

Exercise Sheet 8

Machine Learning 2, SS16

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Exercise 2

$$y_t = (w * x)_t = \sum_{s \in \mathbb{Z}} w_s x_{t-s}$$

(a)

$$\begin{aligned} \frac{\partial E}{\partial x_k} &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial y_t}{\partial x_k} \\ &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial}{\partial x_k} (w * x)_t \\ &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s \in \mathbb{Z}} w_s \frac{\partial}{\partial x_k} x_{t-s} \\ &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s \in \mathbb{Z}} w_s \mathbb{1}_{\{t-s=k\}} \\ &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} w_{-k+t} \\ &= \left[\frac{\partial E}{\partial y} \star w \right]_{-k} \end{aligned}$$

(b)

$$\begin{aligned} \frac{\partial E}{\partial w_k} &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial y_t}{\partial w_k} \\ &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial}{\partial w_k} (w * x)_t \\ &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s \in \mathbb{Z}} x_{t-s} \frac{\partial}{\partial w_k} w_s \\ &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s \in \mathbb{Z}} x_{t-s} \mathbb{1}_{\{s=k\}} \\ &= \sum_{t \in \mathbb{Z}} \frac{\partial E}{\partial y_t} x_{-k+t} \\ &= \left[\frac{\partial E}{\partial y} \star x \right]_{-k} \end{aligned}$$

Exercise 3

(a)

Euler discretization gives us for every $j = 1, \dots, d$:

$$x_j^t - x_j^{t-1} = 0.1(\tanh(\sum_{i=1}^d x_i^{t-1} w_{ij} + b_j) - x_j^{t-1})$$

The transition function is then component wise defined as

$$\Theta(x)_j = 0.1(\tanh(\sum_{i=1}^d x_i w_{ij} + b_j) - x_j) + x_j$$

(c)

$$\begin{aligned} \frac{\partial x_i^t}{\partial x_j^{t-1}} &= 0.1 \frac{\partial}{\partial x_j^{t-1}} (\tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i) - x_i^{t-1}) + \frac{\partial x_i^{t-1}}{\partial x_j^{t-1}} \\ &= 0.1 \frac{\partial}{\partial x_j^{t-1}} (\tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) + 0.9 \mathbb{1}_{\{i=j\}} \\ &= 0.1(1 - \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) \sum_{k=1}^d \frac{\partial}{\partial x_j^{t-1}} x_k^{t-1} w_{ki} + 0.9 \mathbb{1}_{\{i=j\}} \\ &= 0.1(1 - \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) \sum_{k=1}^d \mathbb{1}_{\{k=j\}} w_{ki} + 0.9 \mathbb{1}_{\{i=j\}} \\ &= 0.1(1 - \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) w_{ji} + 0.9 \mathbb{1}_{\{i=j\}} \end{aligned}$$

(d)

$$\begin{aligned} \frac{\partial x_i^t}{\partial b_j} &= 0.1 \frac{\partial}{\partial b_j} \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i) \\ &= 0.1(1 - \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) \frac{\partial b_i}{\partial b_j} \\ &= 0.1(1 - \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) \mathbb{1}_{\{i=j\}} \end{aligned}$$

$$\begin{aligned} \frac{\partial x_i^t}{\partial w_{jl}} &= 0.1 \frac{\partial}{\partial w_{jl}} \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i) \\ &= 0.1(1 - \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) \sum_{k=1}^d x_k^{t-1} \frac{\partial}{\partial w_{jl}} w_{ki} \\ &= 0.1(1 - \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) \sum_{k=1}^d x_k^{t-1} \mathbb{1}_{\{j=k\}} \mathbb{1}_{\{l=i\}} \\ &= 0.1(1 - \tanh(\sum_{k=1}^d x_k^{t-1} w_{ki} + b_i)) x_j^{t-1} \mathbb{1}_{\{l=i\}} \end{aligned}$$