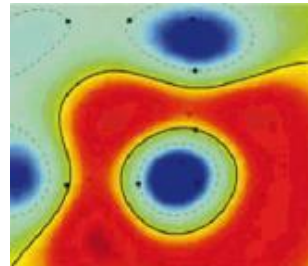


# Dimension Reduction & Visualization

LLE

---

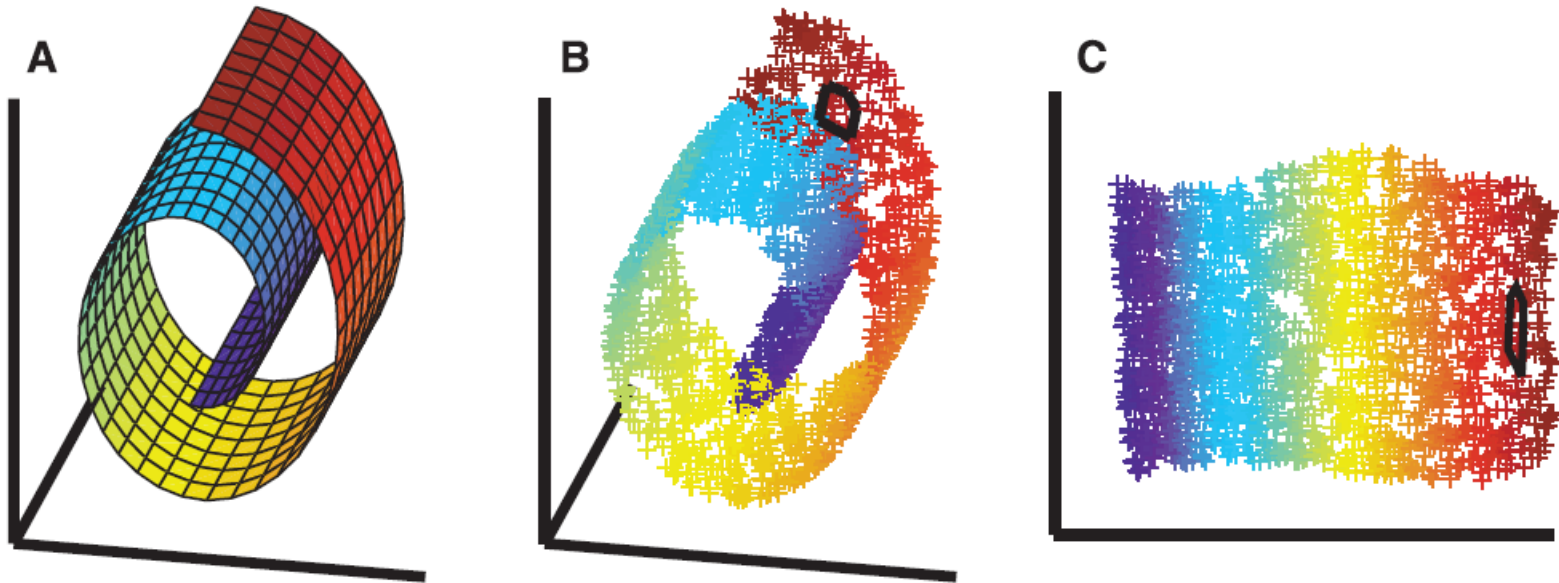


---

Lecture by Klaus-Robert Müller, TUB 2014

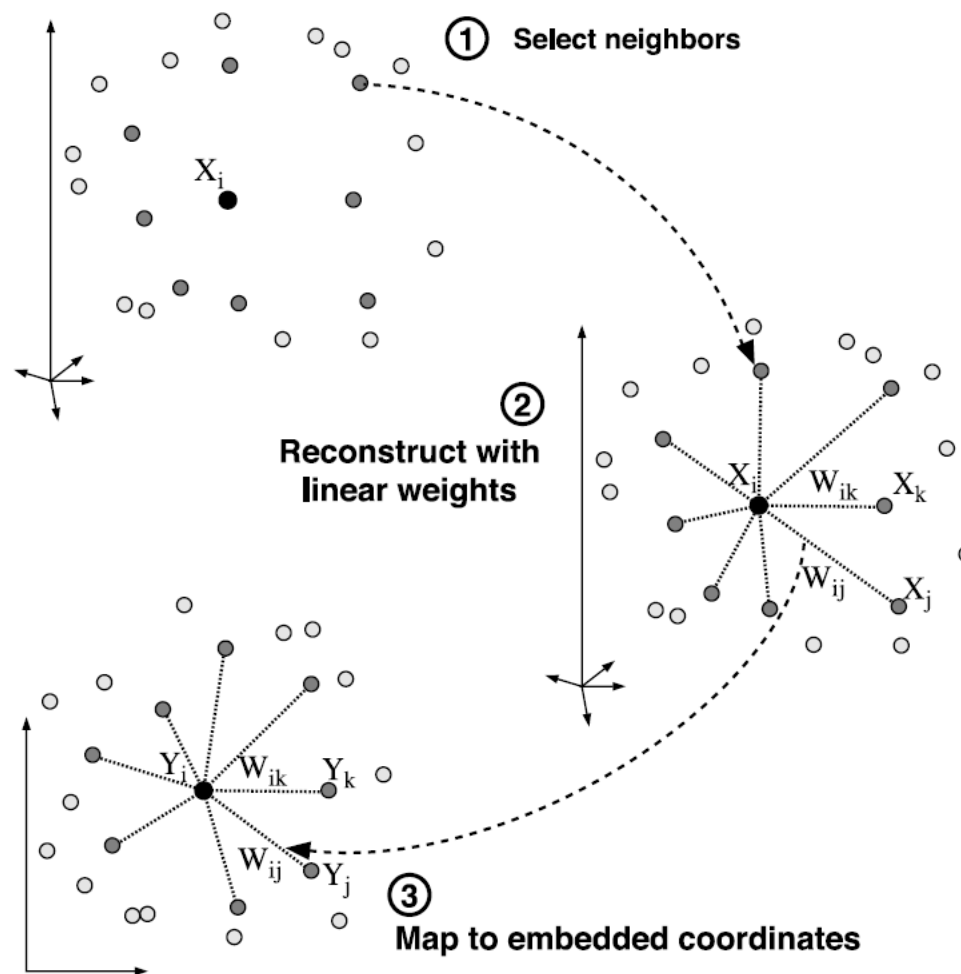
# Local Linear embedding I

---



# Local Linear embedding II

**Fig. 2.** Steps of locally linear embedding: (1) Assign neighbors to each data point  $\tilde{X}_i$  (for example by using the  $K$  nearest neighbors). (2) Compute the weights  $W_{ij}$  that best linearly reconstruct  $\tilde{X}_i$  from its neighbors, solving the constrained least-squares problem in Eq. 1. (3) Compute the low-dimensional embedding vectors  $\tilde{Y}_i$  best reconstructed by  $W_{ij}$ , minimizing Eq. 2 by finding the smallest eigenmodes of the sparse symmetric matrix in Eq. 3. Although the weights  $W_{ij}$  and vectors  $Y_i$  are computed by methods in linear algebra, the constraint that points are only reconstructed from neighbors can result in highly nonlinear embeddings.



# Local Linear embedding III

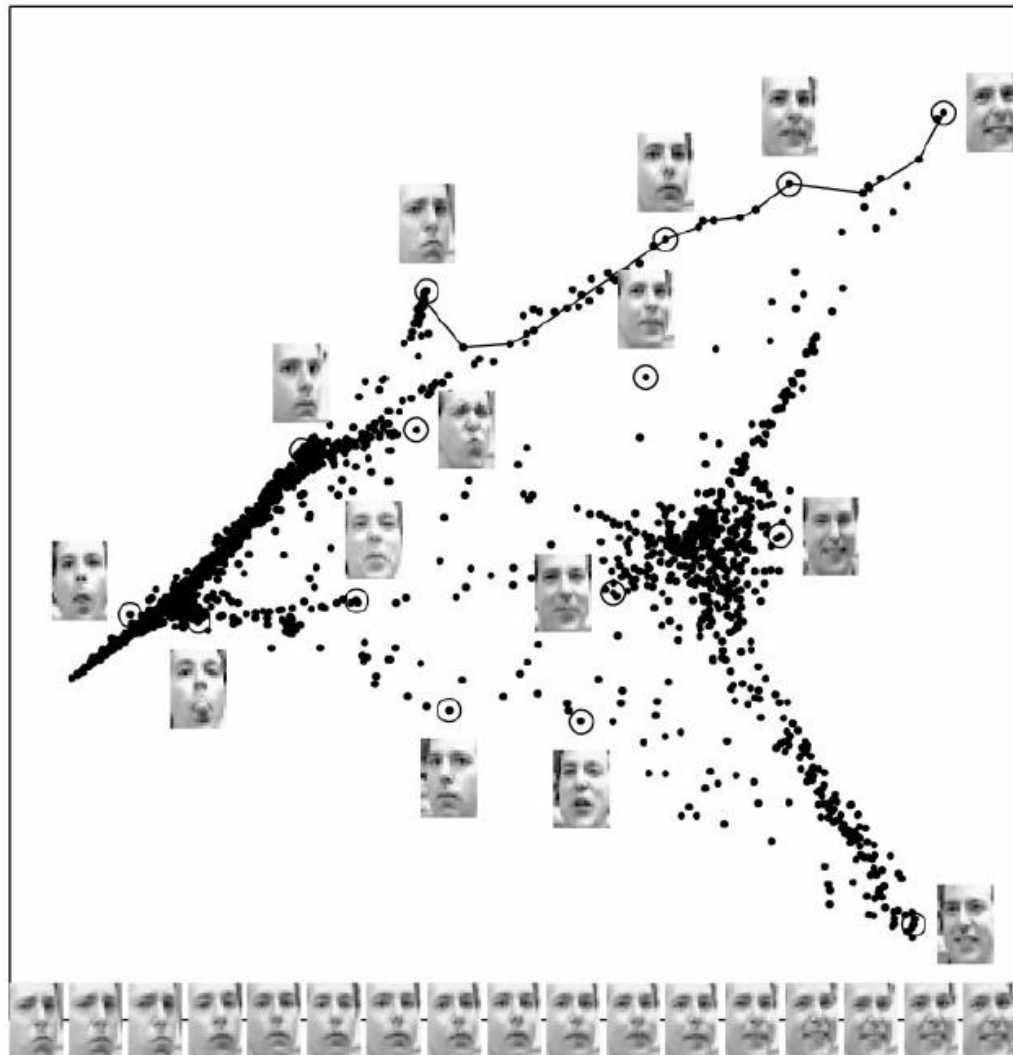


Fig. 3. Images of faces (11) mapped into the embedding space described by the first two coordinates of LLE. Representative faces are shown next to circled points in different parts of the space. The bottom images correspond to points along the top-right path (linked by solid line), illustrating one particular mode of variability in pose and expression.

# Local Linear embedding V

---

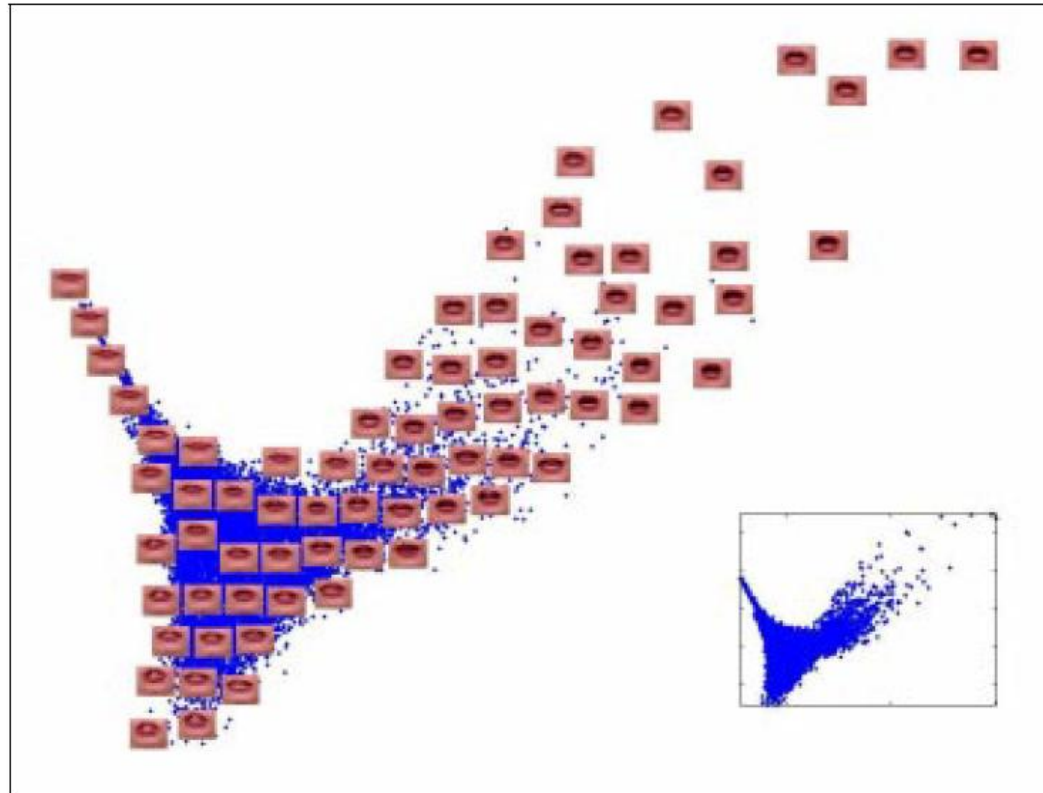


Figure 7: High resolution ( $D=65664$ ) images of lips, mapped into the embedding space discovered by the first two coordinates of LLE, using  $K=24$  nearest neighbors. Representative lips are shown at different points in the space. The inset shows the first two LLE coordinates for the entire data set ( $N=15960$ ) without any corresponding images.

# Local Linear embedding VI

---

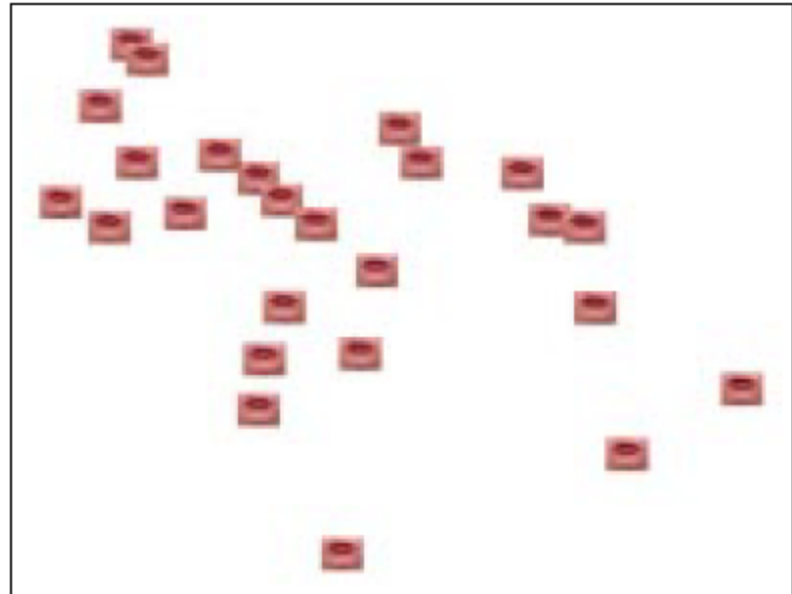
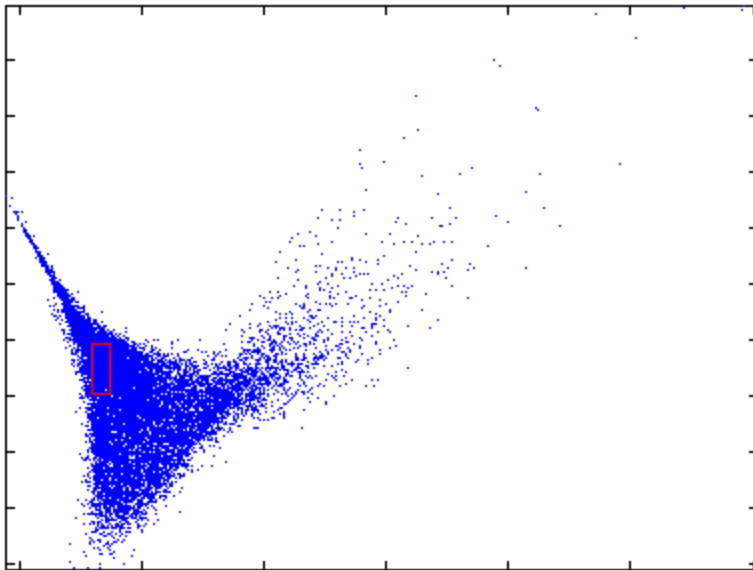


Figure 8: A typical neighborhood of  $K=24$  lip images mapped into the embedding space described by the first two coordinates of LLE. The rectangle in the left plot locates the neighborhood shown on the right in the overall space of lip images.

# Refs

---

- Roweis & Saul Science 2000
- Saul & Roweis JMLR 2003