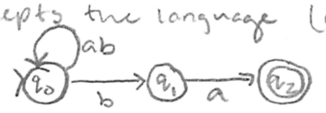
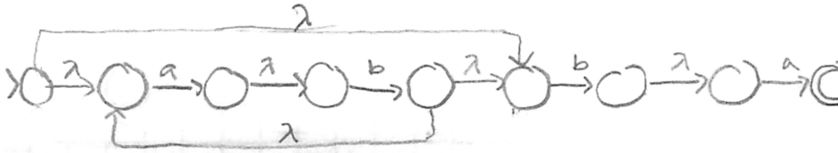
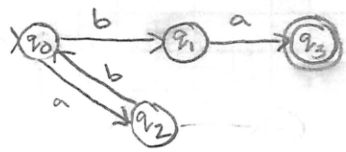
1. Use the technique from section 6.2 to build the state diagram of an NFA-λ that accepts the language $(ab)^*ba$. Compare this with the DFA constructed in Exercise 5.22(...)



Expression Graph

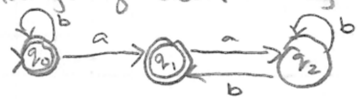




DFA State diagram

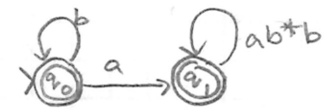
Here the NFA-λ consists of considerably more states than the DFA and Expression graphs. The construction of the NFA-λ allows a more systematic construction of the M_L than the DFA.

2. For each of the state diagrams in Exercise 5.40, use algorithm 6.2.2 to construct a regular expression for the language accepted by the automaton



5.40a w

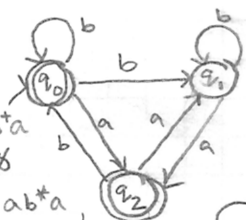
j	k	i	
q_0	q_1	q_2	$w^{\lambda}_{0,2}\ w^{b^*}_{2,2}\ w^{b}_{2,1}$
q_0	q_0	q_2	$w^{\lambda}_{0,2}\ w^{b^*}_{2,2}\ w^{\lambda}_{2,0}$
q_1	q_0	q_2	$w^{a}_{1,2}\ w^{b^*}_{2,2}\ w^{\lambda}_{2,0}$
q_1	q_1	q_2	$w^{a}_{1,2}\ w^{b^*}_{2,2}\ w^{b}_{2,1}\quad w_{1,1} = ab^*b$

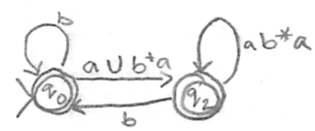


$((b^*)^*a(ab^*b)^*)^*$

5.40b w eliminate q_1

j	k	i	
q_0	q_2	q_1	$w^{b}_{0,1}\ w^{b^*}_{1,1}\ w^{a}_{1,2}\quad w_{0,2} = b^+a$
q_0	q_0	q_1	$w^{b}_{0,1}\ w^{b^*}_{1,1}\ w^{\lambda}_{1,0}\ w_{0,0} = \emptyset$
q_2	q_2	q_1	$w^{a}_{2,1}\ w^{b^*}_{1,1}\ w^{a}_{1,2}\ w_{2,2} = ab^*a$
q_2	q_0	q_1	$w^{a}_{2,1}\ w^{b^*}_{1,1}\ w^{\lambda}_{1,0}\ w_{2,1} = \emptyset$





$(b^*((a \cup b^+a)(ab^*a)^*)^*$

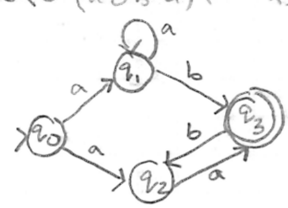
$(b(b^*(a \cup b^+a)(ab^*a)^*)^*)^*$

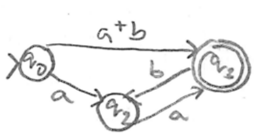
5.40c w eliminate q_1

j	k	i	
q_0	q_0	q_1	$w^{a}_{0,1}\ w^{a^*}_{1,1}\ w^{\lambda}_{1,0}\quad w_{0,0} = \emptyset$
q_0	q_2	q_1	$w^{a}_{0,1}\ w^{a^*}_{1,1}\ w^{\lambda}_{1,2}\quad w_{0,2} = \emptyset$
q_0	q_3	q_1	$w^{a}_{0,1}\ w^{a^*}_{1,1}\ w^{b}_{1,3}\quad w_{0,3} = a^+b$
q_2	q_2	q_1	$w^{\lambda}_{2,1}\ w_{1,1}\ w_{1,2}\ w_{2,2} = \emptyset$
q_2	q_0	q_1	$w^{\lambda}_{2,1}\ w_{1,1}\ w_{1,0}\ w_{2,0} = \emptyset$
q_2	q_3	q_1	$w^{\lambda}_{2,1}\ w_{1,1}\ w_{2,3}\ w_{2,3} = \emptyset$
q_3	q_3	q_1	$w^{\lambda}_{3,1}\ w_{1,1}\ w_{1,3}\ w_{3,3} = \emptyset$
q_3	q_2	q_1	$w^{\lambda}_{3,1}\ w_{1,1}\ w_{1,2}\ w_{3,2} = \emptyset$
q_3	q_0	q_1	$w^{\lambda}_{3,1}\ w_{1,1}\ w_{1,0}\ w_{3,0} = \emptyset$

eliminate q_2

q_0	q_0	q_2	$w^{a}_{0,2}\ w^{\lambda}_{2,2}\ w^{\lambda}_{2,0}\quad w_{0,0} = \emptyset$
q_0	q_3	q_2	$w^{a}_{0,2}\ w^{\lambda}_{2,2}\ w^{a}_{2,3}\quad w_{0,3} = aa$
q_3	q_3	q_2	$w^{b}_{3,2}\ w^{\lambda}_{2,2}\ w^{a}_{2,3}\quad w_{3,3} = ba$
q_3	q_0	q_2	$w^{b}_{3,2}\ w^{\lambda}_{2,2}\ w^{\lambda}_{3,0}\quad w_{3,0} = \emptyset$



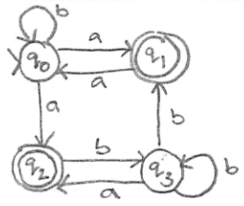


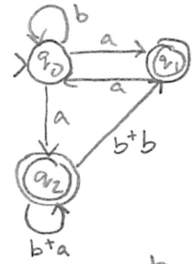
$a^+b \cup aa(ba)^*$

5.40d

w eliminate q_3

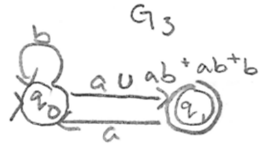
j	k	i				
q_0	q_1	q_3	$w^{\lambda}_{0,3}$	$w_{3,3}$	$w_{3,1}$	$w_{0,1} = \emptyset$
q_0	q_2	q_3	$w^{\lambda}_{0,3}$	$w_{3,3}$	$w_{3,2}$	$w_{0,2} = \emptyset$
q_0	q_0	q_3	$w^{\lambda}_{0,3}$	$w_{3,3}$	$w_{3,0}$	$w_{0,0} = \emptyset$
q_1	q_1	q_3	$w^{\lambda}_{1,3}$	$w_{3,3}$	$w_{3,1}$	$w_{1,1} = \emptyset$
q_1	q_2	q_3	$w^{\lambda}_{1,3}$	$w_{3,3}$	$w_{3,2}$	$w_{1,2} = \emptyset$
q_1	q_0	q_3	$w^{\lambda}_{1,3}$	$w_{3,3}$	$w_{3,0}$	$w_{1,0} = \emptyset$
q_2	q_2	q_3	$w^{b}_{2,3}$	$w^{b^*}_{3,3}$	$w^{a}_{3,2}$	$w_{2,2} = b^+a$
q_2	q_0	q_3	$w^{b}_{2,3}$	$w^{b^*}_{3,3}$	$w^{\lambda}_{3,0}$	$w_{2,0} = \emptyset$
q_2	q_1	q_3	$w^{b}_{2,3}$	$w^{b^*}_{3,3}$	$w^{b}_{3,1}$	$w_{2,1} = b^+b$


G_1


G_2

eliminate q_2 from G_2

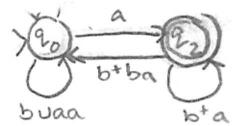
q_0	q_0	q_2	$w^{a}_{0,2}$	$w^{b^+a}_{2,2}$	$w^{\lambda}_{2,0}$	$w_{0,0} = \emptyset$
q_0	q_1	q_2	$w^{a}_{0,2}$	$w^{b^+a}_{2,2}$	$w^{b^+b}_{2,1}$	$w_{0,1} = ab^+ab^+b$
q_1	q_1	q_2	$w^{\lambda}_{1,2}$	$w^{b^+a}_{2,2}$	$w^{b^+b}_{2,1}$	$w_{1,1} = \emptyset$
q_1	q_0	q_2	$w^{\lambda}_{1,2}$	$w^{b^+a}_{2,2}$	$w^{\lambda}_{2,0}$	$w_{1,0} = \emptyset$


G_3

$R_1 =$
$b^*(a \cup ab^+ab^+b)(ab^*(a \cup ab^+ab^+b))$

eliminate q_1 from G_2

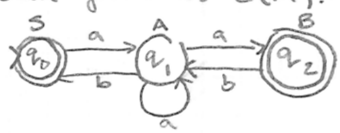
q_0	q_0	q_1	$w^{a}_{0,1}$	$w^{\lambda}_{1,1}$	$w^{a}_{1,0}$	$w_{0,0} = aa$
q_0	q_2	q_1	$w^{a}_{0,1}$	$w^{\lambda}_{1,1}$	$w^{\lambda}_{1,2}$	$w_{0,2} = \emptyset$
q_2	q_2	q_1	$w^{b^+b}_{2,1}$	$w^{\lambda}_{1,1}$	$w^{\lambda}_{1,2}$	$w_{2,2} = \emptyset$
q_2	q_0	q_1	$w^{b^+b}_{2,1}$	$w^{\lambda}_{1,1}$	$w^{a}_{1,0}$	$w_{2,0} = b^+ba$


G_4

$R_2 =$
$(b \cup aa)^* a (b^+a)^* (b^+ba(b \cup aa)^* a (b^+a)^*$

$R_1 \cup R_2$

5. Let M be the NFA
   a) Construct a regular grammar from M that generates L(M).
   b) Give a regular expression for L(M).



$G: S \to aA \mid \lambda$
$A \to aA \mid bS \mid aB$
$B \to bA \mid \lambda$

$R_1 =$
$(a^+b)^*(a^+a(ba^+)^*(ba^+b)(a^+b)^*)$

7. Let L be a regular language over $\{a,b,c\}$. Show that each of the following sets is regular
   a) $\{w \mid w \in L$ and w ends with $aa\}$
   b) $\{w \mid w \in L$ or w contains an $a\}$
   c) $\{w \mid w \notin L$ and w does not contain an $a\}$
   d) $\{uv \mid u \in L$ and $v \notin L\}$

a) $L_1 = (a \cup b \cup c)^* aa \quad L_2 = (a \cup b \cup c)^*$
$L = L_1 \cap L_2$ is regular or $L_0 = L \cap L_1$ is regular

b) $(b \cup c)^*$ describes a string that does not contain an a, therefore $w \in L$;
therefore $L_1 = L \cap (b \cup c)^*$ is regular

c) If L_c describes the set of all strings containing an a because the strings w/o one $\notin L$ then:
L_c
$a^*(b \cup c)^* \cup (b \cup c)^* a^+ \cup (b \cup c)^* a^+ (b \cup c)^* \to L_1 = L \cap L_c$

d) Let $L_1 = u$ and $L_2 = v$
Then $L_d = L_1 \overline{L_2}$ is regular

14. Use the pumping lemma to show that each of the following sets is not regular.
 a) The set of palidromes over $\{a,b\}$
 b) $\{a^n b^m \mid n < m\}$
 c) $\{a^i b^j c^{2j} \mid i \geq 0, j \geq 0\}$
 d) $\{ww \mid w \in \{a,b\}^*\}$
 e) The set of initial sequences of the infinite string

$\Sigma = \{a,b\}$
for $\{z \mid z \in L$ and is a palindrome$\}$ There are k states

$a^{k/2}\, b\, a^{k/2}$ note: z is in L; its a palindrome
z has more than k

$z = uvw$ where
$u = a^{k/2}$ $v = b$ $w = a^{k/2}$ $|vw| \leq k$
$|w| \geq 1$

$a^k b a^k$ to uvw
Let $i = 2$
$uv^2w = a^{k+m} b a^k$
The set of palidromes is not regular

$L_B = \{a^n b^m \mid n < m\}$: intuitively we see that L_B is not regular because it invoves counting the number of states assocated with the LT relation $n < m$ in $a^n b^m$
$z = a^k b^k$ (which is in L) and $|a^k b^k| \geq S$ where $S = \#$ of states
Let $z = uvw$ where $|uv| \leq n \wedge |v| \geq 1$

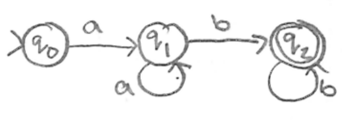
$a^k b^k \in L$ $k \leq k$

$\underbrace{aa\ldots a}_{k}$ $\underbrace{b\ldots b}$ $n = |xyz| = 2k$ $|xy| \leq k$

$w = |\underbrace{xy}_{\leq k}\underbrace{z}_{k}| = 2k$

$a^n b^n$ NR
$a^n b^m$ $n < m$
$[a^n b^n]\, b^j$ $j \geq 1$ $b \in R$
$a^n b^n \notin R$

24. Give the $\equiv_L$ equivalence classes of the language $a^+ b^+$



equiv class		string
$[\lambda]$	=	λ
$[a]$	=	a^+
$[ab]$	=	$a^+ b^+$

27. Use the Myhill-Nerode Theorem to prove that the language $\{a^i \mid i$ is a perfect square$\}$ is not regular.

$S_1 = \{a^i \mid i$ is a perfect square$\}$

let a^k and a^m be arbitrary two different members of the set S_1, where k and m are positive integers and $k \neq m$.

$|y| > 0$, $|xy| \leq n$ and for all $k \geq 0$ the string $xy^k z$ is also in L.

$w = a^k$ or $aaaaa\ldots aa$
$w = \underset{x}{aaaa}\ \underset{y}{a^k}\ \underset{z}{aaa\ldots}$

S_1 should contain the whole equivalence class

So $|xyz| = |xz| + |y| = (n^2 - k) + (k)$
Let $i = 2$
then $|xy^2z| = |xz| + 2|y| = (n^2 - k) + 2k = n^2 + k$
$\Rightarrow n^2 < n^2 + k < (n+1)^2$

8. Prove that the family of regular languages is closed under the operation of set differen
$S_1 = S_2 - S_3$ $\{x \mid x \in S_1 \text{ and } x \notin S_2\}$

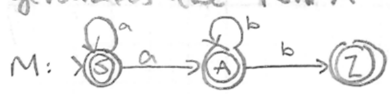
For every pair of regular languages L and L', L − L' is also regular
Let L_1 and L_2 be the set of palindromes over $\{a, b\}$

L_1 and L_2 are not regular
$L_1 - L_2 = \{\}$, which is a regular language
if L_1 is regular, it will contain no irregular strings and it is therefore meaningless to assert $L_r - L_{ir}$ where L_r is regular and L_{ir} is irregular

11. b. Let set $L^R = \{w^R \mid w \in L\}$ of reversals of strings in L.
Choose $w^R \in L$ such that $w^R =$

20. A context-free grammar is called regular if each rule is of the form
$A \to aB$
$A \to a$ — A derivation is terminated by the application of a rule
$A \to \lambda$ — in a form $A \to a$ or $A \to \lambda$

$G: S \to aS \mid aA \mid \lambda$ generates the NFA-M.
$A \to bA \mid b \mid \lambda$



Here it is distinguishable that each regular expression could be accepted by G_M, concluding that right-linear grammars produce regular sets where the regex is
$(a^* b^*)$