

Problem 1. Let $n \geq 0$. Use partial fractions to find an expression for the following coefficient which does not involve summations or ellipses.

$$[x^n] \frac{x^2}{(1-x)(1-x^2)}$$

Problem 2. Solve the following recurrences (i.e. find an explicit formula for a_n as a function of n).

- (a) $a_n = 5a_{n-1} + 6a_{n-2}$ for $n \geq 2$ with initial conditions $a_0 = 2$ and $a_1 = 4$.
- (b) $a_{n+2} + 4a_{n+1} + 4a_n = 0$ for $n \geq 0$ with initial conditions $a_0 = 1$ and $a_1 = 2$.
- (c) $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$ with initial conditions $a_0 = 0$ and $a_1 = 1$.

Problem 3. Find (linear, homogeneous) recurrence equations and initial conditions for

- (a) $a_n = (-3)^n - 4^n$
- (b) $a_n = 3 + 2^{n+1} + 3^n$
- (c) $a_n = (2n^2 + n - 2)5^n - (4n - 1)2^n$