Sheet 11

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$$\Lambda = R^2 + \frac{1}{n\nu}\sum_{i=1}^{n} \xi_i + \sum_{i=1}^{n} \alpha_i \left(\|\phi(x_i) - c\|^2 - R^2 - \xi_i \right)$$

$$\min_{R,c,\xi_i} \max_{\alpha_i} \Lambda = \max_{\alpha_i} \min_{R,c,\xi_i} \Lambda$$

$$\frac{d\Lambda}{dR} = 2R - 2R\sum_{i=1}^{n}\alpha_i = 2\left(1 - \sum_{i=1}^{n}\alpha_i\right)R \overset{!}{=} 0 \Rightarrow \underline{\underline{\sum_{i=1}^{n}\alpha_i = 1}}$$

$$\frac{d\Lambda}{dc} = \sum_{i=1}^{n}\alpha_i \underbrace{\frac{d}{dc}\|\phi(x_i) - c\|^2}$$

$$= \frac{d}{dc}\left(\phi(x_i)^T\phi(x_i) - 2\,\phi(x_i)^T c + c^T c\right)$$

$$= 2c - 2\phi(x_i)$$

$$= 2\left(c - \phi(x_i)\right)$$

$$= 2c\sum_{i=1}^{n}\alpha_i - 2\sum_{i=1}^{n}\alpha_i\,\phi(x_i) = 2\left(c - \sum_{i=1}^{n}\alpha_i\phi(x_i)\right) \overset{!}{=} 0$$

$$\Rightarrow \underline{\underline{\sum_{i=1}^{n}\alpha_i\,\phi(x_i) = c}}$$

$$\frac{d\Lambda}{d\xi_k} = \frac{1}{n\nu} - \alpha_k = 0 \Rightarrow \alpha_k = \frac{1}{n\nu} \quad \forall k \ ???$$

$$\|\phi(x_i) - c\|^2 = \left(\phi(x_i) - \sum\alpha_j\phi(x_j)\right)^T\left(\phi(x_i) - \dots\right)$$

$$= \underbrace{\phi(x_i)^T\phi(x_i)}_{=k(x_i,x_i)} - \underbrace{\phi(x_i)^T\sum\dots - \left(\sum\dots\right)^T\phi(x_i)}_{=2\sum_k\alpha_k k(x_k,x_i)} + \underbrace{\left(\sum_k\alpha_k\phi(x_k)\right)^T\left(\sum\alpha_l\dots\right)}_{=\sum_k\sum_l\alpha_k\alpha_l k(x_k,x_l)}$$

$$\le R^2 + \xi_i$$

2) $P = \left[-k(x_i, x_j) \right]_{i,j=1,\dots,n}$ $\qquad q = \left[k(x_i, x_i) \right]_{i=1,\dots,n}$

$$A = \underbrace{\left[\begin{array}{c|ccc|c} 1 & - & - & - & 1 \\ \hline \phi(x_1) & - & - & - & \phi(x_n) \\ & & & & \end{array}\right]}_{n} \Big\} d+1 \qquad b = \left[\begin{array}{c} 1 \\ \hline c \end{array}\right] \in \mathbb{R}^{d+1}$$

<u>**check:**</u>

$$A\alpha = \begin{bmatrix} \sum \alpha_i \\ \sum \alpha_i \phi(x_i)_1 \\ \sum \alpha_i \phi(x_i)_2 \\ \vdots \\ \sum \alpha_i \phi(x_i)_d \end{bmatrix} \overset{!}{=} \left[\begin{array}{c} \sum \alpha_i \\ \hline \sum_i \alpha_i \phi(x_i) \end{array}\right]$$

$$G = \underbrace{\left[\begin{array}{c} I_n \\ \hline -I_n \end{array}\right]}_{n} \Big\} 2n \qquad h = \begin{bmatrix} 1/n\nu \\ \vdots \\ 1/n\nu \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \Big\} n \\ \Big\} n \end{array} \in \mathbb{R}^{2n}$$