Exercise Sheet 6

Machine Learning 2, SS16

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Exercise 1

(a)
Define

$$y := x^T W - b^T$$

$$\begin{aligned} p_\theta(x) &= \sum_{h \in \{-1,0,1\}^N} p(x,h) \\ &= \sum_{h \in \{-1,0,1\}^N} \frac{1}{Z} \exp(yh + x^T a) \\ &= \frac{1}{Z} \exp(x^T a) \sum_{h \in \{-1,0,1\}^N} \exp(yh) \\ &= \frac{1}{Z} \exp(x^T a) \sum_{h \in \{-1,0,1\}^N} \exp(\sum_{i=1}^{N} y_i h_i) \\ &= \frac{1}{Z} \exp(x^T a) \sum_{h \in \{-1,0,1\}^N} \prod_{i=1}^{N} \exp(y_i h_i) \end{aligned}$$

Because the expression $\exp(y_i h_i)$ only depends on the i'th component of h, we can rewrite the sum and product to get:

$$\begin{aligned} p_\theta(x) &= \frac{1}{Z} \exp(x^T a) \prod_{i=1}^{N} \sum_{h \in \{-1,0,1\}} \exp(y_i h_i) \\ &= \frac{1}{Z} \exp(x^T a) \exp(\log(\prod_{i=1}^{N} \sum_{h \in \{-1,0,1\}} \exp(y_i h_i))) \\ &= \frac{1}{Z} \exp(x^T a) \exp(\sum_{i=1}^{N} \log(\sum_{h \in \{-1,0,1\}} \exp(y_i h_i))) \\ &= \frac{1}{Z} \exp(x^T a) \exp(\sum_{i=1}^{N} \log(1 + e^{y_i} + e^{-y_i})) \\ &= \frac{1}{Z} \exp(x^T a) \exp(\sum_{i=1}^{N} \log(1 + 2cosh(y_i))) \\ &= \frac{1}{Z} exp(x^T a + \sum_{i=1}^{N} \log(1 + 2cosh(w_i x - b_i))) \end{aligned}$$

(a)