# 8.

Dienstag, 14. Juni 2016 15:04

$$(x * x')_t = \sum_{s\in\mathbb{Z}} x_t x'_{t-s}$$

$$\|(x*x')\|^2 = \sum_{t\in\mathbb{Z}} (x*x')_t^2 = \sum_{t\in\mathbb{Z}} \Big(\sum_{s\in\mathbb{Z}} x_t x'_{t-s}\Big)^2$$

$$\sum_{i,j} \alpha_i \alpha_j k(x^i, x^j) = \sum_{i,j} \alpha_i \alpha_j \underbrace{\Big(\sum_{t,s\in\mathbb{Z}} x^i_t x^j_{t-s}\Big)^2}$$

$$= \Big(\sum_{t\in\mathbb{Z}} (\ldots)\Big)^2$$

$$= \sum_{t\in\mathbb{Z}} \Big(\sum_{s\in\mathbb{Z}} x^i_t x^j_{t-s}\Big)^2 + \sum_{\substack{t,t'\in\mathbb{Z}\\ t\neq t'}} \Big(\sum_{s\in\mathbb{Z}} x^i_t x^j_{t-s}\Big)\Big(\sum_{s'} x_{t'} \ldots\Big)$$

$$= \sum_{t,s\in\mathbb{Z}} (x^i_t x^j_{t-s})^2 + \sum_t \sum_{s'\neq s} x^i_t x^j_{t-s'} x^i_t x^j_{t-s} \quad *$$

$$+ \sum_{\substack{t,t'\in\mathbb{Z}\\ t\neq t'}} \Big(\sum_{s\in\mathbb{Z}} x^i_t x^j_{t-s}\Big)\Big(\sum_{s'\in\mathbb{Z}} x^i_{t'} x^j_{t'-s'}\Big)$$

2) $$y_t = (w * x)_t = \sum_{s\in\mathbb{Z}} w_s x_{t-s}$$

$$\frac{\partial E}{\partial x_k} = \sum_{t\in\mathbb{Z}} \frac{\partial E}{\partial y_t} \frac{\partial y_t}{\partial x_k}$$

$$= \sum_{t\in\mathbb{Z}} \frac{\partial E}{\partial y_t} \sum_{s\in\mathbb{Z}} w_s \underbrace{\frac{\partial}{\partial x_k} x_{t-s}}_{= 1_{(k=t-s)}}$$

$$= \sum_t \frac{\partial E}{\partial y_t} \underbrace{w_{t-k}}_{= w_{-k+t}}$$

$$= \left[\frac{\partial E}{\partial y} \star w\right]_{-k}$$

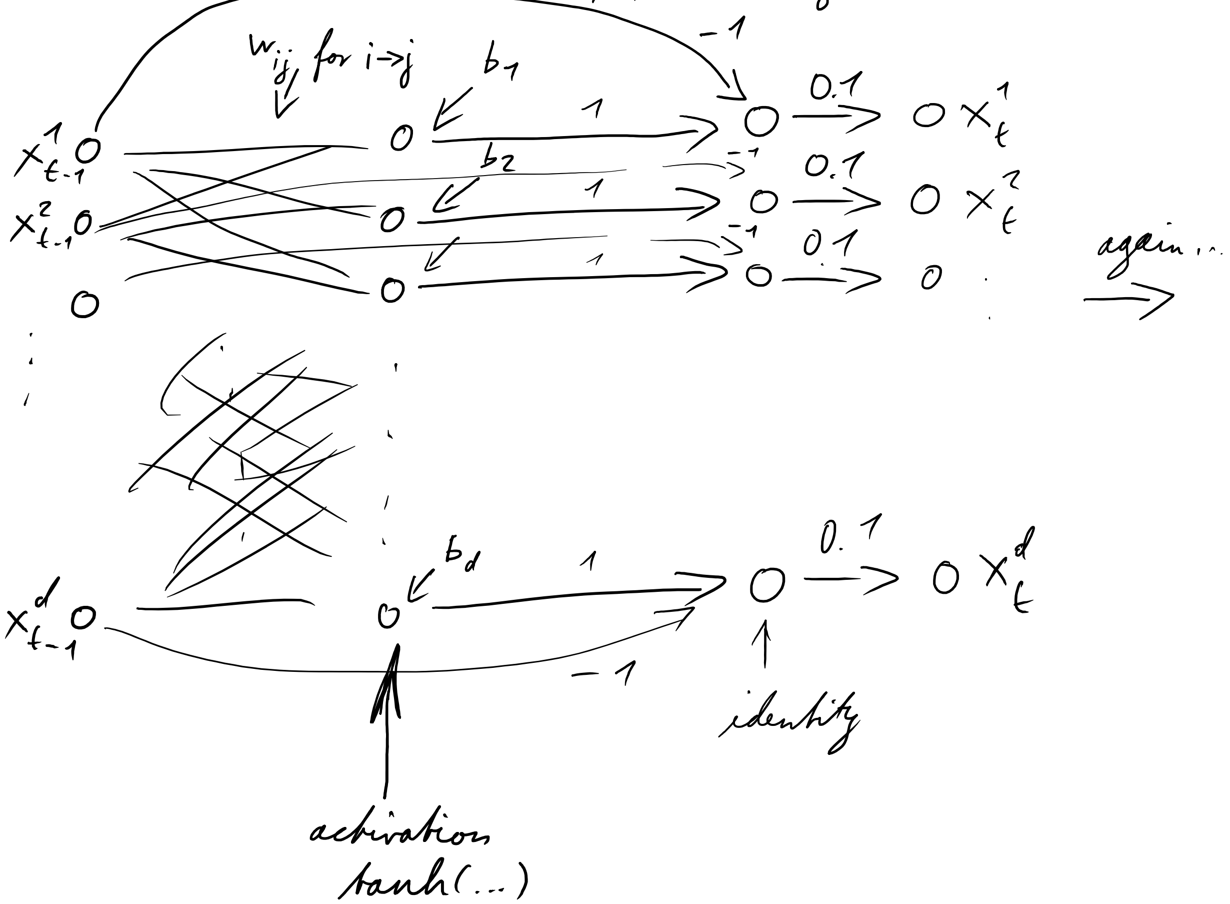
$$\frac{\partial E}{\partial w_k} = \sum_t \frac{\partial E}{\partial y_t} \underbrace{\frac{\partial y_t}{\partial w_k}}_{= \sum_s x_{t-s} \underbrace{\frac{\partial}{\partial w_k} w_s}_{= \mathbb{1}(s=k)}}$$

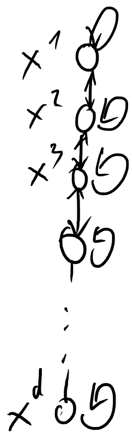
$$= \sum_t \frac{\partial E}{\partial y_t} x_{t-k}$$

$$= \left[\frac{\partial E}{\partial y} \star x\right]_{-k}$$

3) a) $x_t^j - x_{t-1}^j = 0.1\left(\tanh\left(\sum_{i=1}^{d} x_{t-1}^i w_{ij} + b_j\right) - x_{t-1}^j\right)$

b)






c) $$\frac{d}{dx_j^{t-1}} x_i^t = 0.1 \frac{d}{dx_j^{t-1}} \left( \tanh\left( \sum_{k=1}^{d} x_k^{t-1} w_{ki} + b_i \right) - x_i^{t-1} \right) + \frac{d}{dx_j^{t-1}} x_i^{t-1}$$

$$= 0.1 \underbrace{\frac{d}{dx_j^{t-1}} \tanh\left( \sum_k x_k^{t-1} w_{ki} + b_i \right)}_{= (1-\tanh(\dots)) \underbrace{\sum_k w_{ki} \underbrace{\frac{d}{dx_j^{t-1}} x_k^{t-1}}_{=\mathbb{1}_{\{j=k\}}}}_{= w_{ji}}} + 0.9 \underbrace{\frac{d}{dx_j^{t-1}} x_i^{t-1}}_{=\mathbb{1}_{\{i=j\}}}$$

$$= 0.1\, w_{ji} \left(1 - \tanh\left( \sum_{k=1}^{d} x_k^{t-1} w_{ki} + b_i \right)\right) + 0.9\, \mathbb{1}_{\{i=j\}}$$

$$\frac{dx_i^t}{dx_j^{t-s}} = \frac{dx_i^t}{dx_i^{t-1}} \frac{dx_i^{t-1}}{dx_i^{t-2}} \dots \frac{dx_i^{t-s+1}}{dx_j^{t-s}}$$

d) $$\frac{d}{db_j} x_i^t = 0.1 \frac{d}{db_j} \tanh\left( \sum_{k=1}^{d} x_k^{t-1} w_{ki} + b_i \right)$$

$$= 0.1 \left(1 - \tanh\left( \sum_{k=1}^{d} x_k^{t-1} w_{ki} + b_i \right)\right) \underbrace{\frac{db_i}{db_j}}_{=\mathbb{1}_{\{i=j\}}}$$

$$\frac{d}{dw_{jl}} x_i^t = 0.1 \frac{d}{dw_{jl}} \tanh\left( \sum_{k=1}^{d} x_k^{t-1} w_{ki} + b_i \right)$$

$$= 0.1 \left(1 - \tanh\left( \sum_{k=1}^{d} x_k^{t-1} w_{ki} + b_i \right)\right) \sum_{k=1}^{d} x_k^{t-1} \underbrace{\frac{d}{dw_{jl}} w_{ki}}$$

$$= x_k^{t-1} \mathbb{1}_{\{j=k\}} \mathbb{1}_{\{l=i\}}$$

$$= 0.1 \left(1 - \tanh\left(\sum_{k=1}^{d} x_k^{t-1} w_{ki} + b_i\right)\right) x_j^{t-1} \mathbb{1}_{\{l=i\}}$$