

Exercise Sheet 10

Machine Learning 2, SS16

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Exercise 1a

Let $\vec{x}_1, \dots, \vec{x}_n \in \mathcal{X}$ be arbitrary samples and $\vec{v} \in \mathbb{R}^n$ be an arbitrary vector.

$$\begin{aligned} \forall_{l=1}^L \cdot \sum_{i,j=1}^n v_i v_j k_l(x_i, x_j) \geq 0 &\implies \sum_{l=1}^L \sum_{i,j=1}^n v_i v_j k_l(x_i, x_j) \geq 0 \implies \sum_{i,j=1}^n v_i v_j \sum_{l=1}^L k_l(x_i, x_j) \geq 0 \\ &\stackrel{\vec{\beta} \geq 0}{\implies} \sum_{i,j=1}^n v_i v_j \sum_{l=1}^L \beta_l k_l(x_i, x_j) \geq 0 \stackrel{\text{Def. k}}{\implies} \sum_{i,j=1}^n v_i v_j k(x_i, x_j) \geq 0 \end{aligned}$$

We could also proof this for all pairs of datapoints x, x' out of \mathcal{X} , but then the unfold of the definition of positive definiteness wouldn't be that lucid.

Exercise 1b

Let $x, x' \in \mathcal{X}$ be two arbitrary samples.

$$\begin{aligned} \sum_{l=1}^L \beta_l k_l(x, x') &= \sum_{l=1}^L \beta_l \phi_l(x)^\top \phi_l(x') = \sum_{l=1}^L (\sqrt{\beta_l} \phi_l(x)^\top) (\sqrt{\beta_l} \phi_l(x')) \\ &= \underbrace{\begin{bmatrix} \sqrt{\beta_1} \phi_1(x)^\top & \dots & \sqrt{\beta_L} \phi_L(x)^\top \end{bmatrix}}_{\phi(x)^\top} \underbrace{\begin{bmatrix} \sqrt{\beta_1} \phi_1(x') \\ \dots \\ \sqrt{\beta_L} \phi_L(x') \end{bmatrix}}_{\phi(x')} \end{aligned}$$

$\phi(x)^\top$ and $\phi(x')$ are block partitioned matrices, i.e. the ϕ_l 's are simply concatenated together in one very long vector. So, the result is:

$$\phi(x) = \begin{bmatrix} \sqrt{\beta_1} \phi_1(x) \\ \dots \\ \sqrt{\beta_L} \phi_L(x) \end{bmatrix}$$