Z \to AZ \to BBZZ \to bBZABZZ$	$Z \to bBZABZZ$
$Z \to AZ \to BBZZ \to aZABZZ$	$Z \to aZABZZ$
$Z \to AZ \to BBZZ \to bBABZZ$	$Z \to bBABZZ$
$Z \to AZ \to BBZZ \to aABZZ$	$Z \to aABZZ$

$S \to bBZAA \mid aZAA \mid bBAA \mid aAA \mid bBZABZ \mid aZABZ \mid bBABZ \mid aABZ \mid \lambda$

$A \to bBZABZ \mid aZABZ \mid bBABZ \mid aABZ \mid bBZAB \mid aZAB \mid bBAB \mid aAB$

$B \to bBZA \mid aZA \mid bBA \mid aA$

$Z \to bBZABZZ \mid aZABZZ \mid bBABZZ \mid aABZZ$