MATH 239, Winter 2020 Assignment 5 Due: 11:59PM, 14 February 2020 ♡.

Problem 1. For each of the following expressions defining a set of binary strings, describe in words the set of strings it denotes, and state with a brief explanation if the expression is ambiguous. (You may do this by referring to standard unambiguous expressions for the set of all strings that you have seen).

(a) $S_1 = \{00, 10, 01, 11\}^* \cup \{0,1\}\{0,1\}\{0,1\}$.

(b) $S_2 = \{00, 0000, 1\}^*$.

(c) $S_3 = \{0\}^*(\{1\}\{0\}^*)(\{1\}\{0\}^*)^*$

(d) $S_4 = \{111, 11111\}(\{00\}\{1\}\{1\}^*)^*$.

(e) $S_5 = \{1\}^*(\{0\}^*\{1\}\{1\}^*)^*\{0\}^*$.

(f) $S_6 = \{0\}^*(A\{000\}\{0\}^*)^*A$, where A is the set $\{11, 111, 11111, \dots\}$ of strings of 1's whose length is a prime number.

Problem 2. Let $k \geq 2$ be an integer, and let S be the set of all binary strings in which every block of zeroes has length less than k, and every block of ones has length at least k. Find an unambiguous expression for S, and use it to prove that the generating series for S weighted by length is given by

$$\Phi_S(x) = \frac{(1-x^k)(1-x+x^k)}{1-2x+x^2-x^{k+1}+x^{2k}}.$$

(It may be helpful to consider a set U consisting of the allowable blocks of zeroes and a set V containing the allowable blocks of ones, and to compute Φ_U and Φ_V separately before Φ_S.)

Problem 3. Let S be the set of binary strings in which every block b of 0's is followed by a block of 1's, whose length is either equal to the length of b or is one greater than the length of b. (For example, the strings 1111 and 00011100111 are in S, but 00011 is not).

(a) Write an unambiguous decomposition for S of the form $S = A \cup SB$, where A and B are sets of strings that are much simpler to describe than S itself. Explain why the decomposition describes S unambiguously.

(b) Use this decomposition to find $\Phi_S(x)$ (as usual, weighted by length).

(c) State a formula for the number of strings in S of length n for each $n \geq 0$.