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I hereby declare that the thesis submitted is my own unaided work. All direct or indirect sources used are acknowledged as references. This paper was not previously presented to another examination board and has not been published.

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Meschede, 28th September 2023.

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Math stuff for pyspark

1 Simulation of a Dataset

In order to generate a large dataset which fulfills the requirements ($n \gg 10^9, k \gg 10^5$), the generation of the values needs to be done in a distributed fashion. PySpark does not have a pre-defined function to generate an entire dataset suited for OLS, therefore this function is implemented manually. At first, the following values need to be initialized:

- `n` - number of rows/samples
- `k` - number of columns/features
- $\vec{\beta}$ - beta, the coefficients of the function
- `cov` - a covariance vector that determines the covariance to the first column for each column

In this implementation, `n` and `k` need to be set by the user while $\vec{\beta}$ and `cov` are generated randomly by numpy. For generating the actual dataset, `pyspark.mllib.random.RandomRDDs.normalVectorRDD(sc, n, k)` is used. This function creates an `rdd` containing `n` vectors, each containing `k` entries, where each entry is generated from a standard-normal distribution.

After generating this random noise matrix, the user-defined-function `createRow(noise)` is applied to the `rdd`, which returns two values, $\vec{x}$ (1) and y (2).

With `noise` as ϵ and `cov` as c:

$$\vec{x} = (\epsilon_0, \epsilon_0 c_1 + \epsilon_1, \ldots, \epsilon_0 c_i + \epsilon_i) \tag{1}$$

$$y = \vec{x} \cdot \vec{\beta} \tag{2}$$

Applying this function will produce an RDD where the first element is the x-vector while the second element is the target variable.

Therefore, the final outcome is a feature matrix (consisting of n $\vec{x}$ vectors) that consists out of k columns, where each column is linearly dependent on the first column, with additional noise. An example of a distribution is shown in the figure 1.

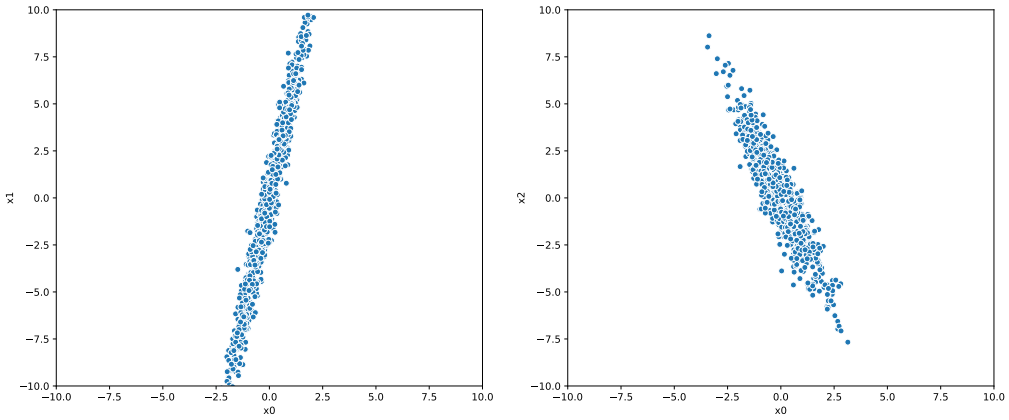

Figure 1: *exemplary generated dataset*