

Exercise Sheet 9

Exercise 1: Hidden Markov Models (10+10+20 P)

Consider the Hidden Markov Model with transition matrix

$$A = \begin{pmatrix} 0.1 & 0.9 \\ 0.5 & 0.5 \end{pmatrix}$$

and emission matrix

$$B = \begin{pmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{pmatrix}$$

The initial probability vector is

$$\pi = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

- (a) *Draw* the graph of the model. As usual, use round shapes for hidden states and square shapes for visible states, and arrows for transitions/emissions; include transition/emission probabilities.
- (b) We interpret the above as a model for an experiment with two hidden (possibly unfair) coins and two visible coins. *Describe* such an experiment which can be modeled by the Markov model given above. Here, heads should correspond to the first indices in A, b, π , tails to the second.
- (c) Using the Bayes formula for conditional probabilities, *compute* for the model above the probability distribution

$$P((q_1, q_2) \mid (O_1, O_2) = (\text{tails}, \text{tails})),$$

where $(q_1, q_2) \in S \times S$ is a sequence of states of length two and $S = \{S_1, S_2\}$ is the set of symbols of the hidden Markov model. That is, for all possible sequences (q_1, q_2) , compute the probability that it were at the origin of the sequence of observations (tails, tails).

Exercise 2: Programming Exercise (60 P)

Download the IPython notebook on ISIS and follow the instructions.