

Exercise 1a

$$\begin{aligned}\frac{\partial C}{\partial q_i} &= \frac{\partial}{\partial q_i} \sum_j p_j \log\left(\frac{p_j}{q_j}\right) \\ &= \sum_j \frac{\partial}{\partial q_i} \left(p_j \log\left(\frac{p_j}{q_j}\right) \right) \\ &= \sum_j \frac{\partial}{\partial q_i} (p_j \log(p_j) - p_j \log(q_j)) \\ &= \sum_j \left(-p_j \frac{\partial}{\partial q_i} \log(q_j) \right)\end{aligned}$$

Since

$$\frac{\partial}{\partial q_i} \log(q_j) = \begin{cases} 0 & , i \neq j \\ \frac{1}{q_i} & , i = j \end{cases},$$

$$\frac{\partial C}{\partial q_i} = -\frac{p_i}{q_i}.$$

Exercise 1b

$$\begin{aligned}
\frac{\partial C}{\partial x_i} &= \frac{\partial}{\partial x_i} \sum_j p_j \log\left(\frac{p_j}{q_j}\right) \\
&= \sum_j \frac{\partial}{\partial x_i} (p_j \log(p_j) - p_j \log(q_j)) \\
&= \sum_j \left(-p_j \frac{\partial}{\partial x_i} \log(q_j) \right) \\
&= \sum_j \left(-p_j \frac{\partial}{\partial x_i} \log\left(\frac{e^{x_j}}{\sum_k e^{x_k}}\right) \right) \\
&= \sum_j \left(-p_j \frac{\partial}{\partial x_i} (x_j - \log(\sum_k e^{x_k})) \right) \\
&= \sum_j \left(-p_j \frac{\partial}{\partial x_i} x_j \right) + \sum_j \left(p_j \frac{\partial}{\partial x_i} \log(\sum_k e^{x_k}) \right) \\
&= \sum_j \left(-p_j \frac{\partial}{\partial x_i} x_j \right) + \sum_j \left(p_j \frac{1}{\sum_k e^{x_k}} \frac{\partial}{\partial x_i} \sum_k e^{x_k} \right) \\
&= \sum_j \left(-p_j \frac{\partial}{\partial x_i} x_j \right) + \sum_j \left(p_j \frac{e^{x_i}}{\sum_k e^{x_k}} \right) \\
&= \sum_j \left(-p_j \frac{\partial}{\partial x_i} x_j \right) + q_i \sum_j p_j \\
&= \sum_j \left(-p_j \frac{\partial}{\partial x_i} x_j \right) + q_i
\end{aligned}$$

Since

$$\frac{\partial}{\partial x_i} x_j = \begin{cases} 0 & , i \neq j \\ 1 & , i = j \end{cases},$$

$$\frac{\partial C}{\partial x_i} = -p_i + q_i$$