

# Exercise Sheet 4

Machine Learning 2, SS16

May 18, 2016

Mario Tambos, 380599; Viktor Jeney, 348969; Sascha Huk, 321249; Jan Tinapp, 0380549

## Exercise 1 - Sparse Coding

(a)

$$\begin{aligned}\frac{\partial E}{\partial W} &= \frac{\partial}{\partial W} \eta \|W\|_F^2 + \frac{\partial}{\partial W} \sum_{i=1}^N (|x^{(i)} - W s^{(i)}|^2 + \lambda |s^{(i)}|_1) \\ &= \eta \sum_l^d \sum_k^h \frac{\partial}{\partial W} (W_{lk})^2 + \sum_{i=1}^N \frac{\partial}{\partial W} (x^{(i)} - W s^{(i)})^\top (x^{(i)} - W s^{(i)}) \\ &= 2\eta W + \sum_{i=1}^N -2(x^{(i)} - W s^{(i)}) s^{(i)\top} = 2\eta W - 2 \sum_{i=1}^N (x^{(i)} - W s^{(i)}) s^{(i)\top}\end{aligned}$$

(b)

$$\begin{aligned}\frac{\partial E}{\partial s^{(i)}} &= \frac{\partial}{\partial s^{(i)}} \eta \|W\|_F^2 + \frac{\partial}{\partial s^{(i)}} \sum_{j=1}^N (|x^{(j)} - W s^{(j)}|^2 + \lambda |s^{(j)}|_1) \\ &= \frac{\partial}{\partial s^{(i)}} (x^{(i)} - W s^{(i)})^\top (x^{(i)} - W s^{(i)}) + \frac{\partial}{\partial s^{(i)}} \lambda |s^{(i)}|_1 \\ &= -2W^\top (x^{(i)} - W s^{(i)}) + \lambda \sum_{k=1}^h \frac{\partial}{\partial s^{(i)}} |s^{(i)}_k| \\ &= -2W^\top (x^{(i)} - W s^{(i)}) + \lambda \left( \frac{s^{(i)}_1}{|s^{(i)}_1|}, \dots, \frac{s^{(i)}_h}{|s^{(i)}_h|} \right)^\top\end{aligned}$$