Exercise Sheet 10

Machine Learning 2, SS16

July 11, 2016

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Exercise 1a

Let $\vec{x}_1, \ldots, \vec{x}_n \in \mathcal{X}$ be arbitrary samples and $\vec{v} \in \mathbb{R}^n$ be an arbitrary vector.

$$\forall_{l=1}^{L}. \sum_{i,j=1}^{n} v_i v_j k_l(x_i, x_j) \geq 0 \Longrightarrow \sum_{l=1}^{L} \sum_{i,j=1}^{n} v_i v_j k_l(x_i, x_j) \geq 0 \Longrightarrow \sum_{i,j=1}^{n} v_i v_j \sum_{l=1}^{L} k_l(x_i, x_j) \geq 0$$

$$\overset{\vec{\beta} \geqq 0}{\Longrightarrow} \sum_{i,j=1}^{n} v_i v_j \sum_{l=1}^{L} \beta_l k_l(x_i, x_j) \geq 0 \overset{\text{Def. k}}{\Longrightarrow} \sum_{i,j=1}^{n} v_i v_j k(x_i, x_j) \geq 0$$

We could also proof this for all pairs of datapoints x, x' out of $\mathcal{X}$, but then the unfold of the definition of positive definiteness wouldn't be that lucid.

Exercise 1b

Let $x, x' \in \mathcal{X}$ be two arbitrary samples.

$$\sum_{l=1}^{L} \beta_l k_l(x, x') = \sum_{l=1}^{L} \beta_l \phi_l(x)^\top \phi_l(x') = \sum_{l=1}^{L} (\sqrt{\beta_l} \phi_l(x)^\top)(\sqrt{\beta_l} \phi_l(x'))$$

$$= \underbrace{\begin{bmatrix} \sqrt{\beta_1}\phi_1(x) & \ldots & \sqrt{\beta_L}\phi_L(x) \end{bmatrix}}_{\phi(x)^\top} \underbrace{\begin{bmatrix} \sqrt{\beta_1}\phi_1(x') \\ \ldots \\ \sqrt{\beta_L}\phi_L(x') \end{bmatrix}}_{\phi(x')}$$

$\phi(x)^\top$ and $\phi(x')$ are block partitioned matrices, i.e. the $\phi_l's$ are simply concatenated together in one very long vector. So, the result is: $\phi(x) = \begin{bmatrix} \sqrt{\beta_1}\phi_1(x) & \ldots & \sqrt{\beta_L}\phi_L(x) \end{bmatrix}^\top$.