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CSF - Seminar

Paper Response 1: Linking Metaphor

Boole's metaphor has had a tremendous effect on how I perceive mathematics and the "categories of thought". Upon my initial reading, I brushed over the inherent significance that this important metaphor encapsulated. George Boole was, so I thought, trying to justify a "more" important conclusion based on the invention of Boolean algebra. The "laws of thought" were an important motivation for Boole in his development of this new, Modulo-based mathematics. Had Boole and his comrade De Morgan realized the implication of their research they would have most certainly disregarded their impossible task of constructing "laws" for neurologic function.

The essence of Boole's metaphor asserts that classes have an algebraic structure. A structure that can be analyzed with arithmetic processes that Boole labored to devise. His school of thought originated from the "Aristotle Prediction Metaphor", which emphasized a class-membership relationship. "Socrates is mortal". "Socrates is a member of the class mortals" (Lakoff & Nunez, 123). Using Aristotelean logic, Boole formulated his metaphor with foundations from the class-membership relationship. The source domain in Boole's first-stage metaphor is represented as a series of arithmetic operators like "Numbers", who's target-domain is "Classes", and "Addition", who's target-domain is "Union" symbolized by 'U'. This is important because operators like "Addition" and "Multiplication" were given symbols that associated them with classes. Which then allowed the categorization object, classes, to be manipulated symbolically as in with arithmetic.

In some ways, Boole's unique way of calculating these new elements was an example of closure, the process by which Boole could conceptualize his system being integrated as a series of manipulatable symbols that contradicted previously known algebraic operations. To justify his new system, Boole had to limit his numbers to 1 and 0; thereby solving his problems $X + X = 2X$ and $X * X = X^2$ as they related to the idempotent laws, $A + A = A$ and $A * A = A$ (126). The construction of these methods within "abstract mathematics" are grounded by "containers schemas" that build a visual conceptualization of what it means to be part of any class in any set. Venn diagrams prove a useful tool in visualizing a physical set "A", which can be differentiated from set "B", an entirely different circle. Performing operations using Boolean algebra can then be seen graphically, as in "A U B" where a set contains all elements of A and B (A*B). Similar processes of set containment must have perpetuated Boole into logical symbolic development.

The metaphor for the source domain does a superb job of replicating arithmetic operations onto a "class"-based system of symbolic manipulation. So much so that it seems paradoxical to evaluate classes in a different manner. Using Aristotelean logic, we can assume the intersection of A and B are the elements that reside both in A and B, exclusively. This makes it easy to calculate percentages of any one class, both classes, or the other class. In this way, Boole's first-stage metaphor creates an easy way to evaluate classes. A intersect B can easily be deciphered. Mapping arithmetic functions to classes was the first step to the construction of Boolean algebra. Of course, Boole's addition table came after his initial work within identifying classes, and his inherent problems were addressed. The creation of the empty set and universal set can be seen as ambiguous at times. Their properties existing only in practice because of apparent need of a supplementation "set" to tie everything together. Even though necessity indeed fuels the proclivity for invention, these can have confusing results in calculation. The idempotent law states that $A * A = A$, whereas the arithmetic $X * X = X^2$ can give an altered perception in terms of what is being "multiplied".

The group of elements in the source domain, Arithmetic (A), and the target domain, classes (C) is the probably one of the most identifiably isomorphic relationships. Even before relating things like "object collection" and "motion domain", which seem dissimilar but are abstractly isomorphic, (A) and (C) are easier to imagine. Boole recognized this, which is why, being an algebraist, he could easily apply arithmetic symbology to a group (C) that seems theoretical at best. We have notation for operations like "and", "or", set "D" and not set "E", that describe a specific output in a less convoluted way.

Fundamentally, these rules developed by Boole (and Augustus De Morgan) allowed for an easier representation of classes. The inherent isomorphic relationship between the arithmetic nature of "number manipulation" and the categorization of classes led to a revolution in how we perceive operations on classes. Boolean algebra, using his addition table, would be a basis for future development in electronics. Indeed, without the elementary foundations of Boolean logic, we might very well still be using circuit designs in computers that filled entire rooms. The significance of Boole's algebraic strategy applied to classes, originally stemmed from the Aristotle prediction metaphor, was a accidental success. George Boole wanted to form the "laws of thought" but ended up creating a unique prescription of laws that dictate how we distinguish classes into the 21st century. Although the human mind is invariably difficult to establish "laws" onto, Boole succeeded in making a system for which we often classify as the law for “set manipulation” IRONY!