Problem 1. How many compositions of n with k parts are there in which each part is at the same time both *greater than* 5 and *even*?

(a) Find a suitable set S whose elements satisfy the above constraints and a weight function w such that an element $\sigma \in S$ is a composition of n if and only if $w(\sigma) = n$.

(b) Find the generating series $\Phi_S(x)$ with respect to w.

(c) Compute the number of compositions explicitly in terms of n and k.

Problem 2. Find a set S, a weight function w and the corresponding generating series $\Phi_S(x)$ such that the coefficient $[x^n]\Phi_S(x)$ is the number of compositions of n in which each part is an odd number and the number of parts is even (but otherwise unspecified). You do not need to compute the coefficient explicitly but the generating series should be written as a simplified rational expression.

Problem 3. Consider the generating series

$$\Phi_S(x) = \frac{x^2}{1 - 3x^2 + x^4}$$

and let $a_n = [x^n]\Phi_S(x)$, $n \geq 0$, be its coefficients.

(a) Show that $a_0 = a_1 = a_3 = 0$, $a_2 = 1$, and find a recurrence for $n \geq 4$.
How can you interpret the sequence of coefficients $(a_0, a_1, a_2, \ldots)$? (Hint: Compute it for a few small n and see if you recognize it. No formal proof needed.)

(b) Show that for even $n \geq 2$

$$a_n = \sum_{j=0}^{\lfloor \frac{m}{2} \rfloor} \binom{m-j}{j} (-1)^j 3^{m-2j}$$

where $m = n/2 - 1$.