Formal Languages | Assignment 5 9, 11, 37 | Kurt Medley | 1

9. Construct a grammar over $\{a,b,c\}$ whose language is $\{a^n b^m c^i \mid 0 \le n+m \le i\}$

$n+m \le i$ S rules

$n+m \ge 0$ λ - cases where $n, m, i = 0$

$i \ge 0$

$S \to ABC \mid \lambda$
$A \to aAC \mid B$
$B \to bBc \mid \lambda$
$C \to Cc \mid \lambda$

no!

if ($n=0, m=1, i=1$) bc; if ($n=1, m=0, i=1$) ac; if ($n=1, m=1, i=1$) abc

acc $S \to ABC \to (aAc)BC \to aAcBC \to a(\lambda)cBC \to ac(\lambda)C \to ac(Cc)$
$\to ac(\lambda)c \to acc$ ✓

bcc $S \to ABC \to (\lambda)BC \to (bBc)C \to b(\lambda)cC \to bc(Cc) \to bc(\lambda)c \to bcc$

aabbcccc $S \to ABC \to (aAC)BC \to a(\lambda)cBC \to X$
$S \to ABC \to (aAc)BC$ X (with nonterminal B as suffix of A rule)

✓
$S \to aSc \mid B$
$B \to bBc \mid C$
$C \to Cc \mid \lambda$

aabbcc $\Rightarrow S \to aSc \to a(aSc)c \to aa(B)cc \to aa(bBc)cc$
$\to aab(bBc)ccc \to aabb(C)cccc \to aabb(\lambda)cccc$
$\to aabbcccc$ where $i >= n+m$

* Rule S allows for $n \ge 0$, where $i \ge n$; the inclusion of terminal c partitioned in between a in S and b in B allows $i \ge n+m$

11. Construct a grammar over $\{a,b\}$ whose language is $\{a^m b^i a^n \mid i = m+n\}$

$S \to abS \mid A$ observations - $m=0, n=1$, then $i=1$; ba
$A \to baA \mid \lambda$ $\to$ derivations - abbaba illegal string

✓
$S \to AB$
$A \to aAb \mid \lambda$ derive $aSb \to a(aSb)b \to aa(aSb)bb \to aaa(A)bbb \to aaa(\lambda)bbb$
$B \to bBa \mid \lambda$ derive $bBa \to b(bBa)a \to bb(\lambda)aa$

sample derivation where $i = m+n$

ab $\Rightarrow S \to AB \to (aAb)B \to a(\lambda)bB \to ab(\lambda) \to ab$

aaabbbbbaa $\Rightarrow S \to AB \to (aAb)B \to a(aAb)bB \to aa(aAb)bbB$
$\to aaa(\lambda)bbbB \to aaabbb(bBa) \to aaabbbb(bBa)a$
$\to aaabbbbb(\lambda)aa \to aaabbbbbaa$ ✓

* The independent rules A preceeding B allows for indepent application of 0 or more aAb, followed by 0 or more bBa, which allows $m \parallel n \ge 0$; thus $i = n+m$.

37. Construct unambiguous grammers for the languages $L_1 = \{a^n b^n c^m \mid n, m > 0\}$ and $L_2 = \{a^n b^m c^m \mid n, m > 0\}$. Construct a grammar G that generates $L_1 \cup L_2$. Prove that G is ambiguous. Ex. of an inherently ambiguous language. Explain intuibively why every grammar generating $L_1 \cup L_2$ must be ambiguous.

Def 3.5.2 A CFG G is ambiguous if there is a string $w \in L(G)$ that can be derived by two distinct leftmost derivations. A grammar that is not ambiguous is called unambiguous

37 (cont) $L_1 = \{a^n b^n c^m \mid n, m > 0\}$

$S \to AB$
$A \to aAb \mid ab$
$B \to cB \mid c$

$abcc \Rightarrow S \to AB \to (ab)B \to ab(cC) \to abc(c) \to abcc$

n is strictly bound to = # of a's and b's > 0; m is disjoint but > 0. This is unambiguous, as there is no 2 ways to generate a string

$L_2 = \{a^n b^m c^m \mid n, m > 0\}$

$S \to AB$
$A \to aA \mid a$
$B \to bBc \mid bc$

$abbcc \Rightarrow S \to AB \to (a)B \to a(bBc) \to ab(bc)c \to abbcc$

m is strictly bound to = # of b's preceeding c's as the suffix of 1 + # of a's. There is only one derivation possible therefore L_2 is unambiguous.

$L_1 \cup L_2$ $S \to AB$
$A \to aAb \mid aA \mid ab \mid a$
$B \to bBc \mid cB \mid bc \mid c$

Let $L_1 \cup L_2 = \{z \mid z \in L_1 \text{ or } z \in L_2\}$ where z is a rule

Sample derivation: $aaabbcc \Rightarrow S \to AB \to (aA)B \to a(aA)B \to aa(ab)B \to aaab(bBc) \to aaabb(c)c \to aaabbcc$ ①

$aaabbcc \Rightarrow S \to AB \to (aA)B \to a(aAb)B \to aa(a)B \to aaa(bBc) \to aaab(bc)c \to aaabbcc$ ②

* clearly $L_1 \cup L_2$ is ambiguous, as we have derived aaabbcc in two different ways. The rules governed by $L_1 \cup L_2$ will always produce more than one way to derive $a^+b^*c^+$.

Find a CFG over $\{a, b\}$ that generates the language consisting of strings that have twice as many a's as b's and prove your grammar correct.

$S \to aabS \mid Sbaa \mid bSaa \mid$ ~~$S \to AB \to (bAaa)B \to b(bAaa)aaB \to bb(aab$~~
$aab \mid baa \mid aba \mid baSa$

$S \to aabS \to aab(bSaa) \to aabb(aab)aa \to aabbaabaa$

For $S \to aabS \mid Sbaa \mid bSaa \mid baSa \mid$
$aab \mid baa \mid aba$

$w \in L$ $a^{2n}b^n$

$S \overset{n-1}{\to} (aab)^{n-1}S \mid S(baa)^{n-1} \mid b^{n-1}S(aa)^{n-1} \mid (ba)^{n-1}Sa^{n-1}$

$S \to (aa)^n(b)^n \mid (b)^n(aa)^n \mid a^n b^n a^n$

$\Downarrow$ $\Downarrow$ $\Downarrow$

$a^{2n}b^n$ $b^n a^{2n}$ $a^n b^n a^n$

$n_a(w) = 2n_b(w)$ a's > b's
$n > \emptyset$

The application of any recursive S rule gives
$n_a(w) + 1 = 2n_b(w) + 1$
and therefore $L(G) \subseteq L$