Exercise Sheet 8

Machine Learning 2, SS16

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Exercise 2

$$y_t = (w * x)_t = \sum_{s \in \mathbb{Z}} w_s x_{t-s}$$

(a)

$$\begin{aligned}
\frac{\partial E}{\partial x_k} &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial y_t}{\partial x_k} \\
&= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial}{\partial x_k} (w * x)_t \\
&= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s \in \mathbb{Z}} w_s \frac{\partial}{\partial x_k} x_{t-s} \\
&= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s \in \mathbb{Z}} w_s \mathbb{1}_{\{t-s=k\}} \\
&= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} w_{-k+t} \\
&= \left[\frac{\partial E}{\partial y} \star w \right]_{-k}
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{\partial E}{\partial w_k} &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial y_t}{\partial w_k} \\
&= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial}{\partial w_k} (w * x)_t \\
&= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s \in \mathbb{Z}} x_{t-s} \frac{\partial}{\partial w_k} w_s \\
&= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s \in \mathbb{Z}} x_{t-s} \mathbb{1}_{\{s=k\}} \\
&= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} x_{-k+t} \\
&= \left[\frac{\partial E}{\partial y} \star x \right]_{-k}
\end{aligned}$$

Exercise 3

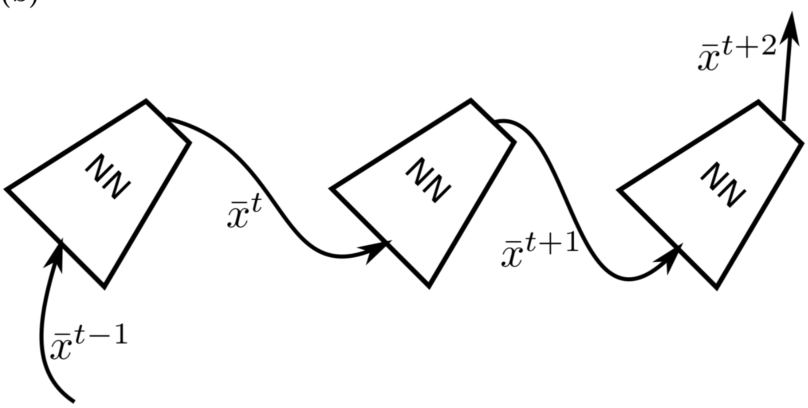
(a)
Euler discretization gives us for every $j = 1, .., d$:

$$x_j^t - x_j^{t-1} = 0.1(\tanh(\sum_{i=1}^{d} x_i^{t-1} w_{ij} + b_j) - x_j^{t-1})$$

The transition function is then component wise defined as

$$\Theta(x)_j = 0.1(\tanh(\sum_{i=1}^{d} x_i w_{ij} + b_j) - x_j) + x_j$$

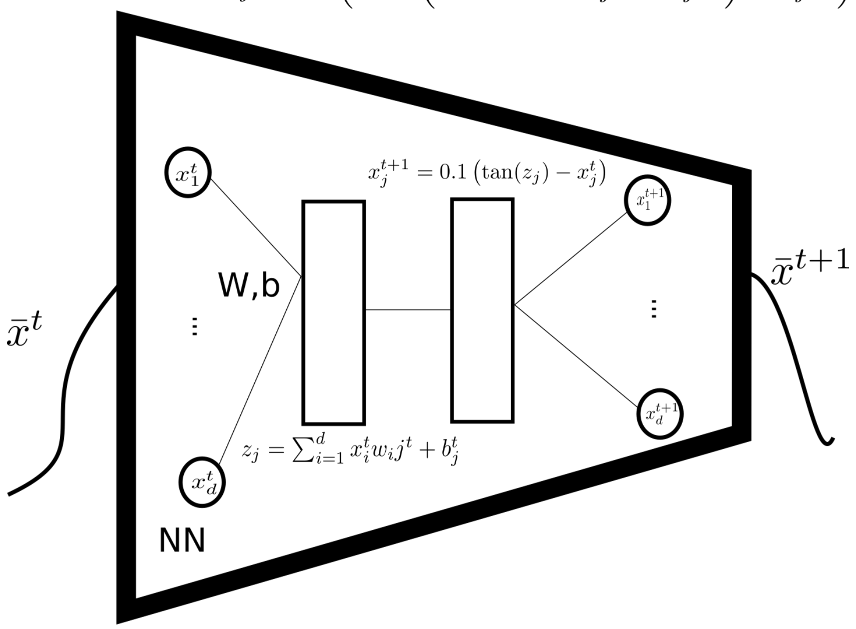
(b)



$\bar{x}^t = [x_1^t, \ldots, x_d^t]^T$

$\forall j \in (1, d) x_j^0 \rightarrow$ initial input

$\forall j \in (1, d), t > 0 x_j^t = 0.1 \left(\tanh\left(\sum_{i=1}^{d} x_i^{t-1} w_{ij}^{t-1} - b_j^{t-1}\right) - x_j^{t-1}\right)$



(c)

if you

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\begin{aligned}
\frac{\partial x_i^t}{\partial x_j^{t-1}} &= 0.1\frac{\partial}{\partial x_j^{t-1}}(\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i) - x_i^{t-1}) + \frac{\partial x_i^{t-1}}{\partial x_j^{t-1}} \\
&= 0.1\frac{\partial}{\partial x_j^{t-1}}(\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i)) + 0.9\mathbb{1}_{\{i=j\}} \\
&= 0.1(1-\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i))\sum_{k=1}^{d}\frac{\partial}{\partial x_j^{t-1}}x_k^{t-1}w_{ki} + 0.9\mathbb{1}_{\{i=j\}} \\
&= 0.1(1-\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i))\sum_{k=1}^{d}\mathbb{1}_{\{k=j\}}w_{ki} + 0.9\mathbb{1}_{\{i=j\}} \\
&= 0.1(1-\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i))w_{ji} + 0.9\mathbb{1}_{\{i=j\}}
\end{aligned}
$$

(d)

$$
\begin{aligned}
\frac{\partial x_i^t}{\partial b_j} &= 0.1\frac{\partial}{\partial b_j}\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i) \\
&= 0.1(1-\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i))\frac{\partial b_i}{\partial b_j} \\
&= 0.1(1-\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i))\mathbb{1}_{\{i=j\}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial x_i^t}{\partial w_{jl}} &= 0.1\frac{\partial}{\partial w_{jl}}\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i) \\
&= 0.1(1-\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i))\sum_{k=1}^{d} x_k^{t-1}\frac{\partial}{\partial w_{jl}}w_{ki} \\
&= 0.1(1-\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i))\sum_{k=1}^{d} x_k^{t-1}\mathbb{1}_{\{j=k\}}\mathbb{1}_{\{l=i\}} \\
&= 0.1(1-\tanh(\sum_{k=1}^{d} x_k^{t-1}w_{ki} + b_i))x_j^{t-1}\mathbb{1}_{\{l=i\}}
\end{aligned}
$$