Assignment 5 Formal Languages	1, 5, 6, 10, 12, 13, 14, 23	Kurt Medley

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1. Give a recursive definition of the length of a string over Σ. Use primitive operation from the definition of string

i) let $\lambda \in \Sigma^*$, then length$(\lambda) = \emptyset$
ii) if $w \in \Sigma^*$ and $a \in \Sigma$, then $wa \in \Sigma^*$ and length$(wa) = n+1$

length corresponds with n elements in Σ; $\{a, b\}$
$n = 2$, 2^k
len$(\lambda) = 0$, len$(a) = 1$, len$(aa) = 2$, len$(aaa) = 3, \ldots$

5. Let L be the set of strings over $\{a, b\}$ generated by
i) Basis: $\boxed{b} \in L$
ii) Rec: if u is in L then $\boxed{ub} \in L$, $\boxed{uab} \in L$, and $\boxed{uba} \in L$ and $\boxed{bua} \in L$
iii) Closure: a string v is in L only if it can be obtained from the the basis by a finite number of iterations of the recursive step
a) $L_0 = \{\lambda\}$, $L_1 = b$, $L_2 = bb, ab, ba$
b) bbaaba in L? yes
③ ba → ② baab → ④ bbaaba
c) bbaaaabb in L? No
⓪ b → ③ abba → ② bbaab → ① bbaabb
There is no way to generate a string w/ 4 a's inbetween b's

6. Give a recursive def. of the set of strings over $\{a, b\}$ that contain at least one b and have an even number of a's before the first b
$\{aa\}^+ \{a, b\}^+ \cup \{b\}^+ \{a, b\}^*$

Base: $\lambda \in S$
Recur: If $u \in L$ then $ub \in L$, $aaubaL$, $bau \in L$, $aabu \in L$
aaaababababab ✓   bab ✓
aaaababababab ✓   aab
aaaabababa ✓

10. Prove that every string in the language defined in Example 2.2.2. has at least as many a's as b's. Let $n_a(u)$ denote the number of a's in the string u and $n_b(u)$ denote the number of b's in u. The inductive proof should establish the inequality $n_a(u) \geq n_b(u)$.

$\Sigma^* = \{a, b\}$

i) Basis $\lambda \in L$
ii) Recur: If $u \in L$, then ① ua, ② $uab \in L$

Ex. Strings in L using ①: λ, a, aa, aaa, aaaa, ...
" " ②: ab, abab, abababab, aab, aabab, ...

$n_a(a) = 1$
$n_a(ab) = 1$
$n_a(aabab) = 3$

Intuitively I can see that $n_a(u) \geq n_b(u)$ as the recursive step gives the productions ua and uab, thus every b must be constructed with 1 a.

B.C. Let x be the string ab, the string containing 1 a and 1 b, then $n_a(x) = 1$ and $n_b(x) = 1$, thus $n_a(x) \geq n_b(x)$ ✓

I.H. Prove for $n_a(u^k) \geq n_b(k+1)$ where k is generated by k+1 applications of the recursive step

10.(cont.) $K = ab$

$K+1 = (ab)^1$ | $K+1 = (ab)^2$
$= a(ab)^1$ | $= ab(ab)^2$
$= a(a(ab))^1$ | $= ab(ab(ab))^2$
$= a(a(a(ab)))^1$ | $= ab(ab(ab(ab)))^2$

Thus $n_a(K+1) \geq n_b(K+1)$

12. i) Basis: λ and a, for all $a \in \Sigma$, are palindromes
ii) Recur: If w is a palindrome and $a \in \Sigma$, then awa is a palindrome.
PV. $\{w \mid w = w^R\}$ generates the same set

BC. Let P = set generated by recursive def. Let $W = \{w \in \Sigma^* \mid w = w^R\}$
With $\emptyset$ apps of i), the set λ and a, $a \in \Sigma$ demonstrates a palin. $w = w^R$ (every single element reversed = itself.

I.H. $u = awa$ for string w and $a \in \Sigma$, where w is generated by ii.
$u^R = (awa)^R$
$= a^R w^R a^R$
$= a w^R a^R$
$= awa$
$= u$

For $\{w \mid w = w^R\}$
B.C. if $len(u) = 0$, then $w = \lambda$, and $\lambda \in P$, and $len(u) = 1 \in P$
I.H. Let $w \in W$ be a string of len $n+1$, $n \geq 1$, $w = ua$ where $len(u) = n$
$w = w^R = (ua)^R = au^R$
$w^R = (ava)^R = a^R v^R a^R = av^Ra$

13. Let $L_1 = \{aaa\}^*$, $L_2 = \{a,b\}\{a,b\}\{a,b\}\{a,b\}$, and $L_3 = L_2^*$
$L_2^0 = \{\lambda\}$, $L_2^1 = \{aaaa, aaab, aaba, abaa, \ldots\}$
L_2 represents all strings over $\{a,b\}$ of length 4.
$L_3 = (\{a,b\}\{a,b\}\{a,b\}\{a,b\})^*$
L_3 represents all strings over $\{a,b\}$ divisible by 4.
$L_1 \cap L_3$ $\{aaa\}^* \cap (\{a,b\}\{a,b\}\{a,b\}\{a,b\})^*$
represents strings over $\{a\}$ divisible by 12

aaa	aaa	aaa	aaa

14. The set of strings over $\{a,b,c\}$ in which all the a's precede the b's, which in turn precede the c's. It is possible that there are no a's, b's, or c's.
$a^*b^*c^*$

23. The set of strings over $\{a,b,c\}$ that begin with a, contain exactly two b's, and end with cc
$a(a \cup c)^* b (a \cup c)^* b (a \cup c)^* cc$