

## Lesson 2: Put a Label on That Number!

What would you do if your mother approached you, and, in an earnest tone, said,

“Honey.”

“Yes,” you replied.

“One million.”

“Excuse me?”

“One million,” she affirmed.

“One million what?” you inquired.

“Nothing more,” she insisted, “just one million.”

What would you make of this? What would you do? How would you respond?

“DAD!”

Numbers and math are wonderful, but they are useless or even misleading unless we know their relevance. For years now, you have been solving math problems. You started out with  $1 + 1 = 2$ . But this is just arithmetic with no application. However, if we were to say 1 centimeter + 1 centimeter = 2 centimeters, the application would be obvious. *Now* we understand the usefulness of the math. It has a purpose in determining the result of a real problem. All we have done is added **labels** to the numbers and suddenly we have gone from an academic exercise to solving a practical problem. Why is this important? It’s because, in science, no math is meaningless. It always has purpose.

Doing math with meaningful numbers is a little different than doing math with numbers alone. Notice in the simple “ $1 + 1$ ” problem from the previous paragraph that every number in the equation is made meaningful with a label. These labels are critical because, without them, numbers may take on entirely different meanings. For example:

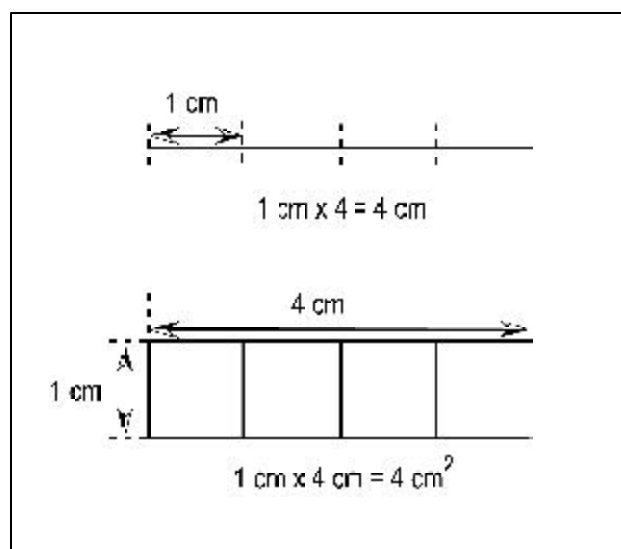
$$1 \text{ cm} \times 4 = 4 \text{ cm}$$

This is just the length of 1 cm multiplied by 4. It answers the question, “What is four times the length of a one-centimeter line on a piece of graph paper?” The

answer is 4 cm. This answer is considerably different from that obtained in the following example:

$$1 \text{ cm} \times 4 \text{ cm} =$$

Notice the only thing that has changed is the insertion of a label on the number four. But what does this do to the answer? Is the answer still 4 cm? No. The answer is no longer just 4 cm, but  $4 \text{ cm}^2$ . This answers the question, “What is the area of a rectangle having one side that is 1 cm long and another side that is 4 cm long?” The following diagram illustrates the difference between the two equations.



*Notice how the simple addition of a label changes the overall meaning of this equation. The first equation has to do only with distance along a straight line, while the second has to do with the area of a rectangle.*

This very subtle difference in the equation will determine whether you arrive at the right answer or the wrong one, so don’t forget it, and never leave off a label accidentally. If a number is left without a label, it must be intentional.

Every number that stands for a count or measure of some quantity has a label that describes the quantity that’s being counted or measured. From now on, we may refer to the label or to the standard of measure it represents (be it meters, grams, liters or any other) as the “**unit of measure**” or just the “**unit**.”

## Adding and Subtracting with Units

When doing meaningful math with labeled numbers, the math itself has to make sense. The following rule governs addition and subtraction using labels:

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Numbers with the same label can be added or subtracted, while numbers with different labels cannot unless the labels are first made to match.

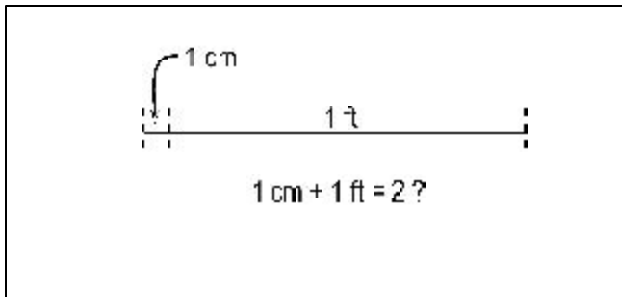
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This is not just an arbitrary, senseless rule. If you think about it, it makes plenty of sense. Just consider the following two problems:

$$1 \text{ cm} + 1 \text{ cm} =$$

$$1 \text{ cm} + 1 \text{ ft} =$$

You can easily add one centimeter and one centimeter to get two centimeters, but what is one centimeter plus one foot? Not only is it impossible to add the labels, but you can no longer add the numbers because those numbers relate to different kinds of things. This is a clear case where one plus one does not simply equal two!



Subtraction and addition are alike when using units. If the quantities are expressed in the same units, you can perform either of these operations. The units in the result will be the same as those in the problem. However, if the quantities are expressed in different units, you can neither add nor subtract them, because there is no name for the result. Notice that the rule says, "...unless the labels are first made to match." In just a while we'll teach you how this can be done, but first we have to learn about multiplying and dividing with labels.

## Multiplying and Dividing with Units

A second reasonable rule governs multiplying and dividing with units:

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To multiply or divide with units, perform the same operation on the labels that you perform on the numbers.

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For a problem such as this one:

$$2 \text{ m} \times 2 \text{ m} =$$

you would first multiply the numbers:

$$2 \times 2$$

Then multiply the units:

$$\text{m} \times \text{m}$$

The completed equation, then, is:

If there is no label on a particular number, the unit of measure is treated as if it were multiplied by one. Of course, anything multiplied by one is unchanged:

One simple way you can work with units confidently is to treat the unit as you would treat an alphabetical variable in an algebraic equation. For example, in the following problem:

$$1 \text{ cm} \times 4 \text{ cm} = 4 \text{ cm}^2$$

replace the cm label with the variable  $x$ :

$$1x \times 4x = 4x^2$$

You see? This latter problem using the variable  $x$  is mathematically equivalent to the previous one having units of cm. [Notice our practice of writing variables in italics to help you distinguish algebraic variables (such as  $x$ ) from units (such as cm).]

The rules of addition and subtraction applied to labels are the same as the rules applied to variables. Just as you could not add two different labels, you cannot add two different variables. For example, we already said that you couldn't perform the following addition:

$$1 \text{ cm} + 1 \text{ ft} =$$

Similarly, you cannot do anything meaningful with:

$$1x + 1y =$$

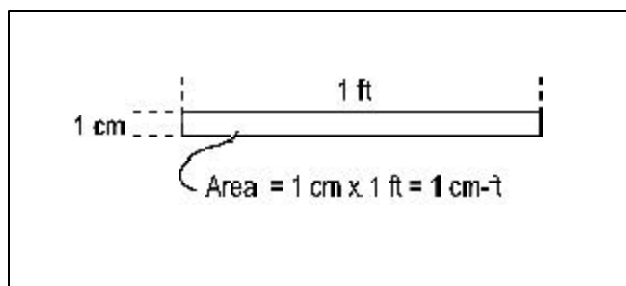
*But, you can multiply* variables that are different, such as:

$$1x \times 1y = 1xy$$

and you *can* (believe it or not) multiply different labels:

$$1 \text{ cm} \times 1 \text{ ft} = 1 \text{ cm-ft}$$

(Unfortunately, the hyphen is often used to show that the units are multiplied. Don't confuse it with a minus sign.) But what *is* a cm-ft? Although it is certainly not a unit of measure that we use every day, it does have meaning. It is the unit of area of a rectangle having one side that is one centimeter in length and an adjacent side that is one foot in length, as shown below.



However, it's such an oddity that we would really prefer to express it in units that are more easily understood. We'll learn to do that in the next lesson.

Similarly, labels are treated just like variables when dividing. Consider the following algebra problem:

$$\frac{1z}{2y} \cdot 14y =$$

You know that you can cancel the  $y$ 's because  $y/y$  is equal to 1. If we replace these with 1, we get:

$$\frac{1z}{2\cancel{y}} \cdot 14\cancel{y} = \frac{z}{2} \cdot 14 = 7z$$

The same is true for the following problem, where  $s$  is the abbreviation for second (the unit of time):

$$\frac{1 \text{ cm}}{2\cancel{s}} \cdot 14\cancel{s} = 7 \text{ cm}$$

Just as  $y/y = 1$ , so it is that second/second = 1.

This equation answers the question, "If an ant traveling at a velocity of one centimeter every two seconds continued traveling for fourteen seconds, what distance would it have traveled?"

Shortly, you'll be asked to complete a set of exercises. To be successful, you will have to remember how to do math with exponents. You must remember that:

$$1/m^2 = m^{-2}$$

and that:

$$y^3/y^2 = y.$$

If you don't understand these two algebraic statements, you will need to review your algebra before continuing.

As with variables, there are several ways to show multiplication of units. Common symbols include the hyphen (as in ft-lb), the asterisk (ft\*lb), the dot (ft·lb), and the "times" sign (ft×lb). Variables are often written side by side to show that they are multiplied (as in  $xy$ ). However, if units are written side by side, you have to leave a space between them so that the reader knows you mean to say "ft lb" instead of "ftlb."

## Exercises

Please perform the specified operation on the labeled numbers in the following exercises and express the answers in their correct units.



*Note: Write the problem, and then write out the solution step by step. This will allow others to check your work. When you review your work, even years later, you will be reminded of your thoughts and will see exactly what you did. Those who don't show their work make mistakes frequently and often fail chemistry courses. For any person looking forward to an occupation in science, showing all of your thoughts on paper will be absolutely necessary.*

(Throughout these exercises, I'll be introducing you to some new units. I don't expect you to memorize or understand them; I just want you to realize that they are not made up. They're for real! I also want you to get used to looking at ugly units. Once you get used to looking at them, you'll realize that they are harmless. Commonly, something as simple as ugly units may scare away a good student. In our curriculum we use only friendly units that don't bite.)

1)  $0.294 \text{ rad} + 0.179 \text{ rad} =$

("rad" is the abbreviation for radian—a measure of distance along a curve.)

2)  $660 \text{ Hz} + 990 \text{ Hz} =$

("Hz" is the abbreviation for hertz—a unit of wave frequency.)

3)  $20.02 \text{ N} + 21.45 \text{ N} =$

("N" is the abbreviation for newton—a unit of force.)

4)  $42 \text{ Pa} - 30 \text{ Pa} =$

("Pa" is the abbreviation for pascal—a unit of pressure.)

5)  $0.025 \text{ J} - 0.003 \text{ J} =$

("J" is the abbreviation for joule—a unit of energy.)

6)  $125 \text{ W} - 100 \text{ W} =$

("W" is the abbreviation for watt—a unit of power.)

7)  $22 \text{ g} \times 2 =$

("g" is the abbreviation for gram—a unit of mass.)

8)  $5 \times 46 \text{ s} =$

["s" is the abbreviation for second—a unit of time (duh) and also a unit of angular distance.]

9)  $0.7 \text{ mm} \times 0.5 \text{ mm} =$

("mm" is the abbreviation for millimeter—a unit of distance equal to 1/1000 meter.)

10)  $3 \text{ cm} \times 34 \text{ cm} \times 92 \text{ cm} =$

("cm" is the abbreviation for centimeter—a unit of distance equal to 1/100 meter.)

11)  $7.402 \text{ m}^2 / 9.706 \text{ m} =$

("m" is the abbreviation for meter.)

12)  $37,546 \text{ cd} \div 2 =$

("cd" is the abbreviation for candela—a unit of light intensity.)

13)  $25 \text{ g} / 16 \text{ g} =$

14)  $4 \text{ mg} / 0.5 \text{ kg} =$

(“mg” is the abbreviation for milligram—a unit of mass equal to 1/1000 g. “kg” is the abbreviation for kilogram, a unit of mass equal to 1000 g.)

15)  $25 \frac{\text{m}^2 \text{ kg}}{\text{s}^2} \cdot 5 \frac{1}{\text{kg}} =$

workspace


16)  $1.064 \frac{\text{m kg}}{\text{s}^2} \times 32.51 \text{ s} =$


17)  $1 \text{ s}^4 \text{ A}^2 \text{ m}^2 \text{ kg}^{-1} \times 1 \text{ m}^2 \text{ kg s}^{-3} \text{ A}^{-2} =$

[The unit “A” is ampere—a unit of electric current. The first of these quantities (the one having the units  $\text{s}^4 \text{ A}^2 \text{ m}^2 \text{ kg}^{-1}$ ) is a measure of electrical capacitance (surface charge). These ugly units are usually replaced with a single unit called *farad* (F). The second of these quantities (the one having the units  $\text{m}^2 \text{ kg s}^{-3} \text{ A}^{-2}$ ) is a measure of electrical resistance. The units are usually replaced with the simple unit called *ohm* (W). Incidentally, it has nothing to do with meditation.]


18)  $80,100 \text{ m}^2 \text{ kg s}^{-3} \text{ A}^{-1} \div 75 \text{ m}^2 \text{ kg s}^{-3} \text{ A}^{-1} =$

[The units,  $\text{m}^2 \text{ kg s}^{-3} \text{ A}^{-1}$ , are units of electrical potential and are equivalent to volt (V).]
