Exercise Sheet 6

Machine Learning 2, SS16

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Exercise 1 - RBM with Ternary Hidden Units

Model: $(x_i \perp\!\!\!\perp x_j|h)$ and $(h_i \perp\!\!\!\perp h_j|x)$ for all $i \neq j$.

(a) We have to show:

$$\sum_{h\in\{-1,0,1\}^N} e^{x^\top a + x^\top Wh + h^\top b} \quad \propto \quad e^{x^\top a + \sum_{j=1}^N \log(\frac{1}{2} + \cosh(w_j^\top x + b_j))}$$

Proof. Devided both sides by $\exp(x^\top a)$ and wrote down the multiplication by h more explicitly:

$$\sum_{h\in\{-1,0,1\}^N} e^{\sum_{j=1}^N h_j w_j^\top x + h_j b_j} \quad \propto \quad e^{\sum_{j=1}^N \log(\frac{1}{2} + \cosh(w_j^\top x + b_j))}$$

It follows by laws of exponentiation:

$$\sum_{h\in\{-1,0,1\}^N} \prod_{j=1}^N e^{h_j w_j^\top x + h_j b_j} \quad \propto \quad \cancel{\exp(\log} \prod_{j=1}^N (\frac{1}{2} + \cosh(w_j^\top x + b_j))$$

This expressions are still equivalent (*):

$$\prod_{j=1}^N \sum_{h_j\in\{-1,0,1\}} e^{h_j w_j^\top x + h_j b_j} \quad \propto \quad \prod_{j=1}^N \frac{(1 + 2\cosh(w_j^\top x + b_j))}{2}$$

Unfold the three cases on the left side and pull out the constant factor on the right side:

$$\prod_{j=1}^N \left(e^{w_j^\top x + b_j} + e^{-w_j^\top x - b_j} + 1\right) \quad \propto \quad \frac{1}{2^N} \prod_{j=1}^N (1 + 2\cosh(w_j^\top x + b_j))$$

Use definition of cosh on the left side:

$$\prod_{j=1}^N \left(2\cosh(w_j^\top x + b_j) + 1\right) \quad \propto \quad \frac{1}{2^N} \prod_{j=1}^N (1 + 2\cosh(w_j^\top x + b_j))$$

□

(c)

$\Pr(x) = \frac{1}{Z} \prod_{j=1}^{N} \exp(-W_j x + a^\top x - b_j) + \exp(W_j x + a^\top x + b_j) + \exp(a^\top x)$
$\Pr(x) = \frac{1}{Z} \exp(a^\top x) \prod_{j=1}^{N} \exp(-W_j x - b_j) + \exp(W_j x + b_j) + 1$

$$\Pr(x) = \sum_{h \in \{-1,0,1\}^N} e^{x^\top a + x^\top W h + h^\top b}$$