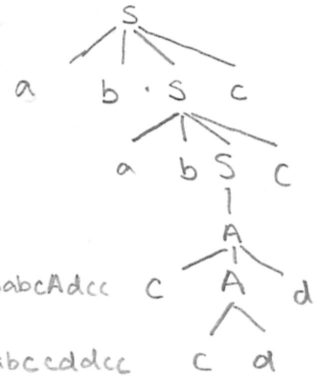
1. Let G be the grammar
$S \to abSc \mid A$
$A \to cAd \mid cd$

a) $S \overset{*}{\Rightarrow} ababccddcc$

$S \Rightarrow abSc$
$\Rightarrow ab(abSc)c$
$\Rightarrow ababScc$
$\Rightarrow abab(A)cc$
$\Rightarrow abab(cAd)cc$
$\Rightarrow ababc(cd)dcc$
$\Rightarrow ababccddcc$

b) DT of part a.

S
$\Rightarrow abSc$
$\Rightarrow ababScc$
$\Rightarrow ababcAdcc$
$\Rightarrow ababccddcc$



c) use set notation to define L(G)
$L(G) = \{(ab)^n c^m d^m c^n \mid n \geq 0, m > 0\}$

2. Let G be the grammar
$S \to ASB \mid \lambda$
$A \to aAb \mid \lambda$
$B \to bBa \mid ba$

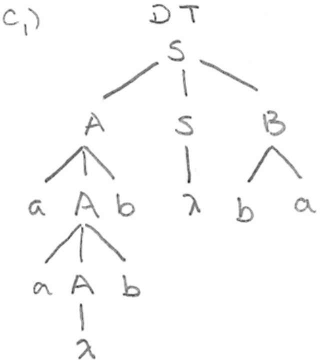
a) Give a leftmost derivation of aabbba

$S \Rightarrow ASB$
$\Rightarrow (aAb)SB$
$\Rightarrow a(aAb)bSB$
$\Rightarrow aa\lambda bbSB$
$\Rightarrow aabb\lambda B$
$\Rightarrow aabbba$

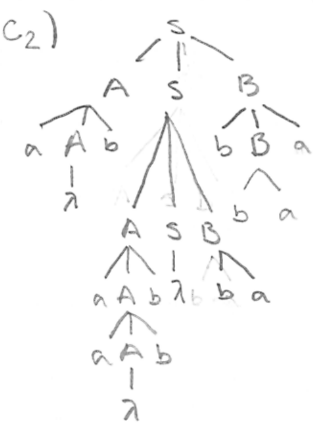
b) Give a rightmost derivation of abaabbbabbaa

$S \Rightarrow ASB$
$\Rightarrow AS(bBa)$
$\Rightarrow ASb(ba)a$
$\Rightarrow A(ASB)bbaa$
$\Rightarrow AAS(ba)bbaa$
$\Rightarrow AA(\lambda)babbaa$
$\Rightarrow A(aAb)babbaa$
$\Rightarrow Aa(aAb)bbabbaa$
$\Rightarrow Aa(\lambda)bbbabbaa$
$\Rightarrow (aAb)aabbbabbaa$
$\Rightarrow a(\lambda)baabbbabbaa$
$\Rightarrow abaabbbabbaa$

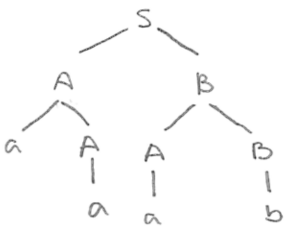
c_1) DT



c_2)



4. Let DT be the derivation tree



$S \to AB$
$A \to aA \mid a$
$B \to AB \mid b$

a) Give a leftmost derivation that generates the tree DT.

$S \Rightarrow AB$
$\Rightarrow (aA)B$
$\Rightarrow a(a)B$
$\Rightarrow aa(AB)$
$\Rightarrow aa(a)B$
$\Rightarrow aaa(b) \Rightarrow aaab$

b) Give a rightmost derivation that generates the tree DT.

$S \Rightarrow AB$
$\Rightarrow A(AB)$
$\Rightarrow AA(b)$
$\Rightarrow A(a)b$
$\Rightarrow (aA)ab$
$\Rightarrow a(a)ab$
$\Rightarrow aaab$

12.) Construct a grammar over $\{a,b\}$ whose language contains precisely the strings with the same number of a's and b's.

$S \to AB$
$A \to aAb \mid \lambda \mid S$
$B \to bBa \mid \lambda \mid S$

$S \Rightarrow aAb\ (\lambda)$ ✓
$S \Rightarrow bBa\ (\lambda)$ ✓
$S \Rightarrow (aAb)B$
$\Rightarrow a(aAb)b\,B$
$\Rightarrow aabb(bBa)$
$\Rightarrow aabbb(bBa)a$
$\Rightarrow aabbbbaa$

15. The set of strings over $\{a,b,c\}$ in which all the a's precede the b's, which in turn precede the c's. It is possible that there are no a's, b's, or c's

$S \to ABC$
$A \to aA \mid \lambda$
$B \to bB \mid \lambda$
$C \to cC \mid \lambda$

$S \Rightarrow ABC$
$\Rightarrow (aA)BC$
$\Rightarrow a(bB)C$
$\Rightarrow ab(cC)$
$\Rightarrow abc$

21. The set of strings over $\{a,b\}$ that do not contain the substring aba

$S \to AB$
$A \to aA \mid b$
$B \to bA \mid aA \mid \lambda$

$S \Rightarrow AB$
$\Rightarrow (aA)B$
$\Rightarrow (ab)B$
$\Rightarrow ab(aA)$
$\Rightarrow abab$

25. The set of strings over $\{a,b\}$ with an even number of a's | odd number of b's

$S \to aO_a \mid bE_a \mid bO_b \mid$
$\to aE_b \mid \lambda \mid \lambda$
$E_a \to aO_a \mid bE_a \mid \lambda$
$O_a \to aE_a \mid bO_a$
$O_b \to aO_b \mid bE_b \mid \lambda$
$E_b \to aE_b \mid bO_b$

$\{a^{2n} b^{2n+1} \mid n \geq 0\}$