

Exercise Sheet 10

Exercise 1: Multiple Kernel Learning (25 P)

Let $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^d$ be two data points. Consider a list of positive semi-definite kernels $(k_l(\mathbf{x}, \mathbf{x}'))_{l=1}^L$. Each of these kernels have an associated feature map $\phi_l : \mathbb{R}^d \rightarrow \mathbb{R}^h$, such that

$$k_l(\mathbf{x}, \mathbf{x}') = \phi_l(\mathbf{x})^T \phi_l(\mathbf{x}').$$

We now consider an MKL kernel defined as:

$$k(\mathbf{x}, \mathbf{x}') = \sum_{l=1}^L \beta_l k_l(\mathbf{x}, \mathbf{x}'),$$

where β_1, \dots, β_L is a list of parameters to be learned.

- (a) *Show* that when $\beta_1, \dots, \beta_L \geq 0$, then the kernel $k(\mathbf{x}, \mathbf{x}')$ is positive semi-definite.
- (b) *Find* a feature map $\phi(\mathbf{x})$ associated to this kernel, such that

$$k(\mathbf{x}, \mathbf{x}') = \phi(\mathbf{x})^\top \phi(\mathbf{x}')$$

for all \mathbf{x}, \mathbf{x}' .

Exercise 2: Structured Output Kernels (25 P)

Let $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^d$ be two data points and $y, y' \in \{1, \dots, C\}$ their respective class, with C the number of classes. Consider the structured output kernel

$$k_{\text{struct}}((\mathbf{x}, y), (\mathbf{x}', y')) = k(\mathbf{x}, \mathbf{x}') \cdot 1_{[y=y']},$$

where $k(\mathbf{x}, \mathbf{x}')$ is a positive semi-definite kernel, with associated feature map $\phi(\mathbf{x})$.

- (a) *Show* that the kernel $k_{\text{struct}}((\mathbf{x}, y), (\mathbf{x}', y'))$ is positive semi-definite.
- (b) *Find* a feature map $\phi_{\text{struct}}(\mathbf{x}, y)$ associated this kernel, such that

$$k_{\text{struct}}((\mathbf{x}, y), (\mathbf{x}', y')) = \phi_{\text{struct}}(\mathbf{x}, y)^T \phi_{\text{struct}}(\mathbf{x}', y')$$

for all (\mathbf{x}, y) and (\mathbf{x}', y') .

Exercise 3: Programming Exercise (50 P)

Download the code for Exercise sheet 10 on ISIS and follow the instructions.