Syntax reference: $[q_j, B] \in \delta(q_i, a, A)$

new state — q_j; new stack top — B; tran. — δ; current state — q_i; current input symbol — a; current stack top — A

1. Let M be the PDA defined by

$Q = \{q_0, q_1, q_2\}$ $\quad \delta(q_0, a, \lambda) = \{[q_0, A]\}$

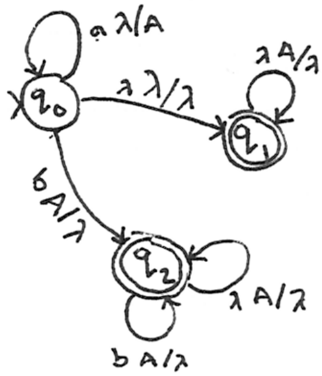
$\Sigma = \{a, b\}$ $\quad \delta(q_0, \lambda, \lambda) = \{[q_1, \lambda]\}$

$\Gamma = \{A\}$ $\quad \delta(q_0, b, A) = \{[q_2, \lambda]\}$

$F = \{q_1, q_2\}$ $\quad \delta(q_1, \lambda, A) = \{[q_1, \lambda]\}$

$\delta(q_2, b, A) = \{[q_2, \lambda]\}$

$\delta(q_2, \lambda, A) = \{[q_2, \lambda]\}$



a) Describe the language accepted by M

$a, \lambda, a^+b \quad \{a^n b^m \mid n \geq 0, \cancel{m \leq n}\}$
$m \leq n$

b) Give the state diagram of M

c) Trace all computations of the strings aab, abb, aba, in M

All (un)successful computations of aab

aab

$[q_0, aab, \lambda]$
$\vdash [q_0, ab, A]$
$\vdash [q_0, b, AA]$
$\vdash [q_1, b, AA]$
$\vdash [q_1, b, A]$
$\vdash [q_1, b, \lambda]$ ×

aab

$[q_0, aab, \lambda]$
$\vdash [q_1, aab, \lambda]$ ×

aab

$[q_0, aab, \lambda]$
$\vdash [q_0, ab, A]$
$\vdash [q_1, ab, A]$
$\vdash [q_1, ab, \lambda]$ ×

aab

$[q_0, aab, \lambda]$
$\vdash [q_0, ab, A]$
$\vdash [q_0, b, AA]$
$\vdash [q_2, \lambda, A]$ ×

aab

$[q_0, aab, \lambda]$
$\vdash [q_0, ab, A]$
$\vdash [q_0, b, AA]$
$\vdash [q_2, \lambda, A]$
$\vdash [q_2, \lambda, \lambda]$ ✓

~~unsuc~~ unsuccessful computations of abb

$[q_0, abb, \lambda]$
$\vdash [q_1, abb, \lambda]$ ×

$[q_0, abb, \lambda]$
$\vdash [q_0, bb, A]$
$\vdash [q_1, bb, A]$ ×

$[q_0, abb, \lambda]$
$\vdash [q_0, bb, A]$
$\vdash [q_2, b, \lambda]$ ×

unsuccessful computations of aba.

$[q_0, aba, \lambda]$
$\vdash [q_1, aba, \lambda]$ ×

$[q_0, aba, \lambda]$
$\vdash [q_0, ba, A]$
$\vdash [q_1, ba, A]$ ×

$[q_0, aba, \lambda]$
$\vdash [q_0, ba, A]$
$\vdash [q_1, ba, A]$
$\vdash [q_1, ba, \lambda]$ ×

$[q_0, aba, \lambda]$
$\vdash [q_0, ba, A]$
$\vdash [q_2, a, \lambda]$ ×

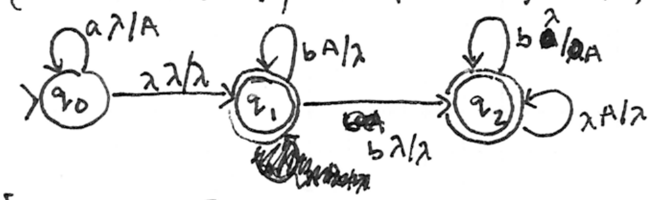
d) show that aabb, aaab $\in L(M)$

$[q_0, aabb, \lambda]$
$\vdash [q_0, abb, A]$
$\vdash [q_0, bb, AA]$
$\vdash [q_2, b, A]$
$\vdash [q_2, \lambda, \lambda]$ ✓

$[q_0, aaab, \lambda]$
$\vdash [q_0, aab, A]$
$\vdash [q_0, ab, AA]$
$\vdash [q_0, b, AAA]$
$\vdash [q_2, \lambda, AA]$
$\vdash [q_2, \lambda, A]$
$\vdash [q_2, \lambda, \lambda]$ ✓

3. Construct PDAs that accept each of the following languages

a) $\{a^i b^j \mid 0 \le i \le j\}$ Sample strings: λ, ab, abb, b^*, aabb, aabbb

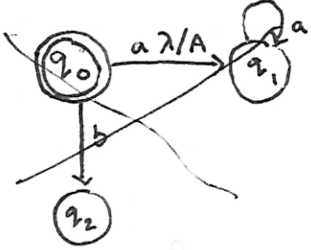


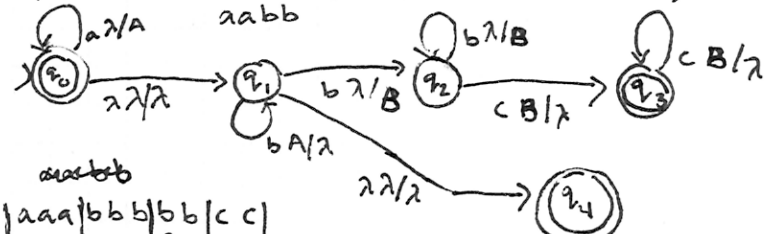
$\vdash[q_0, a, \lambda]$ $[q_0, aabbb, \lambda]$
$\vdash[q_1, a, A]$ $[q_0, abbb, A]$
$\vdash[q_1, a, \lambda]$ $[q_0, bbb, AA]$
$[q_1, bbb, AA]$
$[q_1, bb, A]$
$[q_1, b, \lambda]$
$[q_2, \lambda, \lambda]$

M: $Q = \{q_0, q_1, q_2\}$
$\Sigma: \{a, b\}$
$\Gamma: \{A\}$
$F: \{q_1, q_2\}$

$\delta(q_0, a, \lambda) = \{[q_0, A]\}$
$\delta(q_0, \lambda, \lambda) = \{[q_1, \lambda]\}$
$\delta(q_1, b, A) = \{[q_1, \lambda]\}$
$\delta(q_1, b, \lambda) = \{[q_2, \lambda]\}$
$\delta(q_2, b, \lambda) = \{[q_2, A]\}$
$\delta(q_2, \lambda, A) = \{[q_2, \lambda]\}$

c) $\{a^i b^j c^k \mid i + k = j\}$ Sample Strings: λ, ab, bc, abbc, abbbcc, aabbbbcc, aabb





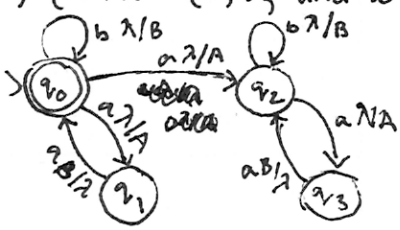
~~aabb~~

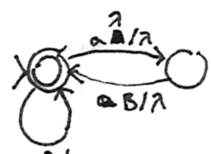
aaa	bbb	bb	cc
AAA	λ	BB	λ

M: $Q = \{q_0, q_1, q_2, q_3, q_4\}$
$\Sigma: \{a, b, c\}$
$\Gamma: \{A, B\}$
$F: \{q_0, q_3, q_4\}$

$\delta(q_0, a, \lambda) = \{[q_0, A]\}$
$\delta(q_0, \lambda, \lambda) = \{[q_1, \lambda]\}$
$\delta(q_1, b, A) = \{[q_1, \lambda]\}$
$\delta(q_1, \lambda, \lambda) = \{[q_4, \lambda]\}$
$\delta(q_1, b, \lambda) = \{[q_2, B]\}$
$\delta(q_2, b, \lambda) = \{[q_2, B]\}$
$\delta(q_2, c, B) = \{[q_3, \lambda]\}$
$\delta(q_3, c, B) = \{[q_3, \lambda]\}$

d) $\{w \mid w \in \{a,b\}^*$ and w has twice as many a's as b's$\}$ aab aba baa, babaaa, babaaabaa bbaaaa

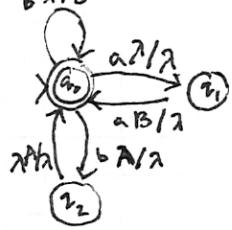


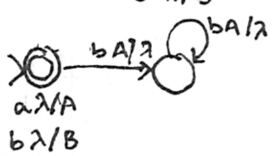


baa

baba
BABA

~~bbaaaa~~
$[q_0, bbaaaa, \lambda]$
$[q_0, baaaa, B]$
$[q_0, aaaa, BB]$
$[q_1, aaa, BB]$
$[q_0, aa, B]$
$[q_1, a, B]$
$[q_0, \lambda, \lambda]$





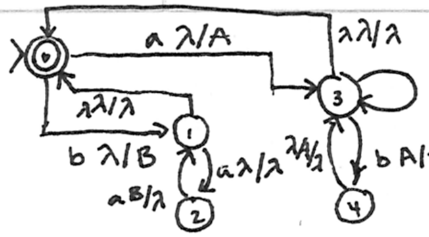
$[q_0, aab, \lambda]$
$[q_0, ab, A]$
$[q_0, b, AA]$
$[q_1, \lambda, A]$
$[q_0, \lambda, \lambda]$

~~$[q_0, babaaa, \lambda]$~~
~~$[q_0, abaaa, B]$~~
~~$[q_1, baaa, B]$~~
$[q_0, babaaa, \lambda]$
$[q_0, abaaa, B]$
$[q_0, baaa, AB]$

$[q_0, aaa, BAB]$
$[q_1, aa, AB]$
$[q_0, a, AB]$

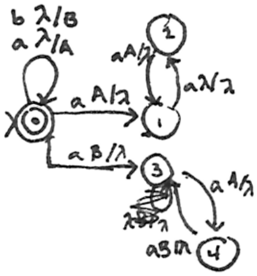
$[q_0, aba, \lambda]$ $\vdash[$
$\vdash[q_0, ba, A]$
$\vdash[q_0, a, BA]$

d) continued..



$[q_0, babaaa, \lambda]$
$\vdash [q_1, abaaa, B]$
$\vdash [q_2, baaa, B]$
$\vdash [q_1, aaa, \lambda]$

$[q_0, babaaa, \lambda]$
$\vdash [q_1, abaaa, B]$
$\vdash [q_0, abaaa, B]$
$\vdash [q_3, baaa, AB]$
$\vdash$



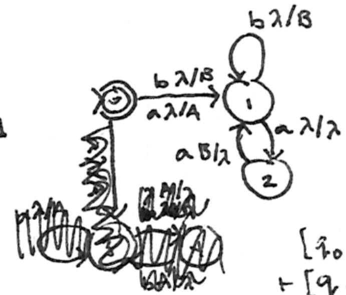
$[q_0, ~~babaaa~~, \lambda]$
$\vdash [q_0, ba, A]$
$\vdash [q_0, a, BA]$

$[q_0, babaaa, \lambda]$
$\vdash [q_0, abaaa, B]$
$\vdash [q_0, baaa, AB]$
$\vdash [q_0, aaa, BAB]$
$\vdash [q_3, aa, AB]$

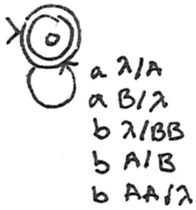
$\vdash [q_4, a, B]$
$\vdash [q_3, \lambda, \lambda]$

$[q_0, aaaabb, \lambda]$
$\vdash [q_0, aaabb, A]$
$\vdash [q_0, aabb, AA]$

$\vdash [q_1, abb, A]$
$\vdash [q_2, bb, \lambda]$



$[q_0, aba, \lambda]$
$\vdash [q_1, ba, A]$



$M: Q = \{q_0\}$
$\Sigma: \{a, b\}$
$\Gamma: \{A, B\}$
$F: \{q_0\}$

$\delta(q_0, a, \lambda) = \{[q_0, A]\}$
$\delta(q_0, a, B) = \{[q_0, \lambda]\}$
$\delta(q_0, b, \lambda) = \{[q_0, BB]\}$
$\delta(q_0, b, A) = \{[q_0, B]\}$
$\delta(q_0, b, AA) = \{[q_0, \lambda]\}$

j) The set of palindromes over $\{a, b\}$

$[q_0, abba, \lambda]$
$\vdash [q_0, bba, A]$
$\vdash [q_0, ba, BA]$
$\vdash [q_1, ba, BA]$
$\vdash [q_1, a, A]$
$\vdash (q_1, \lambda, \lambda]$

$M: Q = \{q_0, q_1\}$
$\Sigma: \{a, b\}$
$\Gamma: \{A, B\}$
$F: \{q_0, q_1\}$

$\delta(q_0, a, \lambda) = \{[q_0, A]\}$
$\delta(q_0, b, \lambda) = \{[q_0, B]\}$
$\delta(q_0, \lambda, \lambda) = \{[q_1, \lambda]\}$
$\delta(q_1, a, A) = \{[q_1, \lambda]\}$
$\delta(q_1, b, B) = \{[q_1, \lambda]\}$

14. Let M be the PDA

$Q: \{q_0, q_1, q_2\}$
$\Sigma: \{a, b\}$
$\Gamma: \{A\}$
$F: \{q_2\}$

$\delta(q_0, a, \lambda) = \{[q_0, A]\}$
$\delta(q_0, b, A) = \{[q_1, \lambda]\}$
$\delta(q_1, b, \lambda) = \{[q_2, \lambda]\}$
$\delta(q_2, b, A) = \{[q_1, \lambda]\}$

a) Give the state diagram of M.

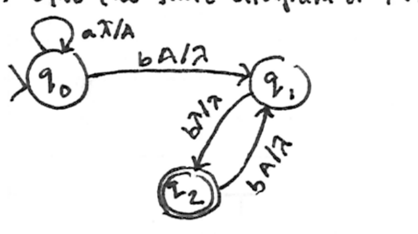


abb
aabbbb

b) Give a set theoretic definition of L(M).

$\{a^n b^{2n} \mid n > 0\}$

d) Trace the computation of aabbbb in M.

$[q_0, aabbbb, \lambda]$
$\vdash [q_0, abbbb, A]$
$\vdash [q_0, bbbb, AA]$
$\vdash [q_1, bbb, A]$
$\vdash [q_2, bb, A]$
$\vdash [q_1, b, \lambda]$
$\vdash [q_2, \lambda, \lambda]$

PDA → CFG

c) Using the technique from Theorem 7.3.2, build a context-free grammar G that generates L(M).

ii 1 $\delta(q_0,a,\lambda)=\{[q_0,A]\} \xrightarrow{+} \delta(q_0,a,A)=\{[q_0,AA]\}$ 6
2 $\delta(q_0,b,A)=\{[q_1,\lambda]\}$ ×
i 3 $\delta(q_1,b,\lambda)=\{[q_2,\lambda]\} \xrightarrow{+} \delta(q_1,b,A)=\{[q_2,A]\}$ 5
4 $\delta(q_2,b,A)=\{[q_1,\lambda]\}$ ×

$Q: \{q_0,q_1,q_2\}$
$\Sigma: \{a,b\}$
$\Gamma: \{A\}$
$F: \{q_2\}$

a Conversion rule – Add a rule $S \to \langle s\lambda f\rangle$ for the start state, s, and each final state, f.

$S \to \langle q_0,\lambda,q_2\rangle$

b Add a rule $\langle q\lambda q\rangle \to \lambda$ for each state q.

$\langle q_0,\lambda,q_0\rangle \to \lambda$; $\langle q_1,\lambda,q_1\rangle \to \lambda$; $\langle q_2,\lambda,q_2\rangle \to \lambda$

c For each transition in the PDA, that pushes a single character (including λ), such as $\delta(q,u,A)=(r,B)$, add rules of the form $\langle qAp\rangle \to u\langle rBp\rangle$ for each state p in the machine. The letter u can be λ (in which case it disappears).

δ q A p u r B p
1 $\langle q_0,\lambda,q_0\rangle \to a\langle q_0,A,q_0\rangle$
1 $\langle q_0,\lambda,q_1\rangle \to a\langle q_0,A,q_1\rangle$
1 $\langle q_0,\lambda,q_2\rangle \to a\langle q_0,A,q_2\rangle$

2 $\langle q_0,A,q_0\rangle \to b\langle q_1,\lambda,q_0\rangle$
2 $\langle q_0,A,q_1\rangle \to b\langle q_1,\lambda,q_1\rangle$
2 $\langle q_0,A,q_2\rangle \to b\langle q_1,\lambda,q_2\rangle$

3 $\langle q_1,\lambda,q_0\rangle \to b\langle q_2,\lambda,q_0\rangle$
3 $\langle q_1,\lambda,q_1\rangle \to b\langle q_2,\lambda,q_1\rangle$
3 $\langle q_1,\lambda,q_2\rangle \to b\langle q_2,\lambda,q_2\rangle$

4 $\langle q_2,A,q_0\rangle \to b\langle q_1,\lambda,q_0\rangle$
4 $\langle q_2,A,q_1\rangle \to b\langle q_1,\lambda,q_1\rangle$
4 $\langle q_2,A,q_2\rangle \to b\langle q_1,\lambda,q_2\rangle$

5 $\langle q_1,A,q_0\rangle \to b\langle q_2,A,q_0\rangle$
5 $\langle q_1,A,q_1\rangle \to b\langle q_2,A,q_1\rangle$
5 $\langle q_1,A,q_2\rangle \to b\langle q_2,A,q_2\rangle$

d For each state in the PDA that pushes two (or more) characters, such as $\delta(q,u,A)=(r,BC)$, add rules of the form $\langle qAp\rangle \to u\langle rBt\rangle\langle tCp\rangle$ for all possible combinations of states p and t in the machine.

6 $\langle q_0,A,q_0\rangle \to a\langle q_0,A,q_0\rangle\langle q_0,A,q_0\rangle$
6 $\langle q_0,A,q_1\rangle \to a\langle q_0,A,q_0\rangle\langle q_0,A,q_1\rangle$
6 $\langle q_0,A,q_2\rangle \to a\langle q_0,A,q_0\rangle\langle q_0,A,q_2\rangle$

6 $\langle q_0,A,q_0\rangle \to a\langle q_0,A,q_1\rangle\langle q_1,A,q_0\rangle$
6 $\langle q_0,A,q_1\rangle \to a\langle q_0,A,q_1\rangle\langle q_1,A,q_1\rangle$
6 $\langle q_0,A,q_2\rangle \to a\langle q_0,A,q_1\rangle\langle q_1,A,q_2\rangle$

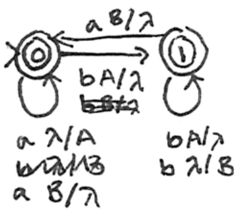
6 $\langle q_0,A,q_0\rangle \to a\langle q_0,A,q_2\rangle\langle q_2,A,q_0\rangle$
6 $\langle q_0,A,q_1\rangle \to a\langle q_0,A,q_2\rangle\langle q_2,A,q_1\rangle$
6 $\langle q_0,A,q_2\rangle \to a\langle q_0,A,q_2\rangle\langle q_2,A,q_2\rangle$

e) Give the derivation of aabbbb in G.

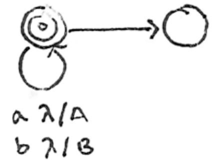
× $S \Rightarrow \langle q_0,\lambda,q_2\rangle$
$\Rightarrow a\langle q_0,A,q_2\rangle$
$\Rightarrow aa\langle q_0,A,q_0\rangle\langle q_0,A,q_2\rangle$
$\Rightarrow aab\langle q_1,\lambda,q_0\rangle\langle q_0,A,q_2\rangle$
$\Rightarrow aabb\langle q_2,\lambda,q_0\rangle$

$S \Rightarrow \langle q_0,\lambda,q_2\rangle$
$\Rightarrow a\langle q_0,A,q_2\rangle$
$\Rightarrow aa\langle q_0,A,q_1\rangle\langle q_1,A,q_2\rangle$
$\Rightarrow aab\langle q_1,\lambda,q_1\rangle\langle q_1,A,q_2\rangle$
$\Rightarrow aab\lambda\langle q_1,A,q_2\rangle$
$\Rightarrow aabb\langle q_2,A,q_2\rangle$
$\Rightarrow aabbb\langle q_1,\lambda,q_2\rangle$
$\Rightarrow aabbbb\langle q_2,\lambda,q_2\rangle$
$\Rightarrow aabbbb\lambda$
$\Rightarrow aabbbb$

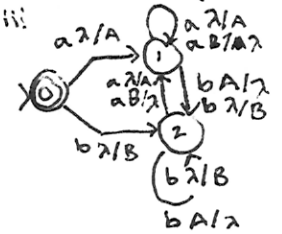
Devise a Deterministic Pushdown Automaton that accepts strings with equally many a's and b's. Convert it to a CFG.



0, ab, λ
0, b, A
1, λ, λ



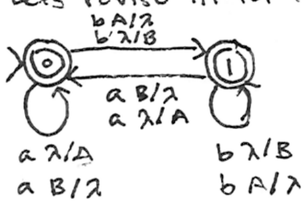
iii



0, aabb, λ
1, abb, A
1, bb, AA
2, b, A
2, λ, λ ✓

Lets revise iii for a less arguous CFG conversion

q_0, abab, λ
q_0, bab, A
q_1, ab, λ
q_0, b, A
$\boxed{q_1, \lambda, \lambda}$



abab
aab

Q: $\{q_0, q_1\}$
Σ: $\{a, b\}$
Γ: $\{A, B\}$
F: $\{q_0, q_1\}$

1 $\delta(q_0, a, \lambda) = \{[q_0, A]\}$ + 9 $\delta(q_0, a, B) = \{[q_0, AB]\}$
2 $\delta(q_0, a, B) = \{[q_0, \lambda]\}$ x
3 $\delta(q_0, b, A) = \{[q_1, \lambda]\}$ x
4 $\delta(q_0, b, \lambda) = \{[q_1, B]\}$ + 10 $\delta(q_0, b, A) = \{[q_1, BA]\}$
5 $\delta(q_1, b, \lambda) = \{[q_1, B]\}$ + 11 $\delta(q_1, b, A) = \{[q_1, BA]\}$
6 $\delta(q_1, b, A) = \{[q_1, \lambda]\}$ x "
7 $\delta(q_1, a, \lambda) = \{[q_0, A]\}$ + 12 $\delta(q_0, a, B) = \{[q_0, AB]\}$
8 $\delta(q_1, a, B) = \{[q_0, \lambda]\}$ x

a) Start State

$S \to \langle q_0, \lambda, q_0\rangle \mid \langle q_0, \lambda, q_1\rangle$

b) λ-rules

$\langle q_0, \lambda, q_0\rangle \to \lambda$; $\langle q_1, \lambda, q_1\rangle \to \lambda$

c) Single char pushes "$\delta(q, u, A) = (r, B)$" add $\langle q A p\rangle \to u\langle r B p\rangle$

1 $\langle q_0, \lambda, q_0\rangle \to a\langle q_0, A, q_0\rangle$
1 $\langle q_0, \lambda, q_1\rangle \to a\langle q_0, A, q_1\rangle$
2 $\langle q_0, B, q_0\rangle \to a\langle q_0, \lambda, q_0\rangle$
2 $\langle q_0, B, q_1\rangle \to a\langle q_0, \lambda, q_1\rangle$
3 $\langle q_0, A, q_0\rangle \to b\langle q_1, \lambda, q_0\rangle$
3 $\langle q_0, A, q_1\rangle \to b\langle q_1, \lambda, q_1\rangle$
4 $\langle q_0, \lambda, q_0\rangle \to b\langle q_1, B, q_0\rangle$
4 $\langle q_0, \lambda, q_1\rangle \to b\langle q_1, B, q_1\rangle$
5 $\langle q_1, \lambda, q_0\rangle \to b\langle q_1, B, q_0\rangle$
5 $\langle q_1, \lambda, q_1\rangle \to b\langle q_1, B, q_1\rangle$
6 $\langle q_1, A, q_0\rangle \to b\langle q_1, \lambda, q_0\rangle$
6 $\langle q_1, A, q_1\rangle \to b\langle q_1, \lambda, q_1\rangle$
7 $\langle q_1, \lambda, q_0\rangle \to a\langle q_0, A, q_0\rangle$
7 $\langle q_1, \lambda, q_1\rangle \to a\langle q_0, A, q_1\rangle$
8 $\langle q_1, B, q_0\rangle \to a\langle q_0, \lambda, q_0\rangle$
8 $\langle q_1, B, q_1\rangle \to a\langle q_0, \lambda, q_1\rangle$

d) double char pushes $\langle q A p\rangle \to u\langle r B t\rangle\langle t C p\rangle$

9 $\langle q_0, B, q_0\rangle \to a\langle q_0, A, q_0\rangle\langle q_0, B, q_0\rangle$
9 $\langle q_0, B, q_1\rangle \to a\langle q_0, A, q_0\rangle\langle q_0, B, q_1\rangle$
9 $\langle q_0, B, q_0\rangle \to a\langle q_0, A, q_1\rangle\langle q_1, B, q_0\rangle$
9 $\langle q_0, B, q_1\rangle \to a\langle q_0, A, q_1\rangle\langle q_1, B, q_1\rangle$
10 $\langle q_0, A, q_0\rangle \to b\langle q_1, B, q_0\rangle\langle q_0, A, q_0\rangle$
10 $\langle q_0, A, q_1\rangle \to b\langle q_1, B, q_0\rangle\langle q_0, A, q_1\rangle$
10 $\langle q_0, A, q_0\rangle \to b\langle q_1, B, q_1\rangle\langle q_1, A, q_0\rangle$
10 $\langle q_0, A, q_1\rangle \to b\langle q_1, B, q_1\rangle\langle q_1, A, q_1\rangle$
11 $\langle q_1, A, q_0\rangle \to b\langle q_1, B, q_0\rangle\langle q_0, A, q_0\rangle$
11 $\langle q_1, A, q_1\rangle \to b\langle q_1, B, q_0\rangle\langle q_0, A, q_1\rangle$
11 $\langle q_1, A, q_0\rangle \to b\langle q_1, B, q_1\rangle\langle q_1, A, q_0\rangle$
11 $\langle q_1, A, q_1\rangle \to b\langle q_1, B, q_1\rangle\langle q_1, A, q_1\rangle$
12 $\langle q_1, B, q_0\rangle \to a\langle q_0, A, q_0\rangle\langle q_0, B, q_0\rangle$
12 $\langle q_1, B, q_1\rangle \to a\langle q_0, A, q_0\rangle\langle q_0, B, q_1\rangle$
12 $\langle q_1, B, q_0\rangle \to a\langle q_0, A, q_1\rangle\langle q_1, B, q_0\rangle$
12 $\langle q_1, B, q_1\rangle \to a\langle q_0, A, q_1\rangle\langle q_1, B, q_1\rangle$

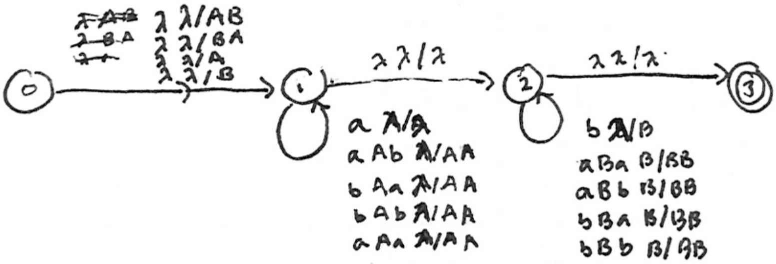
Find a PDA that accepts all strings of a's and b's that are not of the form ww

$S \to AB \mid BA \mid A \mid B$

$A \to aAa \mid aAb \mid bAa \mid bAb \mid a$

$B \to aBa \mid aBb \mid bBa \mid bBb \mid b$

$\delta(q_0, \lambda, \lambda) = \{[q_1, AB], [q_1, BA], [q_1, A], [q_1, B]\}$



17. Proving not context free

$\{a^k \mid k \text{ is a perfect square}\}$

Assume that the language L where $\Sigma = \{a\}$ whose lengths are a perfect square is context free. By the pumping lemma, there is a number k such that every string in L with length k or more can be written uvwxy where

i) $|vwx| \le k$

ii) v and x are not both null

iii) $uv^iwx^iy \in L$ for all $i \ge 0$

$z = a^{k^2}$

$|z| = |uv^2wx^2y|$

$= |uvwxy| + |v| + |x|$

$= k^2 + |v| + |x|$

$\le k^2 + k$

$< (k+1)^2$

$|z| > k^2 < (k+1)^2$

$z \notin L$

Not context free

c) $\{a^ib^{2i}a^i \mid i \ge 0\}$

i) $|vwx| \le k$

ii) v and x are not both null

iii) $uv^iwx^iy \in L$ for all $i \ge 0$

The string $z = a^kb^{2k}a^k$, pumping $uvwxy \to uv^2wx^2y \in L$, we must have that the union of v and x contains both a type and b type of terminals.

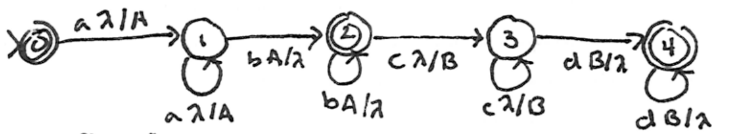
i) $|vwx| \le k$, vwx of z cannot contain a's from both sides of the b's uv^2wx^2y only increases the number of a's either preceding or after b's $uv^2wx^2y \notin L$, L is context free.

d) $\{a^ib^jc^k \mid 0 < i < j < k < 2i\}$

The string $z = a^kb^{k+1}c^{k+2} \in L$. uv^kwx^ky by pumping uvwxy

i) $|vwx| \le k \to$ vwx is a string containing only one type of terminal or the concatenation of either a and b types, or b and c types. If c is not contained in vwx, pumping v and x only increases the # of a's and b's. uv^kwx^ky would have at least $(k+2) + (k-1) = 2k+1$ number of c's while keeping the number of c's the same, i.e. k. $uv^kwx^ky \notin L$, L is not context free

19. a) Prove that the language $L_1 = \{a^i b^i c^j d^j \mid i, j \geq 0\}$ is context-free.



$\delta(q_0, a, \lambda) = \{[q_1, A]\}$
$\delta(q_1, a, \lambda) = \{[q_1, A]\}$
$\delta(q_1, b, A) = \{[q_2, \lambda]\}$
$\delta(q_2, b, A) = \{[q_2, \lambda]\}$
$\delta(q_2, c, \lambda) = \{[q_3, B]\}$
$\delta(q_3, c, \lambda) = \{[q_3, B]\}$
$\delta(q_3, d, B) = \{[q_4, \lambda]\}$
$\delta(q_4, d, B) = \{[q_4, \lambda]\}$

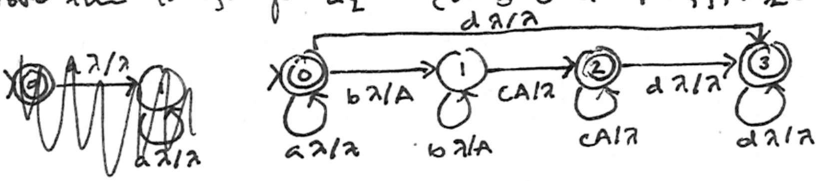
$[q_0, ab, \lambda] \vdash [q_1, b, A]$
$\vdash [q_2, \lambda, \lambda]$

$[q_0, abcd, \lambda] \vdash [q_1, bcd, A]$
$\vdash [q_2, cd, \lambda]$
$\vdash [q_3, d, B]$
$\vdash [q_4, \lambda, \lambda]$

Q: $\{q_{0,1,2,3,4}\}$
Γ: $\{A, B\}$
Σ: $\{a, b, c, d\}$
F: $\{q_{0,2,4}\}$

G: S → ~~aA | λ~~
~~A → aA | bB~~
~~B → bB | cC | λ~~
~~C →~~

b) Prove the language $L_2 = \{a^j b^i c^i d^k \mid i, j, k \geq 0\}$ is context free.



Q: $\{q_{0,1,2,3}\}$
Γ: $\{A\}$
Σ: $\{a, b, c, d\}$
F: $\{0, 3, 2\}$

$\delta(q_0, a, \lambda) = \{[q_0, \lambda]\}$
$\delta(q_0, d, \lambda) = \{[q_3, \lambda]\}$
$\delta(q_0, b, \lambda) = \{[q_1, A]\}$
$\delta(q_1, b, \lambda) = \{[q_1, A]\}$
$\delta(q_1, c, A) = \{[q_2, \lambda]\}$
$\delta(q_2, c, A) = \{[q_2, \lambda]\}$
$\delta(q_2, d, \lambda) = \{[q_3, \lambda]\}$
$\delta(q_3, d, \lambda) = \{[q_3, \lambda]\}$

c) $L_1 = \{a^i b^i c^j d^j \mid i, j \geq 0\}$, $L_2 = \{a^j b^i c^i d^k \mid i, j, k \geq 0\}$

$L_1 \cap L_2 \ \{w \mid w \in L_1 \wedge w \in L_2\}$